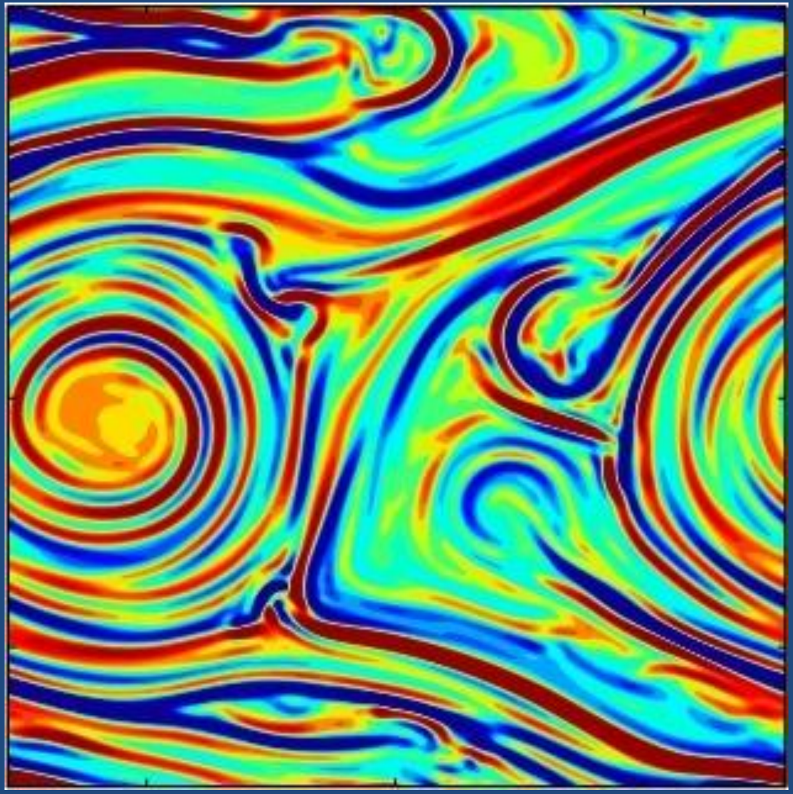
Exoplanets, Magneto-Quasigeostrophy and Compact Vortices

(was originally titled: 2-Layer Magneto-Quasigeostrophic Models of Hot Exosolar Planetary Atmospheres and the Stability of Compact Vortices

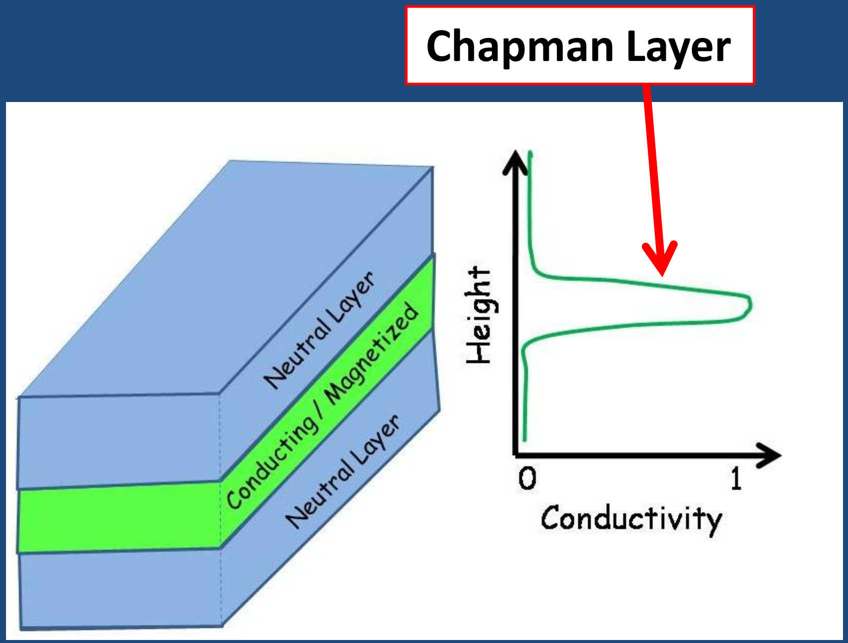
O. M. Umurhan and J. Y-K. Cho

Queen Mary University of London and City College of San Francisco



Magnetized Layers in Hot Jupiter Atmospheres: Background/Motivation

Recent calculations by Koskinen et al. ("Ionization of extrasolar planet atmospheres," to Appear in ApJ) have shown that close-in hot Jupiters are sufficiently irradiated by UV-flux from their parent star to have significant portions of their upper atmospheres fall into the plasma regime. If these layers coincide with the temperature inversion layers inferred from observations then the flow dynamics there will be significantly affected by magneto-hydrodynamic (MHD) effects.



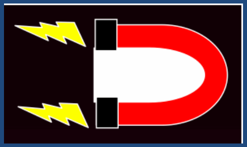
This study is an preliminary exploration of the dynamical flow consequences of such situations. The simplest environment to examine these MHD effects on an atmospheric level is the so-called quasigeostrphic limit.

Relevant Non-dimensional Numbers Governing Dynamics

$$\underline{\text{Rossby Number}} \quad \longleftrightarrow \quad \mathrm{Ro} \equiv \frac{\mathcal{U}}{2\Omega_0 \mathcal{L}},$$

$$\underline{\text{Cowling Number}} \quad \longleftrightarrow \quad C \equiv \frac{\mathcal{B}^2}{4\pi\rho}\frac{1}{\mathcal{U}^2},$$

$$\underline{\text{Burger Number}} \quad \longleftrightarrow \quad \mathrm{Bu}^2 \equiv \frac{g\mathcal{H}}{4\Omega_0^2 \mathcal{L}^2}.$$



Rossby Number

If the velocity scales of the storms are `$\mathcal{U}$' and the synoptic scales are '$\mathcal{L}$' and if the planetary Coriolis is $2\Omega_0$ then the Rossby number measures the ratio of the rotation time of the planet to that of the storm. On Jupiter this number is 1/20th or smaller. On the upper layers of exoplanets this is unknown but likely to be small too.

Cowling Number

If the typical scales of the magnetic fiels are given by `$\mathcal{B}$' and the typical density of the layer(s) is `ρ' then the Cowling number measures the ratio of the magnetic energy content of the fluid's kinetic energy. This is currently unknown for expolanets as there are no measurements of magnetic field strengths of their upper atms.

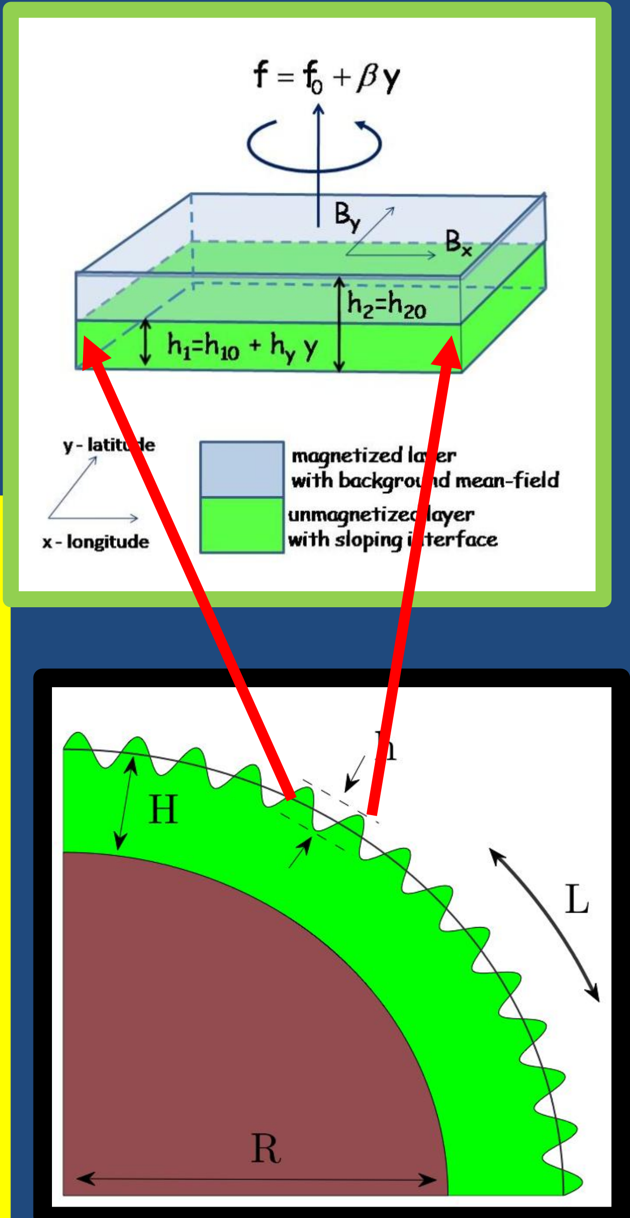
Burger Number

If the atmospheres are stably stratified (i.e. they have stratospheres) and the entropy scale height is `$\mathcal{H}$', and if the planet's gravity is `g' , then the Burger number measures the ratio of the rotation time to the gravity wave time. On the solar system planets this number is order unity. It is probably unity for exoplanets too.

Scalings leading to **Magneto-Quasigeostrophy** (mQG)

$$\mathcal{O}\left(\frac{\mathcal{L}}{R}\right) \sim \mathcal{O}\left(\frac{\mathcal{H}}{\mathcal{L}}\right) \sim \mathcal{O}\left(\frac{\tilde{h}}{\mathcal{H}}\right) \sim \underline{\mathcal{O}\left(\mathrm{Ro}\right) \ll 1}.$$

$$C \sim \mathcal{O}\left(1\right), \qquad \mathrm{Bu} \sim \mathcal{O}\left(1\right).$$



Magneto-quasi-geostrophy

Quasi-geostrophy is a model reduction in which the Rossby numbers are small. As a model set, the equations capture the essence of the dynamics of vortical flows in stably stratified atmospheres by filtering out gravity and sound waves. Flows in these simplifications are "nearly" 2D.

With planetary radius 'R' and the fluctuation scale given by 'h', together with the synoptic and entropy scale heights as given, the scaling relationships above lead to the equations of mQG. Cowling Numbers of order 1 means that the effects of magnetic induction and Lorentz forcing are preserved and Alfven waves are present in the dynamics (see below).

Two-layer mQG equations
(uniform densities in each layer)

$$(\partial_t + \mathbf{u}_1 \cdot \nabla) Q_1 + \beta v_1 = \varphi_1 C \left(q_1^{(m)} + \mathbf{b}_1 \cdot \nabla \right) J_1,$$

$$(\partial_t + \mathbf{u}_2 \cdot \nabla) Q_2 + \beta v_2 = \varphi_2 C \left(\frac{\rho_1}{\rho_2} \right) \left(q_2^{(m)} + \mathbf{b}_2 \cdot \nabla \right) J_2,$$

$$(\partial_t + \mathbf{u}_1 \cdot \nabla)\mathbf{b}_1 = (\mathbf{b}_1 \cdot \nabla)\mathbf{u}_1,$$

$$(\partial_t + \mathbf{u}_2 \cdot \nabla)\mathbf{b}_2 = (\mathbf{b}_2 \cdot \nabla)\mathbf{u}_2.$$

The equation set developed for this study are for two layers – upper magnetized and the lower one non-magnetized (denoted by subscripts).

$$Q_1 = \nabla^2 \psi_1 - \frac{1}{L_{10}^2} \left(\psi_1 - \frac{\rho_2}{\rho_1} \psi_2 \right),$$

$$Q_2 = \nabla^2 \psi_2 - \frac{1}{L_{21}^2} (\psi_2 - \psi_1),$$

$$u_i = -\partial_y \psi_i, \qquad v_i = \partial_x \psi_i,$$

$$J_i \equiv \partial_x b_{yi} - \partial_y b_{xi},$$

$$q_i^{(m)} \equiv \nabla \cdot \mathbf{b}_i = \partial_x b_{xi} + \partial_y b_{yi}.$$

Q_i denote the potential vorticity in each layer. The velocity fields are derived from it through the streamfunctions ψ_i.

J_i denote the current in each layer. It influences the evolution of the potential vorticity. The velocity flow induces evolution on the magnetic fields.

q_i denote the horizontal divergence of the magnetic field. In mQG classes of solutions depend upon whether this is zero at the outset. In this study it is assumed that $q_i = \mathbf{0}$.

The Rossby Radius of Deformation

$$L_{10}^2 \equiv \frac{g\mathcal{H}H_{10}}{4\Omega_0^2\mathcal{L}^2}\left(1-\frac{\rho_2}{\rho_1}\right), \qquad L_{21}^2 \equiv \frac{g\mathcal{H}(H_{20}-H_{10})}{4\Omega_0^2\mathcal{L}^2}\left(1-\frac{\rho_2}{\rho_1}\right),$$

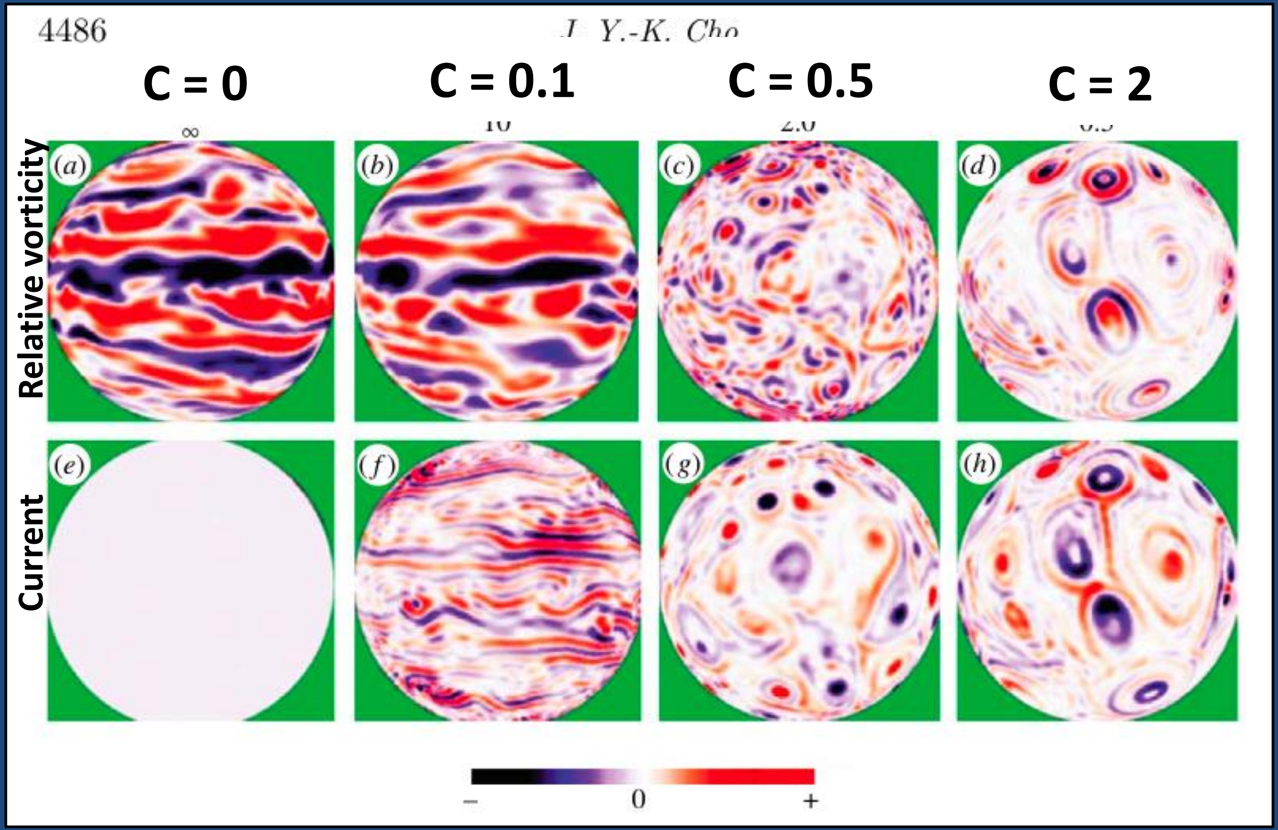
The Deformation Radius determines the dynamical reach and influence of a geostrophic vortex structure : a small value means that the vortex's <u>horizontal</u> reach is very small as it determines the exponential length scale. Large values mean that vortices will merge. <u>On exoplanets this number is predicted to be large</u> implying that planetary scale vortices are a natural outcome for these situations. (see review of Showman, Menou & Cho, 2010)

Antecedents to these equations

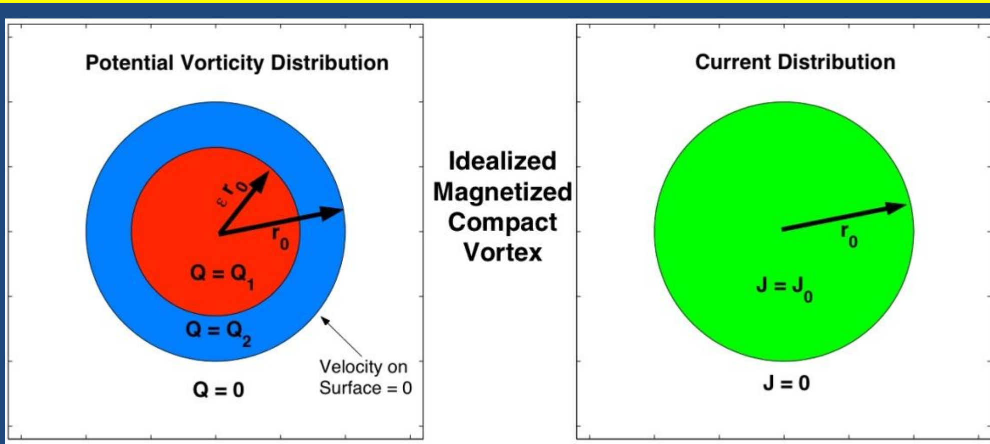
Gilman in 1967-8 (e.g. see "Stability of baroclinic flows in a zonal magnetic field: part I.", J. Atm. Sci, 24, 1967) derived equations like these to study the dynamics of magnetic shear layers. A version of these called shallow-water MHD has been redone by Gilman (e.g. see "Magnetohydrodynamic "shallow water" equations for the solar tachocline", ApJ 544 L54, 2000) <u>to study the dynamics of the solar tachochline</u>.

More Background. Single Magnetized Layer: Shallow Water MHD

Recent calculations by Cho (2008) show that when solving the shallow water MHD equations in a single layer beginning with a random potential vorticity distribution and a zero mean magnetic field: for increasing Cowling numbers the flow tends to develop into **isolated magnetic vortices** instead of familiar geostrophic jets. These structures are examined here.



The Isolated ``Compact" Magnetic Vortex



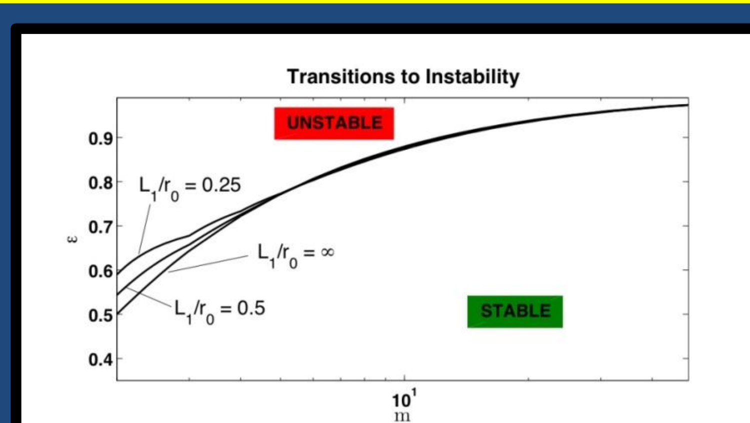
- Surface Speed ≈ 0
- Overturning Jet
- e ↔ "annular measure"

Non-magnetized Linear Theory (C = 0)

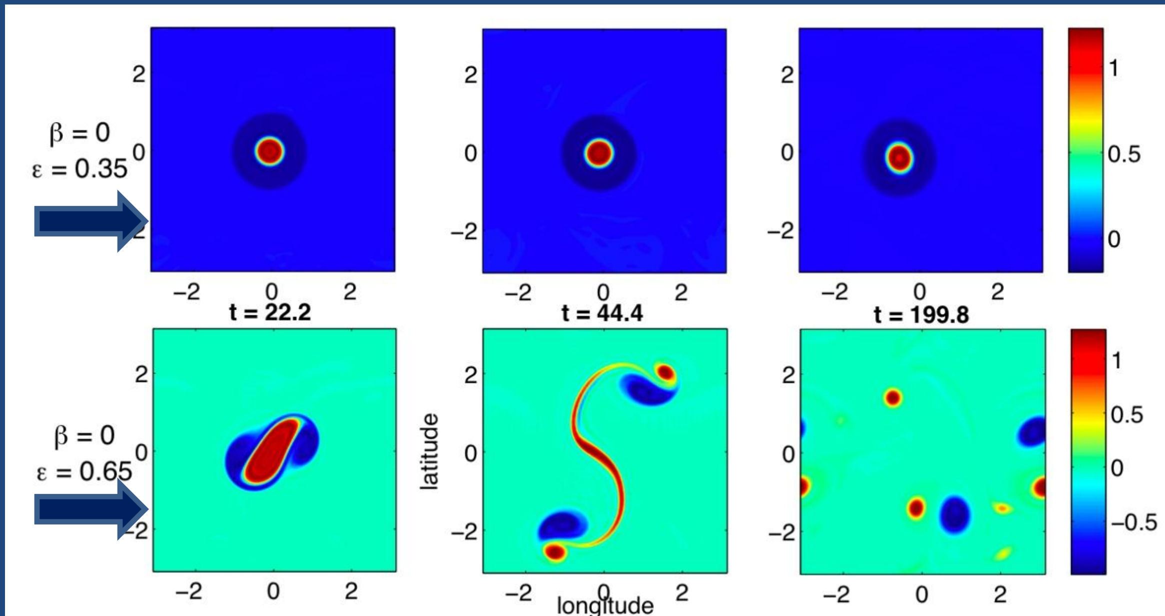
Normal mode perturbations of the form $e^{i(\omega t + m\theta)}$ are adopted and the stability of the isolated vortex shown above is analyzed in the limit where C=0. The vortex is unstable to the instability of Counterpropagating Rossby waves (e.g. Flierl 1988) for values of the annular measure exceeding 1/2. m=1 mode also shows a double root structure suggesting algebraic instability

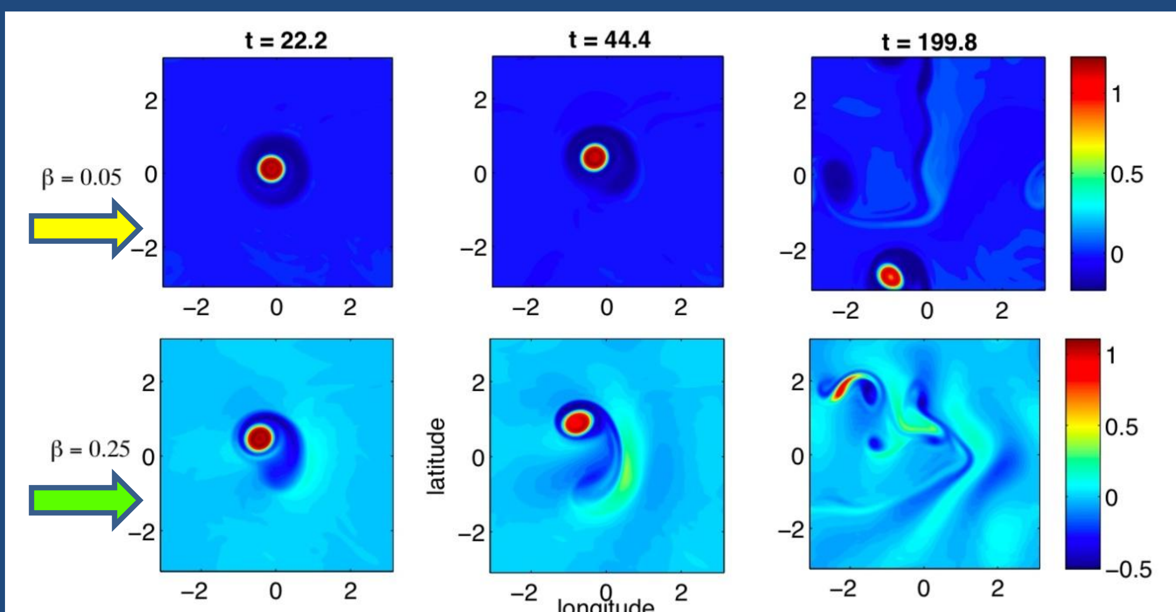
$$\omega_{0m} = -\bar{Q}_1 \frac{1}{2}\left(\frac{m-1}{2m} \pm \sqrt{\Delta}\right),$$

$$\Delta = \frac{\left[1+\epsilon^2+m(\epsilon^2-1)\right]^2 - 4\epsilon^{2(m+1)}}{4m^2(\epsilon^2-1)^2},$$



Non-magnetized evolution of Potential Vorticity





Numerical Solutions

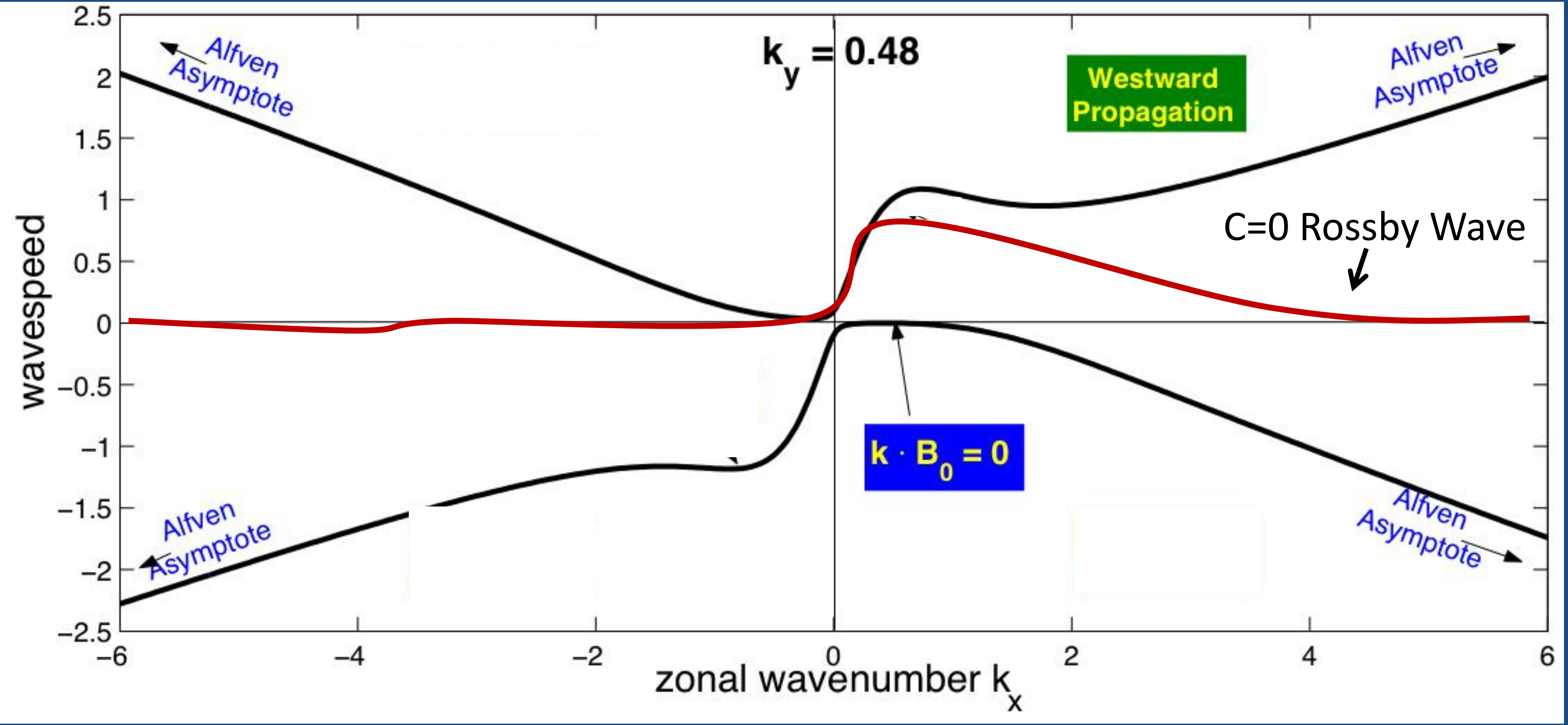
Isolated vortices are laid down and studied using a pseudospectral numerical code using 128x128 to 256x256 resolution with hyperviscosity to kill grid-scale instabilities. The parameter α measures the amplitude of the perturbations imposed on the structures. The planetary 'β' determines where on the planet one is on. In these runs $\alpha = 0.01$.

The example shown on the top is a stable vortex for C=0/. The next one is an unstable configuration – the counter-propagating Rossby wave instability tears the structure apart.

The bottom two panels show an algebraically unstable vortex for two values of the planetary beta. The isolated vortex goes thru a period of drift of their centers and then undergoes a secondary instability which is akin to the counter-propagating wave instability above.

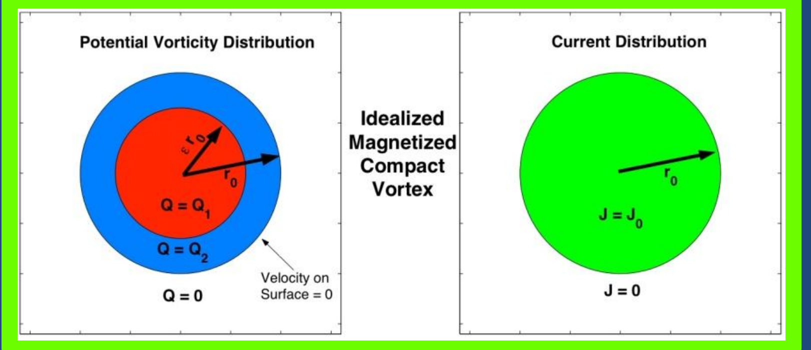
Magnetized theory (C ≠ 0) in presence of background B-field: Rossby-Alfven Waves

Simple linear theory with an imposed background magnetic field predicts Rossby-Alfven waves as generic wavemodes for this system. Normal mode ansatz of $\exp i(c\, k_x\, t + k_x\, x + k_y\, y)$ are assumed. Small k_x modes behave similarly to Rossby waves while large k limit looks like Alfven waves. All modes are stable in this case.



For details see: Acheson & Hide (1973) "Hydromagnetics of rotating fluids"

Linear Theory: C ≠ 0

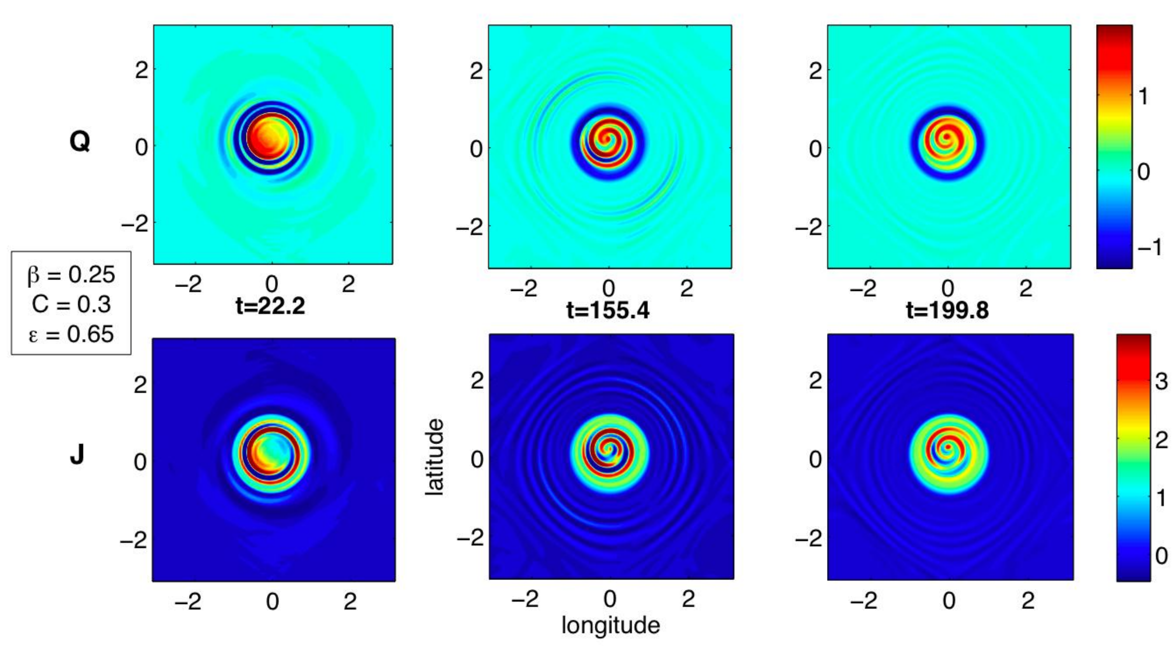


- Limiting form for $\varepsilon \rightarrow 1$ (analytically tractable)

$$\omega_{\pm} = -\frac{1}{2}\left(1-\frac{1}{m}\right) \pm \left[-\frac{1}{4}\left(1-\frac{1}{m^2}\right)+\frac{C}{2}\left(1-\frac{1}{m}\right)\right]^{1/2}.$$

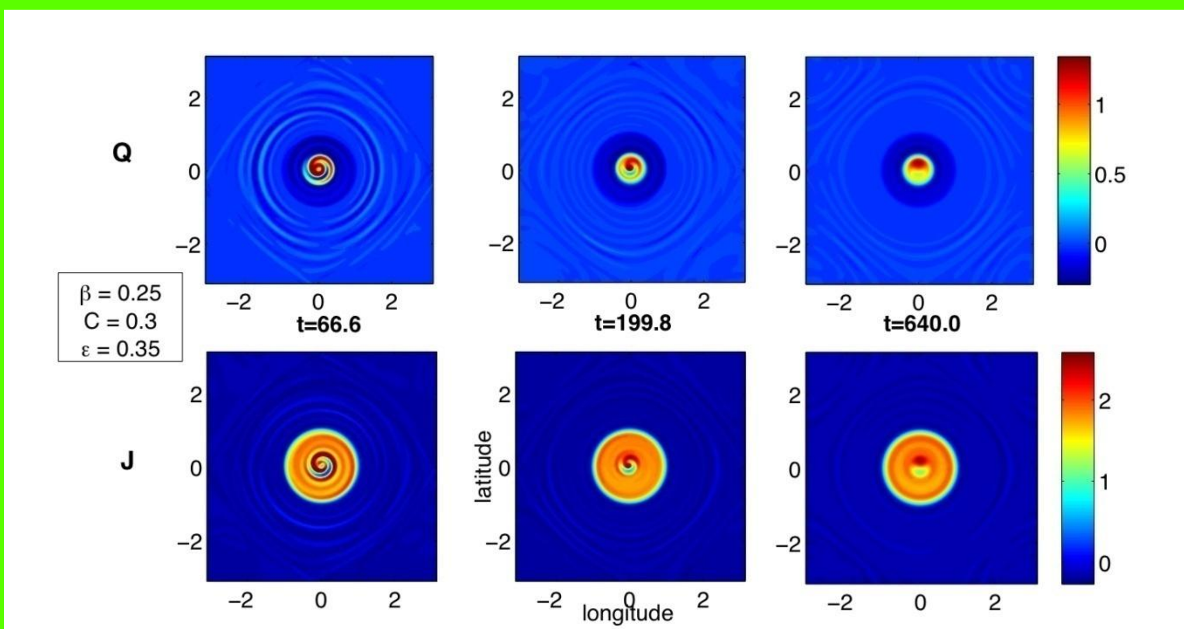
- $C \sim O(1)$ $\leftrightarrow$ Counterpropagating Rossby Modes Stabilize

Known instability for plane-parallel configuraations: D. H. Michael "The stability of a combined current and vortex sheet in a perfectly conducting fluid." Proc . Camb. Phil. Soc. 51 (1955)



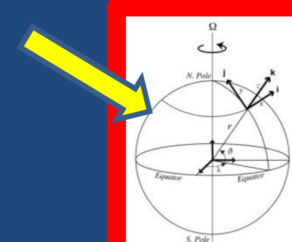
ε = 0.65, C = 0.3, β=0.25

The example shown on the top is a vortex that is unstable in the non-magnetized limit. And located in the mid-latitude zone. The structure goes through a period of winding but remains stable. Rossby-Alfven waves are shed.



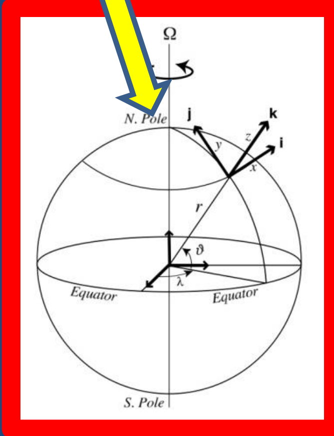
ε = 0.35, C = 0.3, β=0.25

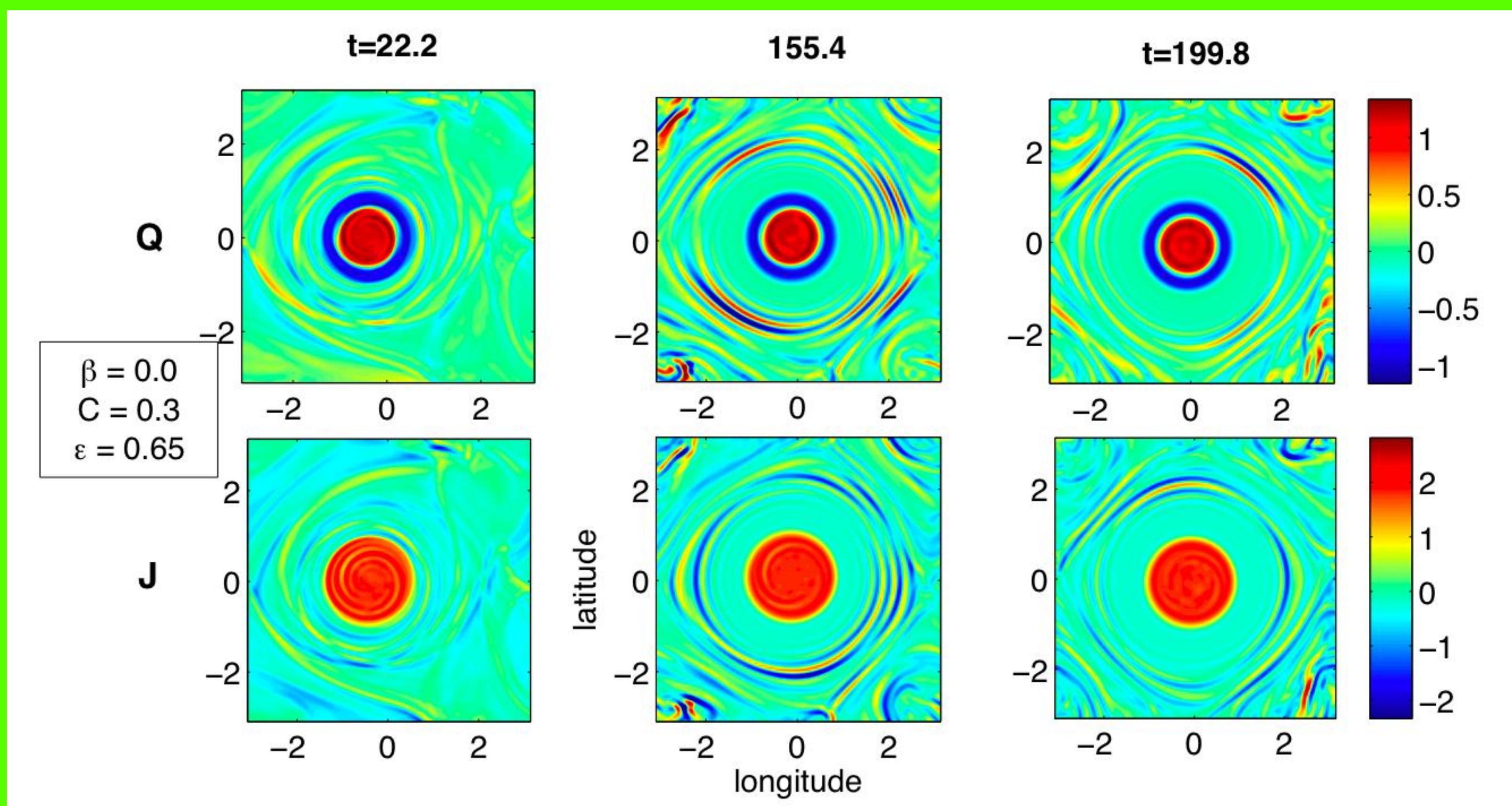
The example shown to the left is algebraically unstable in the non-magnetized limit (C=0). The structure is stabilized at long times.



ε = 0.65, *C* = 0.3, β=0

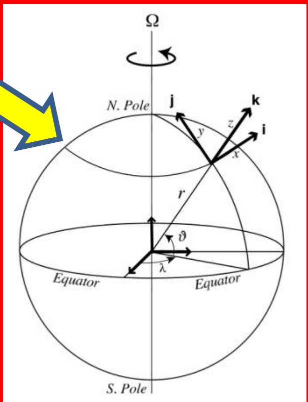
The example shown is a vortex that is unstable in the non-magnetized limit located in the polar region. No strong winding occurs like seen for isolated vortices found in lower latitudes. Strong amplitude Rossby-Alfven waves are shed and form domain boundaries (because this simulation is doubly periodic). These waves carry potential vorticity with them which means that these structures can non-locally transmit angular momentum while adjusting and settling down.

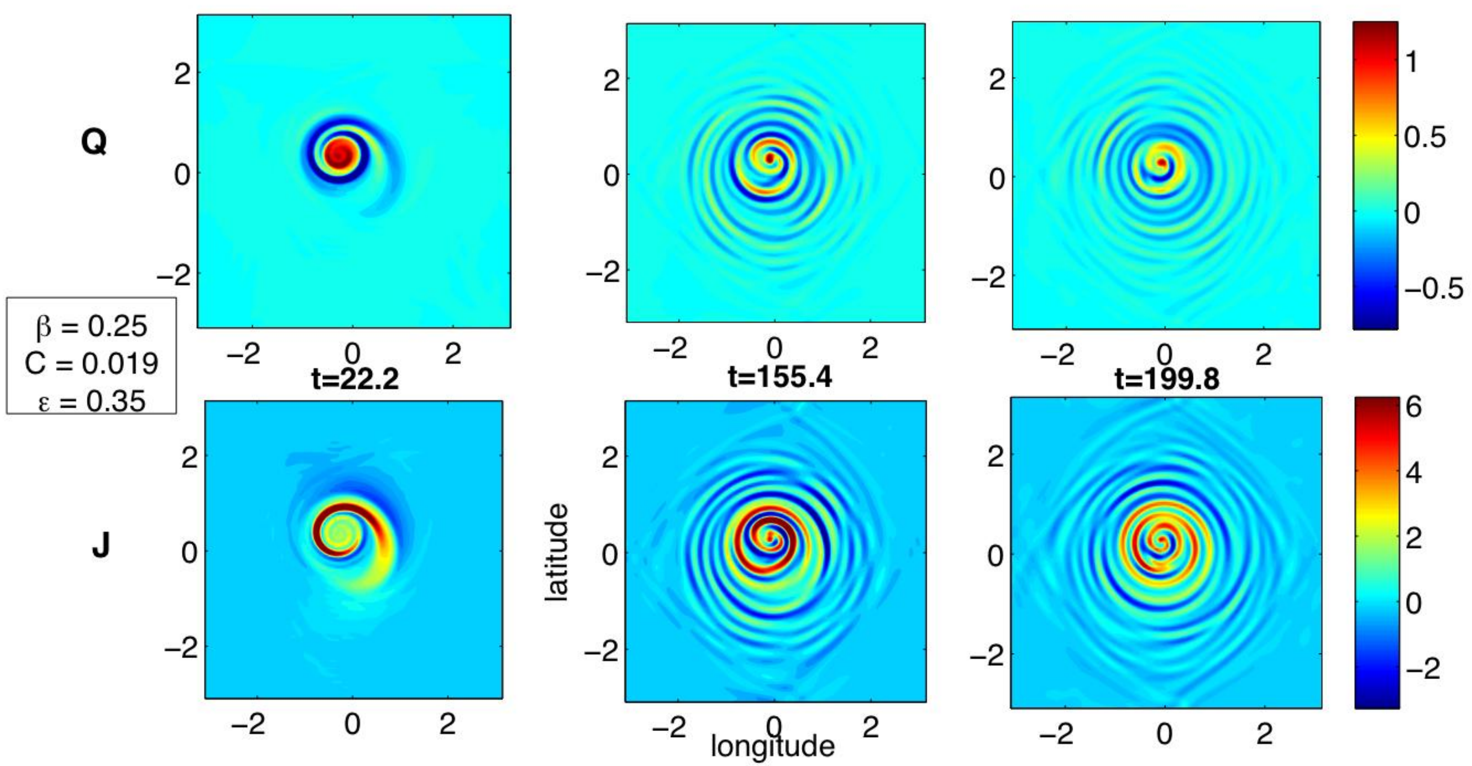




ε = 0.65, *C* = 0.02, β=0.25

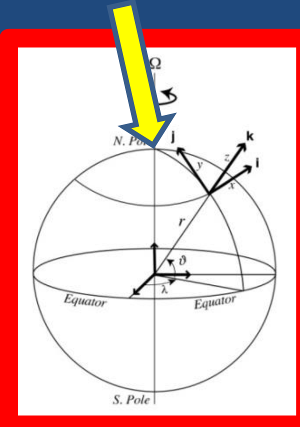
This mid-latitude isolated vortex is linearly unstable in the magnetized limit. The vortex disintegrates by undergoing an m=1 instability. The structure winds apart. The remnants are indistinguishable from the Rossby-Alfven waves seen in other runs. The broken pieces of current and vorticity remain contained in the vicinity of the original structure..

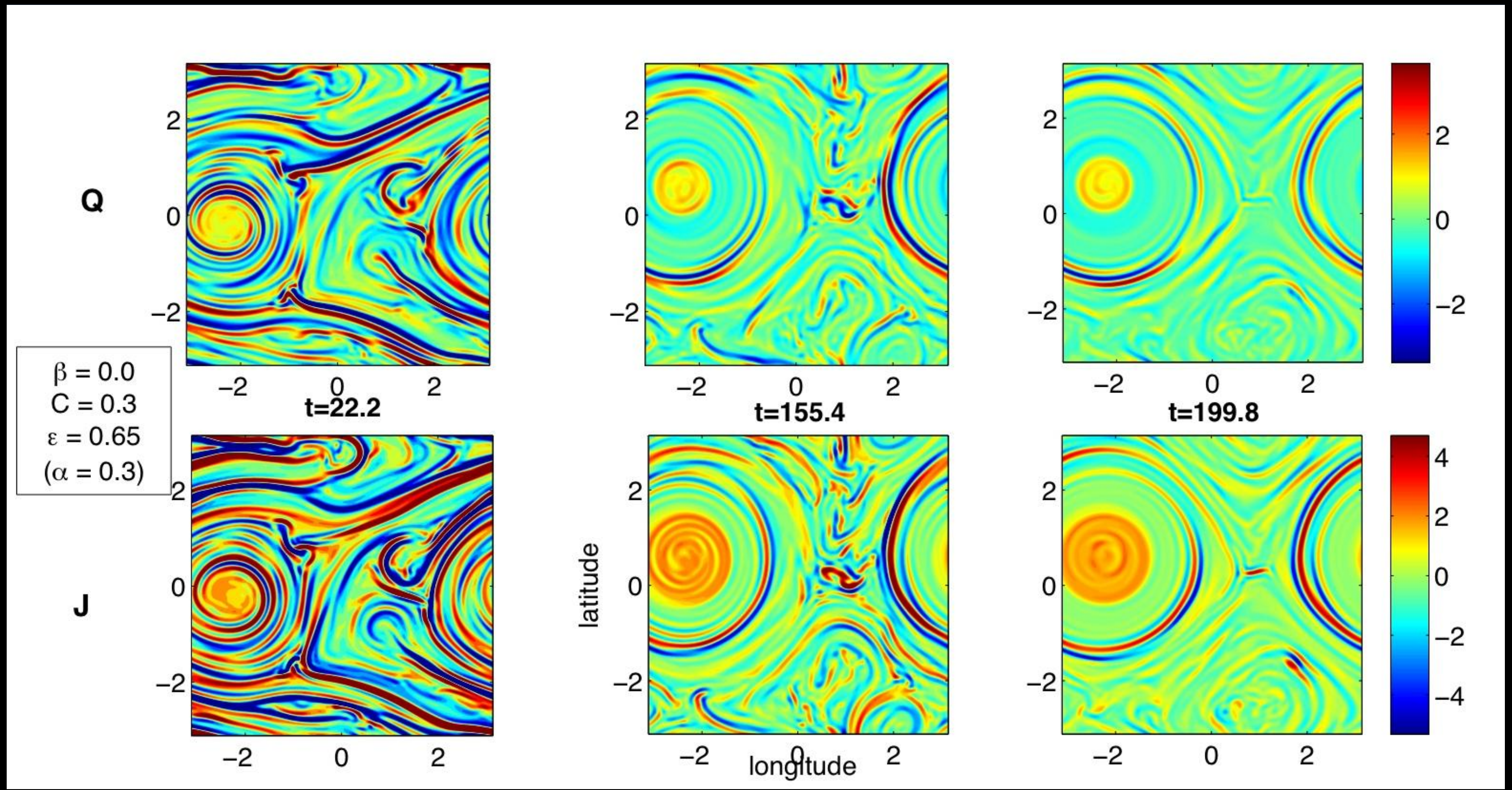




ε = 0.65, C = 0.3, β=0 (Large Amplitude Perturbations)

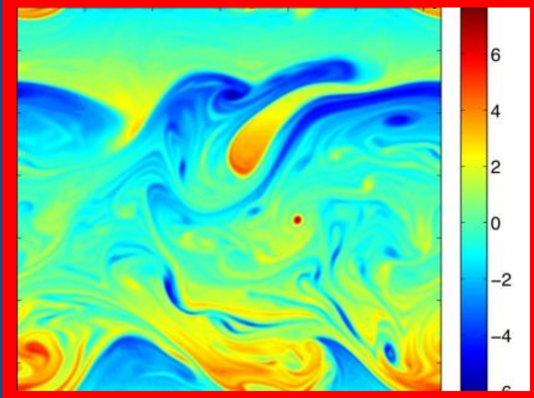
The example shown is a stable isolated vortex located near the pole. This structure is subjected to strong amplitude forcing (whose velocity fields are comparable to the Alfven wave speeds). The long-time behavior shows stability after shedding copious amounts of Rossby-Alfven waves. The energy in these waves are dissipated by the artifical viscosity of the runs.

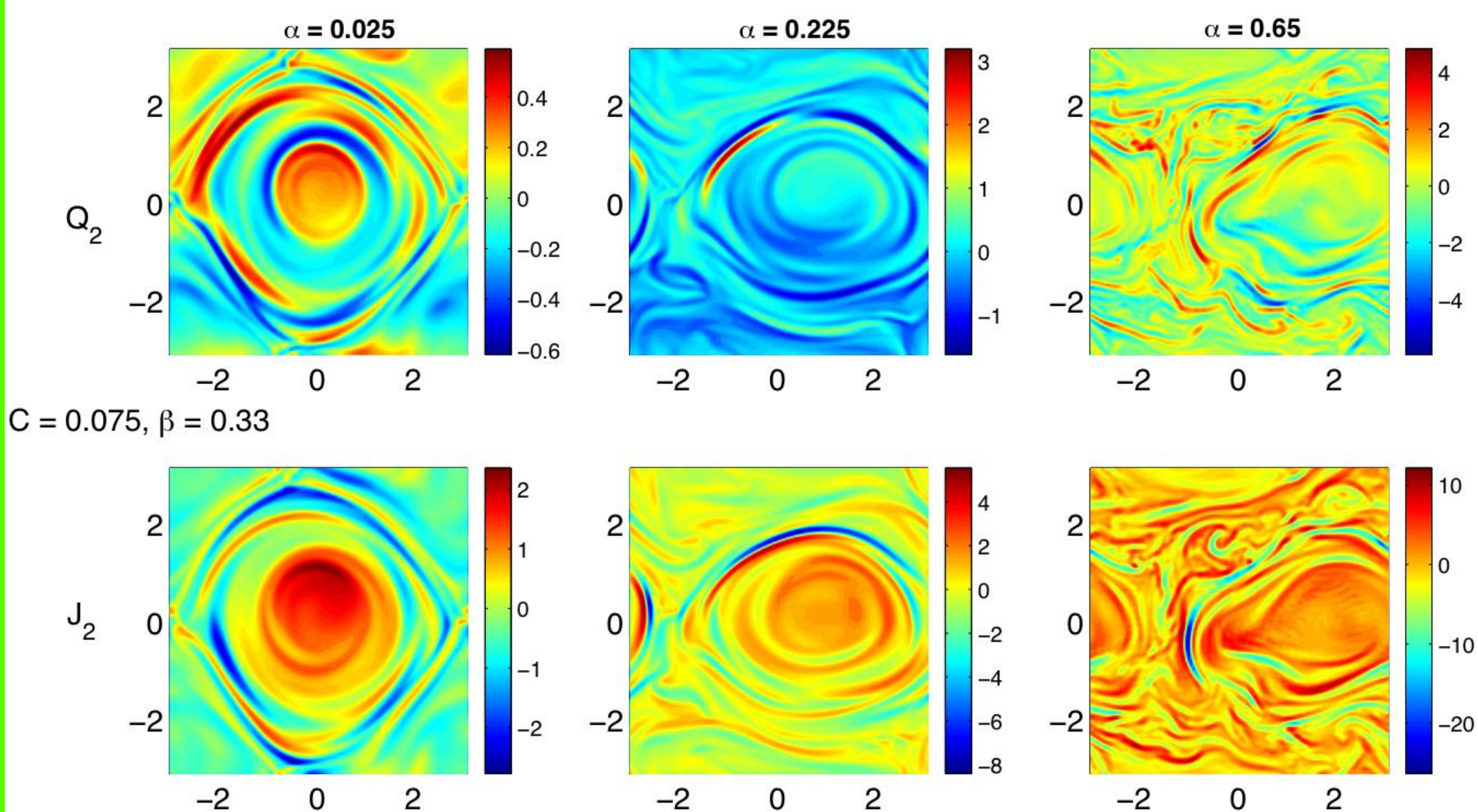




Two Layer Calculation.

A two layer configuration in which the lower layer is non-magnetized and seeded with the potential vorticity field on the right with amplitude α. The upper magnetized layer is seeded with an isolated vortex. The robustness of these structures to driving from below is examined. The three columns below show the fate of the isolated vortex for three values of the amplitude of the turbulent velocity field below. For values of α>0.65 (not shown) the vortex is finally destroyed.





Conclusions and the Future

- Irradiated exoplanet atmospheres which are ionized will need to be addressed by MHD considerations. The development of a quasi-geostrophic reduction which includes MHD effects on a primitive level provides a tool to start understanding what to expect on the synoptic scales of the ionized layers of exoplanets.
- There are many unknowns at this stage: What the are the mean and rms magnetic field strengths figure most prominently (i.e. what are the Cowling Numbers in their stratospheres) as well as the atmosphere entropy scale heights.
- Simulations of stably stratified MHD atmospheres show the emergence of isolated vortices which are stable under a wide variety of conditions but fall apart once magnetization vanishes.
- Current/Future work include (i) understanding how jet flows are affected by the presence of overlying magnetized layers (ii) understanding if the emergence of isolated vortices might provide an effective `Rayleigh-Drag' on flows and determine its parametrization (iii) understand the emergence and evolution of baroclinic instability under magnetic conditions.