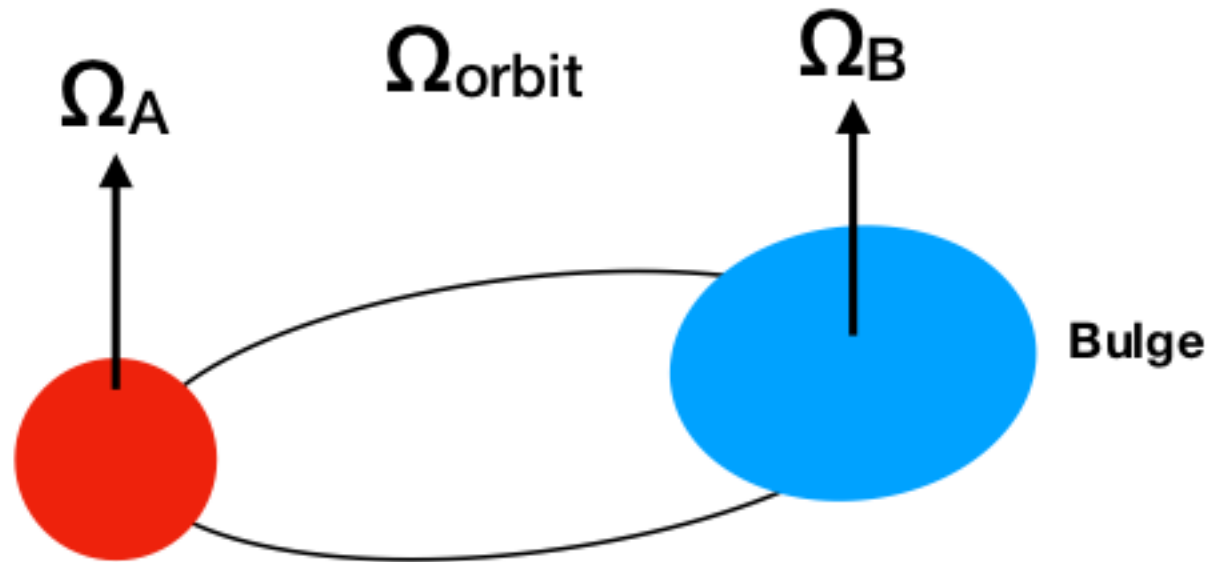


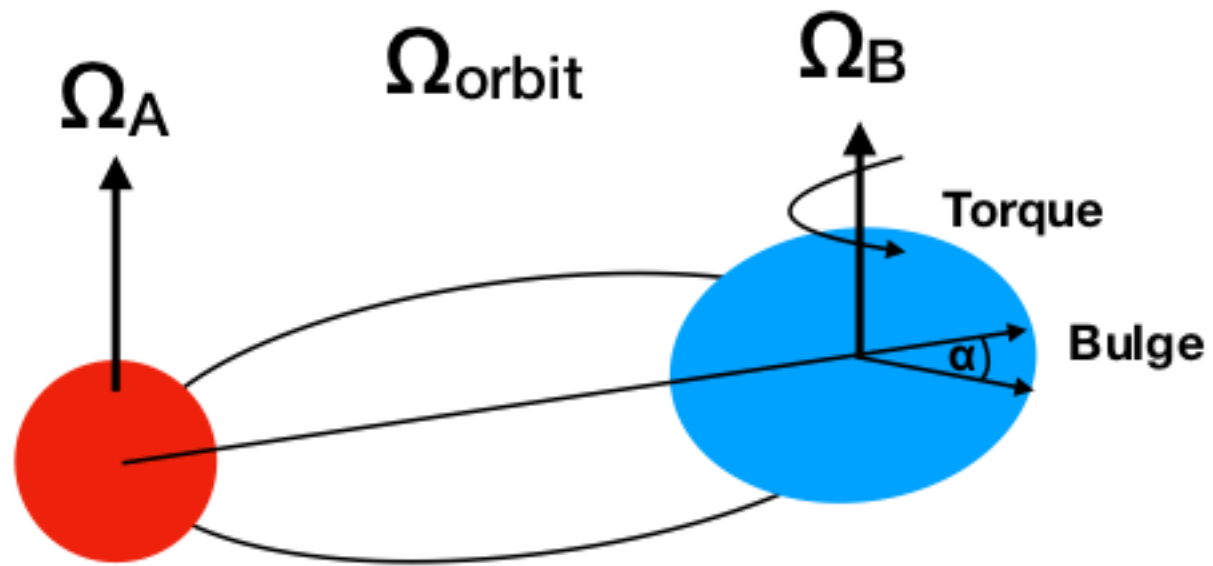
Tides, Differential Rotation, and Eclipsing Binaries

Adam Jermyn
with Jamie Tayar and Jim Fuller

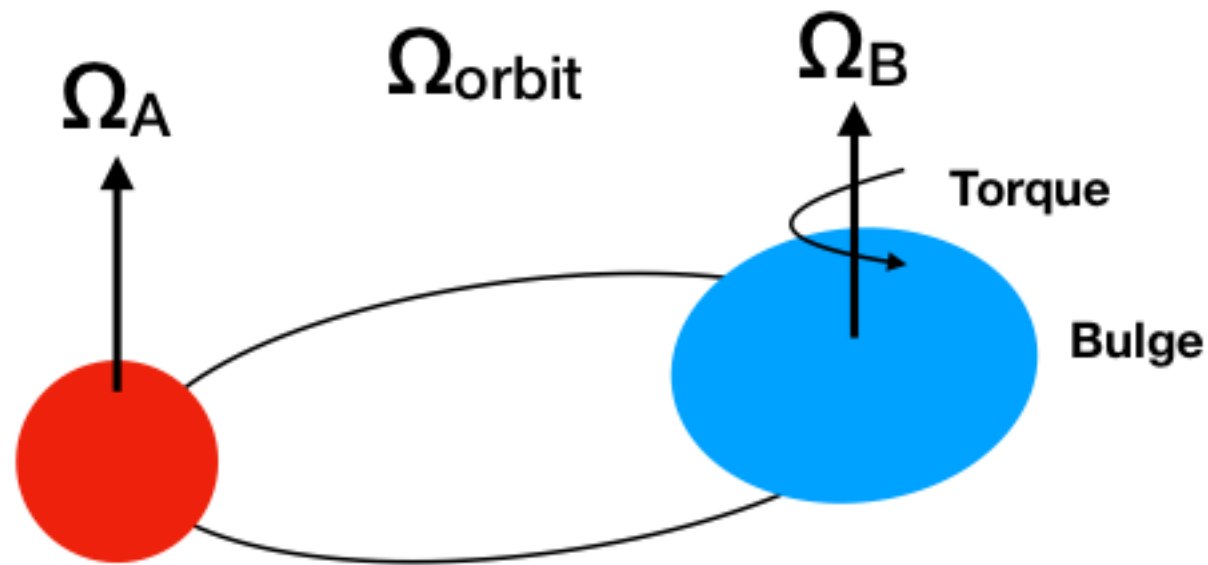
Tides



Tides

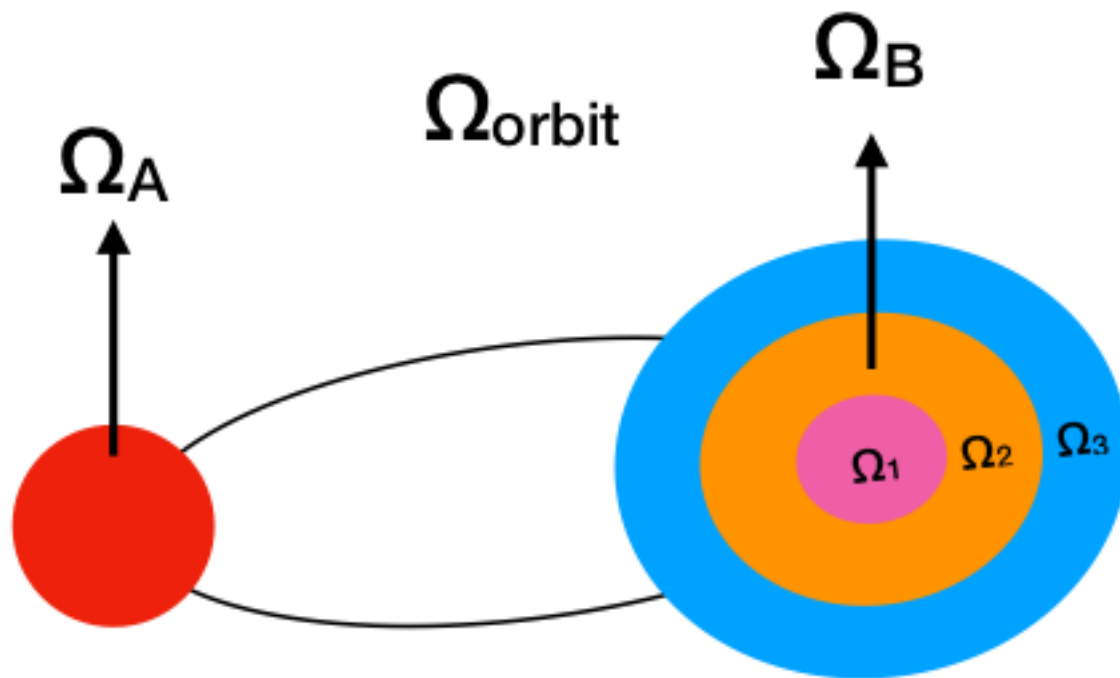


Tides



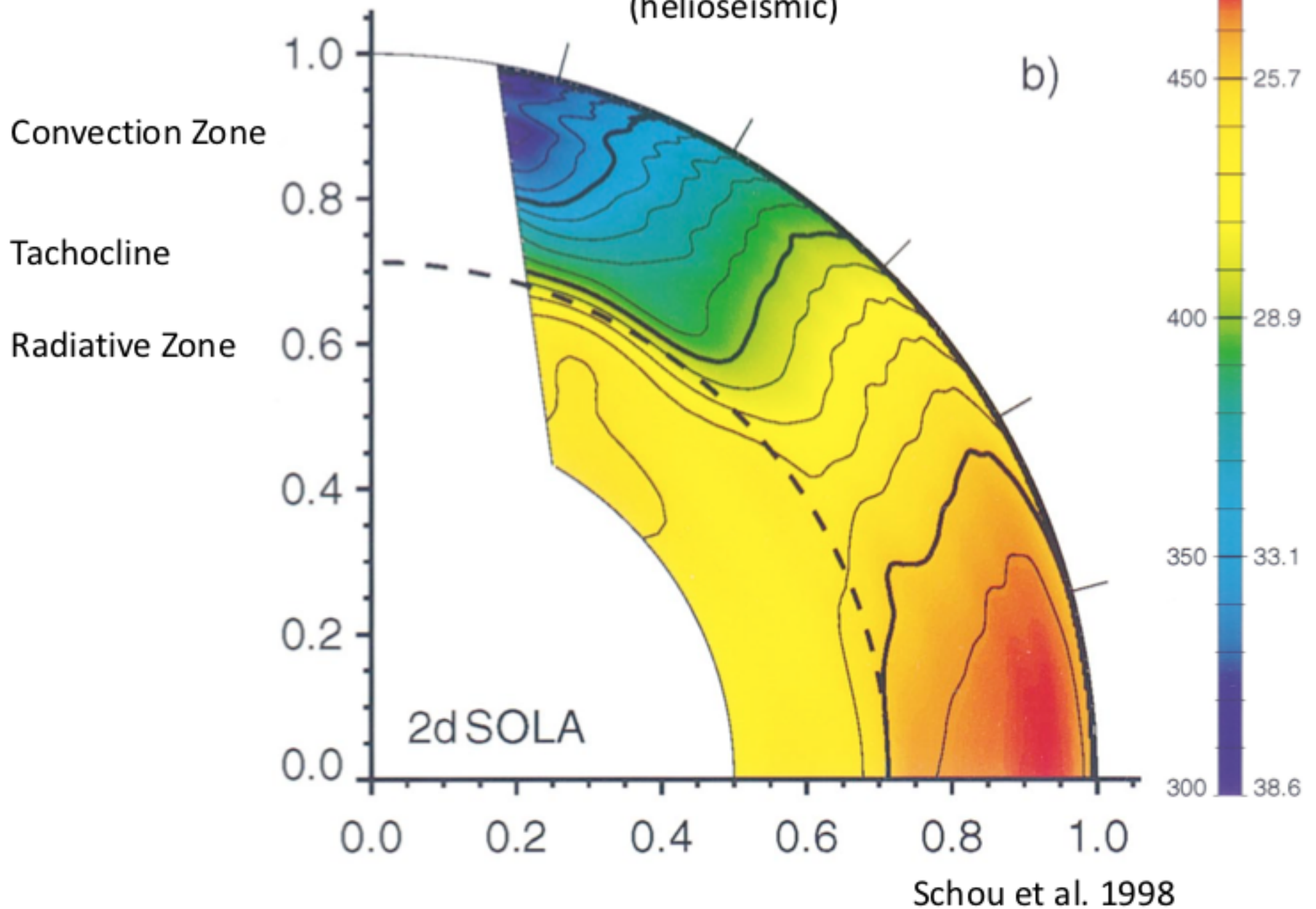
Over time, $\Omega_{A/B}$ synchronize to Ω_{orbit}

Now with differential rotation...



1. Does differential rotation survive a tidal torque?
2. If it does, what does synchronizing mean?

Solar Rotation Profile (helioseismic)



Does differential rotation survive?

$t_{\text{transport}} \gg t_{\text{tides}}$: Tides control the shear

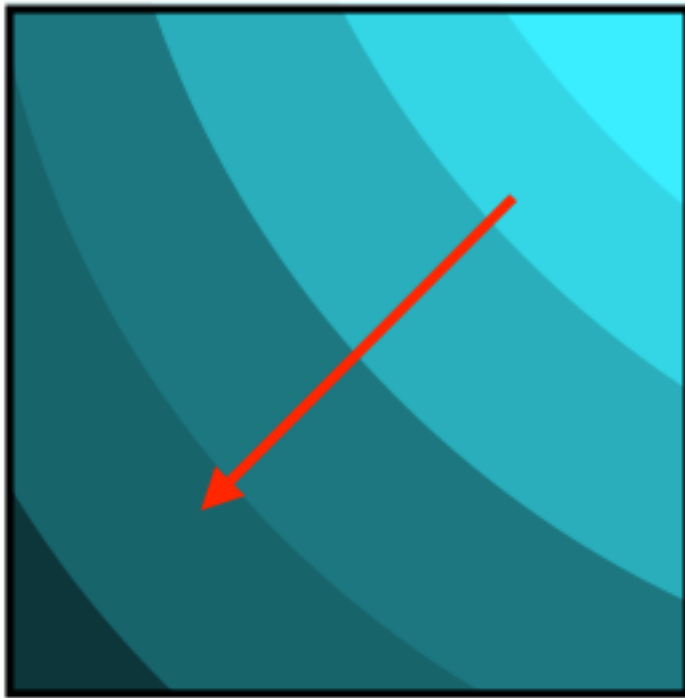
$t_{\text{transport}} \ll t_{\text{tides}}$: Intrinsic shear wins

Depends on what transports angular momentum!

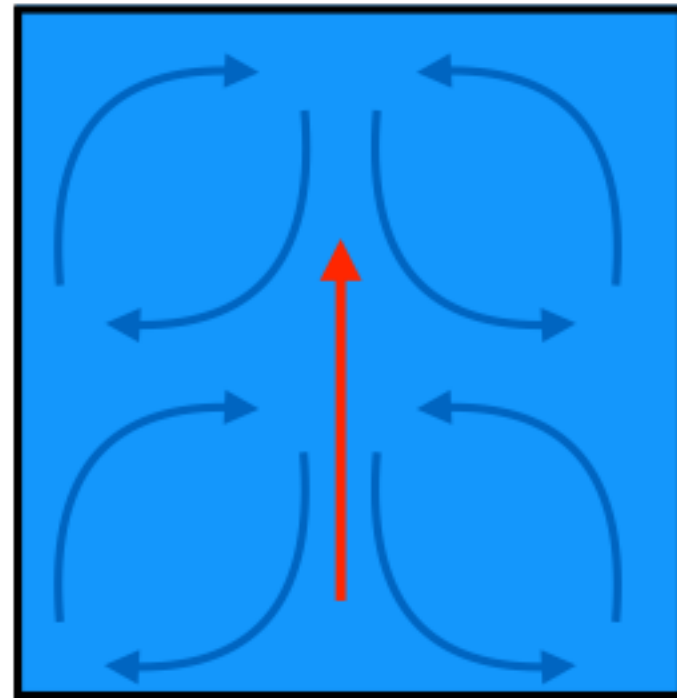
Does differential rotation survive?

Depends on what transports angular momentum!

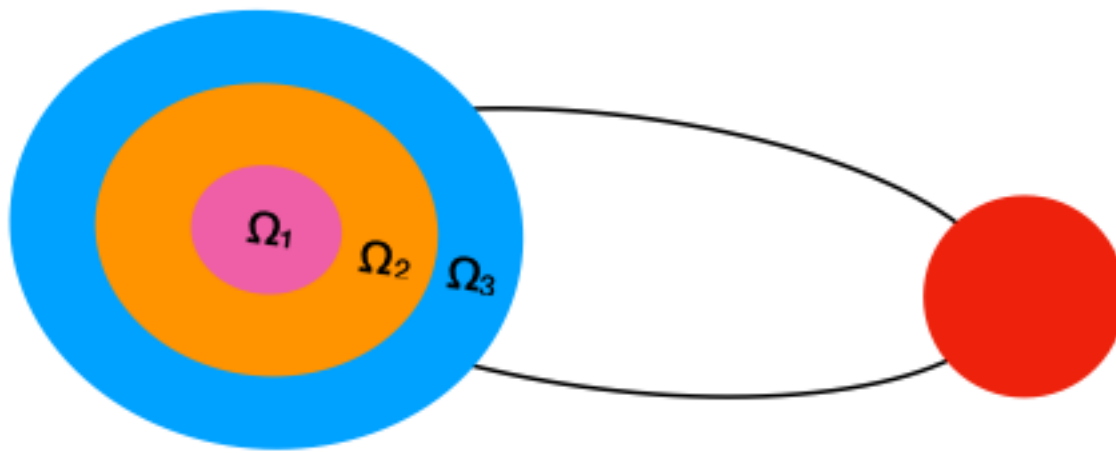
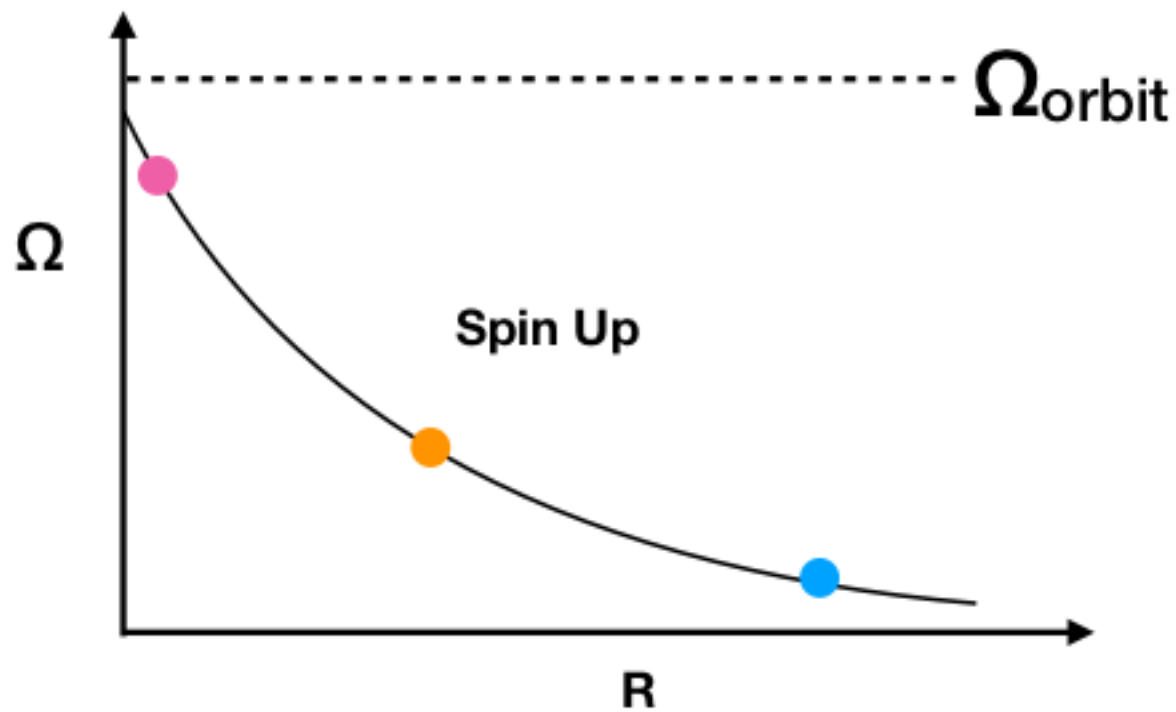
Radiative zones... unclear.



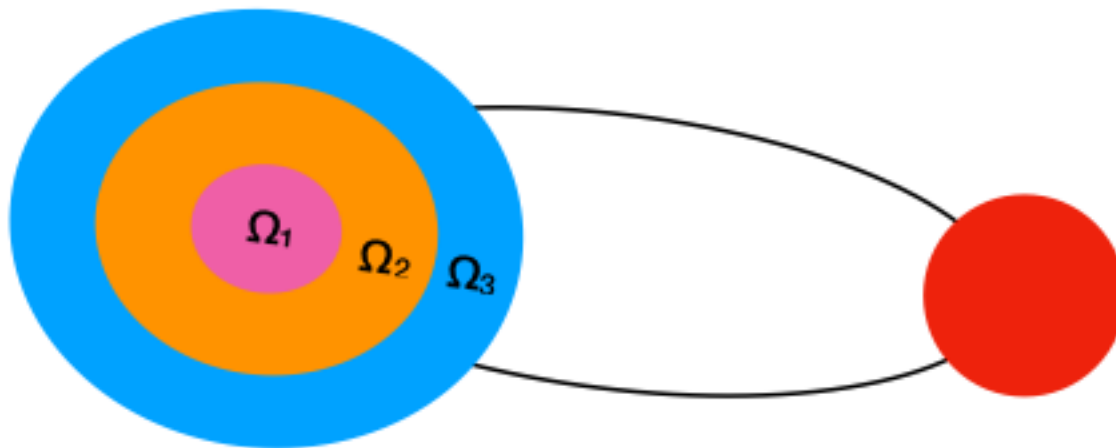
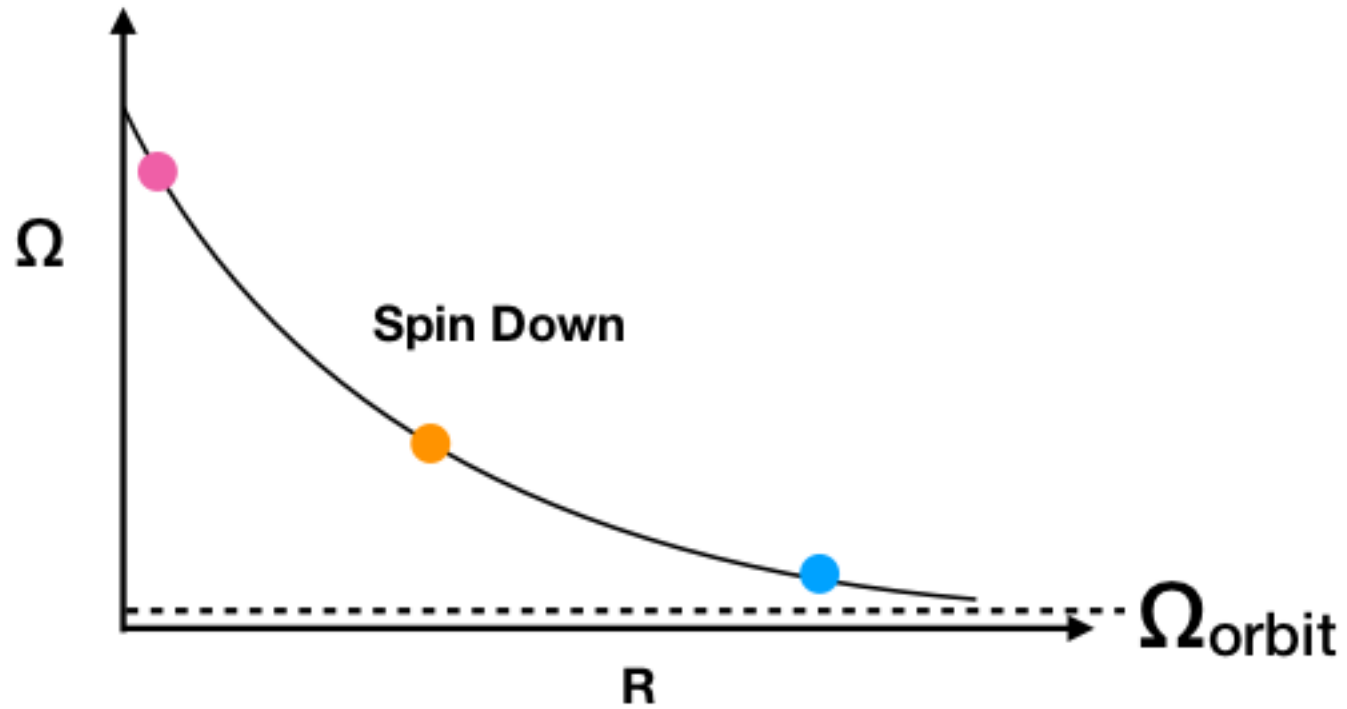
Convection zones... yes!
($t_{\text{conv}} \ll t_{\text{tides}}$)



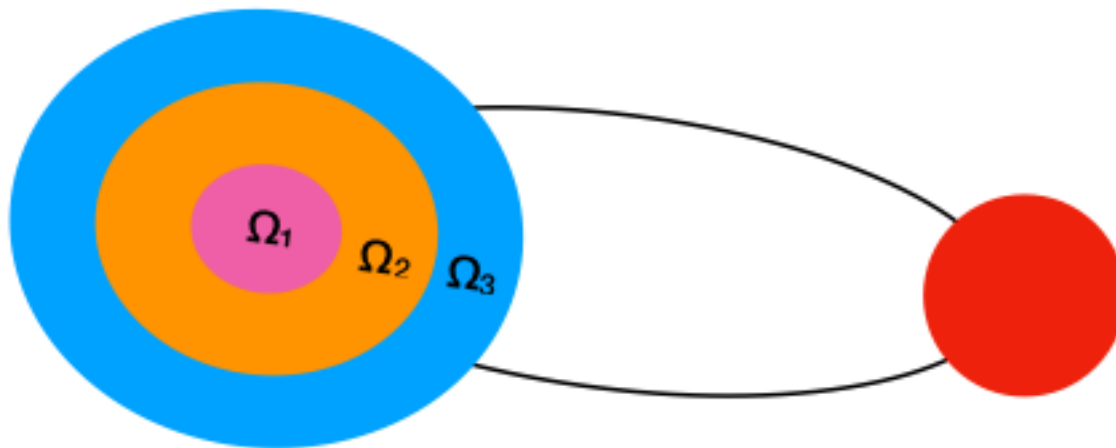
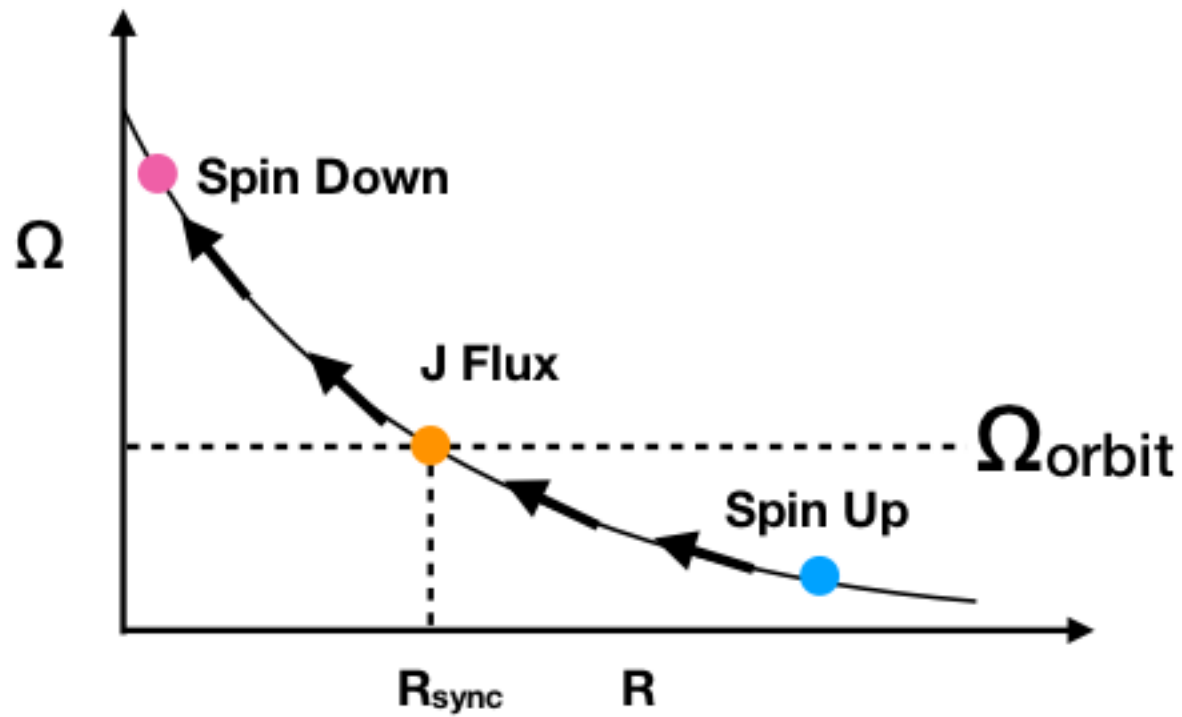
Tides in convecting stars



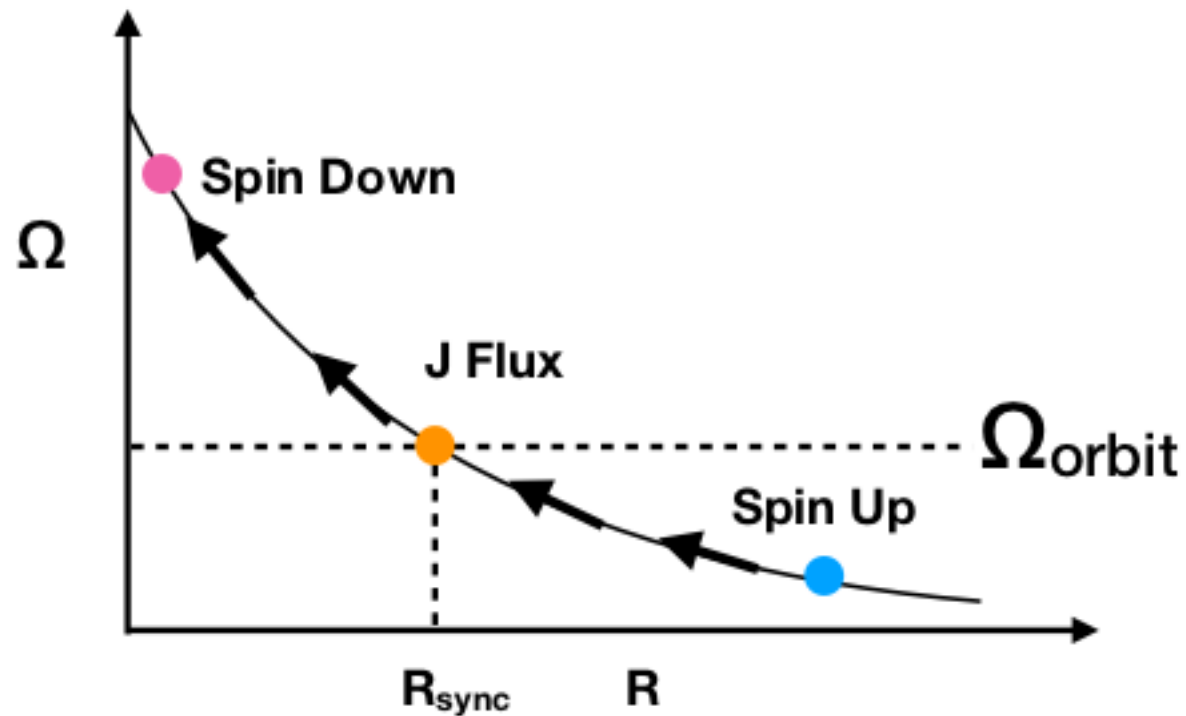
Tides in convecting stars



Tides in convecting stars

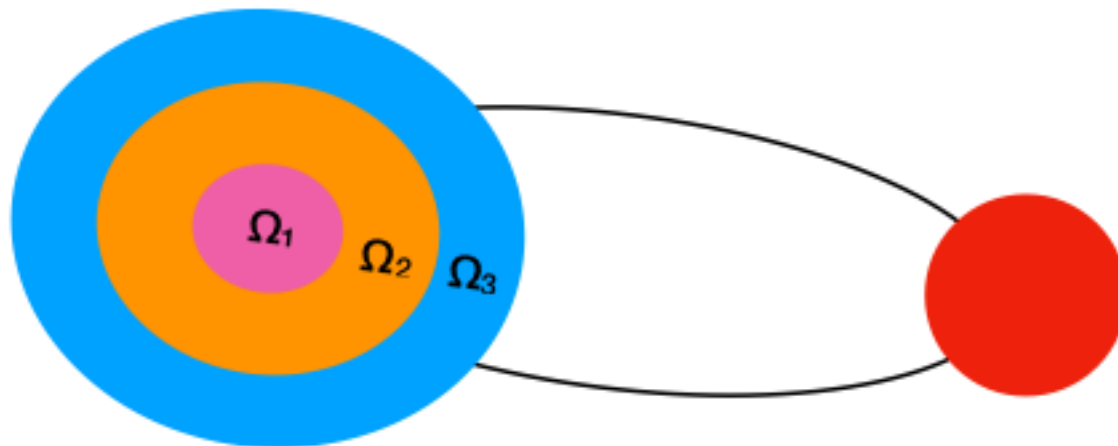


Tides in convecting stars

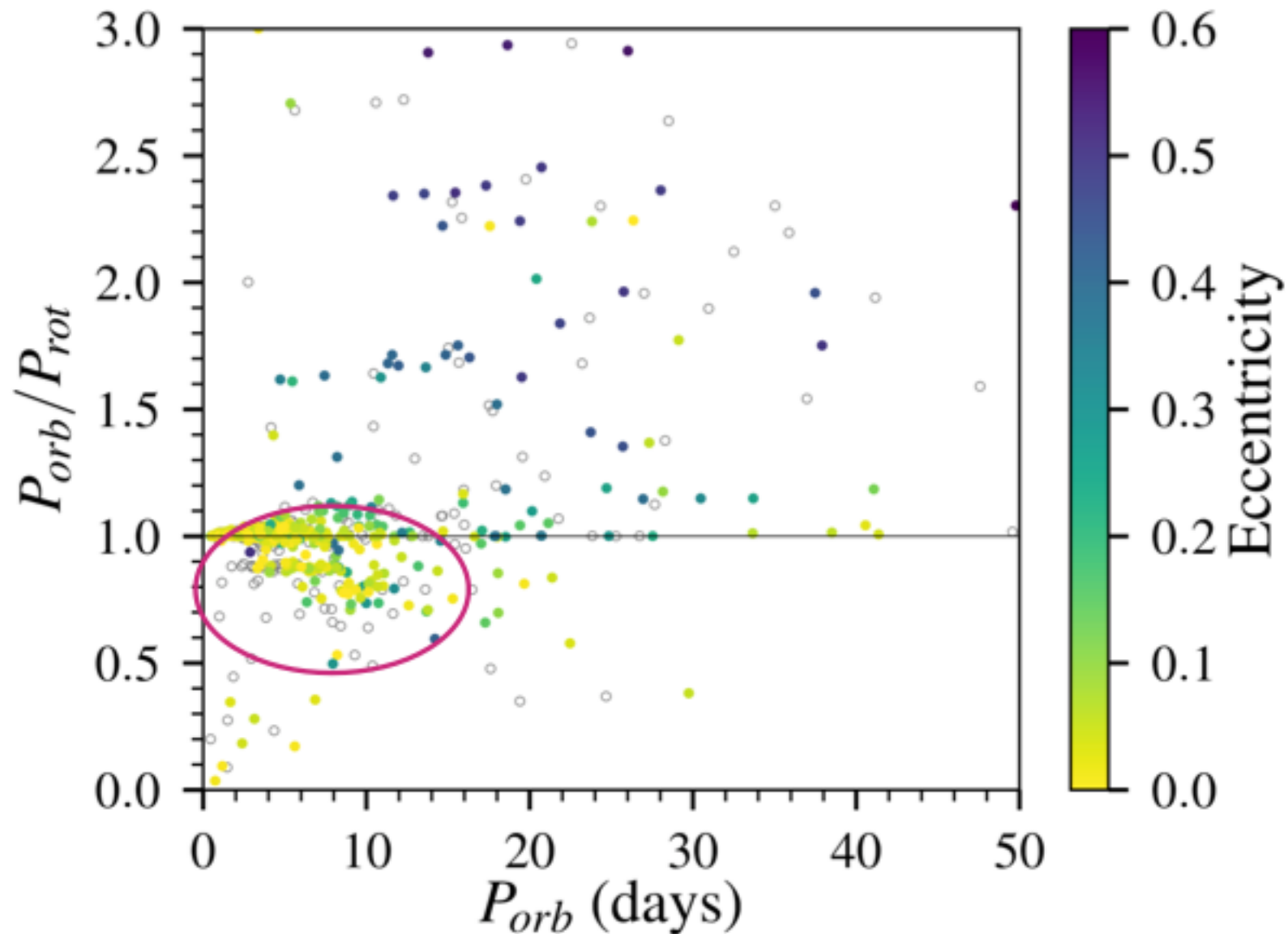


Synchronized:

- Net torque on each star is zero.
- Surface period may not equal orbital period!



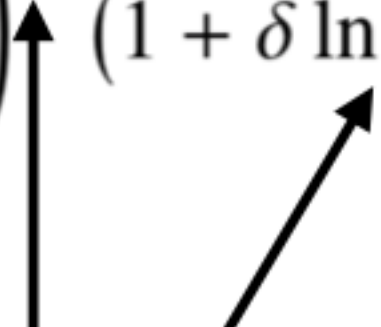
Eclipsing Binaries



Lurie et al. 2017

Inference Problem

Toy Model: $\Omega(r, \theta) = \Omega_0 \left(\frac{r}{R_\star} \right)^\beta (1 + \delta \ln \Omega P_2(\cos \theta))$



Two free parameters

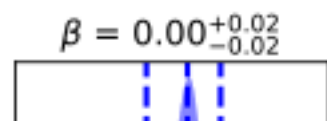
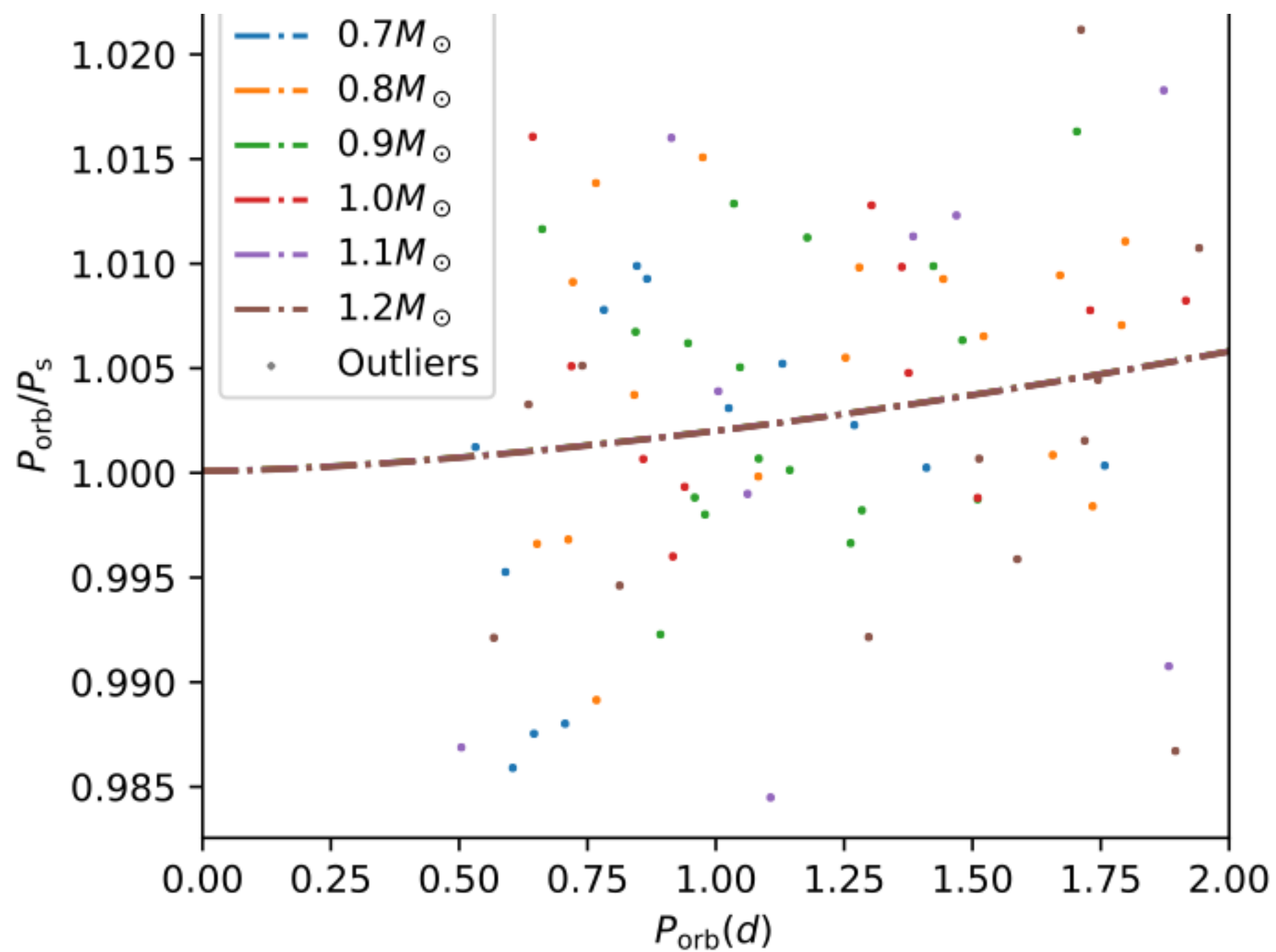
Inference Problem

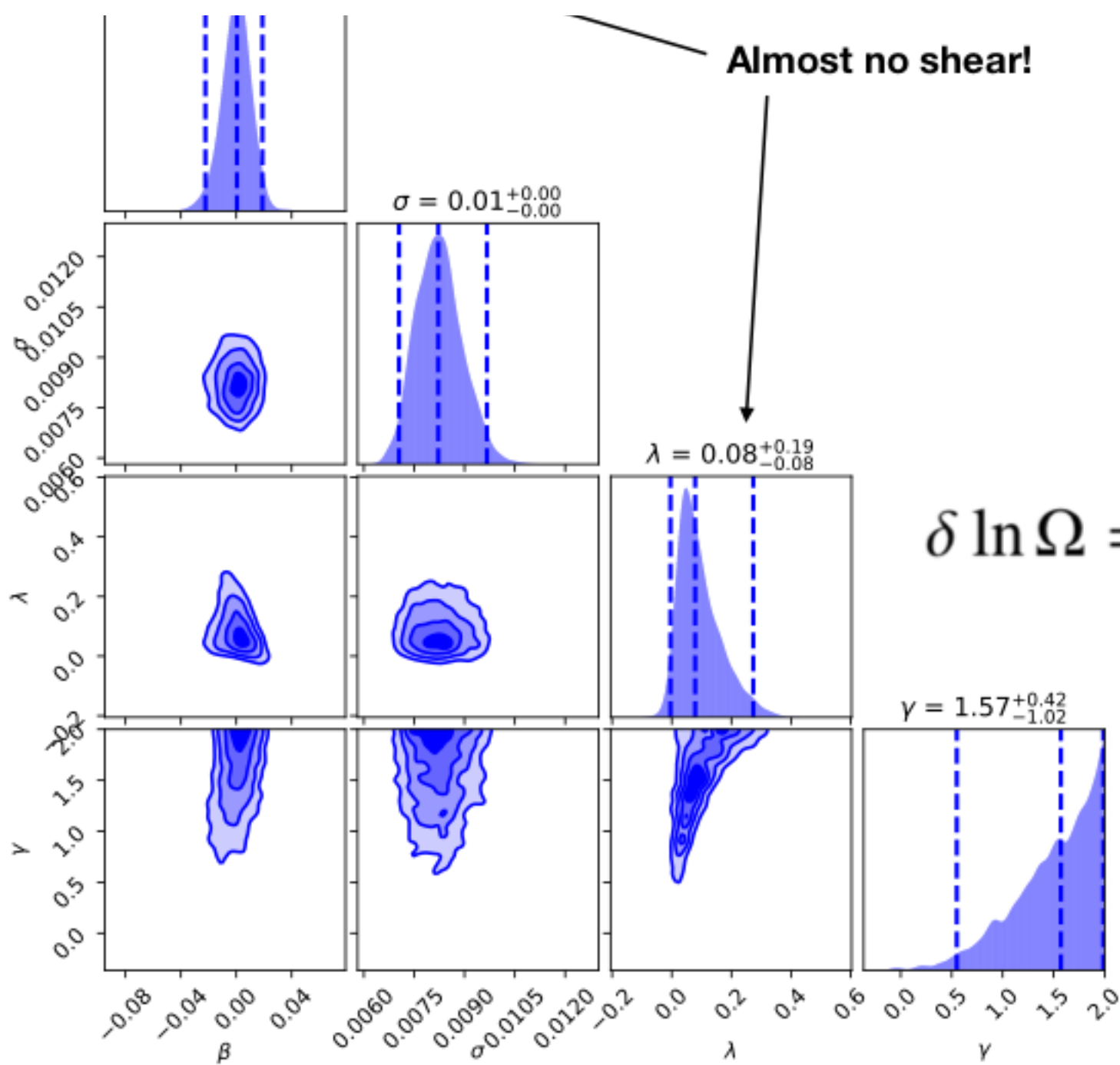
Toy Model: $\Omega(r, \theta) = \Omega_0 \left(\frac{r}{R_\star} \right)^\beta (1 + \delta \ln \Omega P_2(\cos \theta))$

Prediction: $\frac{P_{\text{orb}}}{P_s} \approx k_\star^{-1}(\beta) \frac{1 + \frac{5}{8} \delta \ln \Omega}{1 - \frac{2}{7} \delta \ln \Omega}$ (from tidal integrals)



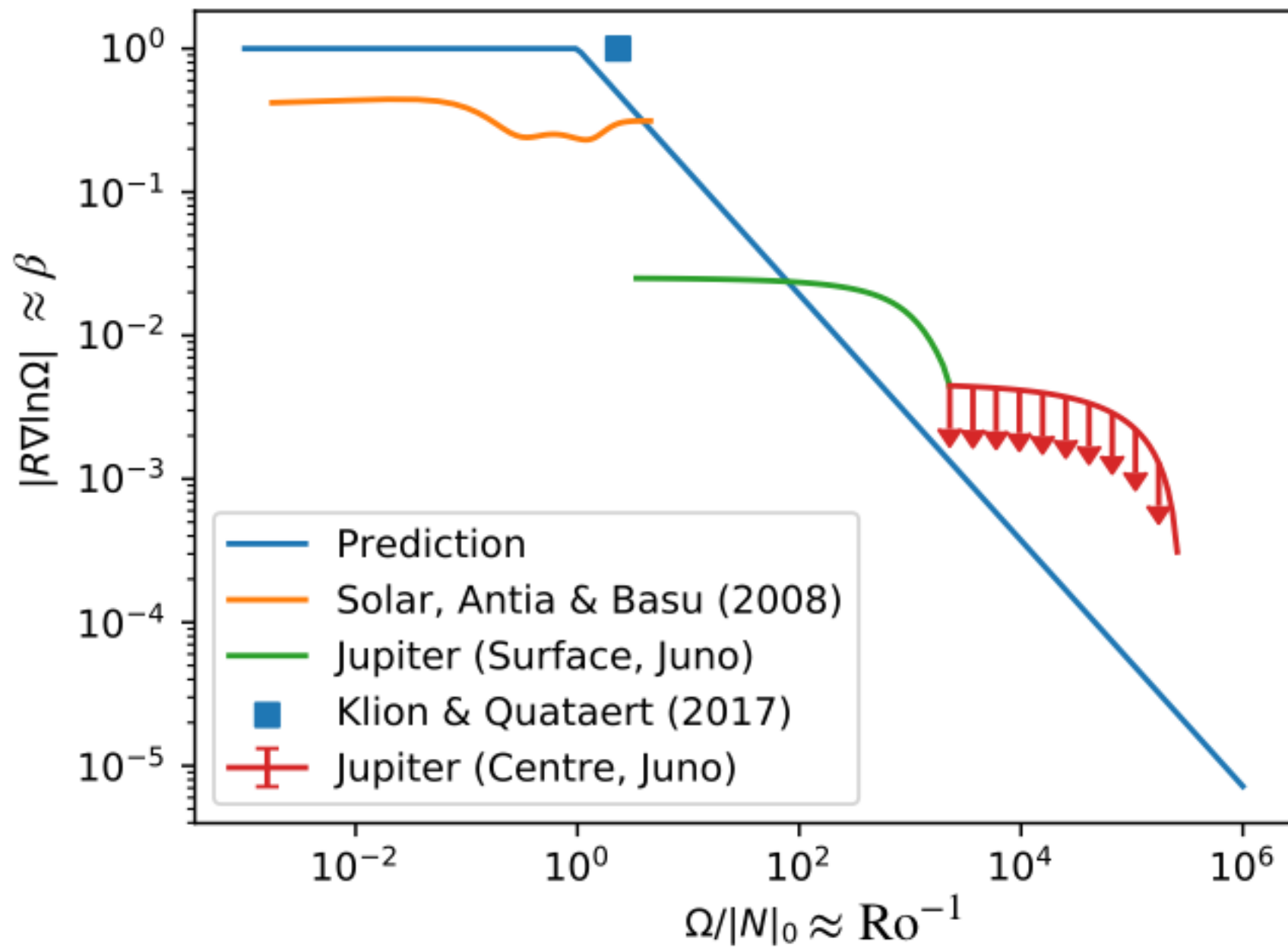
- Depends on stellar structure and β .
- Assume MS, pick mass to match Kepler temperature

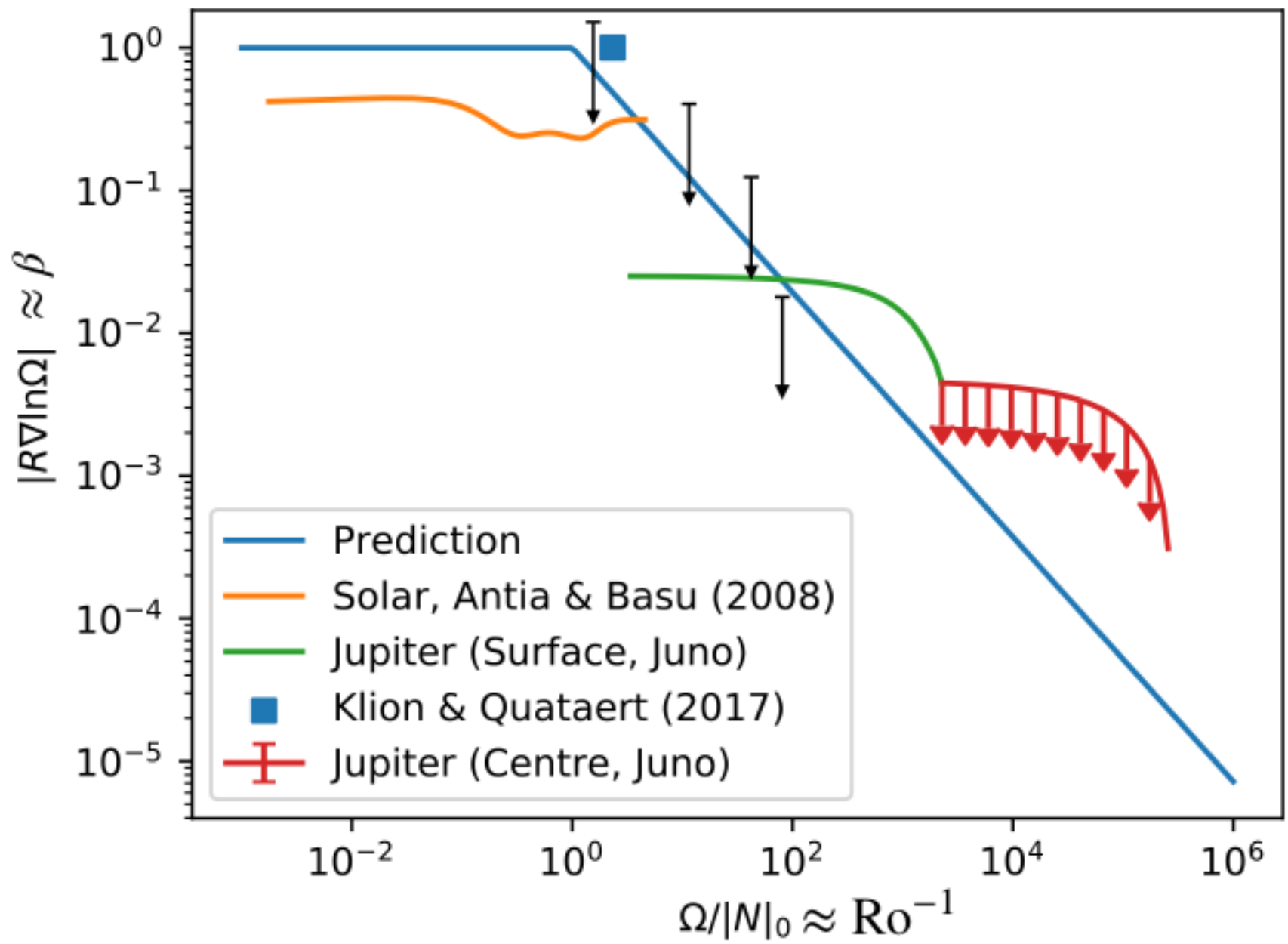




$$\delta \ln \Omega = \lambda \left(\frac{P_s}{10d} \right)^\gamma$$

$P_{\text{orb,min}}$	$P_{\text{orb,max}}$	β Radial Shear	$\beta_{1-\sigma}$	$\beta_{2-\sigma}$
0	50	0.152	0.251 -0.482	0.337 -0.632
0	2	0.000	0.010 -0.010	0.019 -0.022
2	6	0.031	0.091 -0.252	0.139 -0.346
6	10	0.066	0.373 -0.330	0.677 -1.186
10	50	0.264	1.329 -1.911	2.267 -2.778

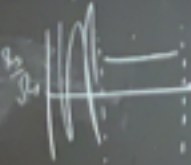




Jermyn+2020 arXiv:2008.09125

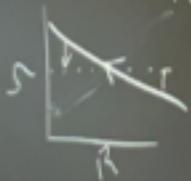
Questions?

$$\begin{aligned}
 &= \int (R-r)^2 r^3 \left(\Omega_s \left(\frac{r}{R} \right)^{\beta} - \Omega_0 \right) dr \\
 &= -\Omega_0 \int r^3 (R-r) dr + \Omega_s \int \frac{r^{3+\beta}}{R^{\beta}} (R-r)^2 dr \\
 &= R^5 \left[-\frac{1}{2} \Omega_0 + \frac{1}{10} \Omega_s \right] \\
 &= \frac{R^5}{10} \left[\frac{10\Omega_s}{10} - \frac{5\Omega_0}{10} \right] \rightarrow \Omega_s = 1 - \frac{7}{10} \beta
 \end{aligned}$$



$$c_0 = \int_0^1 (1-x)^2 x^3 dx = \int_0^1 x^3 - 2x^4 + x^5 dx = \frac{1}{4} - \frac{2}{5} + \frac{1}{6} = \frac{30+20-40}{120} = \frac{10}{120} = \frac{1}{12}$$

$$c_1 = \int_0^1 x^{2\beta} (1-x)^2 dx = \int_0^1 x^{2\beta} (1-2x+x^2) dx = \frac{1}{2\beta+1} - \frac{2}{2\beta+2} + \frac{1}{2\beta+3} = \frac{(2\beta+2)(2\beta+3) - 2(2\beta+1)(2\beta+3) + (2\beta+1)^2}{(2\beta+1)(2\beta+2)(2\beta+3)}$$



$$\begin{aligned}
 &= \frac{30 - 11\beta + 10 - 11\beta + 10\beta + 20\beta}{120} \int dm = \frac{50 - 11\beta + 20\beta}{120} \int dm = \frac{50 + 9\beta}{120} \int dm \\
 &= \frac{2}{120} + \frac{1}{60} = \frac{2 + 2\beta}{120} = \frac{1 + \beta}{60} \\
 &= \frac{1}{60} \int_0^R \frac{1}{a} \left(\frac{\Omega - \Omega_0}{\Omega_0} \right) \left(\frac{6M_0^2}{r} \right) r^2 dr \\
 &= \int_0^R \frac{1}{60} \frac{1}{a} \left(\frac{\Omega - \Omega_0}{\Omega_0} \right) \left(\frac{6M_0^2}{r} \right) r^2 dr \\
 &= \int_0^R \frac{1}{60} \frac{1}{a} \left(\frac{\Omega - \Omega_0}{\Omega_0} \right) 6M_0^2 r dr \\
 &= \int_0^R \frac{1}{60} \frac{1}{a} \left(\frac{\Omega - \Omega_0}{\Omega_0} \right) 6M_0^2 r dr
 \end{aligned}$$

$$\begin{aligned}
 &V_c \sim (F/g)^{2/3} \\
 &\rightarrow hV \sim \left(\frac{F}{g} \right)^{2/3} \\
 &P \sim \rho g^{5/3} \cdot h^2 \sim \frac{\rho g^{5/3}}{5} \sim R \cdot \Omega \\
 &\rightarrow \frac{1}{Q} \rightarrow \frac{1}{Q} \sim \frac{h^2}{R} \sim \frac{(R-r)^2}{R} \\
 &\sim \left(\frac{F}{g} \right)^{2/3} \sim \left(\frac{F}{g} \right)^{2/3} \sim \frac{T^{4/3}}{g^{2/3} R} \\
 &\sim \frac{T^{4/3}}{g^{2/3}} \sim \left(\frac{1}{2} \right)^{2/3} \sim \left(\frac{1}{2} \right)^{2/3}
 \end{aligned}$$