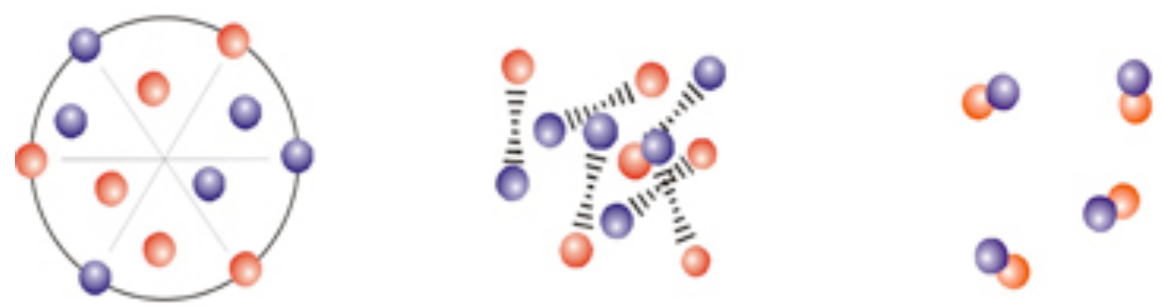
One + many fermions,
or
how do we swim in muddy waters?

Pietro Massignan

ICFO
Institut de Ciències Fotòniques

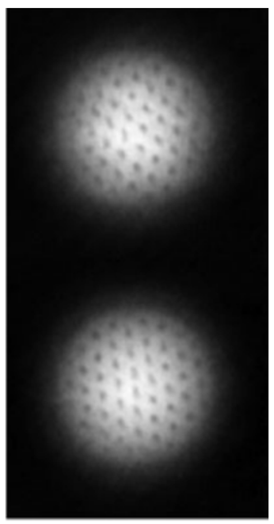
Attractive Fermi Mixtures

N=N at T=0: BCS-BEC crossover



lack of a small parameter ⇒ hard problem!

Population-imbalanced attractive Fermi Mixtures



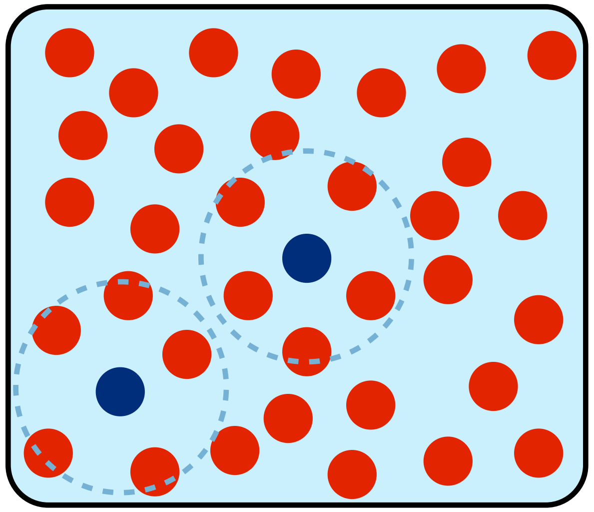
N=N

SF

Zwierlein et al., Nature 2005

Very imbalanced Fermi mixtures

N>>N

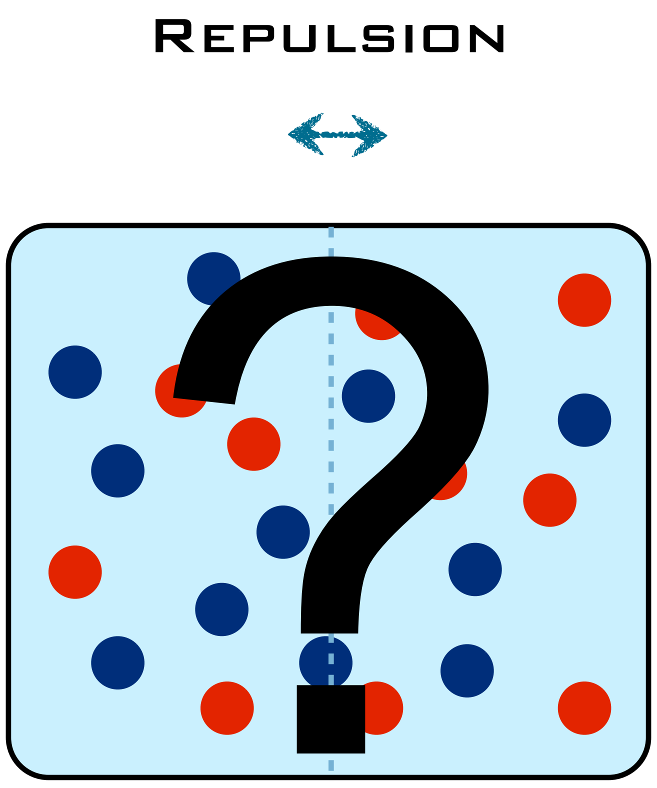


Schirotzek et al., PRL 2009

polarons

lack of a small parameter ⇒ hard problem!

Repulsive Fermi Mixtures



repulsion vs. Fermi pressure

Stoner's Itinerant Ferromagnetism

predicted in 1933, not yet fully understood..

Motivation

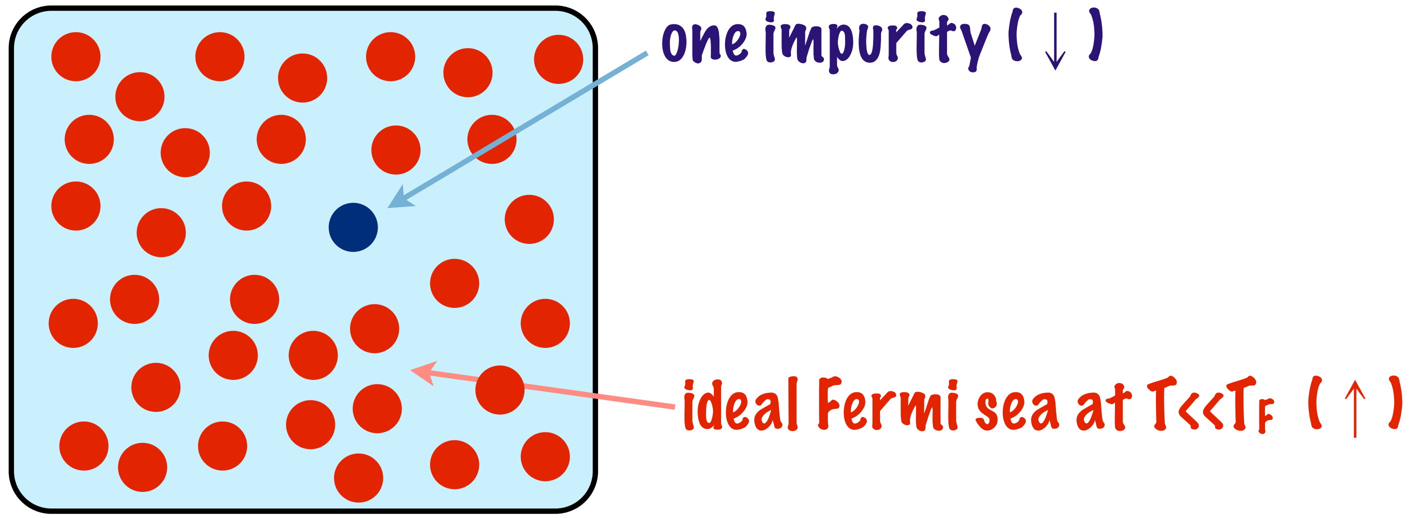
Studying a single impurity in a Fermi sea provides insight on:

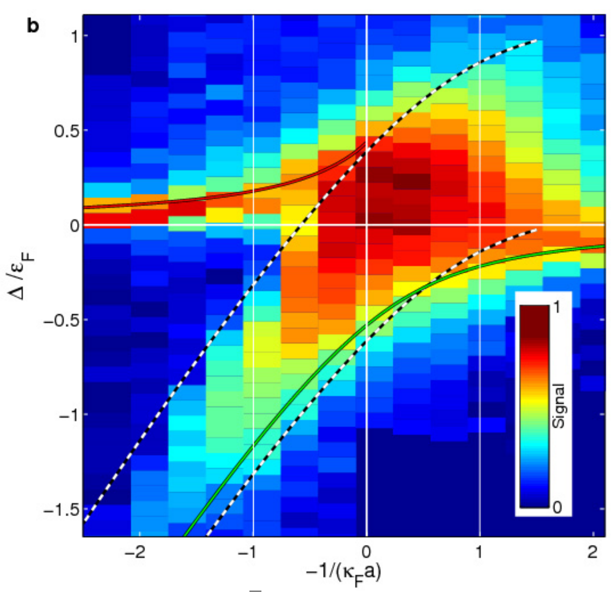
- static, dynamic and coherence properties of fundamental quasiparticles
- phase diagram of imbalanced Fermi gases
- decay mechanisms
- routes towards Itinerant Ferromagnetism?

Very quick few-to-many convergence in one dimension

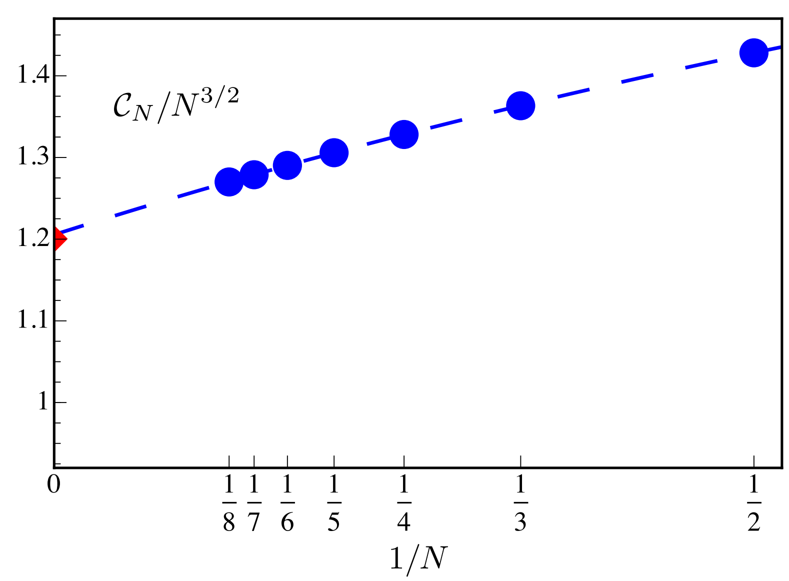
How can few-body calculations contribute further to our understanding?

Outline of this talk





three dimensions

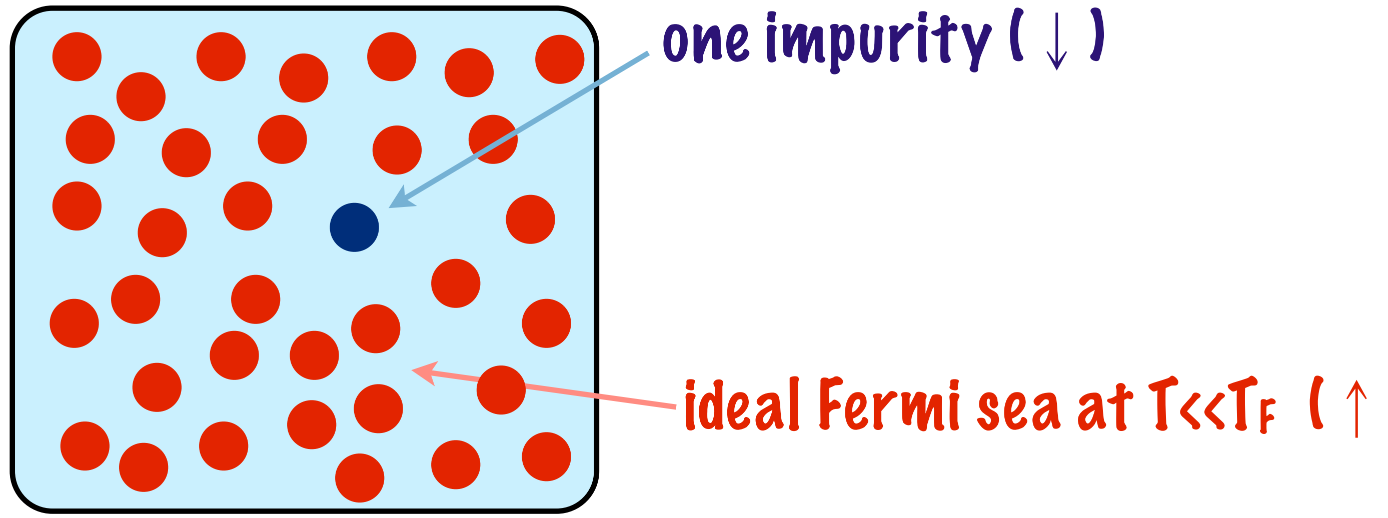


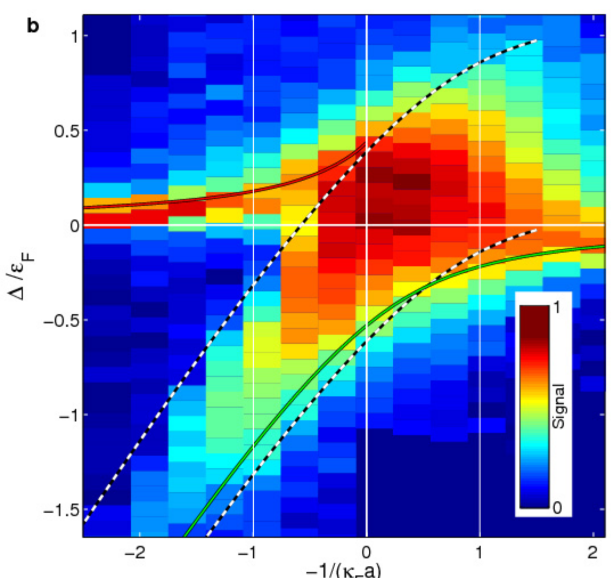
one dimension

?

few → many

Outline of this talk





three dimensions

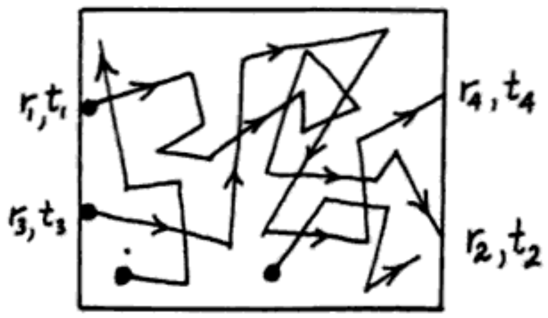
one dimension

?

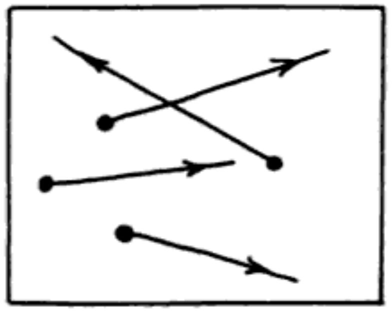
few → many

Quasi-Particles

Landau's idea:
only fools will care about real particles!



Of importance are the excitations, which often behave as quasi-particles!



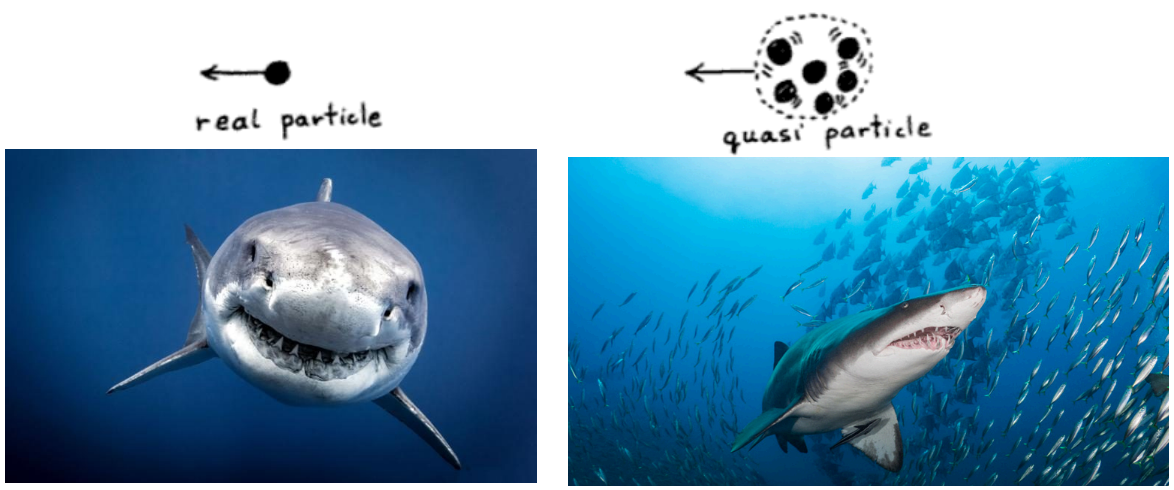
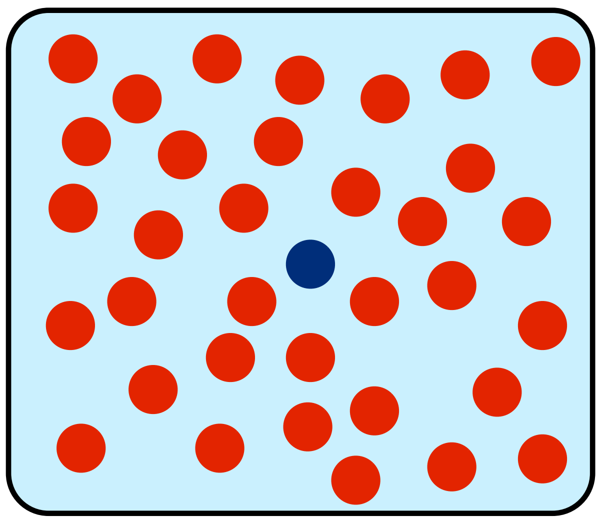


Fig. 0.4 *Quasi Particle Concept*

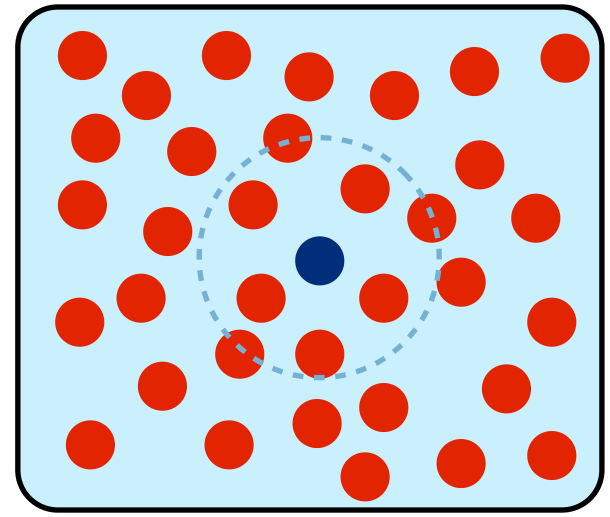
a QP is a "free" particle with:
@ q. numbers (charge, spin, ...)
@ chemical potential
@ renormalized mass
@ shielded interactions
@ lifetime

The impurity problem

Switch on interactions

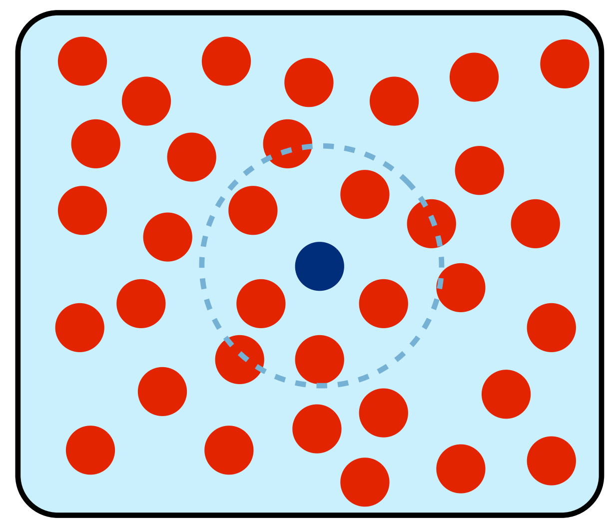


repulsive

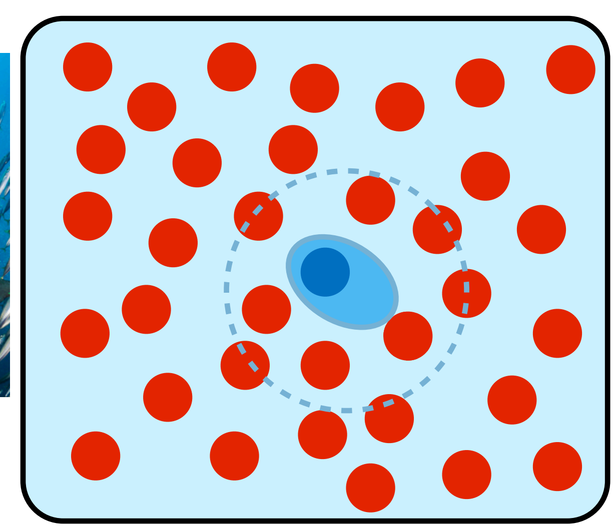


Repulsive polaron

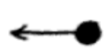
attractive



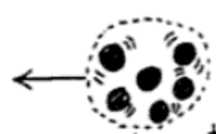
Attractive polaron



Dressed Molecule-Hole



real particle



quasi particle

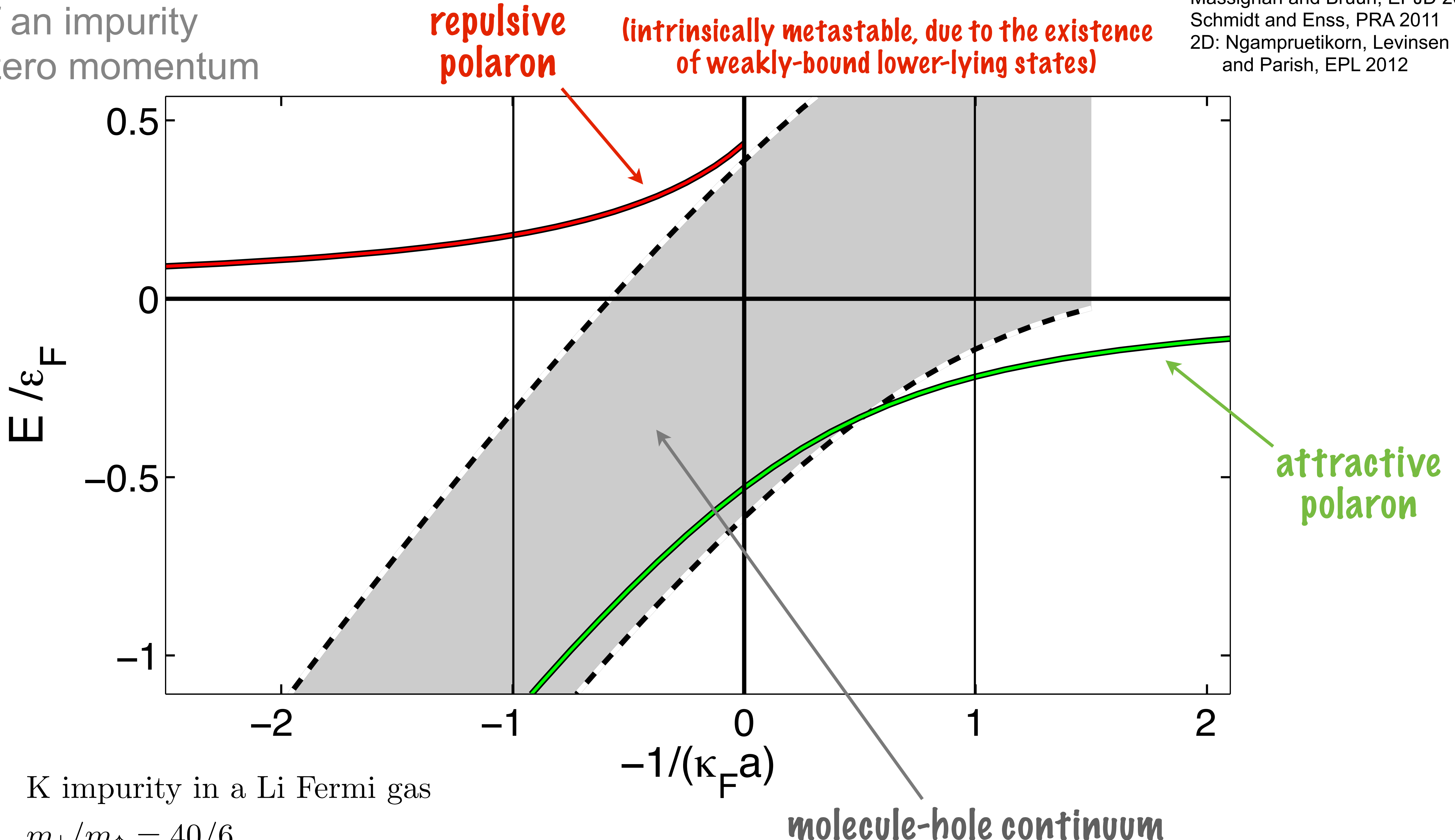
The impurity problem

Energy spectrum,
of an impurity
with zero momentum

new quantum toy!
a gas with strong repulsive interactions

(intrinsically metastable, due to the existence of weakly-bound lower-lying states)

Cui and Zhai, PRA 2010
Massignan and Bruun, EPJD 2011
Schmidt and Enss, PRA 2011
2D: Ngampruetikorn, Levinsen and Parish, EPL 2012



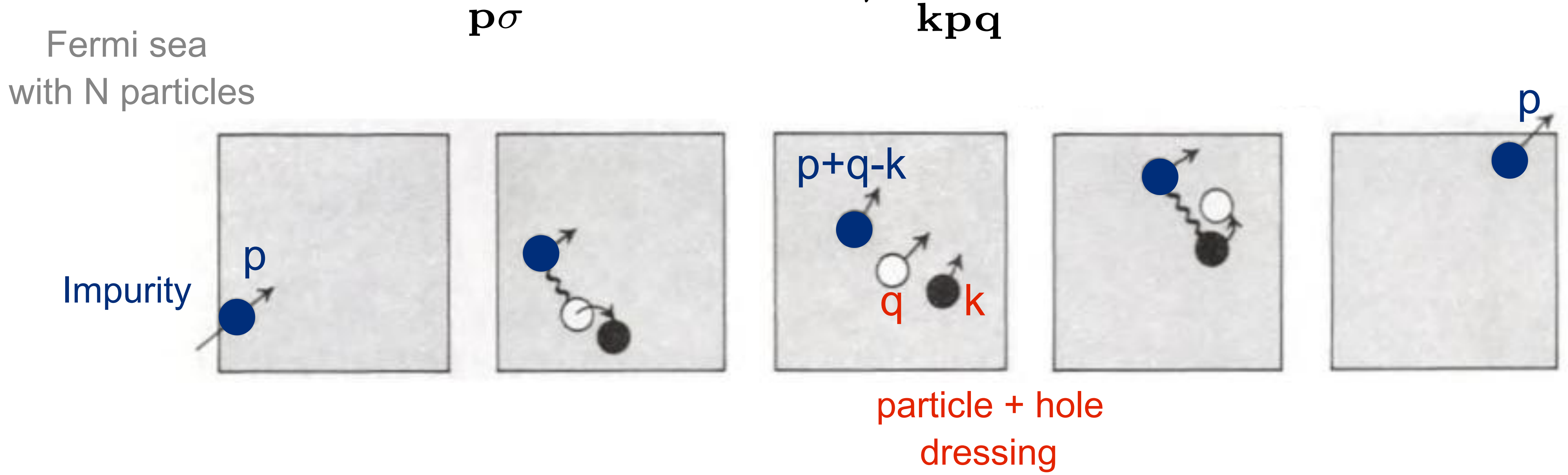
K impurity in a Li Fermi gas

$m_\downarrow/m_\uparrow = 40/6$

$k_F R^* \sim 1$

The polaron: a dressed impurity

$$H = \sum_{\mathbf{p}\sigma} \epsilon^\sigma_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \frac{g}{V} \sum_{\mathbf{kpq}} c^\dagger_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{p}\uparrow} c_{\mathbf{p+q}\uparrow} c_{\mathbf{k-q}\downarrow}$$



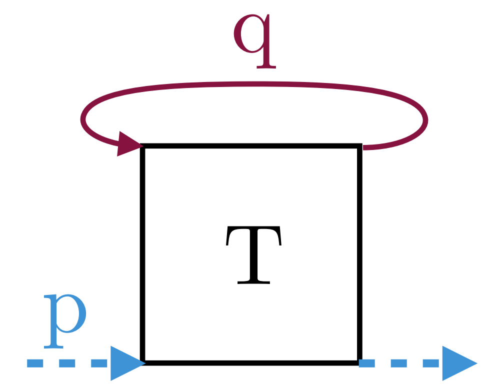
$$|\psi_{\mathbf{p}}\rangle = \phi_{\mathbf{p}} c^\dagger_{\mathbf{p}_\downarrow} |FS_N\rangle + \sum_{q<k_F}^{k>k_F} \phi_{\mathbf{pqk}}\, c^\dagger_{\mathbf{p+q-k}_\downarrow}\, c^\dagger_{\mathbf{k}_\uparrow}\, c_{\mathbf{q}_\uparrow} \, |FS_N\rangle$$

variational Ansatz ➜ strict upper bound

Chevy, PRA (2006)

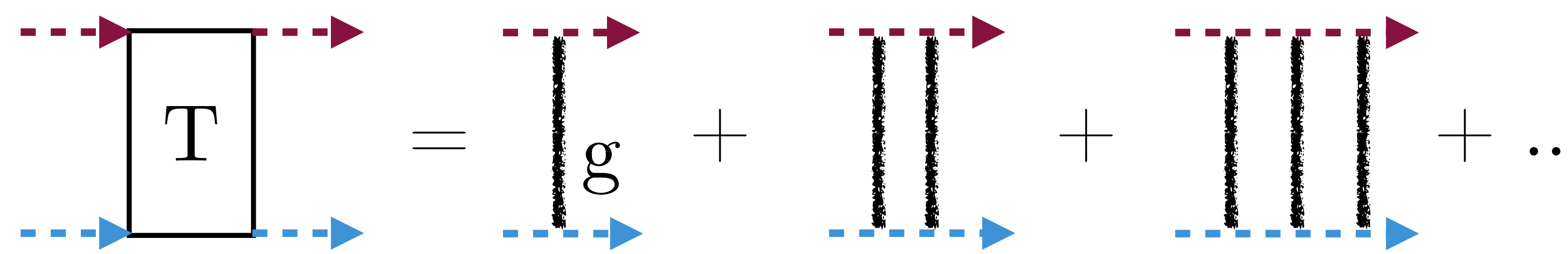
Diagrammatic formalism

impurity Green's function: $G(\mathbf{p},\omega) = \dfrac{1}{\omega - p^2/2m - \Sigma(\mathbf{p},\omega) + i0}$



self-energy of the impurity: $\Sigma_{\mathrm{P}}(\mathbf{p},E) = \sum_{q<k_F} T(\mathbf{p}+\mathbf{q}, E+\xi_{q\uparrow})$

T-matrix:

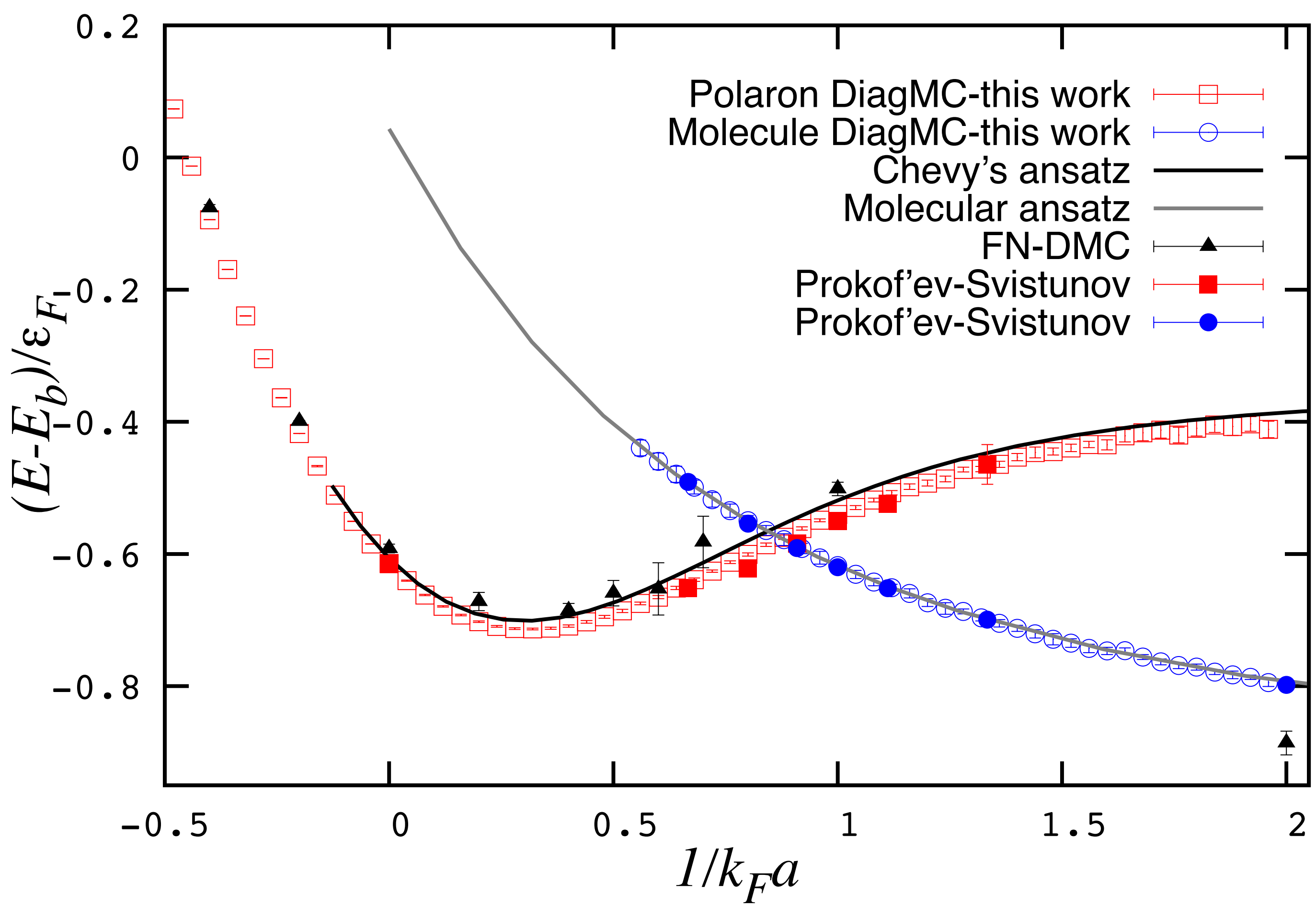


energies of the two polarons: $E_\pm = \Re[\Sigma_{\mathrm{P}}(\mathbf{p}, E_\pm + i0^+)]$

residues: $Z_\pm = \dfrac{1}{1 - \partial_\omega \Re(\Sigma_{\mathrm{P}})}$

effective masses: $\dfrac{m^*}{m_\downarrow} = \dfrac{1}{Z_\pm}\left[1 + \dfrac{\partial \Re(\Sigma_{\mathrm{P}})}{\partial(p^2/2m_\downarrow)}\right]^{-1}$

Comparison with Diagrammatic QMC



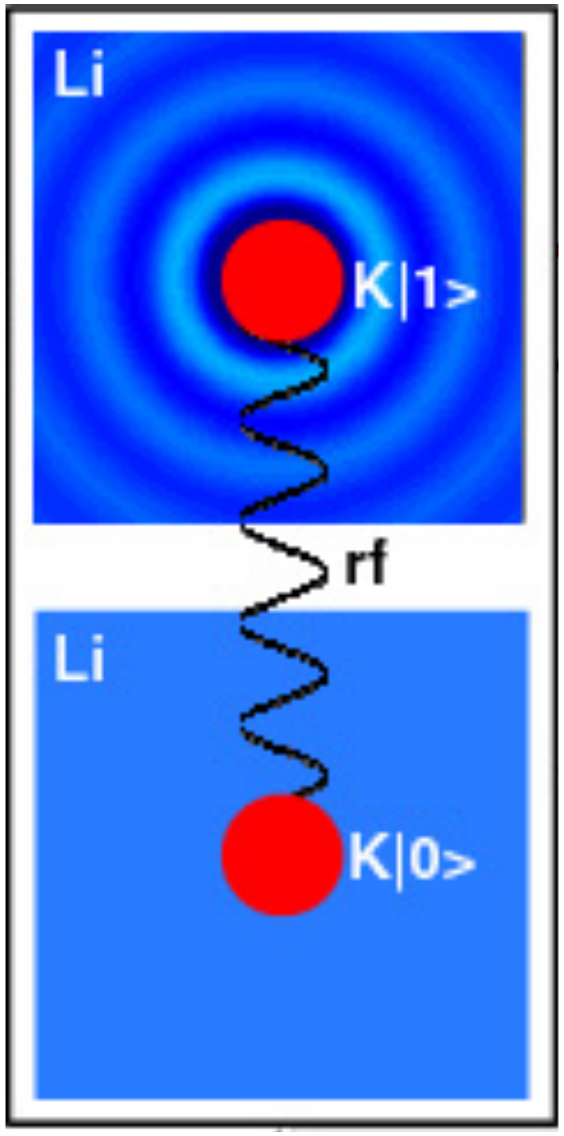
Vlietinck et al., PRB (2013)

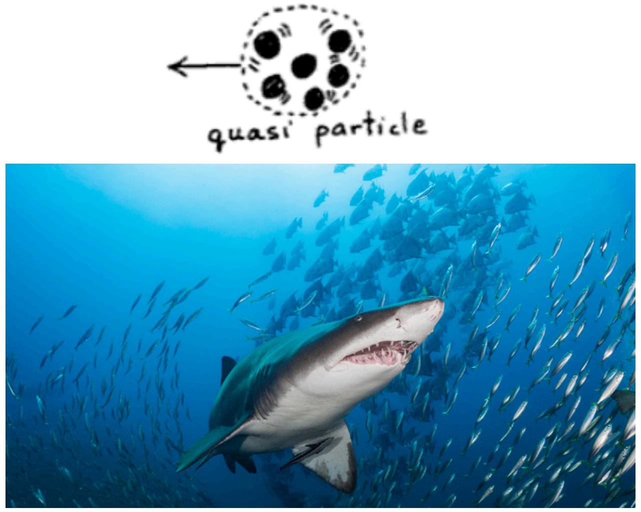
	1PH	QMC
E/E_F	0,606	0,615
m*/m	1,17	1,197
Z	0,78	0,759
$C/N_{\uparrow}k_F$	4,74	?

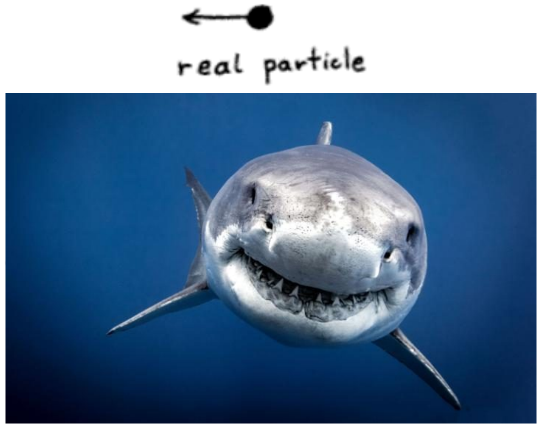
and in 2D, the 2PH Ansatz finds lower energies than diag-QMC!

Parish and Levinsen, PRA (2013)

RF spectroscopy





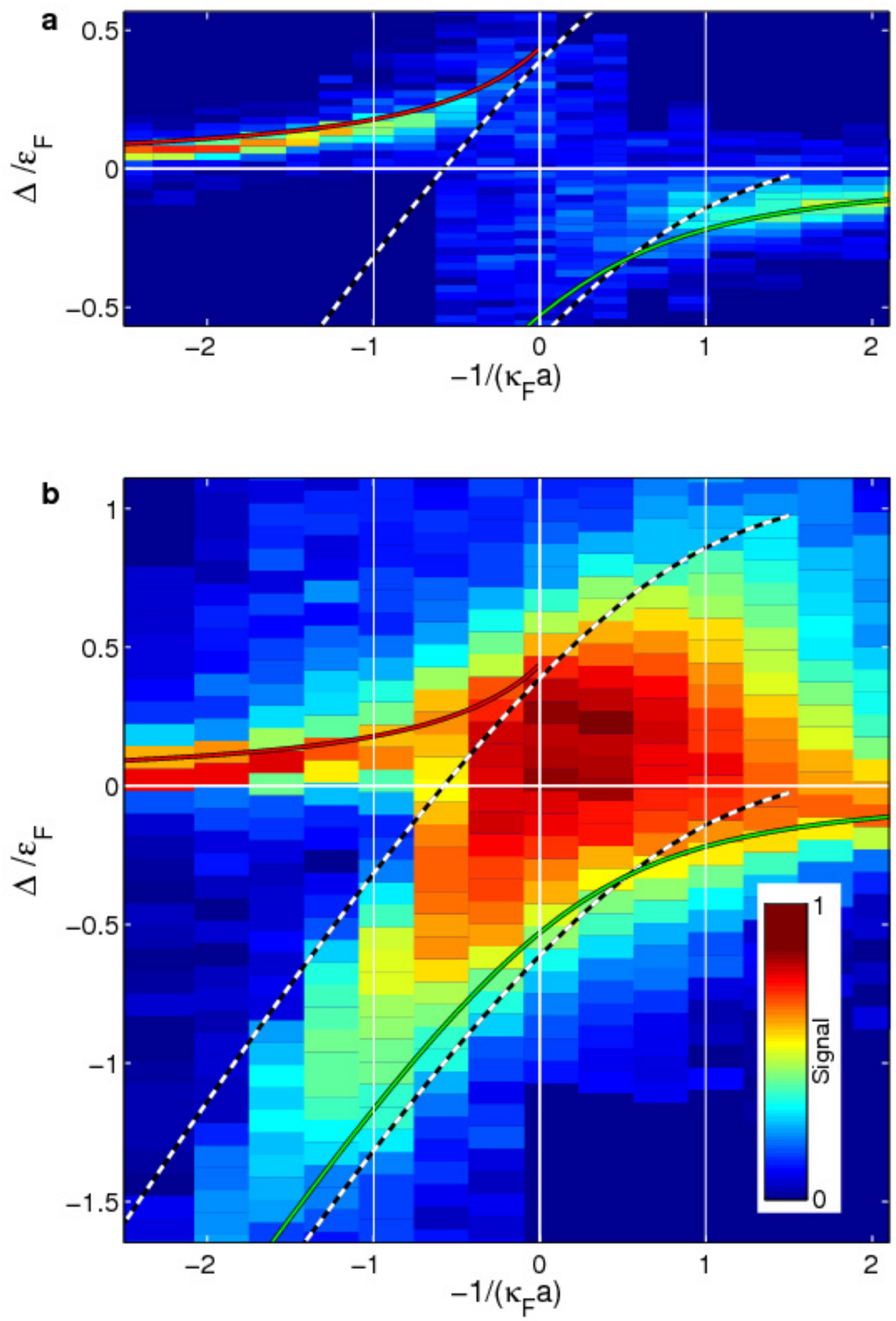


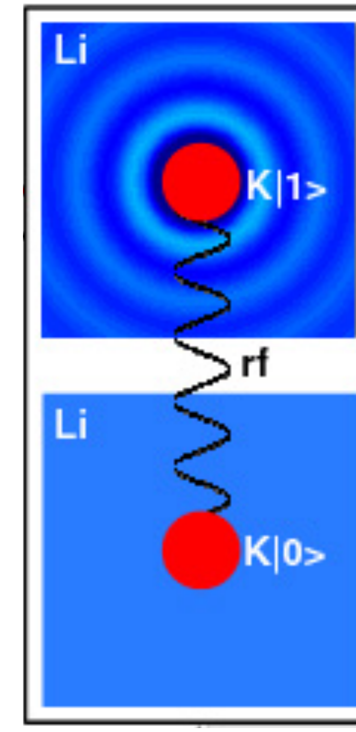
RF spectroscopy

low power RF:

high power RF:

high power is needed to
couple to the MH continuum,
due to a small FC overlap





repulsive polarons exist
as well-defined quasiparticles
even in the strongly-interacting regime

repulsive pol.
attractive pol.
molecule+hole
continuum

Kohstall, Zaccanti, Jag, Trenkwalder, PM, Bruun, Schreck & Grimm, Nature (2012)

Rabi oscillations

$$\hat{R} \propto \Omega_0 \sum_{\mathbf{q}} (\hat{a}^\dagger_{1\mathbf{q}} \hat{a}_{0\mathbf{q}} + h.c.)$$

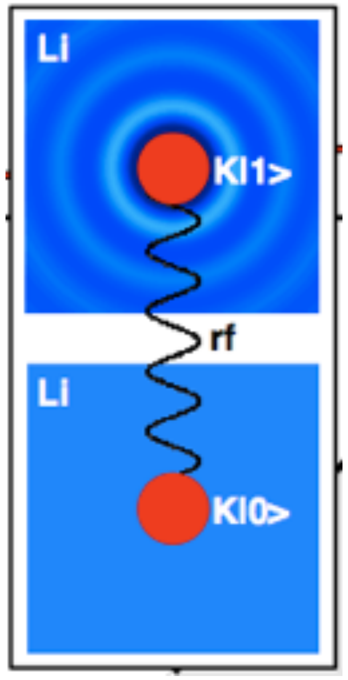
$$|I\rangle = \hat{a}^\dagger_{0\mathbf{q}=0} |\mathrm{FS}\rangle$$

$$|F\rangle = \sqrt{Z} \hat{a}^\dagger_{1\mathbf{q}=0} |\mathrm{FS}\rangle + \sum_{p<\hbar\kappa_F<q} \phi_{\mathbf{q},\mathbf{p}} \hat{a}^\dagger_{1\mathbf{p}-\mathbf{q}} \hat{b}^\dagger_{\mathbf{q}} \hat{b}_{\mathbf{p}} |\mathrm{FS}\rangle + \ldots$$

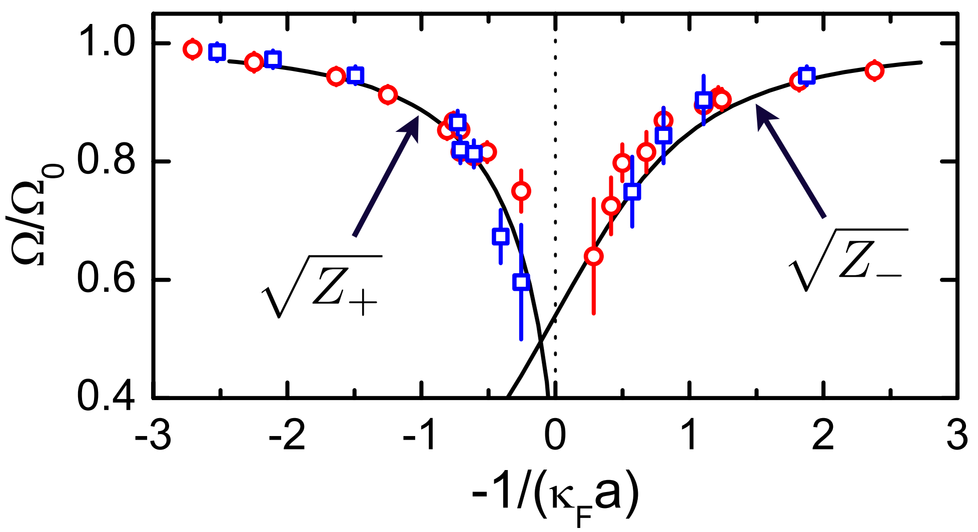
$$\langle F|\hat{R}|I\rangle = \sqrt{Z}\Omega_0$$

regime of very high RF power,
well beyond linear response regime:
fast oscillations, and quasiparticle decay
may be ignored

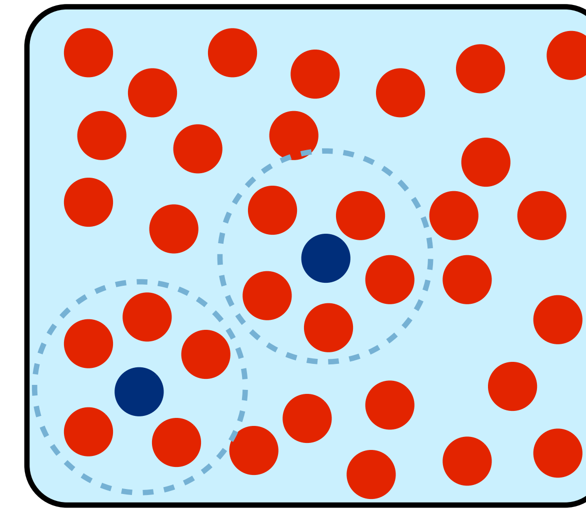
collision-induced decoherence
is the main damping mechanism



Rabi frequency as a measure of polaron residues



Equation of state



A strongly-interacting system, described as
an ensemble of weakly-interacting quasi-particles
(a Fermi liquid)

$$E = \frac{3}{5} E_F N_\uparrow + \left[E_{\text{pol}} + \frac{3}{5} E_F \frac{m}{m^*} \left(\frac{N_\downarrow}{N_\uparrow} \right)^{2/3} + \dots \right] N_\downarrow$$

mean kinetic energy
of the Fermi sea

chemical potential
of one polaron

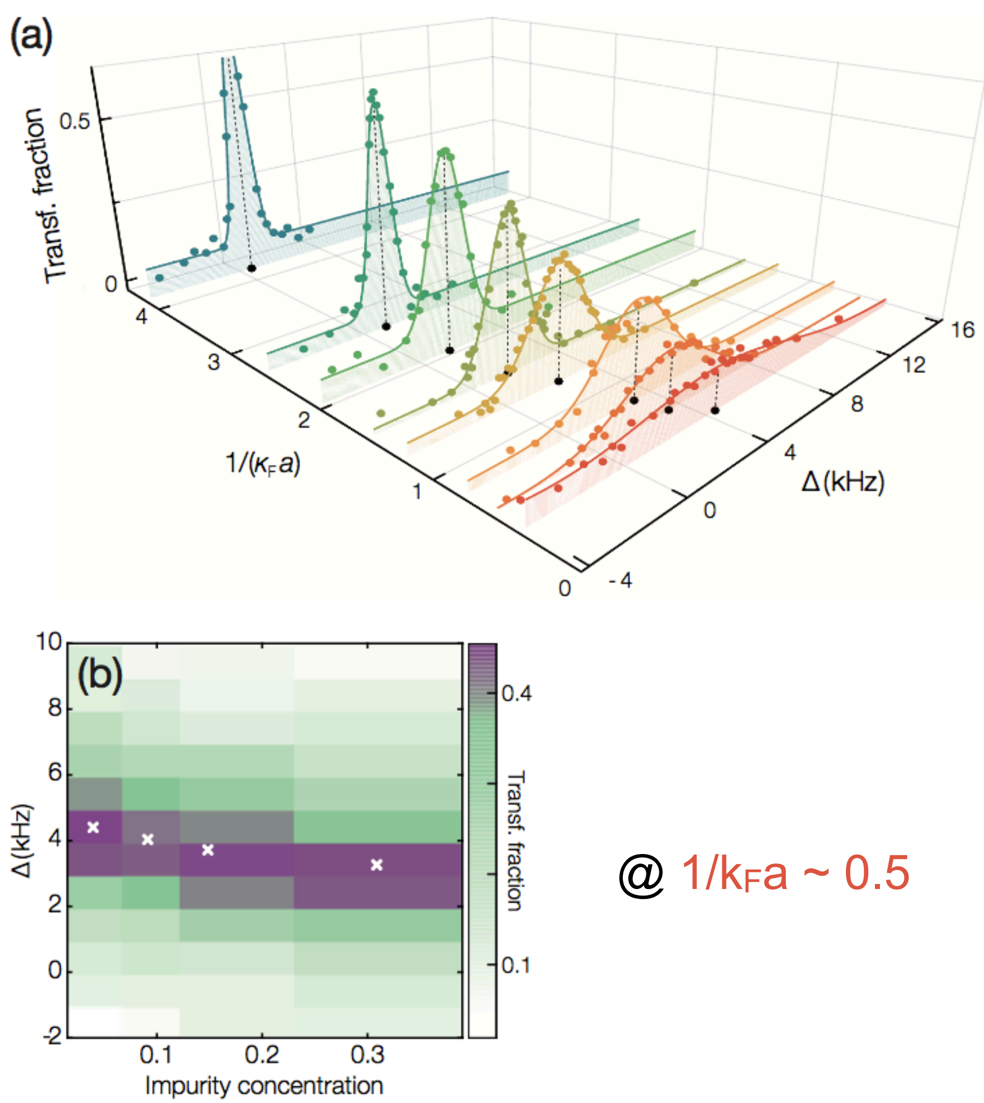
mean kinetic energy
of the polarons
(m* is their effective mass)

polaron-polaron interactions,
and more …

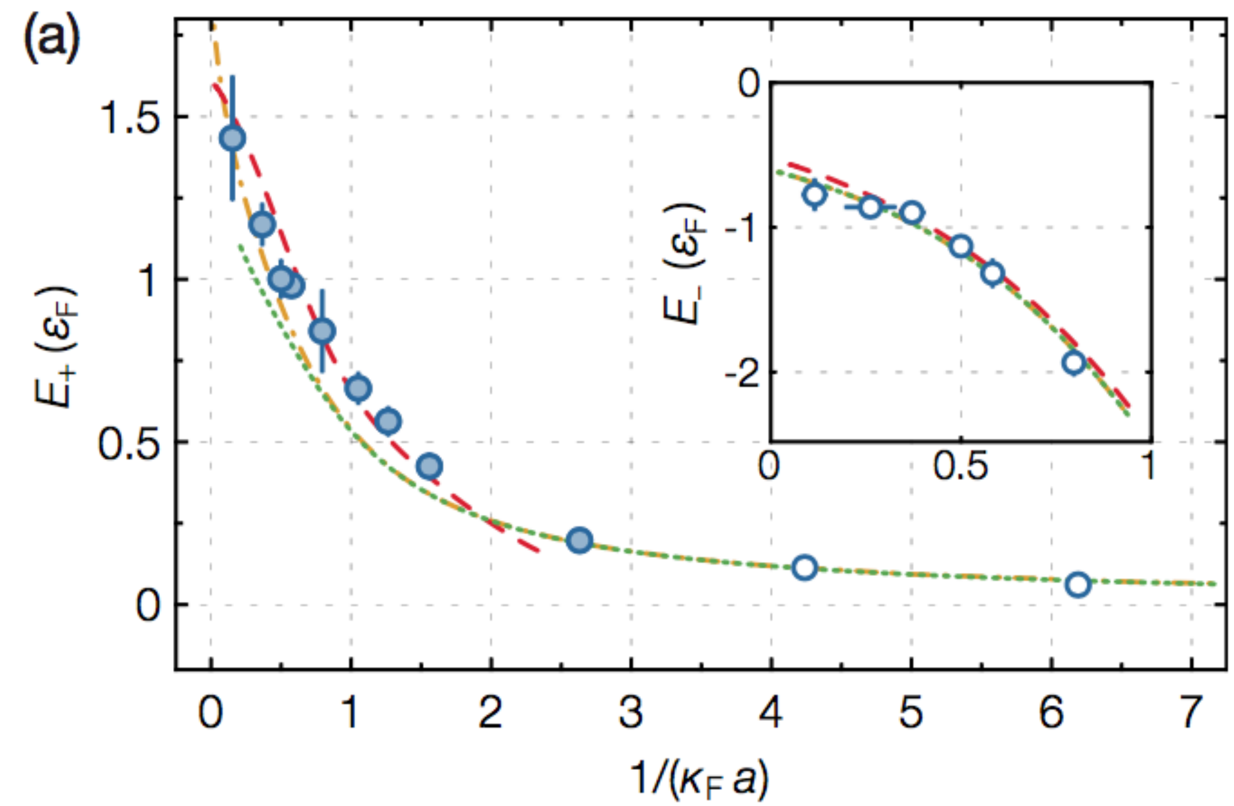
New LENS experiment

Scazza et al, arXiv:1609.09817

Equal masses FF (Li-Li) mixture at broad resonance



@ $1/k_F a \sim 0.5$

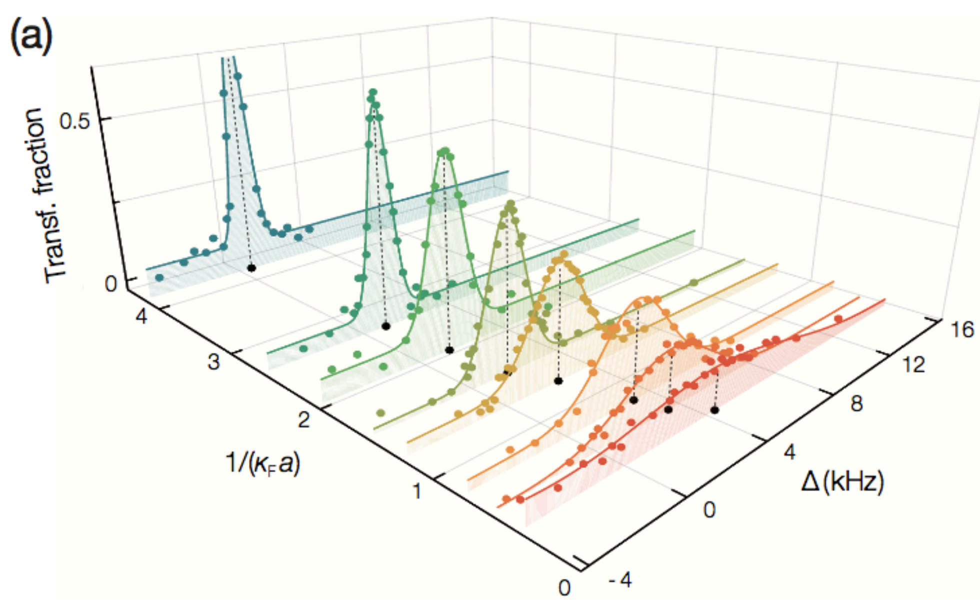


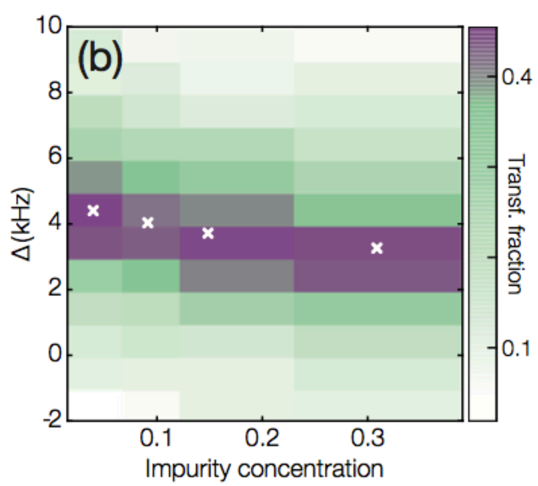
E_+ exceeds the Fermi energy when $1/(k_F a) \lesssim 0.6$:
IFM phase becomes energetically favored

Cui and Zhai, PRA 2010
Massignan and Bruun, EPJD 2011
Schmidt and Enss, PRA 2011

Effective mass m*

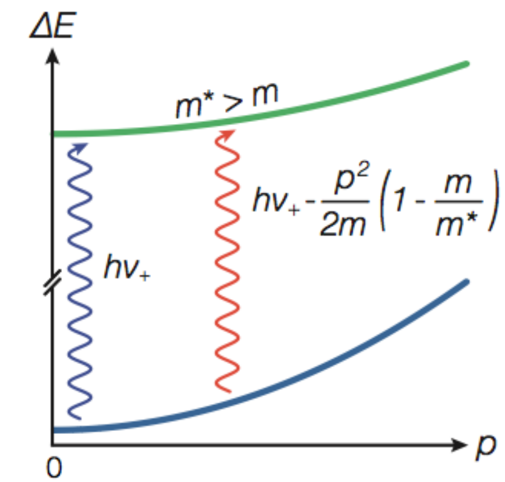
Scazza et al, arXiv:1609.09817

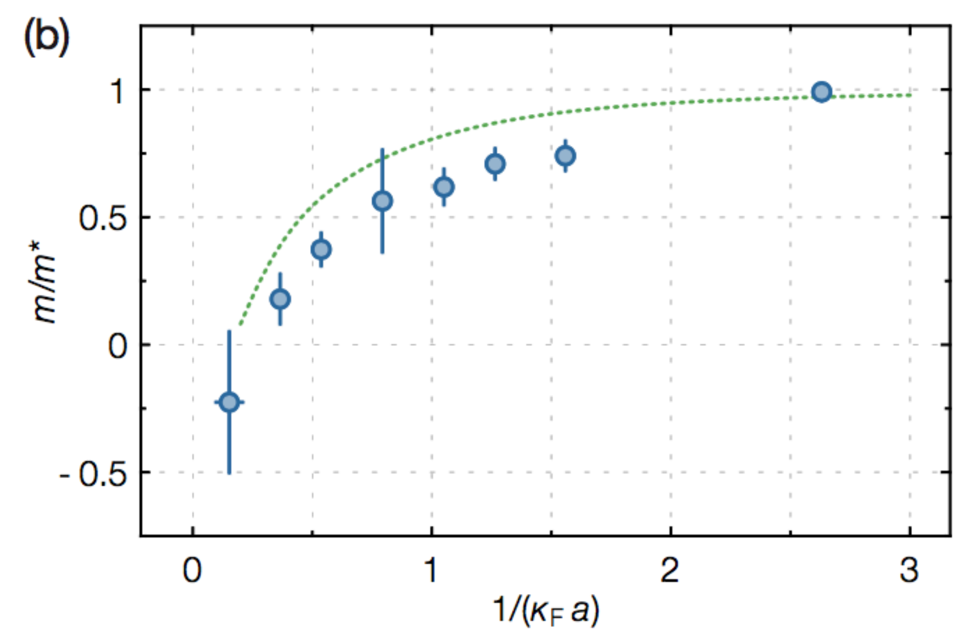




$\bar{\varepsilon} = 0.42\varepsilon_F$ → $\bar{\varepsilon} \sim 0.8\varepsilon_F$

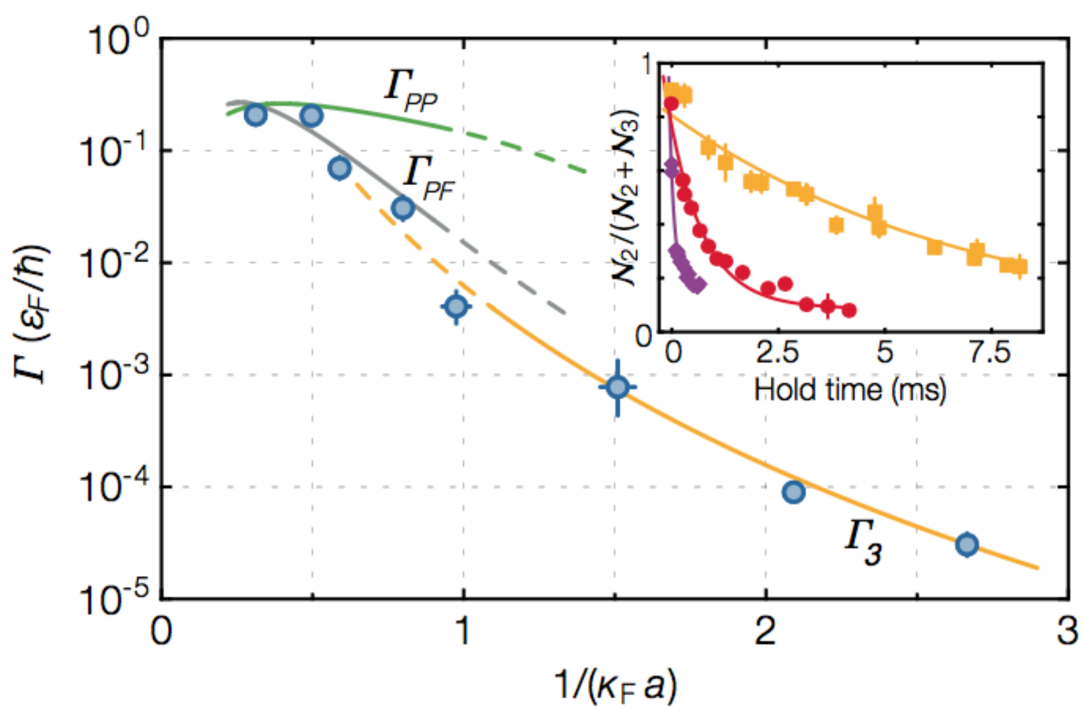
increasing concentration





m*<0 when 1/(kFa)≲0.2:
the paramagnetic phase
becomes thermodynamically unstable

Decay of repulsive polarons

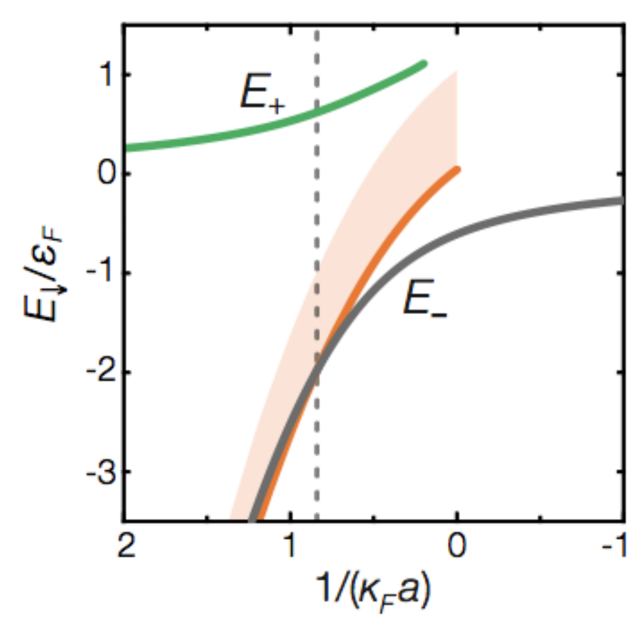


theory for decay into

- $\uparrow + \uparrow +$ attractive polaron
- $\uparrow + \uparrow +$ free (fast) atom
- $\uparrow +$ mol (3-body decay) [Petrov 2003]

long lifetimes!

many-body physics possible at strong repulsion



3D and 1D

IOP Publishing
Reports on Progress in Physics
Rep. Prog. Phys. **77** (2014) 034401 (26pp)
doi:10.1088/0034-4885/77/3/034401

Report on Progress

Polarons, dressed molecules and itinerant ferromagnetism in ultracold Fermi gases

Pietro Massignan[1], Matteo Zaccanti[2] and Georg M Bruun[3]

2D

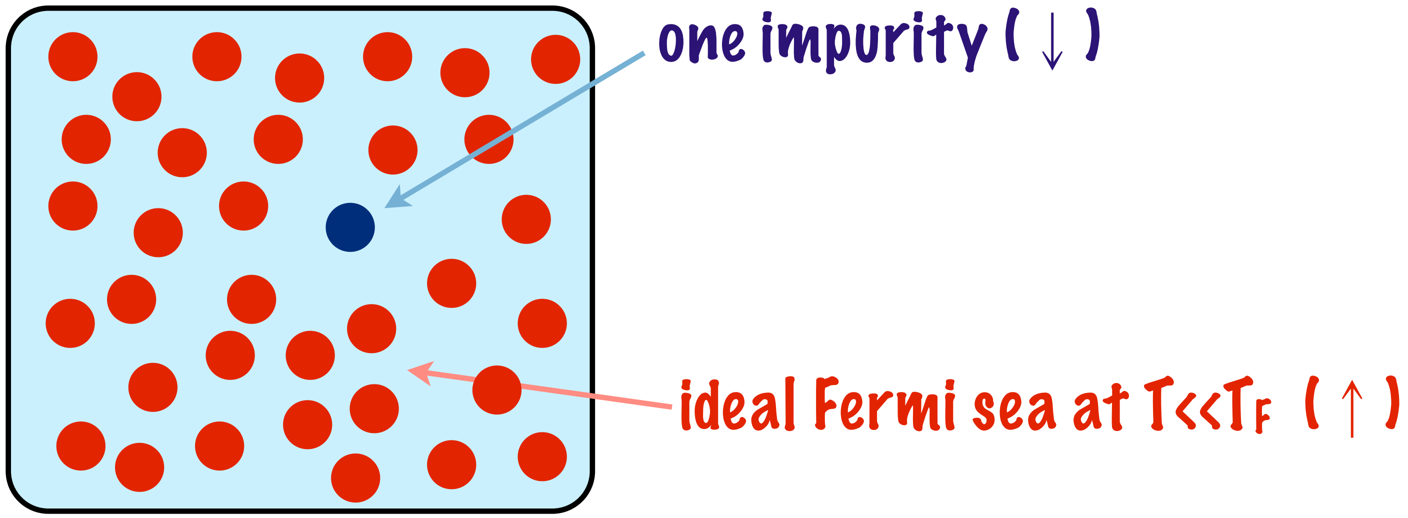
Annual Review of Cold Atoms and Molecules (2015)

STRONGLY INTERACTING TWO-DIMENSIONAL FERMI GASES

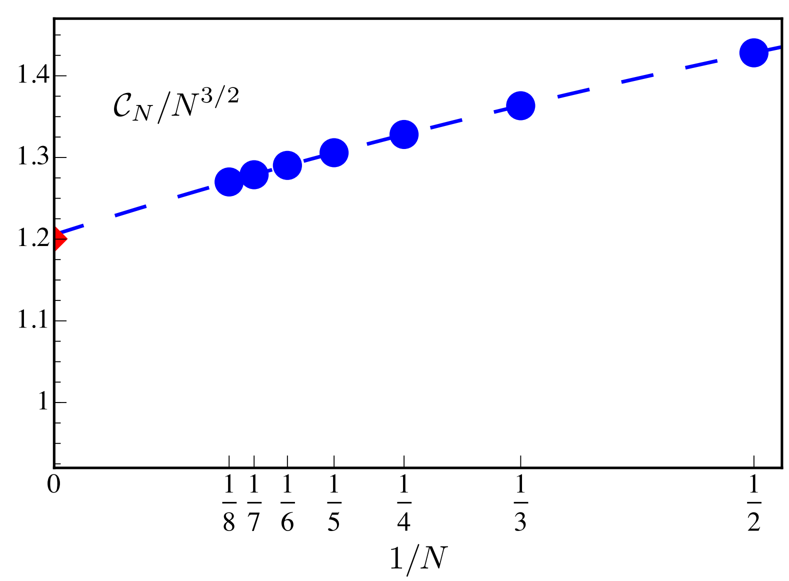
Jesper Levinsen* and Meera M. Parish†

- theoretical methods
- experimental probes and results
- mass imbalance
- reduced dimensionality
- decay processes

Outline of this talk



three
dimensions



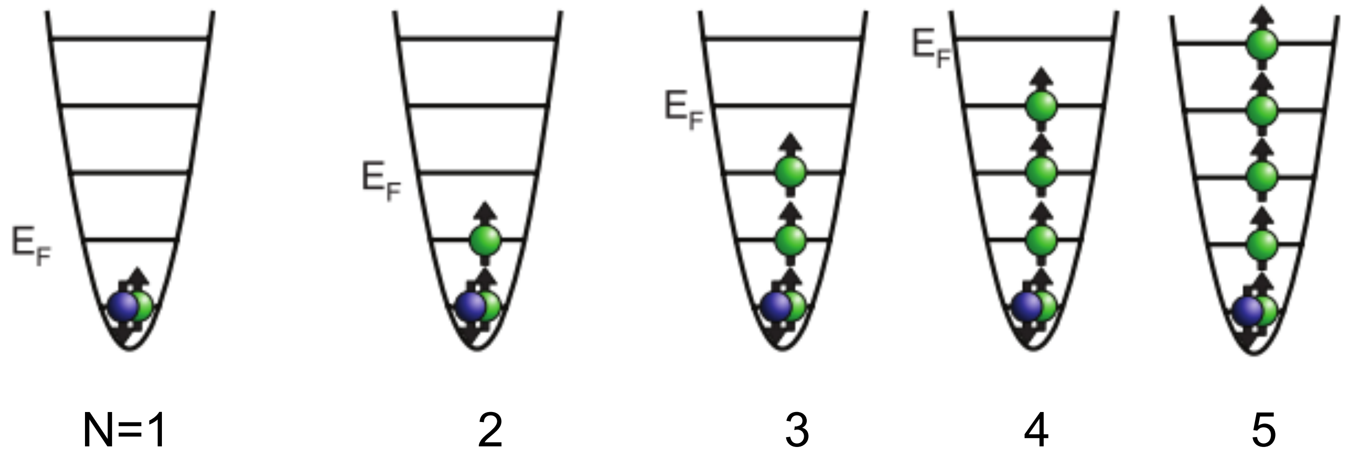
one
dimension

?

few → many

N+1 fermions in a 1D trap

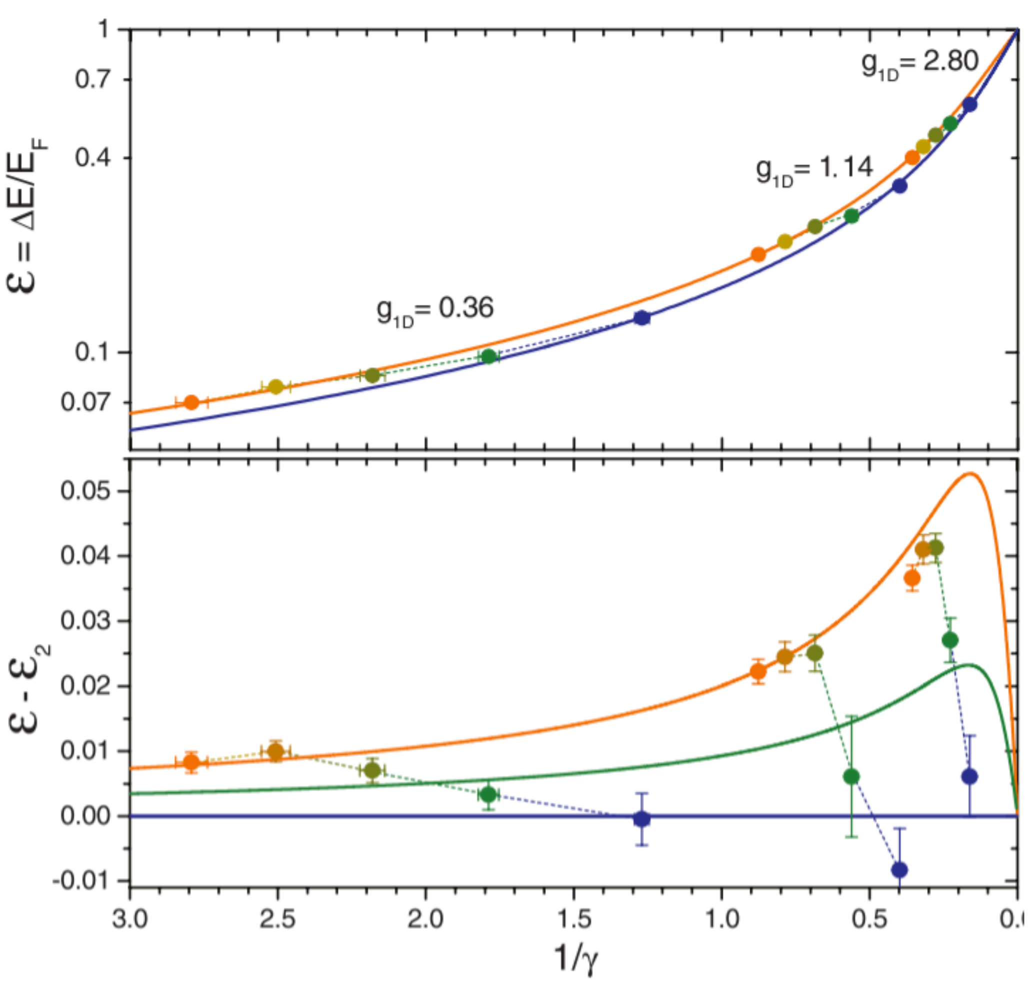
Wenz et al., Science 2013



The energy of an impurity in a 1D trapped "few-body Fermi sea":

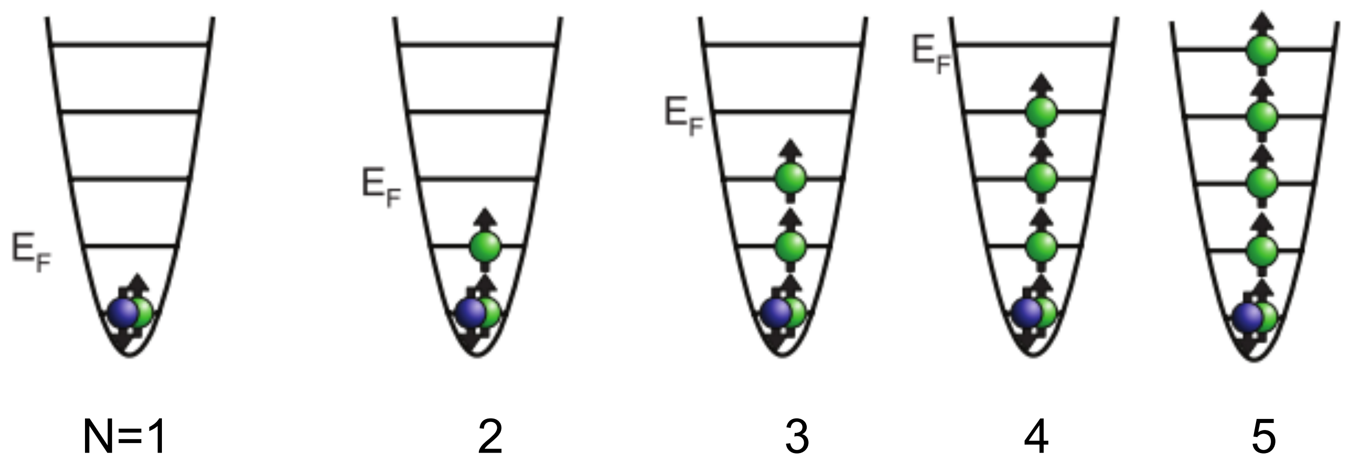
many-body (McGuire+LDA)

2-body



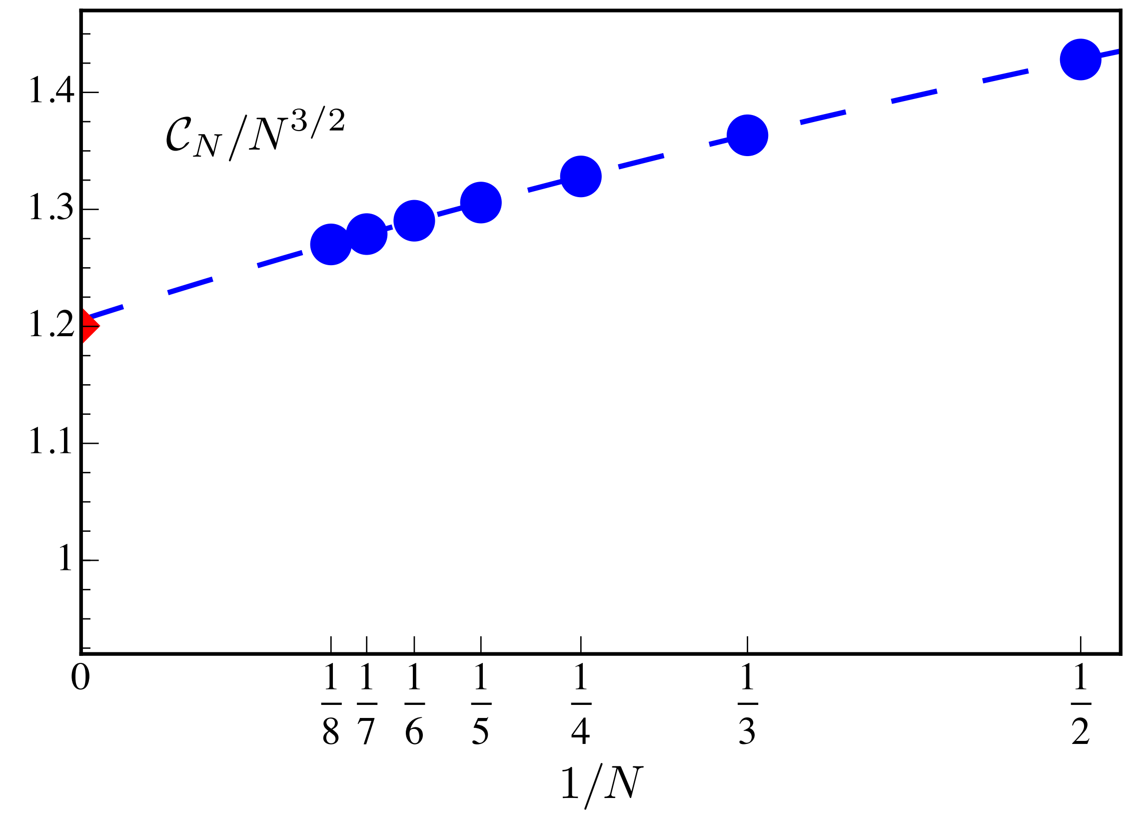
interaction parameter: $\gamma = \frac{\pi m}{\hbar^2} \frac{g_{1D}}{k_F}$

N+1 fermions in a 1D trap



Contact of an impurity at the TG point (1/g=0):

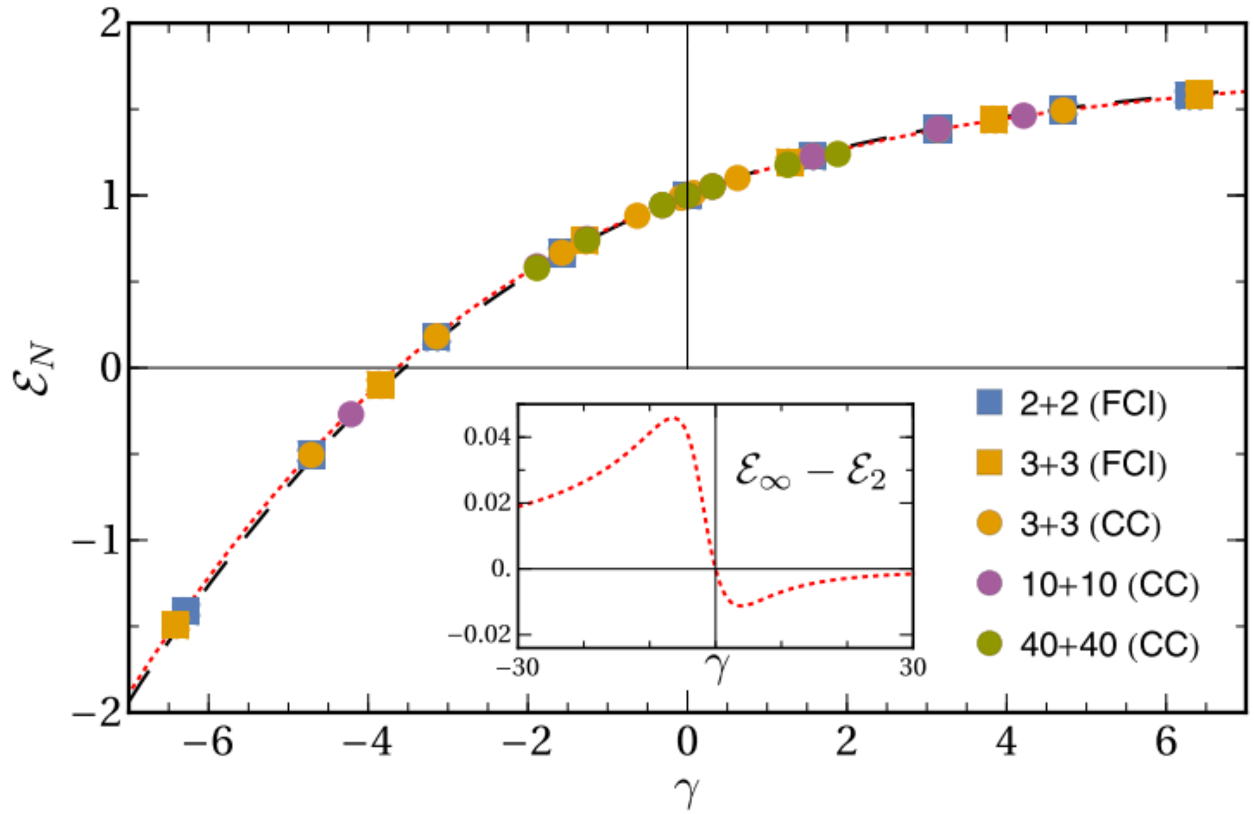
many-body (McGuire+LDA)



Levinsen, Massignan, Bruun, and Parish, Science Advances 2015

$N_\uparrow = N_\downarrow = N/2$ fermions

Grining et al., PRA 2016



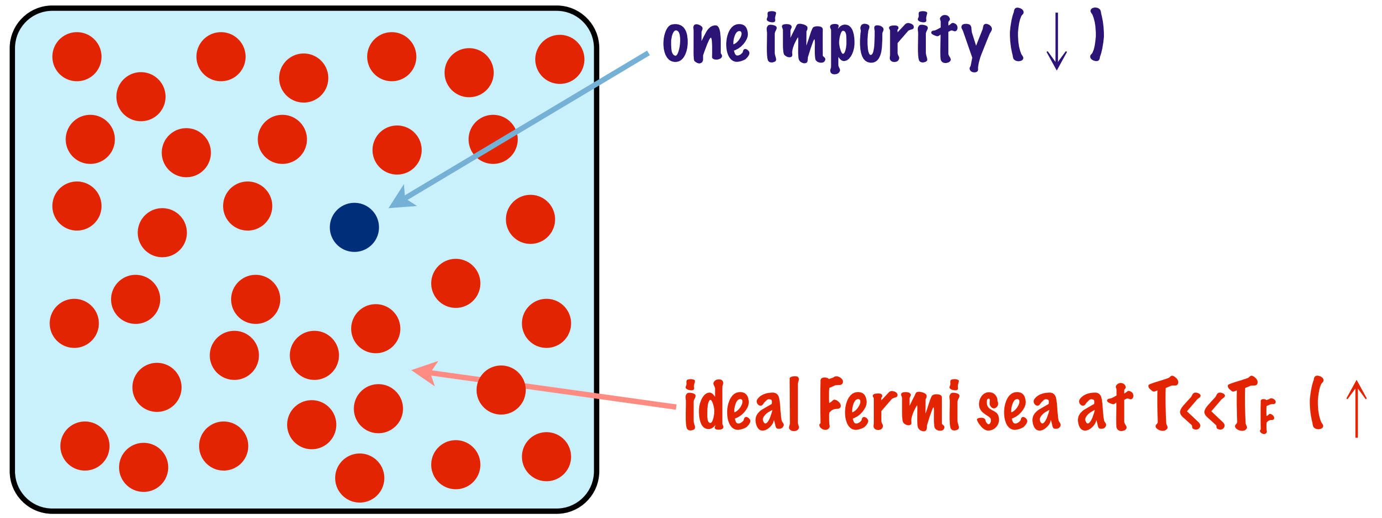
many-body (BA+LDA)

2-body (Busch)

rescaled energy: $\varepsilon_N = \dfrac{E_N}{E_N^{(0)}}$ vs. interaction parameter: $\gamma = \dfrac{\pi}{\hbar \omega a_z} \dfrac{g_{1\mathrm{D}}}{\sqrt{N}}$

The energy of a 1D balanced gas in a harmonic trap converges **<u>INCREDIBLY</u>** quickly to the many-body limit, for every interaction strength!
(and therefore also the contact does)

Outline of this talk



three dimensions

one dimension

?

few → many

few ➜ many

How many is many? Well, it depends…

- two-body properties converge quickly (energy, contact, …)
- more complex quantities converge at slower rates, in particular close to phase transitions (e.g., the pairing gap)

It surely helps to have:
- short-range interactions
- harmonic trapping (separation of c.o.m. motion, viral theorem, …)

What can we learn from few-body calculations?
- controlled and accurate, so provide stringent bounds

Reverse-LDA: compute in a trap, and predict properties of uniform space

e.g., 4^{th} virial coefficient by Yan and Blume [PRL 2016], Endo and Castin, [J. Phys A 2016]

what about the 3D unitary Fermi gas?

e.g., for zero-range unitary interactions the hyper-radius and hyper-angular equations separate exactly in both the trap and in uniform space

many ➜ thanks!