Do halo nuclei exhibit universality?

Daniel Phillips,
Institute of Nuclear and Particle Physics and
Department of Physics and Astronomy
Ohio University, Athens, Ohio

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Outline

- The system: halo nuclei
- The tool: Effective field theory for short-range interactions
- Application 1: dipole response of a two-body system in the universal regime (^{19}C)
- Application 2: Radii of three-body systems in the universal regime (Radii of two-neutron halos)
- Application 3: E1 response of three-body systems in the universal regime (E1 response of ^{11}Li)
- Conclusion

Ordinary vs. halo nuclei

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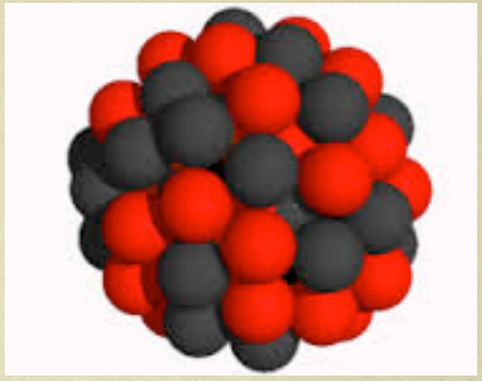

http://alternativephysics.org

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- Nuclear binding energies are on the order of 8 MeV/nucleon

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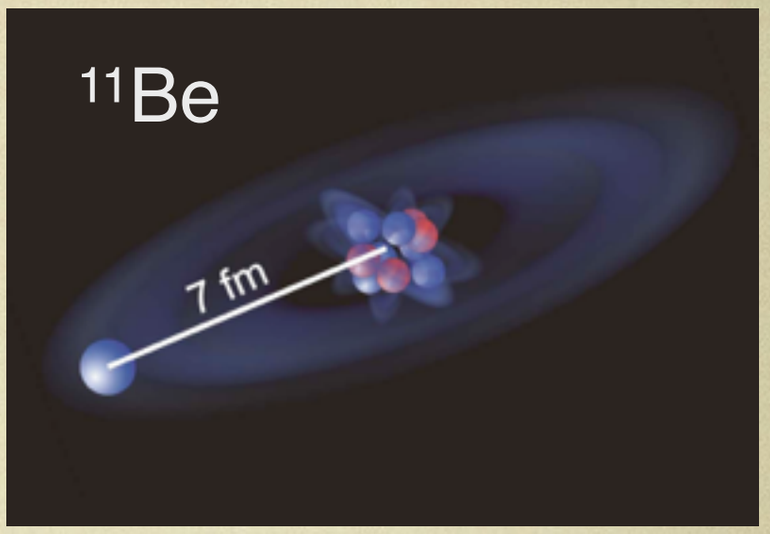

http://www.uni-mainz.de

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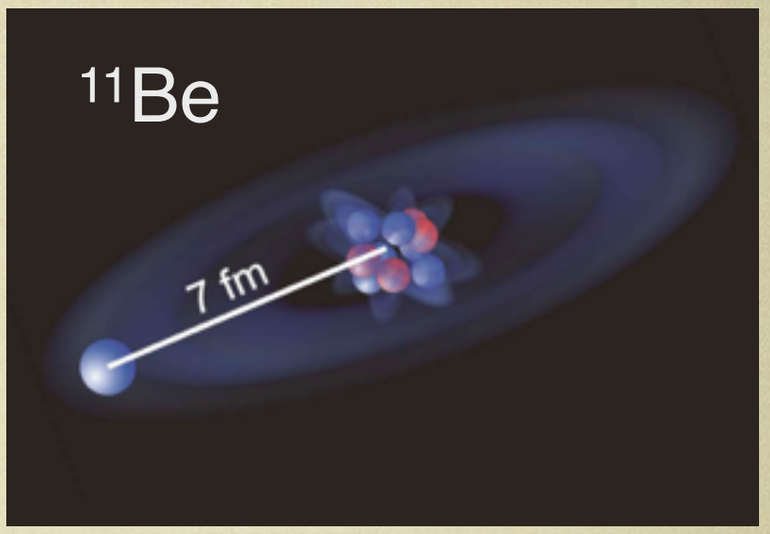

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- Halo nuclei: the last few nucleons “orbit” far from the nuclear “core”
- Characterized by small nucleon binding energies, large radii, large interaction cross sections, large E1 transition strengths.

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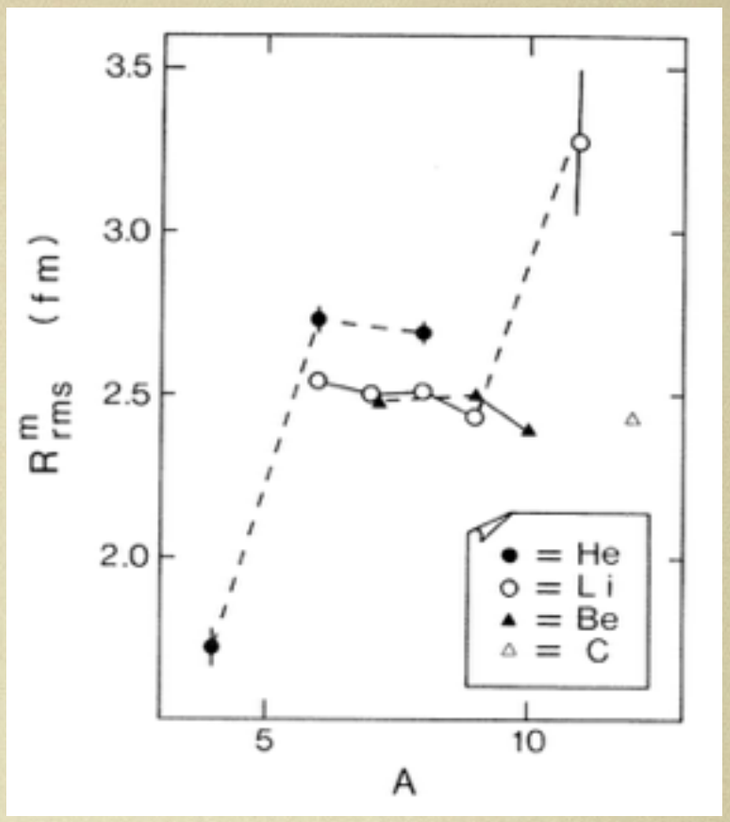

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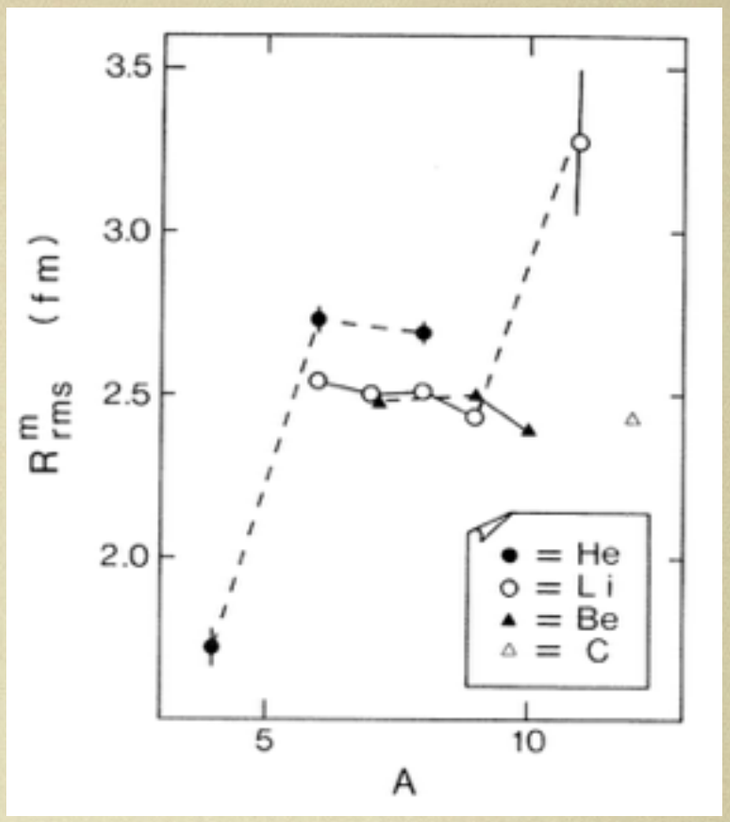

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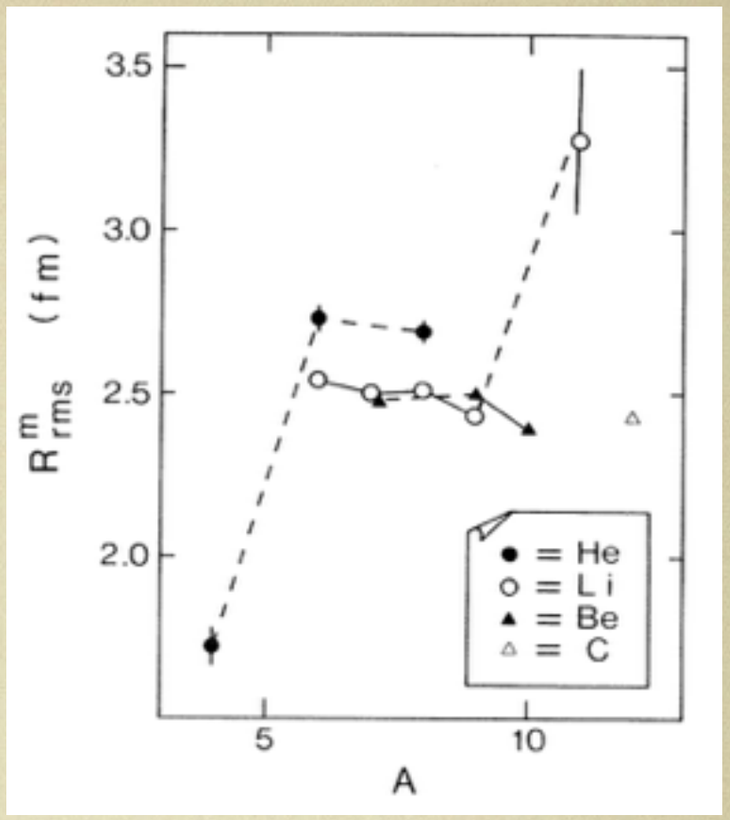
- Often Borromean systems

- Understanding essential to modeling of neutron-rich nuclei



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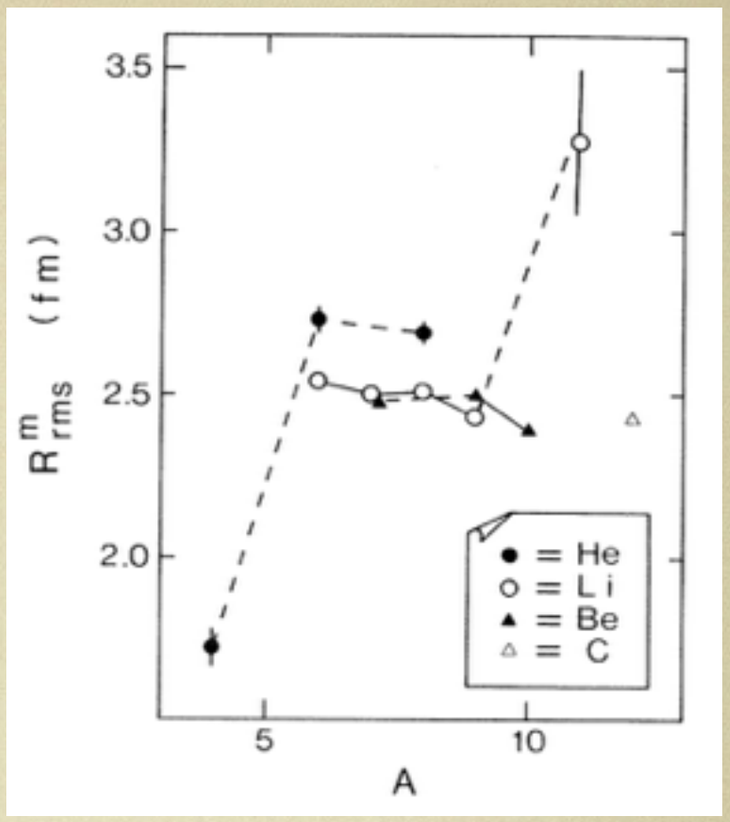

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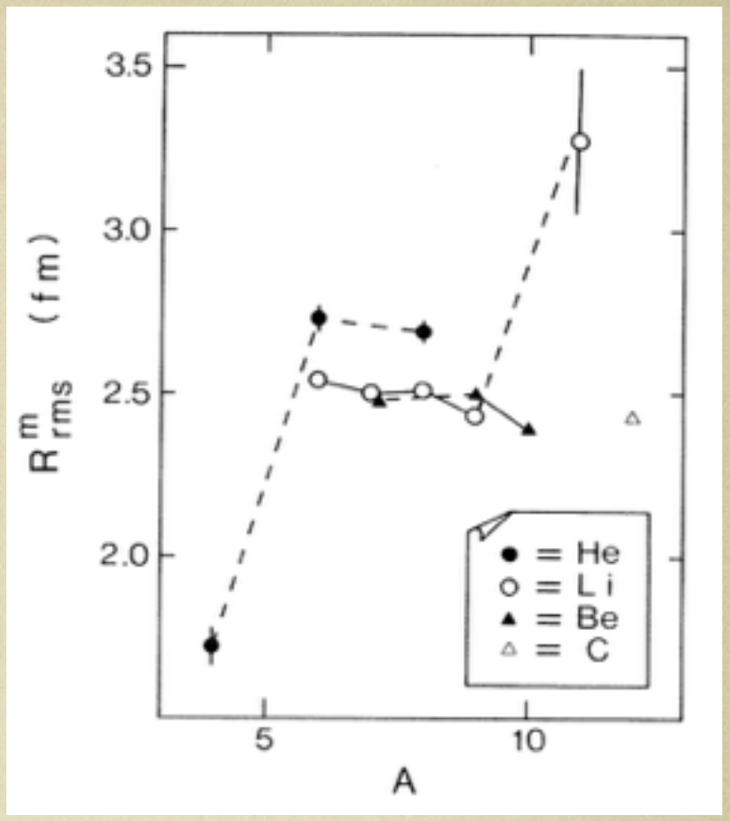

- Often Borromean systems
- Understanding essential to modeling of neutron-rich nuclei
- “Physics beyond mean field”/“Open quantum systems”
- Unversality: common features of weakly-bound quantum systems

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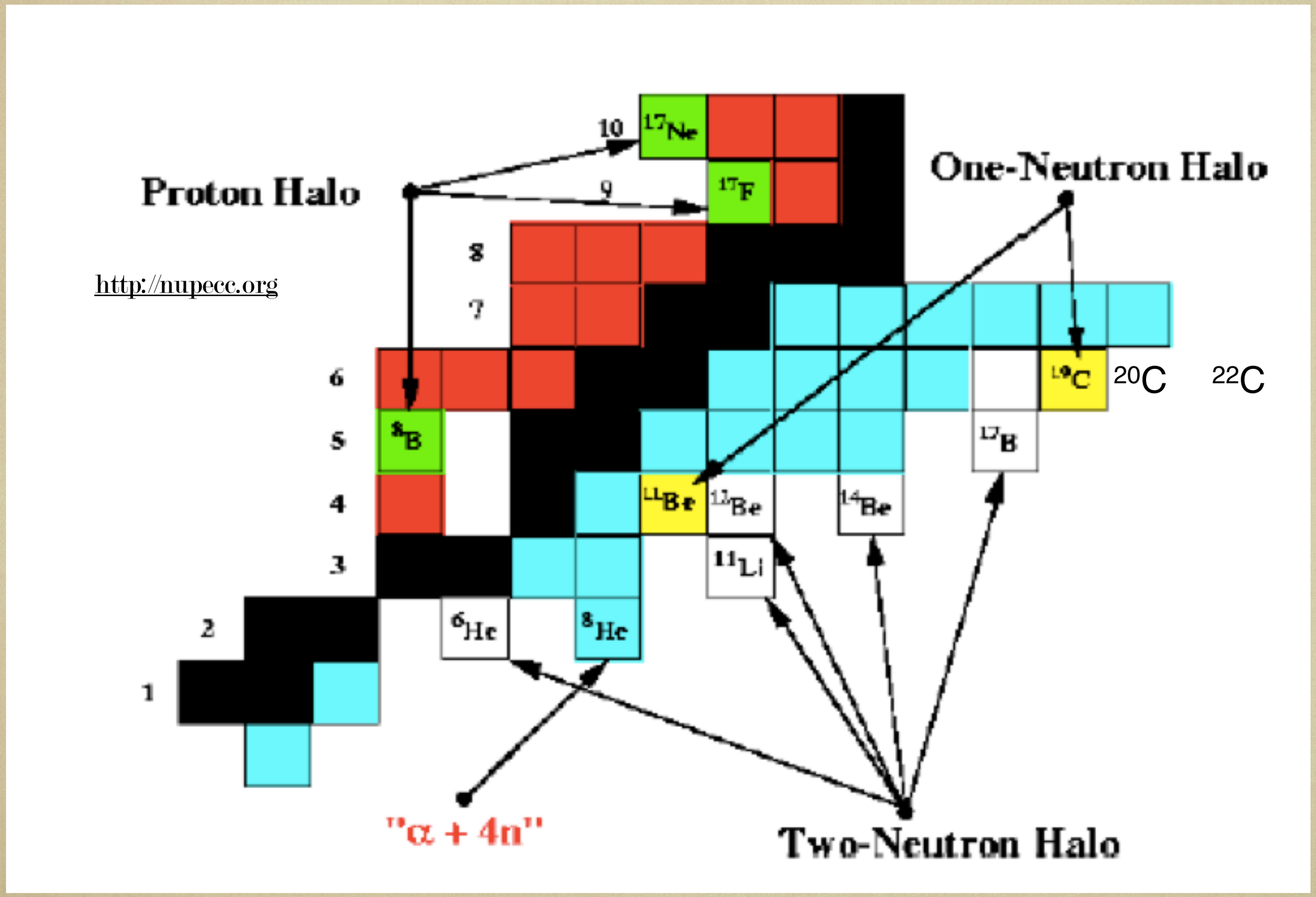

Universality

Systems with $|a| \gg R$ exhibit the same correlations between low-energy observables

System	R	lal	Observables
He atom clusters	7 Å	104 Å	Binding energies, distributions
Cold atoms	100 a_B	Varies	Bound states; recombination
X(3872)	1.5 fm	~10 fm	Spectrum, decays
Halo nuclei	3 fm	~10 fm	Spectrum, scattering, EM excitation
NN, NNN, ...	1.7 fm	5.4 fm	Phase shifts; EM props...

EFT formulation for two-body

$$-\frac{\hbar^2}{2m_R}\nabla^2\psi + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

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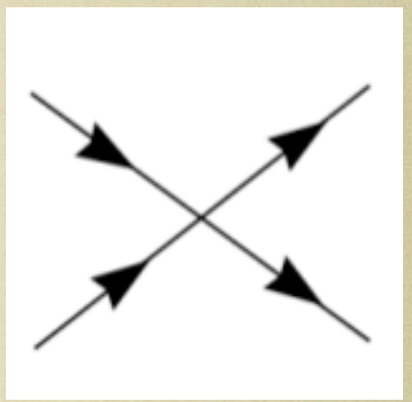

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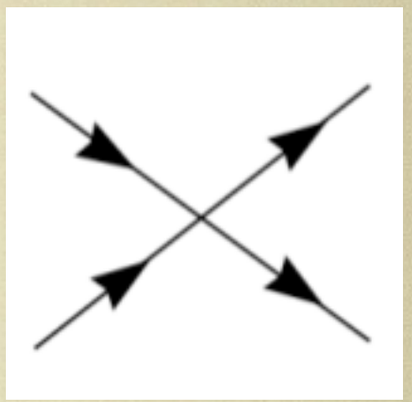

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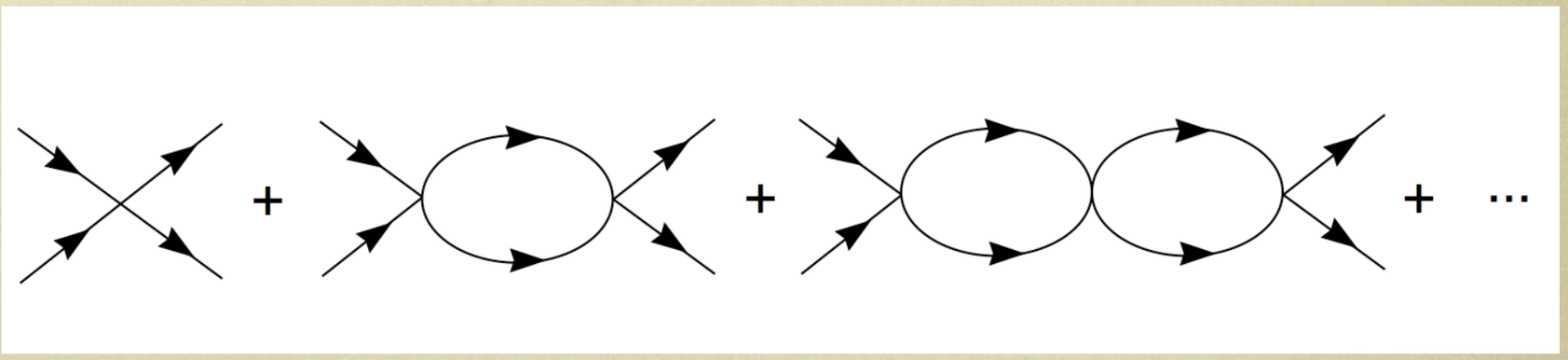
Quantum corrections essential

$$t_0^{2B} = \frac{4\pi a}{m}\frac{1}{1+iak}$$



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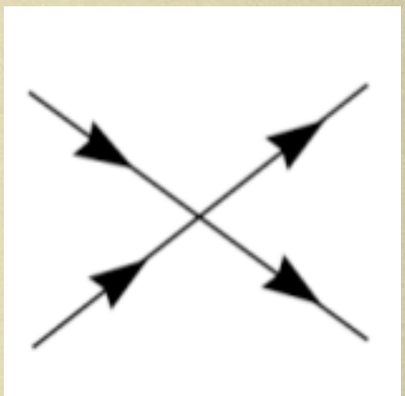

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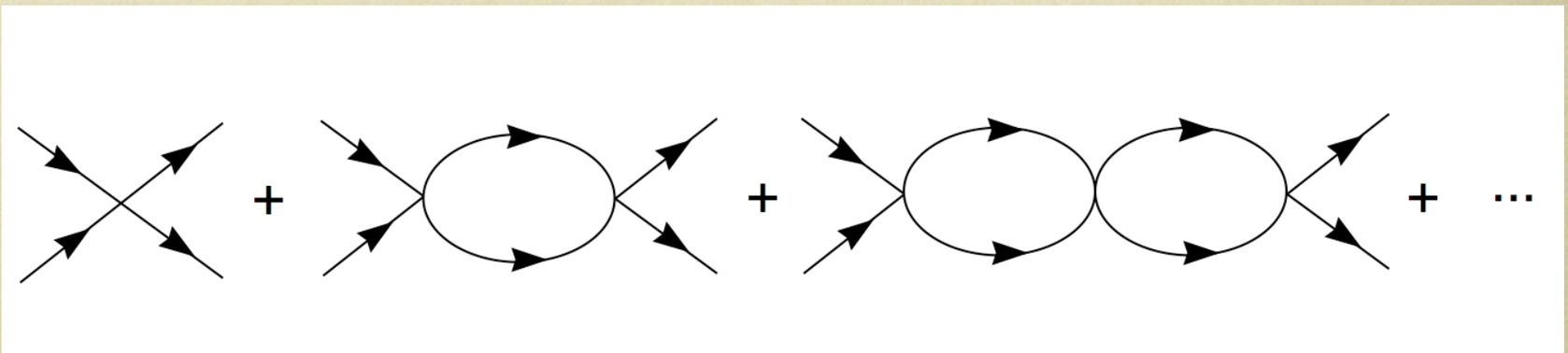

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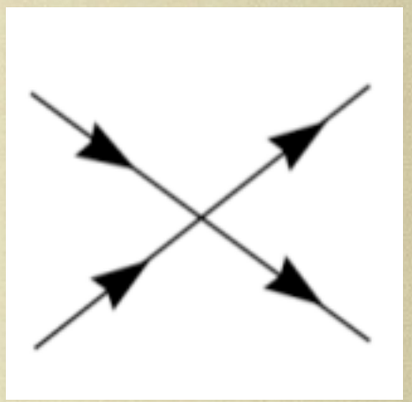

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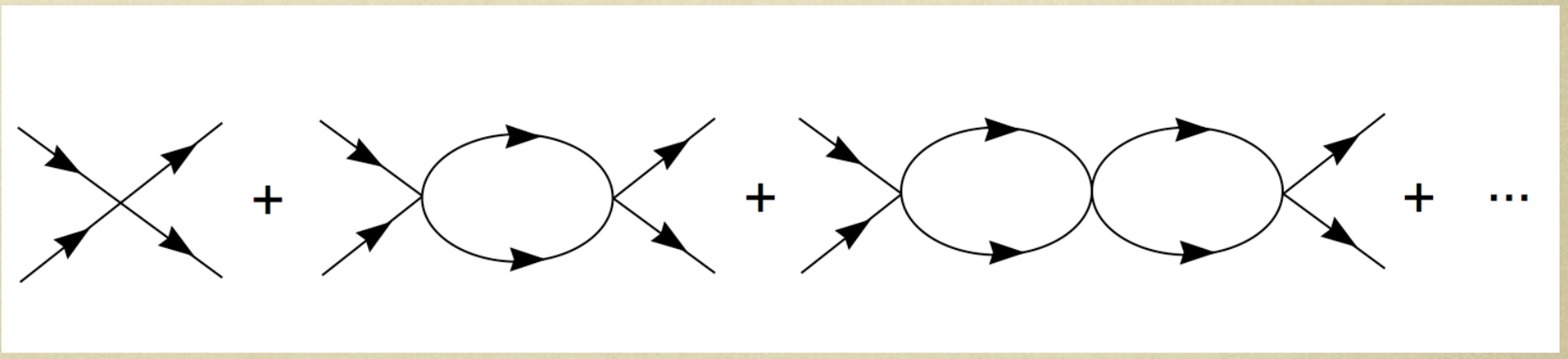

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Leading in systematic EFT expansion⇒Estimate theory uncertainty

Two-body t beyond LO

$$t(E) = -\frac{4\pi}{m}\frac{1}{k\cot\delta(k) - ik}; \quad k = \sqrt{mE}$$

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$$t(E) = \frac{4\pi a}{m}\frac{1}{1+iak}\left[1 + \frac{1}{2}\frac{rk^2}{1/a + ik} + O\left(\frac{r^2}{a^2}\right)\right]$$

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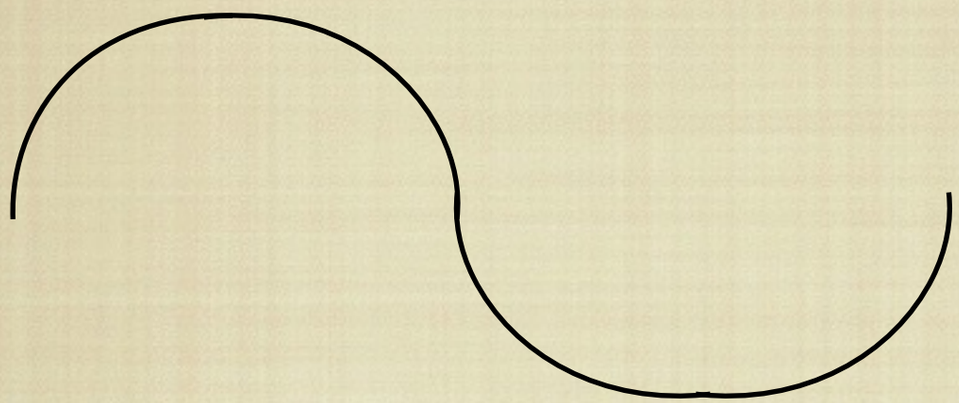
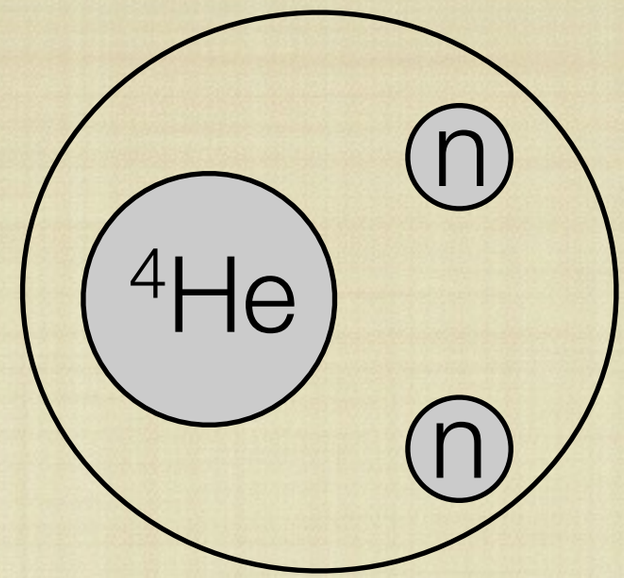
LO NLO

- ...provided k~1/a. As good as ERE?

Scales in halo nuclei

$\lambda \gg R_{core}$; $\lambda \lesssim R_{halo}$





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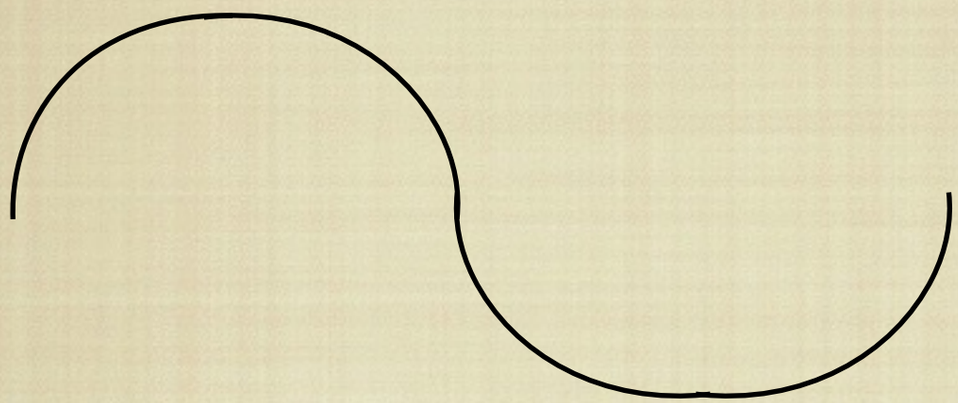

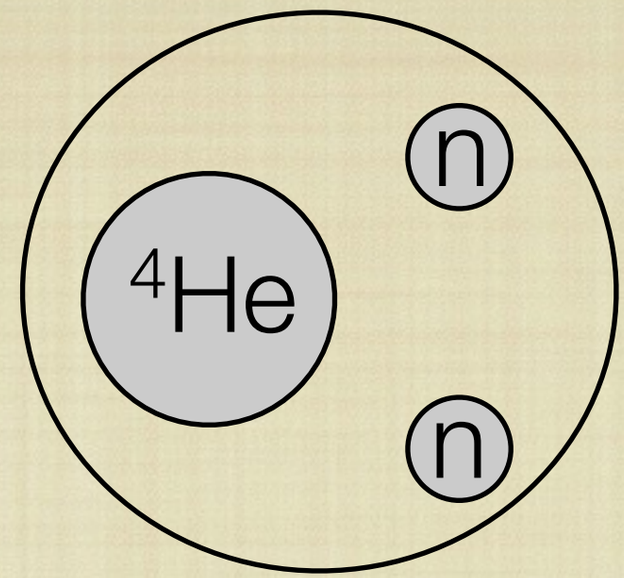

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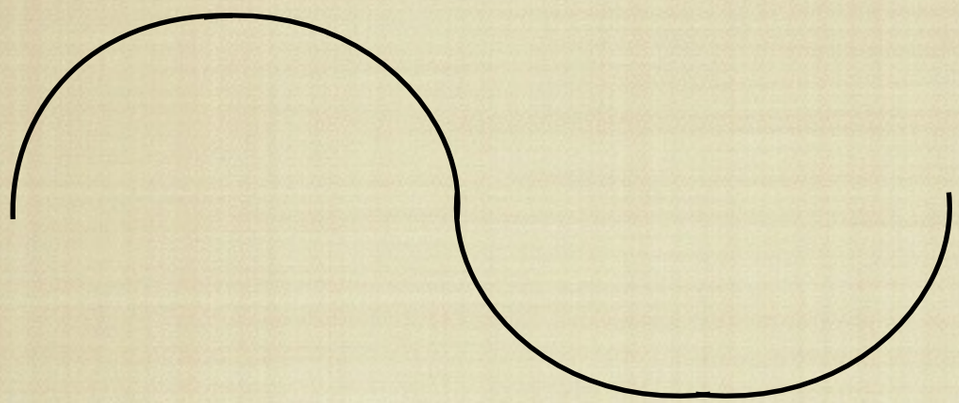

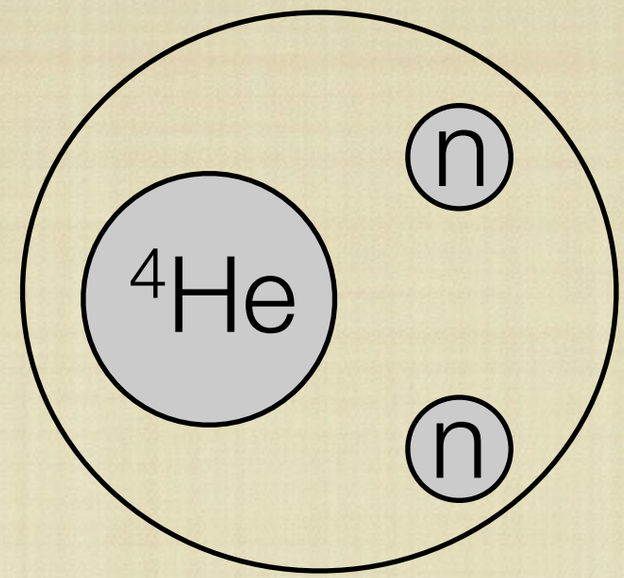

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- By this definition the deuteron is the lightest halo nucleus, and few-nucleon systems are a specific case of halos

Lagrangian for shallow bound states

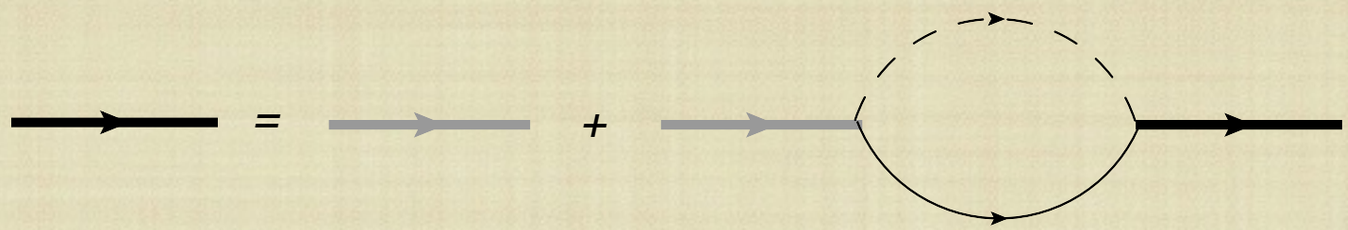
$$
\begin{aligned}
\mathcal{L} \;=\;& c^\dagger\left(i\partial_t+\frac{\nabla^2}{2M}\right)c+n^\dagger\left(i\partial_t+\frac{\nabla^2}{2m}\right)n \\
&+\sigma^\dagger\left[\eta_0\left(i\partial_t+\frac{\nabla^2}{2M_{nc}}\right)+\Delta_0\right]\sigma+\pi_j^\dagger\left[\eta_1\left(i\partial_t+\frac{\nabla^2}{2M_{nc}}\right)+\Delta_1\right]\pi_j \\
&-g_0\left[\sigma n^\dagger c^\dagger+\sigma^\dagger nc\right]-\frac{g_1}{2}\left[\pi_j^\dagger(n\; i\overleftrightarrow{\nabla}_j\; c)+(c^\dagger\; i\overleftrightarrow{\nabla}_j\; n^\dagger)\pi_j\right] \\
&-\frac{g_1}{2}\frac{M-m}{M_{nc}}\left[\pi_j^\dagger\; i\overrightarrow{\nabla}_j\;(nc)-\; i\overleftrightarrow{\nabla}_j\;(n^\dagger c^\dagger)\pi_j\right]+\ldots\,,
\end{aligned}
$$

- c, n: “core”, “neutron” fields. c: boson, n: fermion
- σ, π_j: S-wave and P-wave fields
- Minimal substitution generates leading EM couplings

Dressing the S-wave state

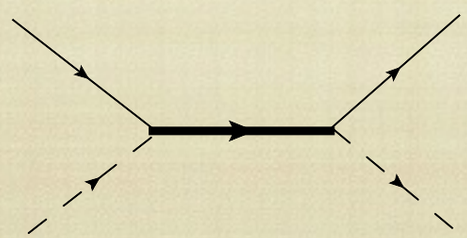
Kaplan, Savage, Wise; van Kolck; Gegelia;
Birse, Richardson, McGovern

- σnc coupling g_0 of order R_{halo}, nc loop of order $1/R_{halo}$. Therefore need to sum all bubbles:



$$D_\sigma(p) = \frac{1}{\Delta_0 + \eta_0[p_0 - \mathbf{p}^2/(2M_{nc})] - \Sigma_\sigma(p)}$$

$$\Sigma_\sigma(p) = -\frac{g_0^2 m_R}{2\pi}\left[\mu + i\sqrt{2m_R\left(p_0 - \frac{\mathbf{p}^2}{2M_{nc}} + i\eta\right)}\right] \qquad \text{(PDS)}$$



$$t = \frac{2\pi}{m_R}\frac{1}{\frac{1}{a_0} - \frac{1}{2}r_0 k^2 + ik}$$

$$D_\sigma(p) = \frac{2\pi\gamma_0}{m_R^2 g_0^2}\frac{1}{1 - r_0\gamma_0}\frac{1}{p_0 - \frac{\mathbf{p}^2}{2M_{nc}} + B_0} + \text{regular}$$

Counting in S waves:
$a_0 \sim R_{halo} \sim 1/\gamma_0$; $r_0 \sim R_{core}$.
$r_0 = 0$ at LO.

Radii of s-wave 1n halos

Wave function: $u(r)=C\exp(-\gamma_0 r) \Rightarrow \langle r_{nc}^2\rangle^{1/2} = \frac{C}{2}\left(\frac{A+1}{2AM_N S_{1n}}\right)^{3/4}$

$C=(2\gamma_0)^{1/2}$ at LO: $C>(2\gamma_0)^{1/2}$ at NLO

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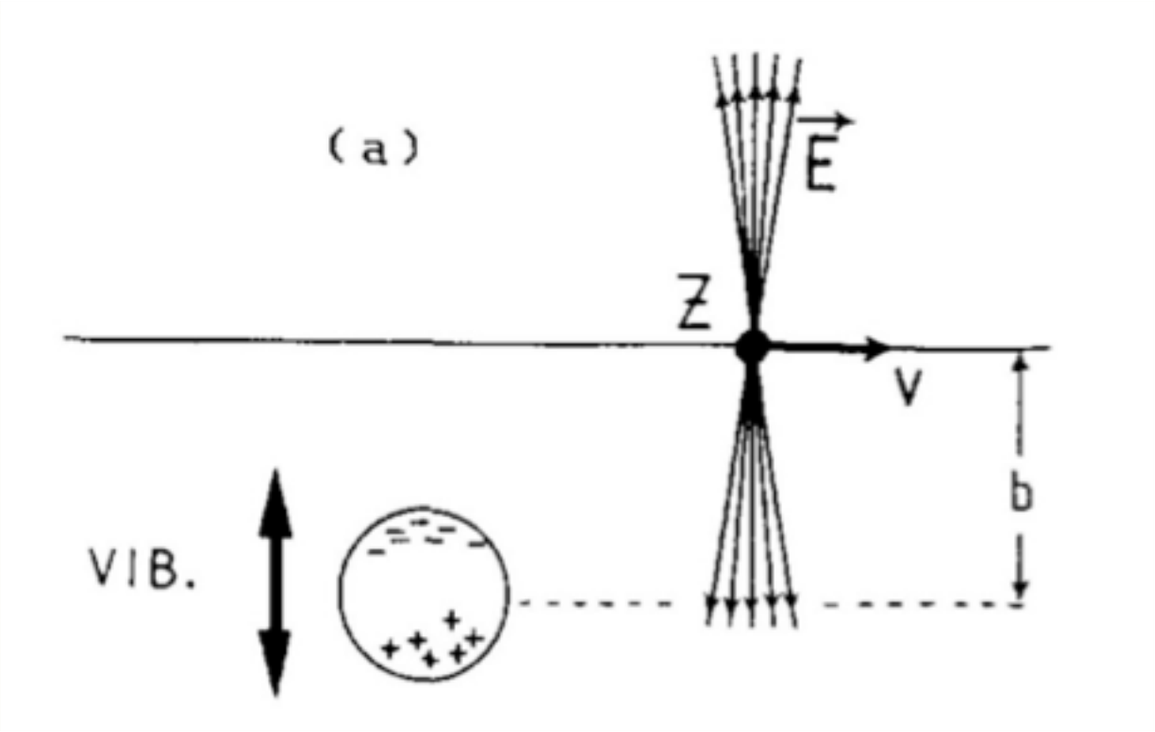
	S_{1n} (MeV)	R_{core}/R_{halo}	$\langle r_{nc}^2\rangle^{1/2}$ (fm) NNLO	$\langle r_{nc}^2\rangle^{1/2}$ (fm) Expt
^{2}H	2.224573(2)	0.33	3.954	3.9270(90)
^{11}Be	0.50164(25)	0.4	6.16	5.7(4)
^{15}C	1.2181(8)	0.45	4.93	4.5(5)
^{19}C	0.58(9)	0.33	5.72	6.8(7)

All radii are substantially smaller at LO: range corrections are crucial to obtaining agreement with experiment

Photodissociation: experiments

Bertulani, arXiv:0908.4307

- Coulomb dissociation: collide halo (peripherally?) with high-Z nucleus
- Do with different Z, different nuclear sizes, different energies to test systematics
- C.f. trimer photoassociation



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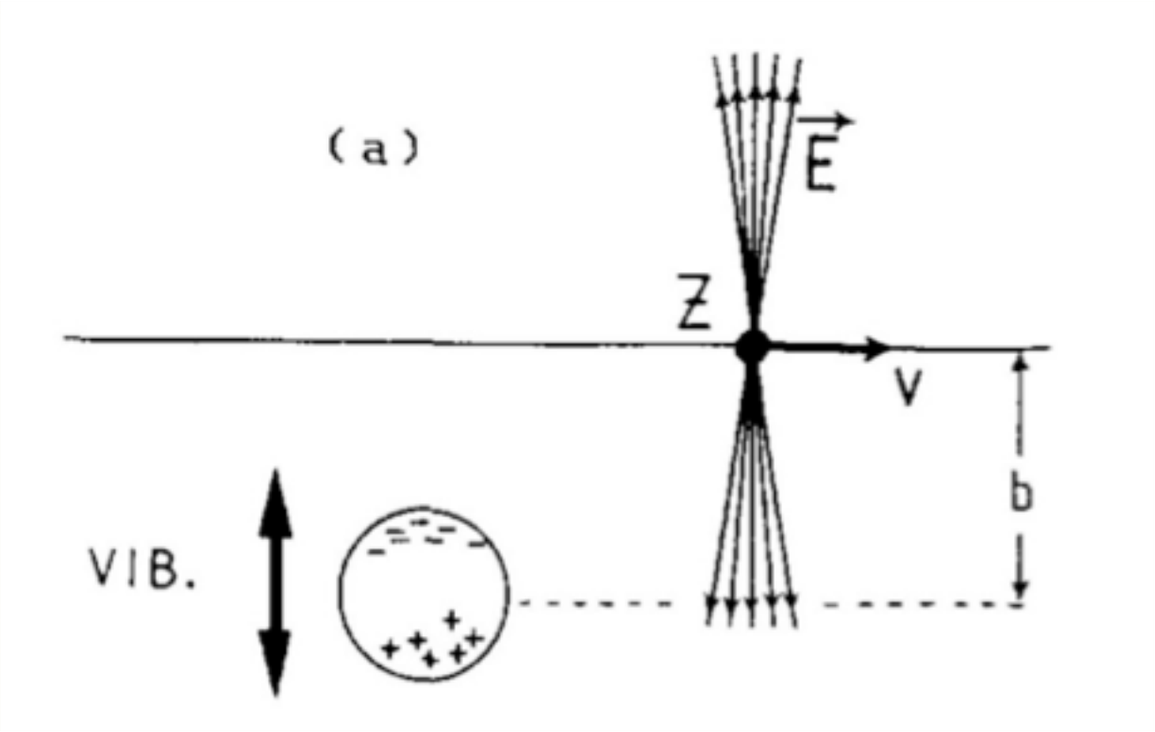

Bazak, Liverts & Barnea, PRL (2012), PRA (2013), Bazak & Barnea, arXiv:1502.07119

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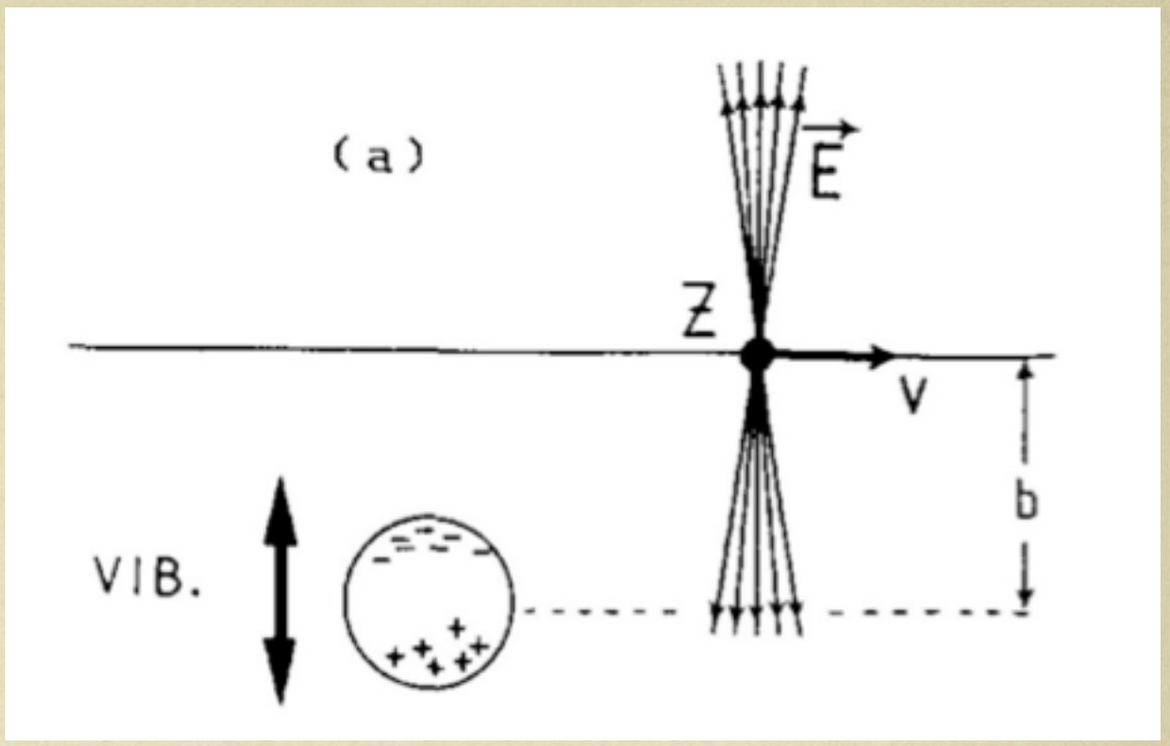

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- Coulomb excitation dissociation cross section (p.v. b≫R_{target})

$$\frac{d\sigma_C}{2\pi b db} = \sum_{\pi L} \int \frac{dE_\gamma}{E_\gamma} n_{\pi L}(E_\gamma, b) \sigma_\gamma^{\pi L}(E_\gamma)$$

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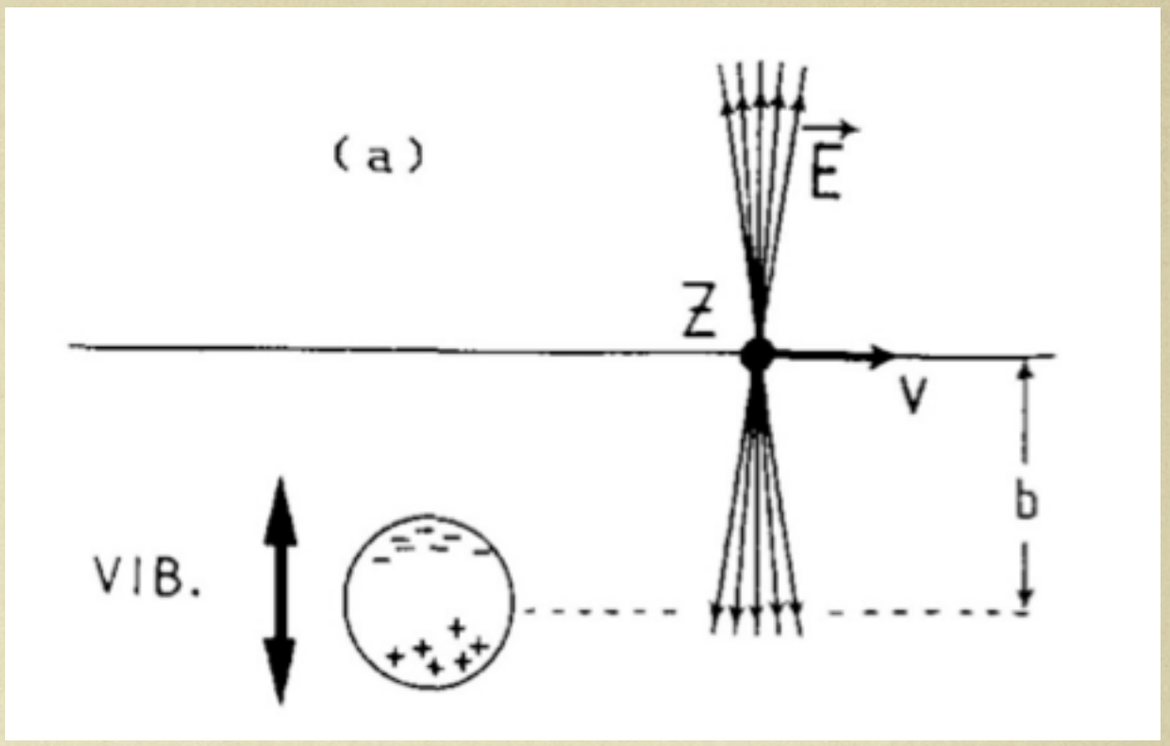


Bazak, Liverts & Barnea, PRL (2012), PRA (2013), Bazak & Barnea, arXiv:1502.07119

- Coulomb excitation dissociation cross section (p.v. b≫R_{target})

$$\frac{d\sigma_C}{2\pi b db} = \sum_{\pi L} \int \frac{dE_\gamma}{E_\gamma} n_{\pi L}(E_\gamma, b) \sigma_\gamma^{\pi L}(E_\gamma)$$

- $n_{\pi L}(E_\gamma, b)$ virtual photon numbers, dependent only on kinematic factors. Number of equivalent (virtual) photons that strike the halo nucleus.
- $\sigma_\gamma^{\pi L}(E_\gamma)$ can then be extracted: it's the (total) cross section for dissociation of the nucleus due to the impact of photons of multipolarity πL.

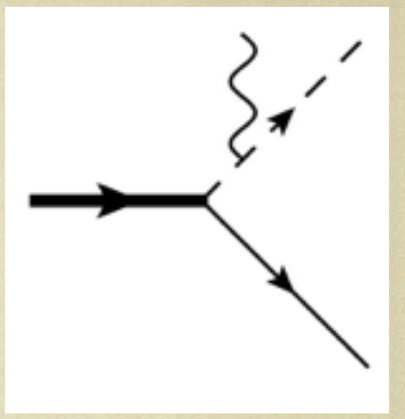
Universal dissociation

Chen, Savage (1999)

- Leading order: no FSI, $r_0=0 \Rightarrow \gamma_0$ is only free parameter

$\gamma_0^2 = 2\, m_R\, S_{1n}$

$$\mathcal{M} = \frac{eQ_c g_0 2m_R}{\gamma_0^2 + \left(\mathbf{p}' - \frac{m}{M_{nc}}\mathbf{k}\right)^2}$$



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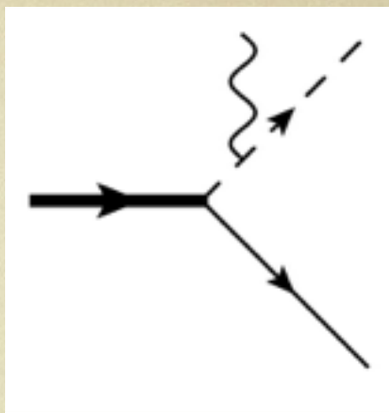

Up to NNLO: $$\frac{dB(E1)}{e^2 dE} = \frac{12 m_R}{\pi^2} Z_{eff}^2 \frac{\gamma_0}{1 - r_0\gamma_0} \frac{p^3}{(\gamma_0^2 + p^2)^4}$$

$Z_{eff} = Z/(A+1)$

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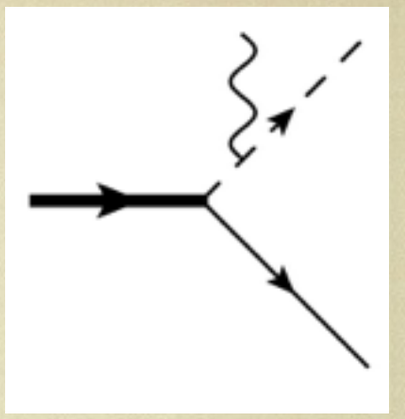

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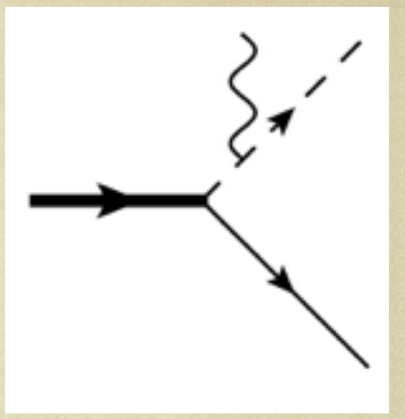

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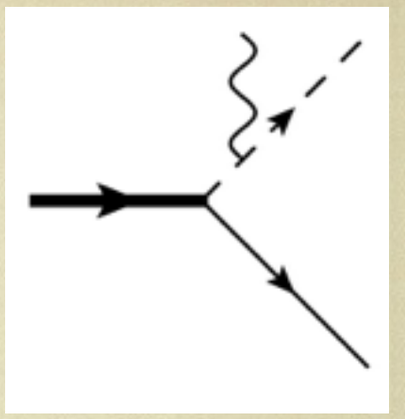

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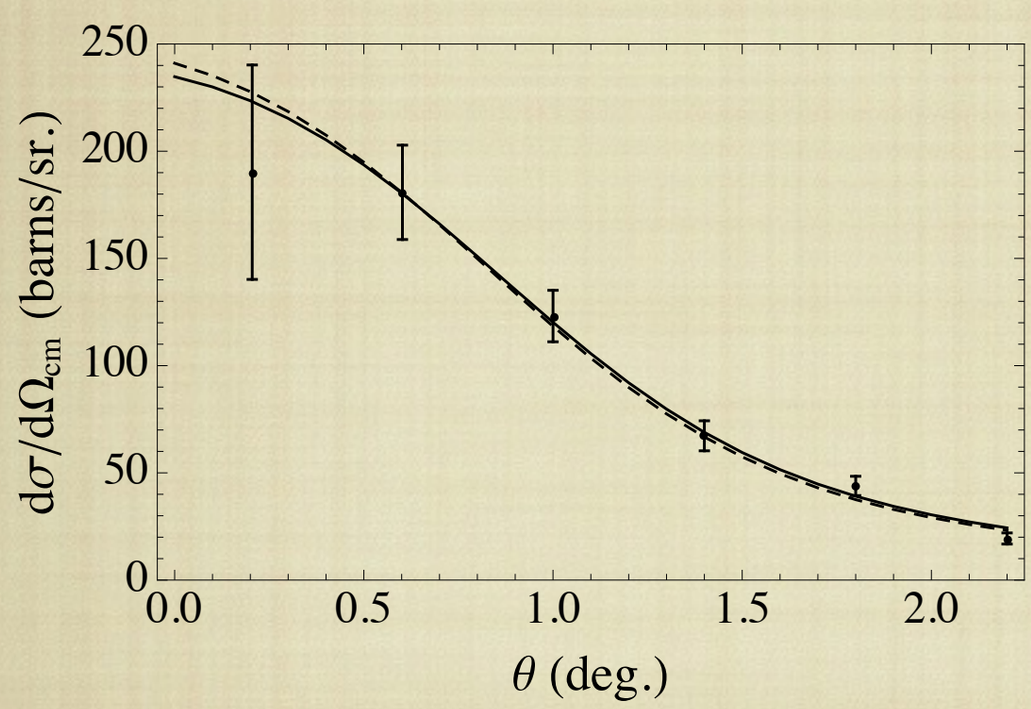
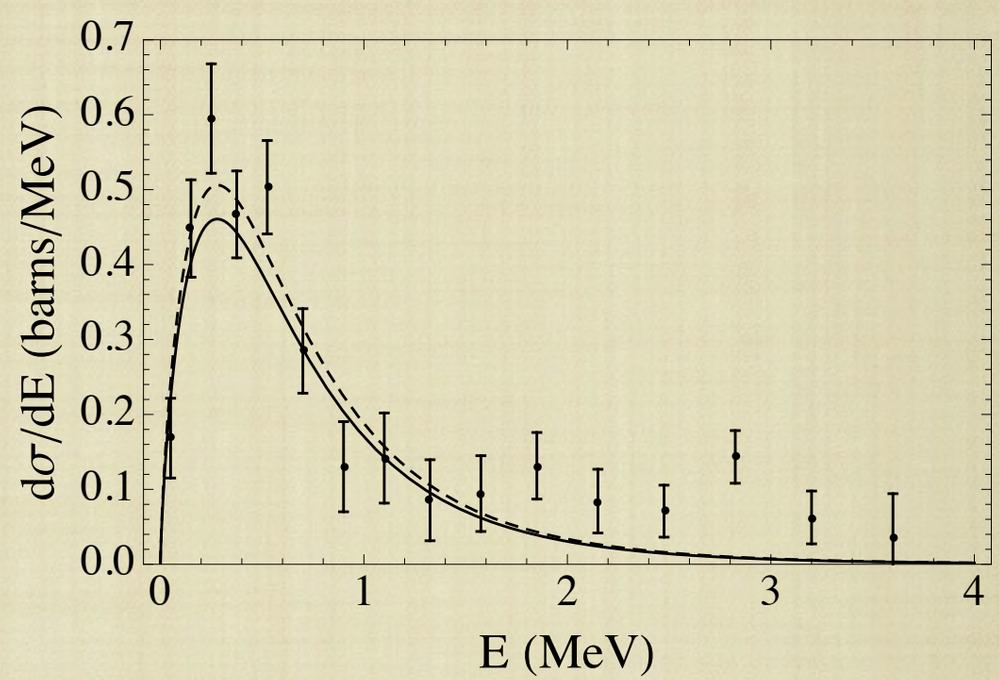
- Final-state interactions suppressed by $(R_{core}/R_{halo})^3$
- First gauge-invariant contact operator: $L_{E1}\sigma^\dagger \mathbf{E} \cdot (n \overleftrightarrow{\nabla} c) + \text{h.c.}$

Results

Data: Nakamura et al., 1999, 2003
Analysis: Acharya, Phillips. 2013
cf. Singh et al., 2008

- Integrate this E1 strength for transition to a core + neutron state, per unit energy per unit solid angle, as function of energy of the outgoing nc pair over differential photon numbers and over angle.





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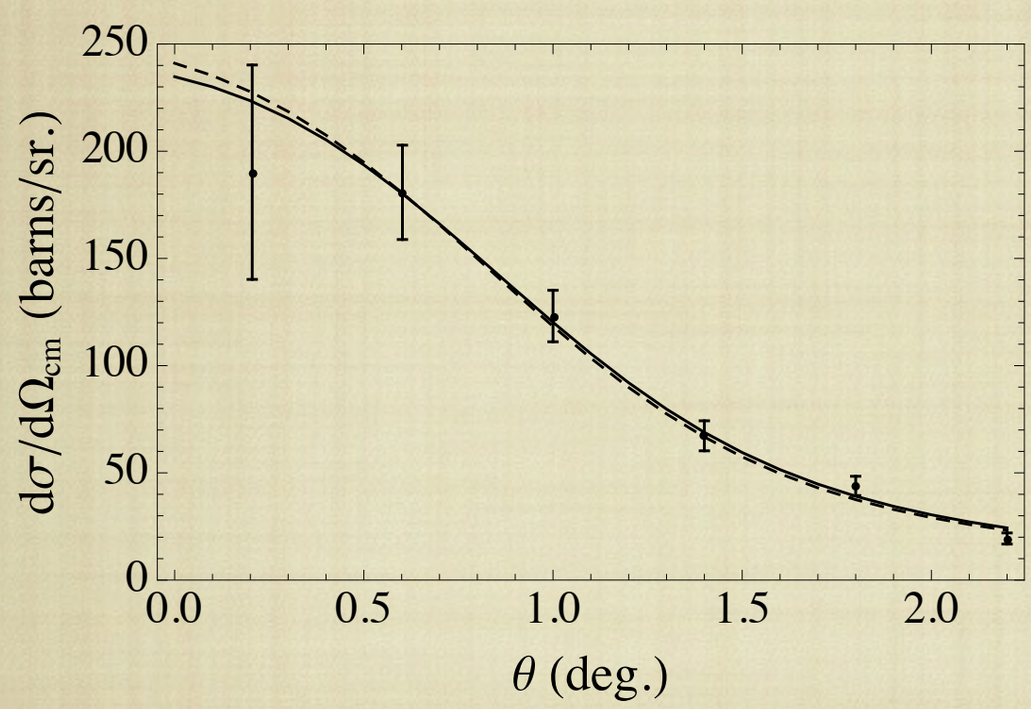

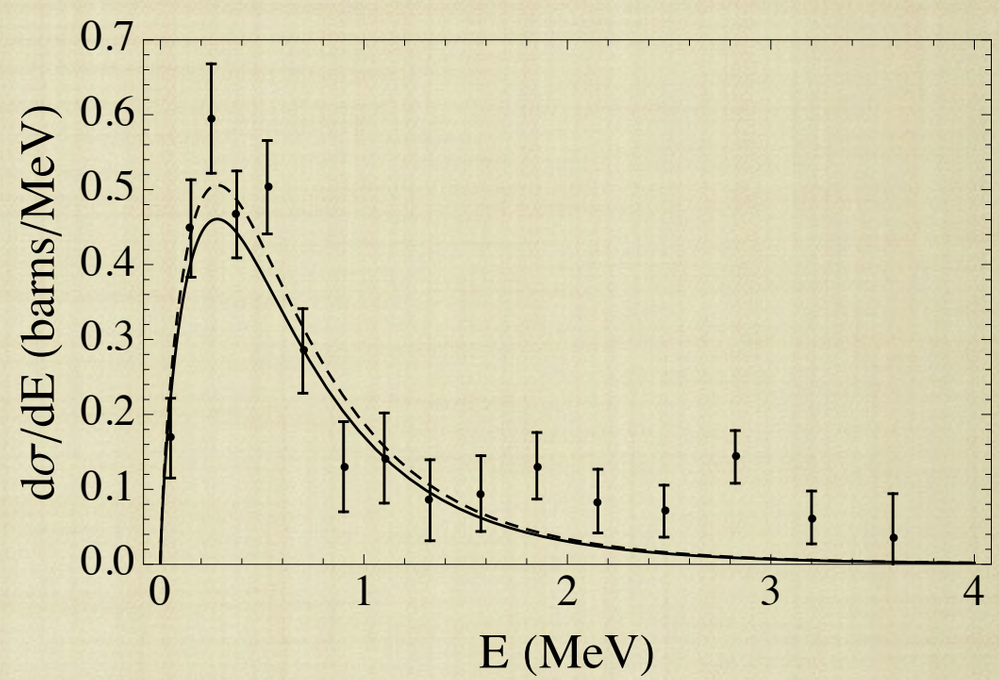

•$\gamma_0 \equiv a$ determines peak position and fall off of angular distribution

- r_0 fixed from fitting height of peak

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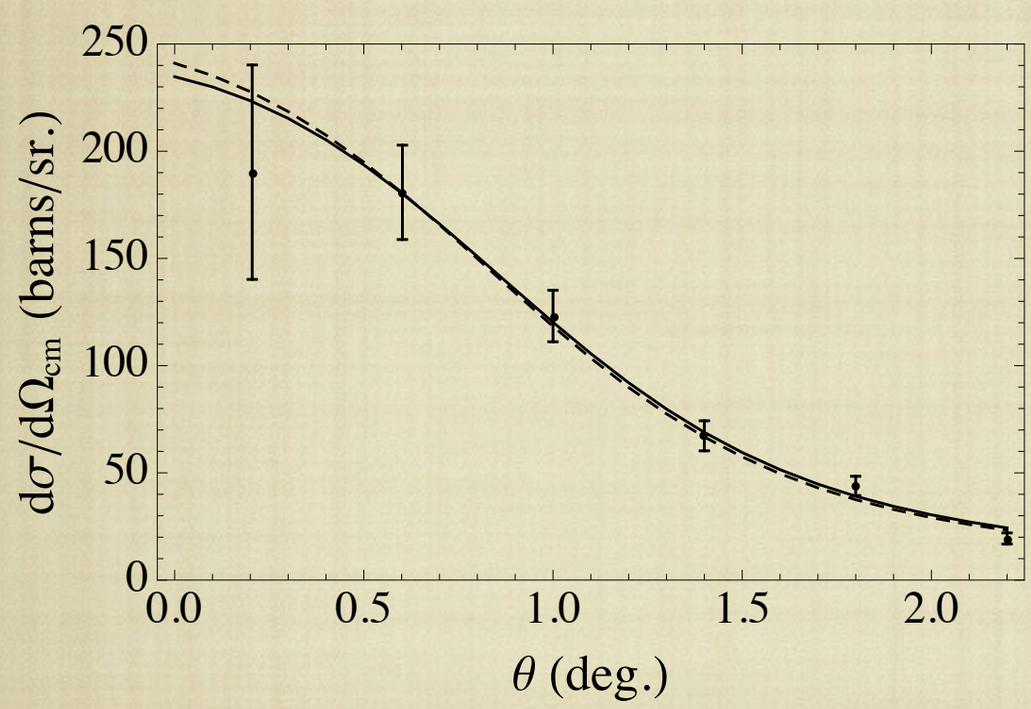

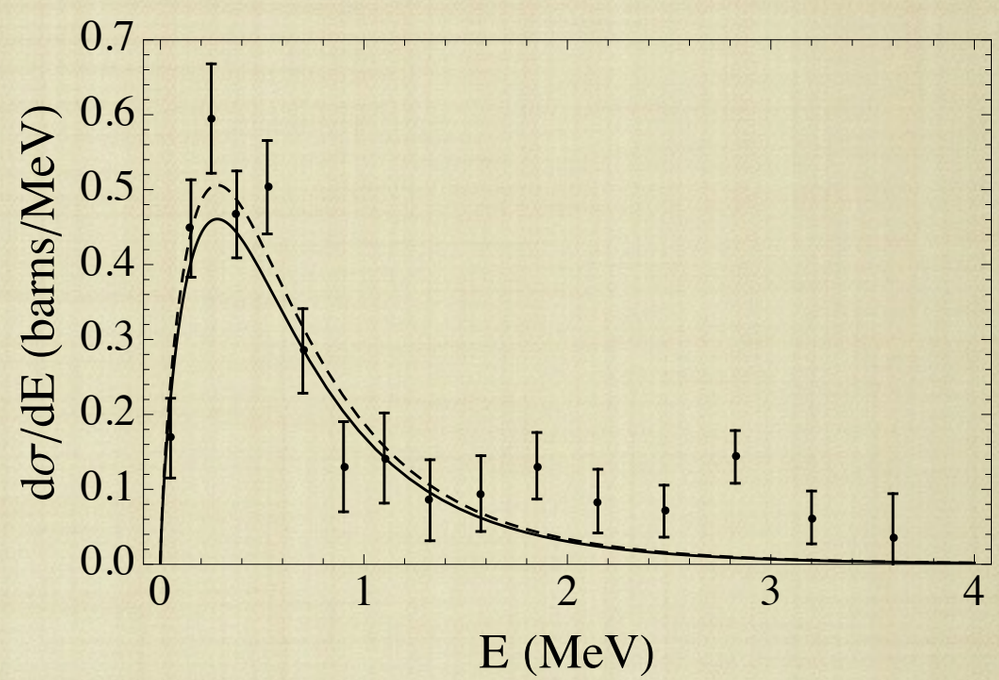

$$a = (7.75 \pm 0.35(\text{stat.}) \pm 0.3(\text{EFT}))\ \text{fm};$$
$$r_0 = (2.6^{+0.6}_{-0.9}(\text{stat.}) \pm 0.1(\text{EFT}))\ \text{fm}.$$

Determine S-wave ^{18}C-n scattering parameters⇔ANCs from dissociation data.

P-waves: $\gamma_{E1} + {}^{11}Be \rightarrow {}^{10}Be + n$

Typel & Baur, Phys. Rev. Lett. 93, 142502 (2004); Nucl. Phys. A759, 247 (2005); Eur. Phys. J. A 38, 355 (2008)

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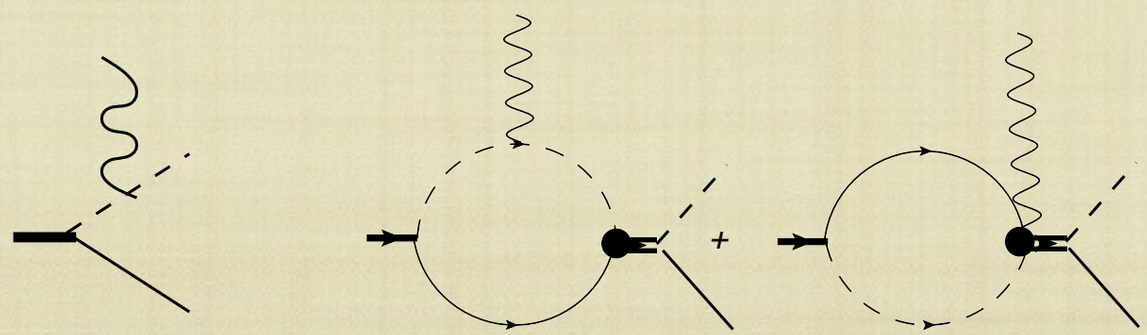
- FSI in spin-1/2 channel: stronger, but "kinematic" nature of P-wave state means it's perturbative away from resonance.

Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)



LO NLO

- Need γ_1 and $r_1 \equiv A_1$ at NLO in this observable. Coulomb dissociation of ^{11}Be

How-to: three-body system

$$-\frac{\hbar^2}{2m_1}\nabla_1^2\Psi - \frac{\hbar^2}{2m_2}\nabla_2^2\Psi - \frac{\hbar^2}{2m_3}\nabla_3^2\Psi + \sum_{i<j} V_{ij}(\mathbf{r}_{ij})\Psi(\mathbf{r}_{ij},\mathbf{r}_{ij,k})$$
$$+V_{123}(\mathbf{r}_{ij},\mathbf{r}_{ij,k})\Psi(\mathbf{r}_{ij},\mathbf{r}_{ij,k}) = E\Psi(\mathbf{r}_{ij},\mathbf{r}_{ij,k})$$

- Remember: most of Ψ occurs outside range of V's
- Construct two-body and three-body potentials as limiting sequence of functions: you can take whatever's easiest to solve!
- Strength of V_{ij} set to a, strength of V_{ijk} set to lowest 3B binding energy
- Favorite solution method
- Perturbative evaluation of R/a (or kR) corrections: EFT expansion

Universal three-body relations

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- Energies of two states: $B_{n+1} = e^{-2\pi/s_0} B_n$
- Features on the Efimov plot: $a_{0,n} = -0.210\ a_{-,n}$

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- Unification via universality: in what ways are all halo nuclei similar?

- Diagnosing via universality: determine unmeasured properties of halo nuclei through universal relationships

Equations for s-wave 2n halo

Canham, Hammer (2008)

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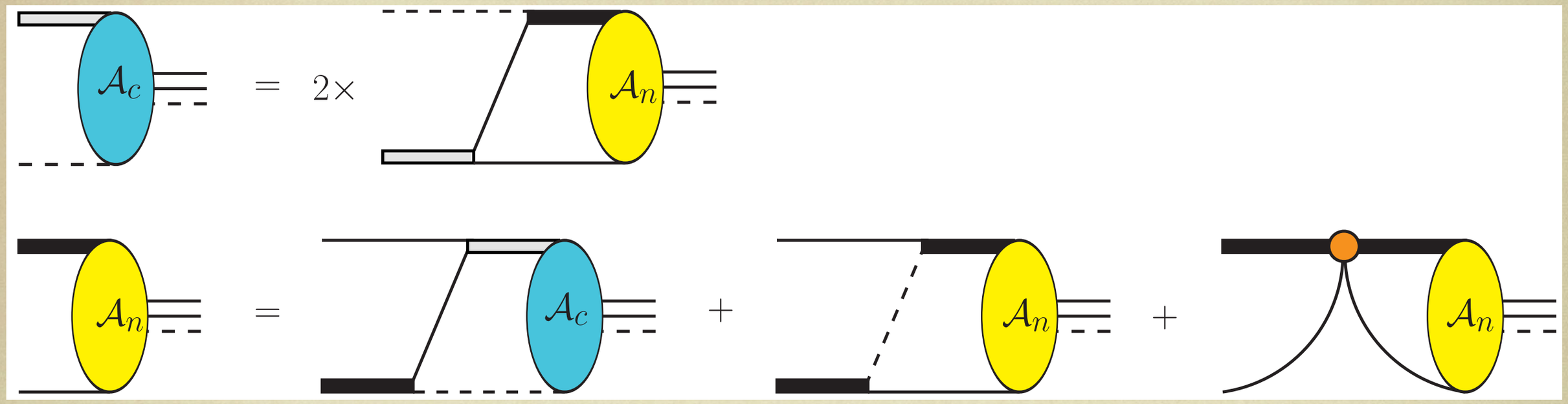

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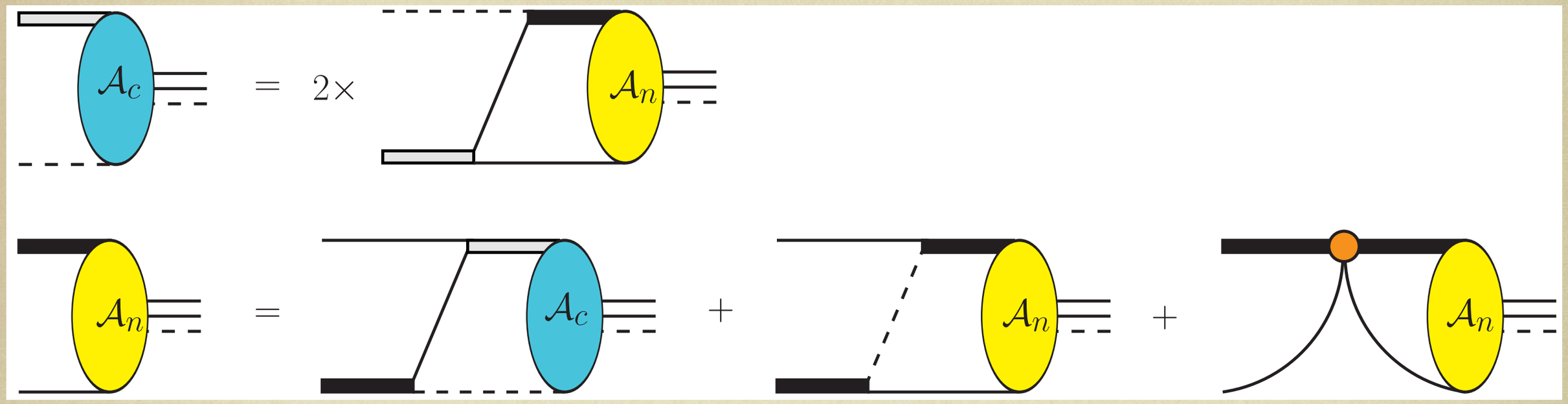

- (cn)-n contact interaction to stabilize three-body system

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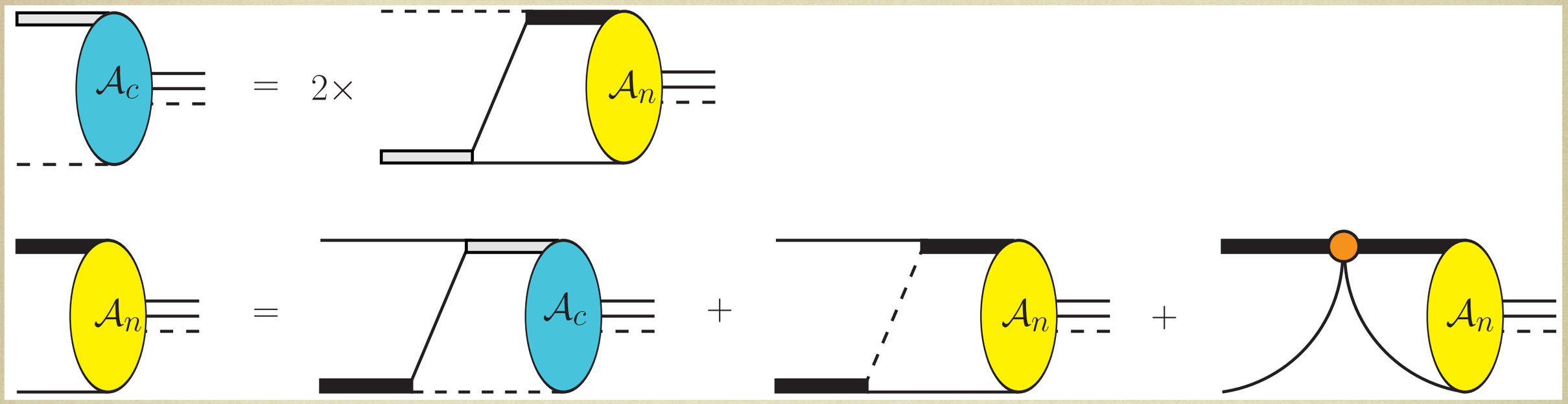

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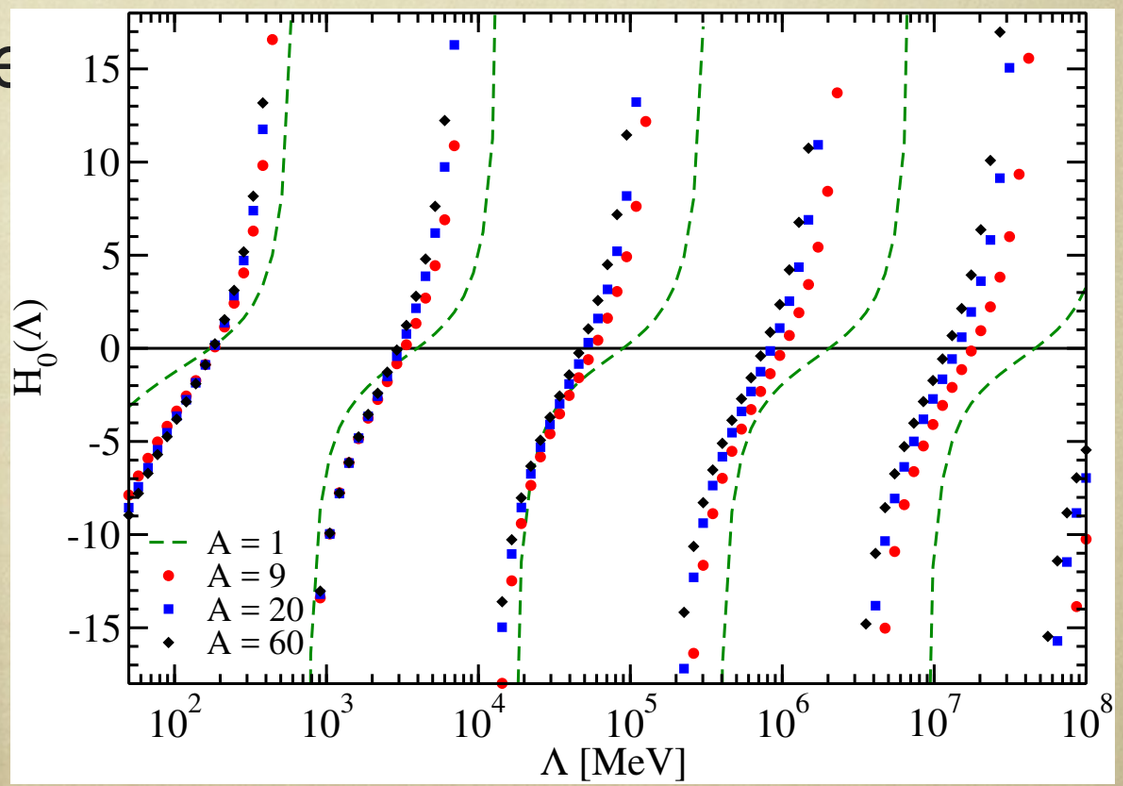

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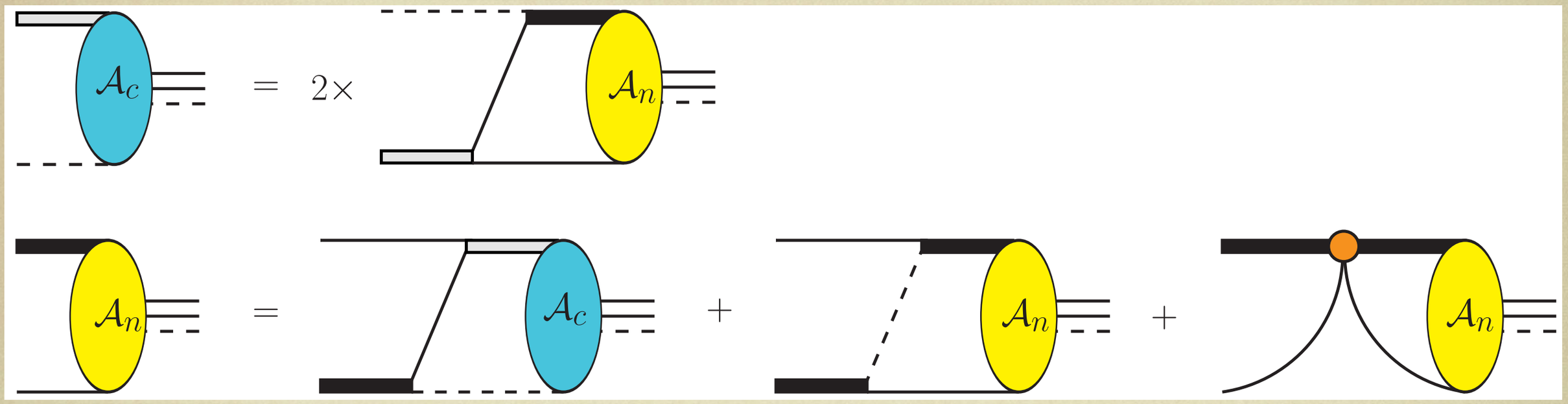

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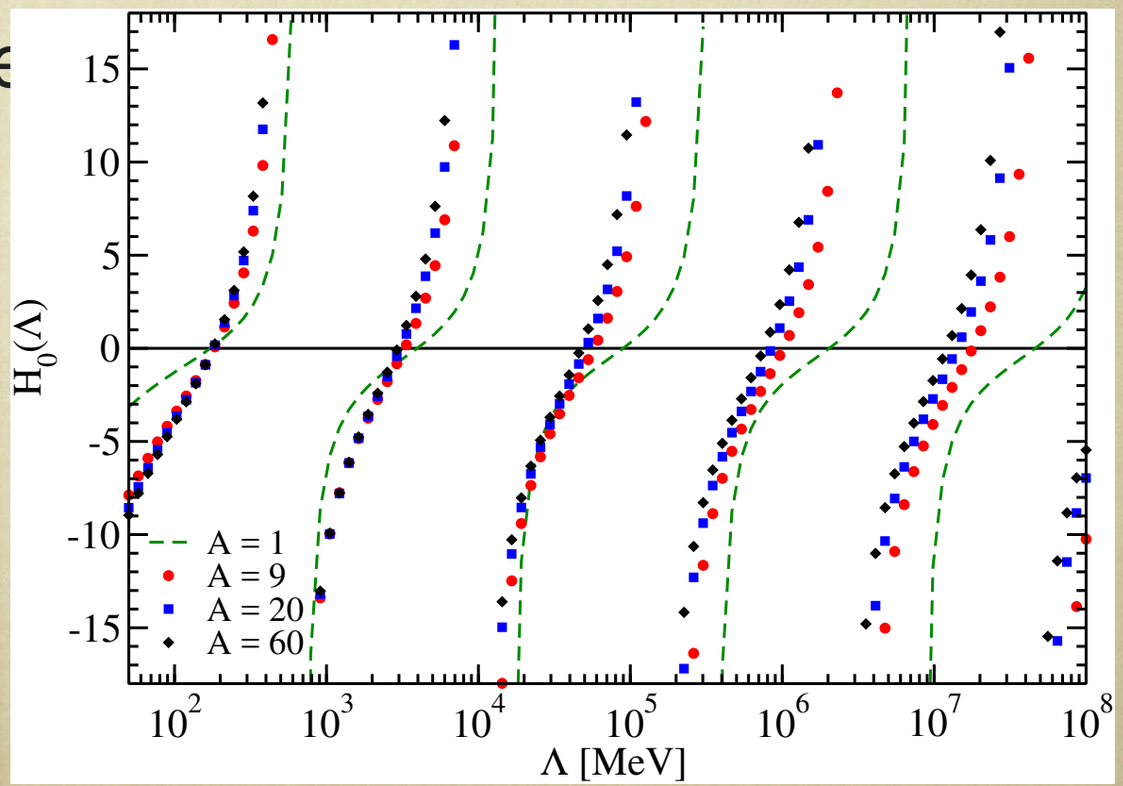

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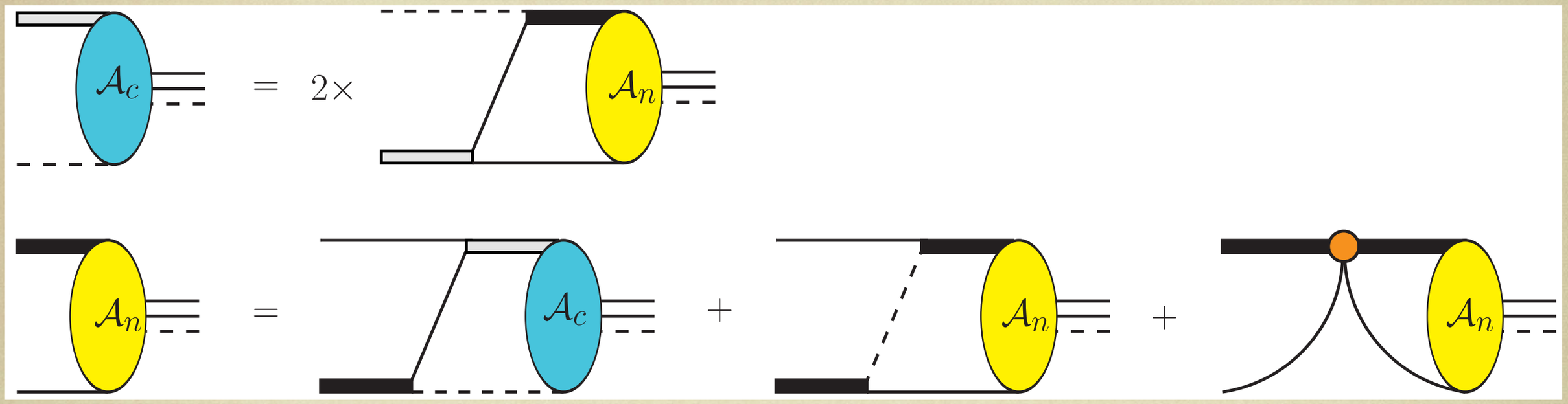

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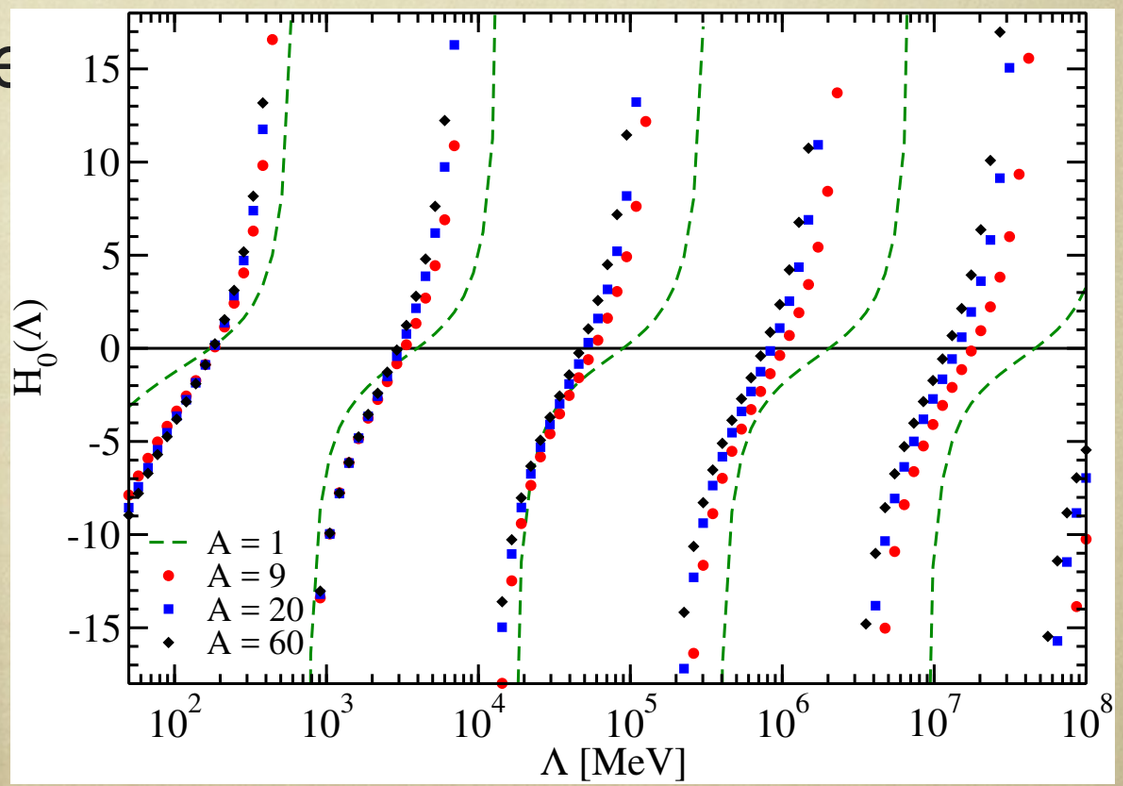

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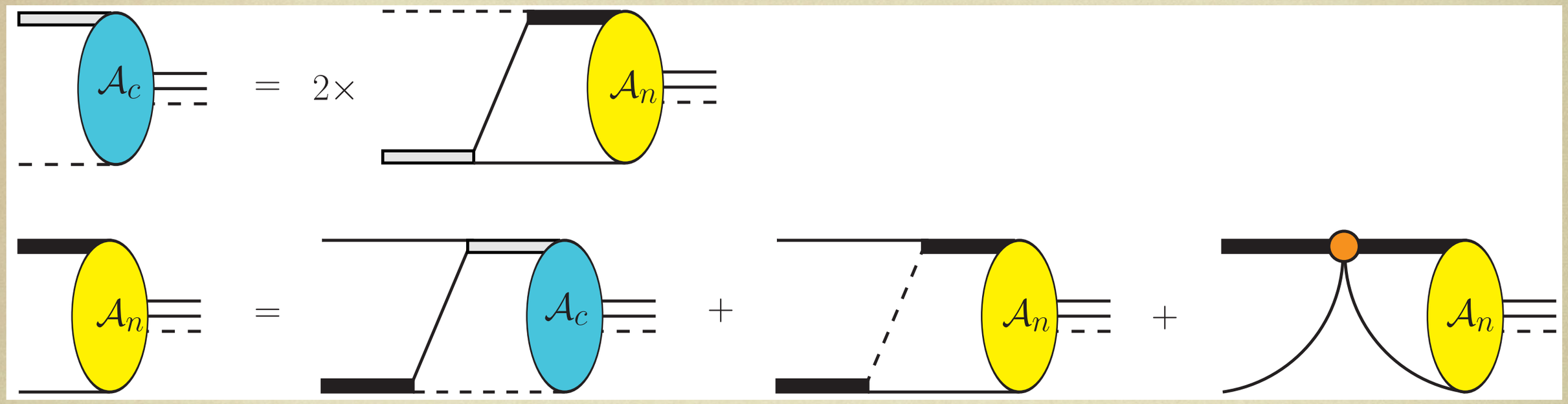

- (cn)-n contact interaction to stabilize three-body system
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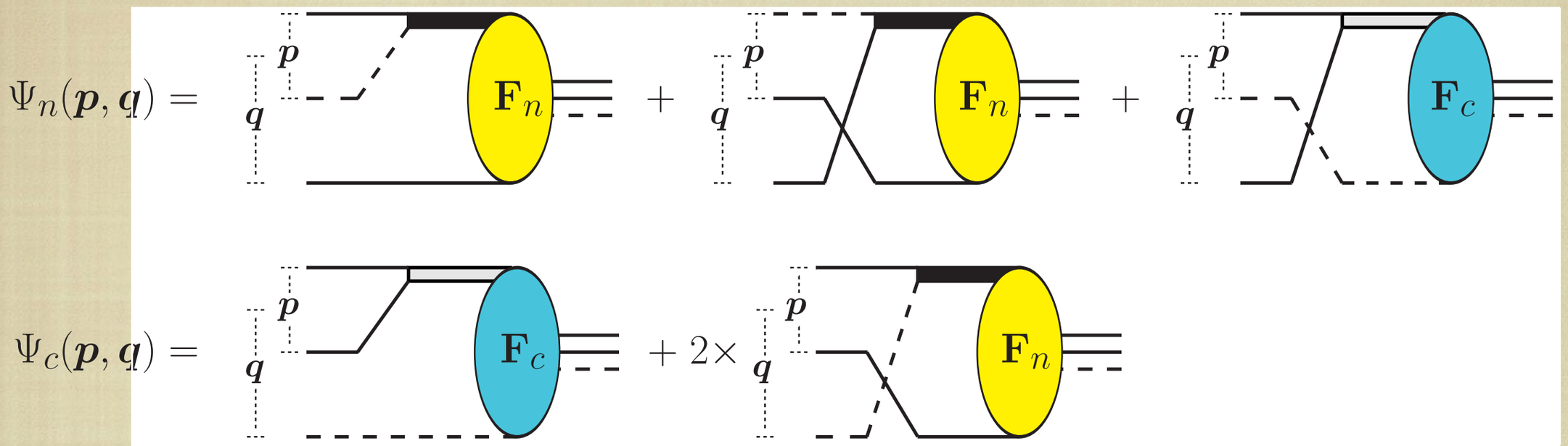
- Output: everything. Up to R_{core}/R_{halo} corrections.

Matter radii of 2n s-wave halos



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- One-body form factors:

$$\mathcal{F}_x(k^2) = \int_0^\infty \mathrm{d}p\; p^2 \int_0^\infty \mathrm{d}q\; q^2 \int_{-1}^{1} \mathrm{d}\left(\hat{q}\cdot\hat{k}\right) \Psi_x(p,q)\; \Psi_x(p,|\vec{q}-\vec{k}|).$$

- Radii: $\mathcal{F}_x(k^2) = 1 - \frac{1}{6}\langle r_x^2\rangle k^2 + O(k^4)$
- Matter radius: $\langle r_0^2\rangle = \frac{2(A+1)^2}{(A+2)^3}\langle r_n^2\rangle + \frac{4A}{(A+2)^3}\langle r_c^2\rangle$

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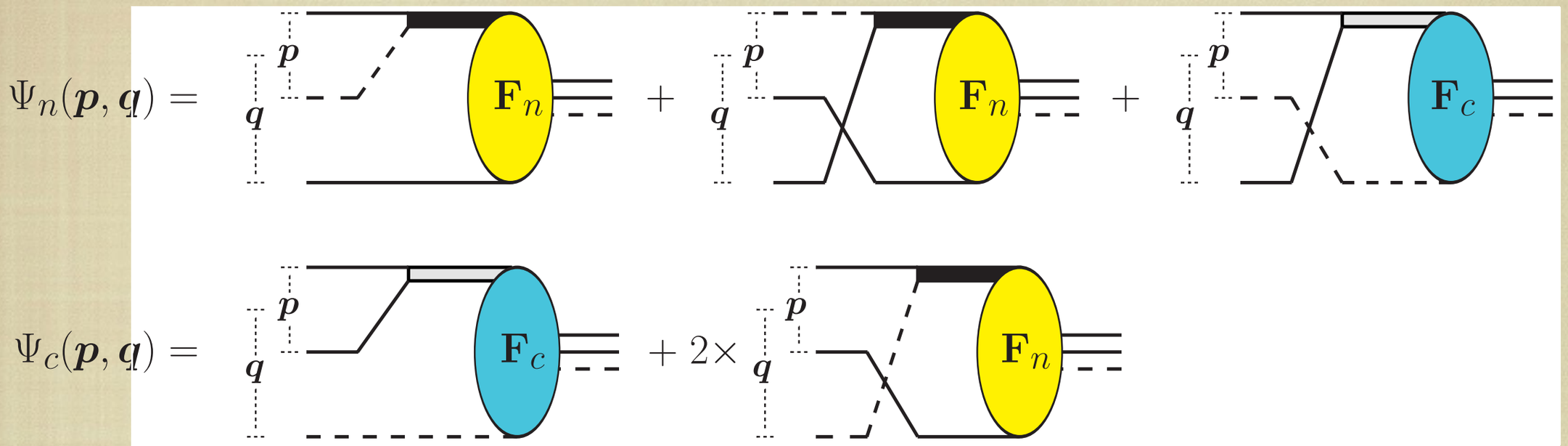

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Output: all radii

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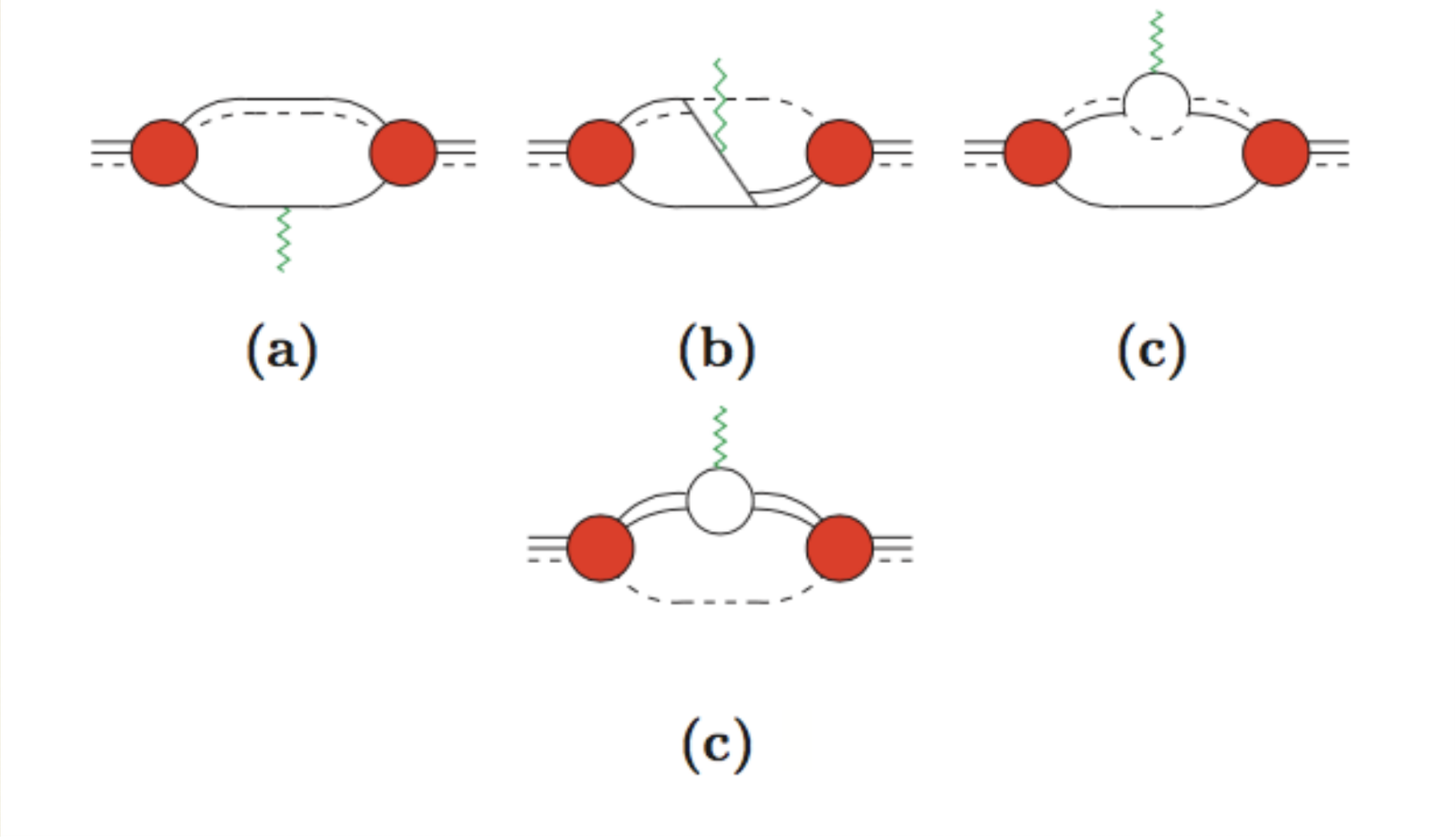
…or use field theory

Hagen et al., (2013), Vanasse arXiv:1609.08552



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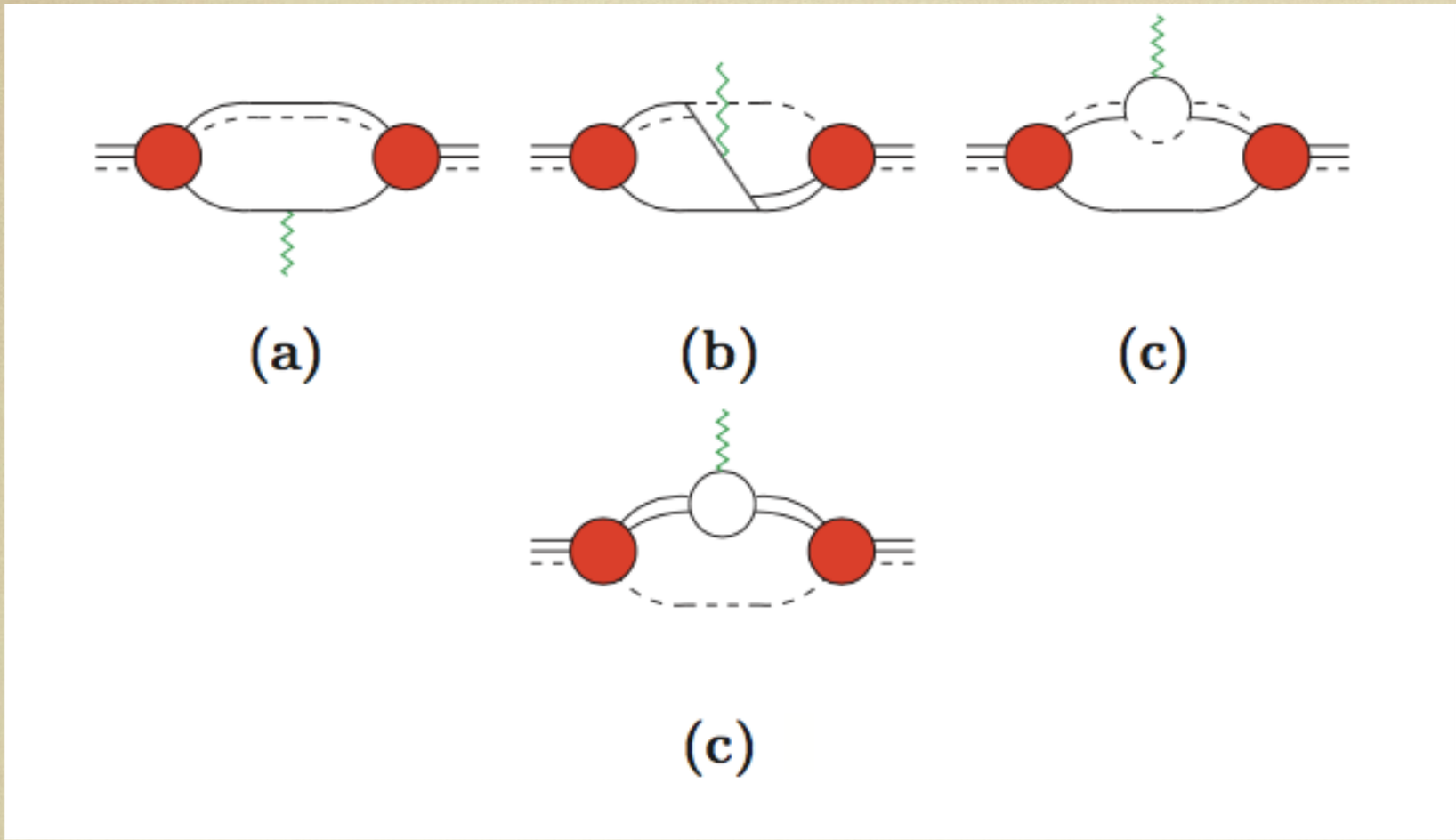

Should be equivalent to QM formulation

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Matter radii of 2n halos at LO

Acharya, Ji, Phillips (2013)

- Define: $f\left(\frac{E_{nn}}{B}, \frac{E_{nc}}{B}; A\right) \equiv mB\langle r_0^2\rangle$

- Unitary limit, $E_{nn}=E_{nc}=0$: f becomes a number depending solely on A

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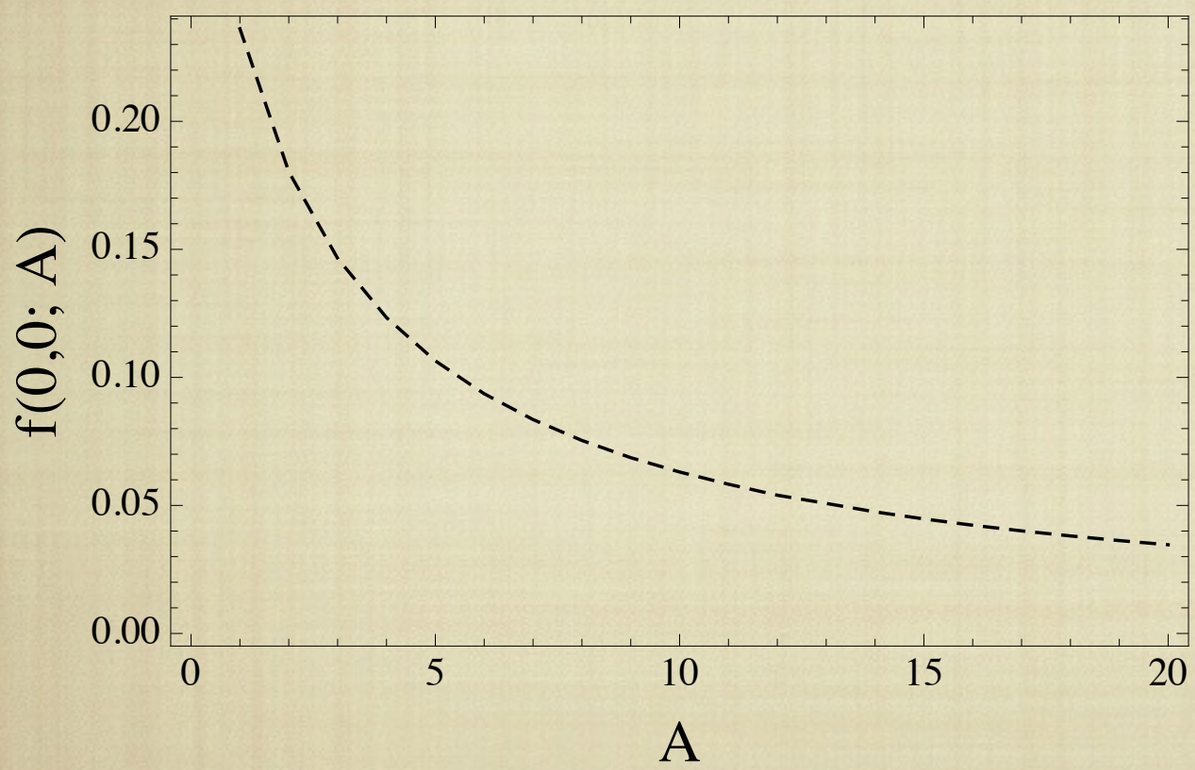

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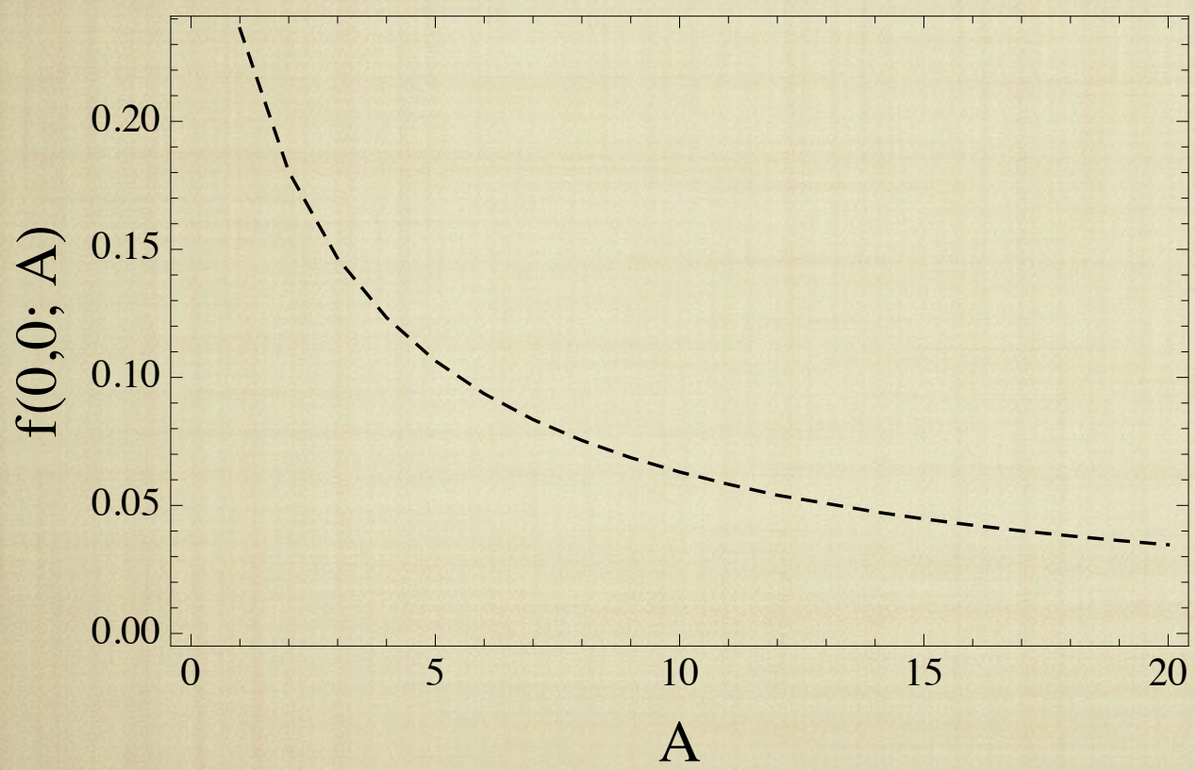

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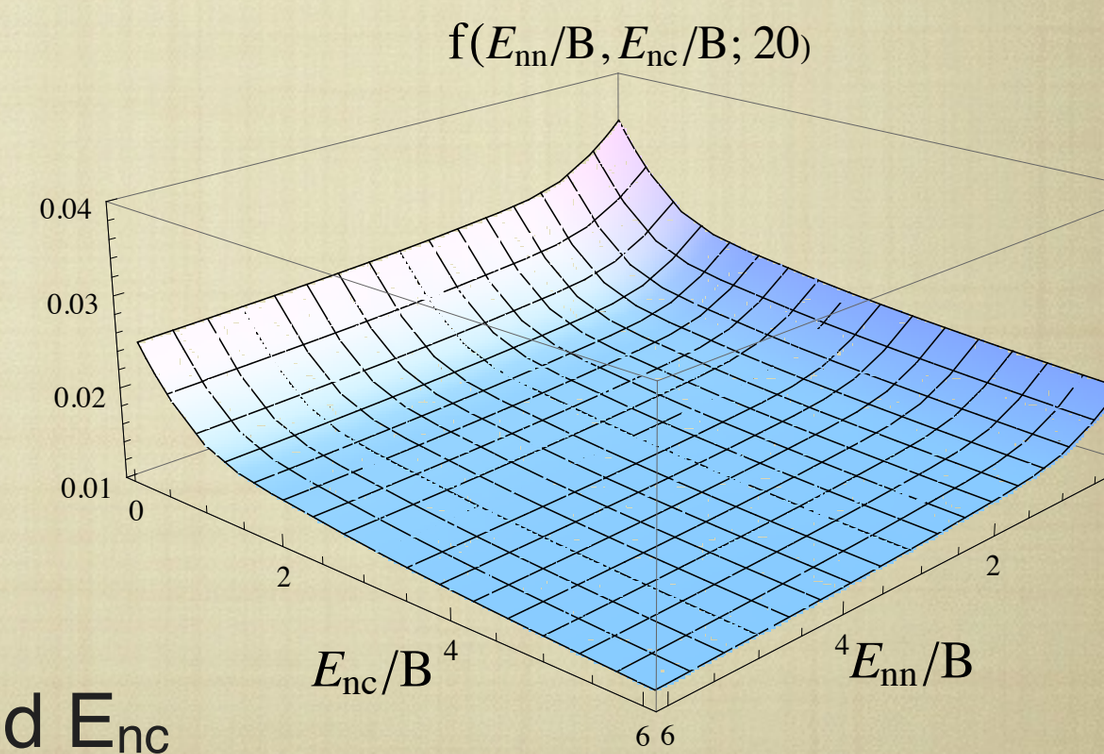

- Fix A=20, plot f as a function of E_{nn} and E_{nc}

LO results for radii of 2n halos

Canham, Hammer (2011); Hagen, Platter, Hammer (2014); Acharya, Ji, Phillips (2013)

Vanasse (2016)

Are these systems "universal enough"?

	E_{nc} (MeV)	S_{2n} (MeV)	R_{core}/R_{halo}	$\langle r_0^2 \rangle$ (fm^2) LO	$\langle r_0^2 \rangle$ (fm^2) Expt
^{11}Li	-0.026(13)	0.3693(6)	0.37	5.76 ± 2.13	5.34 ± 0.15
^{14}Be	-0.510	1.27(13)	0.78	1.23 ± 0.96	4.24 ± 2.42 2.90 ± 2.25
^{22}C	-0.01(47)	0.11(6)	0.26	3.99-∞	21.1 ± 9.7 3.77 ± 0.61

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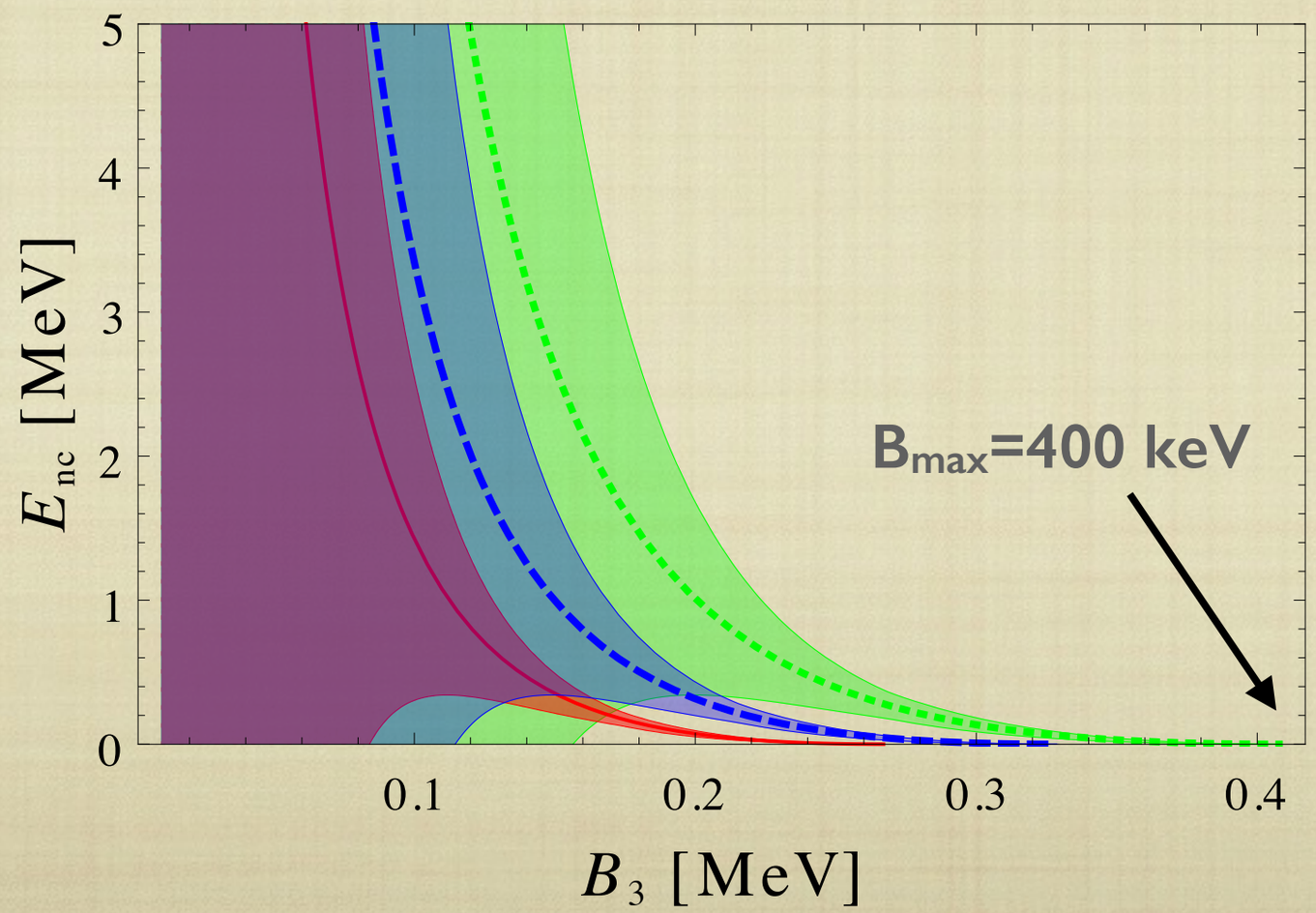
Errors tend to be dominated by EFT uncertainty⇒need ranges to become more accurate

Application to ^{22}C

- Include finite size of ^{20}C

- Consider uncertainty due to NLO effects:

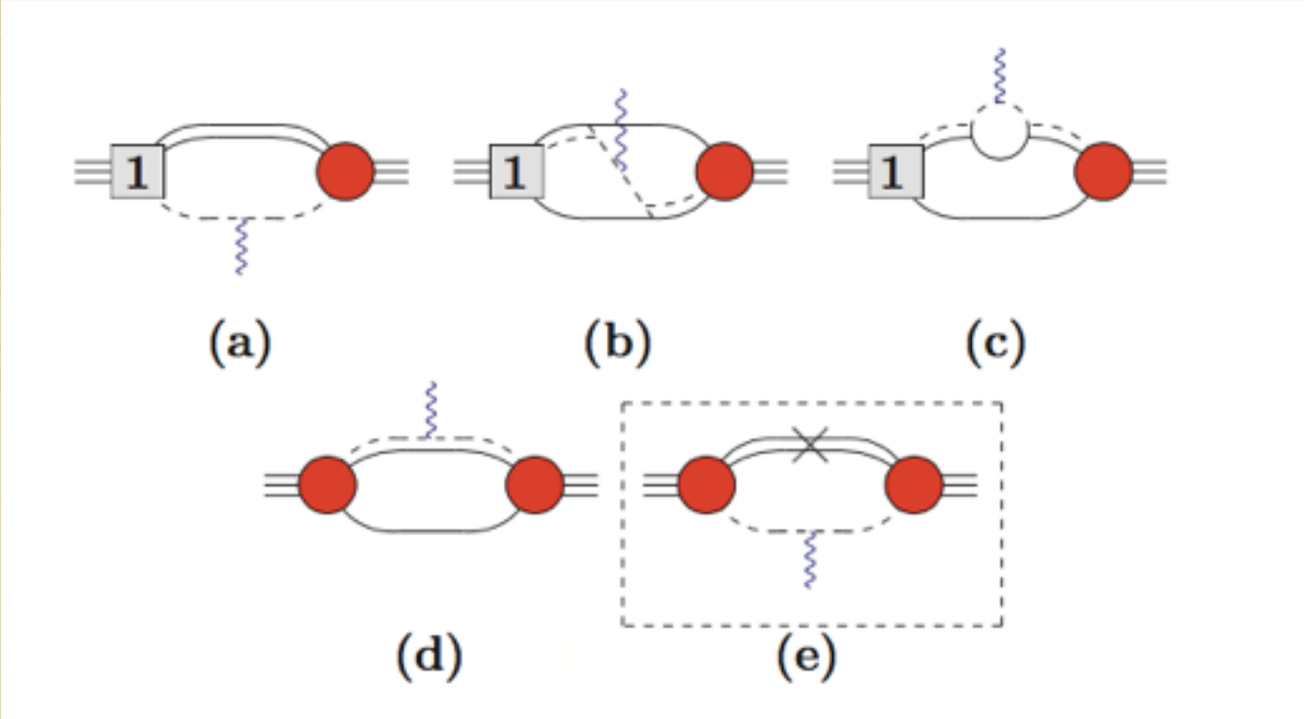
Relative size ~ largest of $(mE_{nn})^{1/2} R_{core}$; $(2mE_{nc})^{1/2} R_{core}$; $(2mB)^{1/2} R_{core}$



cf. Yamashita et al. (2011); Fortune & Sherr (2012)

Next-to-leading order

Vanasse (2016)



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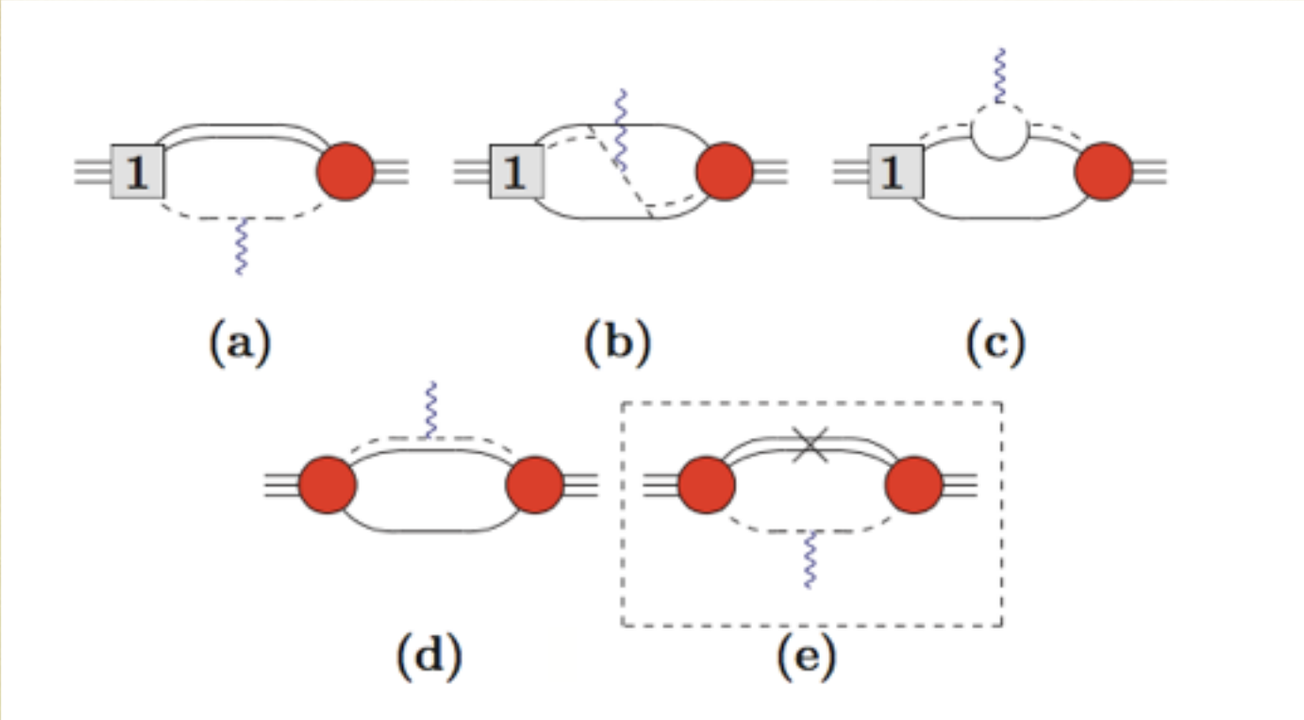

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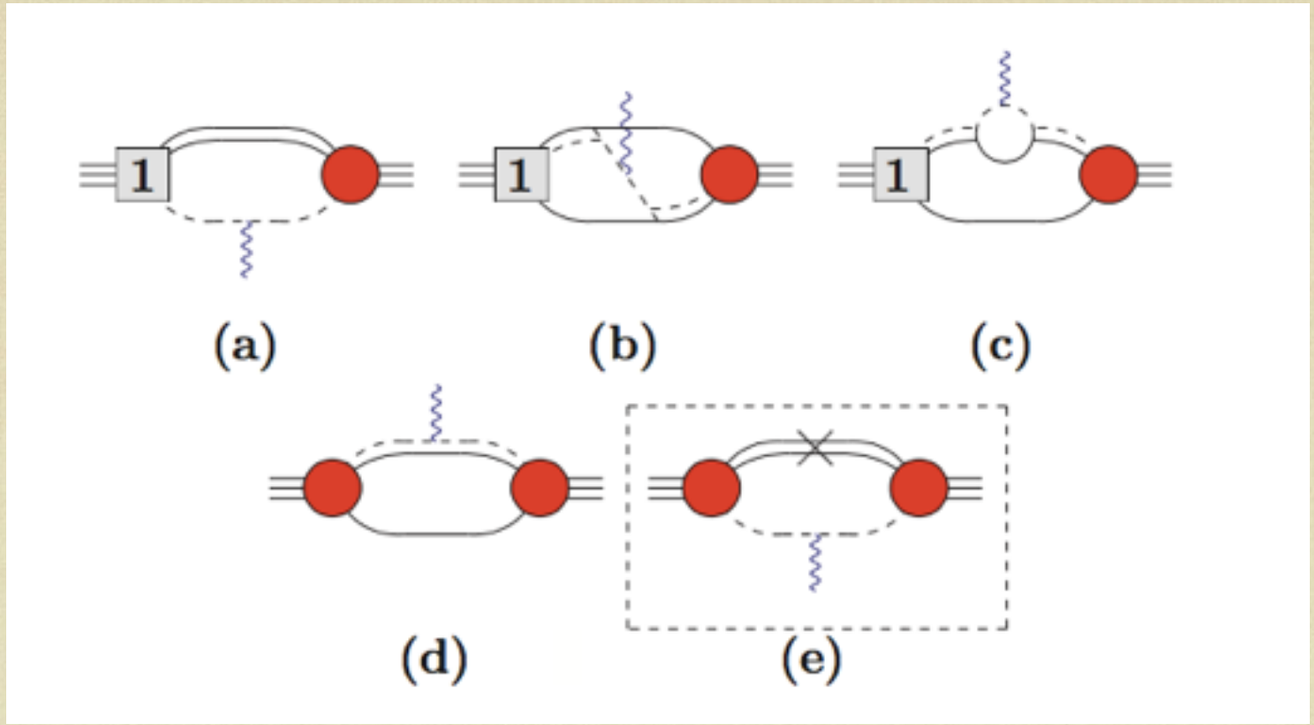

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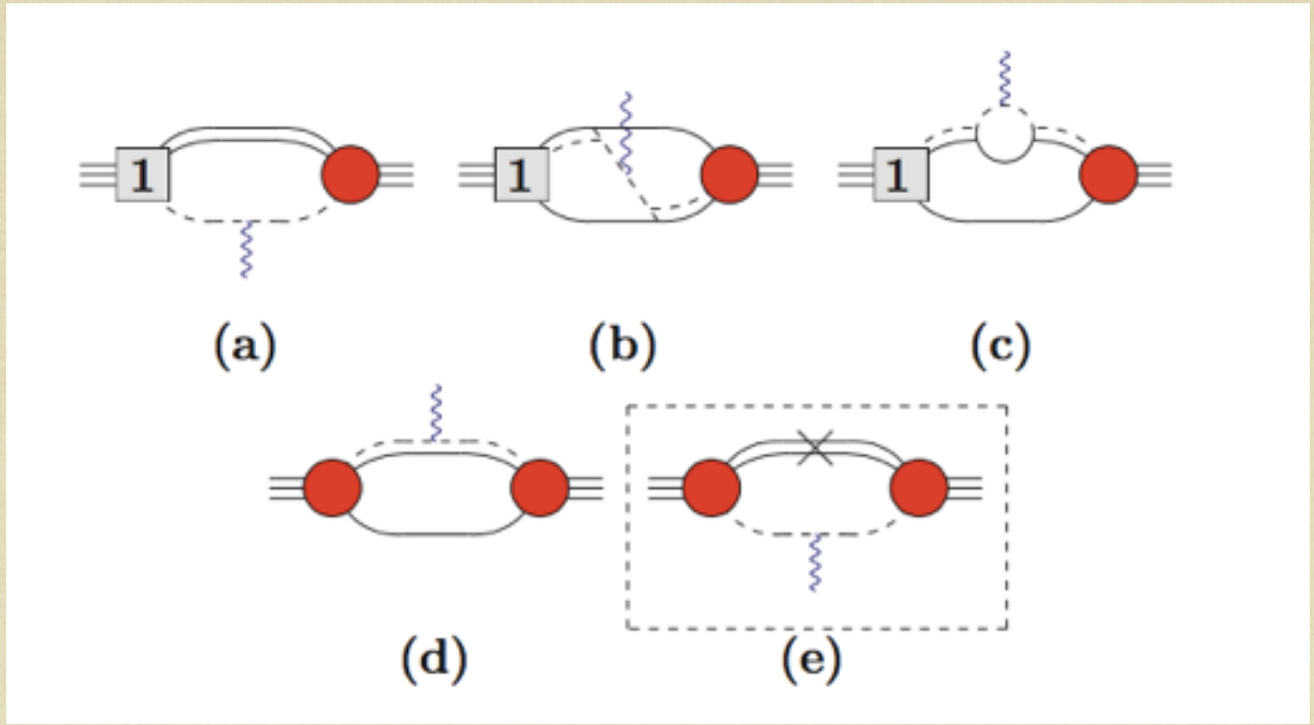

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LO **NLO** **NNLO**

Experiment:

$$\langle r^2_{^3\mathrm{H}}\rangle_{\mathrm{pt}} = 1.598(40) \text{ fm}$$

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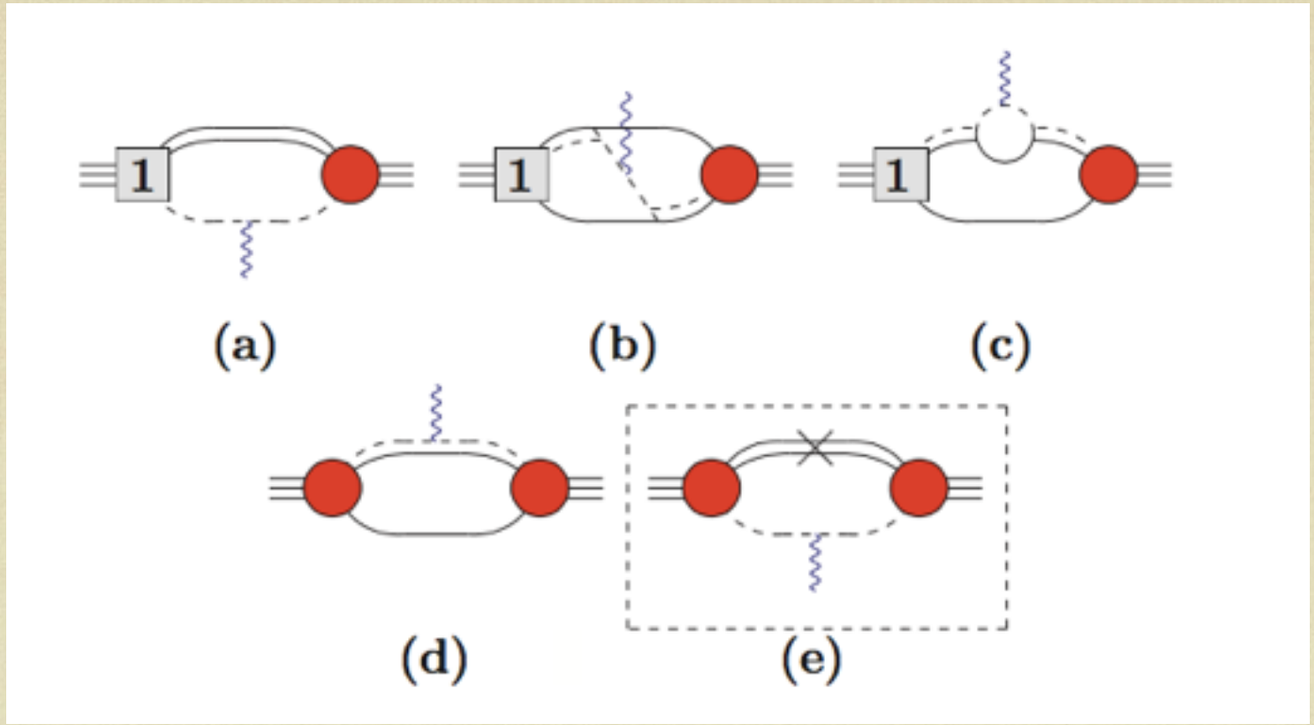

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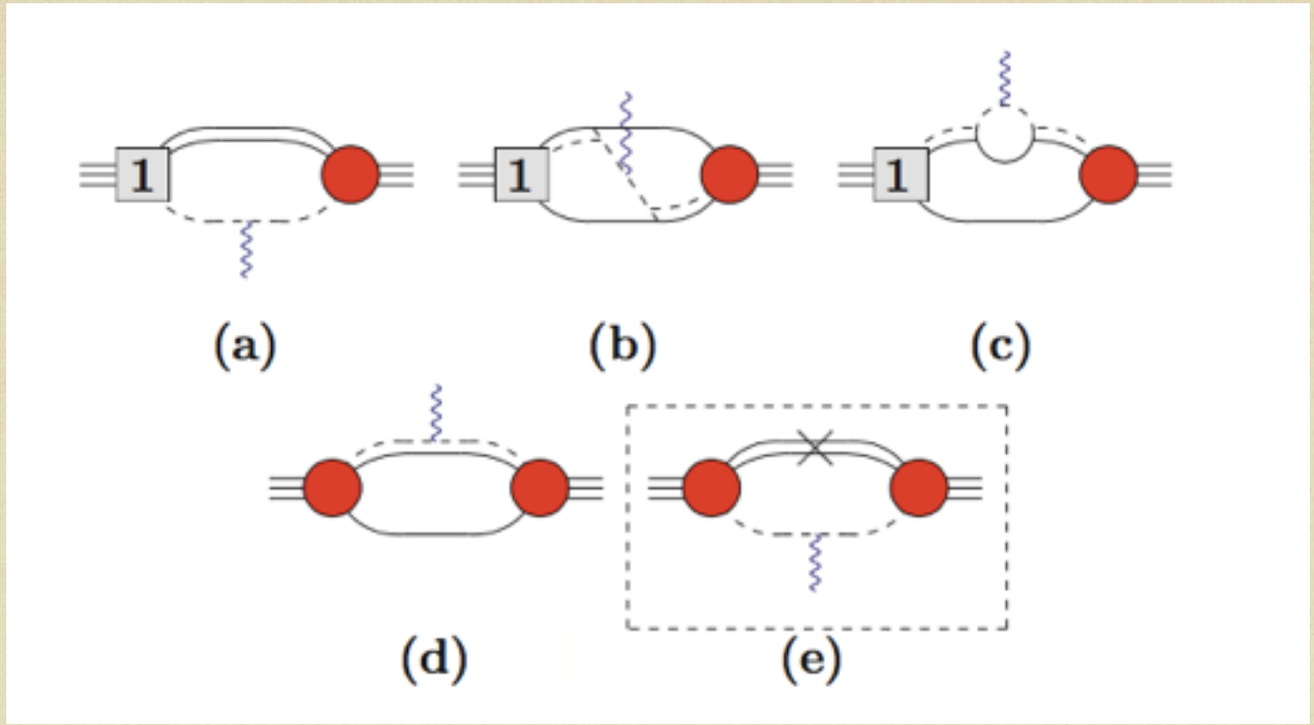

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- Need to know ranges for ^{11}Li, ^{14}Be, ^{22}C; estimates mostly move EFT prediction closer to data.

Photodissociation of trimers

Acharya, Hagen, Hammer, Phillips , in preparation

- Look at E1 dissociation of Borromean core-neutron-neutron system into three particles

- Go to unitary limit $E_{nn}=E_{nc}=0$. Only scales are B and energy of outgoing particles

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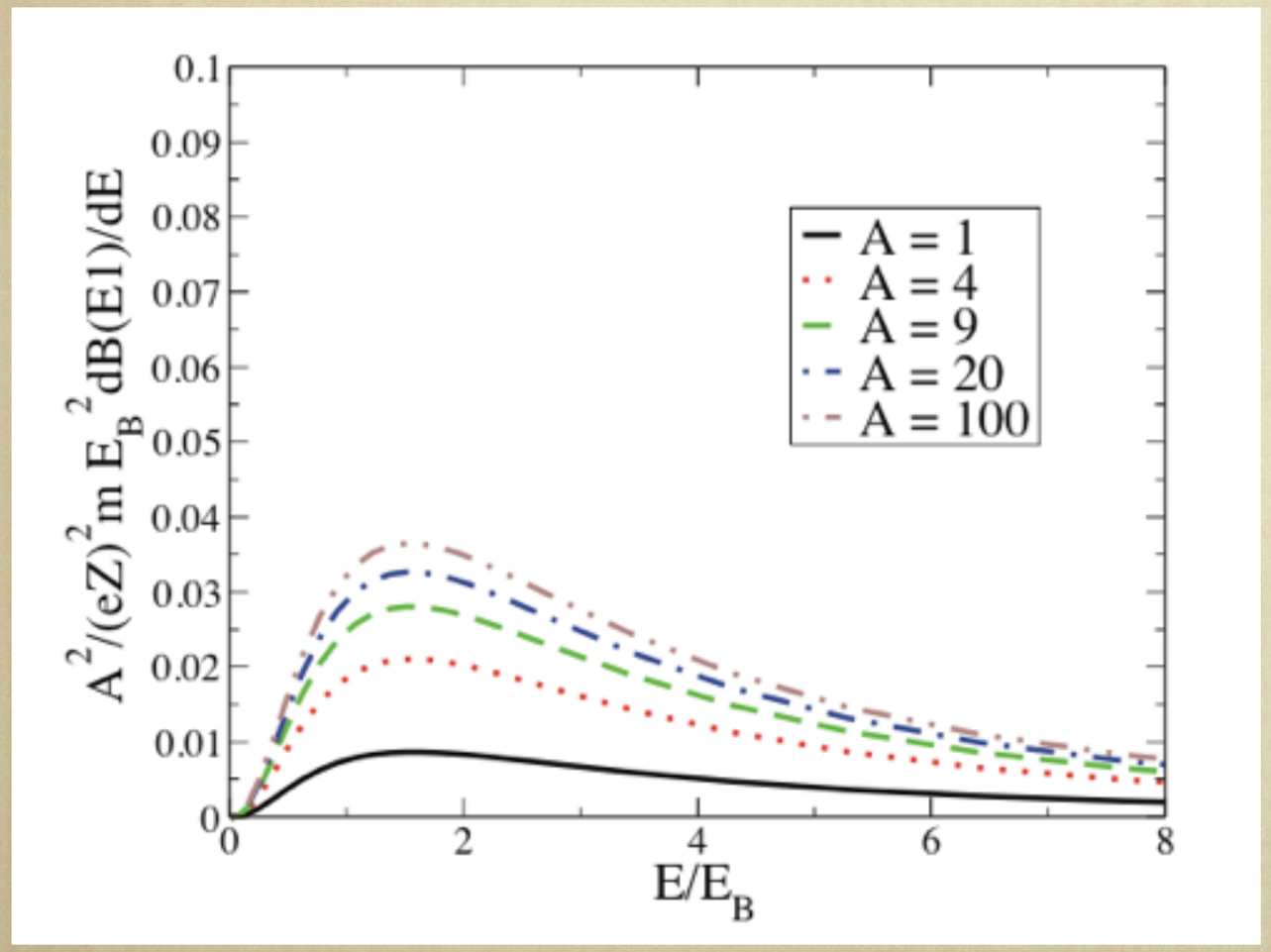

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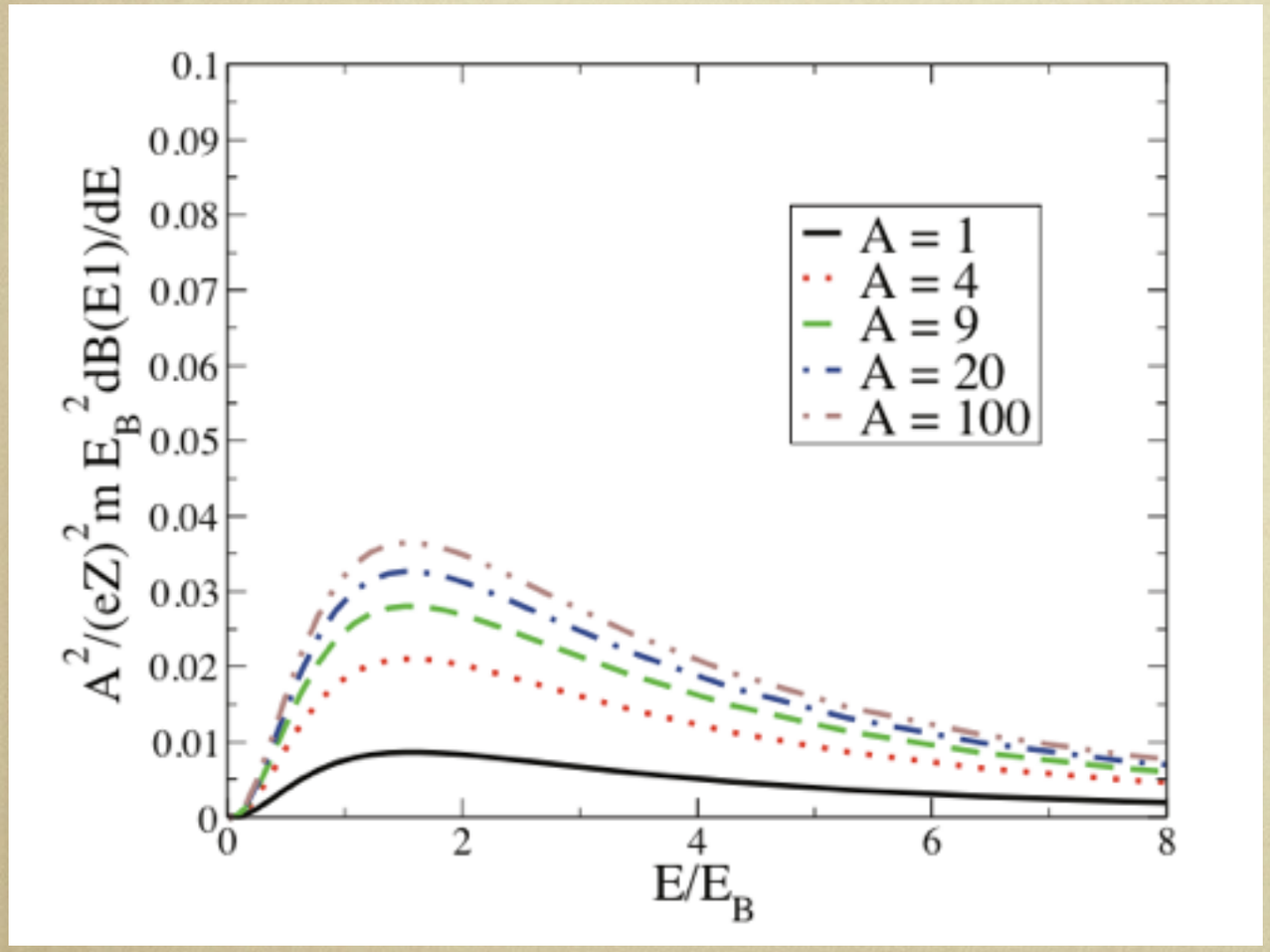

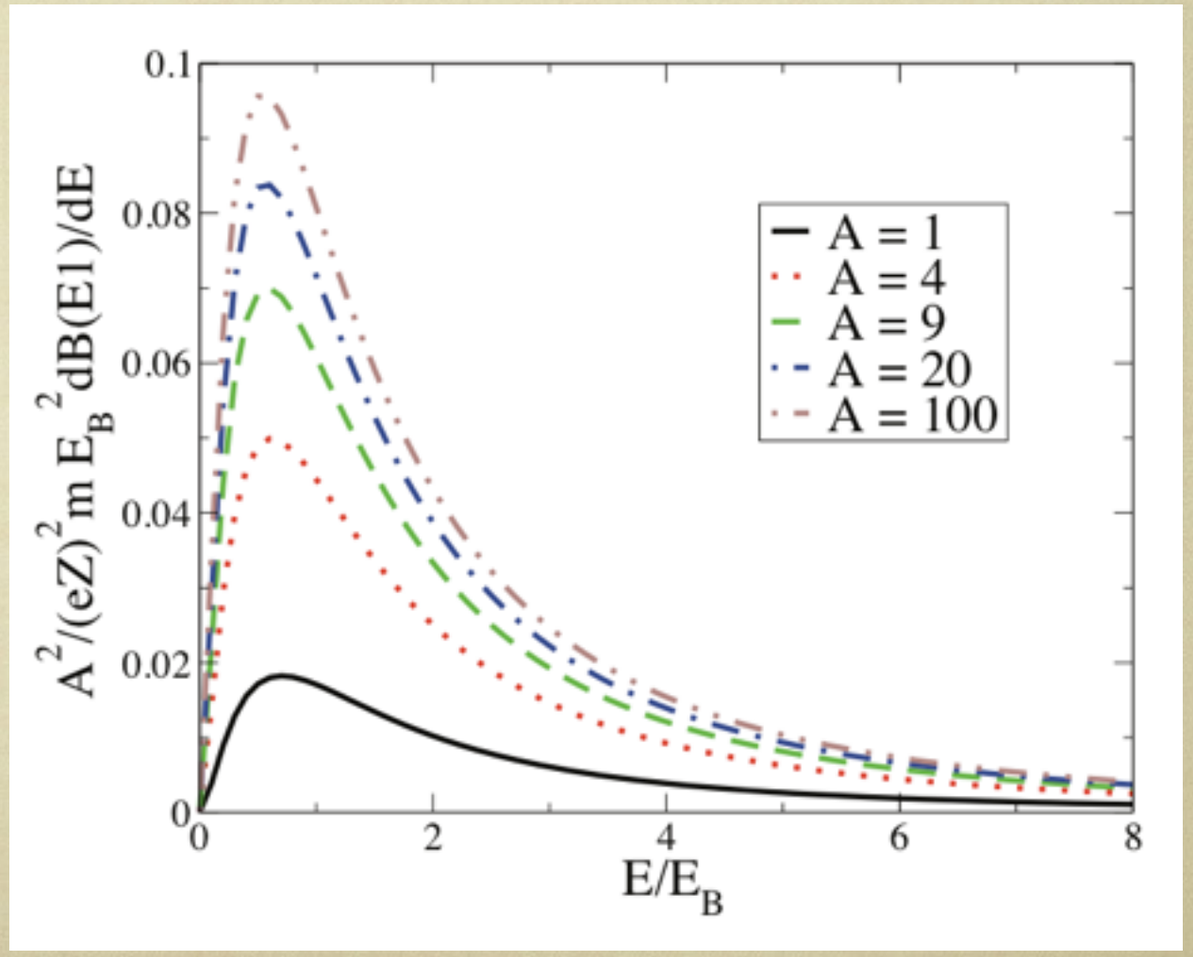

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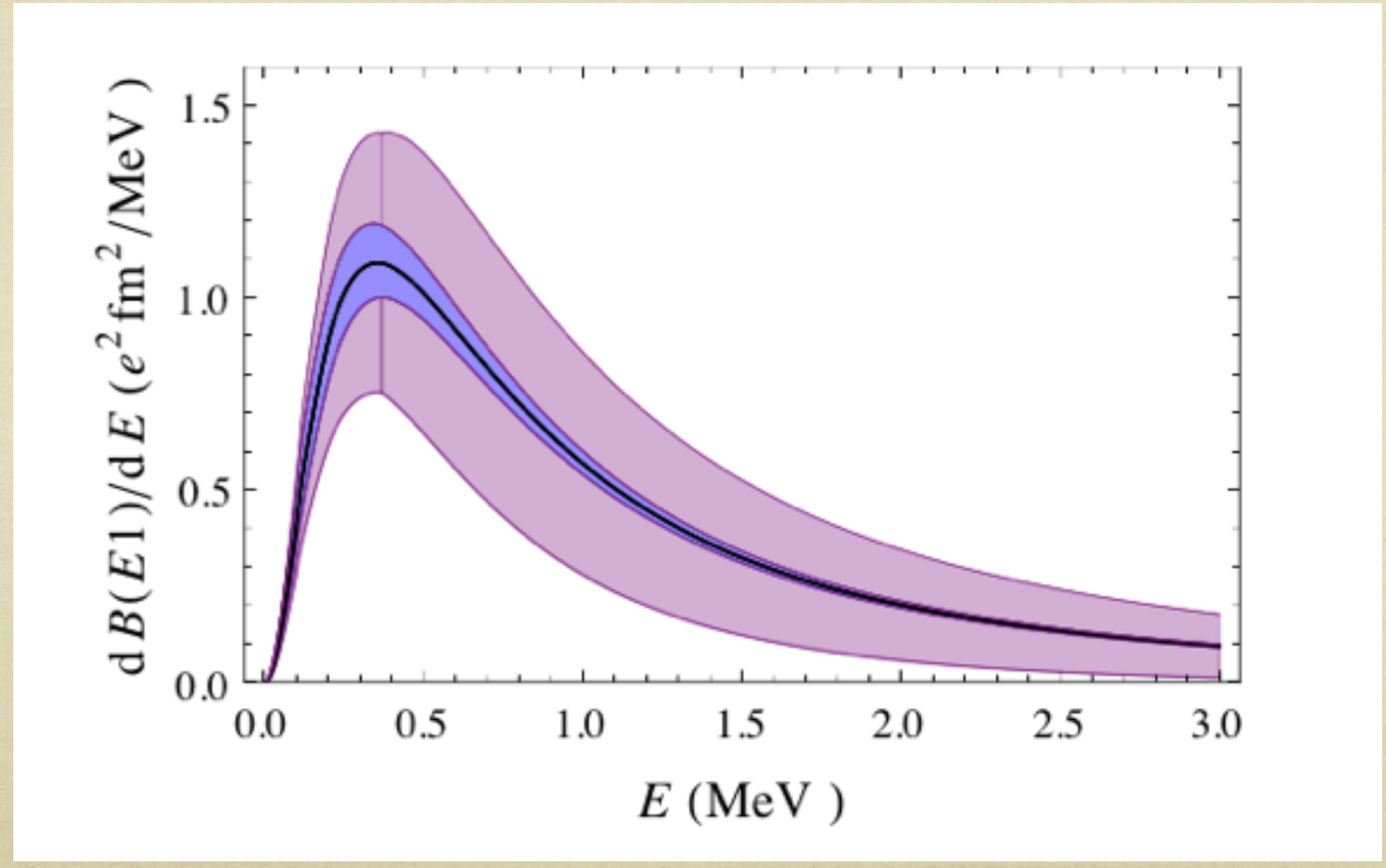

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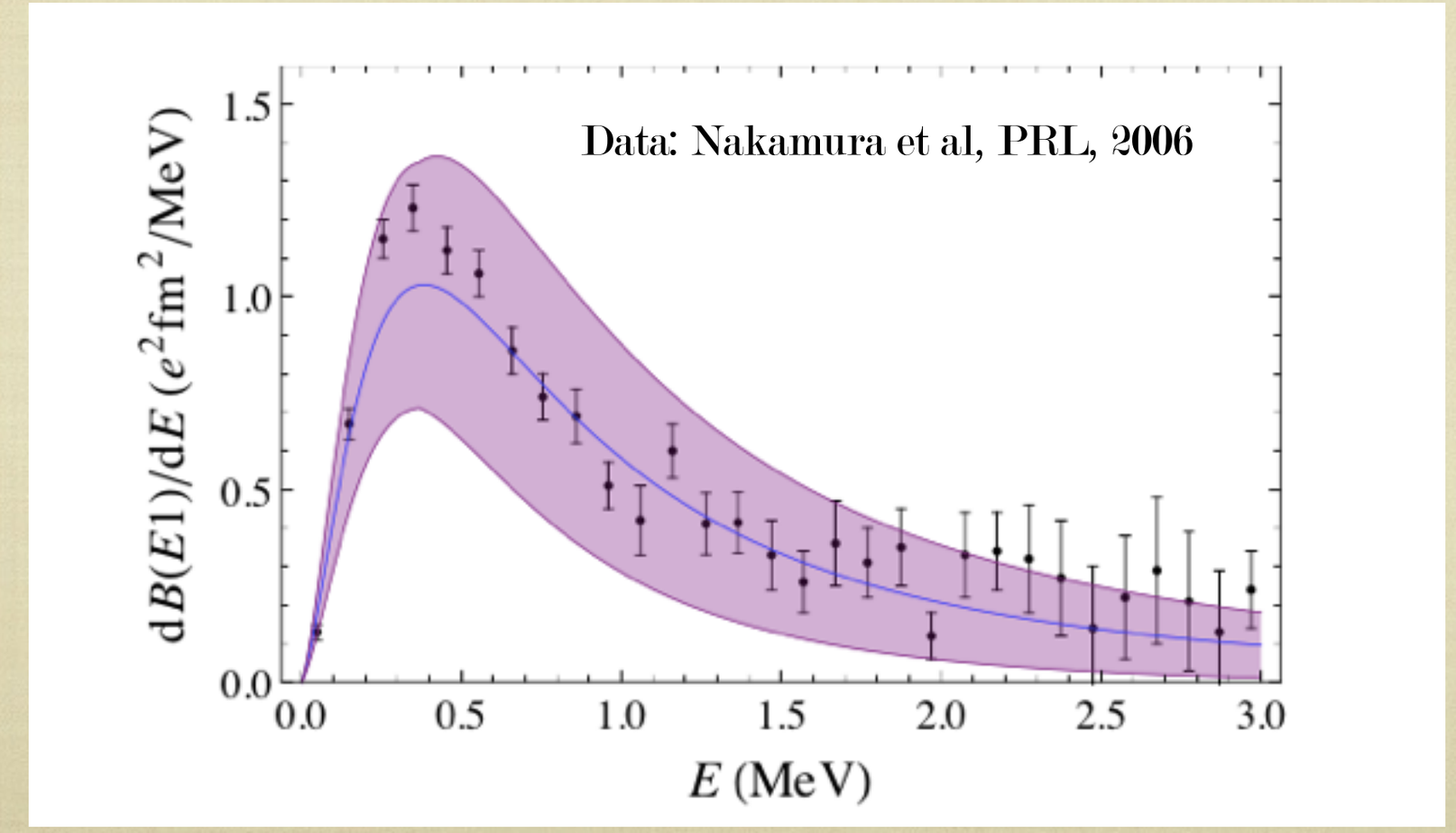

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Conclusion

- Universality: quantum few-body systems with $R \ll |a|$ that differ in scale by orders of magnitude exhibit the same correlations

Correlations between three- and four-body systems…and beyond

- Short-range EFT: expand observables in r/a, kr

- Two-body: compute matter radii, photodissociation cross sections

- Three-body: halo radii in terms of B, E_{nc}, E_{nn}: ^{3}H (NNLO), ^{22}C (LO)

- Photodissociation (aka Coulomb excitation): do halo nuclei approach the E1 response of universal trimers?

- Range effects are sizable: do such systems still exhibit universality?

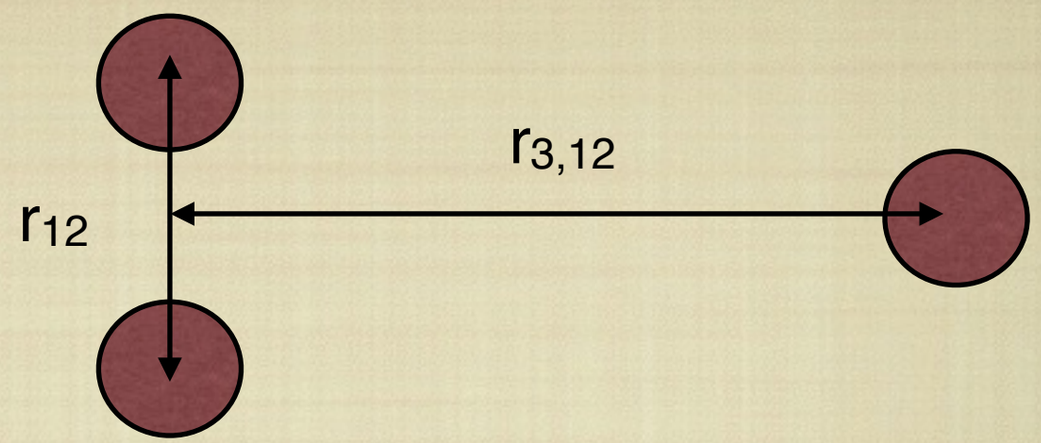
- p-waves are another (controllable) source of universality violation

Backup slides: Efimov effect

Efimov effect

Efimov, 1970

Consider three-body problem in limit $R \to 0$, $|a| \to \infty$



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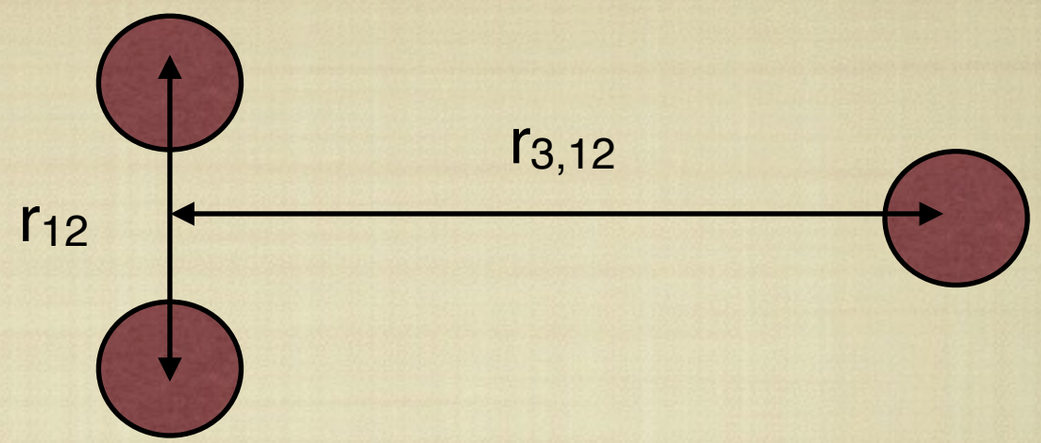

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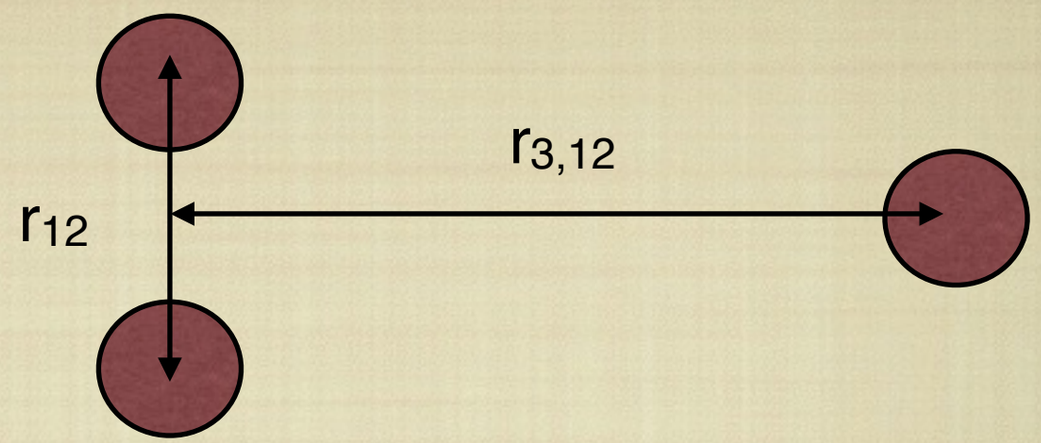

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$\tan \alpha = r_{3,12}/r_{12}$

3B Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{d\rho^2} + V_{\text{eff}}(\rho)\Psi(\rho) = E\Psi(\rho)$$

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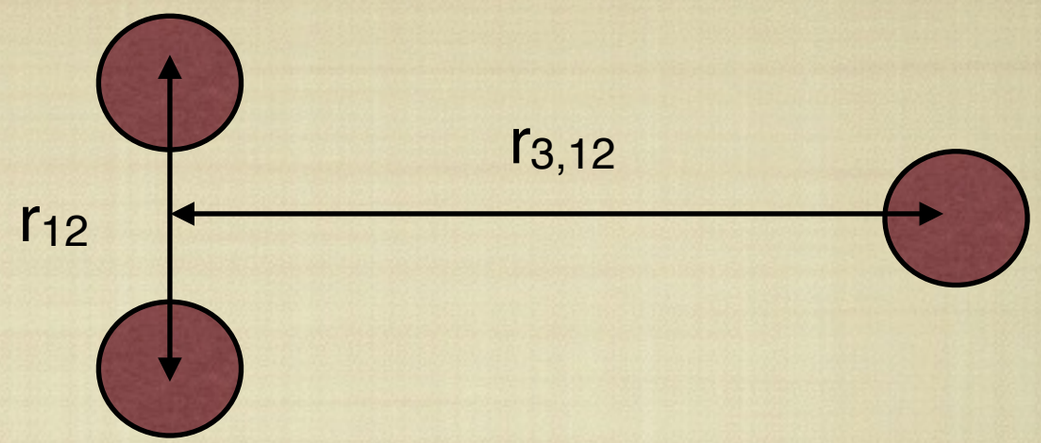

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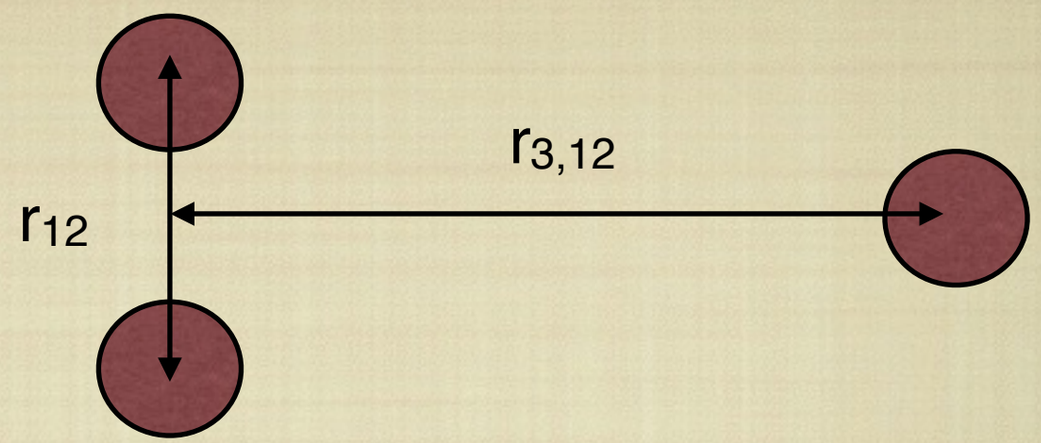

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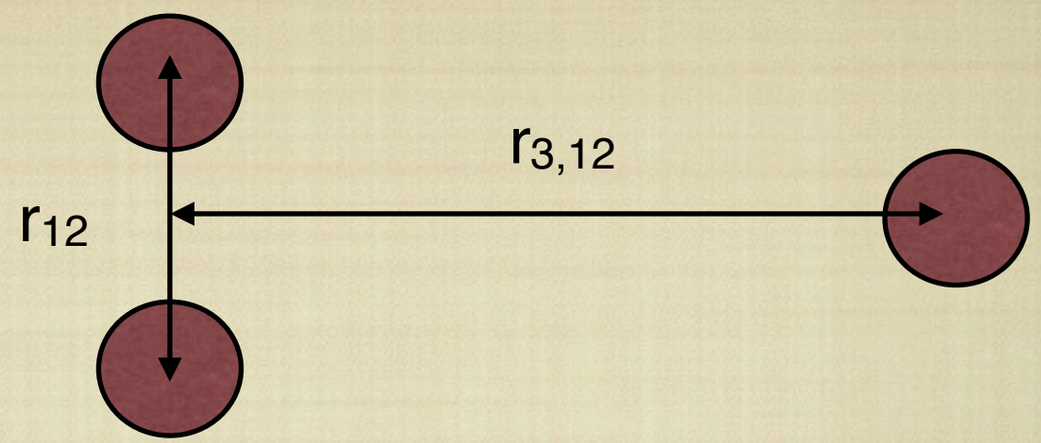

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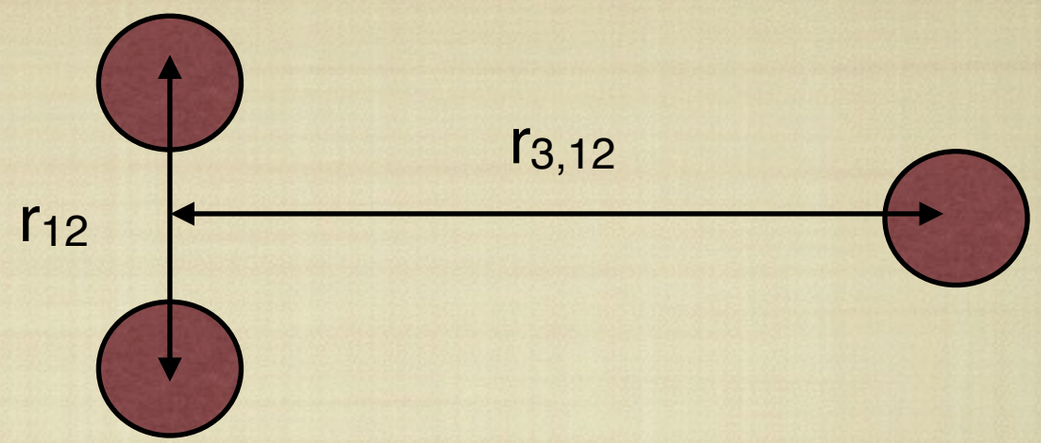

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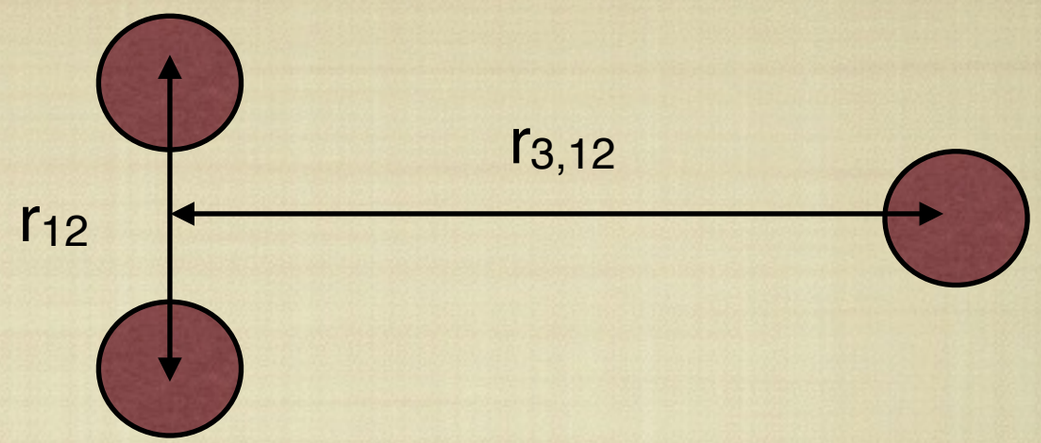

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- Energy of lowest state set by short-distance dynamics $\sim \dfrac{\hbar^2}{mR^2}$

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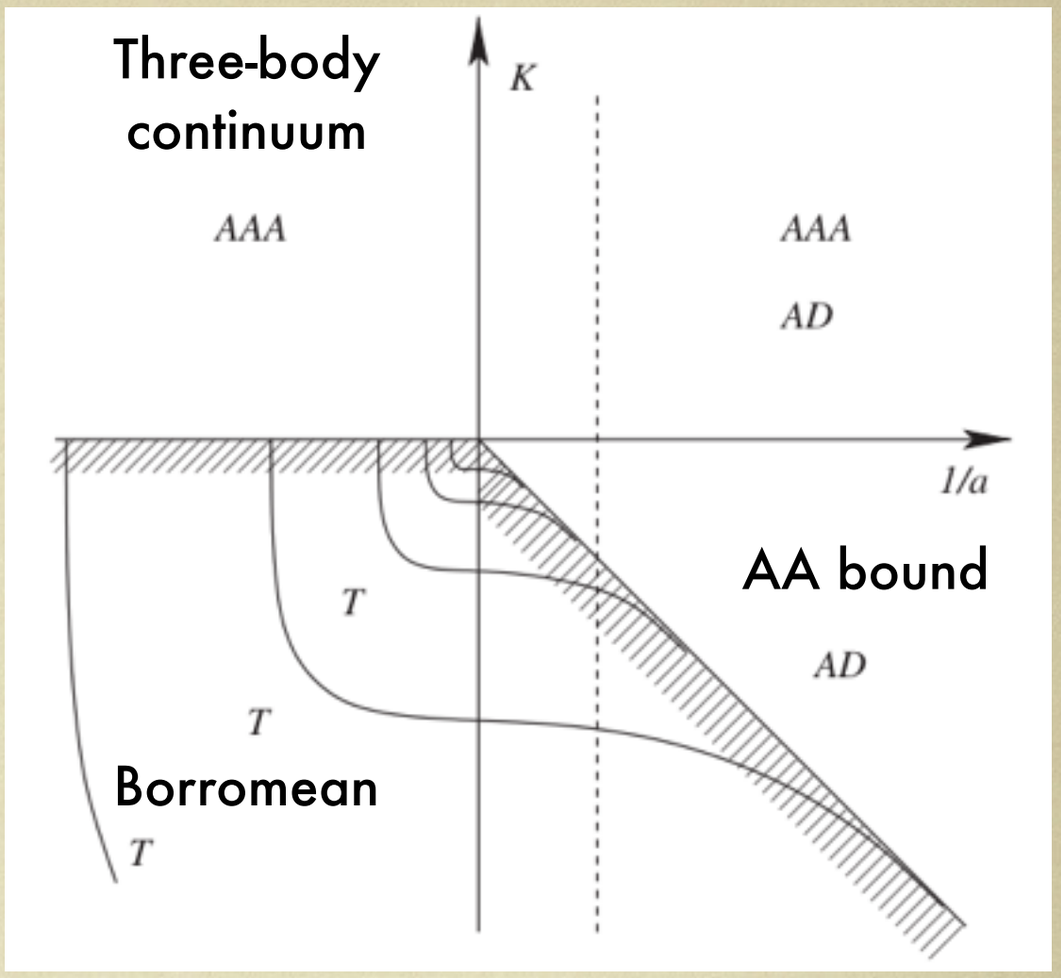
Efimov, Yad. Fiz., 1970



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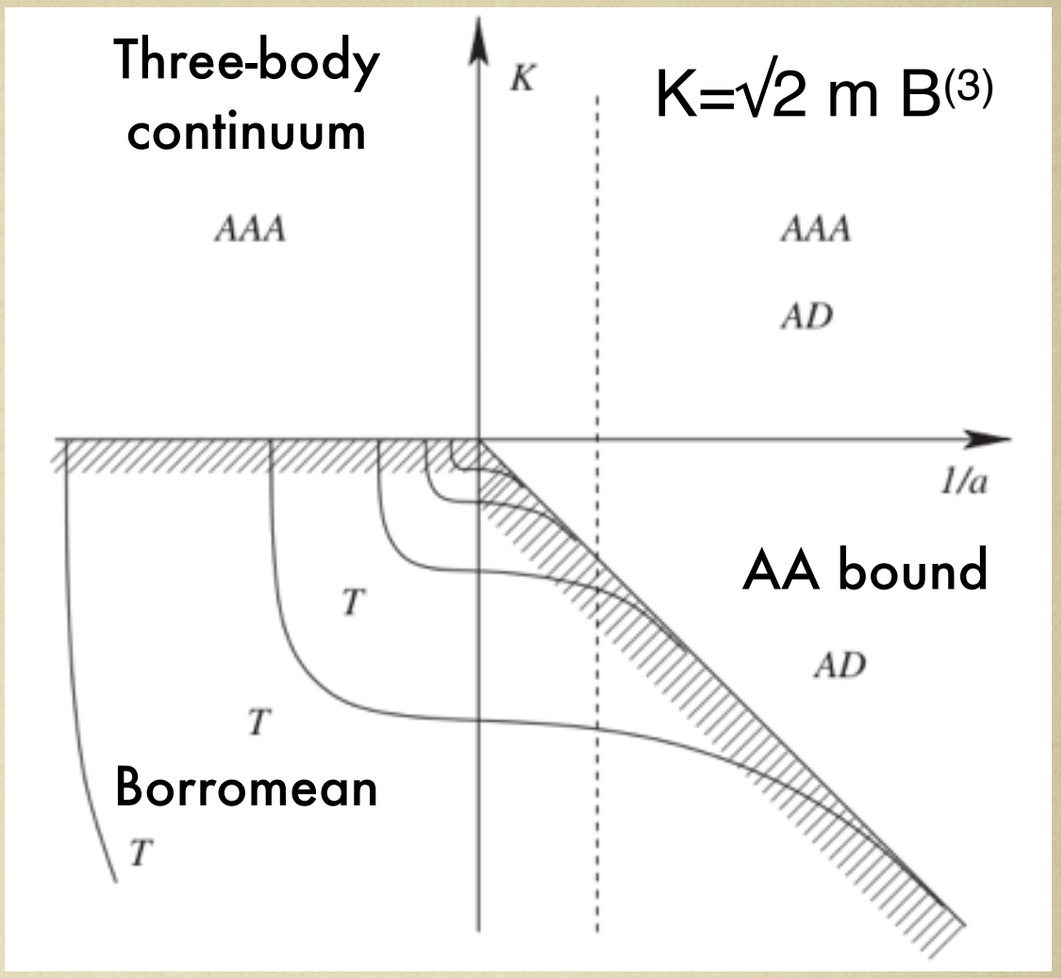

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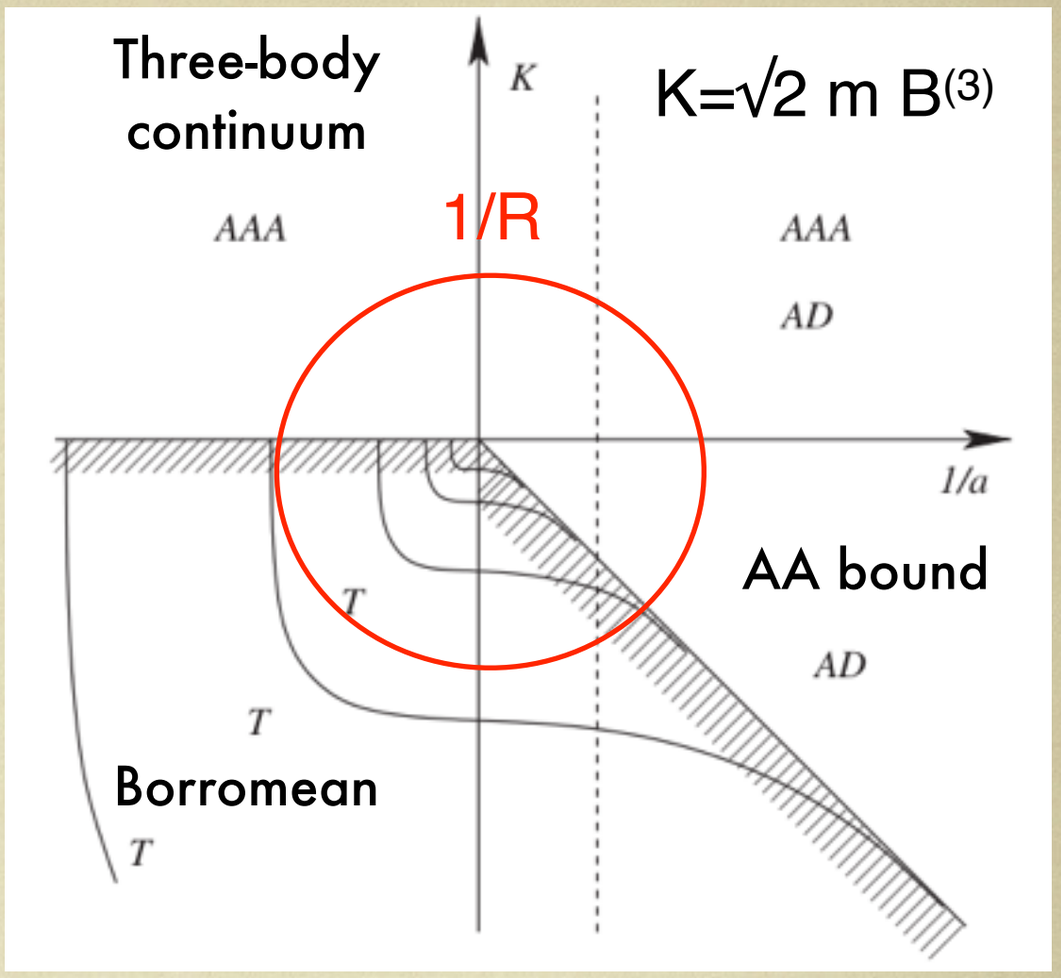

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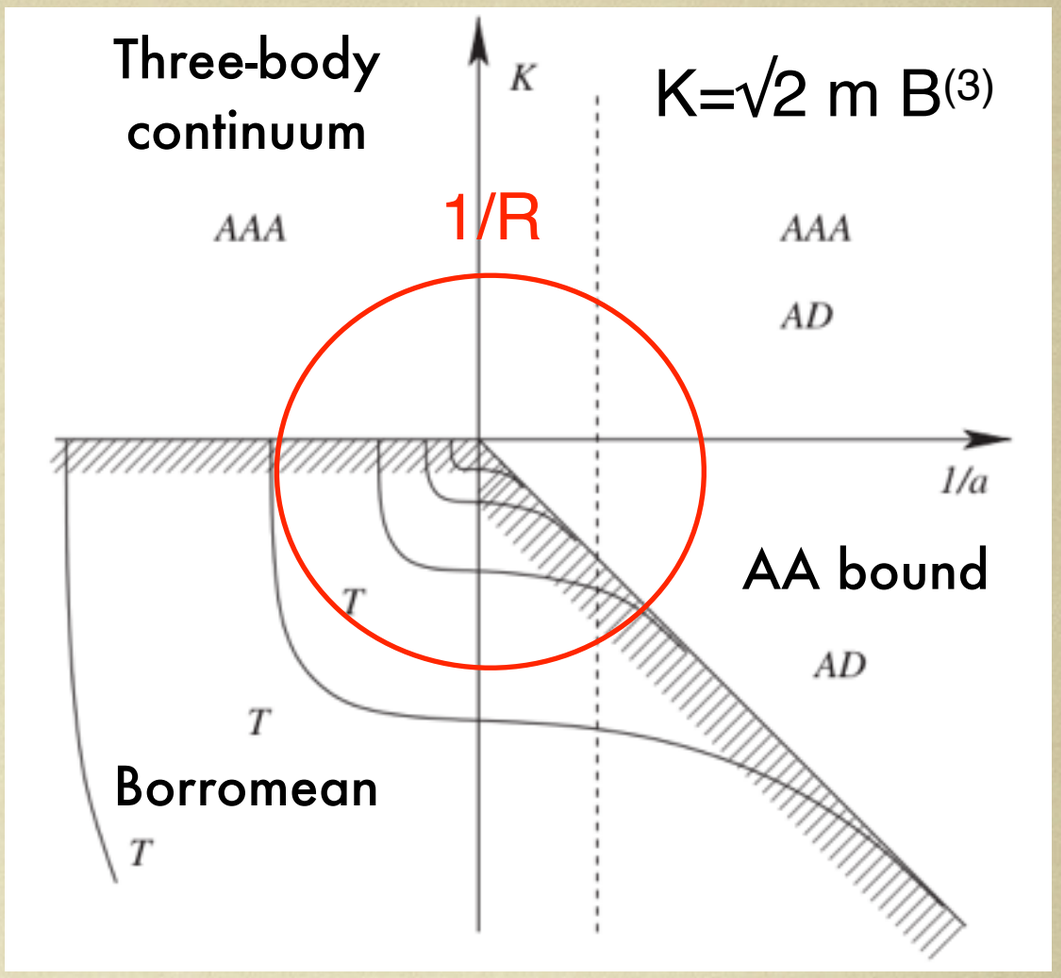

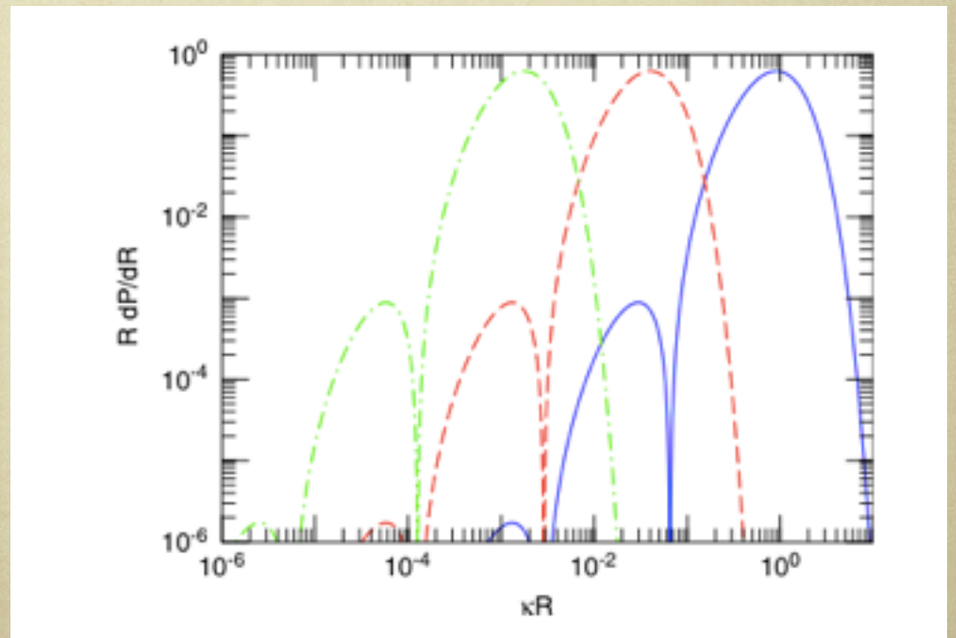

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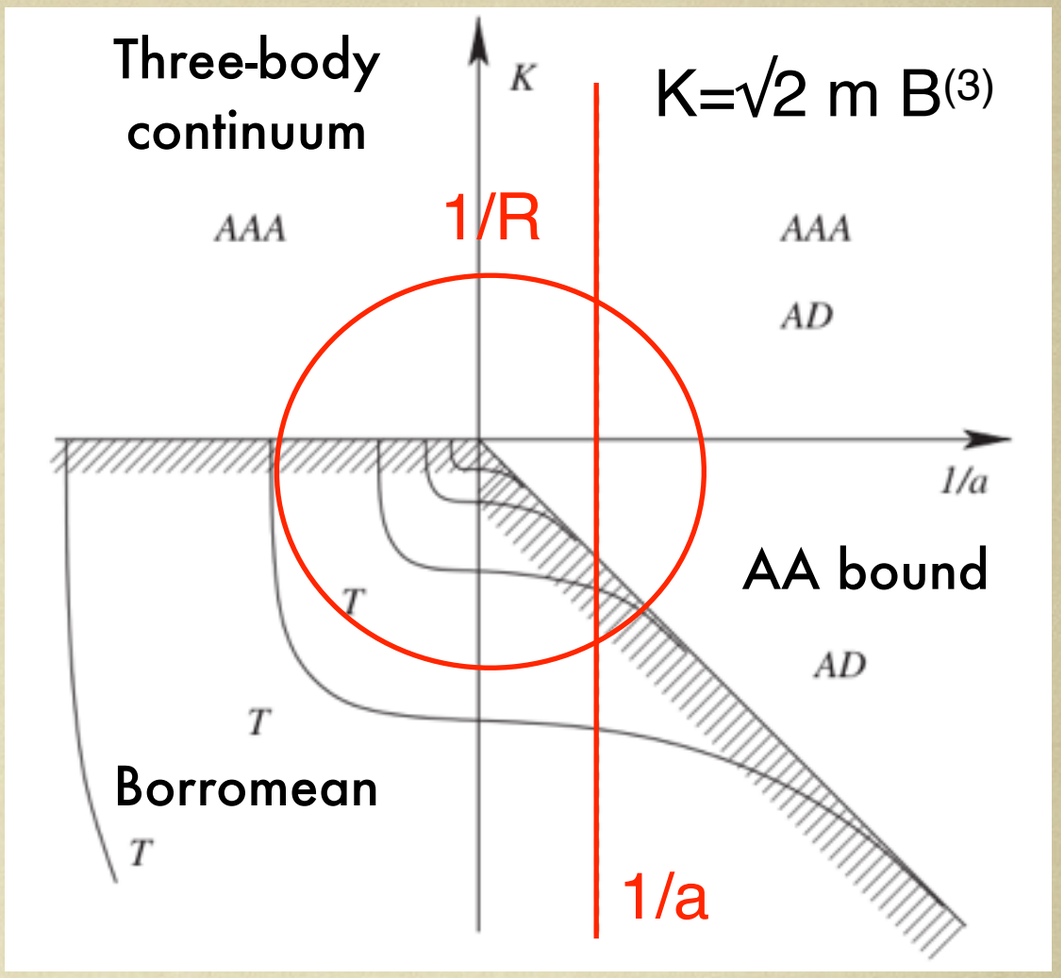

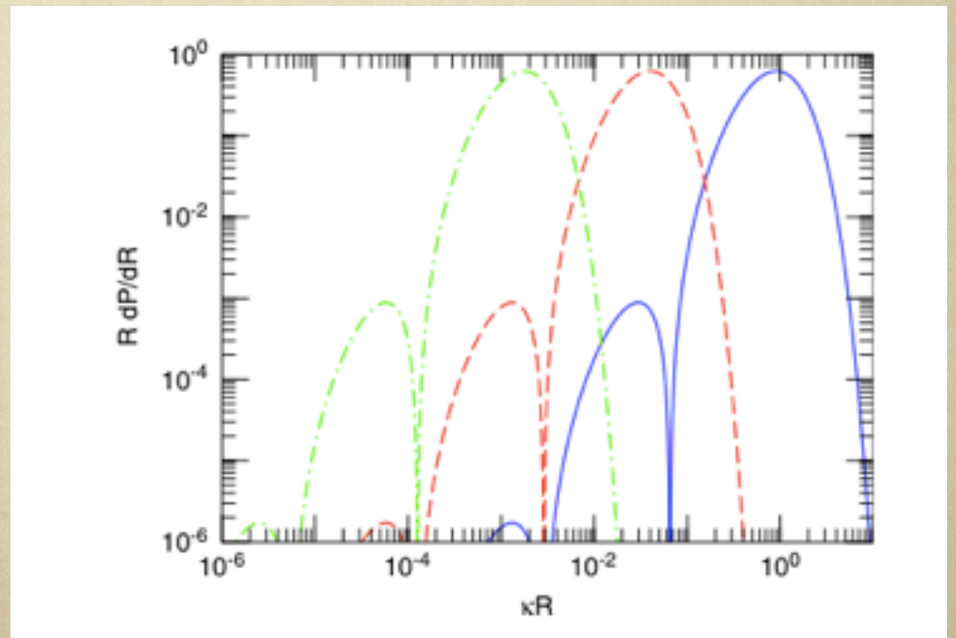

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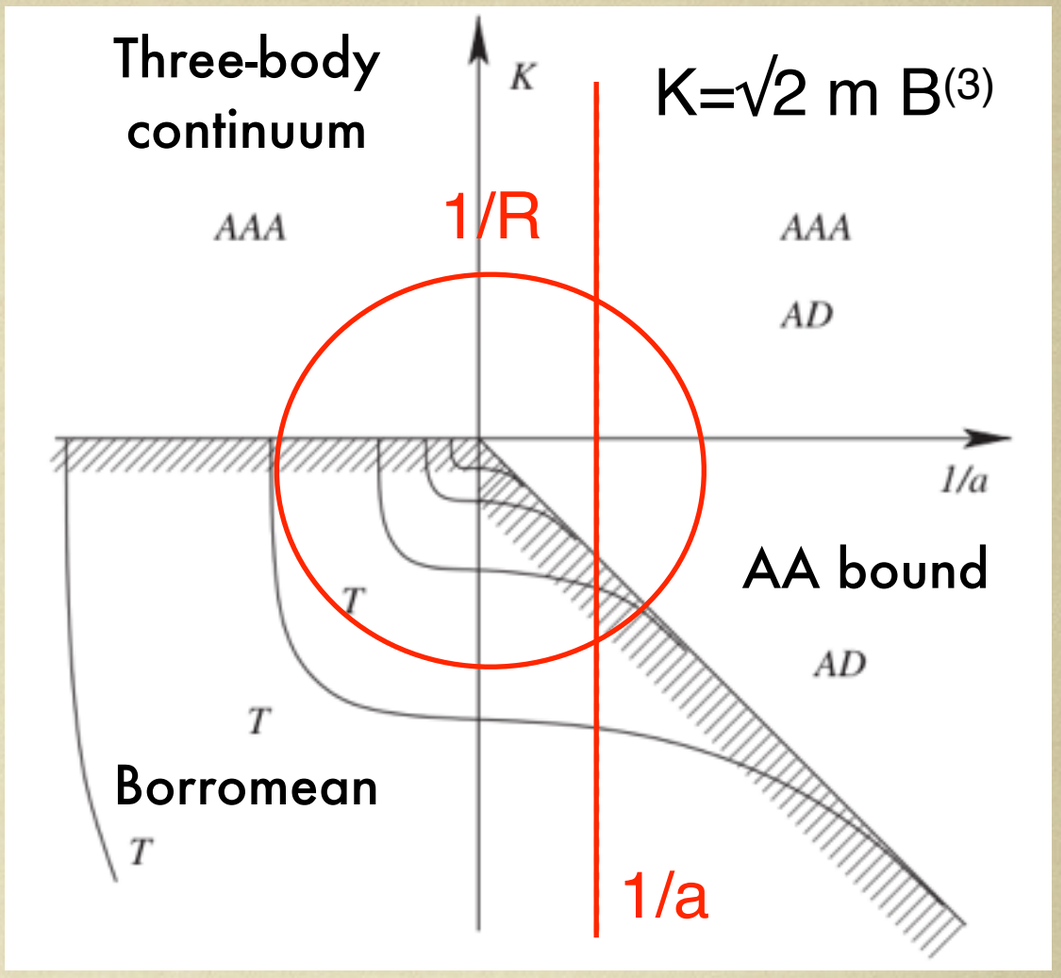

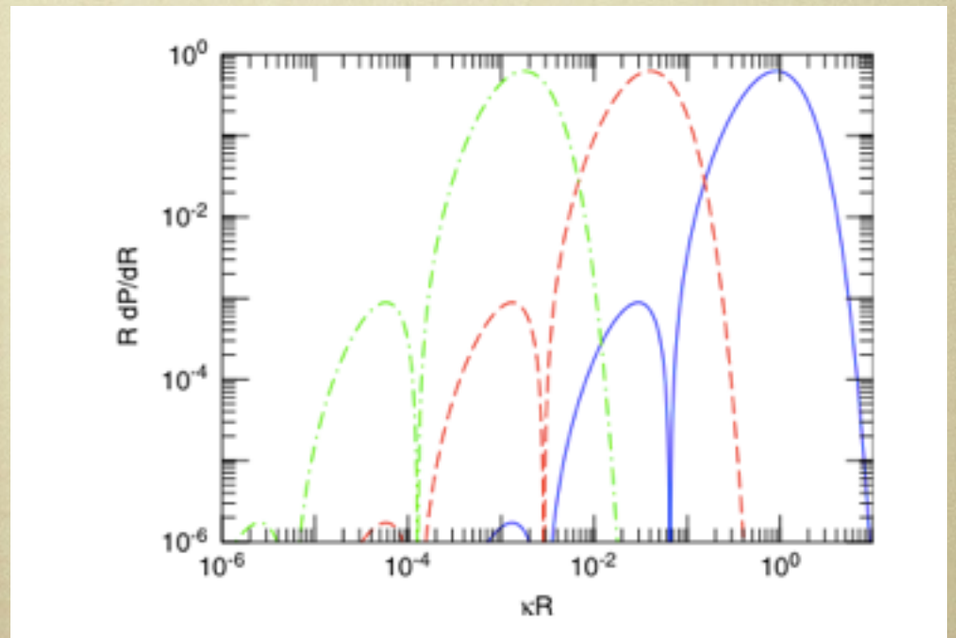

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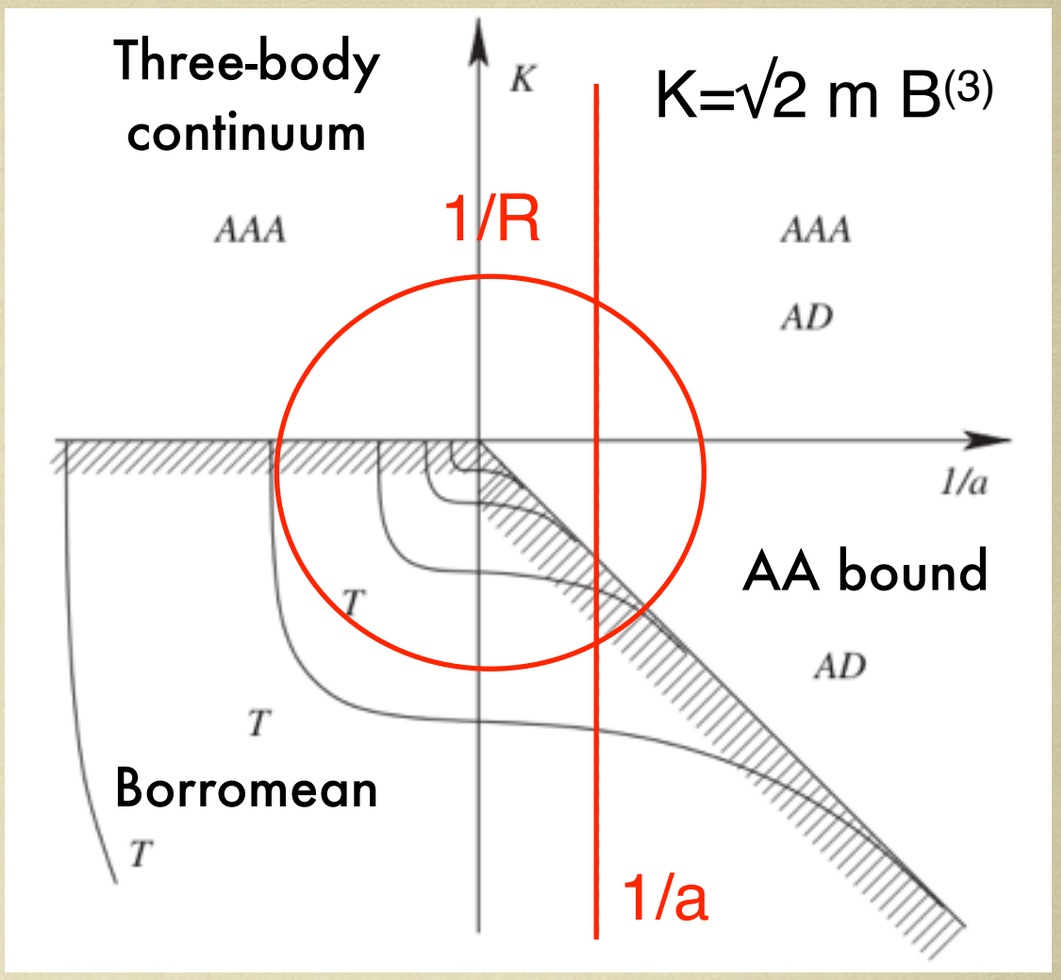
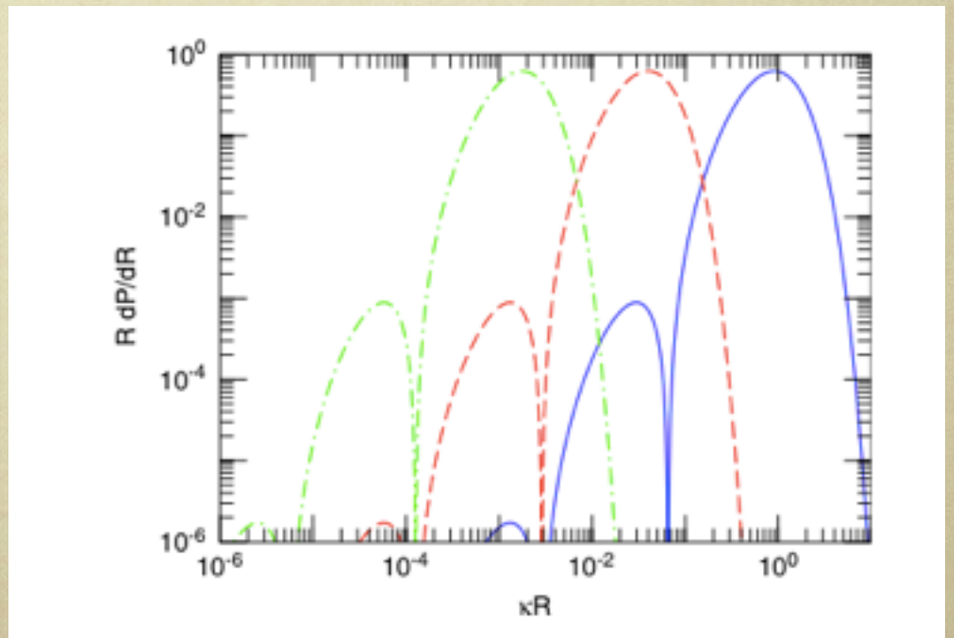
Observed in Innsbruck experiment, 2014





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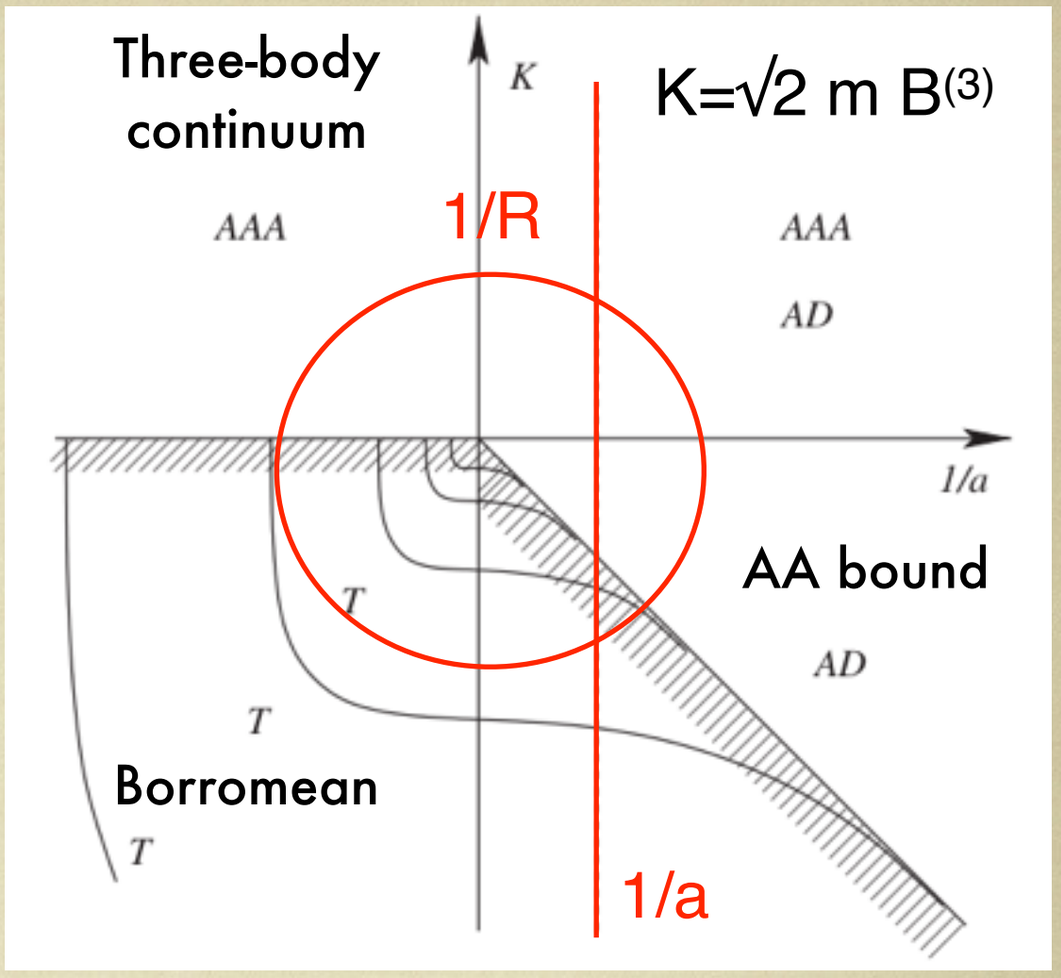

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Observed in Innsbruck experiment, 2014

- Correlations between different recombination features on an Efimov branch (or different branches): universal relations

Efimov states in halo nuclei?

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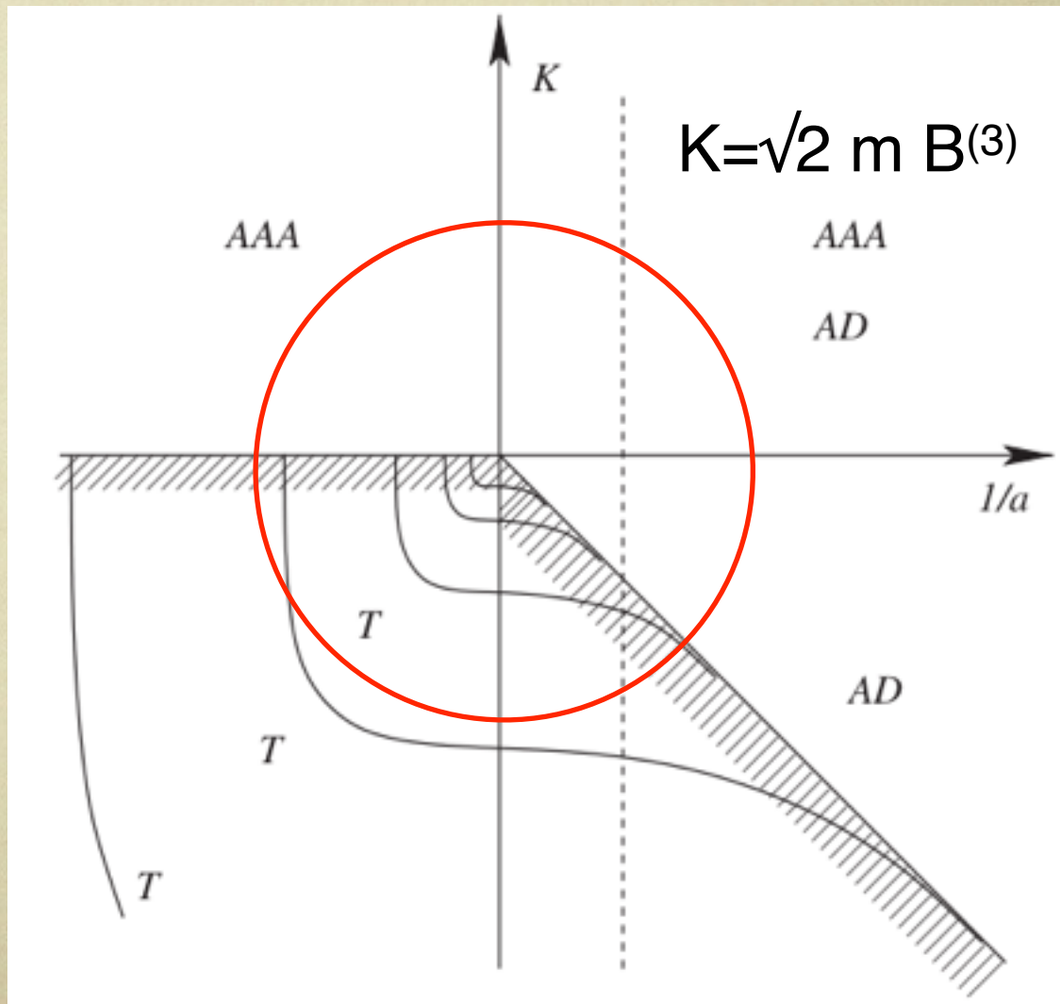

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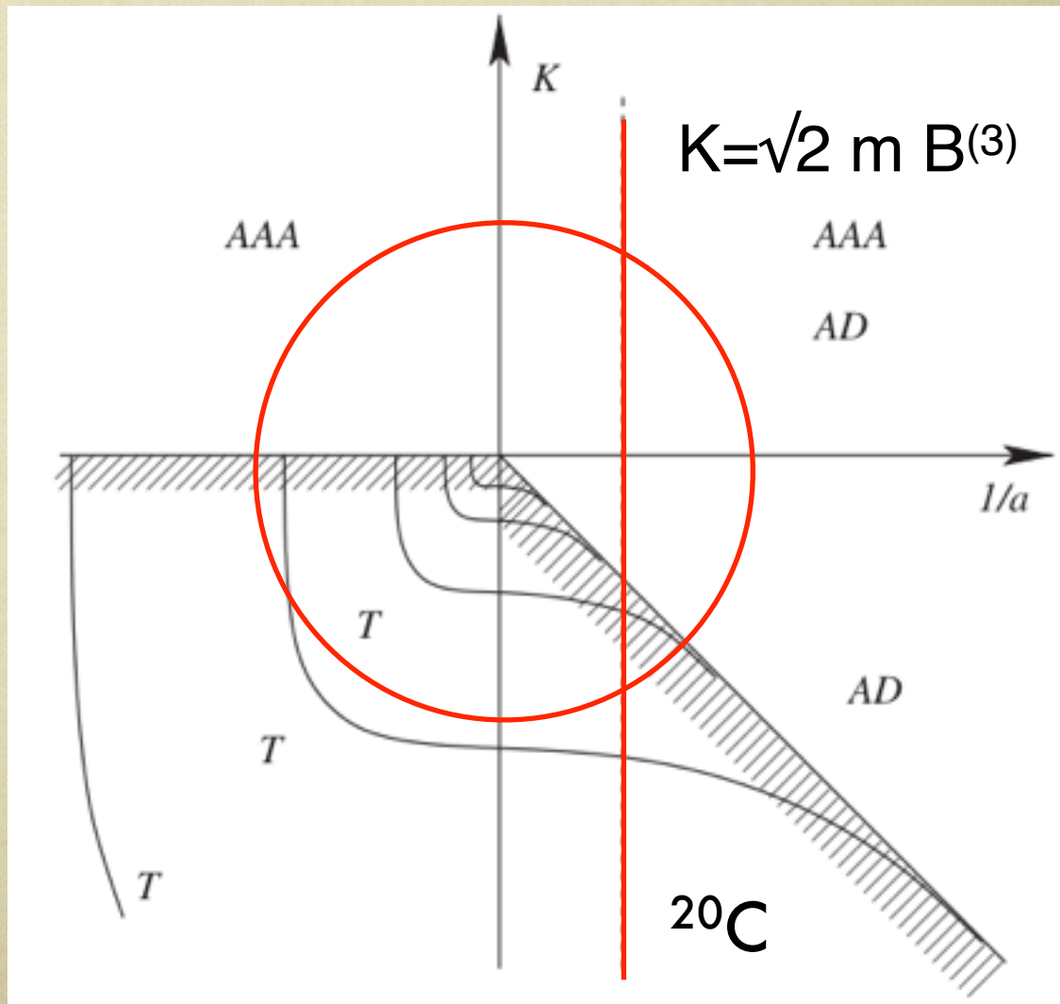

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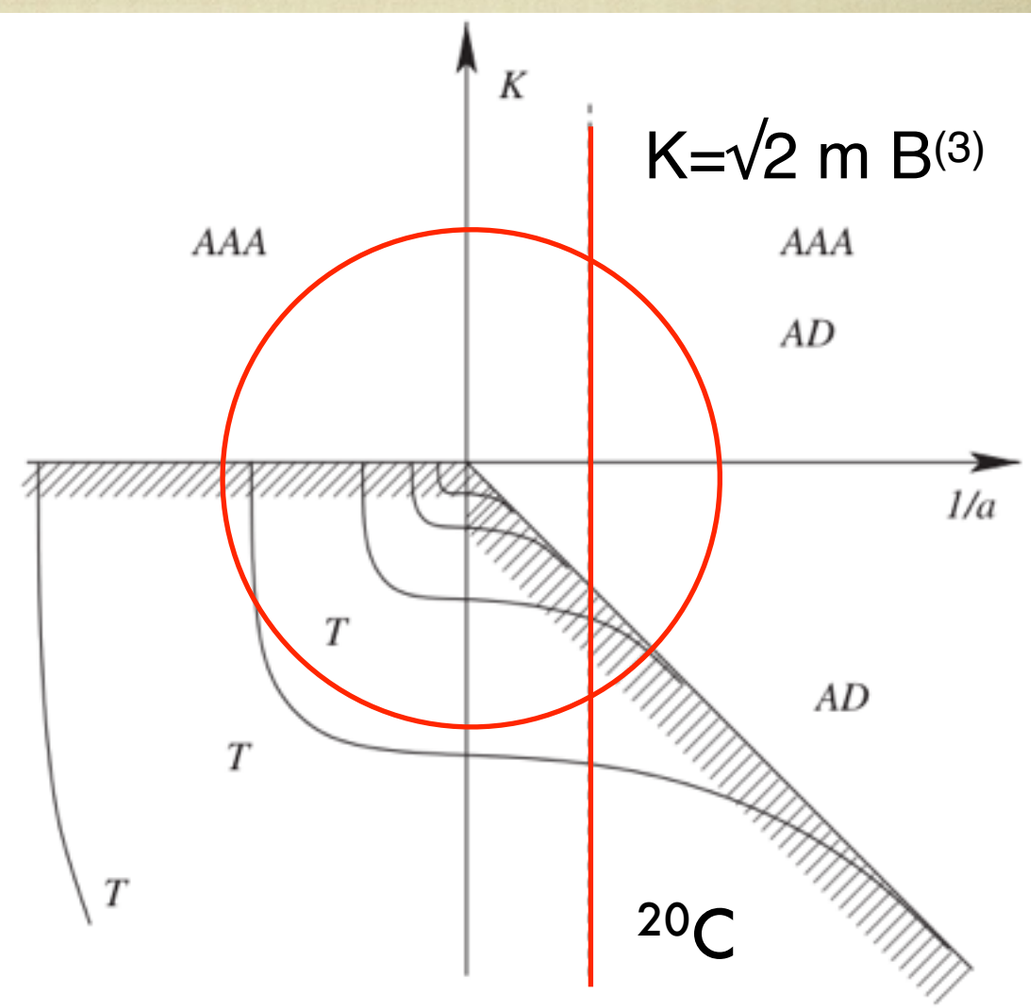

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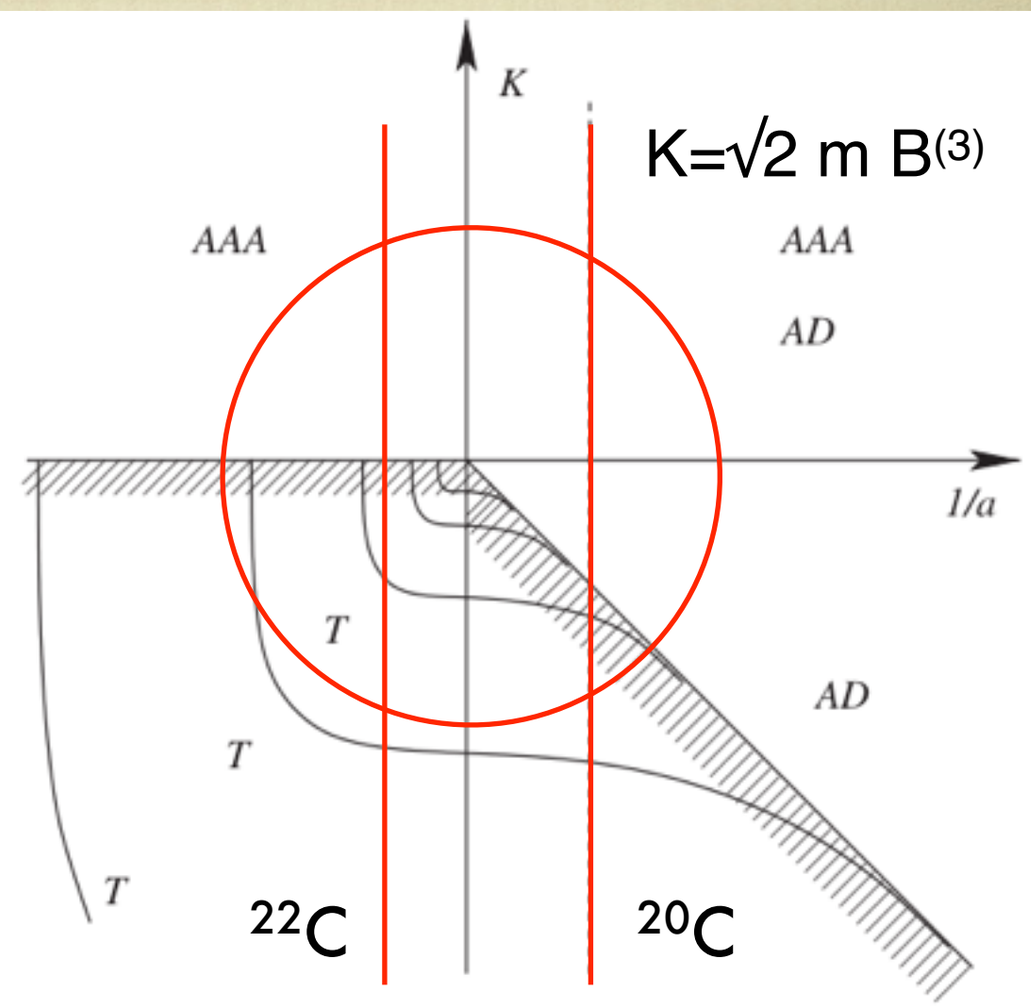

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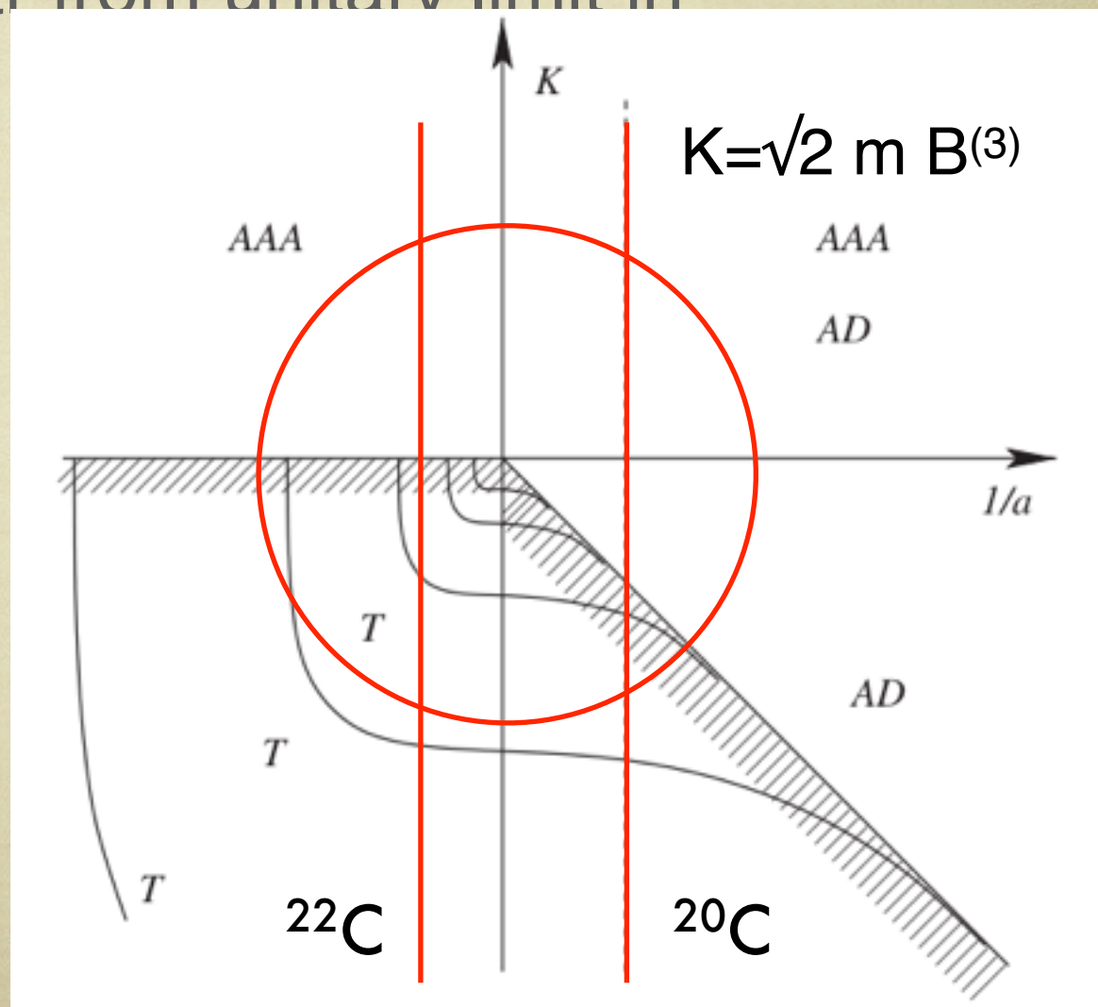

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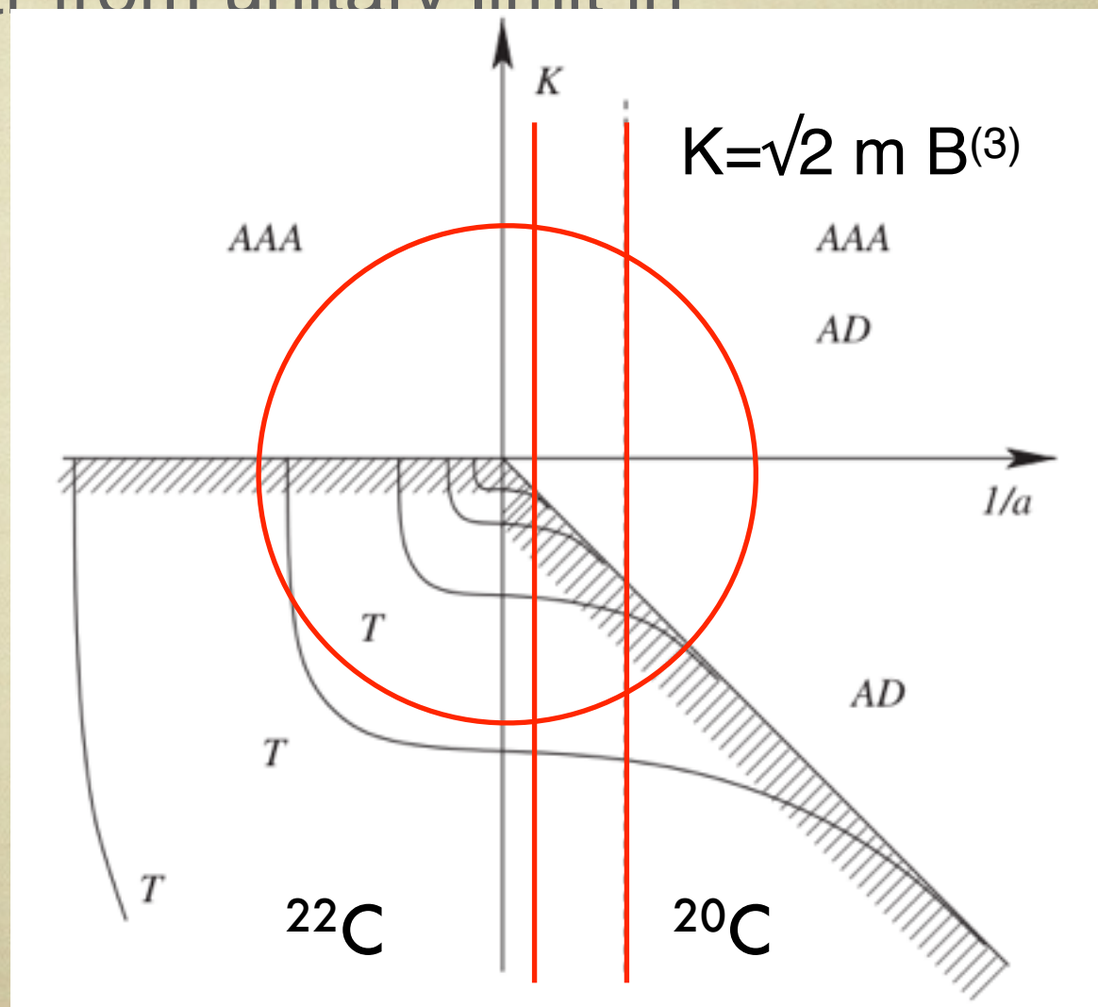

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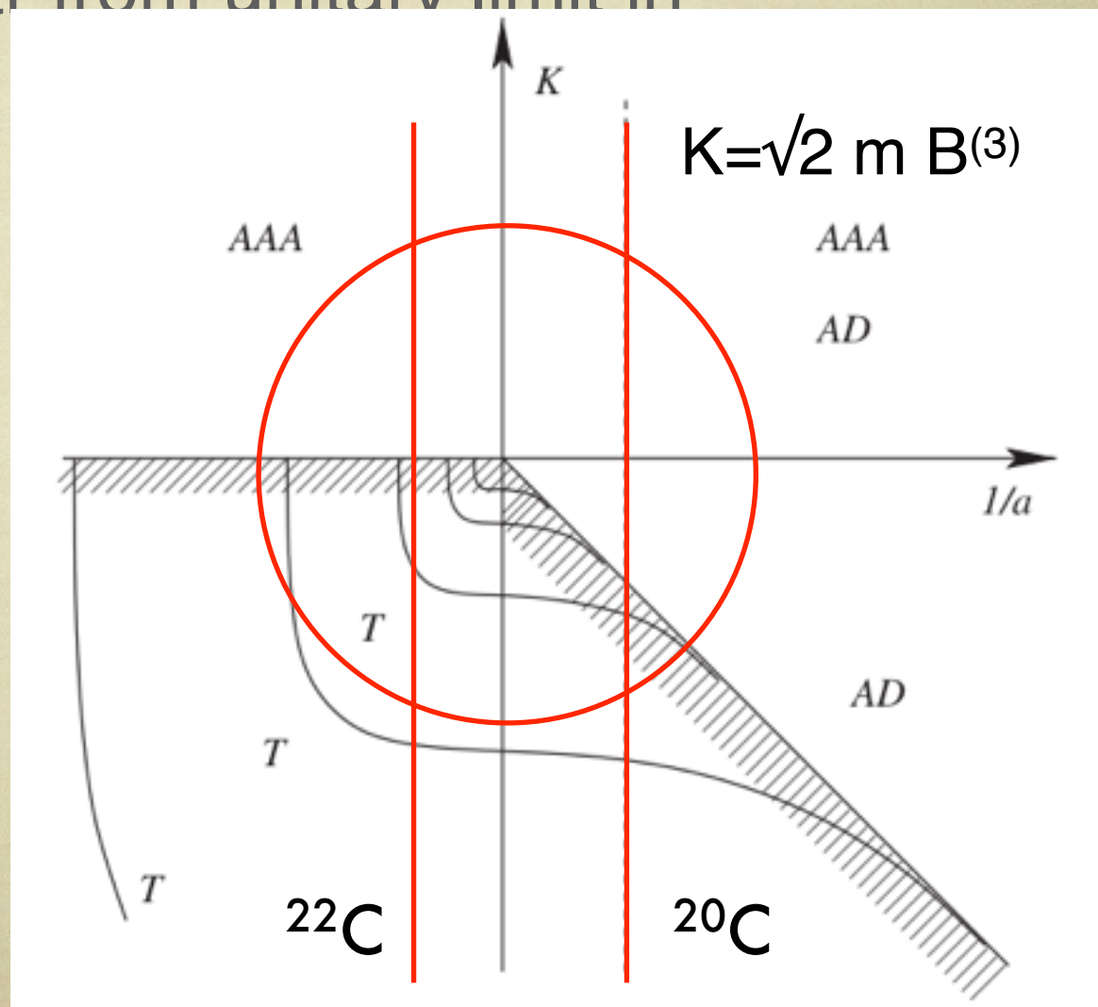

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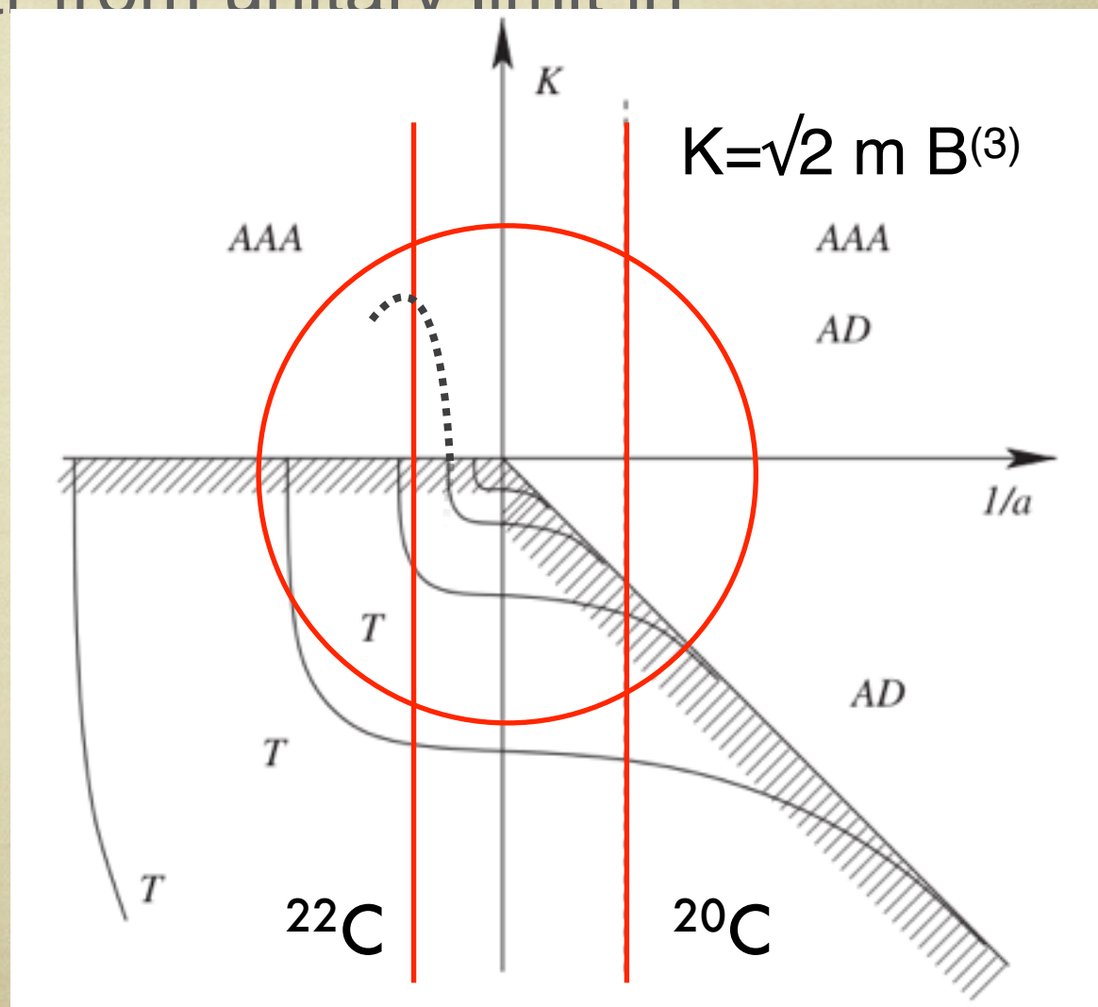
  Grigorenko & Zhukov, arXiv:1503.03186

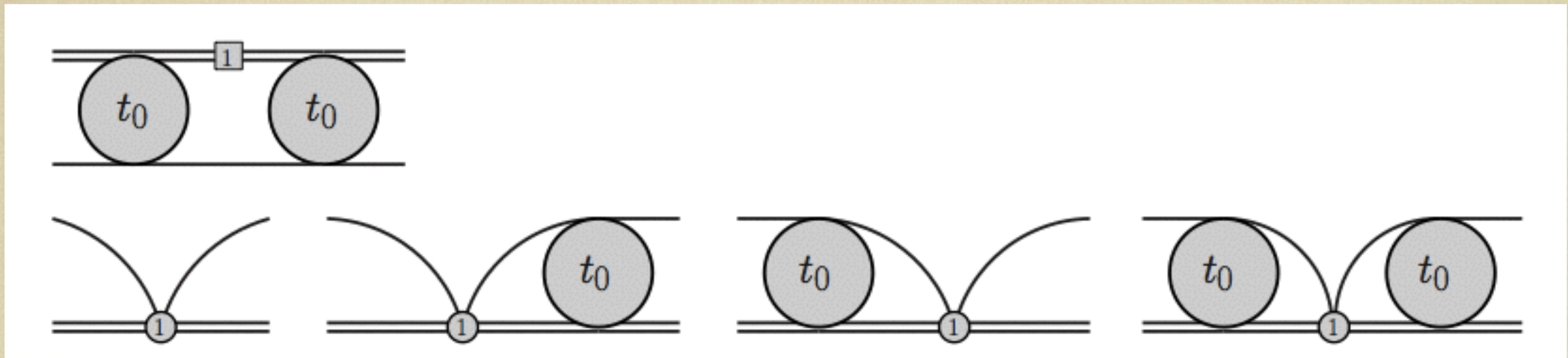


Backup slides: NLO and NNLO in three-body systems

Perturbation theory at NLO

Hammer, Mehen (2001); Ji, Phillips, Platter (2009, 2010)

- Insert t_1^{2B} in first-order perturbation theory between LO wfs



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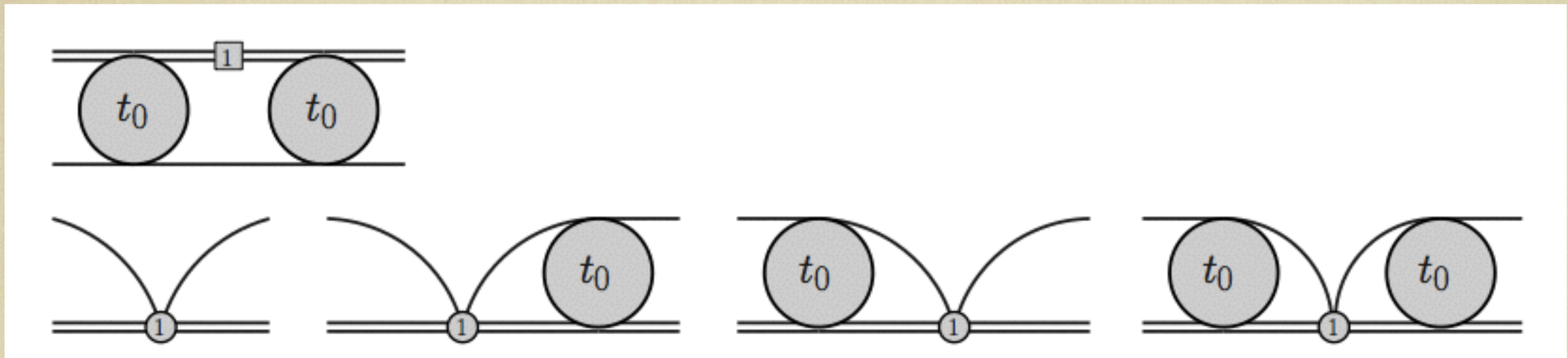

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$$t_1^{(b)}(k,k;E) = \frac{2H_1(\Lambda)}{\Lambda^2}\left[1+\frac{2}{\pi}\int dq\, \frac{q^2}{-1/a+\sqrt{3q^2/4-mE}} t_0(k,q)\right]^2$$

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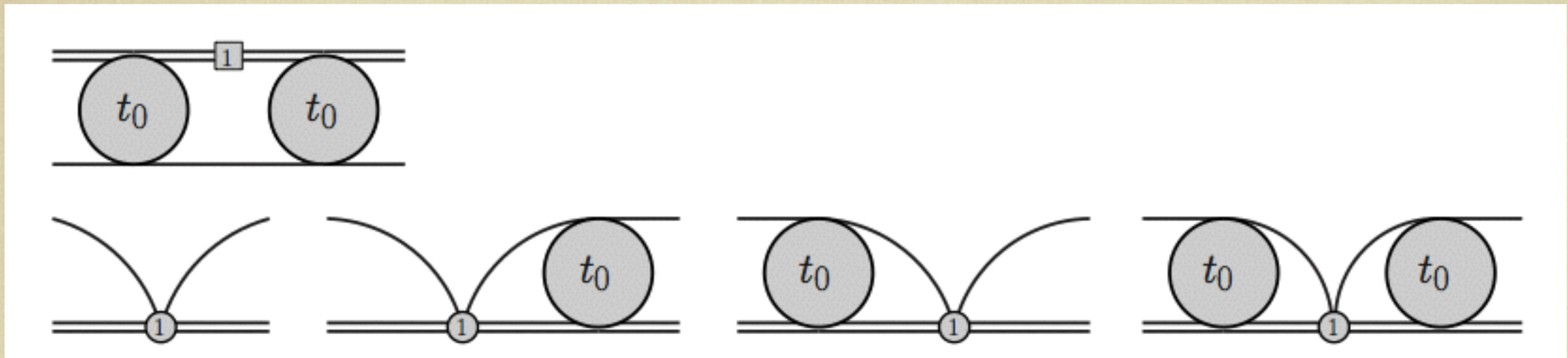

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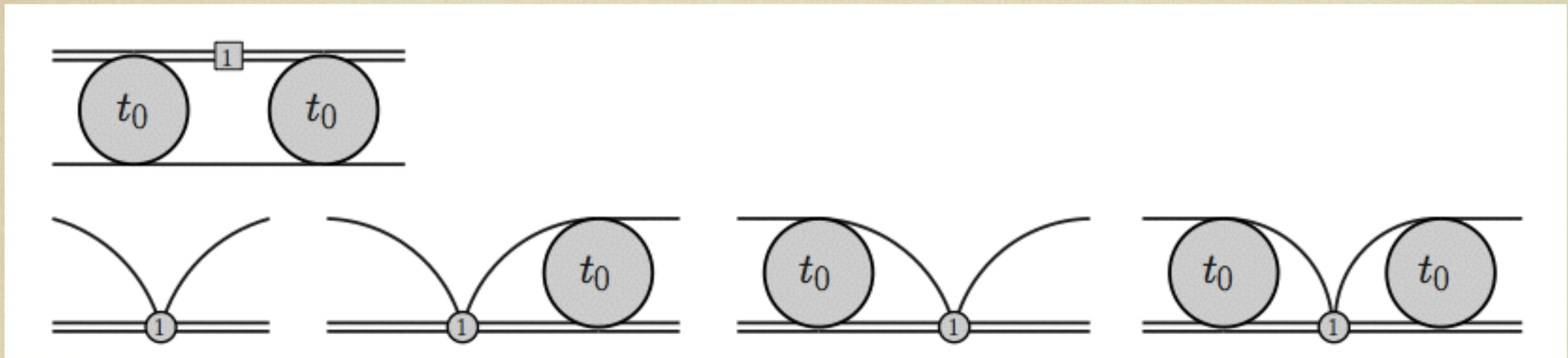

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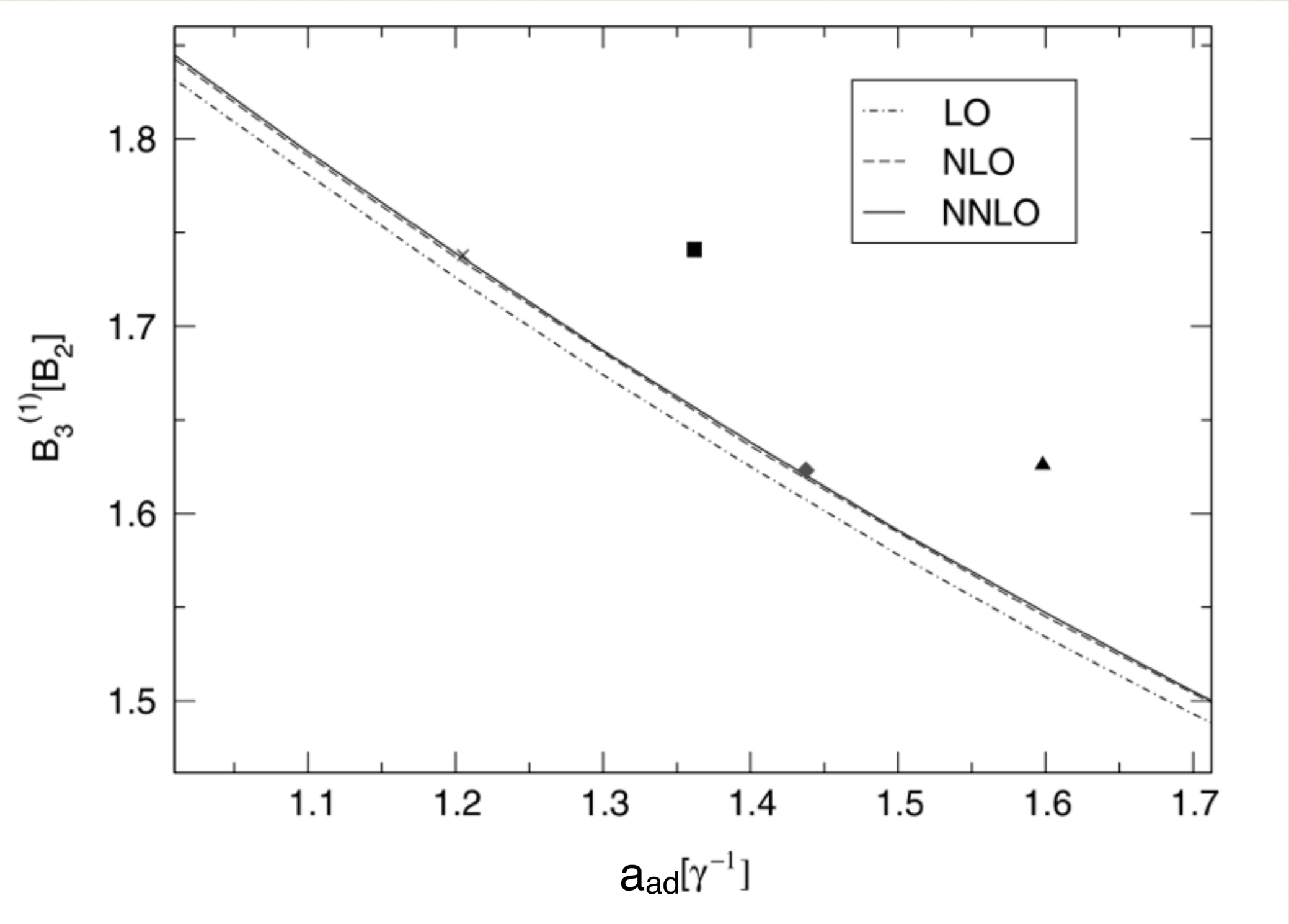
No new 3B datum needed at NLO **at fixed a**.

Corrections to universality



Platter, Phillips (2006)

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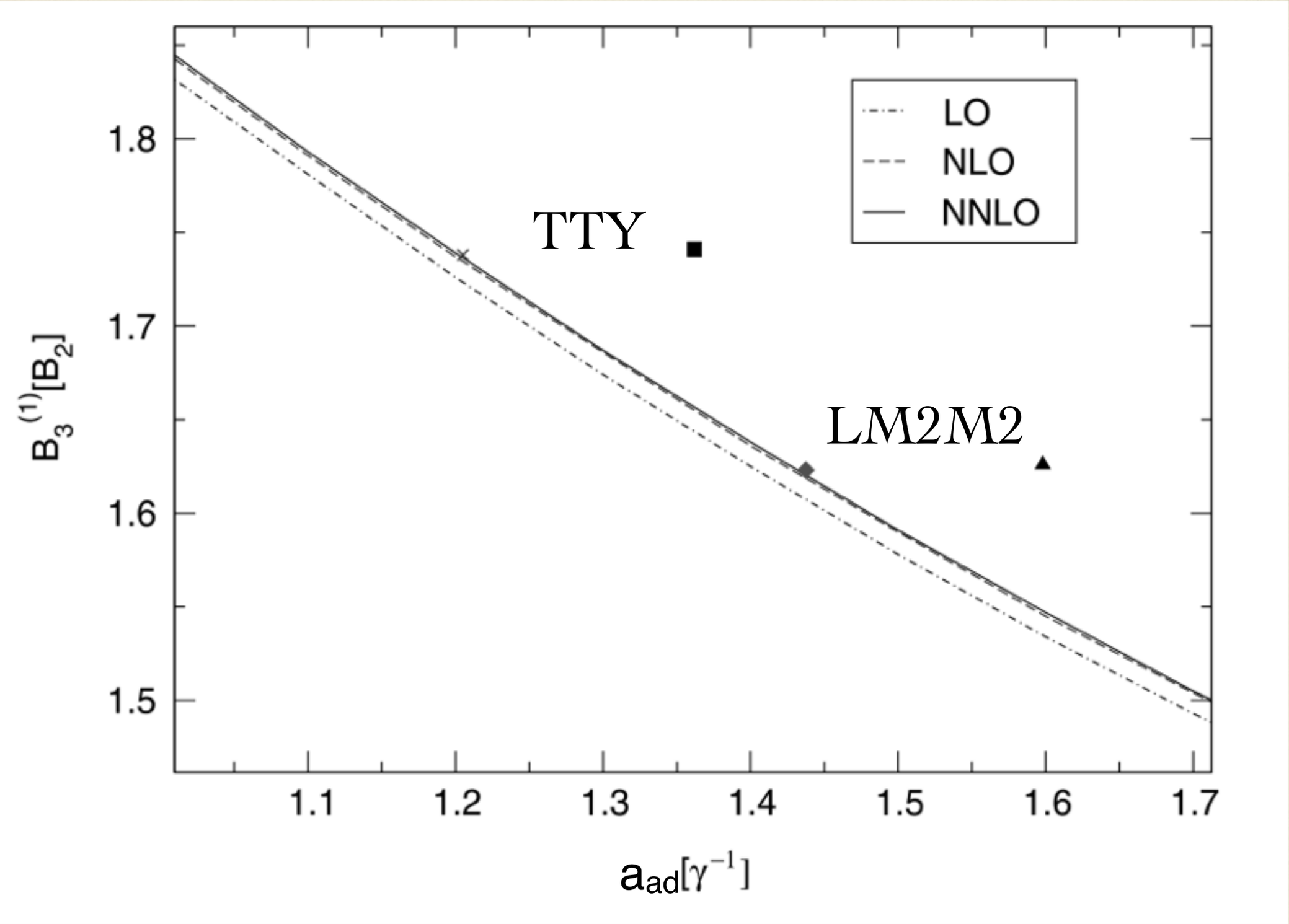

Platter, Phillips (2006)

He-4 trimers at NLO

Platter, Phillips (2006); Ji, Phillips (2012)

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	$B_t^{(1)}$ [B_d]	$B_t^{(0)}$ [B_d]	a_{ad} [γ]	r_{ad} [$1/\gamma$]
TTY	**1.738**	**96.33**	**1.205**	**?**

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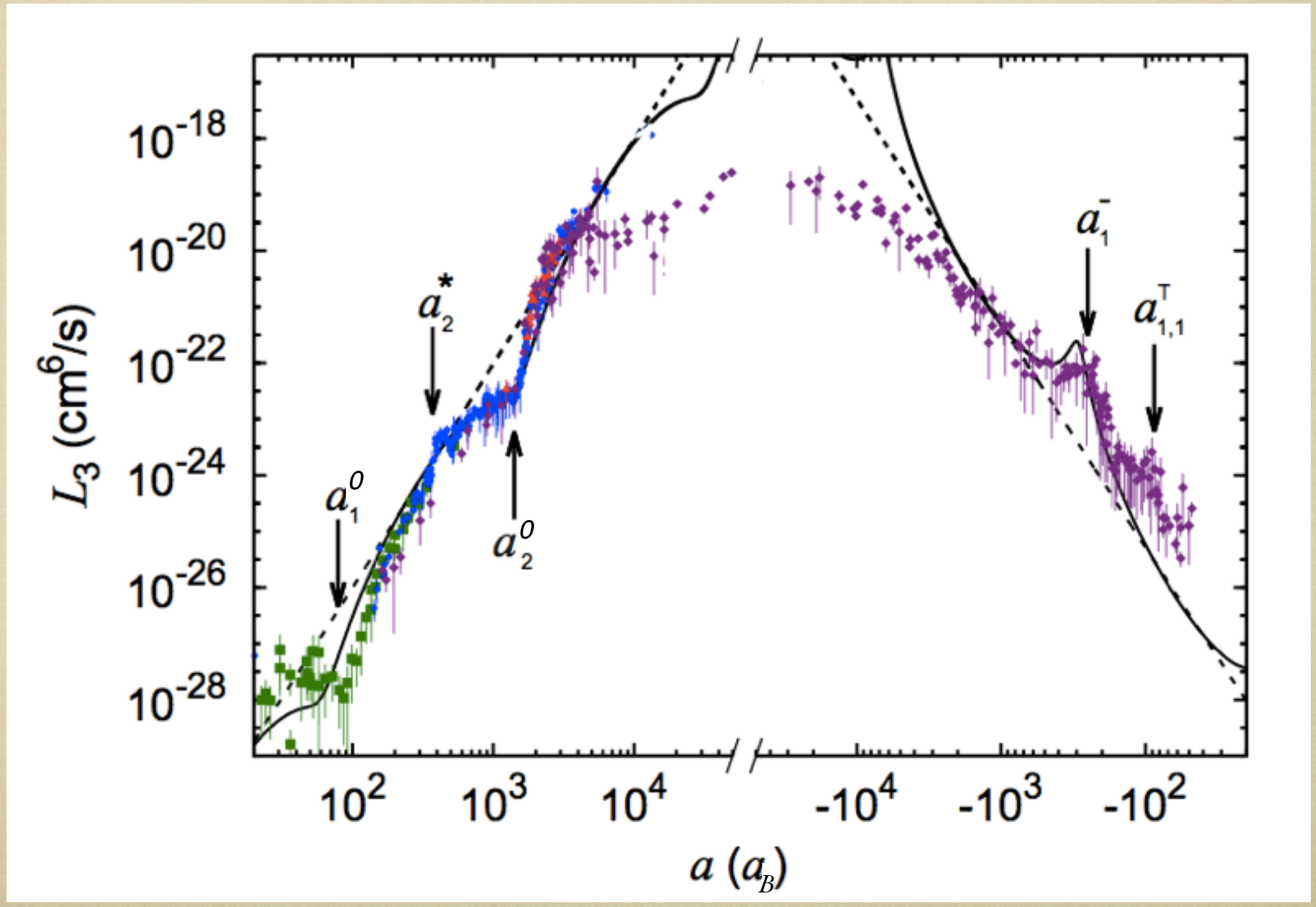
LO, $B_t^{(1)}$	1.738	99.37	1.178	0.8752
NLO, $B_t^{(1)}$	1.738	89.77	1.201	0.913

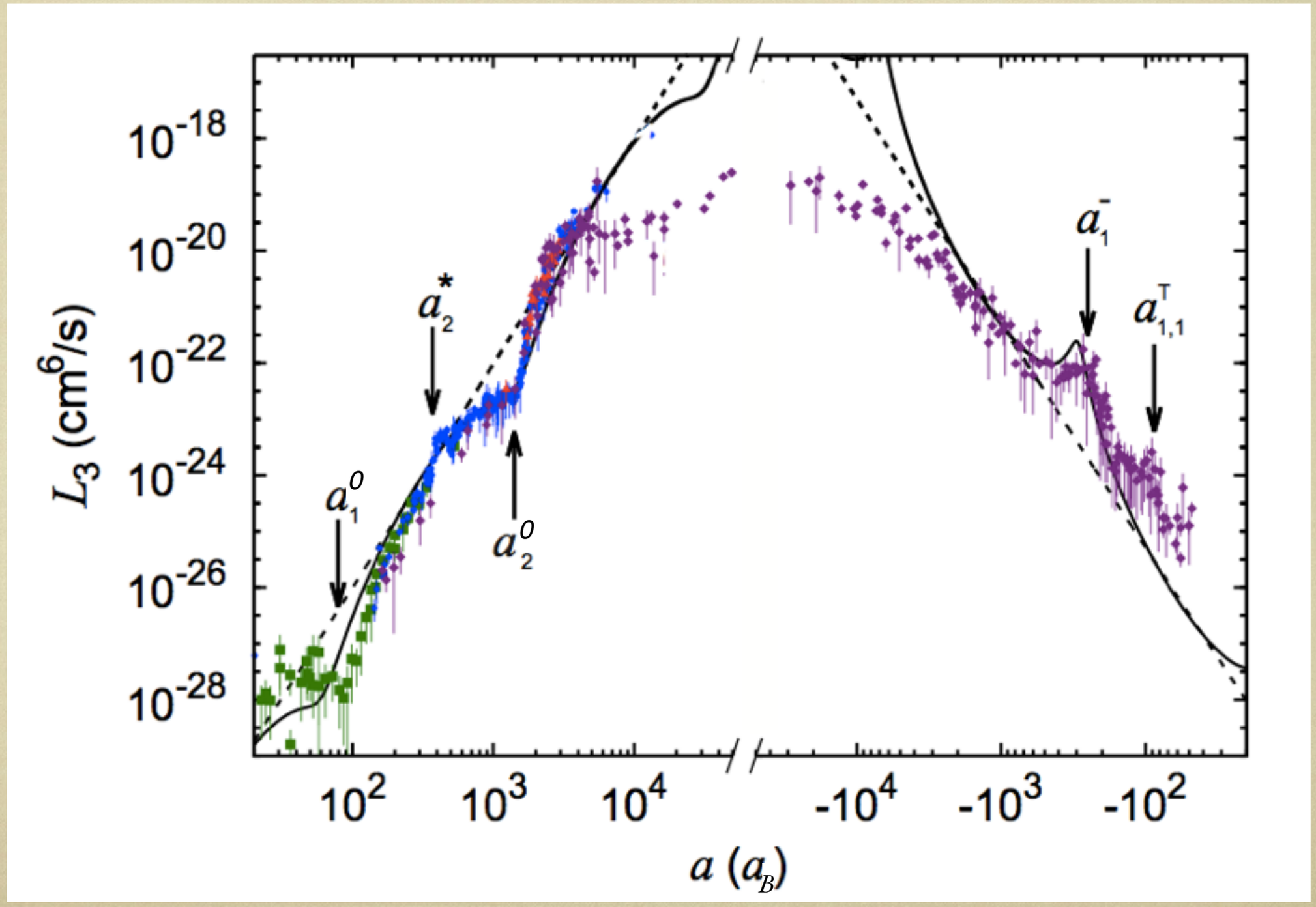
Semi-universal relations in ^{7}Li



Semi-universal relations in ^{7}Li

Dyke, Pollack, Hulet (2013)

F=1, m_F=1



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$$\frac{1}{a_*} = -0.939\gamma_-^{(-)} - 0.309 r(a_*)\frac{1}{a_-^{(-)}} + 7.17\frac{r(a_*)}{r(a_0)}\left(\frac{1}{a_0} + 0.210\frac{1}{a_-^{(-)}}\right)$$

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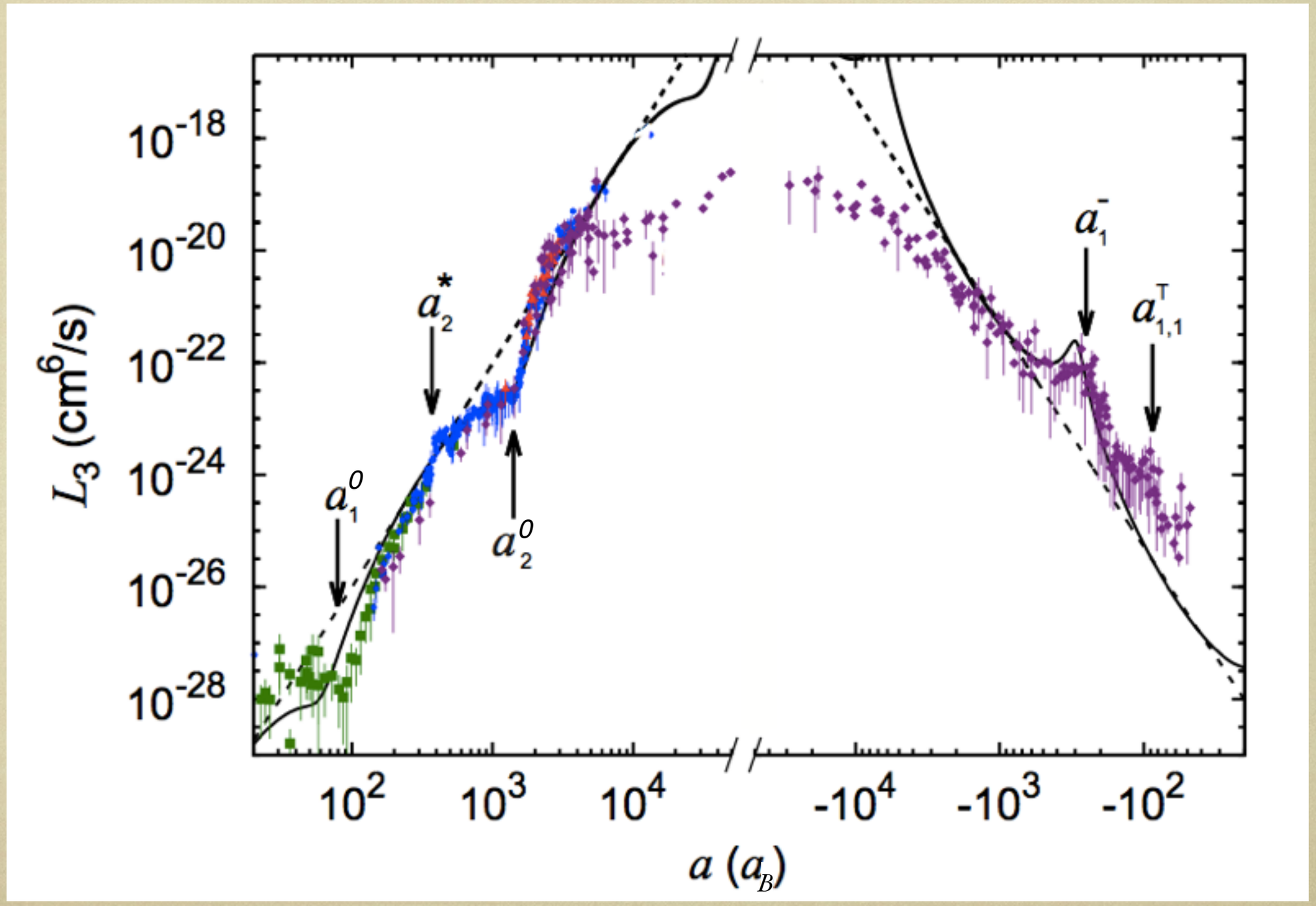

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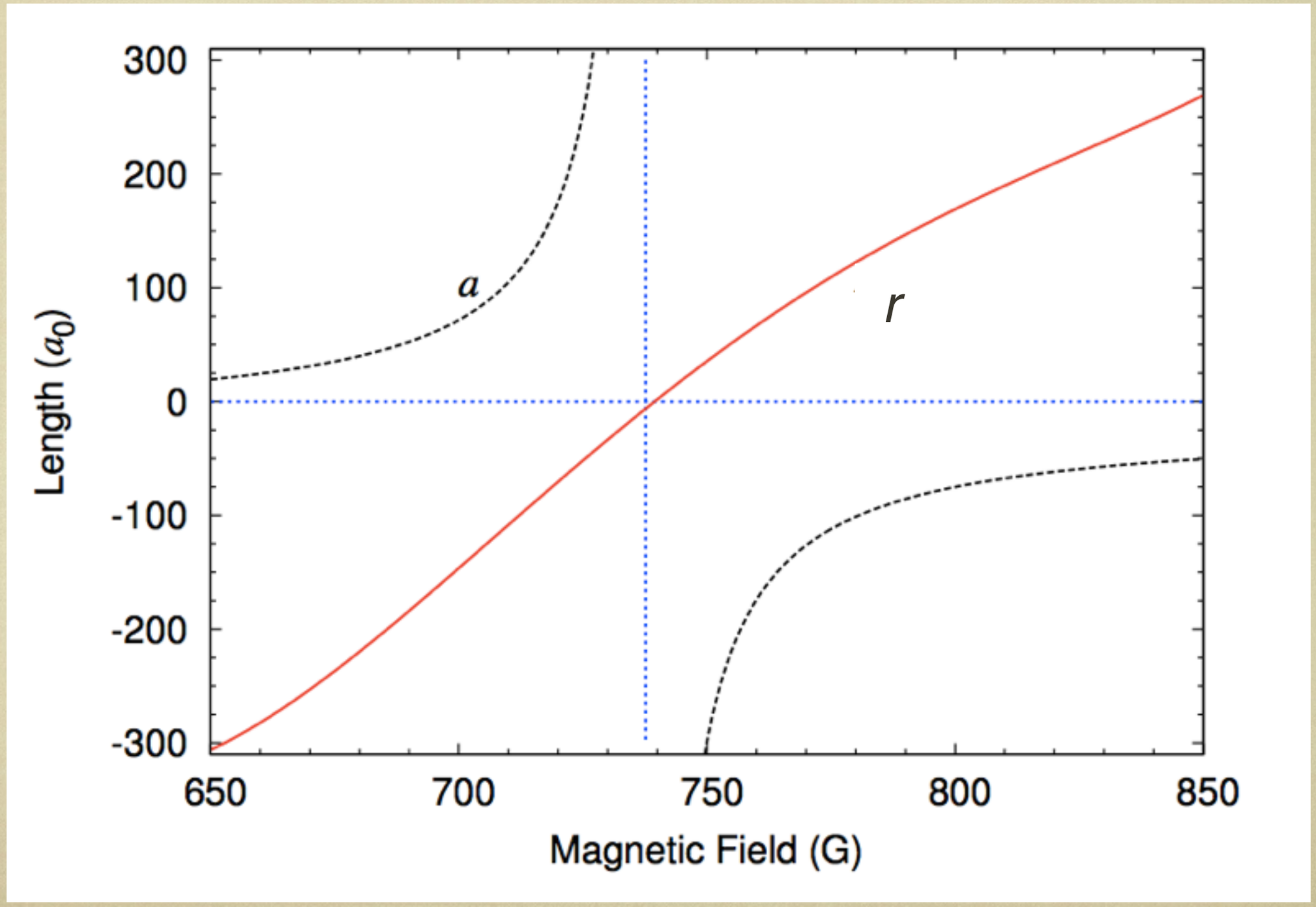

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Experiment: a_*=(196 ± 4) a_B

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Averaging: a_*=(339 ± 65) a_B

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Experiment: a_*= (426 ± 20) a_B

N^2LO results using STM equation

Bedaque, Greisshammer, Hammer, Rupak (2002)

- Insert $t_0^{2B} + t_1^{2B} + t_2^{2B}$ in STM eqn, solve to get $t_0 + t_1 + t_2$

  Only reliable for $\Lambda\ell << 1$ see Platter & Phillips (2005), Platter (2006)

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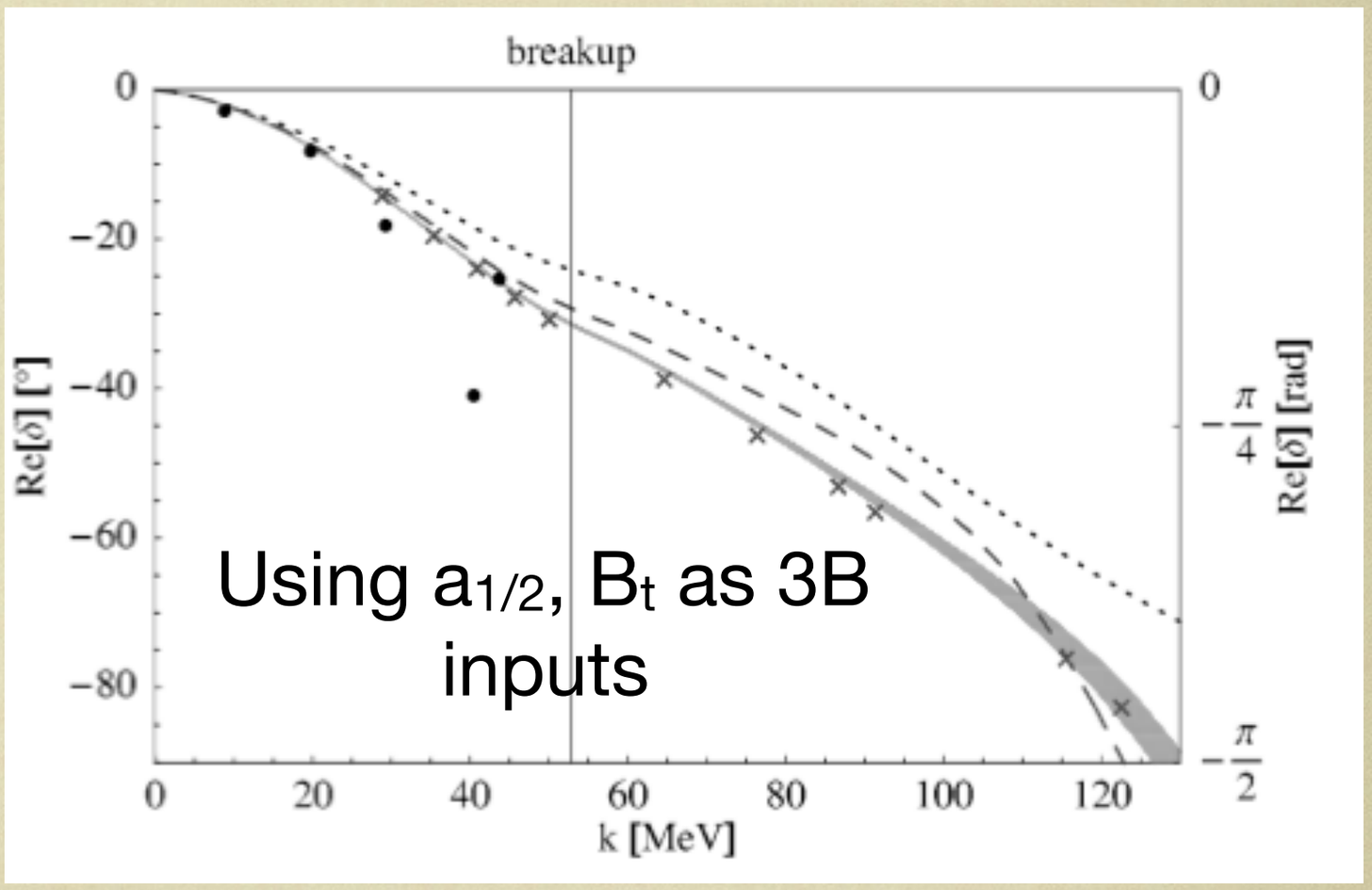
Analysis of eqn. shows that another 3B input needed at NNLO.



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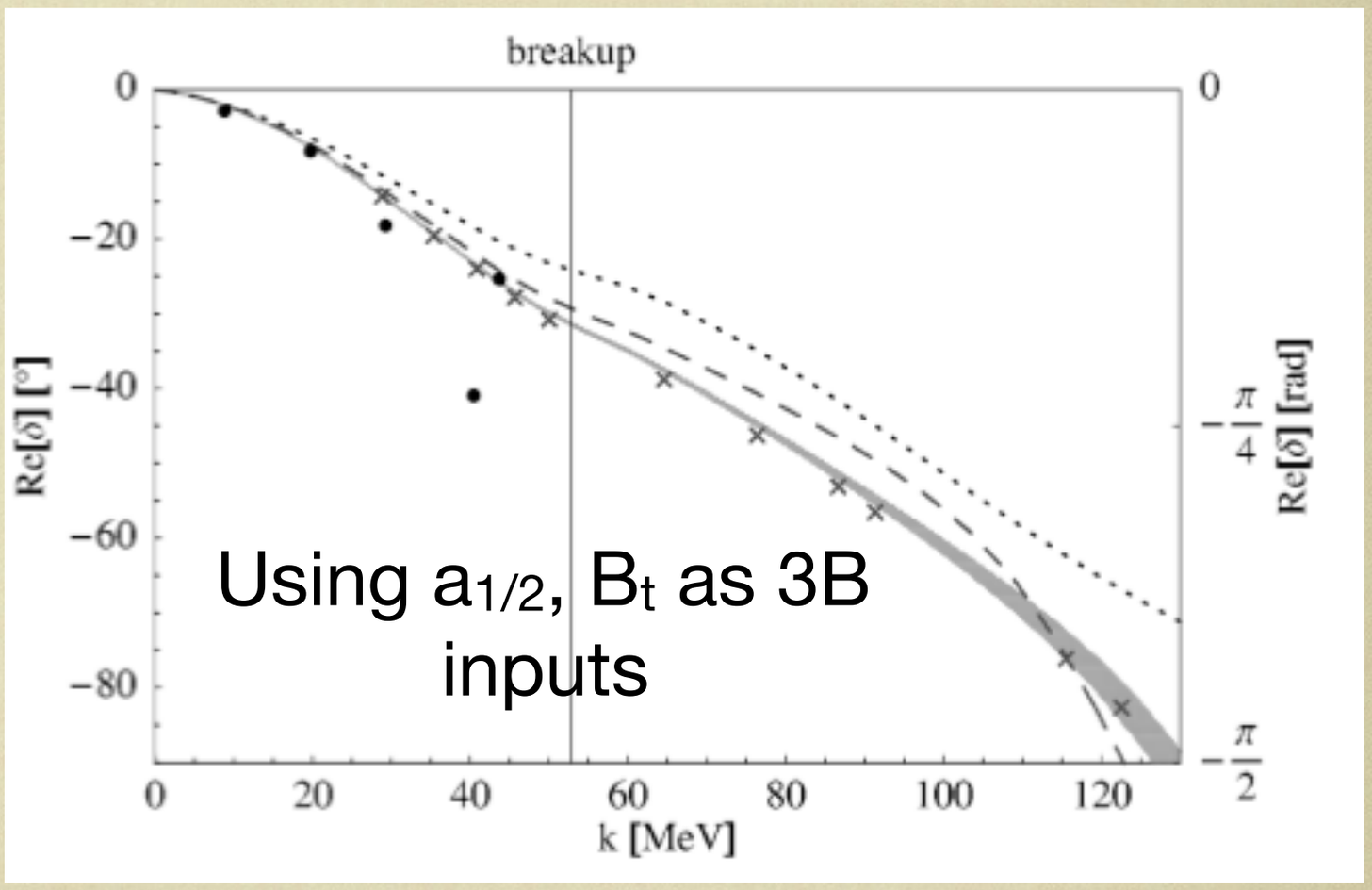
Just six numbers...



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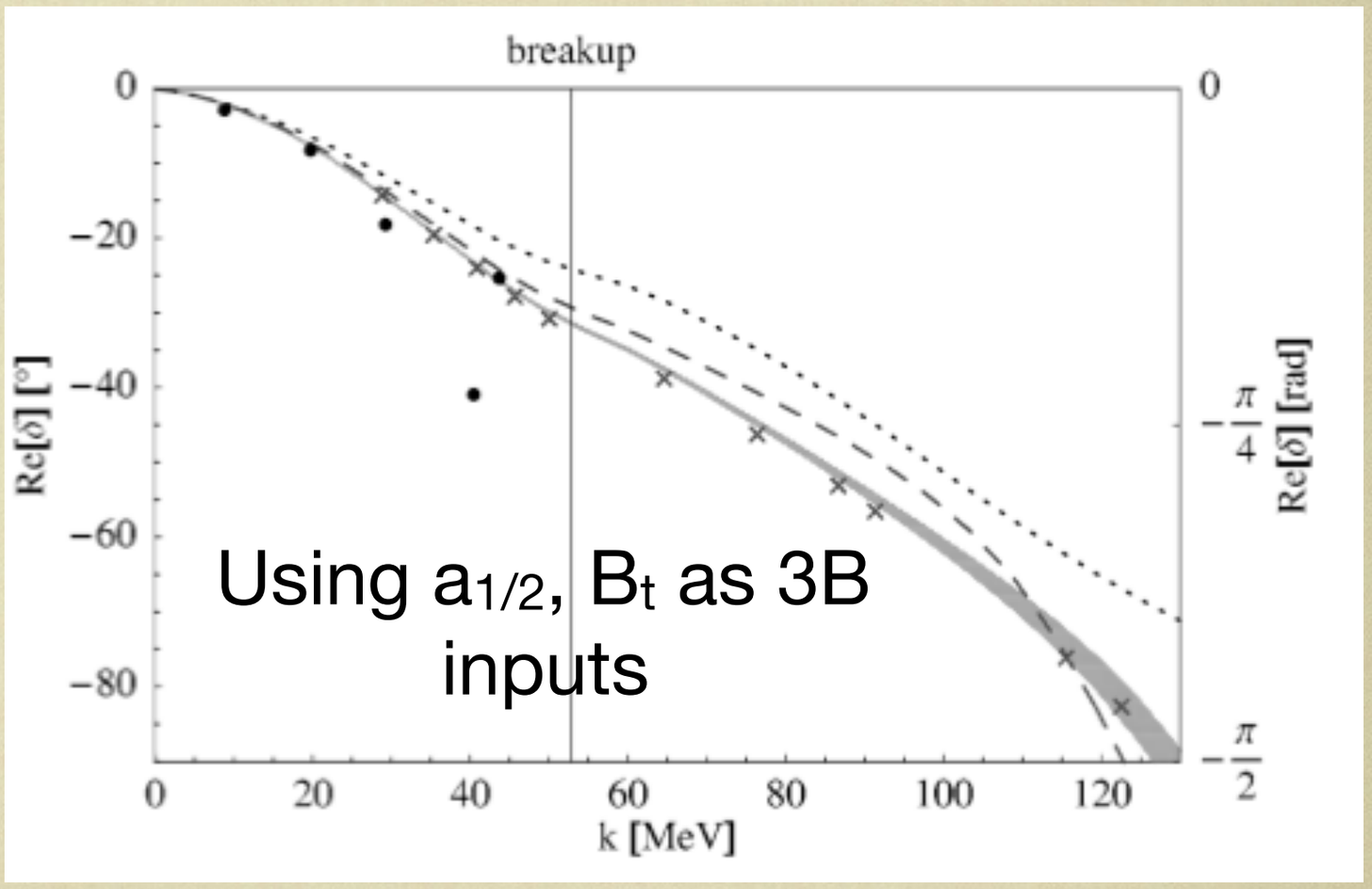
Just six numbers...



- Alternative: replace $-\frac{1}{a} \to -\frac{1}{a} + \frac{1}{2}rk^2$ in t^{2B}, solve integral equation. Caution required!

Bedaque, van Kolck (1999)

Results at N²LO

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	$B_t^{(1)}$ [B_d]	$B_t^{(0)}$ [B_d]	a_{ad} [γ]	r_{ad} [$1/\gamma$]
TTY	**1.738**	**96.33**	**1.205**	**?**
LO, a_{ad}	1.723	97.12	1.205	0.8352
NLO, a_{ad}	1.736	89.72	1.205	0.9049
N^2LO, a_{ad},$B_t^{(1)}$	1.738	116.9	1.205	0.9132

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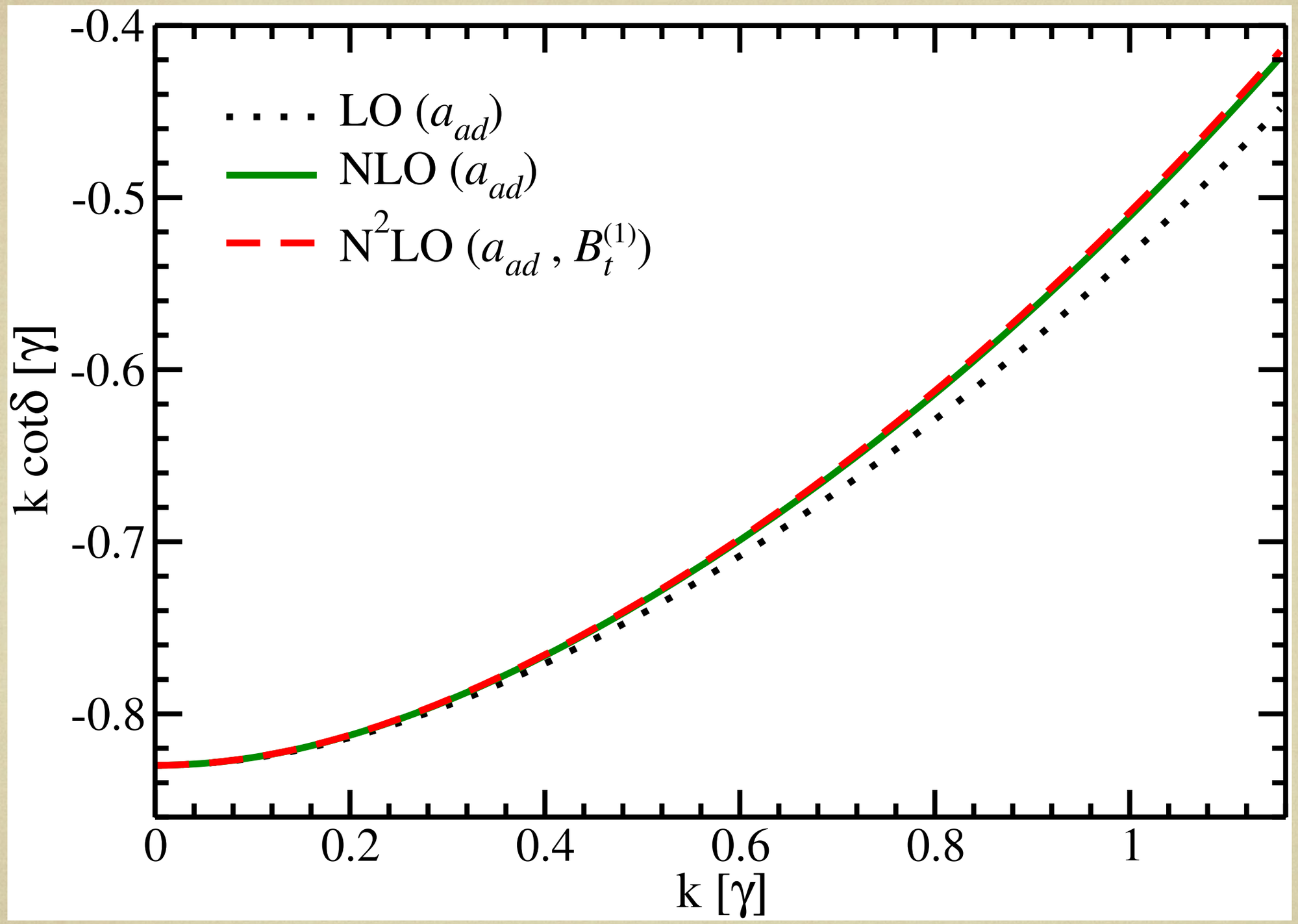
LO, $B_t^{(1)}$	1.738	99.37	1.178	0.8752
NLO, $B_t^{(1)}$	1.738	89.77	1.201	0.913
N^2LO,$B_t^{(1)}$, a_{ad}	1.738	115.9	1.205	0.9135

Results at N²LO



Phase shifts predicted to better than 0.2% at N²LO

For application to n-d system see: Bedaque, Greisshammer, Hammer, Rupak (2002)