BEYOND UNIVERSALITY?
P-WAVE INTERACTIONS IN HALO NUCLEI

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Research supported by the US Department of Energy

Outline

- A one-slide review of p-waves
- p-wave amplitudes in Halo EFT
- How universal are electromagnetic processes with p-wave states?
- Three bodies: Efimov effect? Remnant thereof?
- Outstanding issues

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- A one-slide review of p-waves
- p-wave amplitudes in Halo EFT — Helium-5
- How universal are electromagnetic processes with p-wave states? — Beryllium-11
- Three bodies: Efimov effect? Remnant thereof? — Helium-6
- Outstanding issues

One-slide p-wave review

$$\langle \mathbf{k}|t_1|\mathbf{k}'\rangle = -\frac{6\pi}{m_R}\frac{\mathbf{k}\cdot\mathbf{k}'}{-\frac{1}{a_1}+\frac{1}{2}r_1k^2-ik^3}$$

Bethe (1949)

One-slide p-wave review

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- So low-energy resonance/bound state would seem to have to arise due to cancellation between $-1/a_1$ and $1/2\ r_1 k^2$ terms.

- $a_1 \sim R/M_{lo}^2$ gives $k_R \sim M_{lo}$

Bedaque, Hammer, van Kolck (2003)

Lagrangian for s- and p-wave states

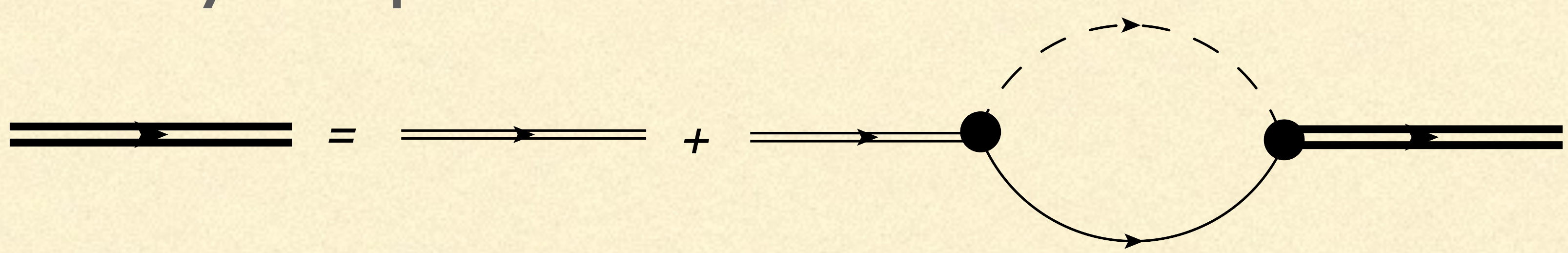
$$\begin{aligned}\mathcal{L} &= c^\dagger\left(i\partial_t+\frac{\nabla^2}{2M}\right)c+n^\dagger\left(i\partial_t+\frac{\nabla^2}{2m}\right)n\\ &+\sigma^\dagger\left[\eta_0\left(i\partial_t+\frac{\nabla^2}{2M_{nc}}\right)+\Delta_0\right]\sigma+\pi_j^\dagger\left[\eta_1\left(i\partial_t+\frac{\nabla^2}{2M_{nc}}\right)+\Delta_1\right]\pi_j\\ &-g_0\left[\sigma n^\dagger c^\dagger+\sigma^\dagger nc\right]-\frac{g_1}{2}\left[\pi_j^\dagger(n\, i\overleftrightarrow{\nabla}_j\, c)+(c^\dagger\, i\overleftrightarrow{\nabla}_j\, n^\dagger)\pi_j\right]\\ &-\frac{g_1}{2}\frac{M-m}{M_{nc}}\left[\pi_j^\dagger\, i\overrightarrow{\nabla}_j\,(nc)-i\overleftrightarrow{\nabla}_j\,(n^\dagger c^\dagger)\pi_j\right]+\ldots,\end{aligned}$$

- c, n: "core", "neutron" fields. c: boson, n: fermion
- σ, π_j: S-wave and P-wave fields
- Minimal substitution generates leading EM couplings

Dressing the p-wave state

Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)

- Proceed similarly for p-wave state as for s-wave state



$$D_\pi(p) = \frac{1}{\Delta_1 + \eta_1[p_0 - \mathbf{p}^2/(2M_{nc})] - \Sigma_\pi(p)}$$

- Here both Δ_1 and g_1 are mandatory for renormalization at LO

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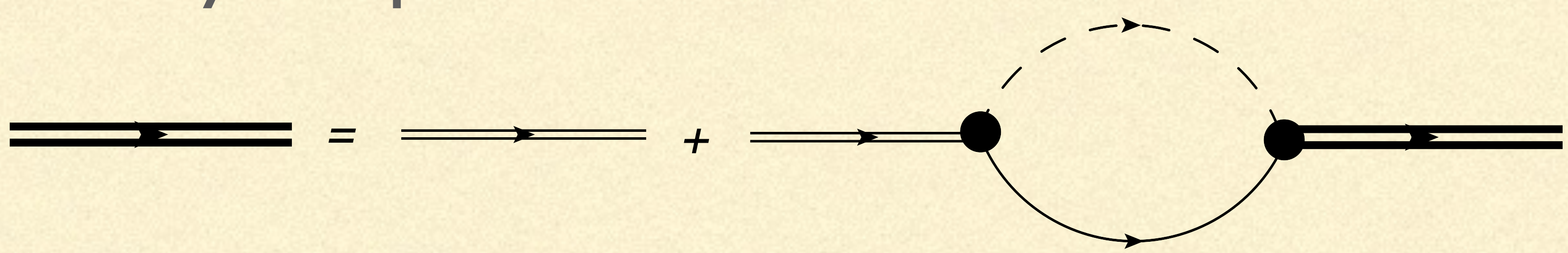

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- If $a_1 > 0$ then pole is at $k=i\gamma_1$ with $B_1=\gamma_1^2/(2m_R)$:

$$D_\pi(p) = -\frac{3\pi}{m_R^2 g_1^2}\frac{2}{r_1 + 3\gamma_1}\frac{i}{p_0 - \mathbf{p}^2/(2M_{nc}) + B_1} + \text{regular}$$

Semi-universal narrow p-wave resonance

Bertulani, Hammer, van Kolck (2002)

Bedaque, Hammer, van Kolck (2003)

Semi-universal narrow p-wave resonance

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- So, off resonance, $\mathrm{Re}[t^{-1}] > \mathrm{Im}[t^{-1}]$: phase shifts are $O(M_{lo}R)$ and scattering is perturbative away from resonance cf. Pascalutsa, DP (2003)

$$\langle \mathbf{k}|t_1|\mathbf{k}'\rangle = -\frac{12\pi}{m_R r_1}\frac{\mathbf{k}\cdot\mathbf{k}'}{k^2 - k_R^2} \qquad k_R^2 = \frac{2}{a_1 r_1}$$

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- Resonance width is $\sim E_R\, k_R/r_1$, so it is parametrically narrow. Need to resum width if $k^2 - k_R^2$ gets small

p-wave pole positions, e.g., ^{5}He

$$\langle \mathbf{k}|t_{n\alpha}|\mathbf{k}'\rangle = -\frac{6\pi}{\mu_{n\alpha}}\frac{\mathbf{k}\cdot\mathbf{k}'}{k^3\cot\delta_1 - ik^3} \qquad k^3\cot\delta_1 = -\frac{1}{a_1} + \frac{1}{2}r_1k^2$$

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Two possibilities:

1. $a_1 \sim 1/M_{lo}^3$, $r_1 \sim M_{lo}$
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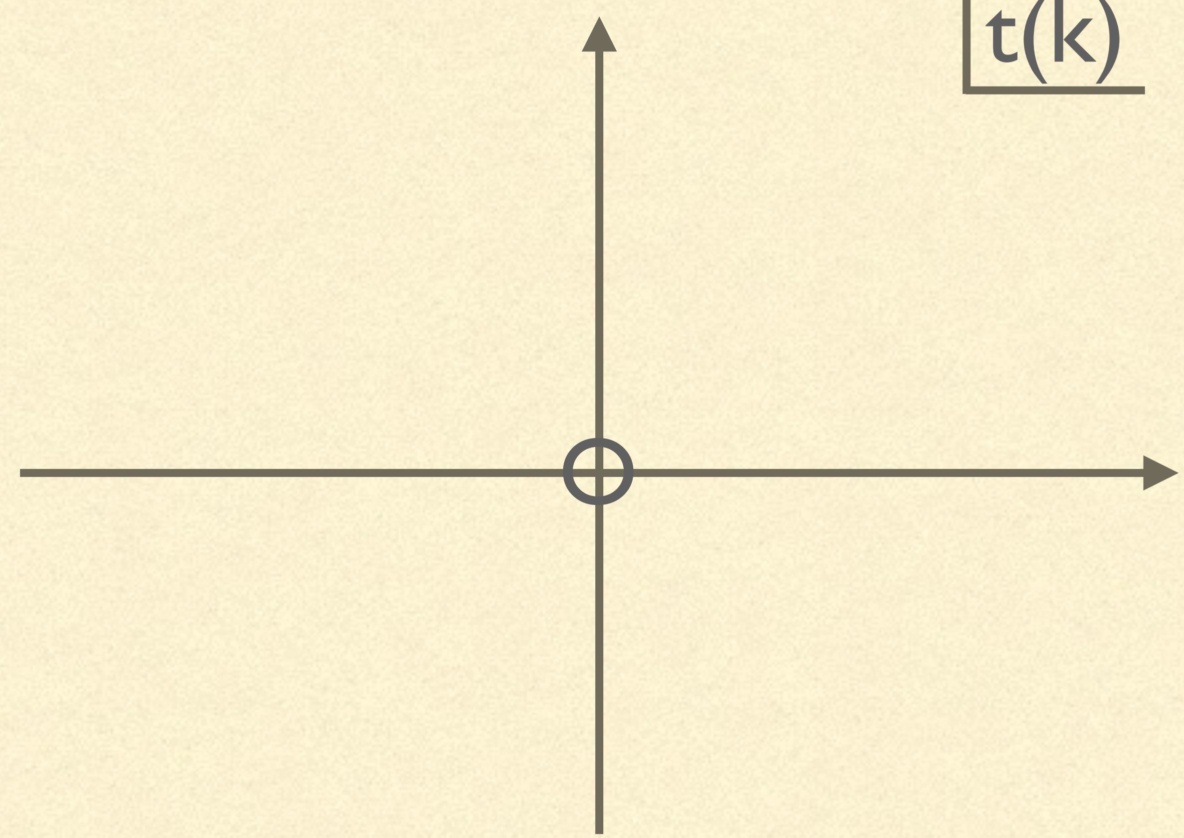
t(k)



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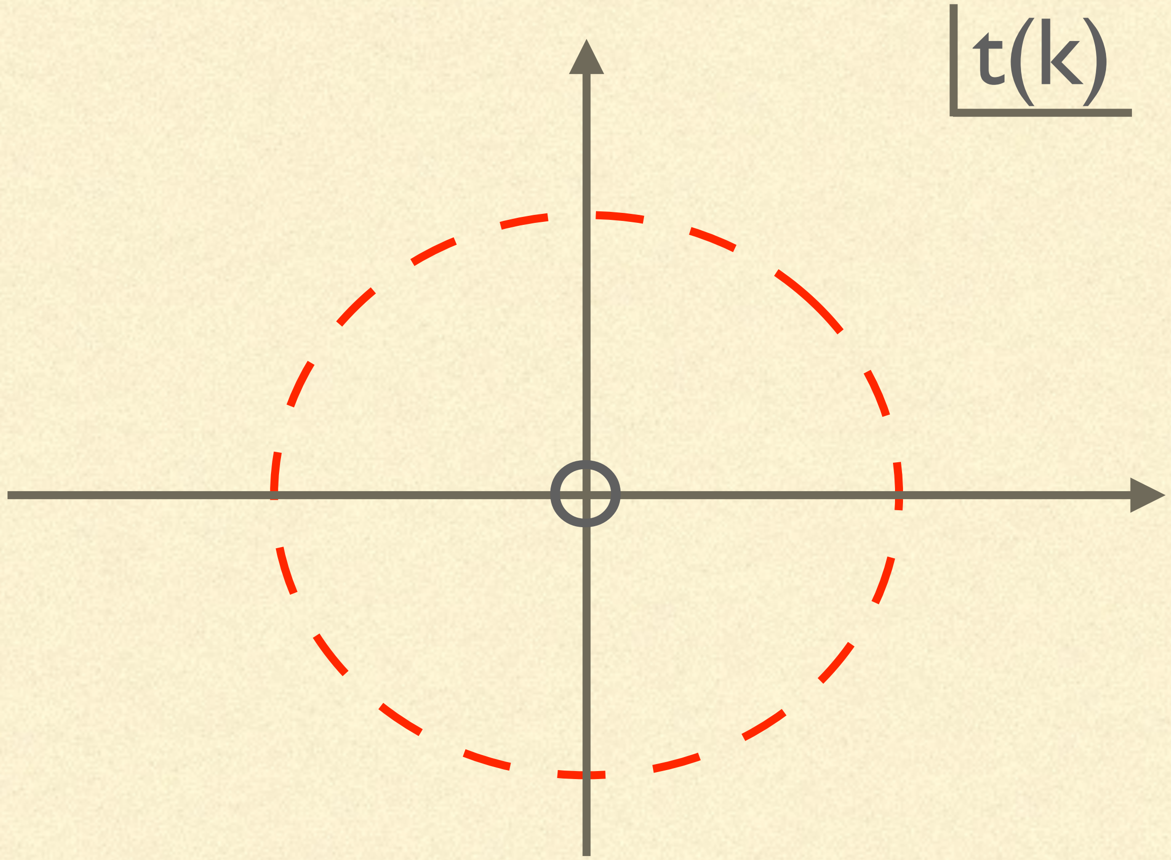

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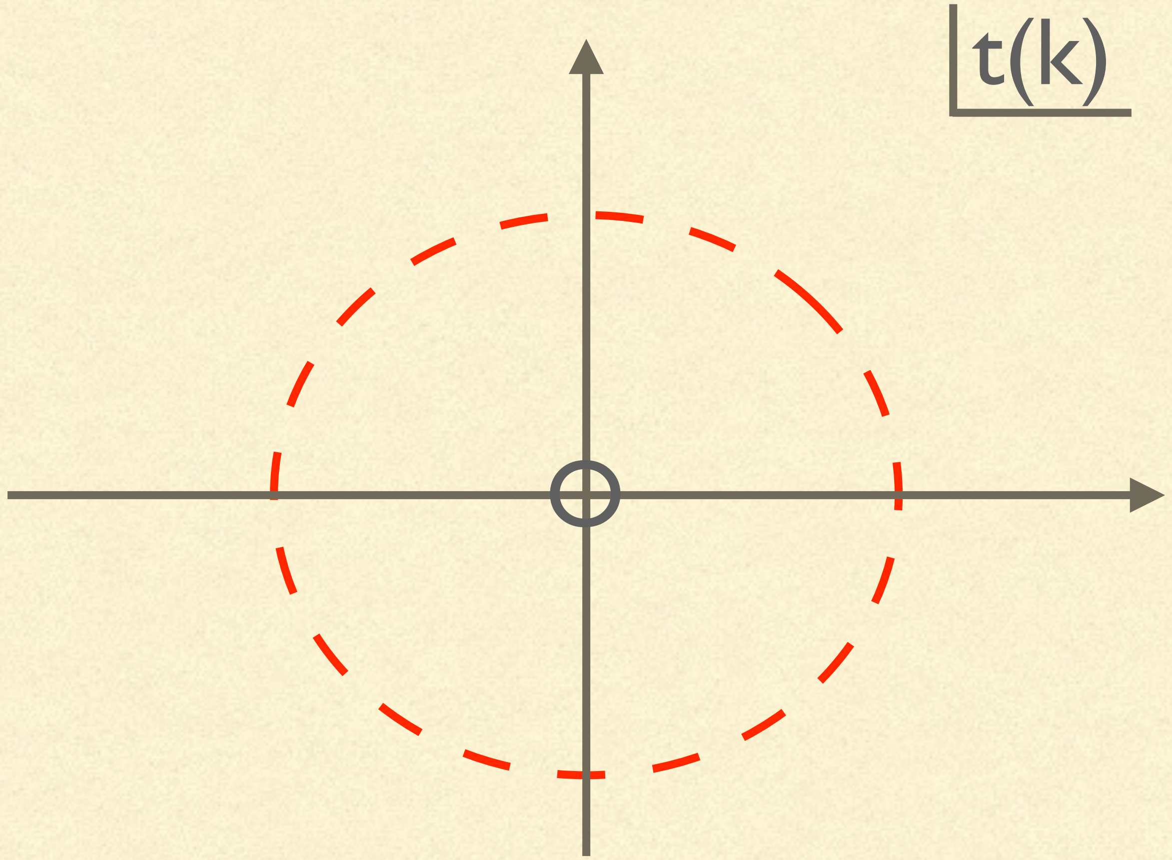

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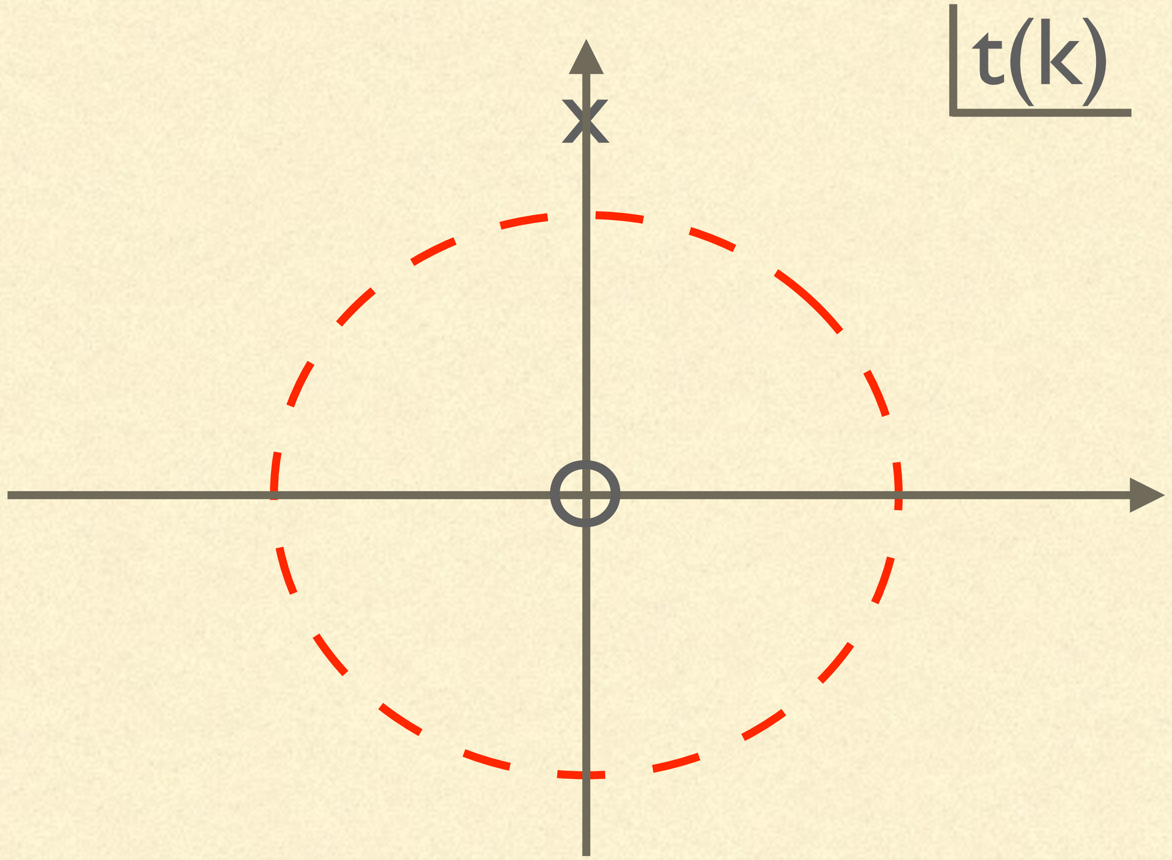

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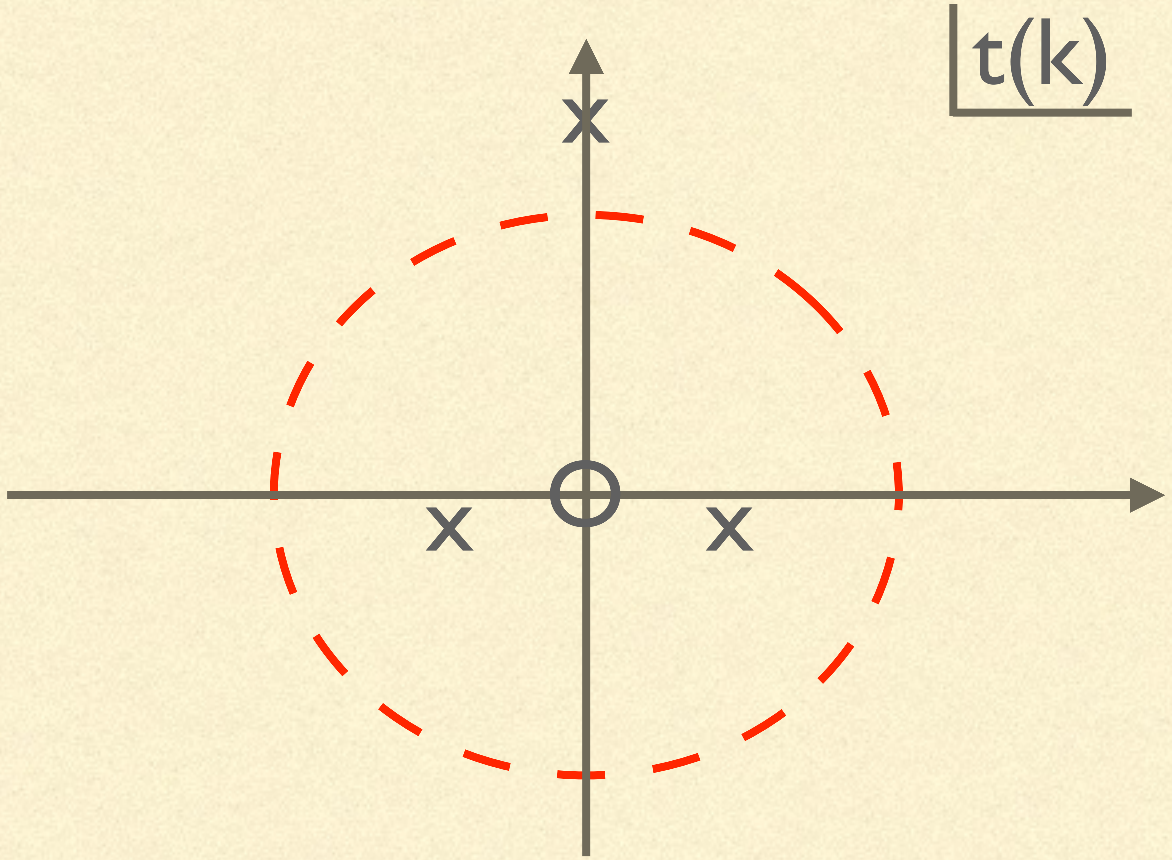
Narrow resonance



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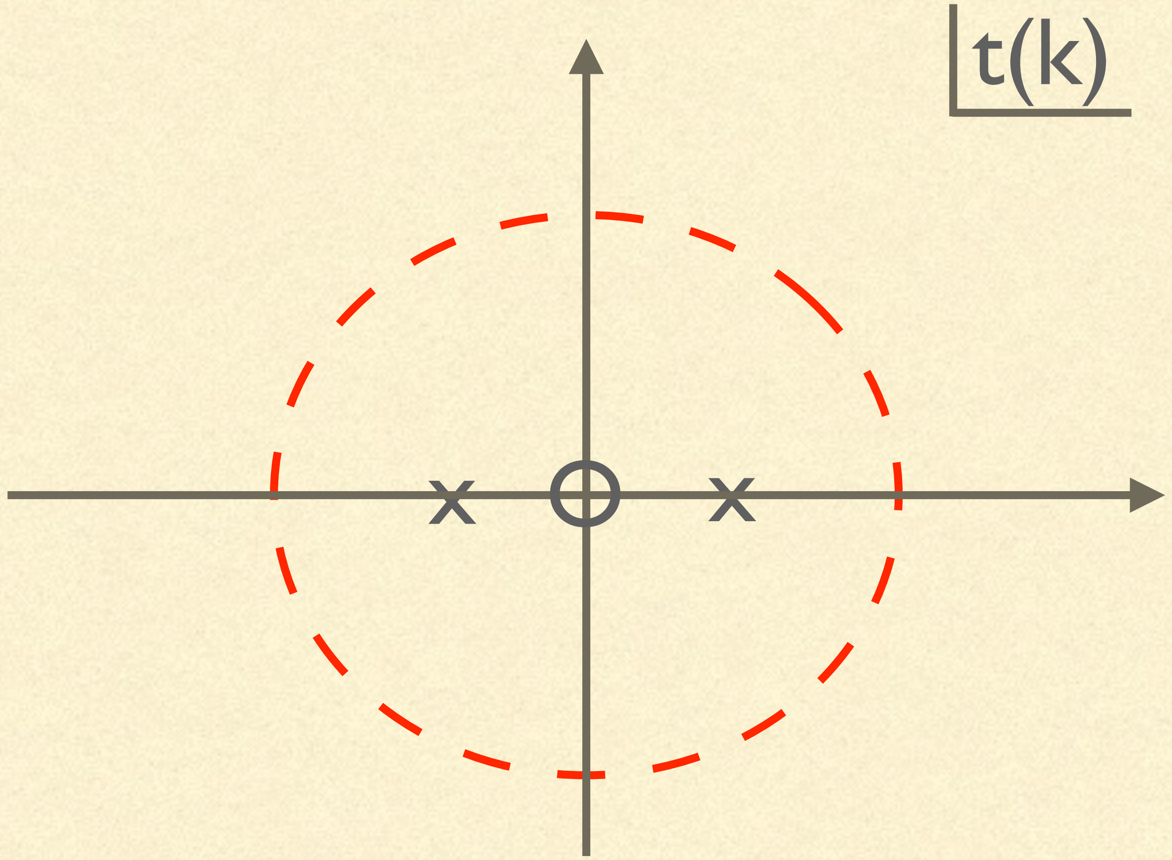
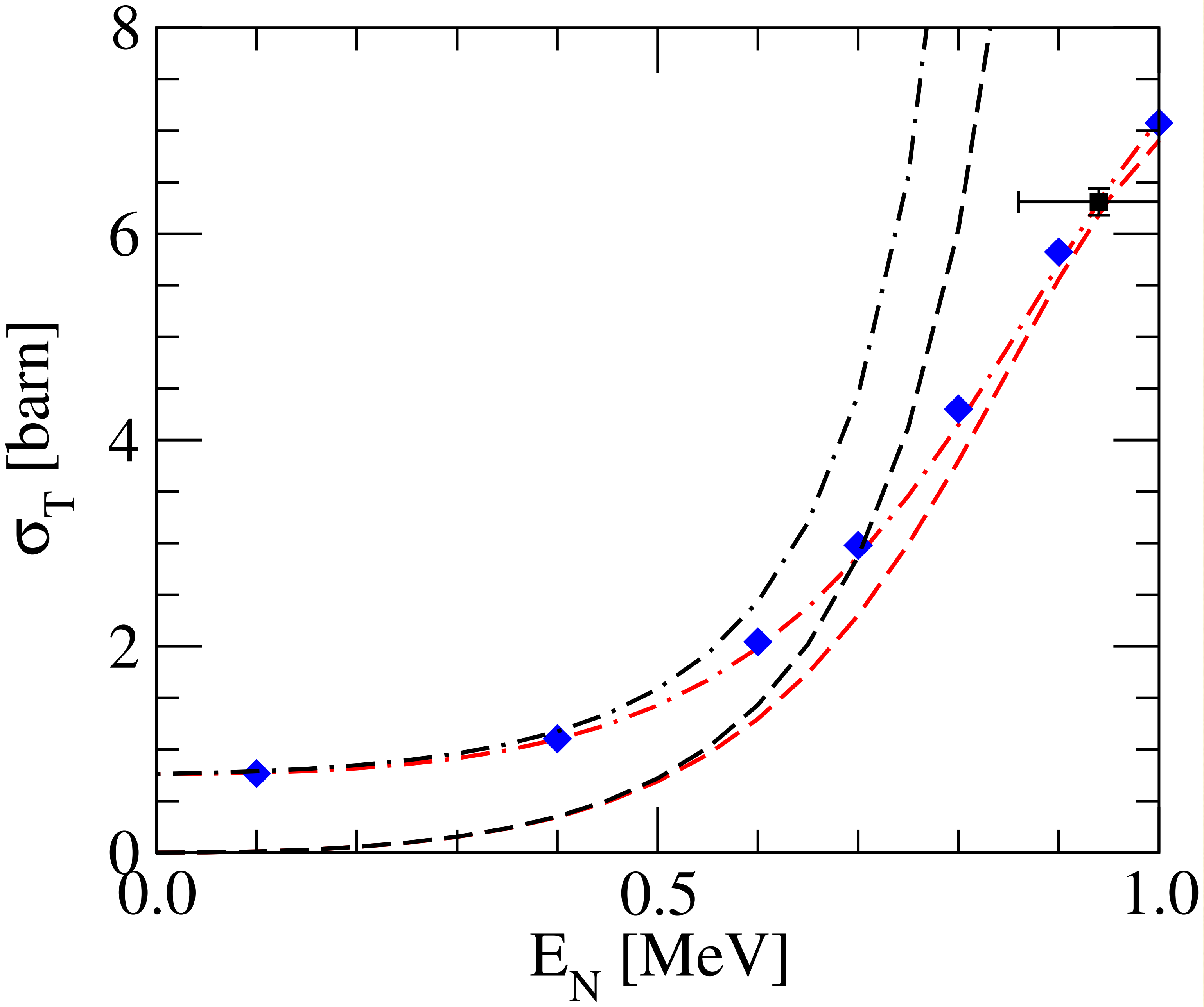


Application to ^{5}He

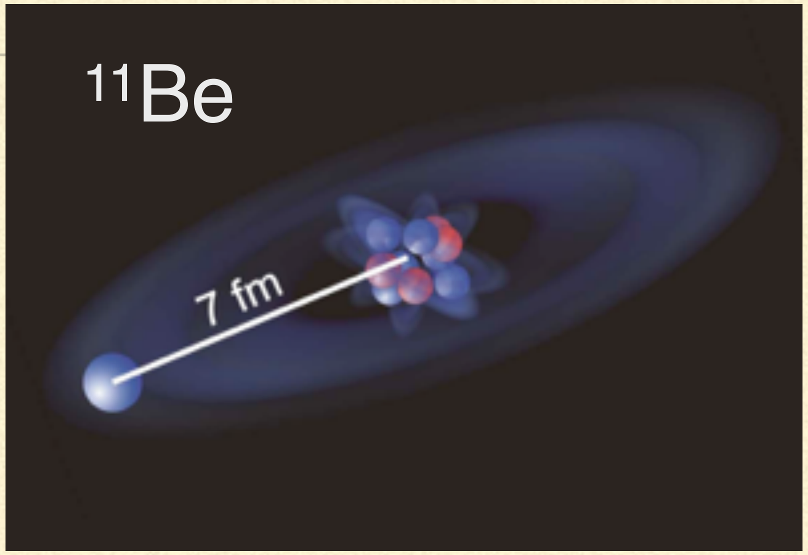
adapted from Bedaque, Hammer, van Kolck (2003)

- Data on neutron-^{4}He scattering
- Note role of s-wave contributions near threshold. They're ~R in amplitude and taken as NLO here
- Red: resummed (needed near resonance)
- Black: not resummed



E1 transitions: $\gamma_{E1} + {}^{11}Be \rightarrow {}^{11}Be^*$

- ${}^{11}Be$: $1/2^+$ (s-wave) state bound by 504 keV, $1/2^-$ (p-wave) state bound by 184 keV
- $1/2^-$ bound state: "on resonance": two parameters at LO, γ_1 and r_1



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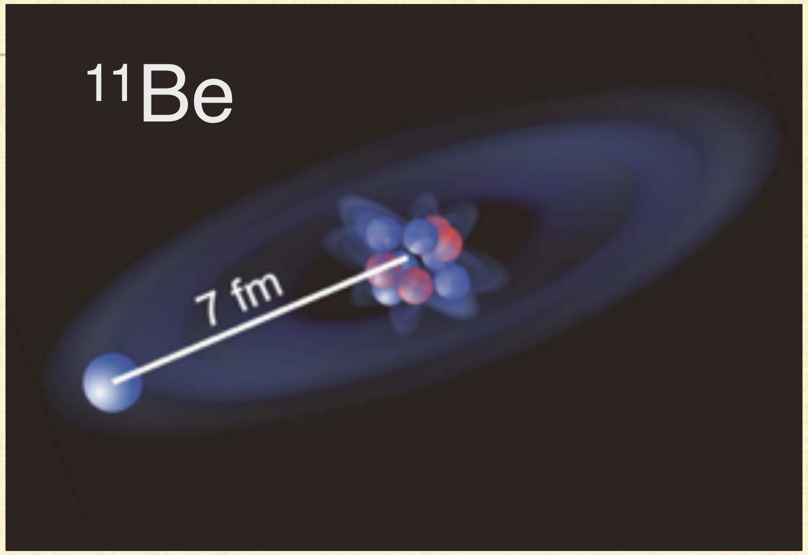

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Typel & Baur (2004, 2005, 2008); Hammer & DP (2011)

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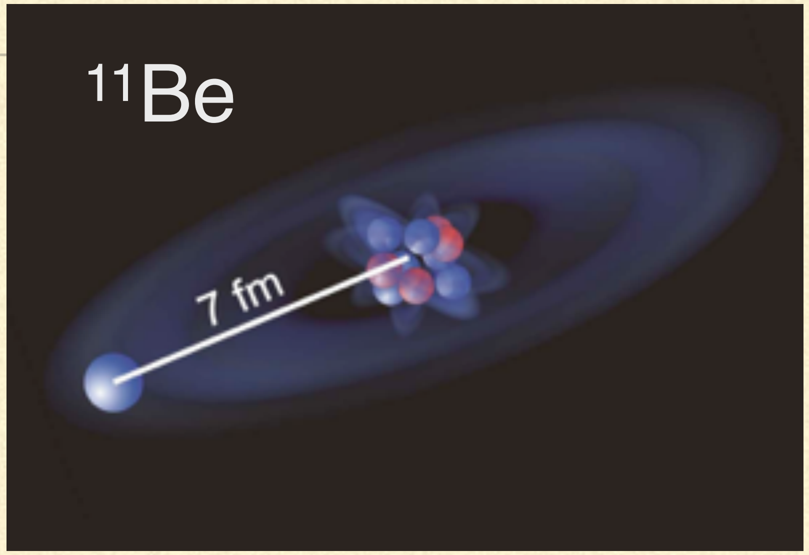

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- Universal relation $\mathrm{B(E1)} = \frac{2e^2 Q_c^2}{15\pi} \langle r_E^2 \rangle x \left[\frac{1+2x}{(1+x)^2}\right]^2 \; ; \; x = \sqrt{\frac{B_1}{B_0}}$

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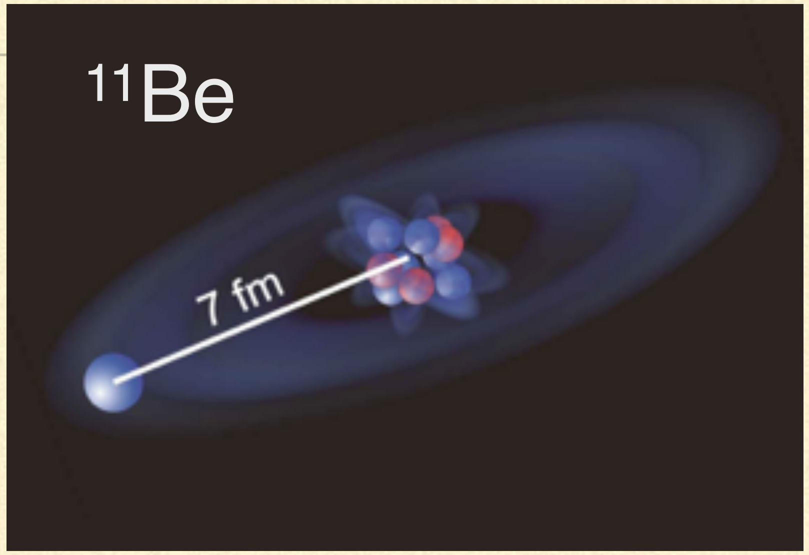

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Predictions for γ_{E1} + ^{11}Be→^{10}Be + n: there p-waves perturbative;
Calculations of p + ^{7}Be →^{8}B + p   Zhang, Nollett, DP (2014, 2015); Ryberg et al. (2015)

^{6}He as a 2n halo

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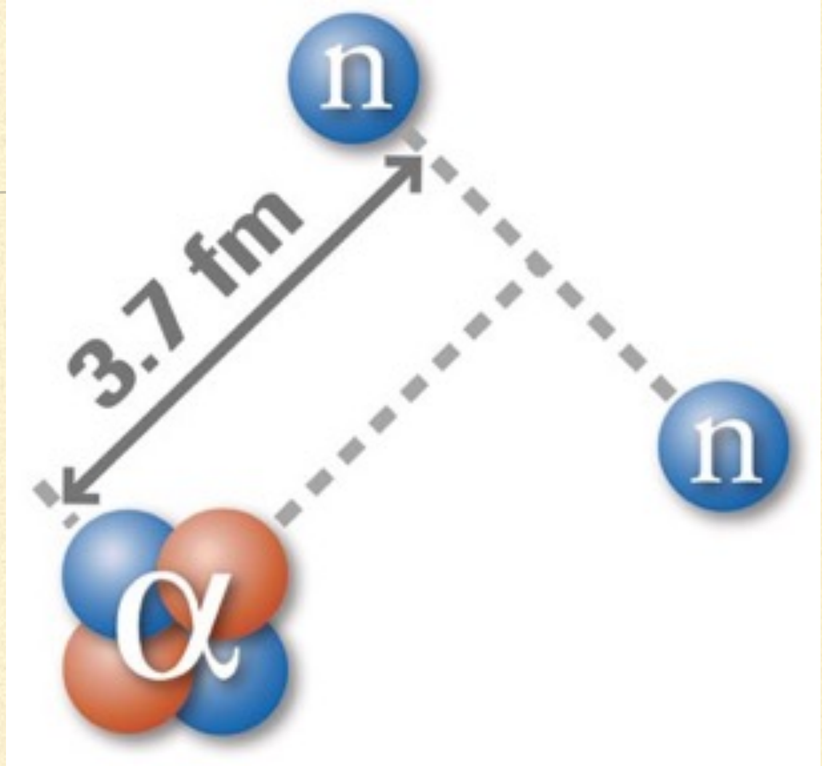

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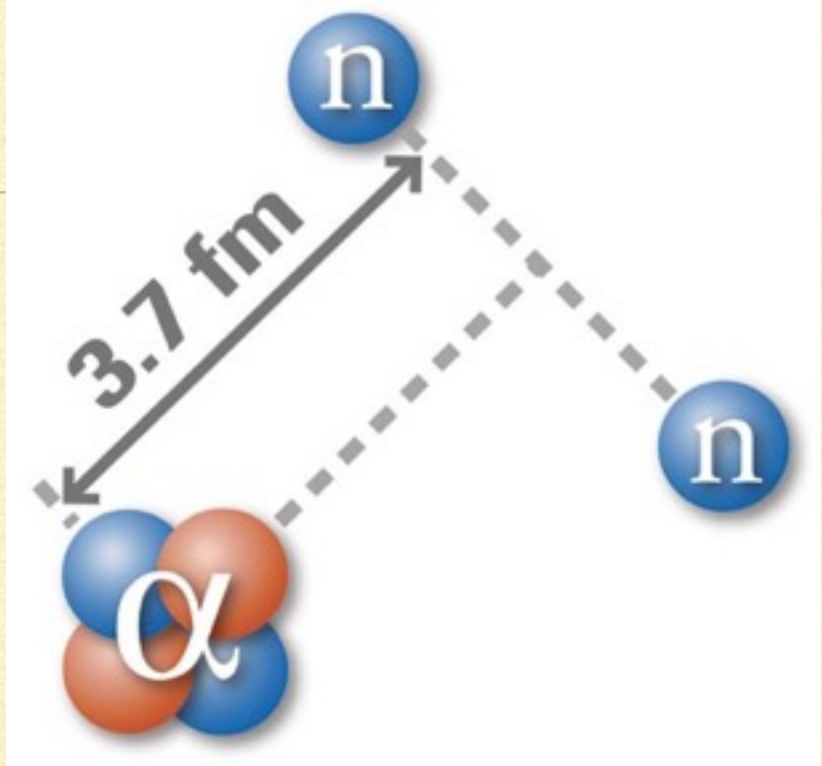

- R≈1.5 fm; M_{lo}≈40 MeV

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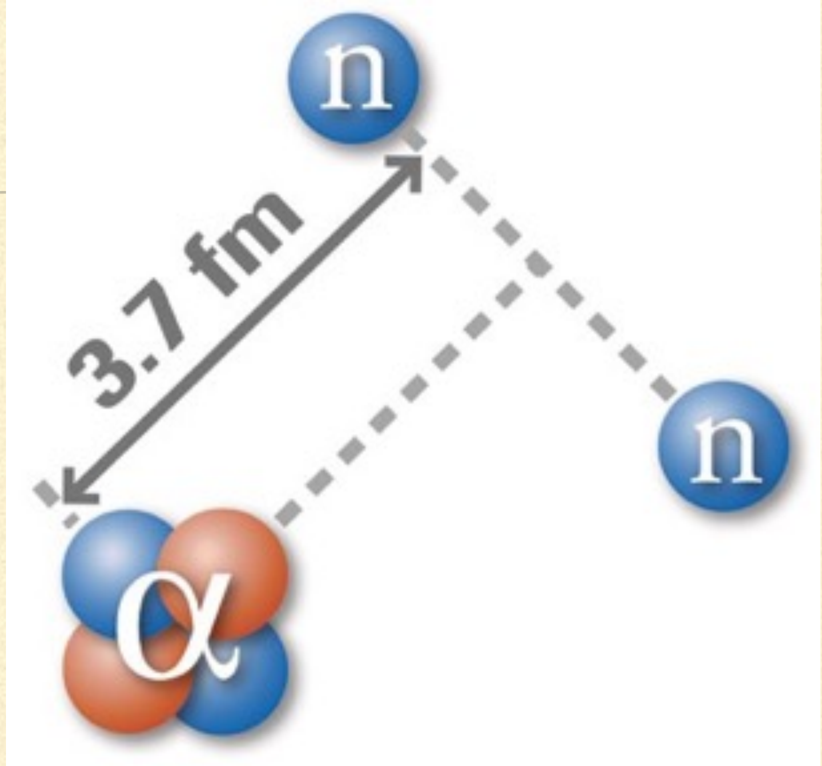
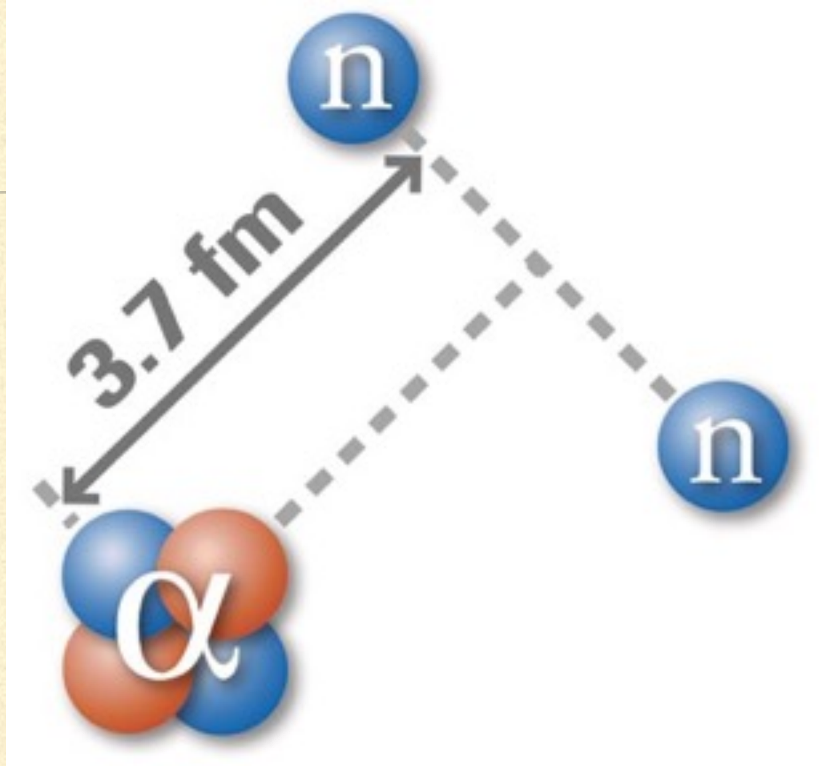


- $R \approx 1.5$ fm; $M_{lo} \approx 40$ MeV
- ^{4}He-n interaction: $^2P_{3/2}$ resonance

$$\langle \mathbf{k} | t_{n\alpha} | \mathbf{k}' \rangle = -\frac{6\pi}{\mu_{n\alpha}} \frac{\mathbf{k} \cdot \mathbf{k}'}{-\frac{1}{a_1} + \frac{1}{2} r_1 k^2}$$

^{6}He as a 2n halo

http://www.anl.gov



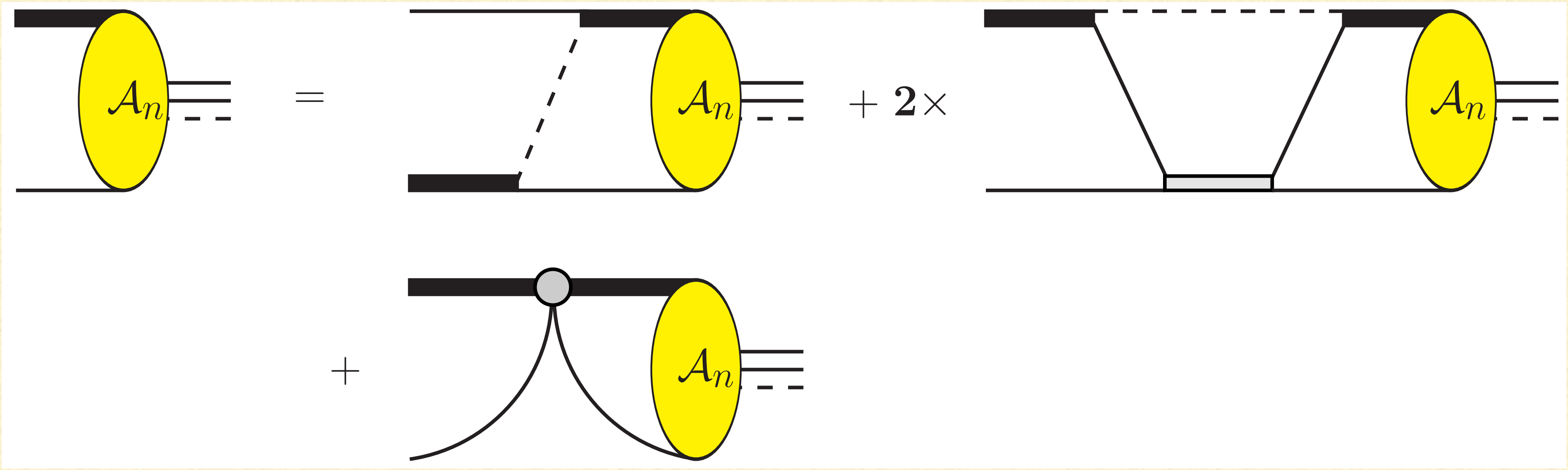
- R≈1.5 fm; M_{lo}≈40 MeV
- ^{4}He-n interaction: $^{2}P_{3/2}$ resonance

$$\langle \mathbf{k} | t_{n\alpha} | \mathbf{k}' \rangle = -\frac{6\pi}{\mu_{n\alpha}} \frac{\mathbf{k} \cdot \mathbf{k}'}{-\frac{1}{a_1} + \frac{1}{2} r_1 k^2}$$

- "Standard" counting for nn: a_0 at leading order, r_0 at NLO
- $^{2}P_{3/2}$: at NLO unitarity piece/width included perturbatively
- $^{2}S_{1/2}$: NLO effect: a_0=2.46 fm is "natural"
- p-wave power counting only valid when not near the $^{2}P_{3/2}$ resonance
- $^{2}P_{1/2}$ resonance?

“STM” equation for ^{6}He

Ji, Elster, DP (2014)



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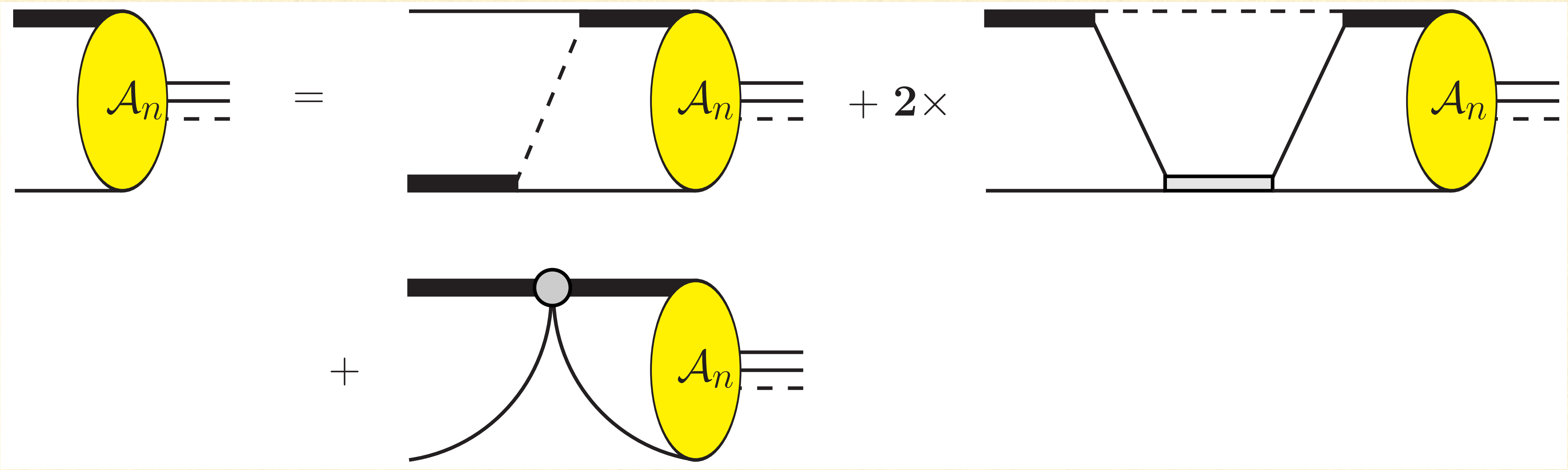

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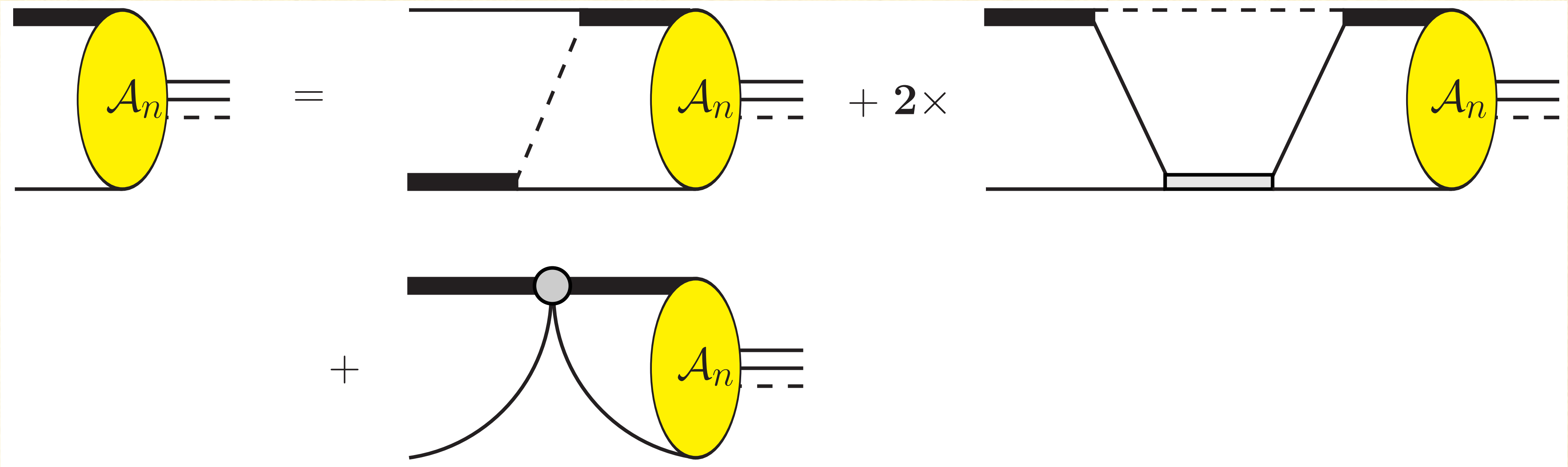

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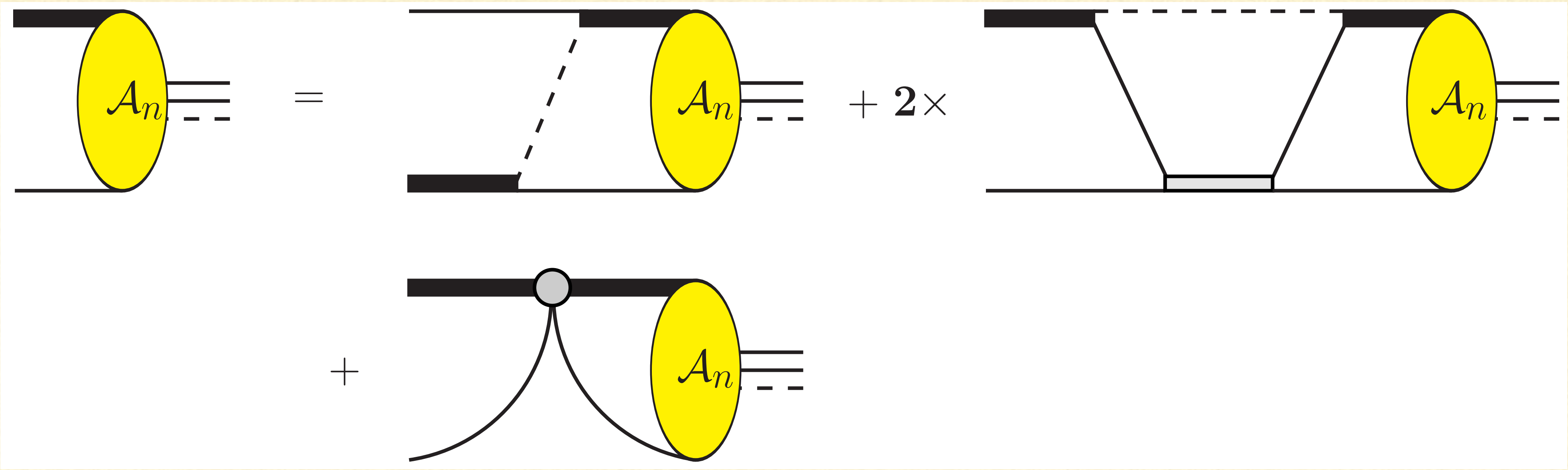

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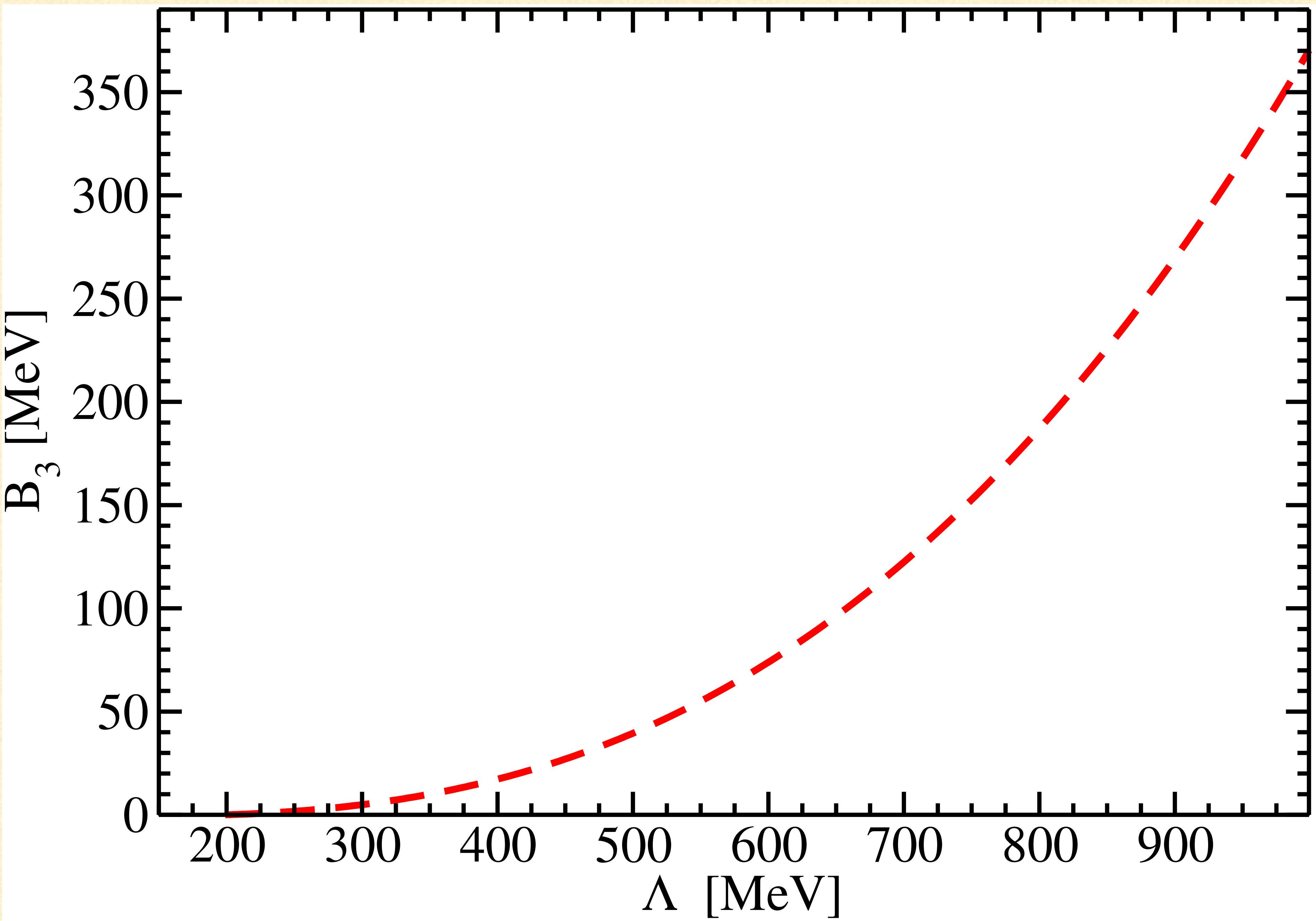
- Is three-body force necessary at LO? No analytic results, so numerics

Renormalizing ^{6}He



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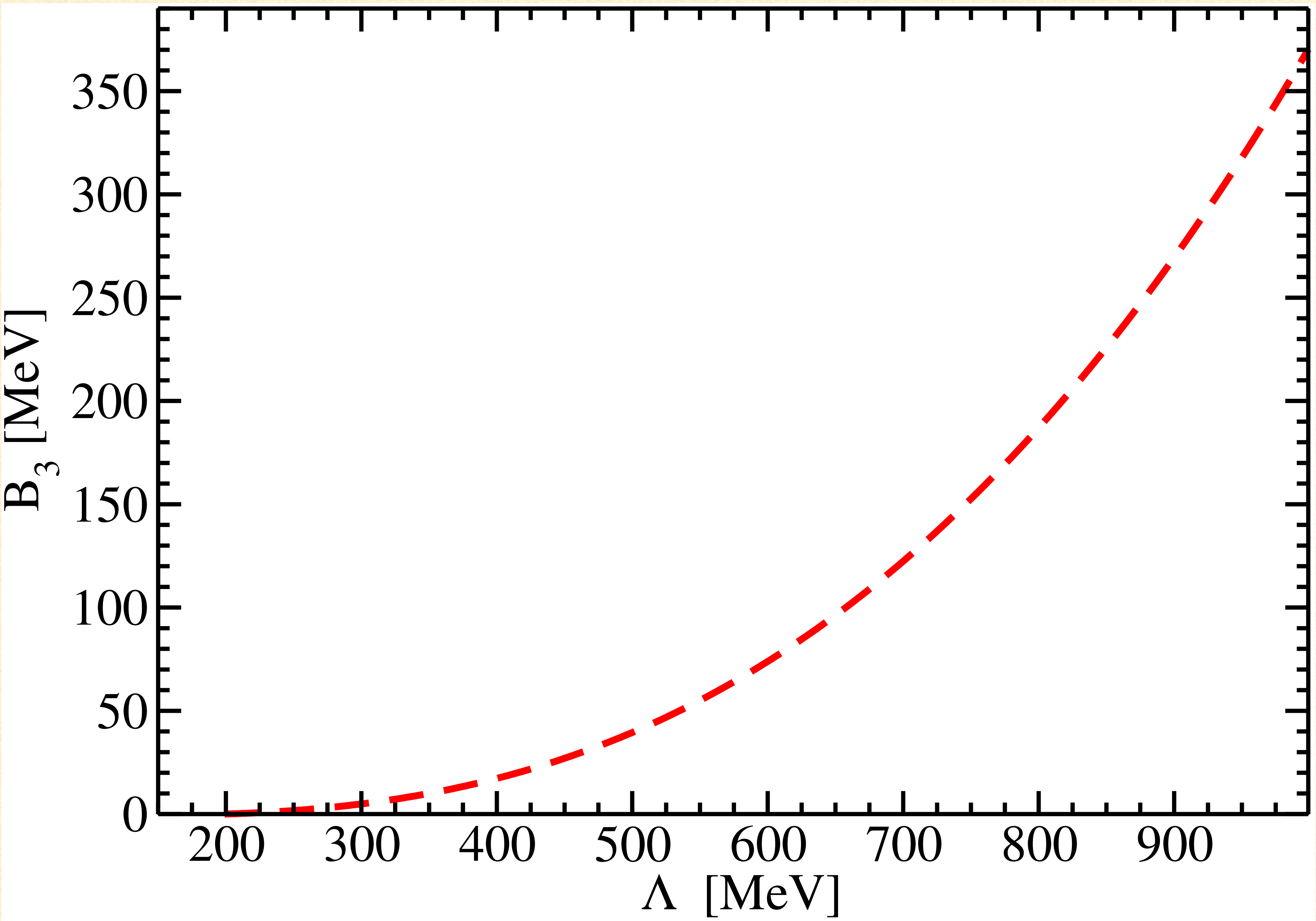

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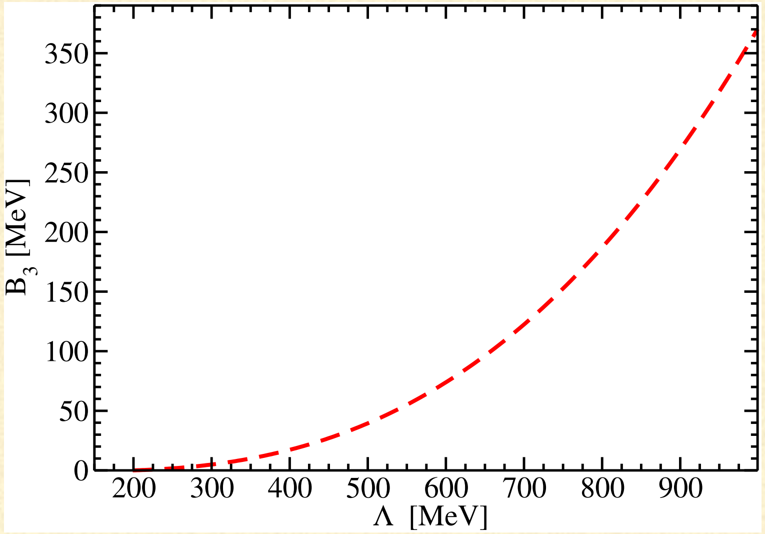

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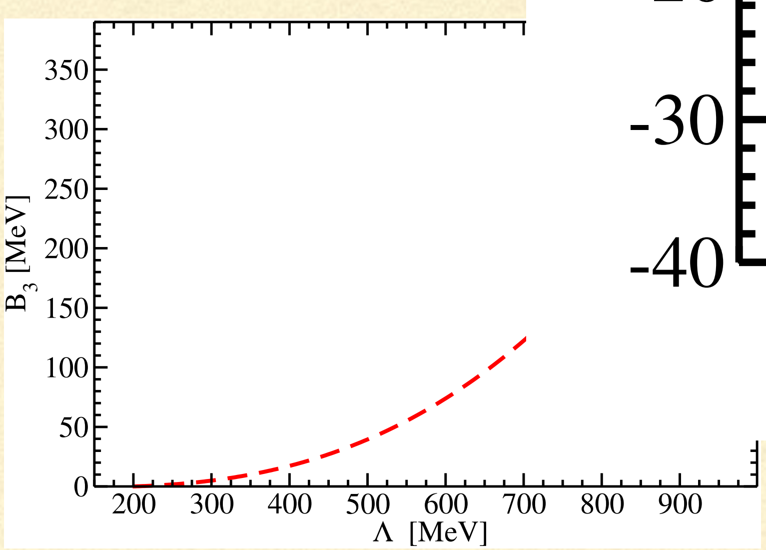

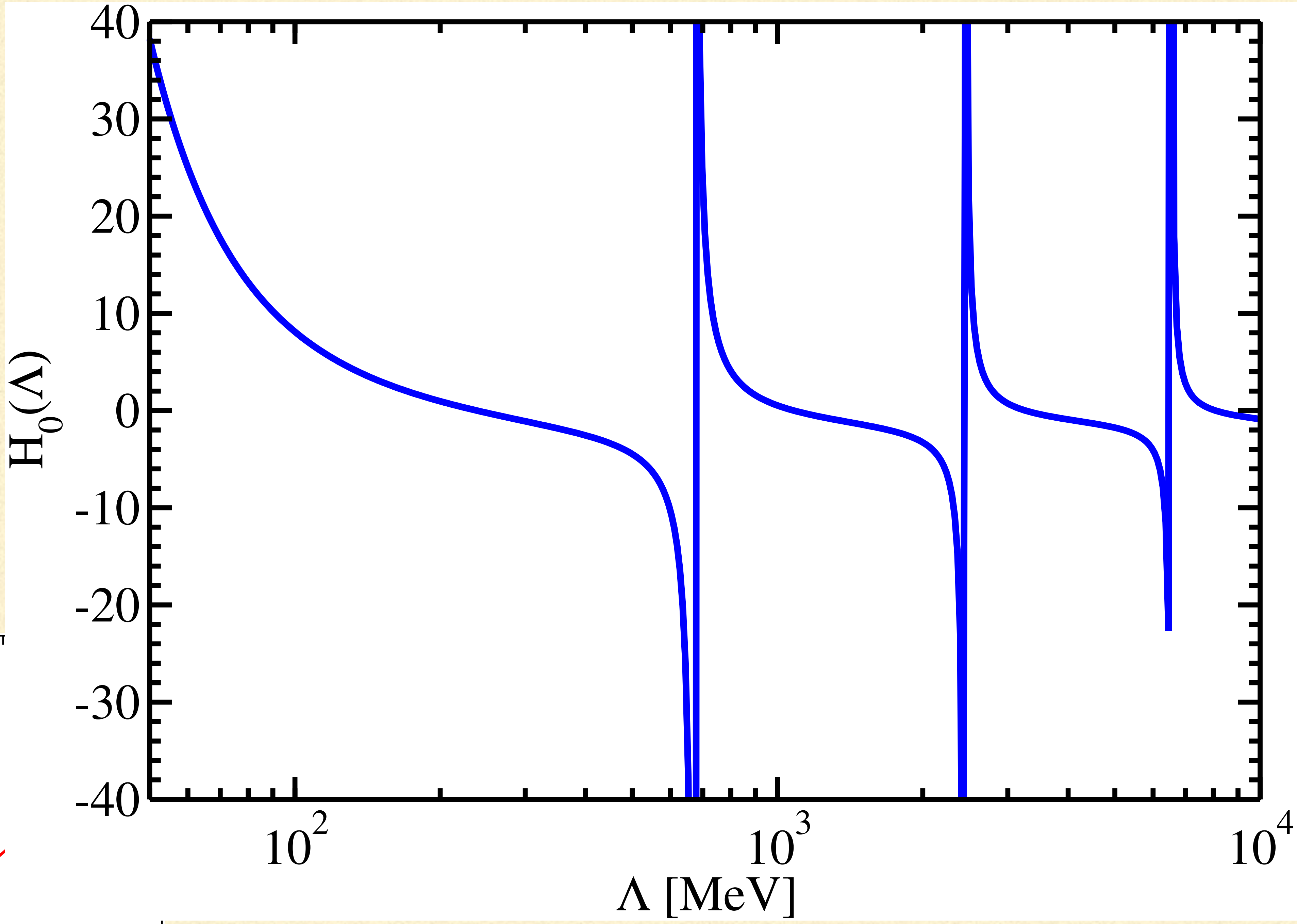

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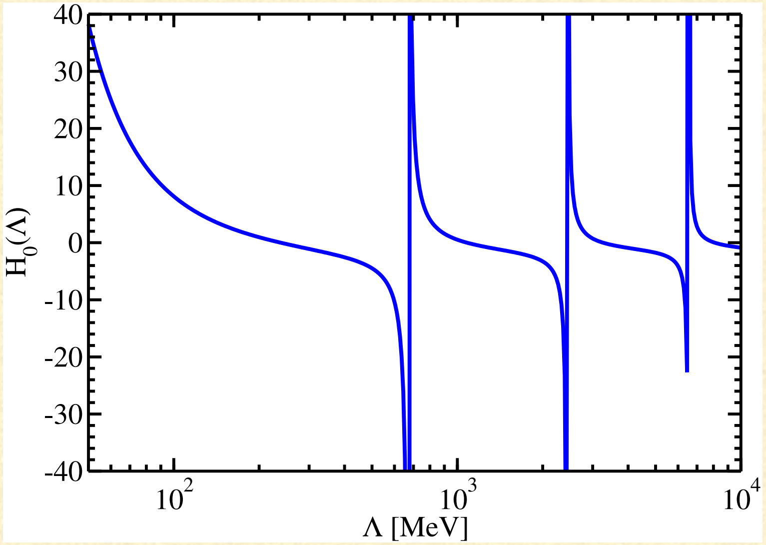

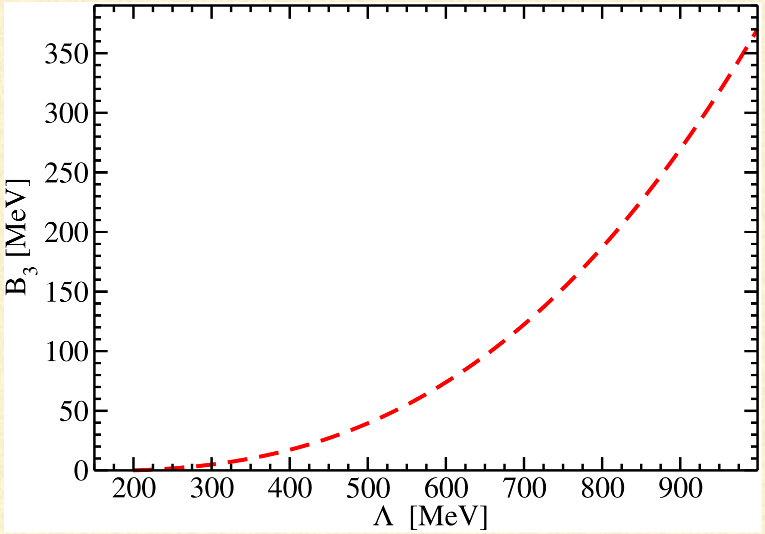

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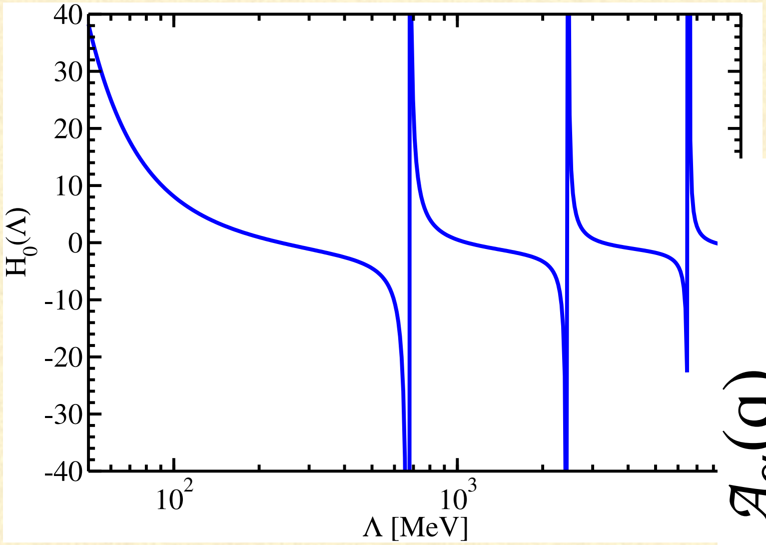

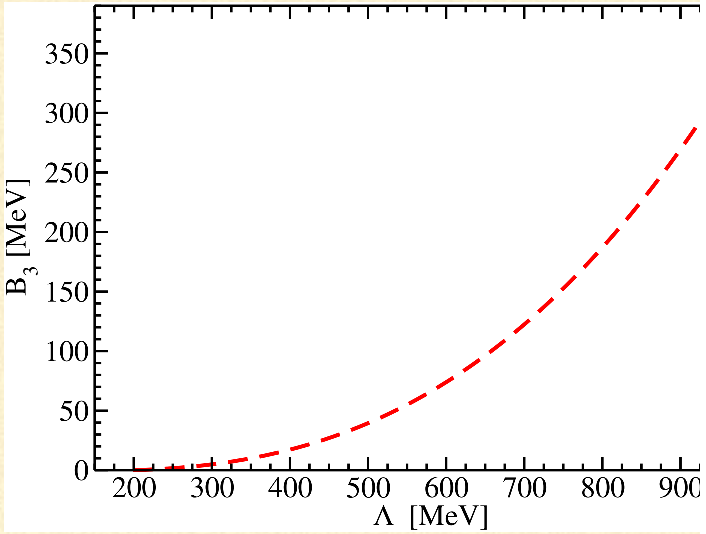

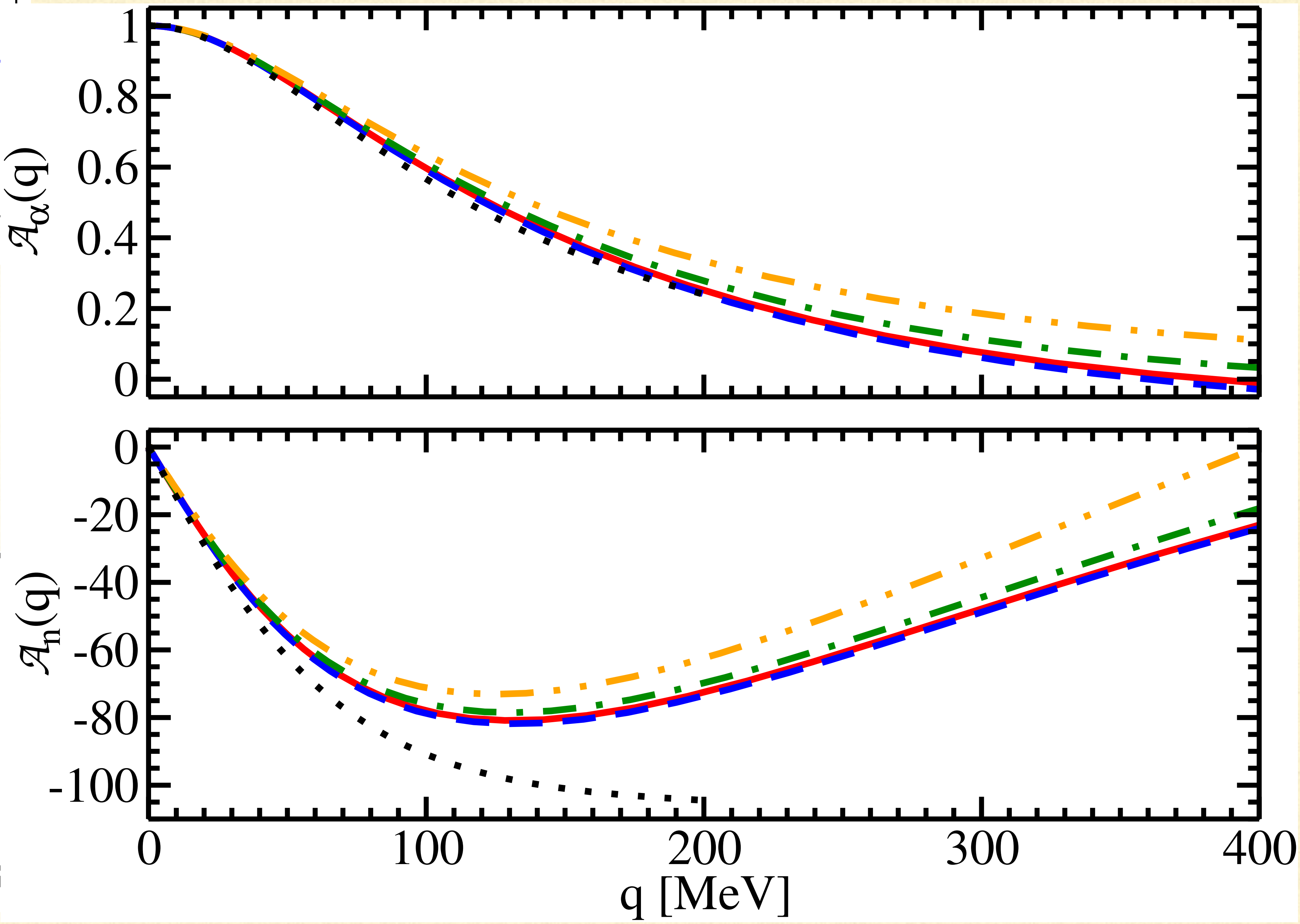

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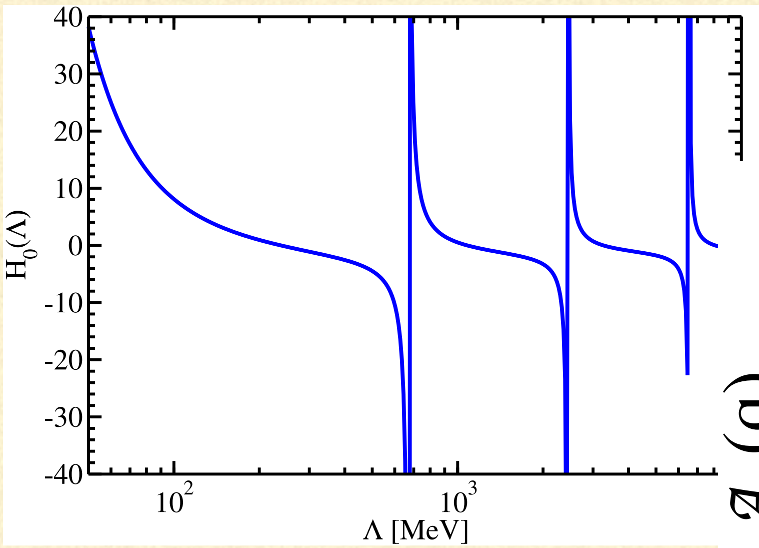
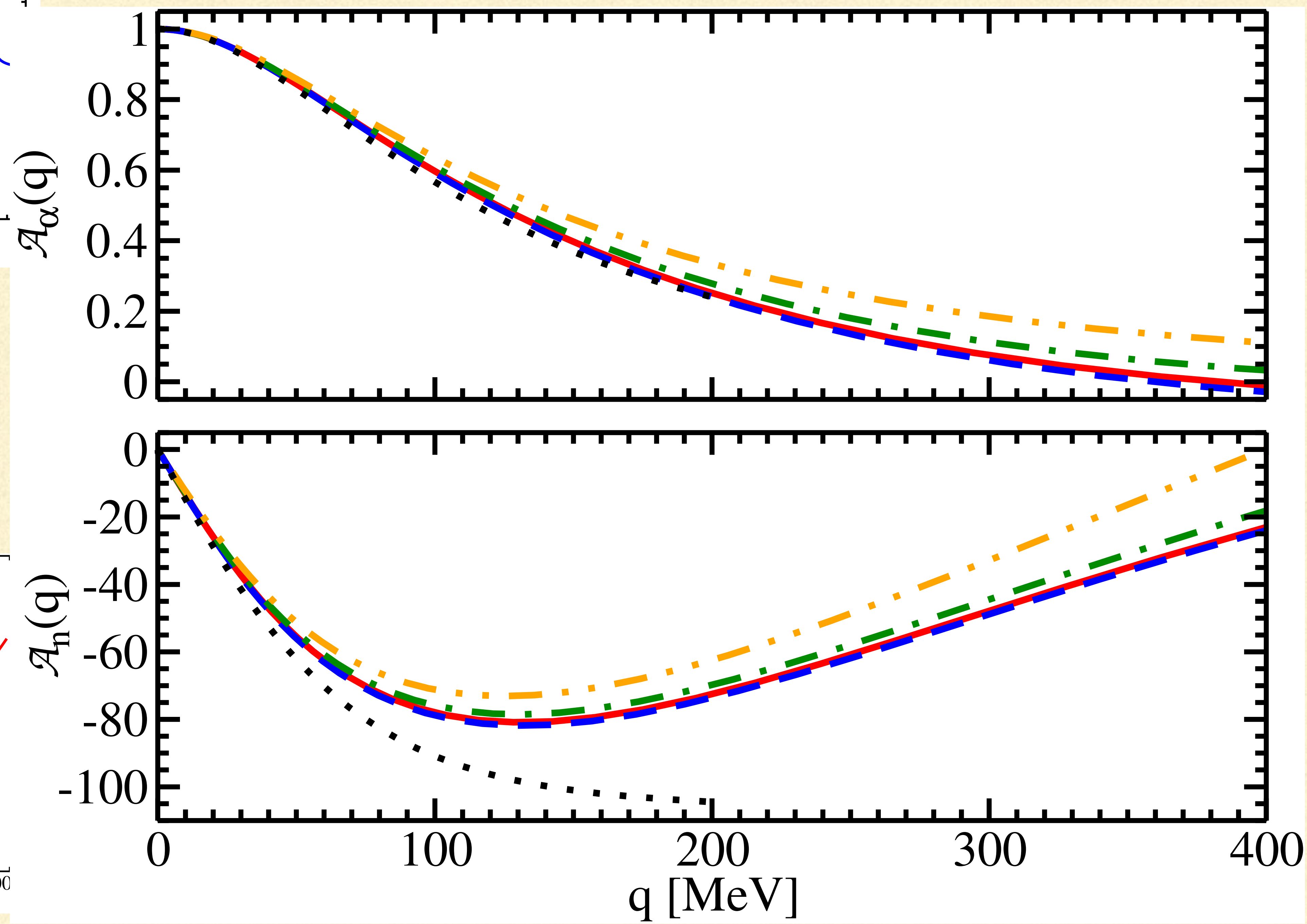
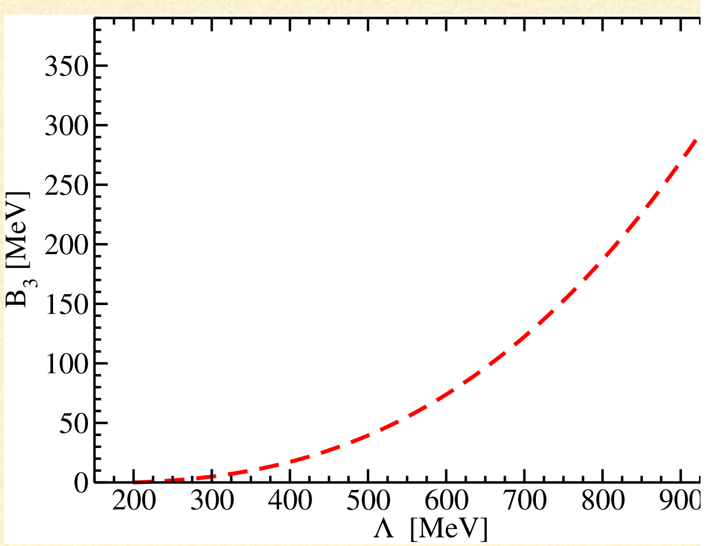
Ji, Elster, DP (2014)

cf. Rotureau and van Kolck (2013)

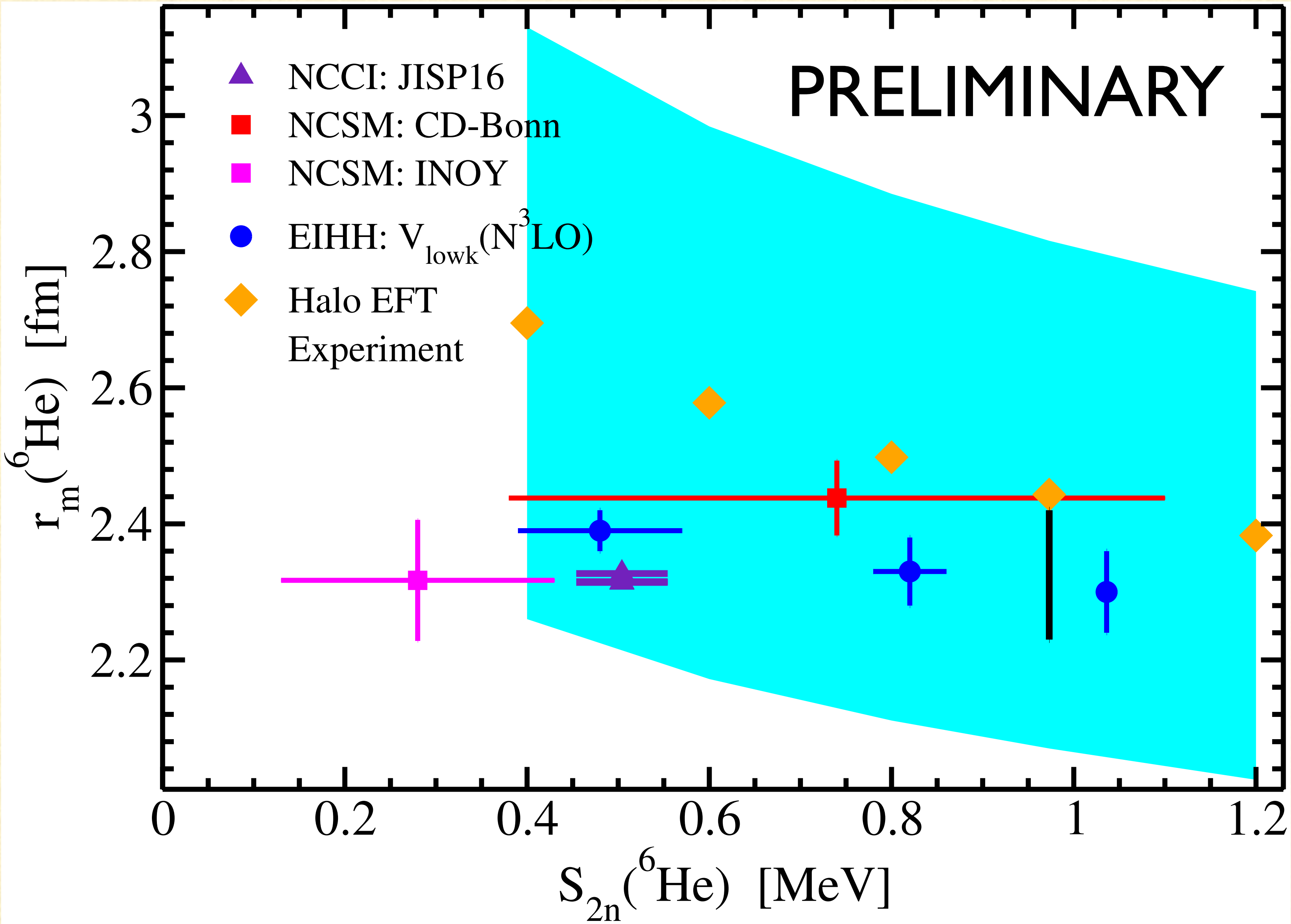






^{6}He matter radius

Ji, Elster, DP (in preparation)



Energy-dependent potential already at leading order unless unitarity/width treated perturbatively→not as simple as just "get wave function and use quantum mechanics"

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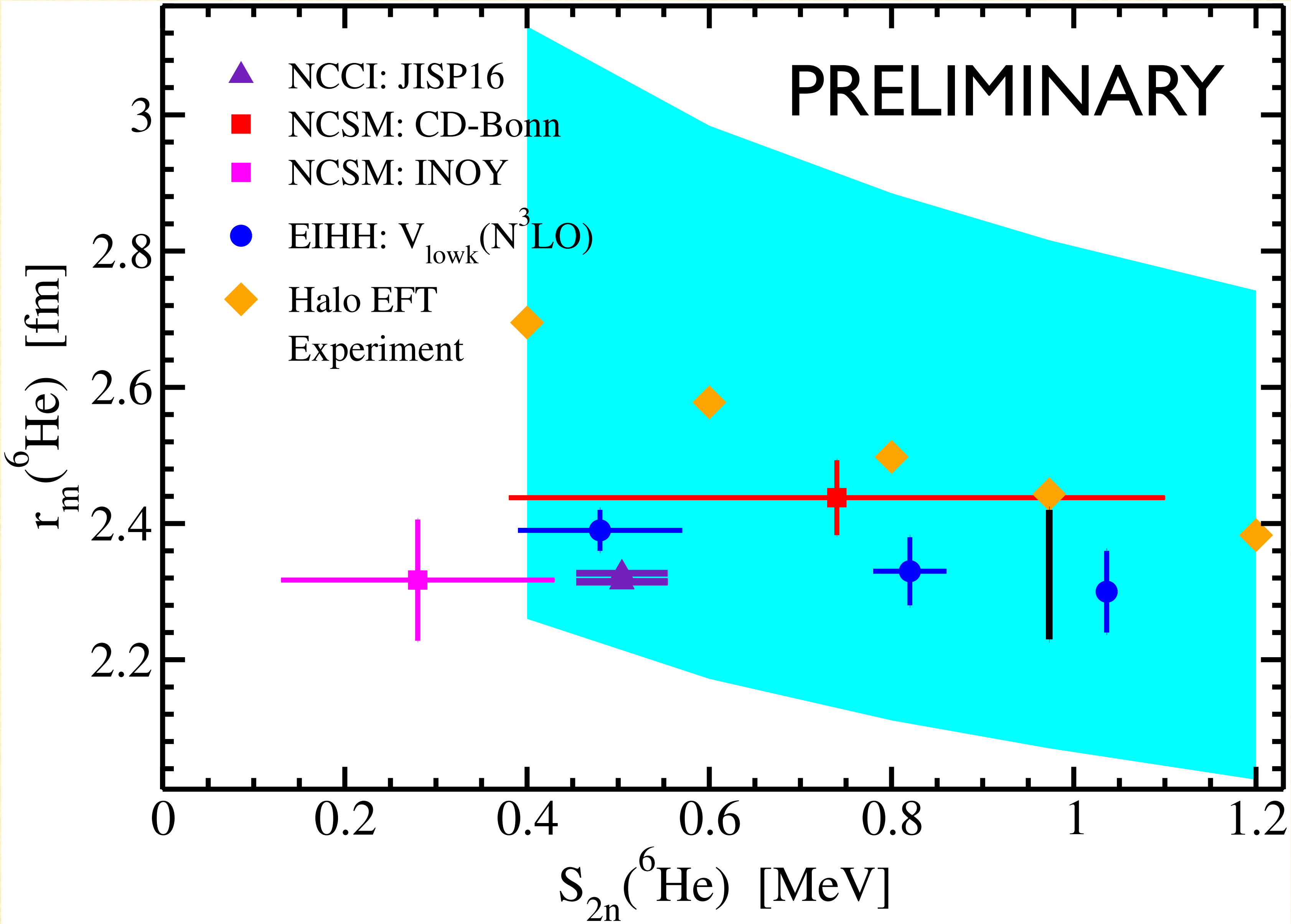
Helium-6 matter radius as a function of B



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- Impact of short-distance operators at higher orders in EFT expansion? Helpful to have asymptotic form of "STM" solution

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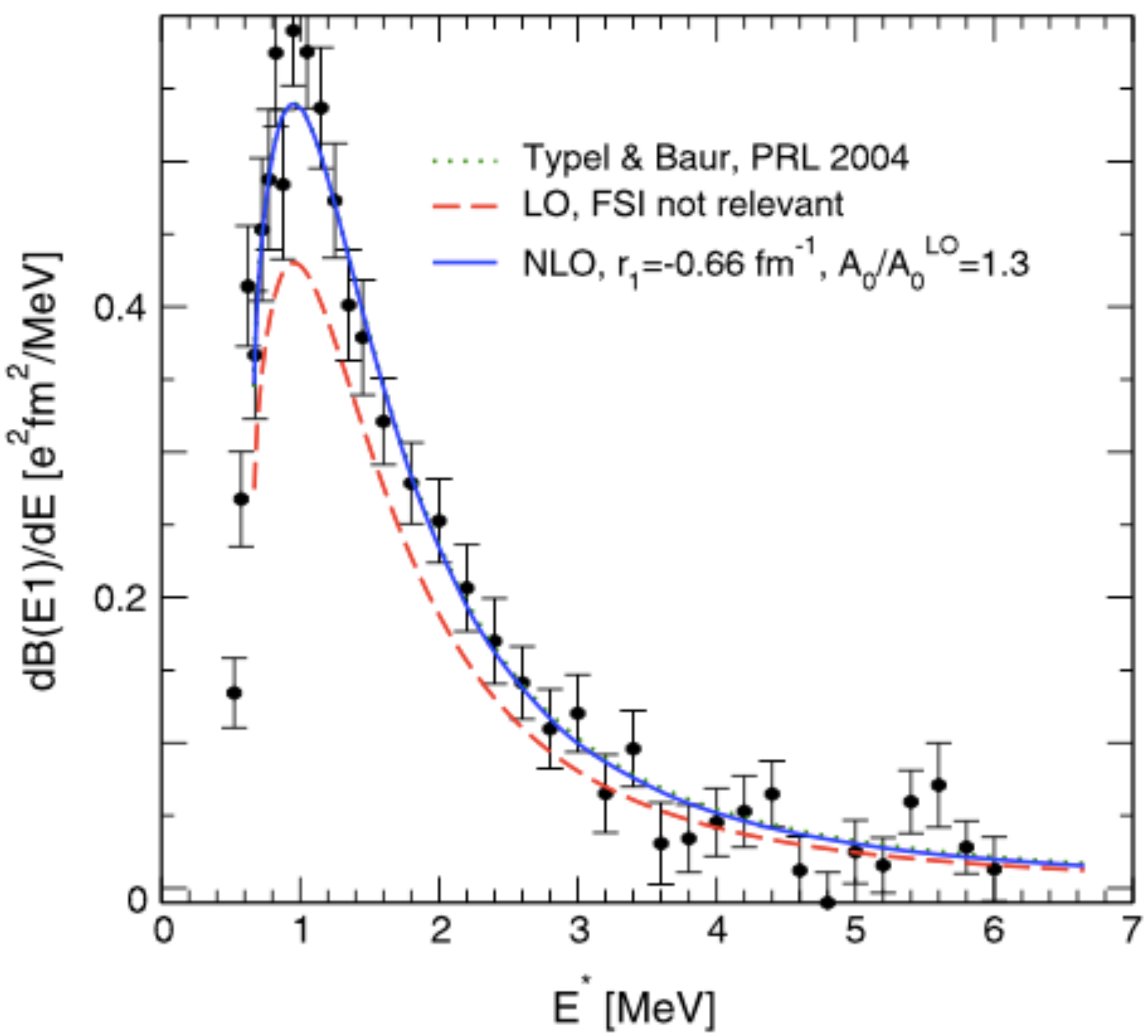
- What about higher partial waves?

Backup slides: Coulomb dissociation

Coulomb dissociation: result



Data: Palit et al., 2003
Analysis: Hammer, Phillips. NPA, 2011

- Reasonable convergence

- Information on value of r_0 through fitting of A_0:

r_0=2.7 fm

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- Here value of r_1 used to fit $B(E1{:}1/2^+ \rightarrow 1/2^-)$ works.

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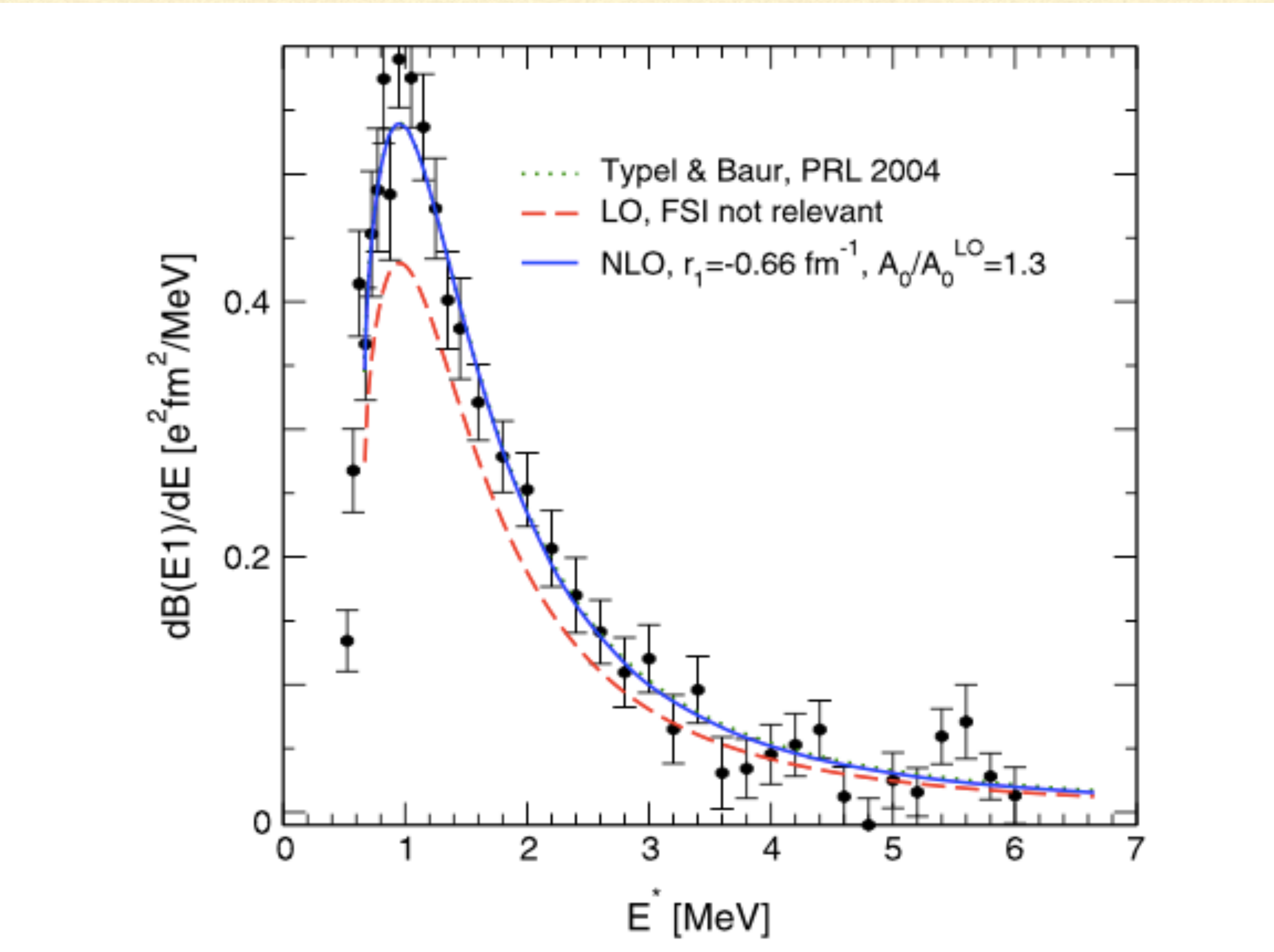

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Other ANC/r_1 measurements? Tests of p-wave universal relations?

Coulomb dissociation: formulae

c.f. Rupak & Higa arXiv:1101.0207

- Straightforward computation of diagrams yields:

$$\frac{d\mathrm{B(E1)}}{dE} = e^2 Z_{eff}^2 \frac{m_R}{2\pi^2} A_0^2 \left(\frac{p'^3 [2p'^3 \cot(\delta^{(1/2)}(p')) + \gamma_0^3 + 3\gamma_0 p'^2]^2}{[p'^6 + p'^6 \cot^2(\delta^{(1/2)}(p'))](p'^2 + \gamma_0^2)^4} + \frac{8p'^3}{(p'^2 + \gamma_0^2)^4} \right)$$

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- Higher-order corrections to phase shift at NNLO. Appearance of S-to-$^2P_{1/2}$ E1 counterterm also at that order.

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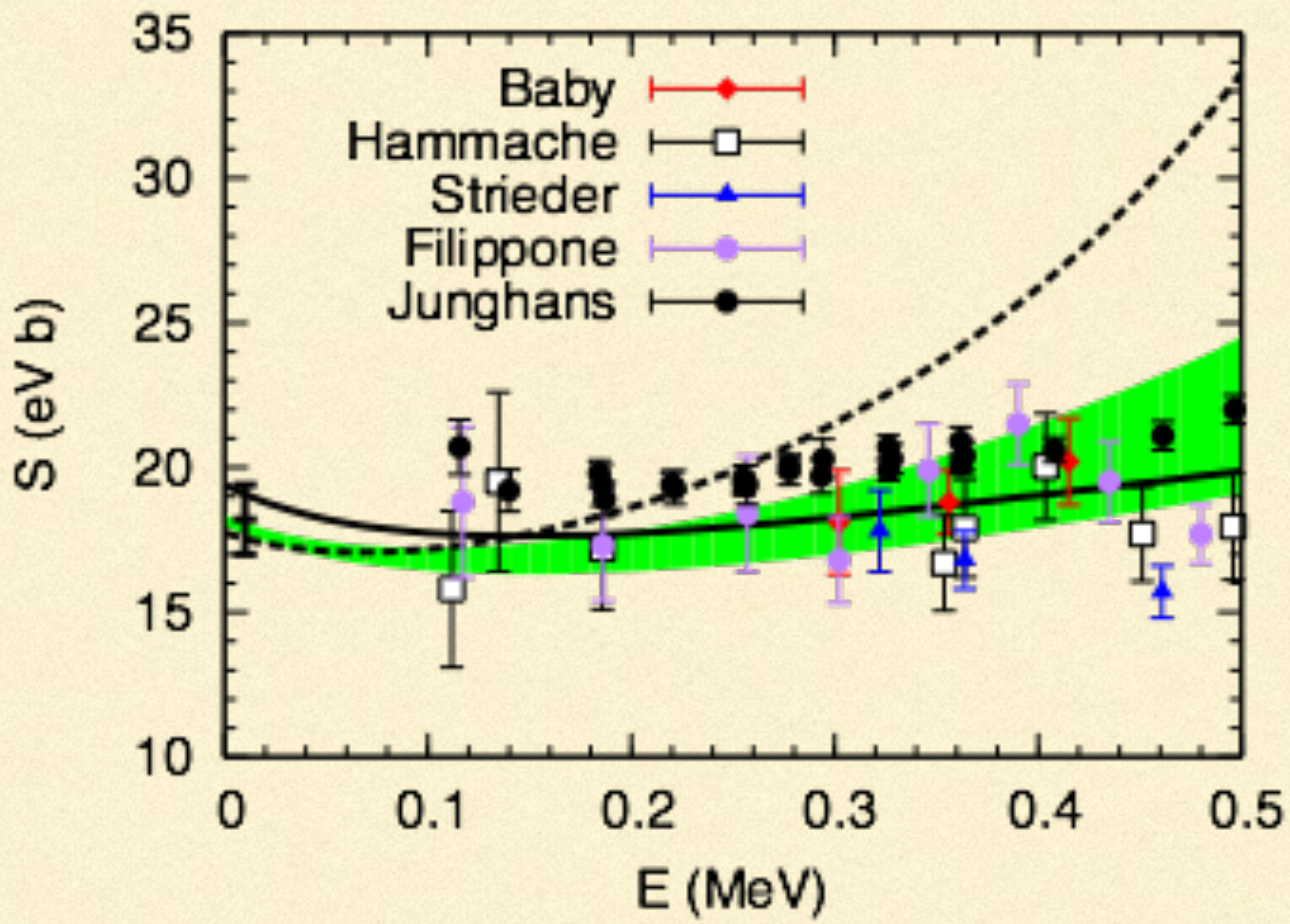

Proton capture on ^{7}Be: results

- ANCs from *ab initio* consistent with estimated 1/R
- S(0) controlled by p-wave ANCs
- Scattering parameters play key role at higher energies
- Approved TRIUMF experiment on p + ^{7}Be elastic scattering
- NLO calculation: fit short-distance contribution to data