Boson droplets close to the unitary limit

Mario Gattobigio

INPHYNI
Institut de Physique de Nice

KITP, 23 May 2022

UNIVERSITÉ CÔTE D'AZUR

cnrs

Outline

Universal Window

- Zero-range universality
- Finite-range universality

^{4}He Effective Potential Description

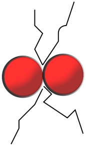
- LO Gaussian Potential - Two Body Force
- LO Gaussian Potential - Two+Three Body Force
- NLO Gaussian Potential - Two Body Force
- NLO Gaussian Potential - Two Body Force + LO Three Body
- NLO Gaussian Potential - Additional Many Body

Conclusions

Universality

@ Low Energy
$\ell \ll \ell_{\text{de Broglie}}$ $\Rightarrow$ Physics governed by the scattering length a

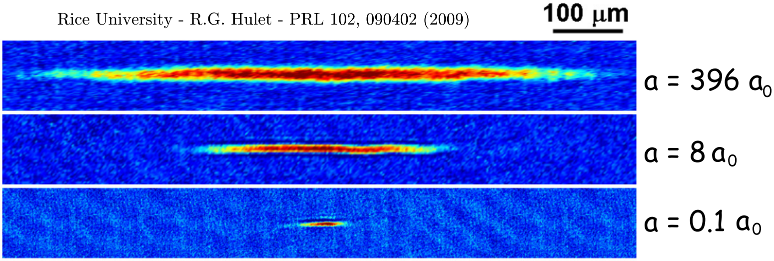
- $a \sim \ell$ Perturbative weak-coupling regime



$$V(r) \propto a\delta(r)$$

  - Gross-Pitaevskii ($V(r) \propto an^2$) for BEC

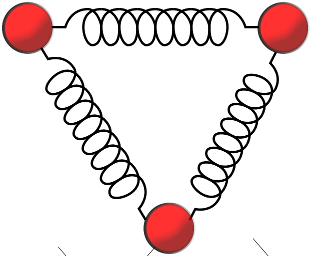
Rice University - R.G. Hulet - PRL 102, 090402 (2009)



Universality

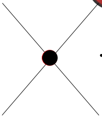
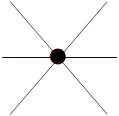
@ Low Energy
$\ell \ll \ell_{\text{de Broglie}}$ $\Rightarrow$ Physics governed by the scattering length a

- $a \gg \ell$ Non-perturbative regime



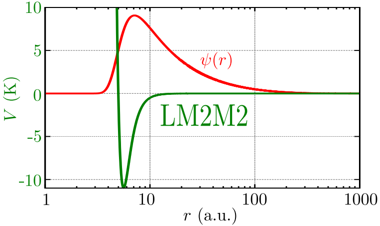
$$E_2 \approx -\frac{\hbar^2}{ma^2} \text{ for } a > 0$$

Efimov effect
$\Rightarrow$
New three-body parameter

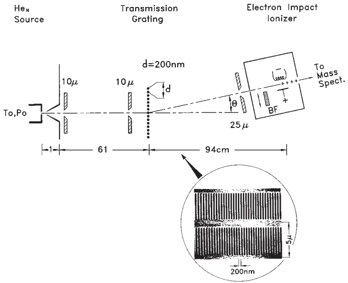
V $\propto$ a  + λ_3 

Natural fine tuning

Atomic Physics - ^{4}He



Schöllkopf and Toennies
J. Chem. Phys. 104, 1155 (1995)

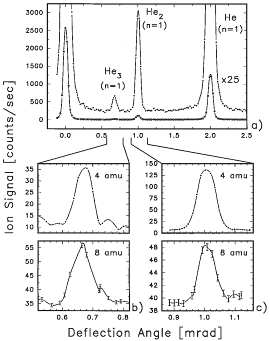


$$\ell_{vdW} \approx 10 a.u.$$

$$a \approx 190 \text{ a.u.}$$

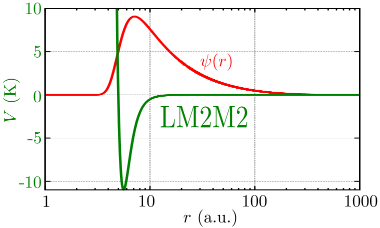
$$E_2 \approx -1.30 \text{ mK} \approx \hbar^2/ma^2$$

$$E_3^{(0)} \approx -126 \text{ mK and } E_3^{(1)} \simeq -2.3 \text{ mK}$$



Natural fine tuning

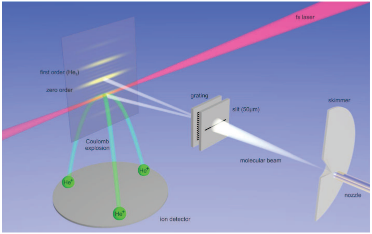
Atomic Physics - ^{4}He



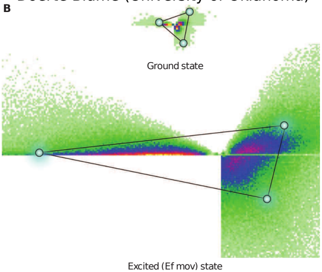
$$\ell_{vdW} \approx 10 a.u.$$
$$a \approx 190 \text{ a.u.}$$
$$E_2 \approx -1.30 \text{ mK} \approx \hbar^2/ma^2$$
$$E_3^{(0)} \approx -126 \text{ mK and } E_3^{(1)} \simeq -2.3 \text{ mK}$$

M. Kunitski et al., Science **348**, 551 (2015)

Reinhard Doerner - University of Frankfur



Doerte Blume (University of Oklahoma)



Finite range

- Effective Range Expansion with one shallow two-body state

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r_e k^2 + \sum_{n=2} P_n r_e^{2n-1} k^{2n}$$

- Simplest S-matrix

$$S(k) = \frac{k + i/a_B}{k - i/a_B} \frac{k + i/r_B}{k - i/r_B}$$

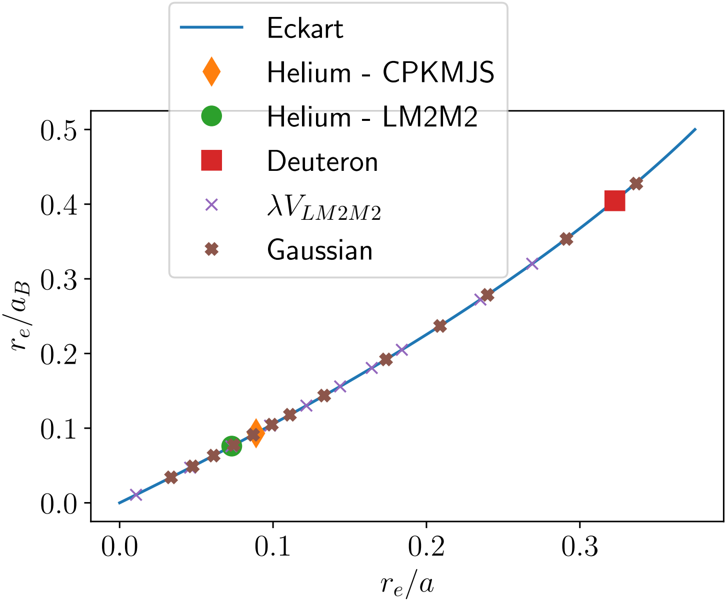
where

$$E_2 = -\frac{\hbar^2}{m a_B^2} \quad \text{and} \quad r_B = a - a_B \quad \text{with} \quad a r_e = 2 a_B r_B$$

- Eckart potential

$$V(r) = -2\beta\lambda^2 \frac{\mathrm{e}^{-\lambda r}}{(1 + \beta \mathrm{e}^{-\lambda r})^2}$$

Finite range universality



Finite range universality

- Universality for two-particle low-energy observables
  - $r_B = a - a_B =$ Constant
  - Asymptotic Constant

$$C_a^2 = \frac{2}{a_B} e^{2r_B/a_B}$$

  - Mean Square Radius

$$\langle r^2 \rangle = \frac{a_B^2}{8} e^{2r_B/a_B}$$

  - Probability for the particle to be outside the interaction region

$$P_e = e^{-2r_B/a_B}$$

- Effective Description using Gaussian Potential

$$V(r) = V_0 \, e^{-(r/r_0)^2}$$

Effective Gaussian Description of ^{4}He

- "*Reference*" ^{4}He given by LM2M2 potential

$$\bar{a} = 189.415\, a_0, \bar{r}_e = 13.845\, a_0, \text{and} \;\; r_B = 7.194\, a_0$$

N	$\bar{E}_N$(mK)	$\bar{E}^*_N$(mK)
2	-1.30348	
3	-126.40	-2.2706
4	-558.98 [Hiyama 2012]	-127.33 [Hiyama 2012]
5	-1300 [Bazak 2020]	
6	-2315 [Bazak 2020]	
7	-3571 [Bazak 2020]	

- Effective Gaussian Potential

$$V_{\text{LO}}(r) = V_0\, e^{-(r/r_0)^2}$$

- Small parameter

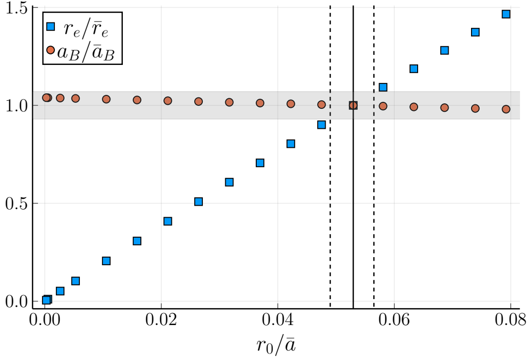
$$\varepsilon = \bar{r}_e/\bar{a} \approx 7\%$$

Two Body

- Effective Gaussian Potential

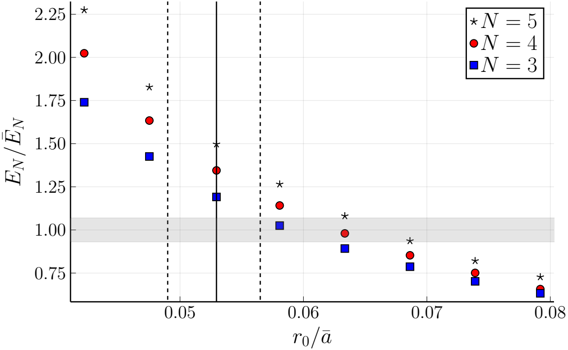
$$V_{\text{LO}}(r) = V_0\, e^{-(r/r_0)^2}$$

- Fix only $\bar{a}$



- Look for $\varepsilon = \bar{r}_e/\bar{a} \approx 7\%$ description also for r_e

Few Body



- Not inside the $\varepsilon = 7\%$ band
- Collapse as $N \to \infty$

$$\frac{E_N}{N} = \frac{V_0}{2} N$$

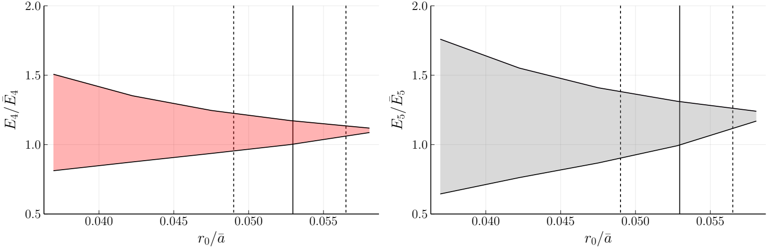
- Need for a three-body force

Few Body

- Three-body force

$$W_{\text{LO}} = W_0\, e^{-(r_{12}^2 + r_{13}^2 + r_{23}^2)/\rho_0^2}$$

- A family of values (W_0, ρ_0) which fix $\bar{E}_3$
- Variation in $\bar{E}_N$

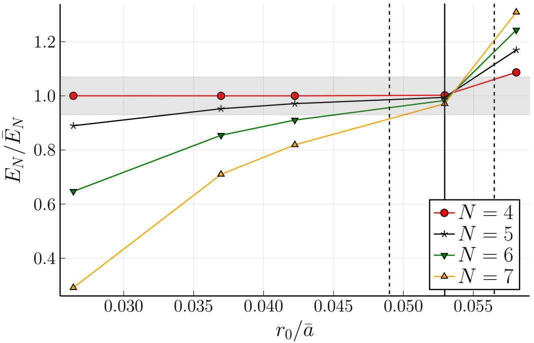


- We can use (W_0, ρ_0) to best fix $\bar{E}_4$

LO Gaussian Description

- LO Potential

$$V_0\, e^{-(r/r_0)^2} + W_0\, e^{-(r_{12}^2+r_{13}^2+r_{23}^2)/\rho_0^2}$$



- Best point is where we reproduce $\bar{a}$, $\bar{a}_B$, and r_e!!

LO Gaussian Description

- Description within the ε-LO band up to liquid

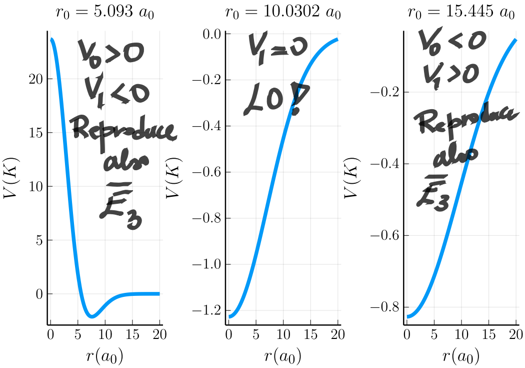
	Physical point	
	SGP	HFD-HE2
$r_0[a_0]$	10.0485	
V_0[K]	1.208018	
$\rho_0[a_0]$	8.4853	
W_0[K]	3.011702	
E_4[K]	0.536	0.536
E_5[K]	1.251	1.266
E_6[K]	2.216	2.232
$E_{10}/10$[K]	0.792(2)	0.831(2)
$E_{20}/20$[K]	1.525(2)	1.627(2)
$E_{40}/40$[K]	2.374(2)	2.482(2)
$E_{70}/70$[K]	3.07(1)	3.14(1)
$E_{112}/112$[K]	3.58(2)	3.63(2)
$E_N/N(\infty)$[K]	7.2(3)*	7.14(2)
HFD-B [K]		7.33(2)

NLO Gaussian Description - Two body

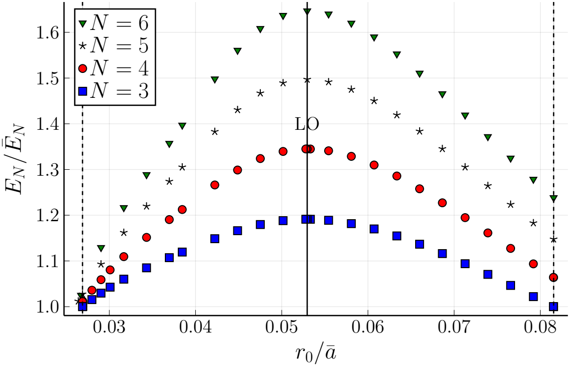
- NLO two-body force

$$V_{\text{NLO}}(r) = V_0\, e^{-(r/r_0)^2} + V_1\, \frac{r^2}{r_0^2} e^{-(r/r_0)^2}$$

- We fix both $\bar{a}$ and $\bar{r}_e$



NLO Two body - Few-body energies



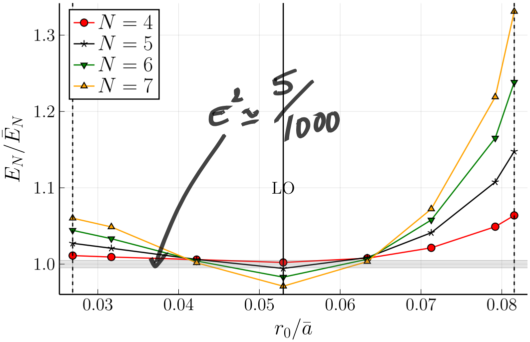
- Without 3-body force the system is unstable

$$\frac{E_N}{N} \propto N$$

NLO Two body + LO Three body

- With the LO 3-body force

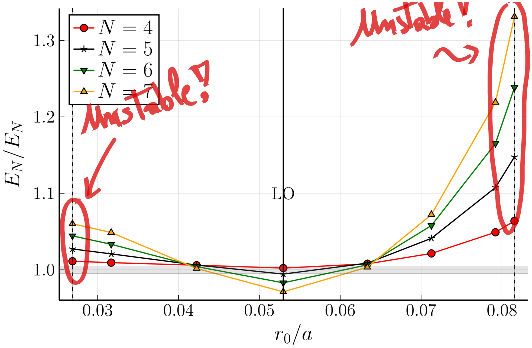
$$W_{\rm LO} = W_0 \, e^{-(r_{12}^2 + r_{13}^2 + r_{23}^2)/\rho_0^2}$$



NLO Two body + LO Three body

- With the LO 3-body force

$$W_{\text{LO}} = W_0\, e^{-(r_{12}^2+r_{13}^2+r_{23}^2)/\rho_0^2}$$

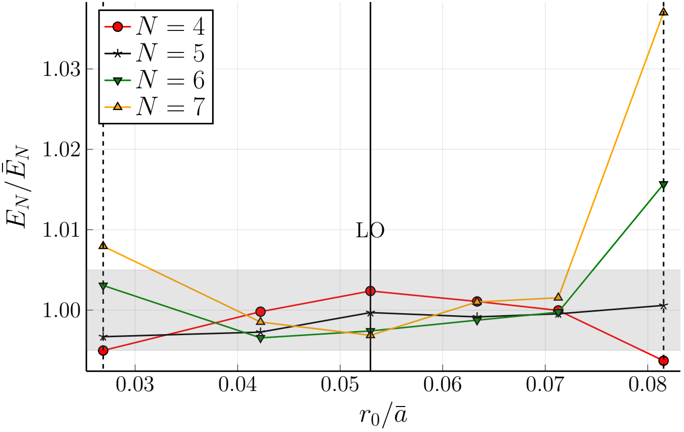


- Need another force at NLO!!!

Analysis with NLO 3-Body

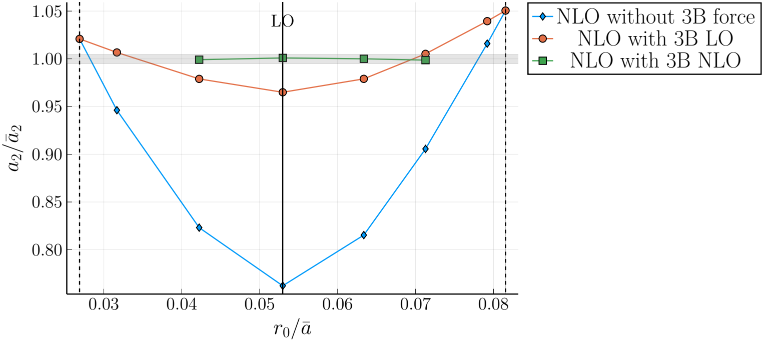
- NLO Three-body force

$$W_{\text{NLO}} = W_0 e^{-r_{123}^2/\rho_0^2} + W_1 \left(\frac{r_{123}}{\rho_0}\right)^2 e^{-r_{123}^2/\rho_0^2}$$



Analysis with NLO 3-Body

- What happens to different three-body observables?
- Atom Dimer scattering length $\bar{a}_2 = 218\ a_0$

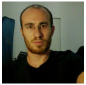


- Different 3-Body potential strengths

Conclusion

- Potential Effective Description of states inside Unitary Window
- Expansion inspired by EFT
  - Small parameter $\varepsilon = \bar{r}_e/\bar{a}$
- Rôle of the potential range(s)
- Stability as $N \to \infty$
  - Hierarchy of forces

Collaborators



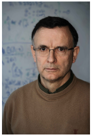
Paolo Recchia

Alejandro Kievsky

Michele Viviani

Luca Girlanda

Natalia Timofeyuk



Artur Polls

Bruno Julia Diaz