Sheets shaping liquids and liquids shaping sheets

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It takes a village...

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0. Wrapping a droplet with a thin solid
1. Indenting a floating film
2. Beyond the “gross shape”
3. Towards solid surfactants

Image credit: Rene Sorensen
We wrap objects in thin sheets to conceal, protect, or enhance them.

(b) Chen et al., Nano Lett. 12 (2012)
(c) Py, Bico, Roman, et al. (2007)
(d) Image courtesy of NASA

What happens for a film that is 1000 times thinner? Bending energies (~t^3) are then ×10^9 smaller!
1 mm circular polystyrene sheet

silicone oil

fluorinated oil

needle
t = 80 nm circular sheet
How do thin sheets respond to confinement?

...they wrinkle
Smooth deformation

...they fold
Localized deformation

...they crumple
Localized deformation and stress

Subtle physics, nonlinear sheet equations

Wrapping: all of these in highly curved geometry!
The Plan:

actual shape

=Gross shape

+“small-scale features”

(wrinkles, crumples, folds)

…which allow in-plane compression at negligible cost

Gross shape can compress but not stretch:

$|f(x) - f(y)| \leq |x - y|$

where $f$: planar sheet $\rightarrow$ gross shape

Geometric model: $U = \gamma A_{\text{free}}$ (System seeks gross shape that minimizes $A_{\text{free}}$)

JDP, Démery, et al., Nat. Mater. 2015
Test case 1: Large droplet (gross shape is axisymmetric)
Global minimum of $U = \gamma A_{\text{free}}$ among all axisymmetric configurations.

Sheet: elliptic integral (enabled by small-scale wrinkling, not shown)

Drop: sphere

Single parameter: $W/R$

$W$: sheet radius

$(4\pi/3) R^3$: drop volume

Comparing geometric model \((U = \gamma A_{\text{free}})\) with experiment

Experiment:

- Sheet: elliptic integral
- Drop: sphere

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Optimization:

Maximize volume for given surface area

Maximize volume for given arclength

Solution:

Sphere

Elliptic integral:

\[ f(x) = \int_x^a \frac{t^2}{\sqrt{a^4 - t^4}} \, dt \]

where \( a = \frac{4r\sqrt{2\pi}}{\Gamma(1/4)^2} \)

Example:

- Liquid droplet, soap bubble
- G.I. Taylor 1919: Parachute
- Paulsen (not me) 1994: Mylar balloon

1. Draw a line starting and ending on z-axis
2. Consider its surface of revolution
What about small drops?
(no longer axisymmetric)
Reduce volume

Minimize exposed surface area

Breaking axial symmetry **improves coverage**

"Surface Evolver" simulations

Optimal axisymmetric

Maximize enclosed volume

Breaking axial symmetry improves coverage


A free

Surface Evolver simulations

Maximize enclosed volume

Reduce volume

$A_{\text{free}} / (4\pi R^2)$

$W/R$
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Non-spherical gross shapes come from simple geometric optimization ($U = \gamma A_{\text{free}}$)

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Indentation:
Holmes & Crosby, PRL 2010
Vella, Davidovitch, et al., PRL 2015, PRE 2018
Paulsen, Hohlfeld, et al., PNAS 2016
Ripp, Démery, Zhang, JDP, Soft Matter 2020

Monica Ripp
Geometric model for poking:

- **Energy functional**, now with gravity
  \[ U = U_{\text{gravity}} + \gamma(\Delta A_{\text{free}}) \]
  \[ = \pi \int_0^\infty \left[ \rho gr\zeta(r)^2 + 2\gamma R \left( \sqrt{1 + \zeta'(r)^2} - 1 \right) \right] dr \]

- Optimal \( \zeta(r) \) satisfies **Euler-Lagrange equation**:
  \[ \zeta''(r) = \frac{r\zeta'(r)}{\ell_{\text{curv}}}^3 \left[ 1 + \zeta'(r)^2 \right]^{3/2} \]
  where \( \ell_{\text{curv}} = \ell_c^{2/3} R^{1/3} \)

- **Small slopes**: Reduces to \( \zeta''(r) = r\zeta'(r)/\ell_{\text{curv}}^3 \) (with same Airy function solution from Dominic’s talk)

- **BONUS**: Geometric model gives access to **large slopes behavior**:
  - Profile: \( \zeta(r) \sim -[3 \log(1/r)/2]^{2/3} \) as \( r \to 0 \)
  - Energy: \( U \sim 2\pi\gamma R\delta \)
  - Force: \( F \sim 2\pi\gamma R \)
“Spring constant” $F/\delta$, felt by indenter

\[ F/\delta \]

\[ \delta^* \quad \delta^{**} \quad \delta^{***} \]

I: linear
II: stiffening
III: pseudo-linear
IV: softening

Force:

I: $F \sim \gamma \delta$
II: $F \sim \sqrt{Y \rho g \delta^2}$
III: $F \approx 4.581 (\gamma R_{\text{film}})^{2/3} (\rho g)^{1/3} \delta$
IV: $F \sim 2\pi \gamma R_{\text{film}}$

Föppl–von Kármán (cf. Benny, Dominic's talks)
Vella, Davidovitch, et al., *PRL* 2015, *PRE* 2018

Geometric model: $U = U_{\text{gravity}} + \gamma (\Delta A_{\text{free}})$
Ripp, Démery, Zhang, JDP, *Soft Matter* 2020
"Spring constant" $F/\delta$, felt by indenter

\[ \delta_* \sim \frac{Y}{\sqrt{ρg}} \]
\[ \delta_{**} \sim R^{2/3} \ell_c^{1/3} \sqrt{Y} \]
\[ \delta_{***} \sim \ell_{\text{curv}} \]

**Force:**

- **I:** $F \sim γ\delta$
- **II:** $F \sim \sqrt{Y ρg} \delta^2$
- **III:** $F \approx 4.581(γR_{\text{film}})^{2/3}(ρg)^{1/3} \delta$
- **IV:** $F \sim 2\pi γR_{\text{film}}$

**Föppl–von Kármán** (cf. Benny, Dominic’s talks)

Vella, Davidovitch, et al., *PRL* 2015, *PRE* 2018

**Geometric model:** \[ U = U_{\text{gravity}} + γ(ΔA_{\text{free}}) \]

Ripp, Démery, Zhang, JDP, *Sott Matter* 2020

Monica Ripp  
Teng Zhang
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   Non-spherical gross shapes come from simple geometric optimization ($U = \gamma A_{\text{free}}$)

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A very reasonable outlook:

The geometric model $U = \gamma A$ is great for explaining gross shapes, forces, **BUT** it doesn’t predict small-scale features (wrinkles, crumplings, folds), *i.e.*, how excess length is stored.

Recent progress:

Annular film on a flat liquid bath:

- Optimal gross shape **implies folds** (vs. wrinkles)

Stamping a curved shell onto a plane:

- Wrinkle direction **fixed** by optimal gross shape

wrinkles ‘free’

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JDP, Démery, Davidovitch, Menon, et al., PRL 118 (2017)

Tobasco, JDP, Katifori, et al. (arXiv 2004.02839)
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   Some folding transitions and wrinkle layouts may be understood as a geometric optimization

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Image credit: Rene Sorensen
Elastic films as “solid surfactants”

- Achieve non-spherical shapes
- Tailor mechanical, optical properties
- Platform for surface patterning (physical, chemical)
- Sequester/protect liquid cargo

Complementary to molecular & particle surfactants

Interesting properties for applications:

Kumar, JDP, Russell, Menon
Science 359 (2018)
Conclusions

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Ultrathin sheets: A tool for tailoring droplets, emulsions, interfaces (shape, rheology, surface chemistry, …)

“Wrapping liquids, solids, and gases in thin sheets”