Classical, Quantum, and Casimir Friction: Energetics and Forces

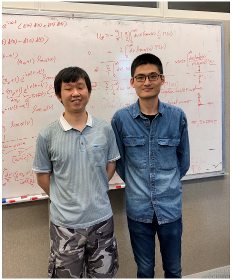
Xin Guo[1], K.A. Milton[1], Gerard Kennedy[2], W. McNulty[3], Yang Li[4], Nima Pourtolami[5]

[1]University of Oklahoma, [2]University of Southampton, [3]Indiana University, [4]Nanchang University, [5]National Bank of Canada

July 12, 2022

KITP Conference: Quantum and Thermal Electrodynamic Fluctuations in the Presence of Matter: Progress and Challenges

Introduction



- We are trying to systemize quantum or Casimir friction, a subject with a long history.
- We use natural units, $\hbar = c = \epsilon_0 = \mu_0 = k_B = 1$.

Publications

- "Electrodynamics friction of a charged particle passing a conducting plate," PRR **2**, 023114 (2020).
- "Self-force on moving electric and magnetic dipoles: dipole radiation, Vavilov-Čerenkov radiation, friction with a conducting surface, and the Einstein-Hopf effect," PRR **2**, 043347 (2020).
- "The energetics of quantum vacuum friction. I," PRD **104**, 116006 (2021) & II, PRD, in press [arXiv:2204.11336]

I. Friction of charged particle above imperfect metal

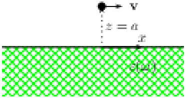


Figure 1: A particle of charge e moving with velocity v in the x direction distance a above a metallic surface.

$$F_x = eE_x = -\frac{e^2}{2\pi i}\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\int\frac{dk_y}{2\pi}g_{xx}(a,a;k_x=\omega/v,k_y,\omega),$$

where ($\varepsilon = \varepsilon(\omega)$, $\kappa = \sqrt{k^2-\omega^2}$, $\kappa' = \sqrt{\kappa^2-\omega^2(\varepsilon-1)}$)

$$g_{xx}(z,z') = \frac{k_y^2}{k^2}\omega^2 g^E(z,z') + \frac{k_x^2}{k^2}\frac{1}{\varepsilon(z)}\frac{1}{\varepsilon(z')}\partial_z\partial_{z'}g^H(z,z').$$

Above a dielectric slab, in the x-y plane,

$$g^{E,H}(z,z') = \frac{1}{2\kappa}\left(e^{-\kappa|z-z'|} + r^{E,H}e^{-\kappa(z+z')}\right),\ r^E = \frac{\kappa-\kappa'}{\kappa+\kappa'},\ r^H = \frac{\kappa-\kappa'/\varepsilon}{\kappa+\kappa'/\varepsilon}$$

Dissipation and frictional force

We use the Drude model to describe an imperfect metal,

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\nu\omega}$$

where for Au: $\omega_p = 9$ eV, $\nu = 0.035$ eV, nominally.
Then defining a dimensionless force by

$$F = -\frac{e^2}{32\pi^2 a^2}\mathcal{F},$$

where the dominant TM contribution is shown in the figure.

TM classical charged-particle friction

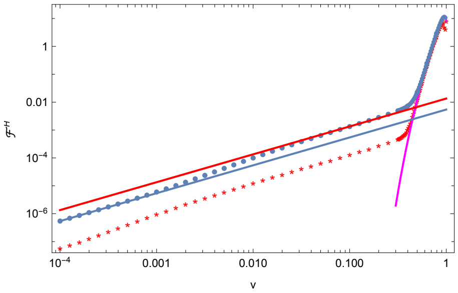


Figure 2: TM friction of a charged particle above a gold surface (dots). Comparison is made with the low velocity limit (blue line), the intermediate velocity behavior (red line), and the high velocity behavior (magenta curve). The friction approaches a finite value, below the peak, for high velocities. The lower dots show the behavior when the damping parameter is reduced by a factor of ten. The friction is reduced by the same factor for low and intermediate velocities, but remains at a nonzero high value as the damping goes to zero. The nonrelativistic limit was first studied by Boyer, PRA **9**, 68 (1974).

II. Permanent dipole moving in vacuum

Classical force density:

$$\mathbf{f}(\mathbf{r},t) = \rho(\mathbf{r},t)\mathbf{E}(\mathbf{r},t) + \mathbf{j}(\mathbf{r},t) \times \mathbf{B}(\mathbf{r},t),$$

where for a time-dependent dipole moving in the x direction

$$\rho(\mathbf{r},t) = -\mathbf{d}(t)\cdot\boldsymbol{\nabla}\delta(x-vt)\delta(y)\delta(z),$$
$$\mathbf{j}(\mathbf{r},t) = -\mathbf{v}\mathbf{d}(t)\cdot\boldsymbol{\nabla}\delta(x-vt)\delta(y)\delta(z) + \dot{\mathbf{d}}(t)\delta(x-vt)\delta(y)\delta(z).$$

For $\mathbf{d}$ polarized parallel to the motion, we get from the vacuum Green's function for the average force times the time of the configuration T

$$\overline{F^X}T = \frac{i}{8\pi^3}\int_{-\infty}^{\infty} d\omega \int dk_x\, dk_y |\tilde{d}(\omega - vk_x)|^2 k_x \frac{(\omega^2 - k_x^2)}{2\kappa},$$

where the i is an instruction to pick out the imaginary part. In vacuum the latter can only come from from κ, where, because we are dealing with retarded Green's functions,

$$\omega^2 > k^2: \quad \kappa = \sqrt{k^2-\omega^2} = -i\,\mathrm{sgn}(\omega)\sqrt{\omega^2-k^2};$$

Friction reflects the dipole radiation emitted

the integral over k_y is

$$\omega^2 - k_x^2 > 0: \quad \int_{-\sqrt{\omega^2-k_x^2}}^{\sqrt{\omega^2-k_x^2}} \frac{dk_y}{\sqrt{\omega^2 - k_x^2 - k_y^2}} = \pi.$$

The force is easily seen to be

$$\overline{F} = -\frac{v\gamma}{6\pi^2 T} \int_0^\infty d\omega\, \omega^4 |\tilde{\mathbf{d}}'(\omega)|^2.$$

This holds for arbitrary orientation of the dipole. Here, $\tilde{\mathbf{d}}'$ is the electric dipole moment in the rest frame of the particle:

$$\tilde{d}'_x(\omega) = \tilde{d}_x(\omega/\gamma), \quad \tilde{d}'_y(\omega) = \frac{1}{\gamma}\tilde{d}_y(\omega/\gamma).$$

This is proportional to the total energy radiated by the time-dependent dipole in the rest frame of the dipole:

$$E'_R = \frac{1}{6\pi^2} \int_0^\infty d\omega\, \omega^4 |\tilde{\mathbf{d}}'(\omega)|^2, \quad \overline{F}T = -v\gamma E'_R.$$

Interpretation

Friction is radiation reaction

There is no force on the dipole, in the rest frame: what this says is that the energy (mass) of the dipole decreases due to the emission of radiation, so its momentum changes correspondingly. Thus, $\overline{F}$ is the radiation reaction force in the moving frame, $\overline{F} = v\gamma\frac{dm_0}{dt}$.

The nonrelativistic limit of this effect was discussed by Sonnleitner, Trautmann, and Barnett, PRL **118**, 053601 (2017).

III. Quantum vacuum friction

If the dipole moments are not permanent, but arise due to quantum or thermal fluctuations, friction can also occur even in vacuum. This is a relativistic generalization of the Einstein-Hopf effect. In terms of symmetrized products, the fluctuation-dissipation theorem states:

$$\langle \mathbf{d}'(t_1')\mathbf{d}'(t_2')\rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t_1'-t_2')} \Im \boldsymbol{\alpha}(\omega) \coth \frac{\beta'\omega}{2},$$

$$\langle \mathbf{E}(\mathbf{r}_1, t_1)\mathbf{E}(\mathbf{r}_2, t_2)\rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t_1-t_2)} \Im \boldsymbol{\Gamma}(\mathbf{r}_1, \mathbf{r}_2, \omega) \coth \frac{\beta\omega}{2},$$

which are written in the rest frame of the particle (primes) and the rest frame of the blackbody radiation (no primes), respectively. The temperature of the particle $(1/\beta')$ is different from that of the vacuum blackbody radiation $(1/\beta)$, both temperatures defined in the respective rest frames.

Einstein-Hopf effect

Following the procedure sketched above, the sum of dipole-fluctuation and field-fluctuation contributions yield the force on an isotropically polarizable particle

$$F^{\text{ISO}} = -\frac{1}{4\pi^2\gamma^2 v^2}\int_0^\infty d\omega \omega^4 \Im\alpha(\omega)\int_{y_-}^{y_+} dy\left(y-\frac{1}{\gamma}\right)\left(\coth\frac{\beta'\omega}{2}-\coth\frac{\beta\omega y}{2}\right)$$

Here, $y_\pm = \gamma(1 \pm v)$. Note that this force could be of either sign.
[Agrees with Dedkov and Kyasov (2010), Pieplow and Henkel (2013), Volokitin and Persson (2017).]
If this is expanded for small v when $\beta = \beta'$ (which we'll see is a nonrelativistic consequence of energy conservation), we obtain the Einstein-Hopf friction

$$F^{\text{EH}} = -\frac{v}{6\pi^2}\int_0^\infty d\omega\, \omega^4 \Im\alpha(\omega)\frac{\beta\omega/2}{\sinh^2 \beta\omega/2}.$$

[Mkrtchian, Parsegian, Podgornik, and Saslow, PRL **91**, 220801 (2003).]

NESS: dipole fluctuations induced by field fluctuations

NESS = Nonequilibrium steady state: The particle energy is conserved. We compute the power, the rate at which the field does work on the moving particle:

$$P(t) = \int (d\mathbf{r})\mathbf{j}(t,\mathbf{r}) \cdot \mathbf{E}(t,\mathbf{r})$$

where the current is as before. The force is also given by the Lorentz force density law. But now the correlations are supposed to be entirely due to field fluctuations. Dipole fluctuations are induced by field fluctuations.

Procedure

- Assume the intrinsic polarizability of the particle is real.
- In the rest frame of the particle, $\mathbf{d}' = \boldsymbol{\alpha}\mathbf{E}'$.
- Expand the free energy out to second order in $\boldsymbol{\alpha}$ using $\mathbf{E} = \boldsymbol{\Gamma} \cdot \mathbf{j}$.
- Use the FDT on $\mathbf{E}$.

Quantization in rest frame of particle

The power and force are obtained by differentiating the free energy $\mathcal{F}$:

$$P = \frac{\partial}{\partial t}\mathcal{F} = \gamma\left(\frac{\partial}{\partial t'} - v\frac{\partial}{\partial x'}\right)\frac{1}{\gamma}\mathcal{F}' = P' + vF',$$

$$F = -\frac{\partial}{\partial x}\mathcal{F} = -\gamma\left(\frac{\partial}{\partial x'} - v\frac{\partial}{\partial t'}\right)\frac{1}{\gamma}\mathcal{F}' = F' + vP'.$$

The NESS condition can then be stated in three equivalent forms:

- $P' = 0$,
- $F' = F$,
- $P = Fv$,

which says that in the rest frame of the particle, the particle energy is conserved, and in the moving frame of the particle, the power is just that required by the motion. The force is the same in both frames.

Force calculated in rest frame of particle

$$F = F' = \int \frac{d\omega}{2\pi} \int \frac{dk_x}{2\pi} k_x \, \mathrm{tr} \left[\boldsymbol{\alpha}(\omega) \Im \boldsymbol{\Gamma}'(\omega, \mathbf{0}, \mathbf{0}) \boldsymbol{\alpha}(\omega) \Im \mathbf{G}'(\omega, k_x) \right] \times \coth \left(\frac{\beta\gamma}{2}(\omega + v k_x) \right) \qquad \text{(general)}$$

or in the vacuum with

$$\Im \boldsymbol{\Gamma}'(\omega; \mathbf{0}, \mathbf{0}) = \int \frac{dk_x}{2\pi} \mathbf{G}'(\omega, k_x) = \frac{\omega^3}{6\pi} \mathbf{1},$$

$$F = \frac{1}{18\pi^3 v\gamma} \int_0^\infty d\omega \, \omega^7 \int_{y_-}^{y_+} dy (y - \gamma) \times \left[(\boldsymbol{\alpha})^2_{xx}(\omega) f^X(y) + \left((\boldsymbol{\alpha})^2_{yy}(\omega) + (\boldsymbol{\alpha})^2_{zz}(\omega) \right) f^Y(y) \right] \frac{1}{e^{\beta\omega y} - 1},$$

where $(\boldsymbol{\alpha})^2_{xx} = \alpha^2_{xx} + \alpha_{xy}\alpha_{yx} + \alpha_{xz}\alpha_{zx}$, etc., and

$$f^X(y) = \frac{3}{4\gamma v} \left[1 - \frac{1}{\gamma^2 v^2} (y - \gamma)^2 \right], \quad f^Y(y) = \frac{3}{4\gamma v} - \frac{1}{2} f^X(y).$$

Independent dipole and field fluctuations

Alternatively, we may allow for independent dipole and field fluctuations, with the FDT applying separately to each, at the corresponding temperatures in the respective rest frames. The formula for the force is as given above. But now we can impose the NESS condition, that $P' = 0$ or $P = Fv$. Then there is a connection between the two temperatures given by $(P = X, Y, \mathrm{ISO})$

$$\int_0^\infty d\omega\, \omega^4 \Im\alpha_P(\omega) \int_{y_-}^{y_+} dy \frac{f^P(y)}{e^{\beta'\omega} - 1} = \int_0^\infty d\omega\, \omega^4 \Im\alpha_P(\omega) \int_{y_-}^{y_+} dy \frac{f^P(y)}{e^{\beta\omega y} - 1}.$$

For an isotropic atom, this reads

$$\int_0^\infty d\omega\, \omega^4 \Im\alpha(\omega) \frac{2\gamma v}{e^{\beta'\omega} - 1} = \int_0^\infty d\omega\, \omega^4 \Im\alpha(\omega) \frac{1}{\beta\omega} \ln \frac{1 - e^{-\beta\omega y_+}}{1 - e^{-\beta\omega y_-}}.$$

NESS condition when $\Im\alpha \propto \omega^n$

The most important monomial dependence is that for the radiation-reaction model,

$$\Im\alpha = \frac{\omega^3}{6\pi}\alpha_0^2.$$

For general n, the NESS temperature condition for the ratio of the temperature of the particle to that of the blackbody radiation reads

$$\frac{T'}{T} = \left[\frac{\gamma^{3+n}\left((1+v)^{4+n} - (1-v)^{4+n}\right)}{2v(4+n)}\right]^{\frac{1}{5+n}}.$$

[n=3: Volokitin and Persson, *Electromagnetic Fluctuations at the Nanoscale*, (Springer, Berlin, 2017)] This is shown in the figure.

Temperature ratio for monomial model

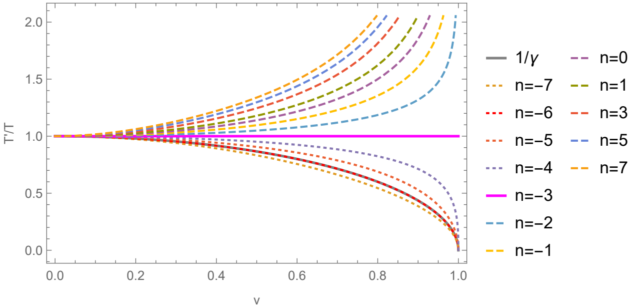


Figure 3: The NESS temperature ratio for the case $\Im\alpha \propto \omega^n$. For $n = -3$ the temperature ratio is 1. Even though $n = -4$ would seem to be required for convergence of the integrals, the results are consistent with the required limit $T'/T > 1/\gamma$ up to $n = -6$.

NESS Frictional force

In general, the friction for arbitrary temperatures of particle and blackbody radiation can be positive or negative. But if the NESS condition is fulfilled, it is always a drag. For example, for an isotropic particle,

$$F^{\rm ISO}_{dd+EE} = \frac{1}{2\pi^2\gamma^2 v^2}\int_0^\infty d\omega\,\omega^4 \Im\alpha(\omega)\int_{y_-}^{y_+}(y-\gamma)\frac{1}{e^{\beta\omega y}-1}.$$

This, and the corresponding formulas for anisotropic particles, coincide with that found above by considering only EE fluctuations, so that the dipole fluctuations are induced by those of the field. The connection is merely a reparameterization of the imaginary part of the polarizability:

$$\Im\alpha_P(\omega) = \frac{\omega^3}{6\pi}\alpha^2_{P,0}(\omega), \quad \alpha_{P,0} \text{ real.}$$

Effective polarizability: $\hat{\boldsymbol{\alpha}} = \boldsymbol{\alpha}_0[1 - i\omega^3\boldsymbol{\alpha}_0/6\pi]^{-1}$, after renormalization.

Radiation reaction model $n = 3$, α_0 constant

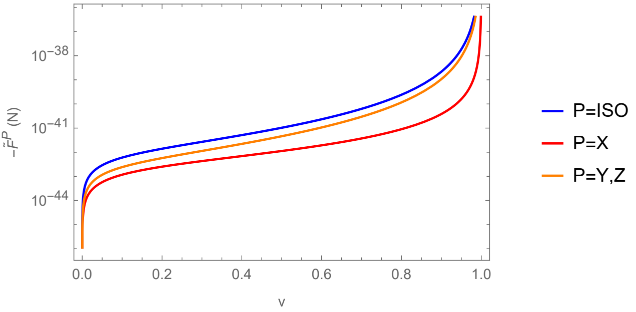


Figure 4: The frictional force for the radiation reaction model in NESS for different polarization states at room temperature using the static polarizability of a gold atom.

Friction force for gold nanosphere using Lorentz model

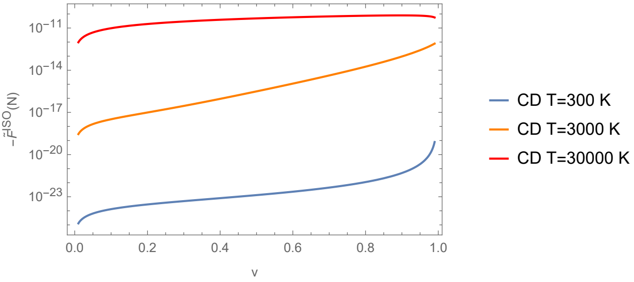


Figure 5: Frictional force for different temperatures, as a function of v.

Here the Lorentz model is used to model the nanosphere:

$$\alpha(\omega) = V \frac{\omega_p^2}{\omega_1^2 - \omega^2 - i\omega\nu}, \quad \omega_1 = \omega_p/\sqrt{3}.$$

Friction for gold nanosphere for different velocities as a function of $T(\mathrm{K})$

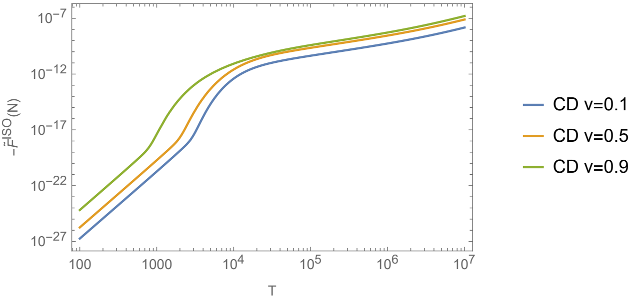


Figure 6: The friction on a gold nanosphere at $v = 0.1, 0.5, 0.9$ as a function of temperature. The low and high-temperature asymptotic limits agree with monomial models ($n = 1$ and $n = -3$, respectively.) (CD in these figures means we are assuming the damping is temperature-independent.)

Relativistic behavior: Comparison of friction on gold nanosphere with resonance and high-v approximations

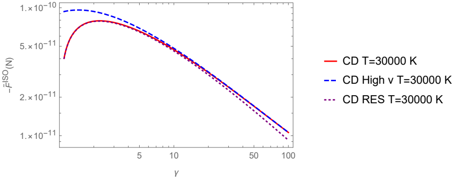


Figure 7: $T = 30,000$ K

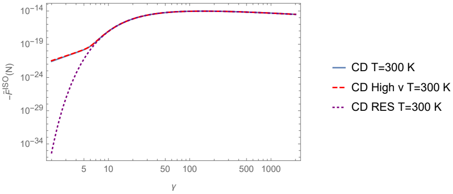


Figure 8: $T = 300$ K

- Resonance Model: $\Im\alpha \propto \delta(\omega - \omega_1)$.
- Notice, that even at room temperature, the friction decreases for high enough velocities.

Temperature ratio of gold nanosphere including temperature-dependent damping

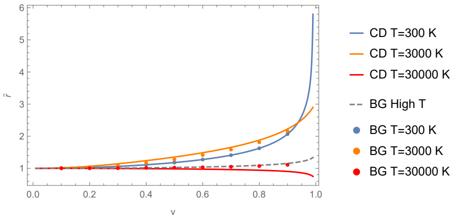


Figure 9: $\tilde{r}$ as a function of v, various Ts

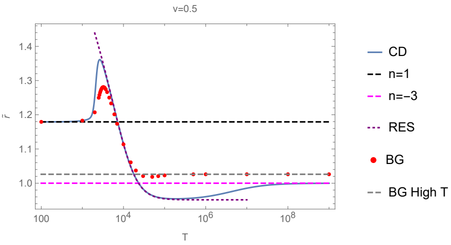


Figure 10: $\tilde{r}$ as a function of T(K) for $v = 0.5$

Bloch-Grüniesen model: $$\nu(T) = \nu_0 \left(\frac{T}{\Theta}\right)^5 \int_0^{\theta/T} dx \frac{x^5 e^x}{(e^x - 1)^2}$$

Effect of BG temperature dependence on friction force

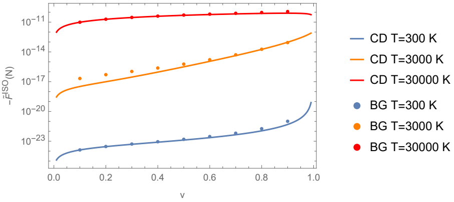


Figure 11: F as a function of v, various Ts

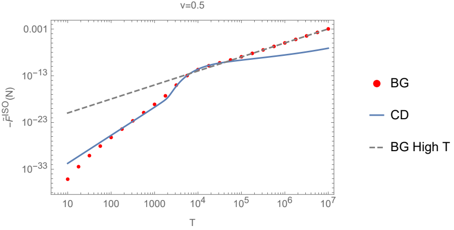


Figure 12: F as a function of T(K) for $v = 0.5$

Out of NESS

So far, we have considered the Non-Equilibrium Steady State condition to be satisfied. This occurs either because the particle has no intrinsic dissipation, or the temperature of the particle has a special ratio r to the environmental temperature. In that case, an external force, F_{ext}, balancing the frictional force F, must be supplied to keep the particle moving with constant velocity:

$$F_{\text{total}} = F_{\text{ext}} + F = 0.$$

But if the NESS condition is not satisfied,

$$F_{\text{total}} = F_{\text{ext}} + F = v\gamma \frac{dm_0}{dt} = v \frac{dm_0}{dt'} = vP',$$

because the internal energy or mass $(m = m_0\gamma)$ of the particle is not constant.

Stability of NESS

If the system is out of NESS, the temperature of the particle, T' is not equal to the NESS temperature $\tilde{T} = rT$. Then, we can easily show

$$\begin{cases} T' < \tilde{T} \Rightarrow P' = \frac{dm}{dt'} > 0 \\ T' = \tilde{T} \Rightarrow P' = \frac{dm}{dt'} = 0 \\ T' > \tilde{T} \Rightarrow P' = \frac{dm}{dt'} < 0 \end{cases}$$

This suggests stability: If the atom is hotter than the NESS temperature, it loses energy and cools off, while if it is cooler than the NESS temperature, it gains energy and heats up.

Observability

There are at least two promising avenues

- Observe deceleration of particle:

$$\text{low } T\text{, NR:} \quad -\frac{32\pi^5\alpha_0^2}{135\beta^8}v = m\frac{dv}{dt} \Rightarrow \Delta t = -\tau \ln \frac{v_f}{v_i},$$

where for a gold atom, $\tau = 1.72 \times 10^{25}$s at room temperature. This would mean that a reduction of the velocity by 10% could occur in less than 6 years if the effect could be observed at 30,000 K! (Still "low temperature" for a gold atom.)

- Observe temperature ratio T'/T. For radiation reaction model

$$\frac{T'}{T} = \left[\gamma^6\left(1 + 5v^2 + 3v^4 + \frac{v^6}{7}\right)\right]^{1/8}.$$

For example, at $v = 0.5$, $\frac{T'}{T} = 1.25$. This effect should be observable.

IV. Formalism is readily extended to interaction with translationally invariant background

A. Dielectric

The above formalism applies to the case of a particle passing above a dielectric surface.

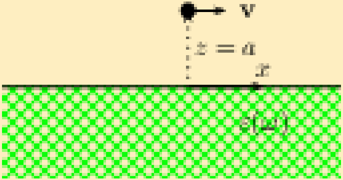


Figure 13: Polarizable particle moving above a dielectric surface

However, in this case the general analysis is somewhat complex, because in general, the imaginary part can arise from κ as well as ε.

NR, zero-temperature approximation

$$\beta \to \infty, \quad v \ll 1$$

Then the frictional force for x polarization is

$$F^X = -\frac{\alpha_{xx}(0)^2}{(2\pi)^5} \int d^2k\, d^2k'\, k'_x \int_{k'_x v}^{k_x v} d\omega\, \Im g_{xx}(k_x, \omega - k_x v) \Im g_{xx}(k'_x, \omega - k'_x v)$$

Since the frequencies are small, only the imaginary part of the H reflection coefficient contributes:

$$\Im g_{xx}(k, \omega - k_x v) \xrightarrow{\kappa = k} \frac{k_x^2}{k\omega_p^2} \nu(\omega - k_x v) e^{-2kz}$$

The friction for x polarization and isotropic polarization are the known results, e.g., Marty Oelschläger, Dissertation, Humboldt-Universität.

$$F^X = -\frac{39}{2\pi^3} \frac{\alpha_{xx}(0)^2 \nu^2 v^3}{\omega_p^4 (2z)^{10}}, \quad F^{\rm ISO} = -\frac{18}{\pi^3} \frac{\alpha_{xx}(0)^2 \nu^2 v^3}{\omega_p^4 (2z)^{10}}.$$

B. Quantum vacuum friction above a perfect conductor: $\varepsilon \to \infty$

PC boundary modifies friction near plate, while far away friction is unchanged.

For $aT \ll 1$, $v \ll 1$:

$$\begin{aligned} F^Z &\sim O(vT^8) = 4F^Z_{\text{vac}}, \\ F^{XZ} &\sim O(va^2T^{10}), \\ F^{Y,X} &\sim O(va^4T^{12}). \end{aligned}$$

The fact that the z-polarization friction is enhanced by 4, while the perpendicular polarizations are suppressed, is due to the PC boundary conditions:

$$\mathbf{E}_\perp(0) = 0, \quad E_z(0) = 2E_{z,\text{vac}}(0)$$

because of the image charge.

Graphs of transition of QVF modified by PC boundary

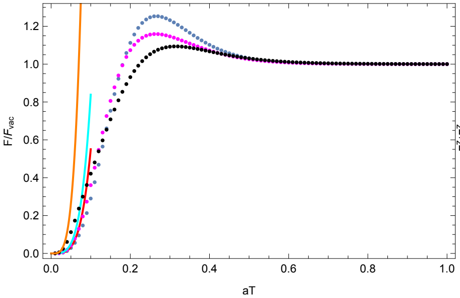


Figure 14: F^X/F^X_{vac} for $v = 0.1$ (blue), $v = 0.5$ (magenta), $v = 0.9$ (black).

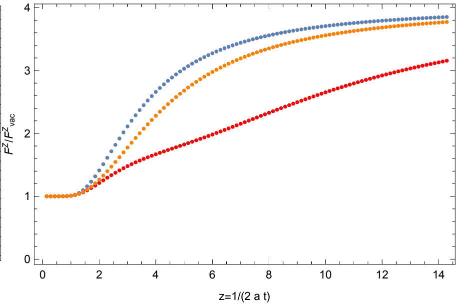


Figure 15: F^Z/F^Z_{vac} for $v = 0.1$ (blue), $v = 0.5$ (orange), $v = 0.9$ (red).

C. Induced Čerenkov friction

If particle travels parallel to a dispersionless dielectric surface faster than the speed of light in the medium, friction occurs, because Čerenkov radiation occurs in the medium. (n=index of refraction, a is distance to surface)

This is particularly simple for a charged particle:

$$F = -\frac{e^2}{2\pi}\frac{1}{(2a)^2}\frac{1}{(n^2-1)\sqrt{\gamma^2-1}}\left[\frac{n^2\gamma^2}{\sqrt{n^2+\gamma^2}} - \sqrt{\gamma^2-1} - \sqrt{n^2-1}\right].$$

This vanishes at threshold, where

$$nv_C = 1, \quad \text{or} \quad \gamma_C = \frac{n}{\sqrt{n^2-1}}.$$

$$\text{For high velocities:} \quad F \to -\frac{e^2}{8\pi a^2}, \quad \gamma \to \infty.$$

Include dispersion: dielectric characterized by $n(\omega)$

Master formula

$$F = -\frac{e^2}{\pi^2}\int_0^\infty \frac{d\omega}{\omega}\int dk_y \Im g_{xx}(\omega, k_x = \omega/v, k_y; z = z' = a),$$

which here leads to

$$F^E = -\frac{e^2}{\pi^2}\int_{n(\omega)v>1} d\omega\,\omega \int_0^1 dy \frac{y^2}{y^2+b^2}\frac{\sqrt{1-y^2}}{1+b^2/\gamma^2}\, e^{-x\sqrt{y^2+b^2/\gamma^2}},$$

$$F^H = -\frac{e^2}{\pi^2}\int_{n(\omega)v>1} d\omega\,\frac{\omega}{n^2v^2}\int_0^1 dy \frac{y^2+b^2/\gamma^2}{y^2+b^2}\frac{\sqrt{1-y^2}\, e^{-x\sqrt{y^2+b^2/\gamma^2}}}{(1-1/n^4)y^2+b^2/\gamma^2+1/n^4}$$

where $b = (n^2v^2-1)^{-1/2}$ and $x = 2\omega a\sqrt{n^2-1/v^2}$.

Example of effect of dispersion on Čerenkov friction

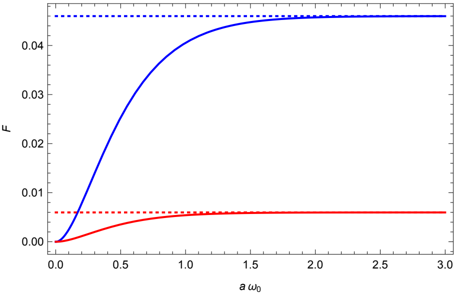


Figure 16: For $n = 2.5$, $v = 0.500$, $-(8\pi a^2)/e^2 F$ plotted versus $\omega_0 a$. Blue denotes TM, red TE. Dotted lines show dispersionless model.

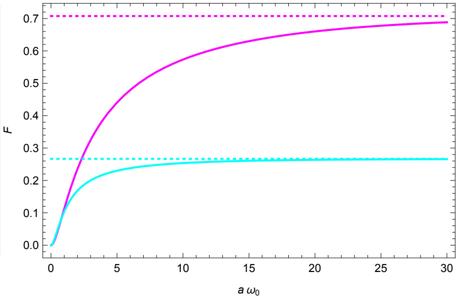


Figure 17: For $n = 2.5$, $v = 0.999$, $-(8\pi a^2/e^2)F$ plotted versus $\omega_0 a$. Magenta is TM, cyan is TE. Dotted lines are dispersionless model.

Crude model:

$$\varepsilon(\omega) - 1 = (n^2 - 1)\theta(\omega_0 - \omega).$$

Much literature on Čerenkov friction

For example:

- Classical: B.M. Bolotovskii, Sov. Phys. Usp. **4**, 781 (1962) [Usp. Fiz. Nauk **75**, 295 (1961)]
- Two slabs, quantum: M. G. Silveirinha, Phys. Rev. A **88**, 043846 (2013).
- Quantum: G. Pieplow and C. Henkel, J. Phys.: Condens. Matter **27**, 214001 (2015). ($T = 0$)

V. Conclusions

1. We systematically studied classical friction of a charged particle with an imperfect metallic surface.
2. Classical dipole friction occurs in vacuum occurs due to dipole radiation.
3. Quantum vacuum friction is due to both dipole and field fluctuations.
   1. If the particle has no intrinsic dissipation, the dipole fluctuations are due to the electromagnetic field fluctuations.
   2. This means the energy of the particle is conserved, the NESS condition.
   3. The NESS condition seems to be stable.
   4. The NESS temperature ratio seems a promising candidate for detecting this effect.
4. We are now generalizing our considerations to examine interactions of a moving particle with surfaces, which may be dissipative (Casimir friction) or nondissipative (quantum) Čerenkov friction. Both have been extensively studied, but we hope to generalize to arbitrary temperatures, velocities, and dispersions.