Critical Casimir forces at work

measuring and tuning femto-Newton forces

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“The theory and practice of fluctuation-induced interactions”
Kavli Institute for Theoretical Physics, UCSB
November 5, 2008

Outline

QED $\longleftrightarrow$ Stat Phys

1. Casimir in Quantum ElectroDynamics (QED)
2. Casimir in Stat Phys/Critical Phenomena
3. Direct measurement of *critical* Casimir forces

$\hookrightarrow$ **Thorough discussion: Dietrich 23 Sep**
24 Sep

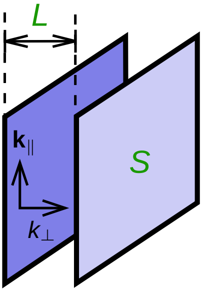
H. B. G. Casimir (1909 – 2000)

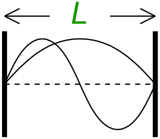
“On the attraction between two perfectly conducting plates”
Proc. Kon. Nederland. Akad. Wetensch. **B51**, 793 (1948)

$\rightsquigarrow$ **Casimir effect**

Casimir in QED

Plates $\Rightarrow \mathbf{E}_{\parallel}, B_{\perp} = 0$, *Boundary conditions!*





$\rightsquigarrow$ allowed *modes*: $k_{\perp} = \dfrac{\pi n}{L}, \quad n = 1, 2, \ldots$

QED: $$\mathcal{E} = \sum_{\text{modes}} \frac{1}{2} \hbar c |\mathbf{k}|_{\text{modes}}$$ c = speed of light

Casimir in QED

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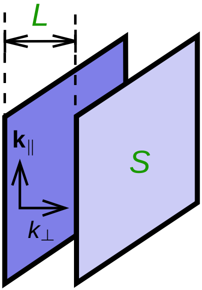

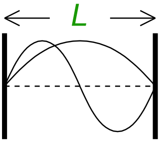

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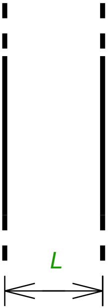
$$\mathcal{E} = \mathcal{E}_{\text{bulk}} + \mathcal{E}_{\text{surf}}^{(L+R)} + S\frac{\hbar c}{L^3}\left[-\frac{\pi^2}{1440} + O((\kappa L)^{-2})\right]$$

$$\sim SL^1 \qquad \sim SL^0 \qquad \mathcal{E}_{\text{Cas}} \sim SL^{-3}$$

Casimir in QED: "Measurement"



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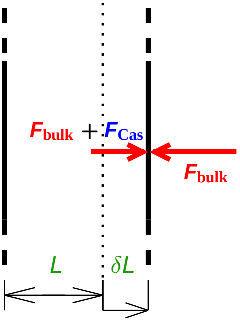


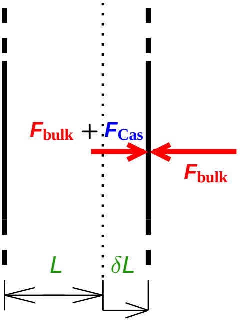
$$\text{force} \equiv -\frac{\delta \mathcal{E}}{\delta L} = F_{\rm bulk} + F_{\rm Cas}$$

$$\boxed{\frac{F_{\rm Cas}}{S} = -\frac{\pi^2}{480}\frac{\hbar c}{L^4} \quad \text{for} \quad L \gg \kappa^{-1}}$$

Universal!

Q: κ^{-1} =?? $c\kappa \simeq \omega_p \simeq 3 \cdot 10^{15}$Hz (Cu) $\rightsquigarrow$ $\boldsymbol{\kappa^{-1} \simeq 0.3\mu}$**m**

Casimir in QED: "Measurement"



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Exp: Sparnaay (1958)

Lamoreaux PRL **78**, 5 (1997)

Bressi *et al.* PRL **88**, 041804 (2002) $\qquad 0.5\mu\text{m} \lesssim L \lesssim 6\mu\text{m}$

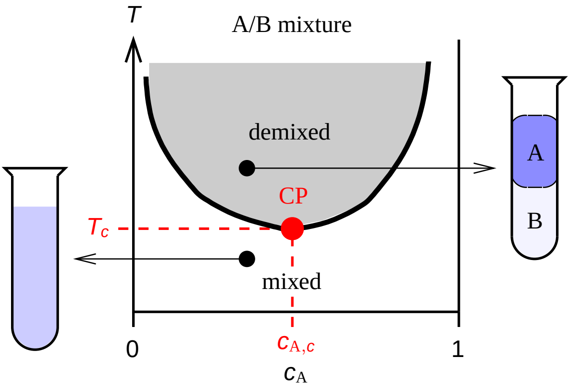
Revs.: Lamoreaux, Physics Today, Feb 2007
Ball, Nature **447**, 772 (2007)

M. E. Fisher and P.-G. de Gennes

"Phenomena at the walls in a critical binary mixture"
C. R. Acad. Sc. Paris **B287**, 207 (1978)

$\rightsquigarrow$ **Casimir-like effect in Stat Phys**

Binary mixtures & Critical points (reminder)



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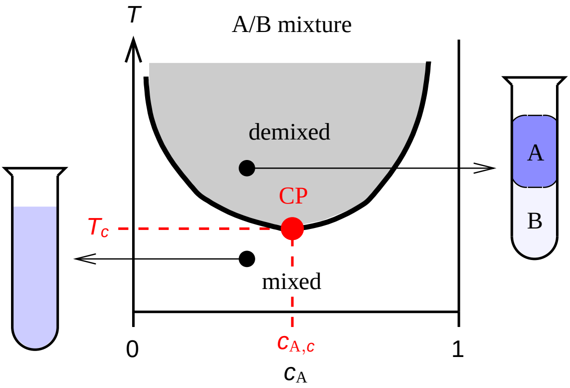


CP Order parameter: $\delta c_A(\mathbf{x}) \equiv c_A(\mathbf{x}) - c_{A,c}$

$$\langle \delta c_A(\mathbf{x}) \delta c_A(\mathbf{y}) \rangle \sim \exp\{-|\mathbf{x} - \mathbf{y}|/\xi\}$$

$$\xi \nearrow \infty \text{ for } (T, c_A) \to (T_c, c_{A,c})$$

Casimir: QED vs. Stat Phys

QED

	QED
fluctuating quant.:	**E**, **B**
excitation:	**Quantum** $\hbar c$ $(T = 0)$
range of fluct.:	∞

$\Downarrow$

Confinement

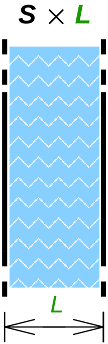
$\Downarrow$

Long-range force

Casimir: QED vs. Stat Phys

	QED	Stat Phys
fluctuating quant.:	**E**, **B**	Order param. ϕ
excitation:	**Quantum** $\hbar c$ $(T = 0)$	**Thermal (classical)** $k_B T$ $(\hbar = 0)$
range of fluct.:	∞	finite: ξ $\xi \nearrow \infty$ at CP!
	$\Downarrow$	$\Downarrow$
	Confinement	**Confinement**
	$\Downarrow$	$\Downarrow$
	Long-range force	**Range: ξ**
		Long-range at CP!

Critical Phenomena: Confined systems



: *critical medium:*

– ^{4}He, ^{4}He/^{3}He
– binary liquid mixt.
– Bose gas
– liquid crystals
– . . . . . .

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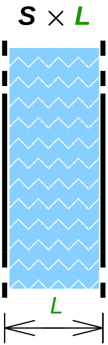

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- ......

$(c_{\rm A} = c_{{\rm A},c};\ L, \xi \gg \ell_{\rm micr})$

$$\mathcal{F} = \mathcal{F}_{\rm bulk} + \mathcal{F}^{(L+R)}_{\rm surf} + S\frac{k_{\rm B}T}{L^2}\Theta_{\|}(L/\xi) + \ldots$$

$$T \to T_c \Longrightarrow \xi \sim \xi_0 |(T - T_c)/T_c|^{-\nu} \nearrow \infty$$

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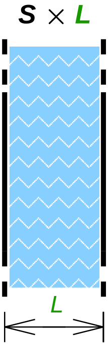

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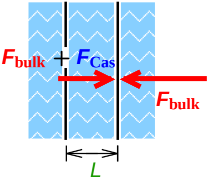

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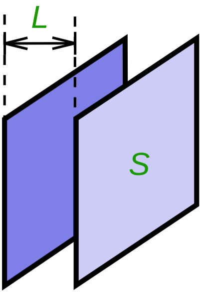
$$\boxed{\frac{F_{\mathrm{Cas}}}{S} = \frac{k_B T}{L^3}\vartheta_{\parallel}(L/\xi) \quad \text{for} \quad L, \xi \gg \ell_{\mathrm{micr}}}$$

Universality!

How large is the Casimir force?



$$S = 1\ \text{cm}^2$$
$$L = 1\ \mu\text{m}$$

QED	Stat Phys
force $\sim \hbar c / L^4 \sim 10^{-7}$N	**force** $\sim k_B T / L^3 \sim 0.2 \cdot 10^{-7}$N

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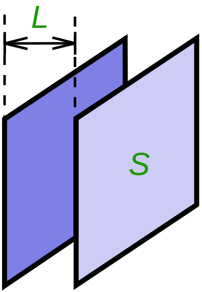

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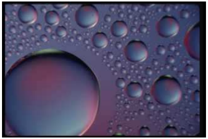
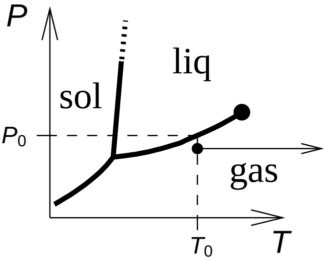


water droplet, diam. $= 0.5$ mm

$\Longrightarrow$ weight $\sim 2 \cdot 10^{-7}$N

Wetting films

[Krech&Dietrich PRA'92]



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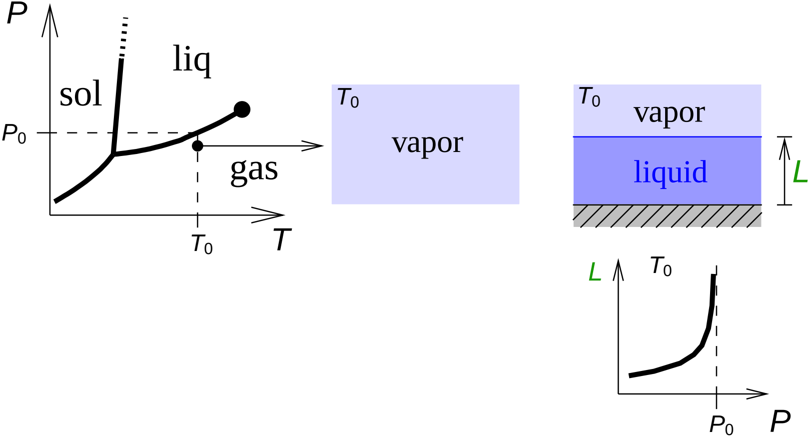

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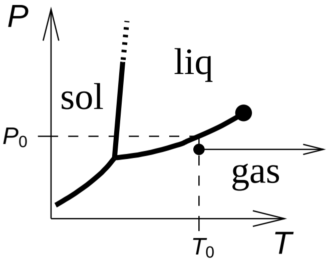
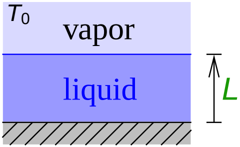
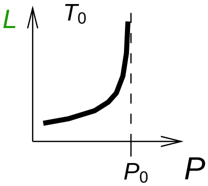
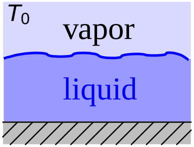




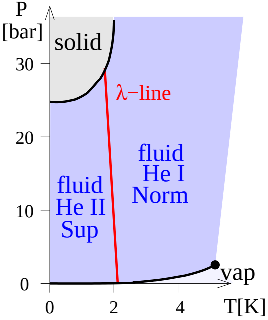


(+) *Alignment!*

(−) *capillary waves:*



^{4}He film & XY model



Norm/Superfl $\overset{\text{universality}}{\longleftrightarrow}$ XY Model

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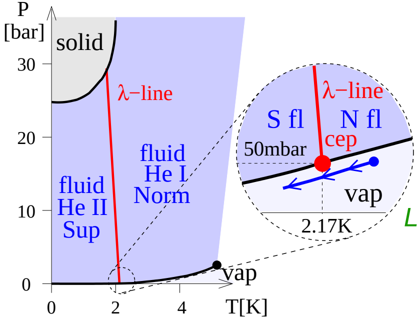

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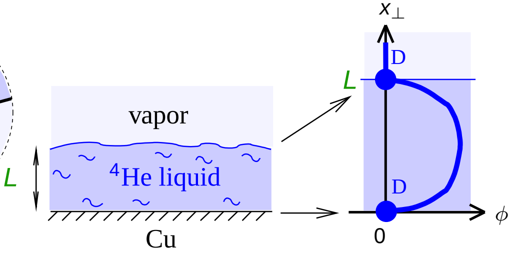

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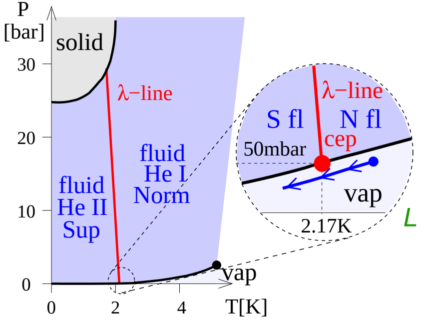

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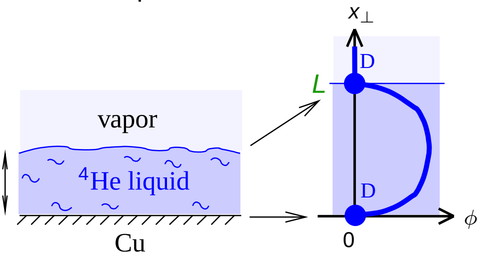
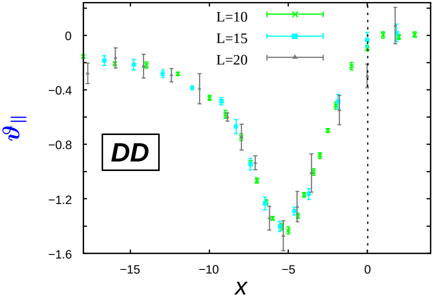




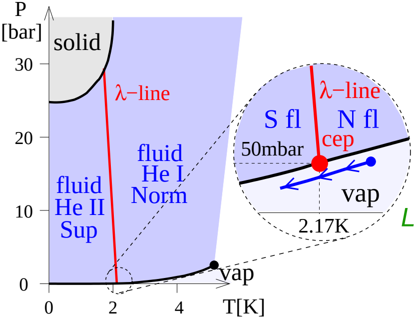
Monte Carlo sim.:

$x = (T/T_\lambda - 1)(L/\xi_0)^{1/\nu}$ $\nu \simeq 0.66$

[Vasilyev, Gambassi,Maciołek&Dietrich EPL'07]

[Hucht PRL'07]

^{4}He film & XY model



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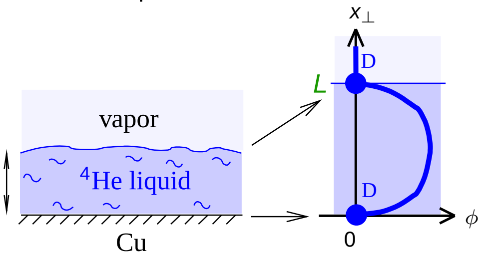

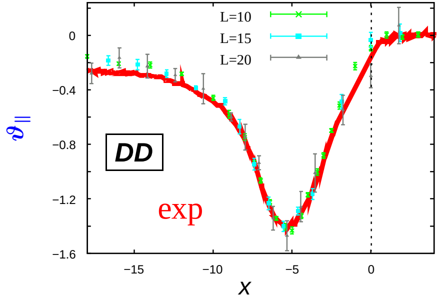

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[Vasilyev, Gambassi,Maciołek&Dietrich EPL'07]
[Hucht PRL'07]

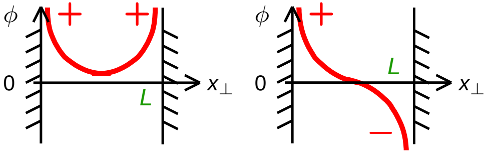
Exp.:
[Garcia&Chan PRL'99]
[Ganshin,Scheidemantel,Garcia&Chan PRL'06]

↪ **Talks: Garcia 4 Nov**
Chan 2 Sep

$T_\lambda = 2.18$K
$L \simeq 200..300$Å

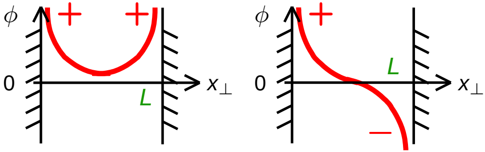
Binary mixtures & Ising model

Binary mixture $\overset{\text{universality}}{\longleftrightarrow}$ Ising Model

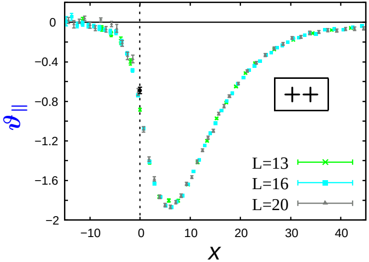


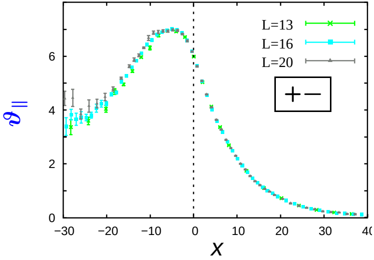
Binary mixtures & Ising model

Binary mixture $\overset{\text{universality}}{\longleftrightarrow}$ Ising Model



Monte Carlo sim.: $x = (1 - T/T_c)(L/\xi_0)^{1/\nu}$, $\nu \simeq 0.63$ [Vasilyev, Gambassi,Maciołek&Dietrich EPL'07]





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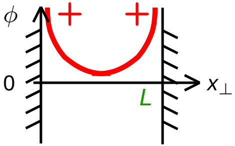

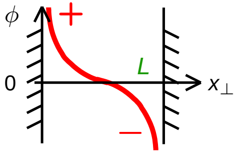

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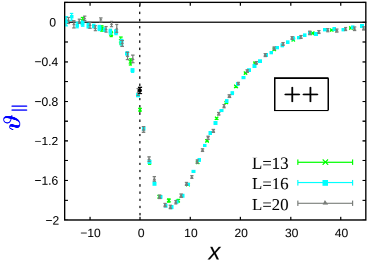
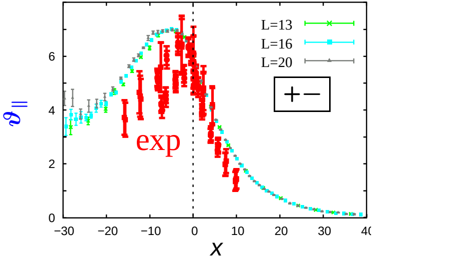
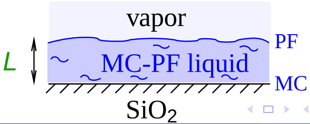




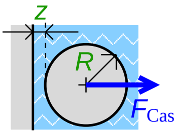
exp: [Rafaï,Bonn&Meunier Physica A'07]

Si substr.
[++] nonane-methanol
[+−] heptane-methanol



exp: [Fukuto,Yano&Pershan PRL'05]
MC = methylcyclohexane
PF = perfluoroMC
$T_c = 42.6^\circ C$
$x_c(\text{PF}) = 0.36$ molar fr.
$L \simeq 100$Å

fN dynamometer: TIRM



$R \gg z$:

$$F_{\mathrm{Cas}} = k_{\mathrm{B}} T \, \frac{R}{z^2} \vartheta_{|\circ}(z/\xi)$$

$\vartheta_{\|} \rightsquigarrow \vartheta_{|\circ}$

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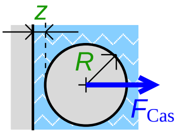

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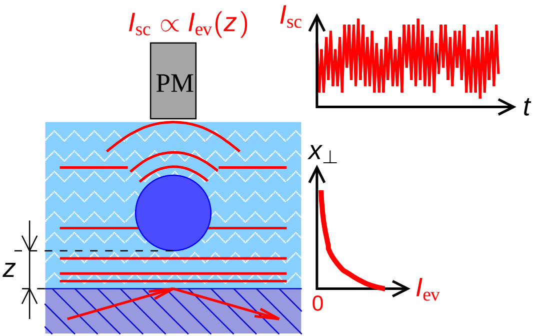
Total Internal Reflection Microscopy ($1\ \mathrm{fN} = 10^{-15}\mathrm{N}$):

Cavendish $\simeq 1.4 \cdot 10^{-7}$N
AFM $\simeq 10^{-12}$N



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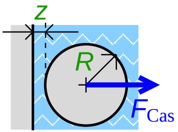

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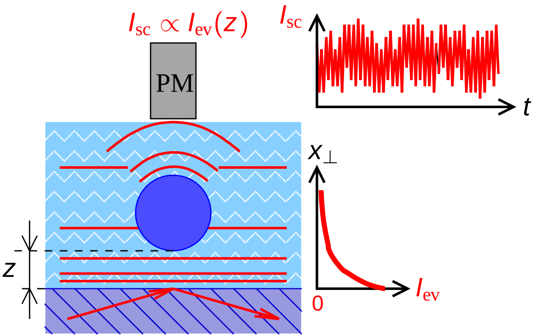
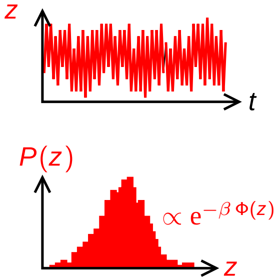
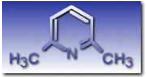
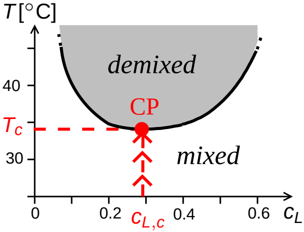




Direct measurement of Casimir forces

water + 2,6-lutidine



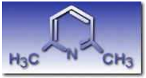


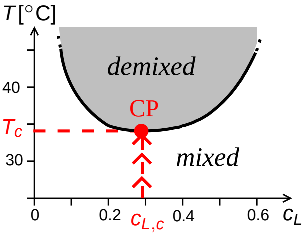
$T_c \simeq 34°\mathrm{C}$
$c_{\mathrm{L},c} \simeq 0.29$ mass fract.

$\phi(x) \equiv c_{\mathrm{L}}(x) - c_{\mathrm{L},c}$

Direct measurement of Casimir forces

water + 2,6-lutidine





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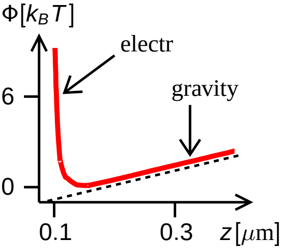

[Hertlein,Helden, Gambassi, Dietrich&Bechinger, Nature'08]

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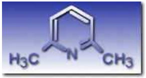

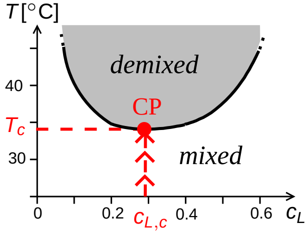

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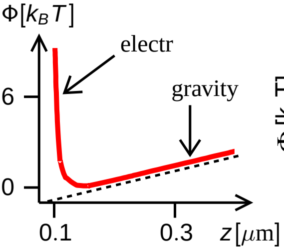
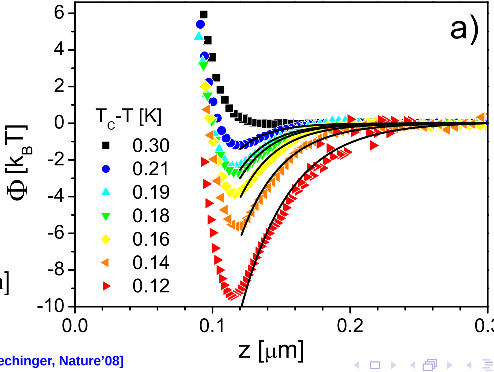
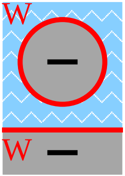




$R = 1.2\mu$m



$F_{Cas} \simeq 600f$N

[Hertlein,Helden, Gambassi, Dietrich&Bechinger, Nature'08]

Direct measurement of Casimir forces

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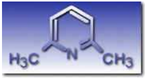

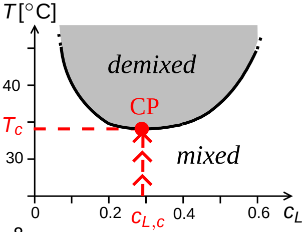

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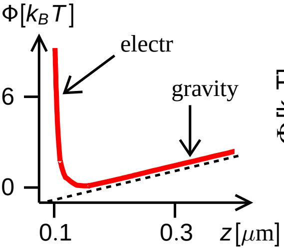
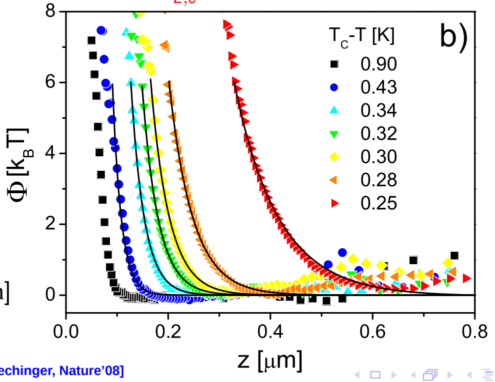
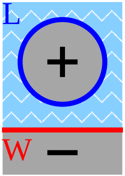




$R = 1.85\mu m$



[Hertlein,Helden, Gambassi, Dietrich&Bechinger, Nature'08]

Conclusions

- Fluctuations + boundaries $\mathcal{B}$

  $\Longrightarrow$ effective (long-ranged) Casimir forces on $\mathcal{B}$

- Universality (**length** $\gg \ell_{\mathbf{micr}}$)

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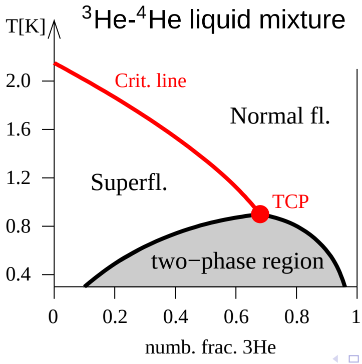
Tricritical Casimir effect:

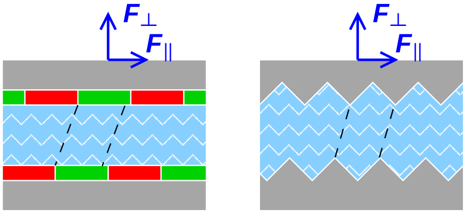
[Garcia&Chan PRL'02]
[Maciołek&Dietrich EPL'06]
[Maciołek,Gambassi&Dietrich PRE'07]
$\hookrightarrow$ **Talks: Garcia 4 Nov**
Chan 2 Sep



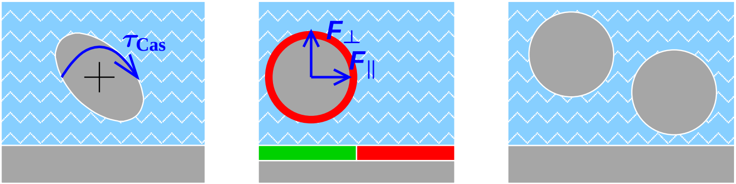
Perspectives

» Non-homogeneous surf.s (chem/phys)

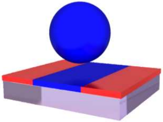
[Sprenger,Schlesener&Dietrich JCP'06]
[Tröndle,Harnau&Dietrich arXiv'08]

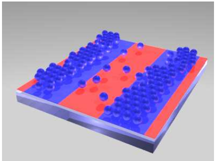


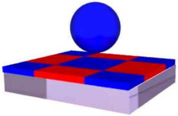
» Geometry (torque, many-body)

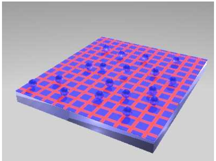


stripes: chessboard:



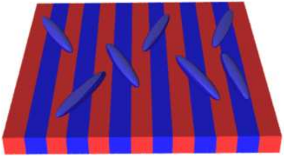


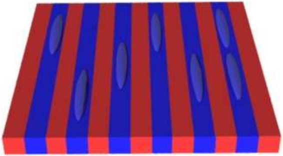




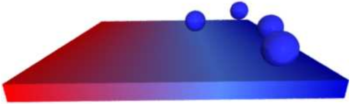
[Soyka,Zvyagolskaya,Hertlein,Helden,Bechinger PRL'08]

play with orientation:



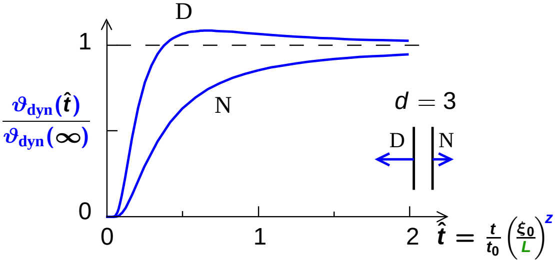


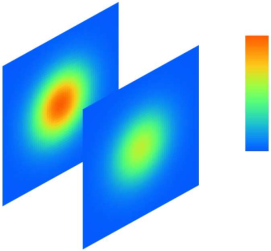
transport:



» Eq./Non-eq. dynamics

[Gambassi&Dietrich JSP'06]





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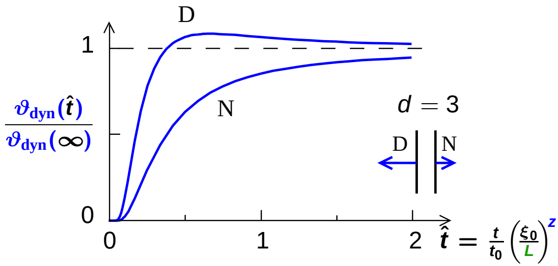
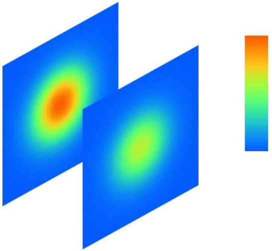
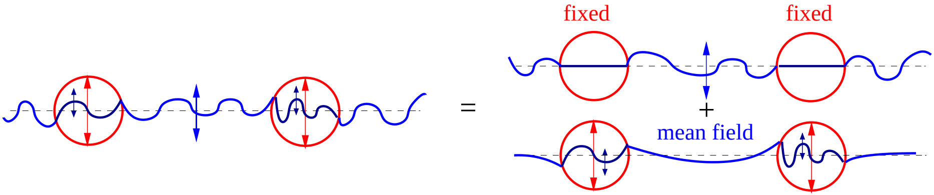




» Capillary waves

[Lehle&Oettel PRE'07]
[Lehle,Oettel&Dietrich EPL'06]



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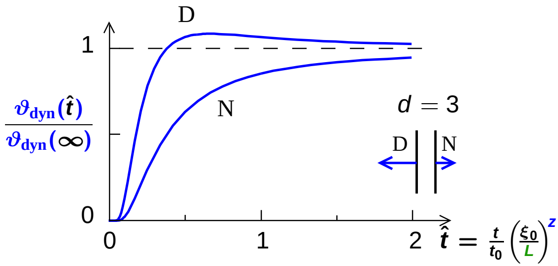

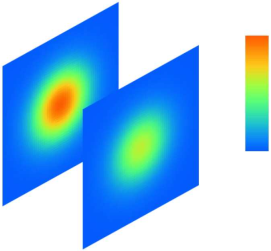

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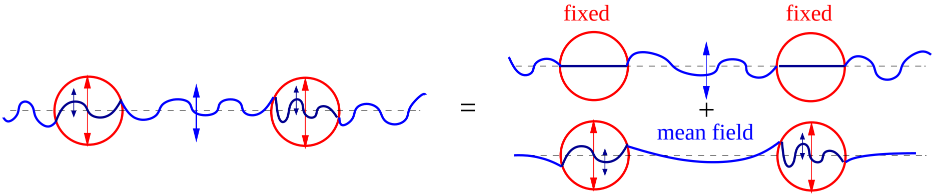

» Colloids as model systems/photonic applications?

» Anti-stiction in MEMS?

Acknowledgments

Theory	*Experiments*
S. Dietrich	**C. Bechinger**
A. Maciolek	**C. Hertlein**
O. Vasilyev	**L. Helden**
L. Harnau	
M. Tröndle	
M. Oettel	
D. Dantchev	

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M. Krech, F. Schlesener, M. Sprenger, H. Lehle