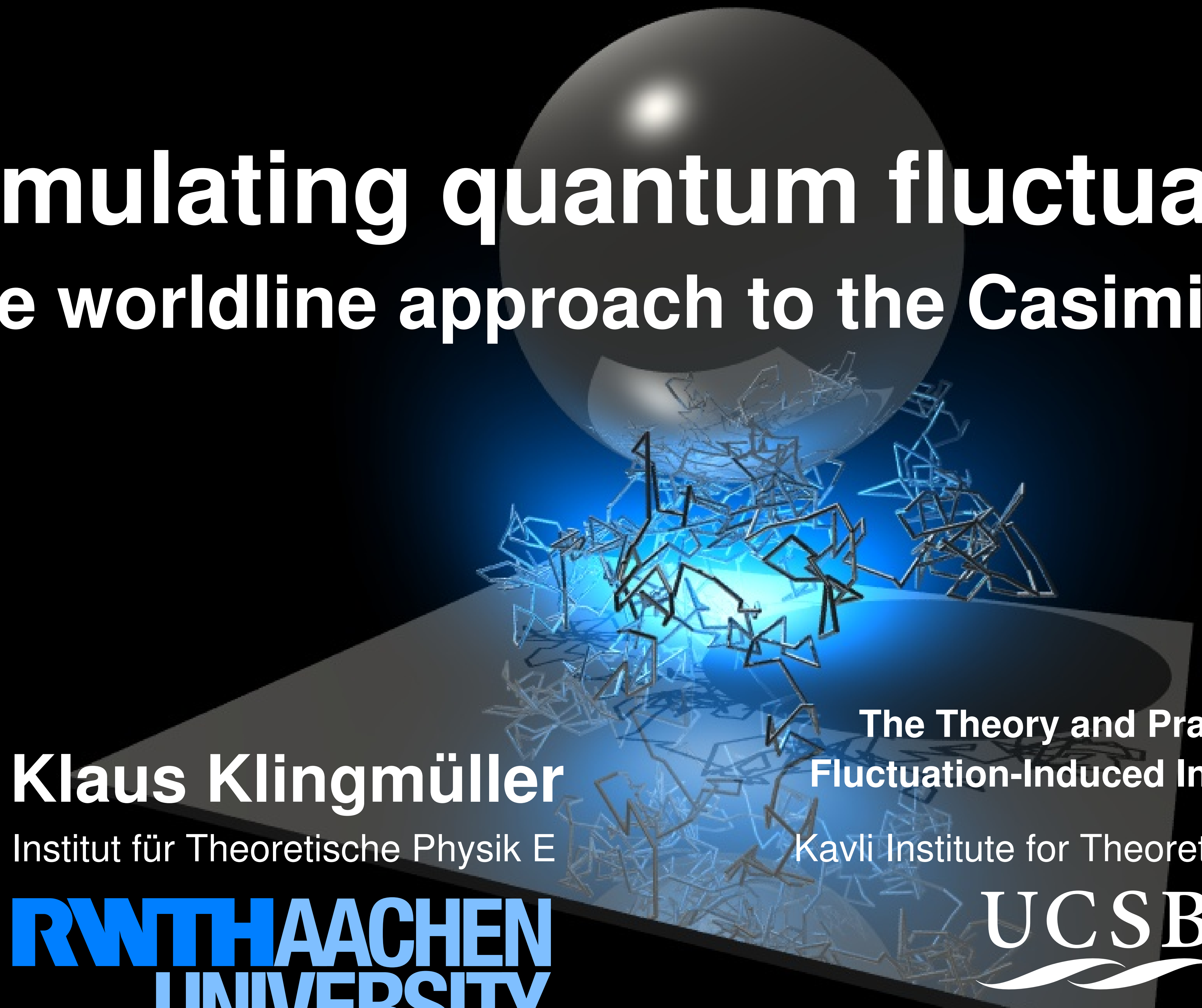
Simulating quantum fluctuations

The worldline approach to the Casimir effect



Klaus Klingmüller

Institut für Theoretische Physik E

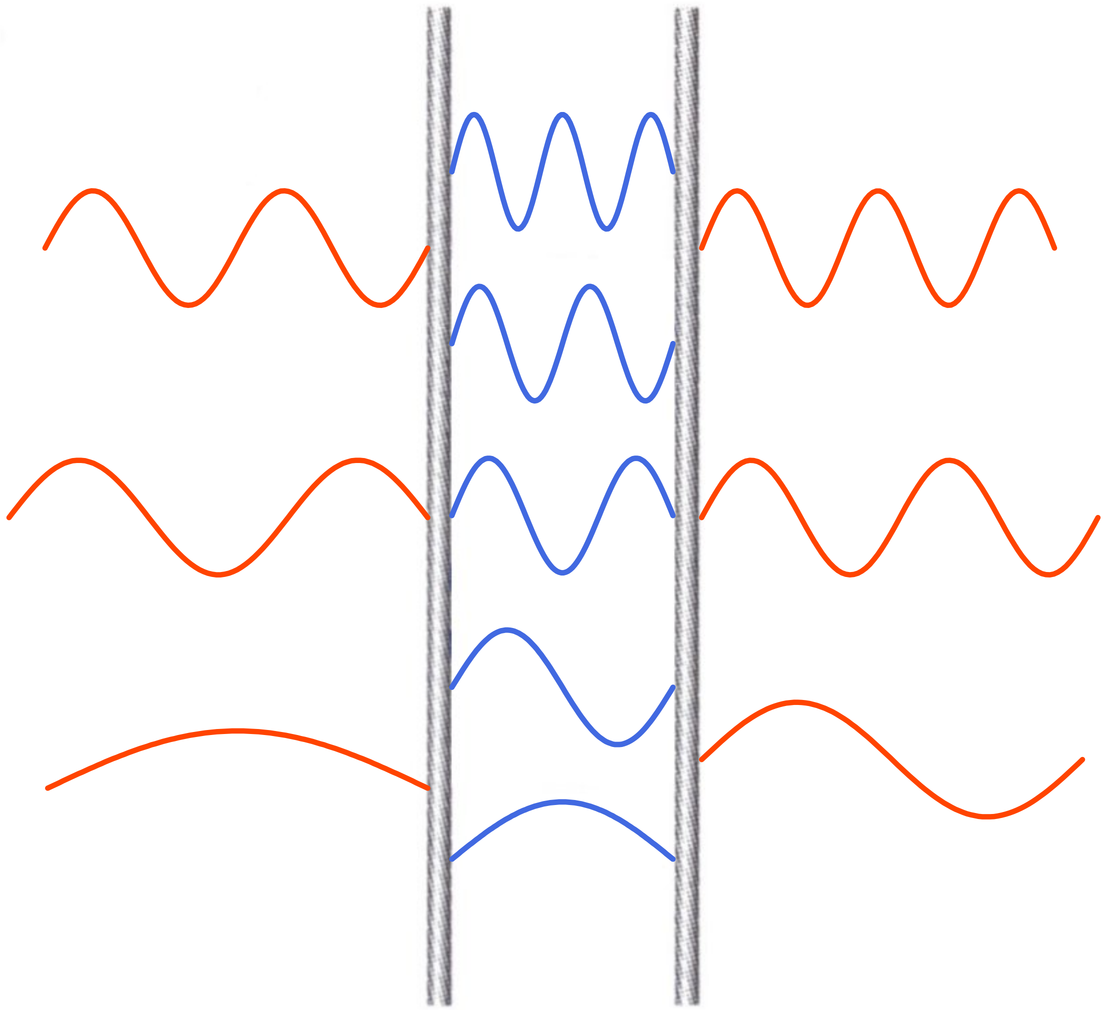
RWTH AACHEN UNIVERSITY

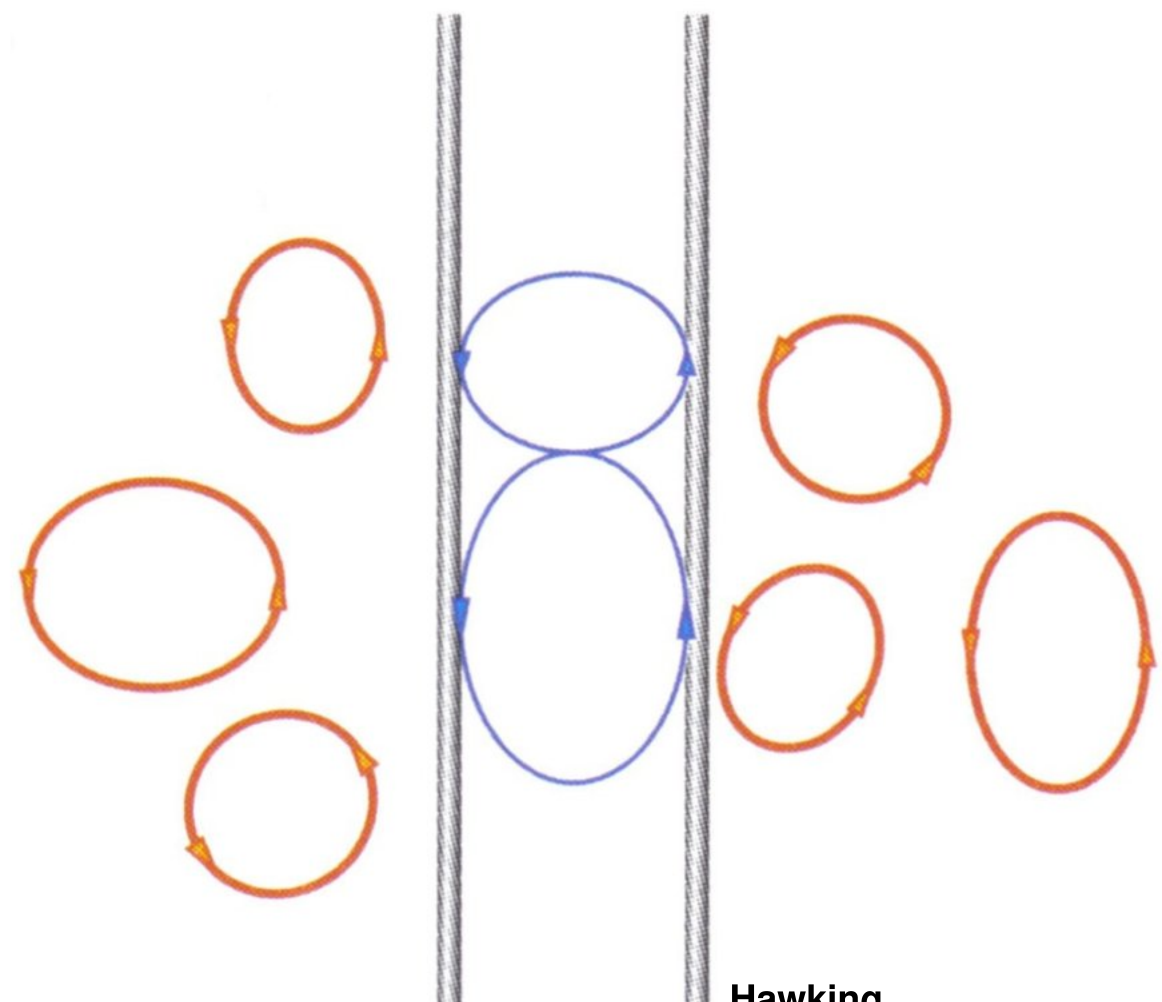
The Theory and Practice of Fluctuation-Induced Interactions

Kavli Institute for Theoretical Physics

UCSB

UNIVERSITY OF CALIFORNIA
SANTA BARBARA

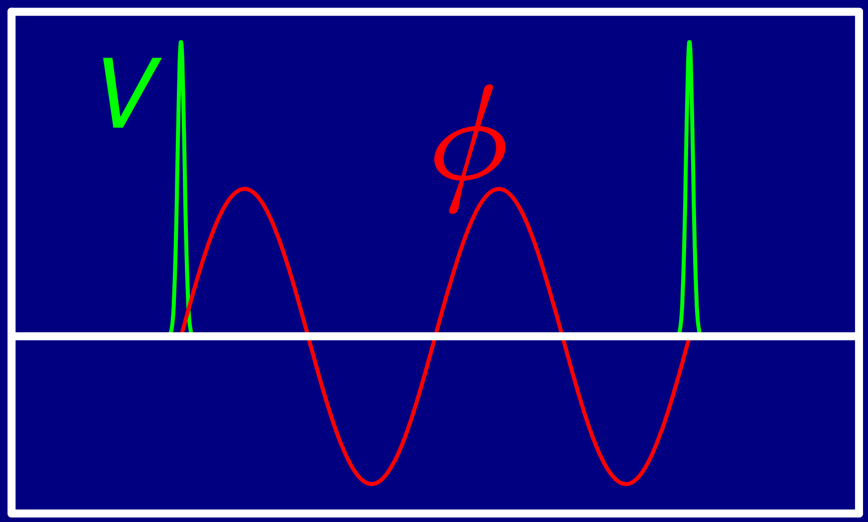




Hawking

Field-theoretic formulation

$$\mathscr{L}_{\text{Euclid.}} = \frac{1}{2}(\partial \phi)^2 + \frac{1}{2} V(x) \phi^2$$



Scalar field, Dirichlet boundaries

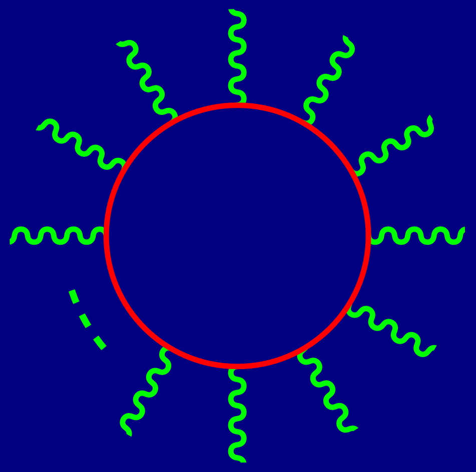
$$E_\Omega = \frac{\Gamma_{\text{Euclid.}}}{\int dx_D} \qquad \Gamma = \Gamma[V] = \Gamma(a, \alpha, \ldots)$$

Bordag, Hennig, Robaschik [J. Phys. A 25 (1992) 4483]
Graham, Jaffe, Khemani, Quandt, Scandurra, Weigel [Nucl. Phys. B 645 (2002) 49]

Worldline formalism

Loop expansion:

$$\Gamma = \frac{1}{2}\mathrm{Tr}\ln\frac{\delta^2 S}{(\delta\phi)^2} =$$



$$= \frac{1}{2}\mathrm{Tr}\ln\left(-\partial^2 + V(x)\right)$$

$$= -\frac{1}{2}\int d^D x \int_{1/\Lambda^2}^{\infty} \frac{dT}{T} \underbrace{\langle x|e^{-T(-\partial^2+V)}|x\rangle}_{\mathcal{N}\int_{x(0)=x(T)=x} \mathscr{D}x\, e^{-\int_0^T d\tau\, (\dot{x}^2/4+V)}}$$

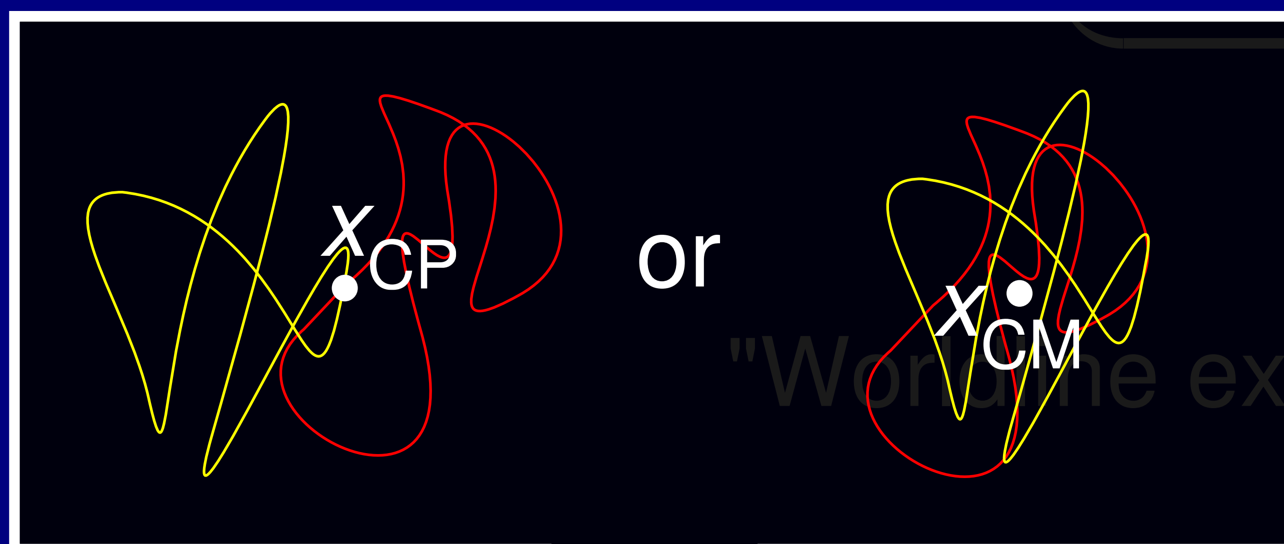
Feynman [Phys. Rev. 80 (1950) 440]
Polyakov [Gauge fields and strings, Harwood, 1987]
Bern, Kosower [Nucl. Phys. B 362 (1991) 389]
Schmidt, Schubert [Phys. Lett. B 318 (1993) 438]

$$\mathcal{N} \int\limits_{x(0)=x(T)=x} \mathcal{D}x \cdots e^{-\int_0^T d\tau \ \dot{x}^2/4} = \frac{1}{(4\pi T)^{D/2}} \underbrace{\frac{\int \mathcal{D}x \cdots e^{-\int_0^T d\tau \ \dot{x}^2/4}}{\int \mathcal{D}x \ e^{-\int_0^T d\tau \ \dot{x}^2/4}}}_{=: \left\langle \cdots \right\rangle_x}$$

"Worldline expectation value"

$$E_\Omega = -\frac{1}{2(4\pi)^{D/2}} \int d^d x \int_{1/\Lambda^2}^{\infty} \frac{dT}{T^{1+D/2}} \left\langle e^{-\int_0^T d\tau \ V} \right\rangle_x$$

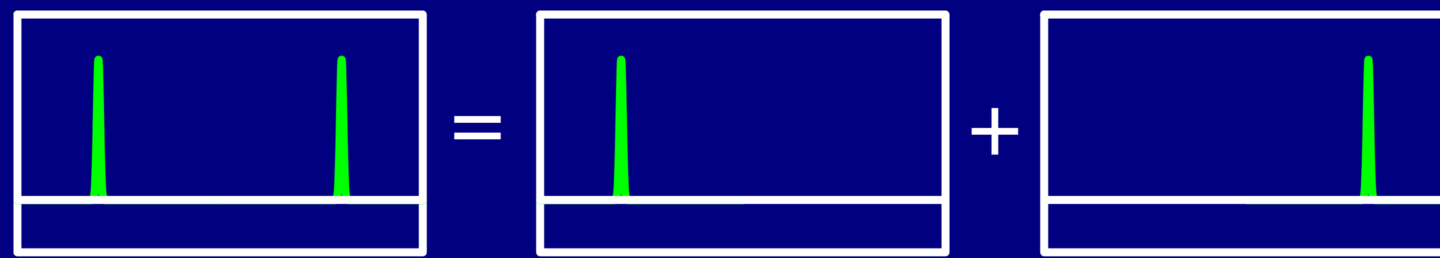
$$\mathcal{N}\int\limits_{x(0)=x(T)=x}\mathcal{D}x\cdots e^{-\int_0^T d\tau\;\dot{x}^2/4} = \frac{1}{(4\pi T)^{D/2}}\;\underbrace{\frac{\int\mathcal{D}x\cdots e^{-\int_0^T d\tau\;\dot{x}^2/4}}{\int\mathcal{D}x\; e^{-\int_0^T d\tau\;\dot{x}^2/4}}}_{=:\,\langle\cdots\rangle_x}$$



"Worldline expectation value"

$$E_\Omega = -\frac{1}{2(4\pi)^{D/2}}\int d^d x\int_{1/\Lambda^2}^{\infty}\frac{dT}{T^{1+D/2}}\,\left\langle e^{-\int_0^T d\tau\; V}\right\rangle_x$$

Casimir interaction energy



$V = V_1 + V_2 + \ldots \quad \rightarrow \quad$ subtract self-energies:

$$E_{\mathrm{Casimir}} = E_\Omega[V] - E_\Omega[V_1] - E_\Omega[V_2] - \cdots$$

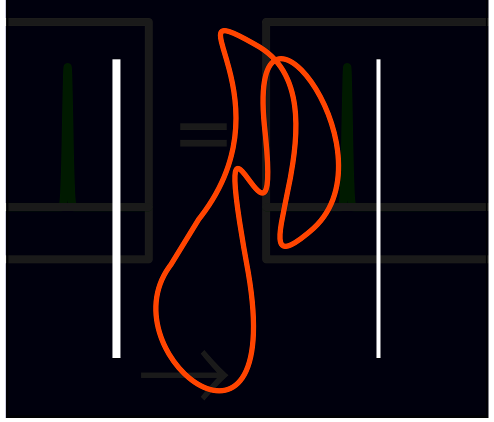
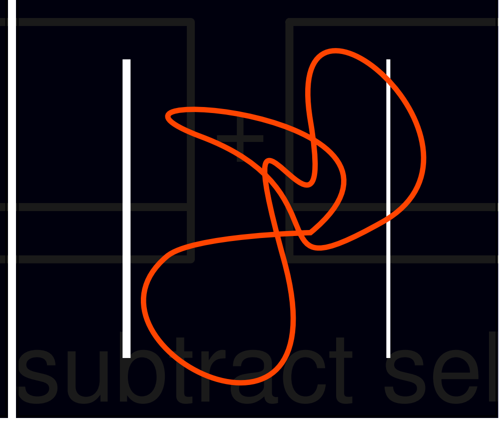
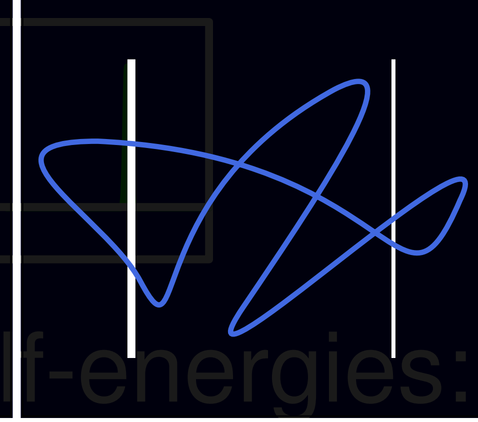
$$e^{-\int_0^T d\tau\ V} - e^{-\int_0^T d\tau\ V_1} - e^{-\int_0^T d\tau\ V_2} + 1 = \Theta_\Sigma[\mathbf{x}(\tau)]$$

$$\boxed{V(\mathbf{x}) = \lambda \int_\Sigma d\sigma\ \delta^{(d)}(\mathbf{x} - \mathbf{x}_\sigma)\ , \quad \lambda \rightarrow \infty}$$

$$E_{\mathrm{Casimir}} = -\frac{1}{2(4\pi)^{D/2}} \int d^d x \int_0^\infty \frac{dT}{T^{1+D/2}} \left\langle \Theta_\Sigma[\mathbf{x}(\tau)] \right\rangle_x$$

Gies, Langfeld, Moyaerts [JHEP 0306:018,2003]

Casimir interaction energy

$\mathbf{x}(\tau)$			
$\Theta_\Sigma[\mathbf{x}(\tau)]$	0	0	1

$V = V_1 + V_2 + \ldots \longrightarrow$ subtract self-energies:

$E_{\text{Casimir}} = E_\Omega[V] - E_\Omega[V_1] - E_\Omega[V_2] \cdots$

$$e^{-\int_0^T d\tau \; V} - e^{-\int_0^T d\tau \; V_1} - e^{-\int_0^T d\tau \; V_2} + 1 = \Theta_\Sigma[\mathbf{x}(\tau)]$$

$$\boxed{V(\mathbf{x}) = \lambda \int_\Sigma d\sigma \;\; \delta^{(d)}(\mathbf{x} - \mathbf{x}_\sigma)\,, \qquad \lambda \to \infty}$$

$$E_{\text{Casimir}} = -\frac{1}{2(4\pi)^{D/2}} \int d^d x \int_0^\infty \frac{dT}{T^{1+D/2}} \left\langle \Theta_\Sigma[\mathbf{x}(\tau)] \right\rangle_x$$

Gies, Langfeld, Moyaerts [JHEP 0306:018,2003]

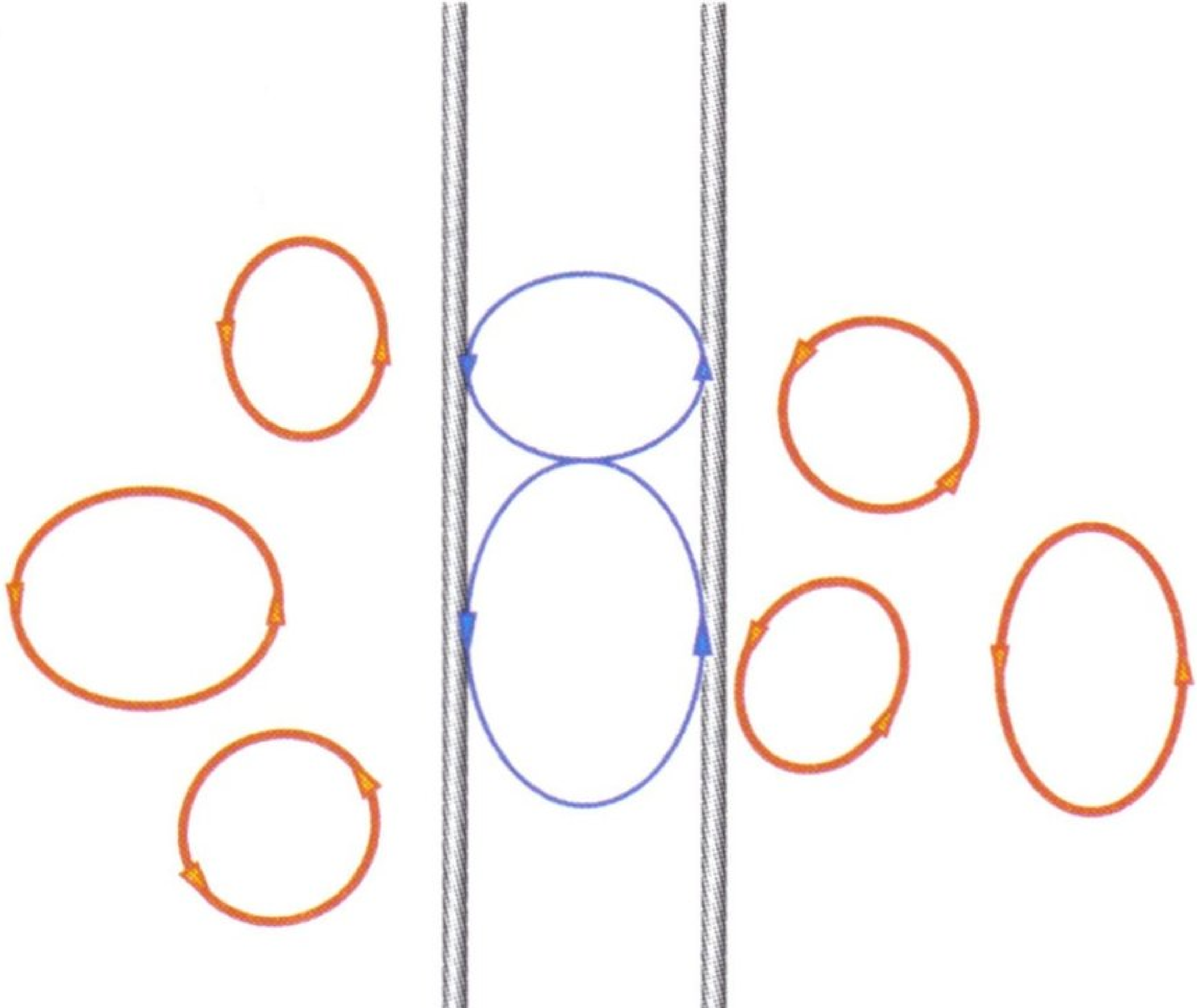
Worldline numerics

Use finite Gaußian ensemble of discrete paths:

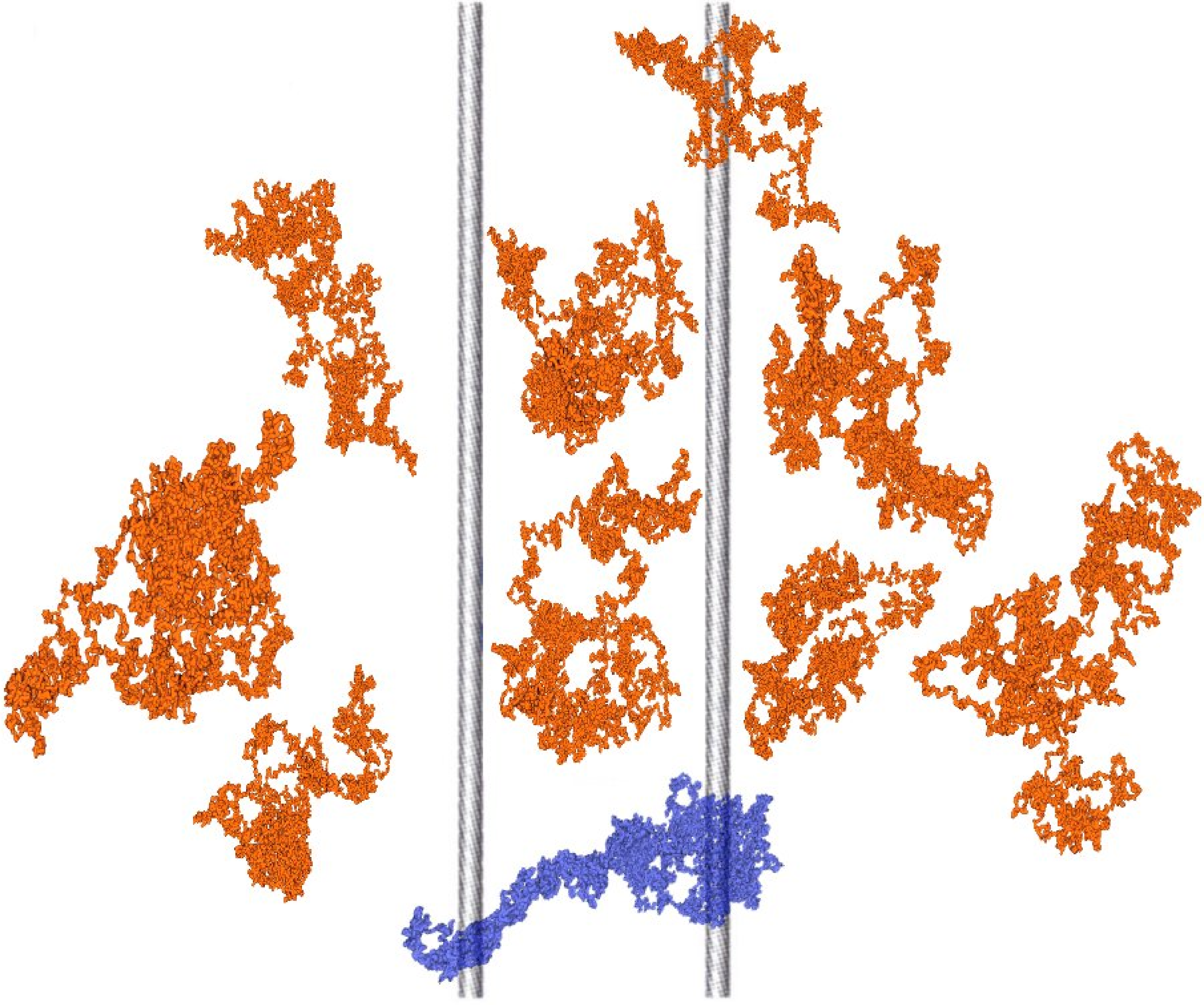
$$\int \mathcal{D}x \cdots e^{-\int_0^T d\tau\ \dot{x}^2/4} \quad \longrightarrow \quad \sum \cdots$$

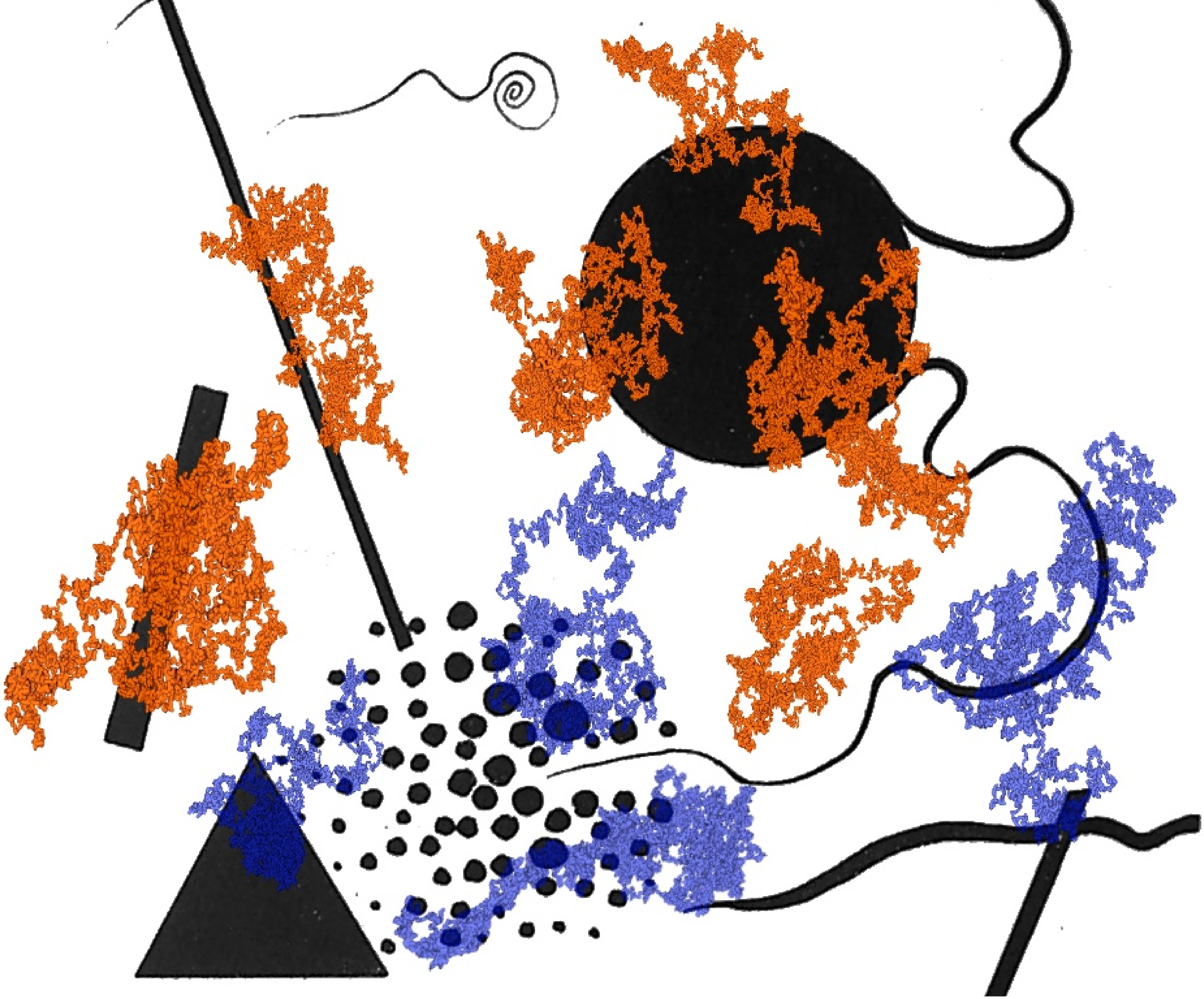
- Generate only "unit loops" with CM/CP $x = 0$ and $T = 1$, shift and rescale to obtain the rest
- *Embarrassingly parallel problem*:
  divide loop ensemble into independently processed sub-ensembles ($\rightarrow$ **MPI**, Open**MP**)
- Exploit symmetries / specialise to (dramatically) reduce CPU time
- Perform (some) integrals analytically

Gies, Langfeld [Nucl.Phys.B613:353-365,2001]

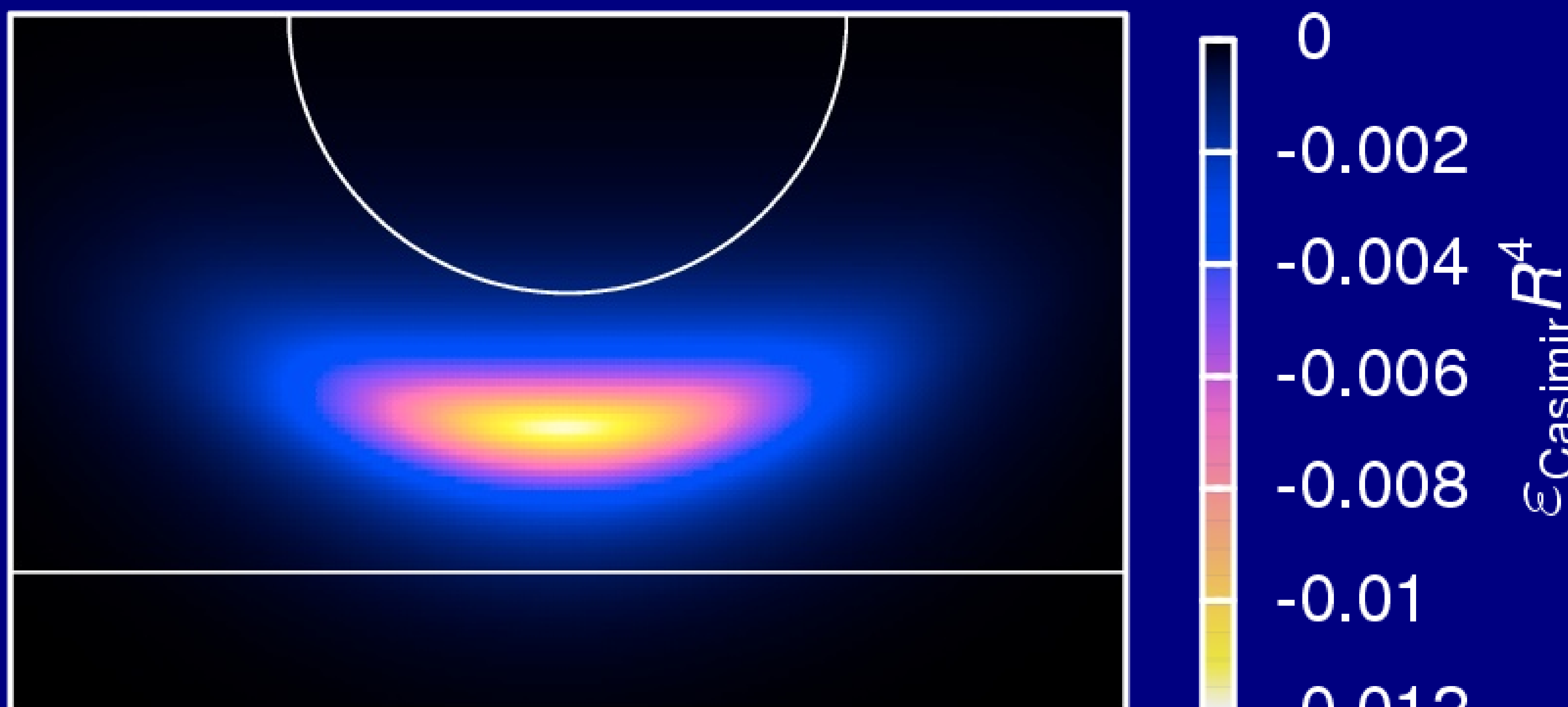


Hawking

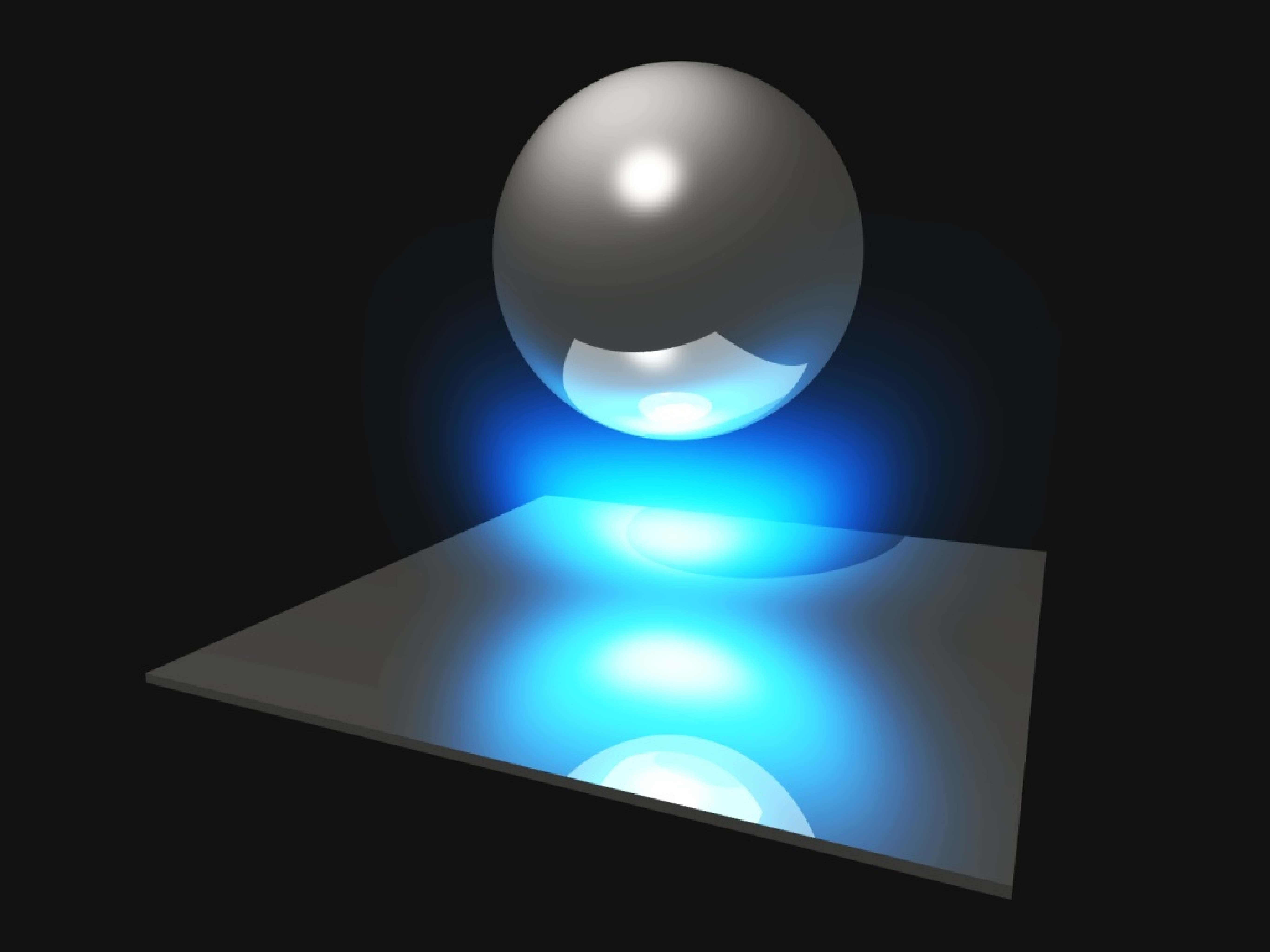


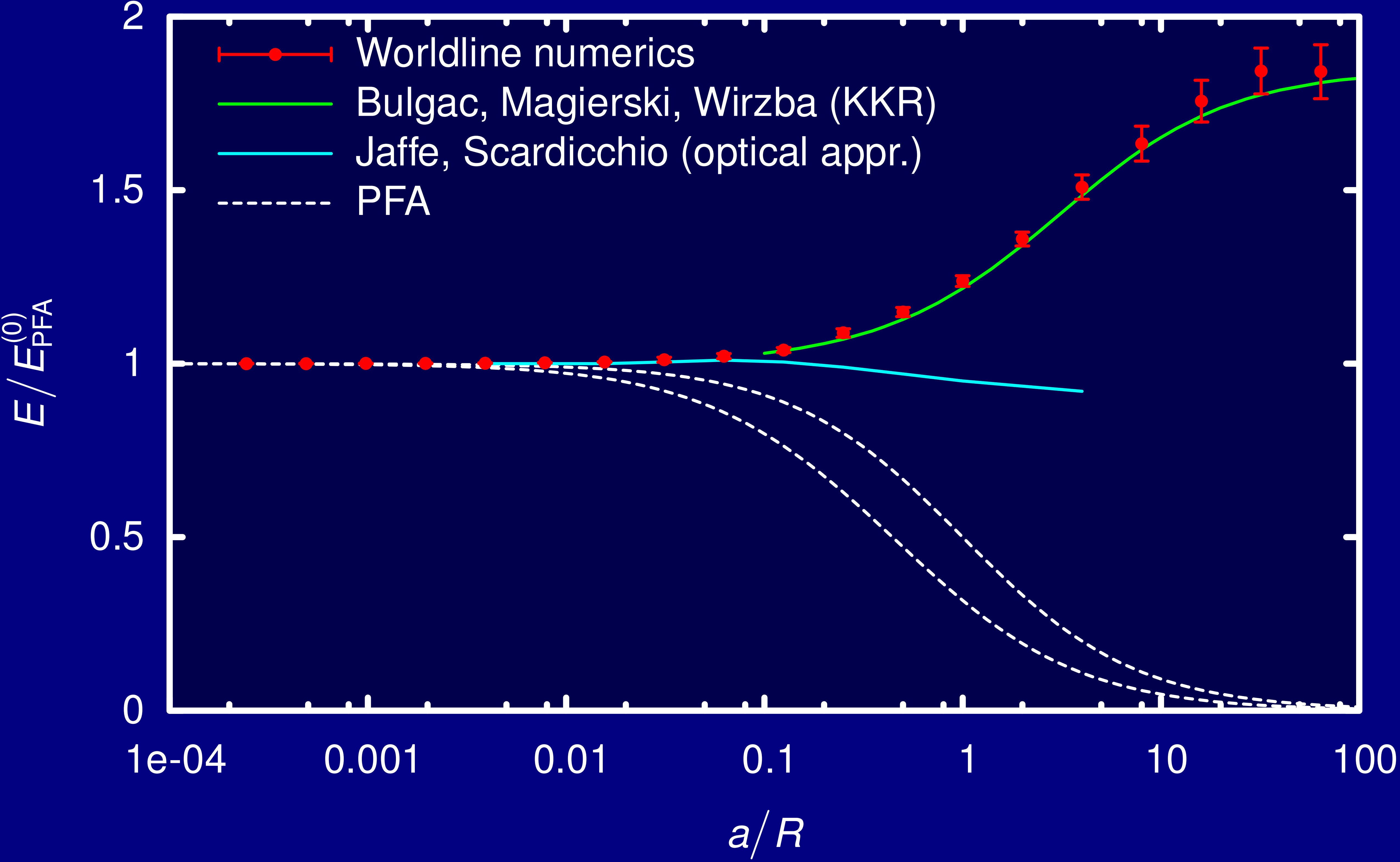


$$\overset{\varepsilon_{\text{Casimir}}}{\cancel{E_{\text{Casimir}}}} = -\frac{1}{2(4\pi)^{D/2}} \cancel{\int d^d x} \int_0^\infty \frac{dT}{T^{1+D/2}} \left\langle \Theta_\Sigma[\mathbf{x}(\tau)] \right\rangle_x$$

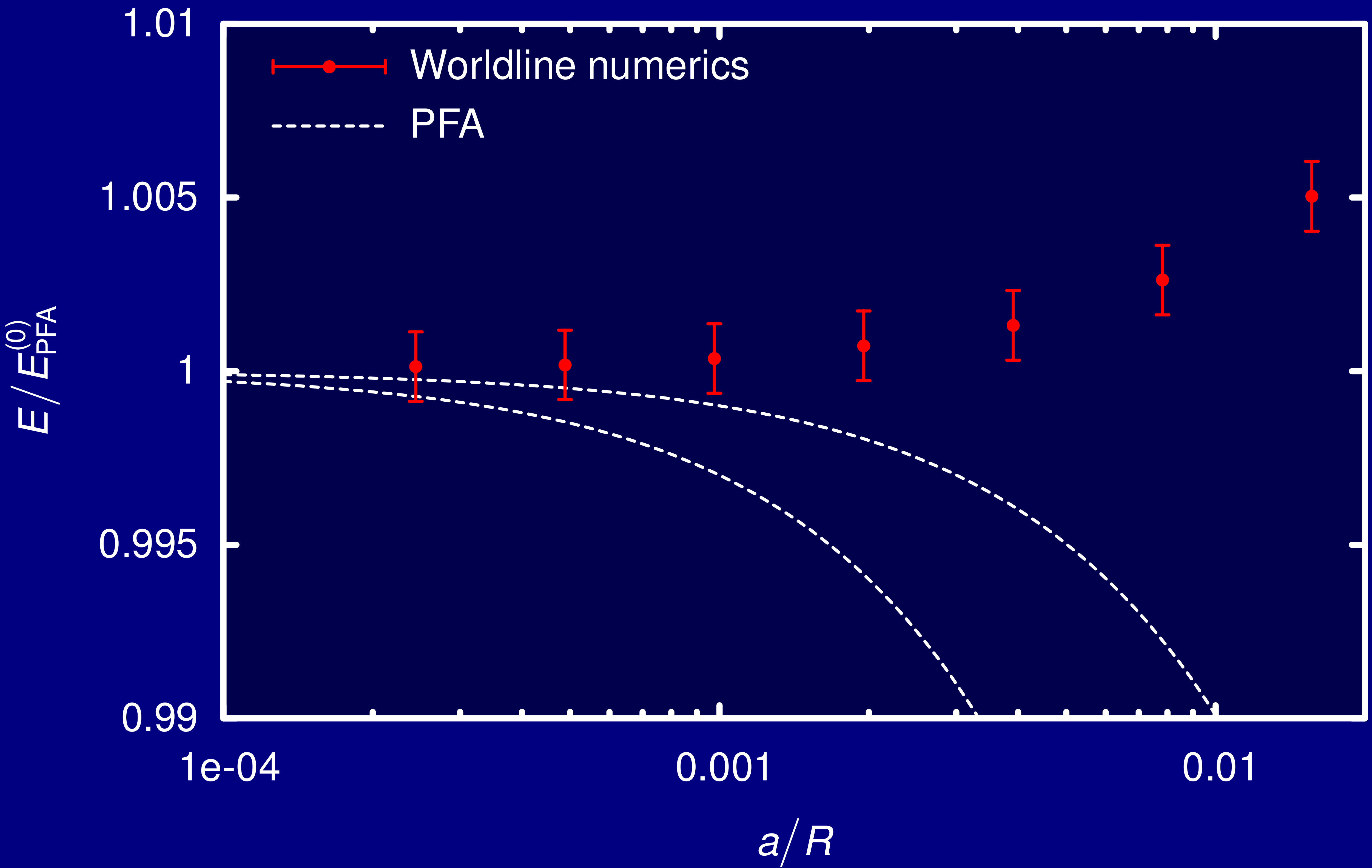


Gies, KK [Phys. Rev. Lett. 96 (2006) 220401]





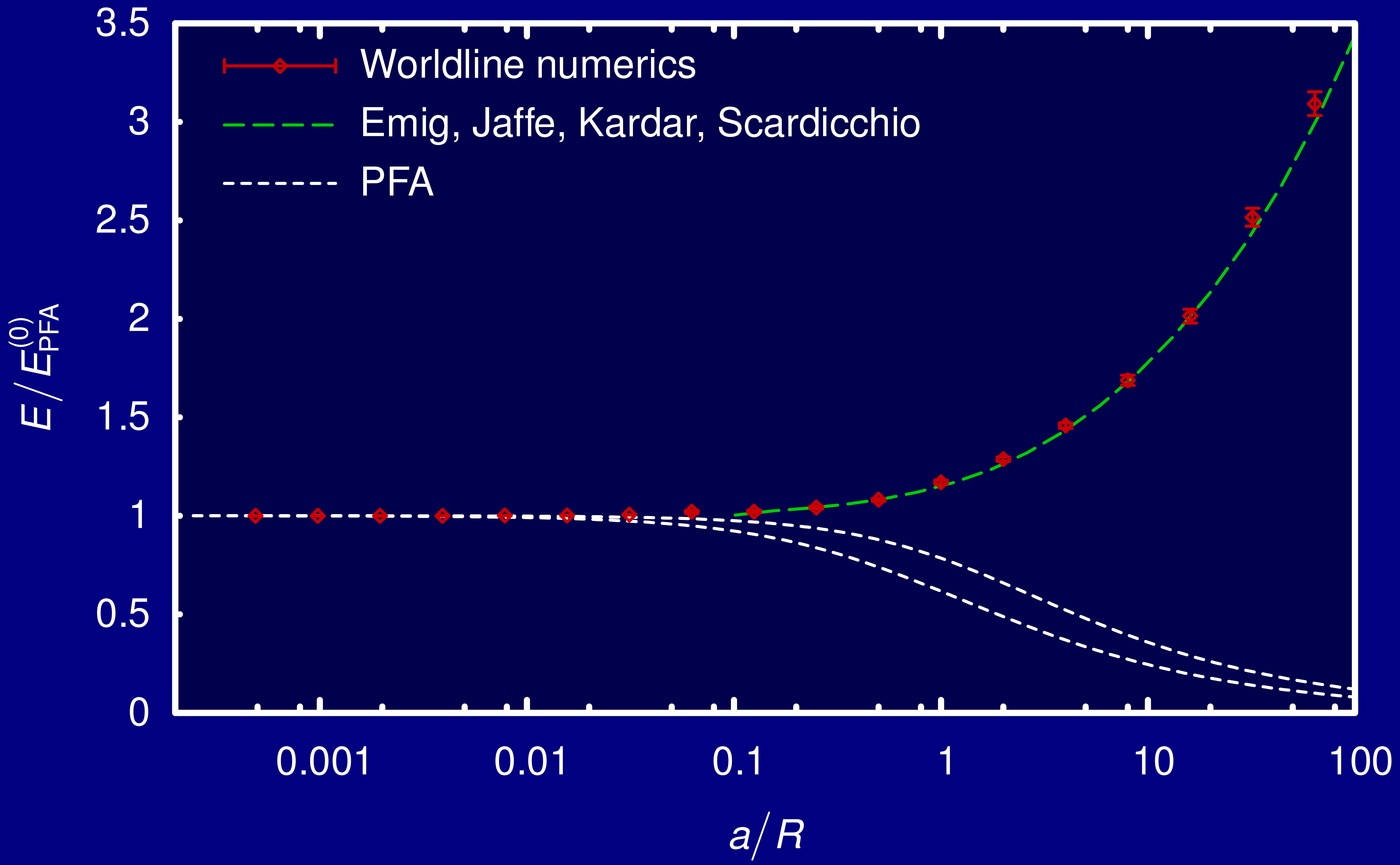
Worldline numerics
Bulgac, Magierski, Wirzba (KKR)
Jaffe, Scardicchio (optical appr.)
PFA
2
1.5
1
0.5
0
1e-04
0.001
0.01
0.1
1
10
100
$E / E^{(0)}_{\mathrm{PFA}}$
a / R



Coefficient of linear term: 0.33 ± 0.06

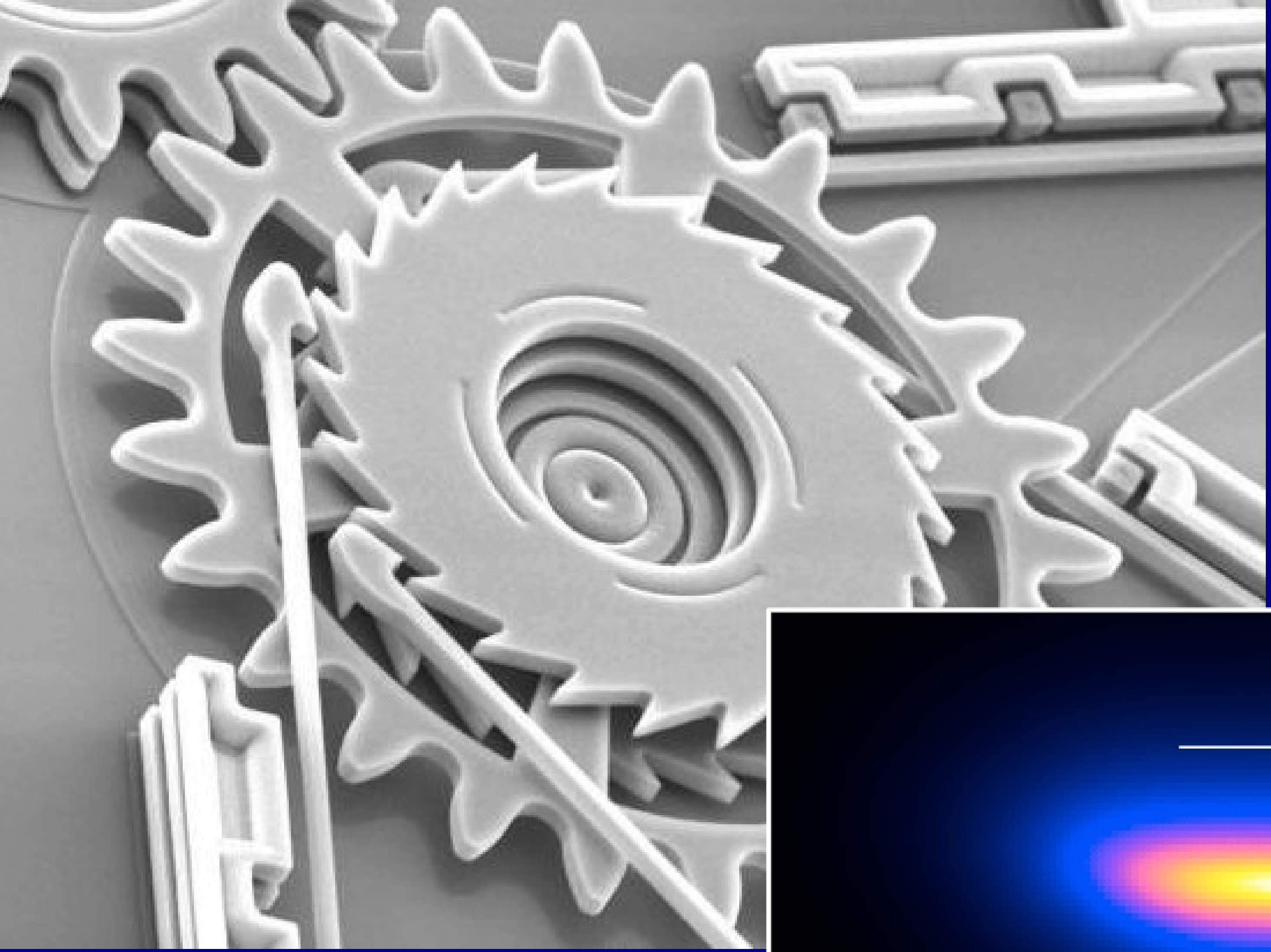
Exact: 1/3 Bordag [J. Phys. A **41** (2008) 164002]

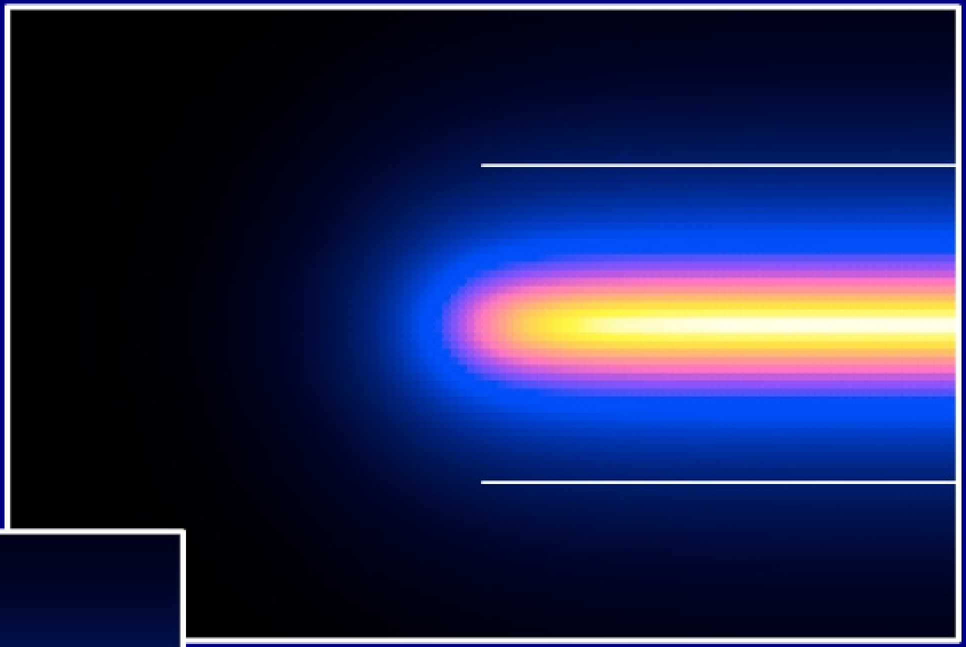
Cylinder-Plate

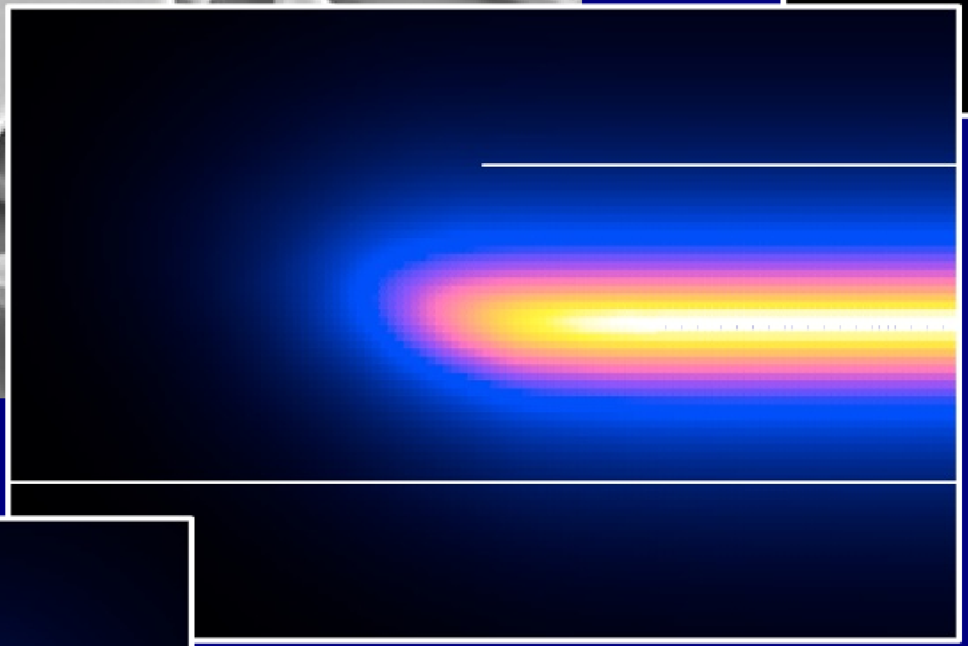


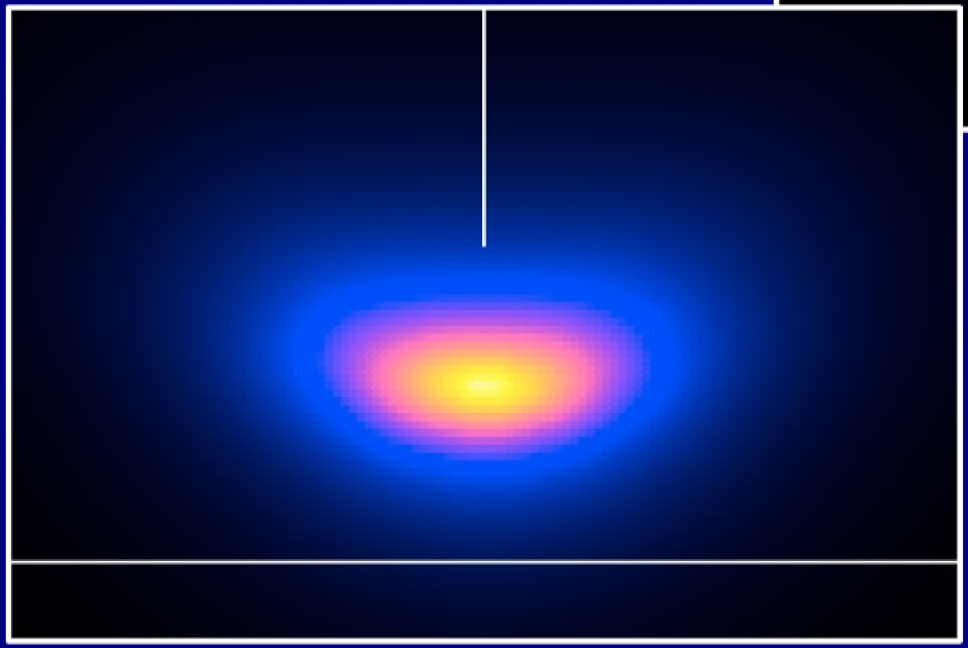
Coefficient of linear term: 0.195 ± 0.028

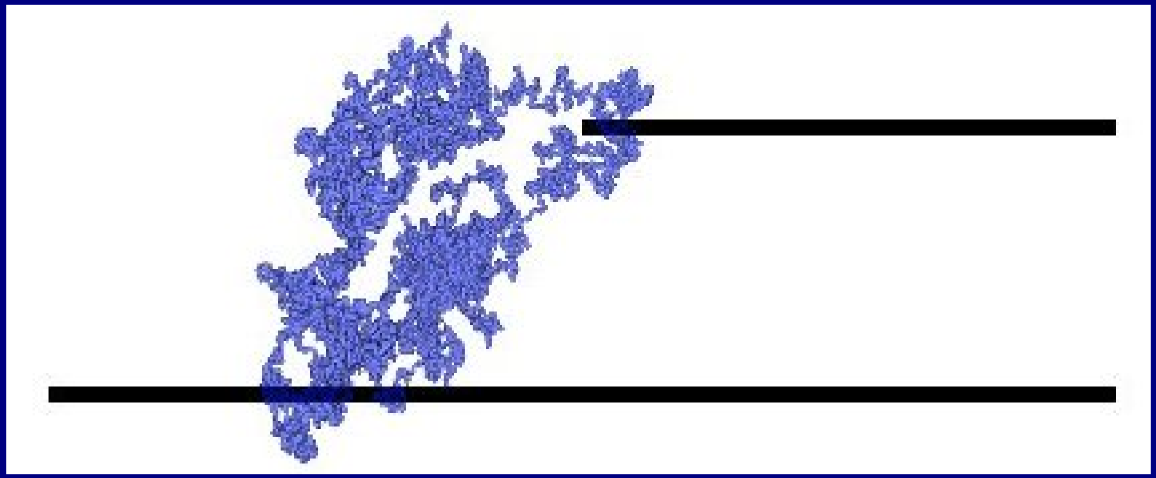
Exact: 0.19444... Bordag [Phys. Rev. D **73** (2006) 125018]



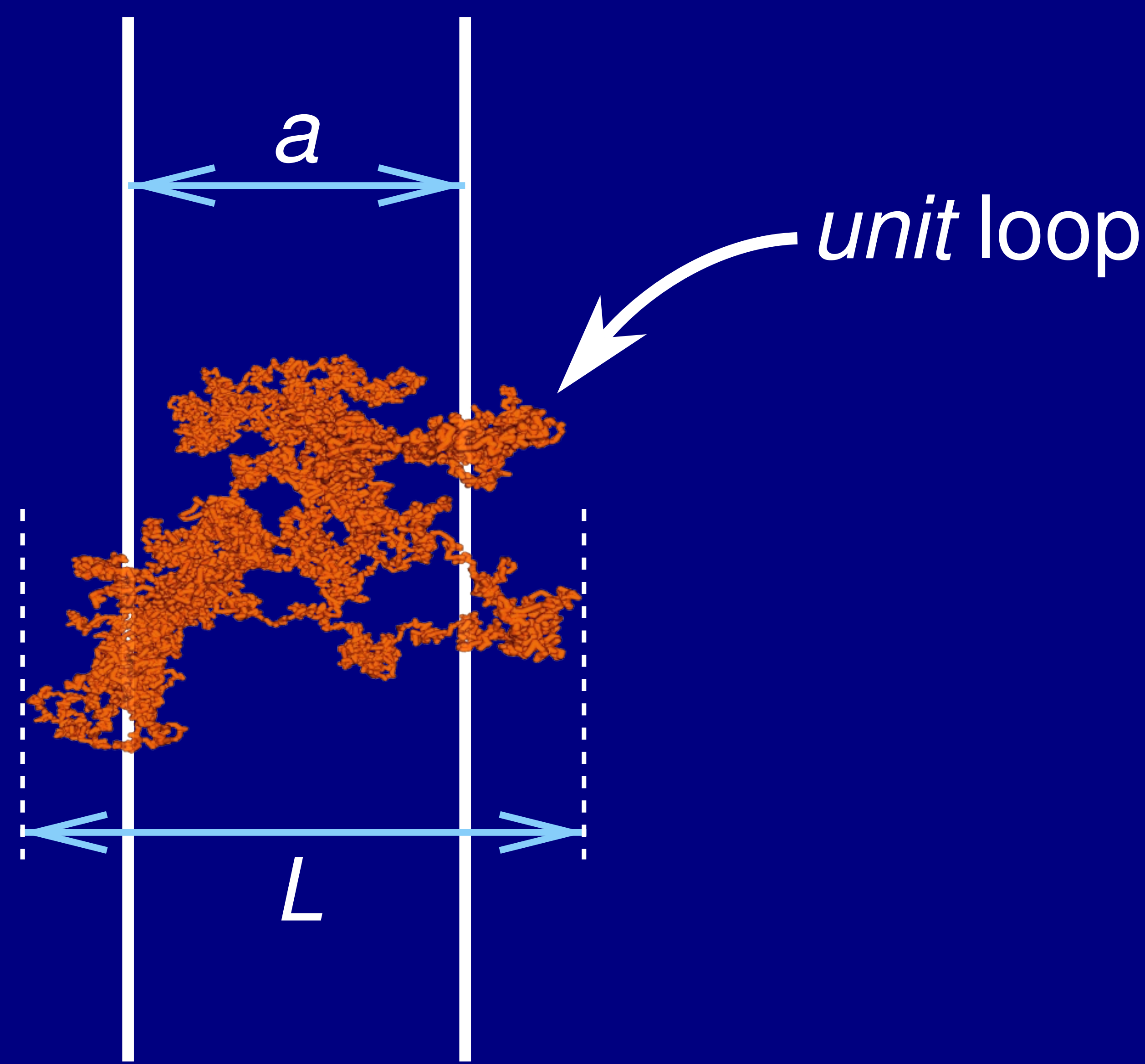








Gies, KK [Phys. Rev. Lett. 97 (2006) 220405]

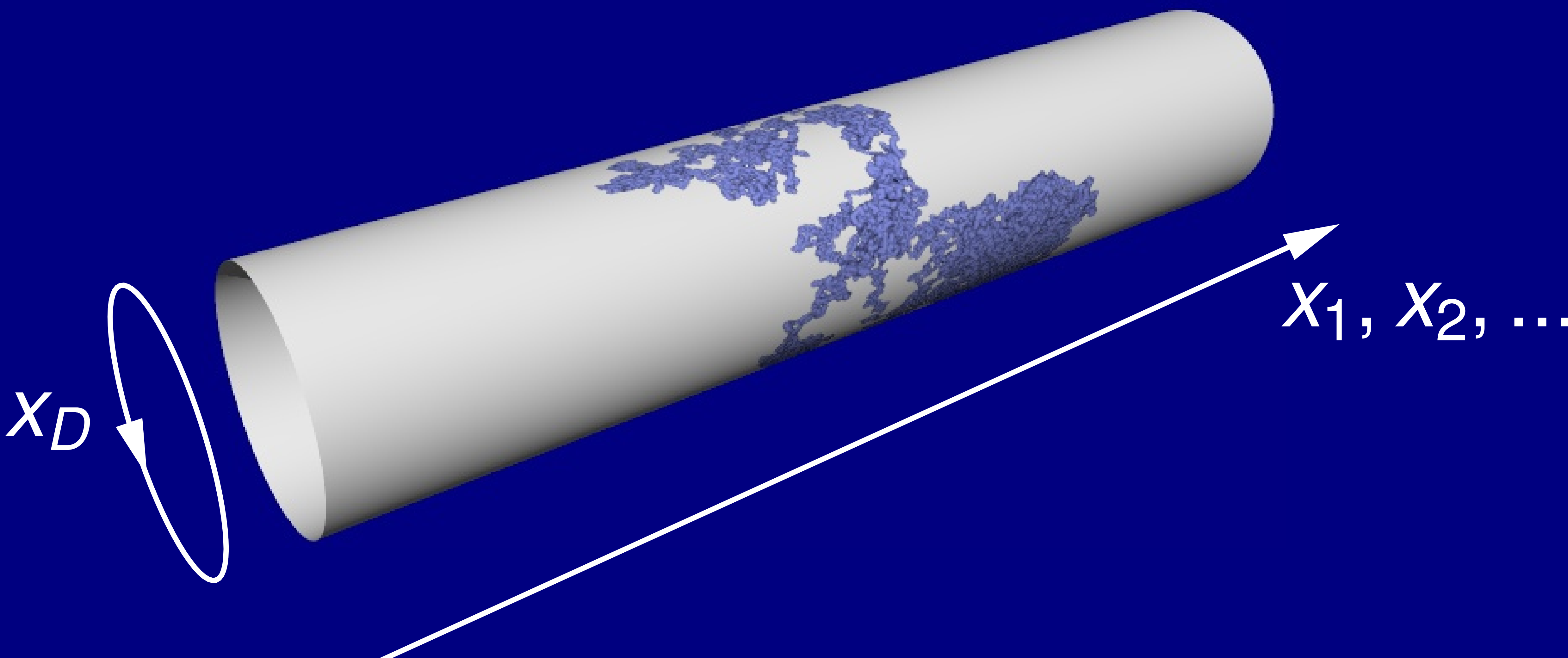


$$E_{\text{Casimir}} = -\frac{A}{D(D-1)(4\pi)^{D/2}a^{D-1}}\langle L^D \rangle$$

$$\langle L^D \rangle = D(D-1)\,\Gamma(D/2)\,\zeta(D)$$

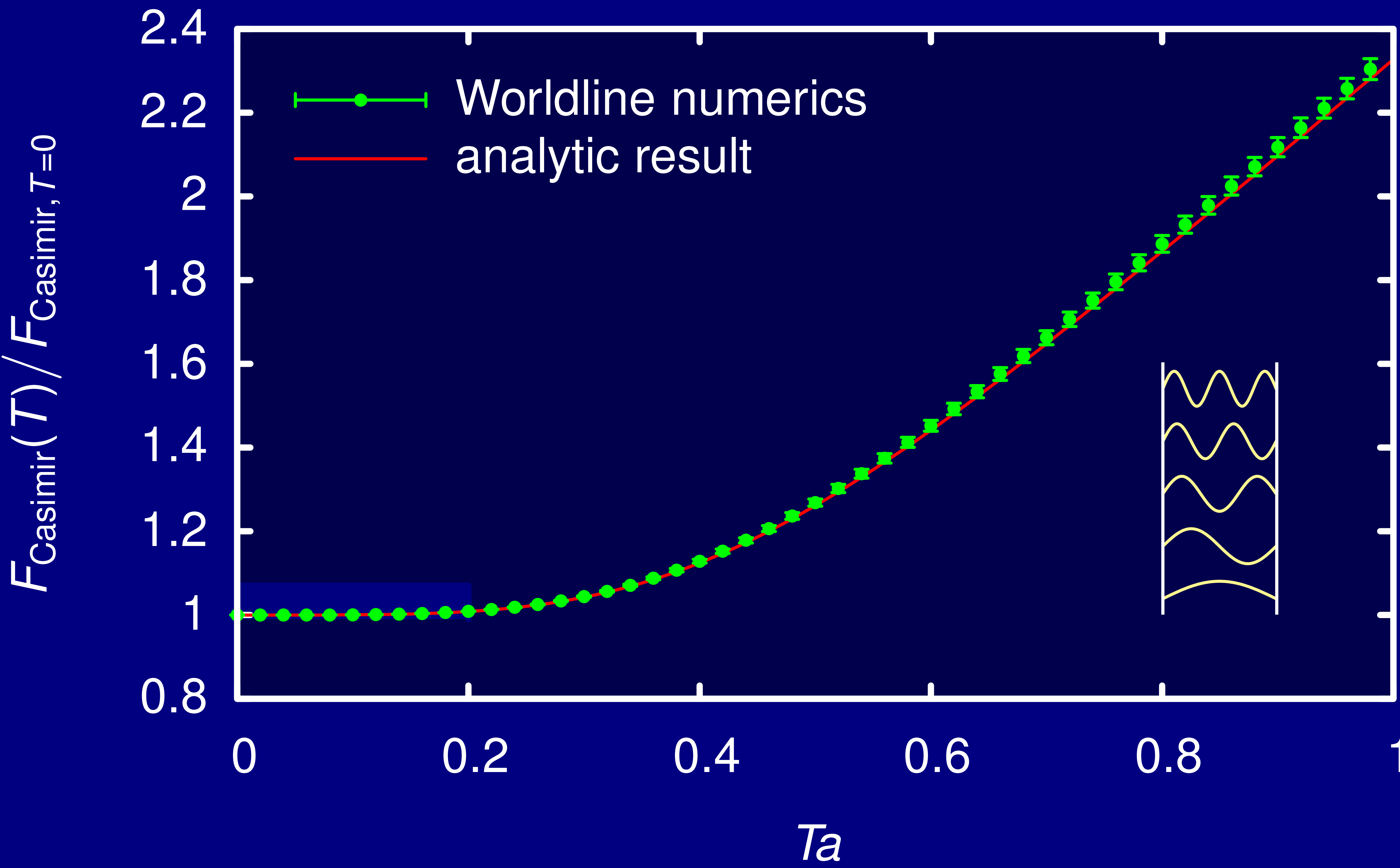
Gies, KK [Phys.Rev.D74:045002,2006]

Geothermal Casimir Phenomena

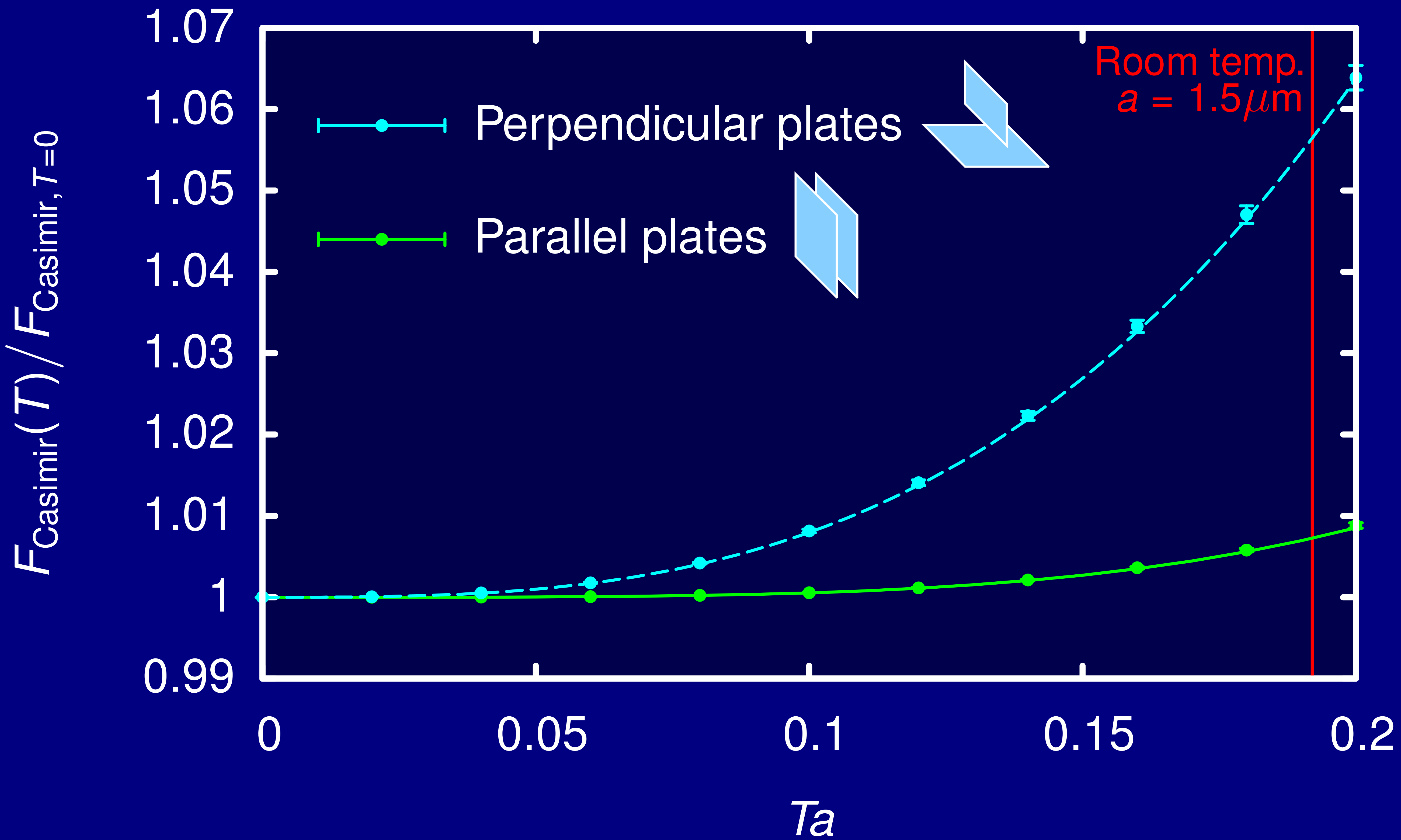


$$\int_{x(0)=x(T)} \mathscr{D}x \ldots \longrightarrow \left(\sum_{n=-\infty}^{\infty} e^{-\frac{n^2\beta^2}{4T}}\right) \int_{x(0)=x(T)} \mathscr{D}x \ldots$$

Infinite parallel plates:

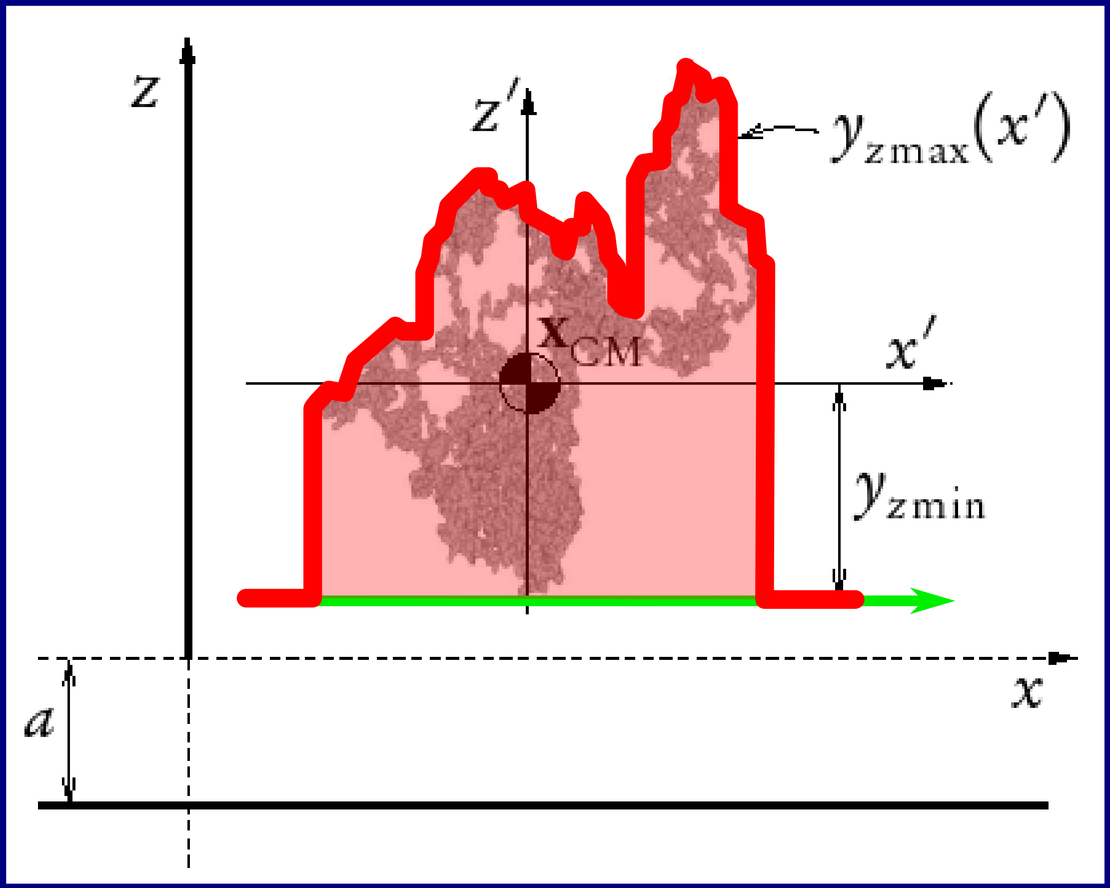


see e.g. Feinberg, Mann, Revzen [Annals Phys.288:103-136,2001]



Gies, KK [J. Phys. A **41** (2008) 164042]
cf. Scardicchio, Jaffe [Nucl. Phys. B 743 (2006) 249]

$$F_{\text{Casimir}} = -\frac{L}{2(4\pi)^2}\frac{1}{a^3}\left\langle \int d\xi \int_{1/l(\xi)^2}^{\infty} \frac{d\mathscr{T}}{\mathscr{T}^{5/2}} \left(\sum_{n=-\infty}^{\infty} e^{-\frac{n^2}{4\mathscr{T}}\frac{\beta^2}{a^2}} \right) \right\rangle$$



Thermal correction (n ≠ 0 terms) for small temperatures:

- $\mathscr{T}$ integral dominated by large $\mathscr{T}$
- ξ integral independent of ξ
- Use $\left\langle \int d\xi \right\rangle \equiv \langle L_\ell^D \rangle|_{D=1} = \sqrt{\pi}$

$$F_{\text{Casimir}}(T) = F_{\text{Casimir},T=0} - \frac{\zeta(3)}{4\pi}\frac{L}{a^3}(aT)^3 + \mathscr{O}(T^4)$$

Conclusions

- Worldline numerics geometry independent
  - Results for curvature and edge effects
- Temperature easily included
  - First studies of “geothermal Casimir phenomena”
  - Particularly relevant for “open” geometries

Outlook

- Temperature effects in experimentally relevant geometries (sphere plate / cylinder plate...)
- Other boundary conditions (Neumann/EM (?))
- Technically similar problems, e.g., Gross-Neveu model

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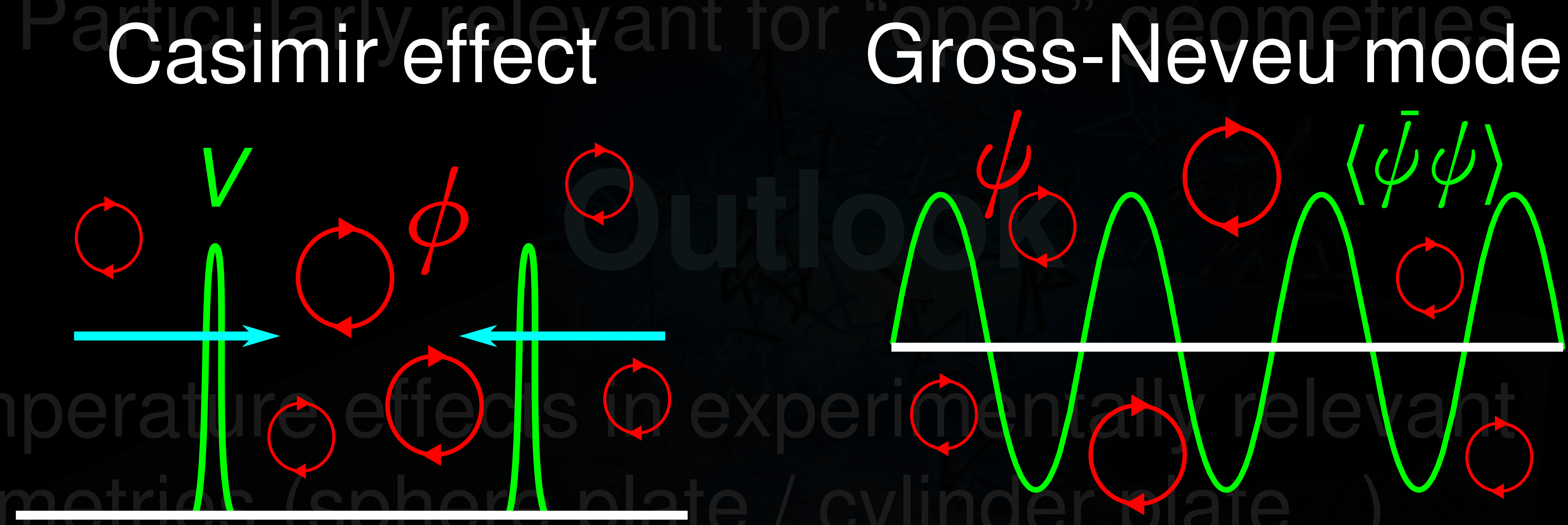

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