# Lattice determination of confining string contributions in the (2+1) dimensional SU(2) $LGT^1$ .

Michele Caselle

Universitá degli Studi di Torino

KITP 20/01/2022



# Summary:

1 Introduction and motivations

- 2 Effective String Theory
- 3 Effective String Theory at Finite Temperature

4 SU(2) simulations



			- `
Muche		lini	
IVITCHE		UIII	101

(日) (四) (日) (日) (日)

э

## Introduction and motivations

- The long distance behaviour of the interquark potential at zero temperature in pure LGTs is well described by the Nambu-Goto Effective String Theory (EST)
- At shorter distances and/or higher temperature corrections beyond the Nambu-Goto action are expected, their study is of great importance to understand confinement: Nambu-Goto can be considered as a sort of mean field model for the interquark potential, the fine details of the gauge theory (the gauge group, the physical mechanisms behind confinement, the physical degrees of freedoms which originate the EST) are encoded in these higher order terms.
- A perfect laboratory to address this issue is the (2+1) dimensional SU(2) model, which can be simulated at a reasonable cost and has a second order deconfinement transition which allows to use universality to constrain the model in the vicinity of the critical point.

イロト イヨト イヨト

# Lattice regularization and quark confinement.

Only a truly non-perturbative approach such as lattice regularization can describe the deconfinement transition and the confined phase of non-abelian gauge theories. For SU(N) pure gauge theories on the lattice the dynamics are described by the standard Wilson action

$$S_W = eta \sum_{p=sp,tp} (1 - rac{1}{N} \operatorname{ReTr} U_p)$$

where  $U_P$  is the product of four  $U_\mu$  SU(N) variables on the space-like or time-like plaquette P and  $\beta = \frac{2N}{g^2}$ . The partition function is

$$Z = \int \prod_{x,\mu} \mathrm{d} U_{\mu}(x) e^{-S_W}$$

the expectation value of an observable A

$$\langle A \rangle = rac{1}{Z} \int \prod_{n,\mu} \mathrm{d} U_{\mu}(n) A(U_{\mu}(n)) e^{-S_W}$$



#### Lattice determination of the interquark potential.

In pure lattice gauge theories the interquark potential is usually extracted from two (almost) equivalent observables

• Wilson loop expectation values < W(R, T) > ("zero temperature potential")

$$V(R) = \lim_{T o \infty} -rac{1}{T} \log < W(R,T) >$$



Michele (bselle (1)	ni Lo I
michele caselle (0	

4 A >

• Polyakov loop correlators  $< P(0)P(R)^{\dagger} >$  ("finite temperature potential")

$$< P(0)P(R)^{\dagger} > \sim \sum_{n=0}^{\infty} c_n e^{-N_t E_n}$$

where  $N_t$  is the inverse temperature, i.e. the length of the lattice in the compactified imaginary time direction

$$V(R, N_t) = -rac{1}{N_t} \log < P(0) P(R)^{\dagger} >$$



b 4 3 4 b

• In the low temperature limit  $(N_t \to \infty) V(R)$  is dominated by  $E_0$  and coincides with the result obtained with the Wilson loop

$$E_0 = V(R) = - \lim_{N_t 
ightarrow \infty} rac{1}{N_t} \log < P(0) P(R)^\dagger >$$

•  $V(R, N_t)$  flattens out as the temperature increases (i.e. as  $N_t$  decreases) and becomes constant for a critical value  $(N_t)_c$ . the Deconfinement Temperature. The Polyakov loop is an order parameter of this phase transition.



Michele Caselle

			€ ≣	► < ≣	► Ξ	:
(UniTo)		KI	TP 2	20/01/20	22	7 / 38

#### The "area law".

• In the large R limit  $\langle P(0)P(R)^{\dagger} \rangle$  is dominated by the "area law"

$$< P(0)P(R)^{\dagger} > \sim e^{-\sigma RN_t}$$

which implies

 $V(R) = \sigma R$ 

where  $\sigma$  is a function of the temperature:  $\sigma(N_t)$ 

•  $\sigma(N_t)$  decreases as the temperature increases and vanishes at the Deconfinement Temperature. I will define the T = 0 value of  $\sigma(N_t)$  as  $\sigma_0 = \lim_{N_t \to \infty} \sigma(N_t)$ .

< □ > < 同 > < 回 > < 回 >

# Effective String Theory: the Nambu-Goto action.

- Confinement is associated to the creation of a thin flux tube joining the quark antiquark pair. In this framework the "area law" represents the classical contribution to the interquark potential on top of which we expect to have quantum corrections. The theory which describes these quantum fluctuations is known as "effective string theory" (EST).
- The simplest choice fulfilling Poincaré invariance for the EST is the Nambu-Goto string in which quantum corrections are evaluated summing over all the possible surfaces bordered by the two Polyakov loops with a weight proportional to their area.
- In the low T (large N<sub>t</sub>) limit the interquark potential is dominated by the lowest state

$$V(R) \sim E_0(R) = \sqrt{\sigma_0^2 R^2 - 2\pi \sigma_0 \frac{D-2}{24}},$$
  
 $V(R) \sim \sigma_0 R - \frac{\pi (D-2)}{24R} - \frac{1}{2\sigma_0 R^3} \left(\frac{\pi (D-2)}{24}\right)^2 + O(1/R^5),$ 

イロト イヨト イヨト

## The Lüscher term.

This correction is in remarkable agreement with numerical simulations. First high precision test in d=4 SU(3) LGT almost twenty years ago. <sup>1</sup>



Figure: The static potential. The dashed line represents the bosonic string model and the solid line the prediction of perturbation theory.

<sup>1</sup> S. Necco and R. Sommer, Nucl.Phys. B622 (2002) 32	8
--	---

Effective String Theory

< □ > < 同 > < 回 > < 回 >

# Beyond the Lüscher term: $1/R^3$ corrections.

High precision fit in the SU(2) case in 2+1 dimensions (A. Athenodorou, B. Bringoltz, M. Teper JHEP 1105:042 (2011) )



Figure 6: Energy of absolute ground state for SU(2) at  $\beta = 5.6$ . Compared to full Nambu-Goto (solid curve) and just the Lüscher correction (dashed curve).

Michele (bselle (1)	ni Lo I
michele caselle (0	

Effective String Theory

KITP 20/01/2022 11/38

< □ > < 同 > < 回 > < 回 >

# Confining String: Low Energy Universality.

- It can be shown that, due to Poincaré and parity invariance, the Nambu-Goto action for d = 3 LGTs is exact up to the order 1/R<sup>5</sup> included <sup>1</sup> : "Low energy universality"
- The Nambu-Goto action is only the leading-order approximation of the actual EST describing the infrared dynamics of a confining gauge theories.
- The terms beyond the Nambu-Goto approximation encode important physical information. They should depend on the particular gauge group of the LGT and could shed light on the mechanisms underlying confinement and on the physical degrees of freedoms from which the confining string arises.
- The first of these terms contributes to the interquark potential at the order  $1/R^7$  and could in principle be detected using Monte Carlo simulations.
- However, this approach is hampered by the existence of boundary terms (which encode the interaction of the flux tube with the Polyakov loops at the boundaries) which contribute at the order  $1/R^4$ .

<sup>&</sup>lt;sup>1</sup>M. Lüscher and P. Weisz, JHEP **0407** (2004) 014

O. Aharony and Z. Komargodski, JHEP 1305 (2013) 118

## The boundary contribution to the interquark potential

The leading boundary correction to the Effective String action turns out to be:

$$S_{b,2}^{(1)} = \int d\xi_0 \left[ b_2 \partial_1 \partial_0 X \cdot \partial_1 \partial_0 X 
ight] \,.$$

Its contribution to the interquark potential can be evaluated performing a gaussian functional integration<sup>1</sup>. In the usual low temperature limit in which  $N_t >> R$  one finds:

$$\langle S_{b,2}^{(1)} \rangle = -b_2 \frac{\pi^3 N_t}{60 R^4} E_4(e^{-\frac{\pi N_t}{R}})$$

where  $E_4$  denotes the fourth-order Eisenstein series:

$$E_4(q)\equiv 1+rac{2}{\zeta(-3)}\sum_{n=1}^\infty rac{n^3q^n}{1-q^n}$$

and  $\zeta(s)$  is the Riemann  $\zeta$  function.

< □ > < 同 > < 回 > < 回 >

<sup>&</sup>lt;sup>1</sup>O. Aharony and M. Field JHEP01(2011)065

## The boundary contribution to the interquark potential

• We end up with the following expression for the interquark potential

$$\mathcal{N}(R) = \sigma_0 R - rac{\pi (D-2)}{24R} - rac{1}{2\sigma_0 R^3} \left(rac{\pi (D-2)}{24}
ight)^2 - b_2 rac{\pi^3 (D-2)}{60R^4} + O(1/R^5) \; ,$$

where  $b_2$  is a new physical parameter, similar to the string tension  $\sigma_0$ , which depends on the theory that we study and should be determined by simulations and comparison with experiments.

- This  $1/R^4$  term shadows the bulk corrections to the Nambu-Goto action and makes it very difficult to detect it in MC simulations.
- The solution to this problem is to use the "open-closed" string duality and look to the interquark potential in the high temperature regime, in the neighbourhood of the deconfinement transition, but still in the confining phase.
   It can be shown that in this regine the boundary term becomes subleading and can be neglected.

イロト イヨト イヨト

## The boundary contribution to the interquark potential

Using the modular properties of the Eisenstein function,

$$E_4\left(e^{-\frac{\pi N_t}{R}}\right) = \left(\frac{2R}{N_t}\right)^4 E_4\left(e^{-\frac{4\pi R}{N_t}}\right),$$

it is easy to see that in the  $R \gg N_t$  ("high-temperature") regime, the boundary correction becomes

$$\langle S_{b,2}^{(1)} 
angle = -b_2 rac{4\pi^3}{15N_t^3} E_4 \left( e^{-rac{4\pi R}{N_t}} 
ight),$$

which does not contain terms linear in R and thus it does not contribute to the temperature-dependent string tension.



< □ > < 同 > < 回 > < 回 >

## Effective String Theory at Finite Temperature

• For a generic Poincaré invariant EST in D = 2 + 1, we have<sup>2</sup> for  $R >> N_t$ 

$$\langle P(0)P^{\dagger}(R)\rangle = \sum_{n=0}^{\infty} |v_n(N_t)|^2 \frac{E_n}{\pi} K_0(E_n R)$$

- In the large *R* limit the sum is dominated by the lowest state and from the exponential decay of the modified Bessel function we may extract the correlation length  $E_0 = \frac{1}{\xi}$
- For the Nambu-Goto string

$$\langle P(0)P^{\dagger}(R)\rangle = \sum_{n=0}^{\infty} \omega_n N_t \sqrt{\frac{\sigma_0}{\pi}} K_0(E_n R)$$
  
with  $E_n = \sigma_0 N_t \sqrt{1 + \frac{8\pi}{\sigma_0 N_t^2} (n - \frac{1}{24})} \Longrightarrow E_0 = \sigma_0 N_t \sqrt{1 - \frac{\pi}{3\sigma_0 N_t^2}}$ 

<sup>2</sup>M. Lüscher and P. Weisz, *JHEP* 2004.07(2004), p. 014

16/38

• • • • • • • • • • •

## Interquark Potential and Deconfinement Transition

- In the large R limit  $\langle P(0)P^{\dagger}(R)\rangle$  is dominated by the lowest state  $E_0$
- For the Nambu-Goto string

$$\langle P(0)P^{\dagger}(R)\rangle = e^{-N_t V(R,N_t)} \sim K_0(E_0R)$$

with 
$$E_0 = \sigma_0 N_t \sqrt{1 - \frac{\pi}{3\sigma_0 N_t^2}}$$
  
 $E_0 = \sigma_0 N_t - \frac{\pi}{6N_t} - \frac{\pi^2}{72\sigma_0 N_t^3} - \frac{\pi^3}{432\sigma_0^2 N_t^5} - \frac{5\pi^4}{10368\sigma_0^3 N_t^7} + \dots$ 

- Low Energy Universality: Corrections to the Nambu-Goto action appear only at the order  $\frac{1}{\sigma_0^3 N_t^7}$ .
- Problem: detecting such a correction requires a very precise determination of  $E_0$ .

イロト イヨト イヨト

# Interquark Potential and Deconfinement Transition

## • Solution: Svetitsky-Yaffe mapping<sup>3</sup>

- For some choices of the gauge group the finite temperature Deconfinement Transition of a pure Yang-Mills Theory is critical and is described by a CFT. This is the case, in particular for the (2+1) dimensional SU(2) LGT whose deconfinement transition belongs to the same universality class of the 2d Ising model.
- The confining phase is mapped into the *symmetric* phase of the perturbed CFT (i.e the high T phase of the Ising realization, even if for the Yang-Mills theory it corresponds to the low T regime!!).
- ⟨P(0)P<sup>†</sup>(R)⟩ in the vicinity of the critical point is equivalent to the spin-spin correlator of the 2d Ising model ⟨s(0)s(R)⟩<sub>t</sub> which is know exactly.

<sup>3</sup>B. Svetitsky and L. G. Yaffe, Nucl. Phys. B210 (1982) 423

18/38

< □ > < 同 > < 回 > < 回 >

# $\langle s(0)s(R)\rangle$ correlator in the 2d Ising model

For the 2d Ising model, thanks to the exact integrability, the  $\langle s(0)s(R)\rangle$  correlator is known exactly<sup>4</sup>. (apart from the non-universal normalization constants  $C_1$  and  $C_2$ ):

• For  $R < \xi$  it coincides with the Conformal Perturbation result:

$$\langle s(0)s(R)\rangle = \frac{C_1}{r^{\frac{1}{4}}} \left[ 1 + \frac{t}{2} \ln\left(\frac{e^{\gamma_E}t}{8}\right) + \frac{1}{16}t^2 + \frac{1}{32}t^3 \ln\left(\frac{e^{\gamma_E}t}{8}\right) + O(t^4) \right]$$
  
where  $t = \frac{R}{\epsilon}$ 

• For  $R>\xi$  is the typical expression for a free isolated particle in d=2 $\langle s(0)s(r)\rangle = C_2 K_0(R/\xi)$ 

where  $K_0$  denotes the modified Bessel function:  $K_0(x) \sim \frac{e^{-x}}{\sqrt{x}}$  for large x

• Notice the shift from  $r^{-\frac{1}{4}}$  at short distance to  $r^{-\frac{1}{2}}$  at large distance

<sup>4</sup>T. T. Wu, B. M. McCoy, C. A. Tracy and E. Barouch, Phys. Rev. B 13 (1976) 316 🗈 K 🗉 🐑 🛬 🛷 🔍

# SU(2) LGT in (2+1) dimensions. Summary of simulations

$N_t \times N_s^2$	β	$T/T_{\rm c}$	$n_{ m conf}$	]	$N_t \times N_s^2$	β	$T/T_{\rm c}$	$n_{ m conf}$
$9 \times 96^2$	11.3048	0.80	$2.5  imes 10^5$		0.002	10.10736	0.80	$2.5\times 10^5$
	11.72873	0.83	$2.5  imes 10^5$		$8 \times 96^2$	10.486386	0.83	$2.5\times 10^5$
$9 \times 160^2$	12.15266	0.86	$2.5  imes 10^5$	]		10.865412	0.86	$2.5\times 10^5$
$7 \times 96^2$	9.228023	0.83	$2.5  imes 10^5$		$6 \times 96^2$	8.258494	0.86	$2.5\times 10^5$
	9.561566	0.86	$2.5 \times 10^5$			8.546581	0.89	$2.5\times 10^5$

$\beta$	$N_t$	$N_s$	$T/T_{\rm c}$	$n_{\rm conf}$	β	$N_t$	$N_s$	$T/T_{\rm c}$	$n_{\rm conf}$		β	$N_t$	$N_s$	$T/T_{\rm c}$	$n_{ m conf}$
	6	160	0.935	$2.0  imes 10^5$		8	240	0.960	$2.0 \times 10^5$	]		9	240	0.947	$2.0\times 10^5$
	7	96	0.801	$2.0 \times 10^5$		9	160	0.853	$2.0 \times 10^5$			10	160	0.852	$2.0\times 10^5$
$\beta = 9$	8	96	0.701	$2.0 \times 10^5$	$\beta=12.15266$	10	96	0.768	$2.0 \times 10^5$		$\beta=13.42445$	11	160	0.775	$2.0\times 10^5$
	9	96	0.623	$2.0 \times 10^5$		11	96	0.698	$2.0 \times 10^5$			12	96	0.710	$2.0\times 10^5$
	10	96	0.561	$2.0 \times 10^5$		12	96	0.640	$2.0 \times 10^5$			13	96	0.655	$2.0\times 10^5$
	11	96	0.510	$2.0 \times 10^5$		13	96	0.591	$2.0 \times 10^5$			14	96	0.609	$2.0  imes 10^5$
	12	96	0.468	$2.0 \times 10^5$		14	96	0.549	$2.0 \times 10^5$			15	96	0.568	$2.0\times 10^5$

Michele Caselle (UniTo)

Effective String Theory

KITP 20/01/2022

20 / 38

# SU(2) LGT in (2+1) dimensions: Scaling test



Michele Caselle (UniTo)

< 1<sup>™</sup> >

# **Testing CPT Predictions**

• 
$$f(x) = \frac{C_1}{x^{\frac{1}{4}}} \left[ 1 + \frac{x}{2\xi} \ln\left(\frac{e^{\gamma}E_X}{8\xi}\right) + \frac{1}{16} \left(\frac{x}{\xi}\right)^2 + \frac{1}{32} \left(\frac{x}{\xi}\right)^3 \ln\left(\frac{e^{\gamma}E_X}{8\xi}\right) \right]$$
  
 $\xi = 22.05(6)$   $\chi^2_{red} = 0.92$  in the range  $R/a = [5:26]$ 



A B + 
 A B +
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A

문 🛌 🖻

# CPT vs EST



KITP 20/01/2022

• • • • • • • • • •

#### Deviations from Nambu-Goto

- Deviations from Nambu-Goto can be better appreciated looking at the *T* dependence of  $E_0$  at fixed  $\beta$ . We chose three values of  $\beta$ :  $\beta = 9$ ,  $\beta = 12.15266$  and  $\beta = 13.42445$  and simulated several values of  $N_t$ .
- The correlation length diverges at the critical point as  $\xi \sim |T T_c|^{-\nu}$ . Two predictions for the critical exponent:

(1) Nambu-Goto:  $\nu = 1/2$ 

$$E_0 = \sigma_0 N_t \sqrt{1 - \frac{\pi}{3\sigma_0 N_t^2}} \to \xi = \xi_0 \sqrt{1 - T^2/T_c^2} = \xi_0 \sqrt{1 - \frac{N_{t,c}}{N_t}}$$

2 Svetitsky-Yaffe conjecture:  $\nu = 1$ 

• For the linear divergent behaviour we fit the string ground state with the function

$$f(N_t) = k_s \left(1 - \frac{N_{t,c}}{N_t}\right)$$

Miche	0 ( 260	ا مال	Ini	<b>1</b> 01
whiche	ic case	ne (		,

(日) (四) (日) (日) (日)

#### Deviations From Nambu-Goto



▶ < E >

A B > 4
 B > 4
 B
 B
 A
 B
 A
 B
 A
 A
 B
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A

#### Deviations from Nambu-Goto

- Due to the Poincarè invariance and to consistency requirements, the first correction to the Nambu-Goto energy spectrum is expected to be of the order ~ <sup>1</sup>/<sub>σ<sup>2</sup>N<sup>7</sup>/2</sub>
- We fit the temperature dependence of the string ground state  $E_0$  with the function

$$\begin{aligned} f(N_t) &= \mathsf{Taylor}_4(E_{0,NG}) + \frac{k_c}{\sigma_0^3 N_t^7} \\ \mathsf{where:} \\ \mathsf{Taylor}_4(E_{0,NG}) &= \sigma_0 N_t - \frac{\pi}{6N_t} - \frac{\pi^2}{72\sigma_0 N_t^3} - \frac{\pi^3}{432\sigma_0^2 N_t^5} - \frac{5\pi^4}{10368\sigma_0^3 N_t^3} \end{aligned}$$

•  $k_c$  is not completely free. It can be constrained using a bootstrap analysis<sup>1</sup>. Setting

$$\gamma_3 = -\frac{225}{32\pi^6}k_c$$

one find:  $\gamma_3 \ge -\frac{1}{768} \simeq -0.0013$ 

## Deviations from Nambu-Goto



• • • • • • • •

27/38

문 🛌 🖻

β	N <sub>t,min</sub>	N <sub>t,max</sub>	k <sub>c</sub>	$\sigma_0$	$\chi^2_{ m red}$	literature
9	6	12	0.040(8)	0.02603(19)	1.60	0.02583(3)
12.15266	8	14	0.054(5)	0.01366(5)	0.89	0.01371(29)
13.42445	9	15	0.053(8)	0.01104(5)	1.33	0.01108(23)

- The three values for  $k_c$  are almost compatible within the errors: the correction shows the correct scaling behaviour in the continuum limit!
- As our final result for  $k_c$ , we have:

 $k_c = 0.050(8)$ 

from which we find

$$\gamma_3 = -\frac{225}{32\pi^6}k_c = -0.00037(6),$$

• which is well inside the bound  $\gamma_3 \geq -\frac{1}{768} \simeq -0.0013$  derived in refs.<sup>1</sup>

<sup>1</sup>J. Elias Miró, A. L. Guerrieri, A. Hebbar, J. Penedones and P. Vieira, Phys. Rev. Lett. **123** (2019) 221602 J. Elias Miró and A. Guerrieri, JHEP **10** (2021), 126

Michele Caselle (UniTo)

Effective String Theory

KITP 20/01/2022 28/38

## Comments

- The value is confirmed by an independent set of simulations<sup>5</sup> of zero momentum correlators at  $\beta = 16$ .
- We start to see a dependence on the gauge group. Independent estimates based on the glueball spectrum<sup>6</sup> suggest a positive value of  $\gamma_3$  for the SU(6) LGT in (2+1) dimensions.

 $<sup>^{5}</sup>$ A. Athenodorou and M. Teper, JHEP **10** (2016), 093

<sup>&</sup>lt;sup>6</sup>S. Dubovsky, R. Flauger and V. Gorbenko, J. Exp. Theor. Phys. 120 (2015), 399-422

## Conclusions

- The interquark potential in the SU(2) LGT in (2+1) dimensions near the deconfinement transition is perfectly fitted by the 2d Ising spin-spin correlator in the whole range of distances that we studied, down to  $0.8T_c$ .
- This allows to reach an increased precision in the determination of the ground state energy  $E_0$  of the string.
- A reliable estimate of the first correction to Nambu-Goto can be extracted from the data. As expected from low energy universality, this correction is proportional to  $1/(\sigma_0^3 N_t^7)$ . Its value is negative and agrees with the bootstrap constraints.
- Using Conformal Perturbation Theory this type of analysis could be extended to other gauge groups and to the (3 + 1) dimensions.

イロト イヨト イヨト

# Acknowledgements

Collaborators:

Fabrizio Caristo<sup>\$</sup>, Alessandro Nada<sup>\$</sup>, Marco Panero<sup>\$</sup>, Antonio Smecca<sup>\$</sup>, Nicodemo Magnoli<sup>\*</sup>, Claudio Bonati<sup>ast\*</sup>, Silvia Morlacchi<sup>\*\*</sup>,

 $^{\diamond}$  Dipartimento di Fisica, Università di Torino

\* Dipartimento di Fisica, Università di Genova

\*\* Dipartimento di Fisica, Università di Pisa

(日) (四) (日) (日) (日)

Michol	0 ( 200		llmilz	- 1
IVITCHE		11E I I		

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ □臣 = のへで

Nucholo		
IVITURE E	- ASCHE	

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ □臣 = のへで

# Coupling SU(2) with bosonic matter<sup>7</sup>.

- It is interesting to see what happens of the EST if we couple the theory with matter
- Fermions are too difficult to handle, but bosonic matter can be simulated easily
- to avoid any subtelty let us start with bosonic matter in the fundamental representation.

$$egin{aligned} \mathcal{S} &= - \,eta_h \sum_{x,\mu} \operatorname{Re} \operatorname{Tr} \left( arphi_x^\dagger U_{x,\mu} arphi_{x+\hat{\mu}} 
ight) \ &- \, rac{eta}{2} \sum_{x,\mu > 
u} \operatorname{Re} \operatorname{Tr} U_{\mu
u}(x) \;. \end{aligned}$$

with the constraint Tr  $\varphi_x^{\dagger}\varphi_x = 1$ .

We have a very simple phase diagram: a confining phase and a Higgs-like pahse separated by a crossover (non phase transition)

34 / 38

<sup>&</sup>lt;sup>7</sup>C. Bonati, M. Caselle and S. Morlacchi, Phys. Rev. D 104 (2021) no.5; 054501 + E + 4 E + E - S - O O O

Phase diagram at  $\beta = 9$ 



★ 注 ト 4 注 ト 注 の Q (ペ KITP 20/01/2022 35/38

< (T) >

## Potential as a function of $\beta_h$



∃ ⊳

• • • • • • • •

# Potential as a function of $\beta_h$

Main result:

Even in the presence of string breaking, the confining part of the interquark potential is well described by the Effective String Theory.

$$V(R) = \sigma_0 R - \frac{\pi(D-2)}{24R} - \frac{1}{2\sigma_0 R^3} \left(\frac{\pi(D-2)}{24}\right)^2 - \frac{b_2 \frac{\pi^3(D-2)}{60R^4}}{60R^4} + O(1/R^5) ,$$

$\beta_h$	$\sigma$	$\tilde{b}_2$	$E_{sb}$	$R_{sb}$
0	0.02583(3)	0.020(3)		
0.93	0.02541(1)	0.019(2)		
0.95	0.02523(3)	0.019(3)		
0.97	0.02493(3)	0.020(1)	$0.683^{+0.018}_{-0.009}$	$\sim 20$
1.00	0.02386(8)	0.019(3)	0.548(2)	$\sim 15$
1.02	0.0214(8)	0.006(15)	0.440(2)	$\sim 10.5$

イロト イヨト イヨト イヨト

37 / 38

э

	 /.		
Nucha		10112	
IVITCHE	11C I.V	211110	

KITP	20/01/2022	38 / 38

◆□ > ◆□ > ◆臣 > ◆臣 > ─ 臣 ─ の < ⊙