Tetraquarks $QQ'\overline{q}\overline{q}'$ in a quark model

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1. Personal Overview of the quark model
2. Tetraquarks $QQ'\overline{q}\overline{q}$

If time allows: Role of the pions for baryons —
3. Holographic model by Sakai-Sugimoto

QCD ==> Quark model ==> Hadrons
and pion (meson) degrees of freedom

1. *Personal* Overview of the quark model

Marek Karliner: Questions to be answered

1. Do they exist?
2. f they do, which ones?
3. What is their internal structure?
4. How best to look for them?

Marek Karliner, QNP proceedings, 2018@Tsukuba
https://journals.jps.jp/doi/book/10.7566/QNP2018

Studying heavy (exotic) hadrons is somewhat similar to investigating the social life of various quarks:

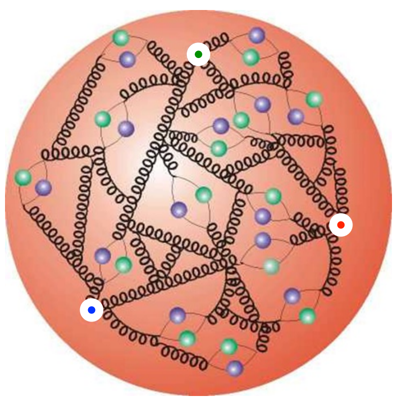
(a) Who with whom?
(b) For how long?
(c) A short episode? or
(d) "Till Death Us Do Part"?

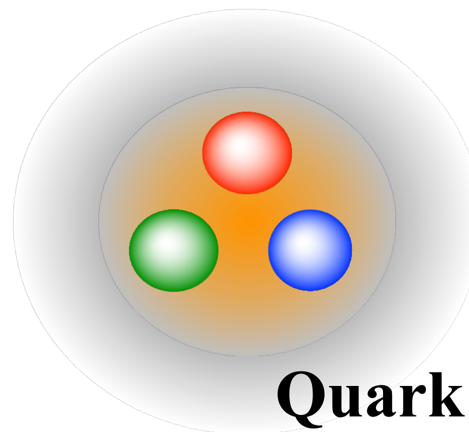
Variable dynamics

QCD shows various aspects with effective degrees of freedom

At High energies: Current quarks, gluons, with perturbation
Systematics

At low energies: Constituent quarks, gluons, diquarks, pions, …
Ad hoc, phenomenological





Quark model

Light quarks and gluons

http://ppssh.phys.sci.kobeu.ac.jp/~yamazaki/lectures/07/modernphys-yamazaki07.pdf

Confined constituent quarks
+ pion cloud

Scenario from QCD

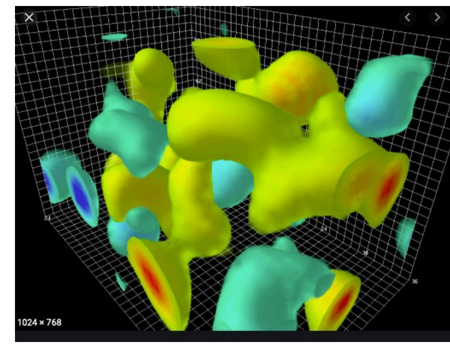
Uniqueness of QCD as a many-body problem → Non-trivial dynamics

QCD vacuum is not empty ~ Instantons are created and annihilated

- Extended (topological) object of gluons, of size ~ 0.2 fm
- QCD vacuum is topologically nontrivial
- Chiral symmetry is broken spontaneously $m \neq 0$

$$\langle \bar{q} q \rangle \sim \int \frac{d^4k}{i(2\pi)^4} \mathrm{tr} \frac{1}{m - \not{k}} \sim \int_{\infty}^{\infty} d\lambda \nu(\lambda) \frac{\mu}{\lambda^2 + \mu^2} |_{\mu \to 0}$$

Banks-Casher, NPB169(1989)193
D. Diakonov, PPNP51(2003)173
Fukaya et al, PRL104.122002 (2010), PRD.83.074501 (2011)

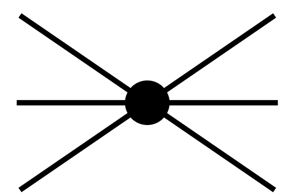


Snapshot of topological densities fluctuating in the vacuum
Derek Leinweber, 2003, 2004
http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/Nobel/index.html

- Instanton Induced Interaction (III) with $U_A(1)$ breaking

Kobayashi-Maskawa_PTP44(1970)1422
G. ’t Hooft, PRL37.8 (1976), PRD14, 3432 (1976)

$$\mathcal{L}_{III} = g_D\left(\det[\bar{q}_i(1-\gamma_5)q_j] + h.c.\right)$$



Systematic study: Hatsuda-Kunihiro: Phys. Repts. 247 (1994) 221-367

Effective theory for hadron physics

Building blocks **Confined constituent quarks + pions around**

Manohar-Georgi' chiral quark model, NPB234 (1984) 189, with III

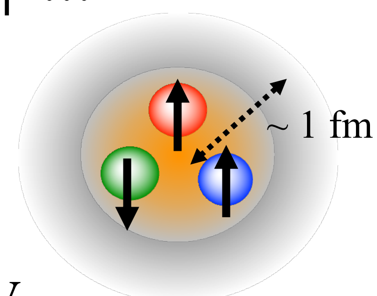
Confinement ~ String?

$$\mathcal{L} = \bar{q}(i\,\partial\!\!\!/ - g G\!\!\!/)q - m\bar{q}q + \mathcal{L}_{III} + \sigma r$$

Quarks

$$+\bar{q}V\!\!\!/ q + g_A \bar{q} A\!\!\!/ \gamma_5 q + \frac{1}{4} f_\pi^2 \mathrm{tr}\, \partial^\mu U\, \partial_\mu U^\dagger - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \cdots$$

Pion-quark **Pions**



Quarks at intermediate distances: **0.2 fm < *r* < 1 fm**

Massive constituent quarks: $m_u \sim m_d \sim 360$, $m_s \sim 540$ MeV

Spin-spin (*SS*) and **spin-orbit (*LS*) forces** from **OGE** and **III**

Meson cloud at long distances, ***r* > 1 fm**

Systematically explored by chiral perturbation theory

Light constituent quarks: $q = u, d, s$

Evidence I: Baryon magnetic moments $\vec{\mu} = \langle B | \sum_q \frac{e_q}{2m_q} \vec{\mu}_q | B \rangle$

SU(6) + $m_{u,d} = 360\text{MeV}, m_s = 540\text{MeV}$

	Non-rel	+ Rel Corr	Exp
p	2.61	2.78	2.79
n	– 1.74	– 1.90	– 1.91
Σ^+	2.51	2.35	2.46
Λ	– 0.58	– 0.61	– 0.61
Σ^-	– 0.97	– 1.15	– 1.16
Ξ^0	– 1.35	– 1.25	– 1.25
Ξ^-	– 0.48	– 0.68	– 0.65
Ω^-	– 1.92	– 2.26	prediction – 2.02
$\chi 2$	0.1	0.01	

Evidence II: Baryon masses

(From T. Kunihiro, Textbook in Japanese:
クォークハドロン物理学入門, サイエンス社, 2013)

$m_{u,d} = 335\text{MeV}, m_s = 527\text{MeV}$

Phenomenological mass formula

$$M_B = M_0 + \sum_i^{uds} \left[m_i + \frac{a}{2m_i} \right] + b \sum_{i<j} \frac{\sigma_i \cdot \sigma_j}{m_i m_j}$$

$a = (175.2\text{MeV})^2$
$b = (176.4\text{MeV})^3$
$M_0 = -56.4\text{MeV}$

Data:	$\Lambda(1116)$	$\Sigma(1193)$	$\Sigma^*(1385)$	$\Xi(1320)$	$\Xi^*(1507)$
Calc.	1114	1186	1372	1332	1519

Evidence III: Axial couplings ~ pion couplings

$$\mathcal{L}_{\pi qq} = \bar{q}Vq + g_A\bar{q}A\gamma_5 q + \frac{1}{4}f_\pi^2 \mathrm{tr}\,\partial^\mu U\,\partial_\mu U^\dagger - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \cdots$$

$$\frac{g_A^q}{2f_\pi}\bar{q}\vec{\tau}\cdot\partial_\mu\vec{\pi}\gamma_5\gamma^\mu q \sim \frac{1}{2m_q}\chi^\dagger\vec{\sigma}\cdot\vec{q}\tau^a\chi\pi^a \to \sigma^i\tau^a$$

$g_A^q = 1$

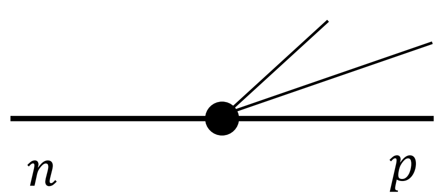
Weinberg, PRL 65, 1177, 1990

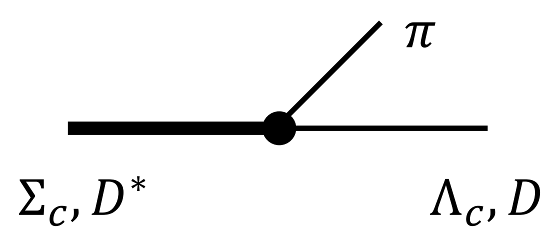
Leading order (*LO*) of NR expansion

	Leading term
$\langle p\vert\sigma\tau\vert n\rangle$ (betadecay)	$\frac{5}{3}$
$\langle\Lambda_c\pi\vert\sigma\tau\vert\Sigma_c\rangle$	1
$\langle D\pi\vert\sigma\tau\vert D^*\rangle$	1

Relativistic corrections (*NLO*) →

About ~ 30 % reduction
Agrees well with exp.

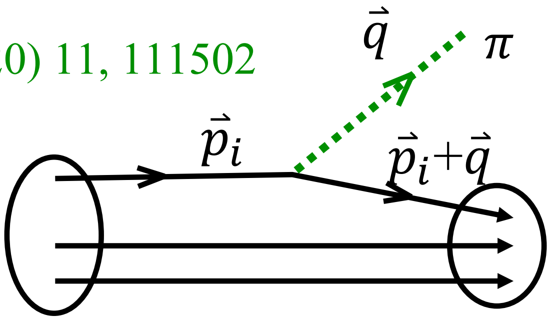




Evidence III': Axial couplings for $N^*(1440) \to N\pi$

Arifi et al, Phys.Rev.D 101 (2020) 11, 111502

N(1440) $1/2^+$:
Radial excitation



N(940) $1/2^+$:
Ground state

- Null in the leading order (*LO*) $\langle N(940)\pi|\sigma\tau|N(1440)\rangle = 0$

  Same spin structure => the orthogonality of radial functions at $q = 0$

- Relativistic corrections (*NLO*) : $2\boldsymbol{\sigma} \cdot (\boldsymbol{q} - 2\boldsymbol{p}_i) \times (\boldsymbol{q} \times \boldsymbol{p}_i)$

  Internal quark motion

In fact, finite q: For the **decay width:**

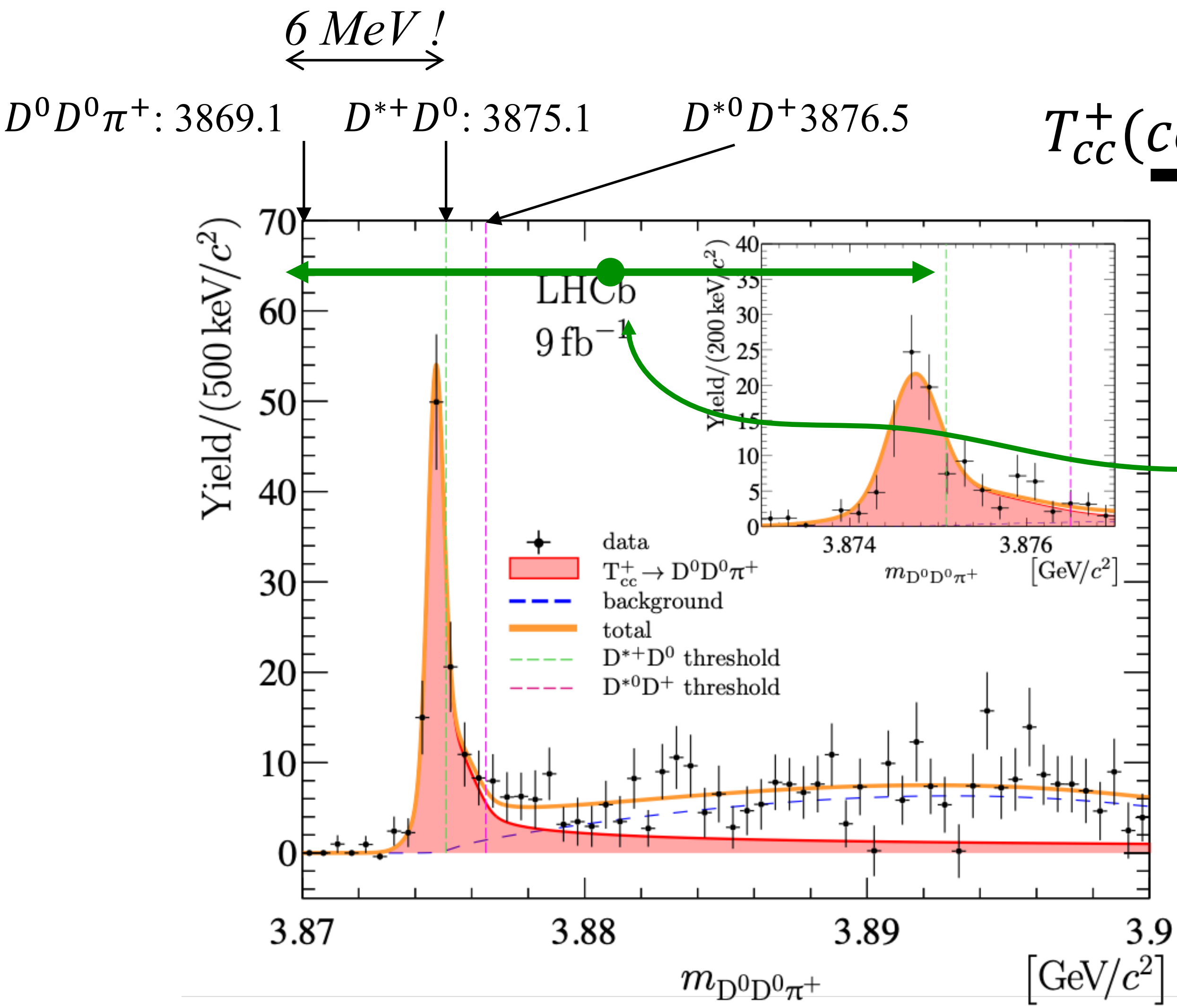
$\langle N(940)\pi|\sigma\tau|N(1440)\rangle$ 12 MeV → 150 MeV (Data: 110 MeV)

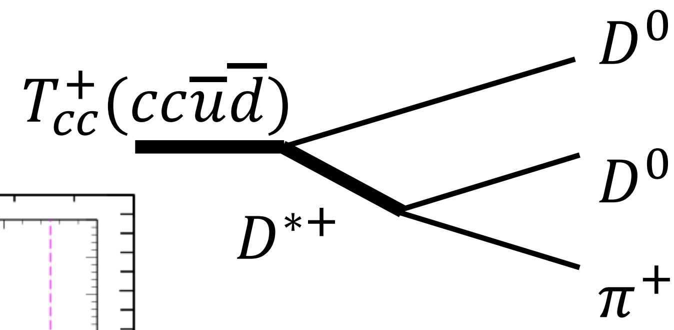
LO *NLO*

Constituent quarks work well for hadrons

2. Tetraquarks $QQ'\overline{qq}'$

LHCb: arXiv: 2109.01056





Karliner, Rosner, PRL119, 202001, 2017

Why T_{cc} is interesting

- Toward answering "Who with whom?

— why not clear evidence to find exotics only with light q's

- The role of heavy vs light quarks

$$\text{Light} \ll \Lambda_{QCD} \ll m_Q$$

Interplay of light and heavy scales of QCD

- Many theoretical models
  Diquarks, triquarks, molecules, hybrid, …
- Are they bound or resonant states?
- Test the quark model; its applicability up to where?

$Q, \overline{Q}$, and $q, \overline{q}$: two distinct scales

$q, \overline{q}$ — Λ_{QCD} — $Q, \overline{Q}$

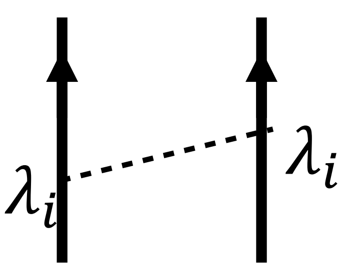
Non-perturbative

Perturbative ~ Color-Coulomb

$$H = \frac{p^2}{2M_Q} - \frac{\alpha}{r} \Rightarrow E_B = \frac{1}{2}\alpha^2 M_Q$$

$QQ: 3\times 3 = \overline{3} + 6$

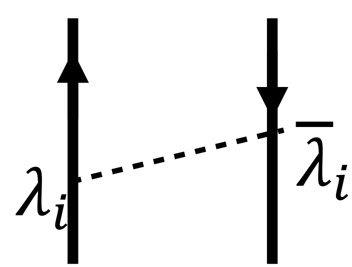
QQ is in color triplet in a baryon



$$-\frac{1}{2}\frac{\alpha}{r}$$

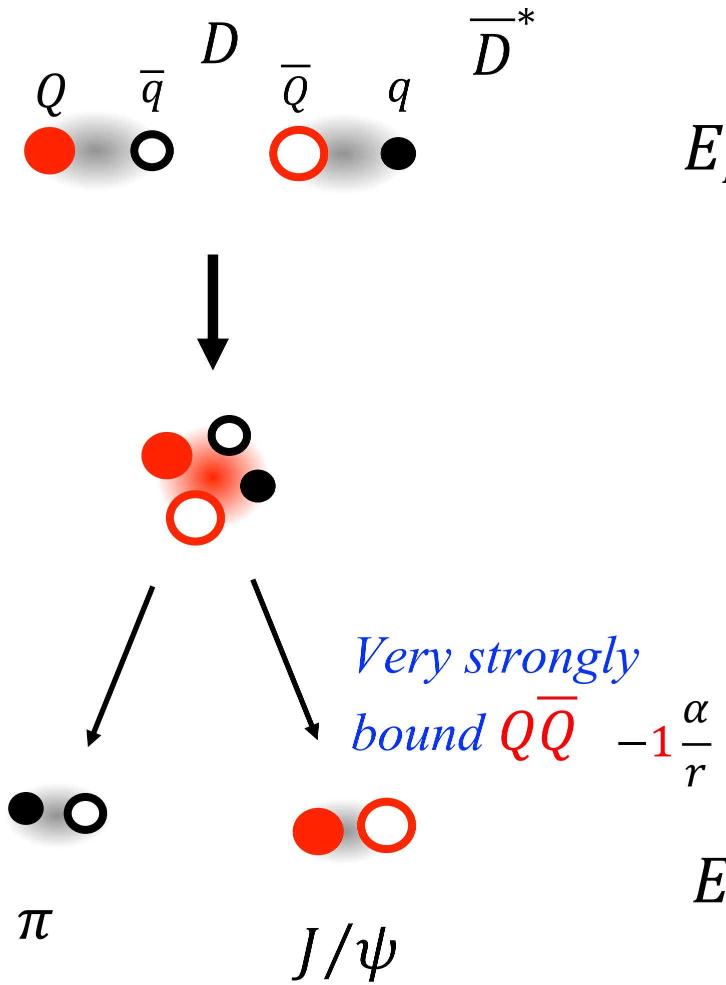
$Q\overline{Q}: 3\times\overline{3} = 1 + 8$

$Q\overline{Q}$ is in color singlet in a meson



$$-1\frac{\alpha}{r}$$

$Q\overline{Q}q\overline{q}$

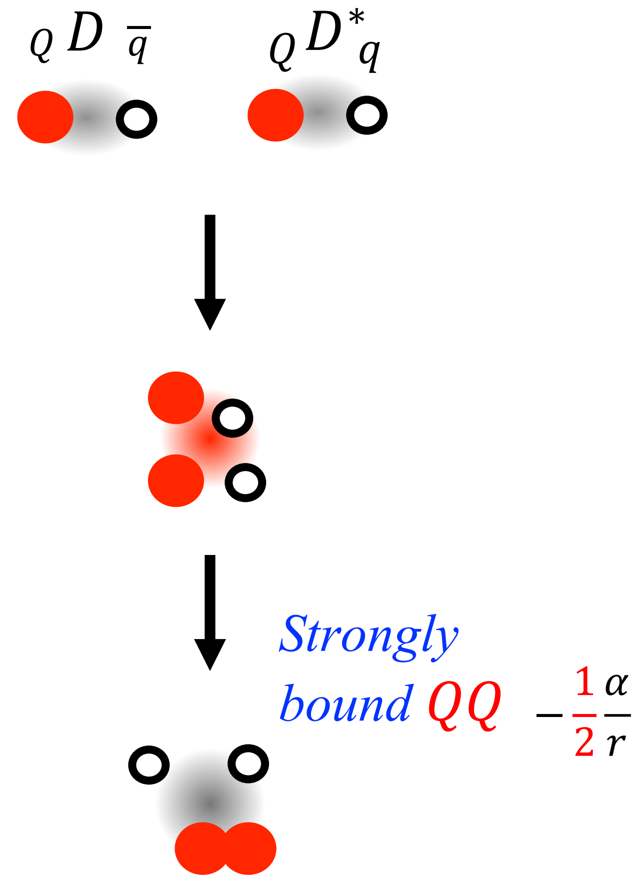


$E_B \sim \Lambda_{QCD}$

$E_B \sim \alpha M_Q$

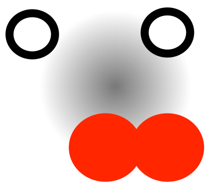
Decay into ordinary mesons

$QQ\overline{q}\overline{q}$



Stays as stable T_{QQ}

Expected J^P



- Orbitally in S-state
- QQ must has $j^P = 1^+$ due to Pauli principle
- $\overline{qq}$ is a good diquark $S = I = 0$

The lowest T_{QQ} has $j^P = 1^+, I = 0$

Verify in the quark model — 4-body calculation

Meng et al, PLB814 (2021) 136095

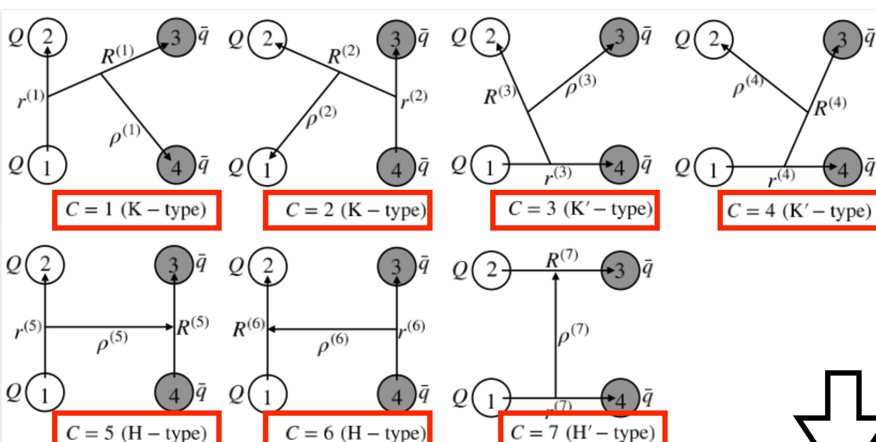
Gauss expansion method ~Hiyama et al, Prog. Part. Nucl. Phys. 51 (2003) 223

Hamiltonian

$$H = \sum_i^4 \left(m_i + \frac{p_i^2}{2m_i} \right) - T_G \qquad V_{ij}(r) = -\frac{\kappa}{r} + \lambda r^p - \Lambda$$

$$-\frac{3}{16} \sum_{i<j=1}^4 \sum_a^8 \left((\lambda_i^a \cdot \lambda_j^a) V_{ij}(r_{ij}) \right) \qquad + \frac{2\pi\kappa'}{3m_i m_j} \frac{\exp(-r^2/r_0^2)}{\pi^{3/2} r_0^3} \sigma_i \cdot \sigma_j$$

Ansatz Expand WF by different combinations of coordinates



$$\Psi_{I,JM} = \sum_{C} \xi_1^{(C)} \sum_\gamma B_\gamma^{(C)} \eta_I^{(C)} \Big[\Big[[[\chi_{\frac{1}{2}} \chi_{\frac{1}{2}}]_s \chi_{\frac{1}{2}}]_\Sigma \chi_{\frac{1}{2}} \Big]_K \times [[\phi_{n\ell}^{(C)}(\mathbf{r}_C) \psi_{NL}^{(C)}(\mathbf{R}_C)]_\Lambda \phi_{\nu\lambda}'^{(C)}(\rho_C)]_G \Big]_{JM}, \quad (3)$$

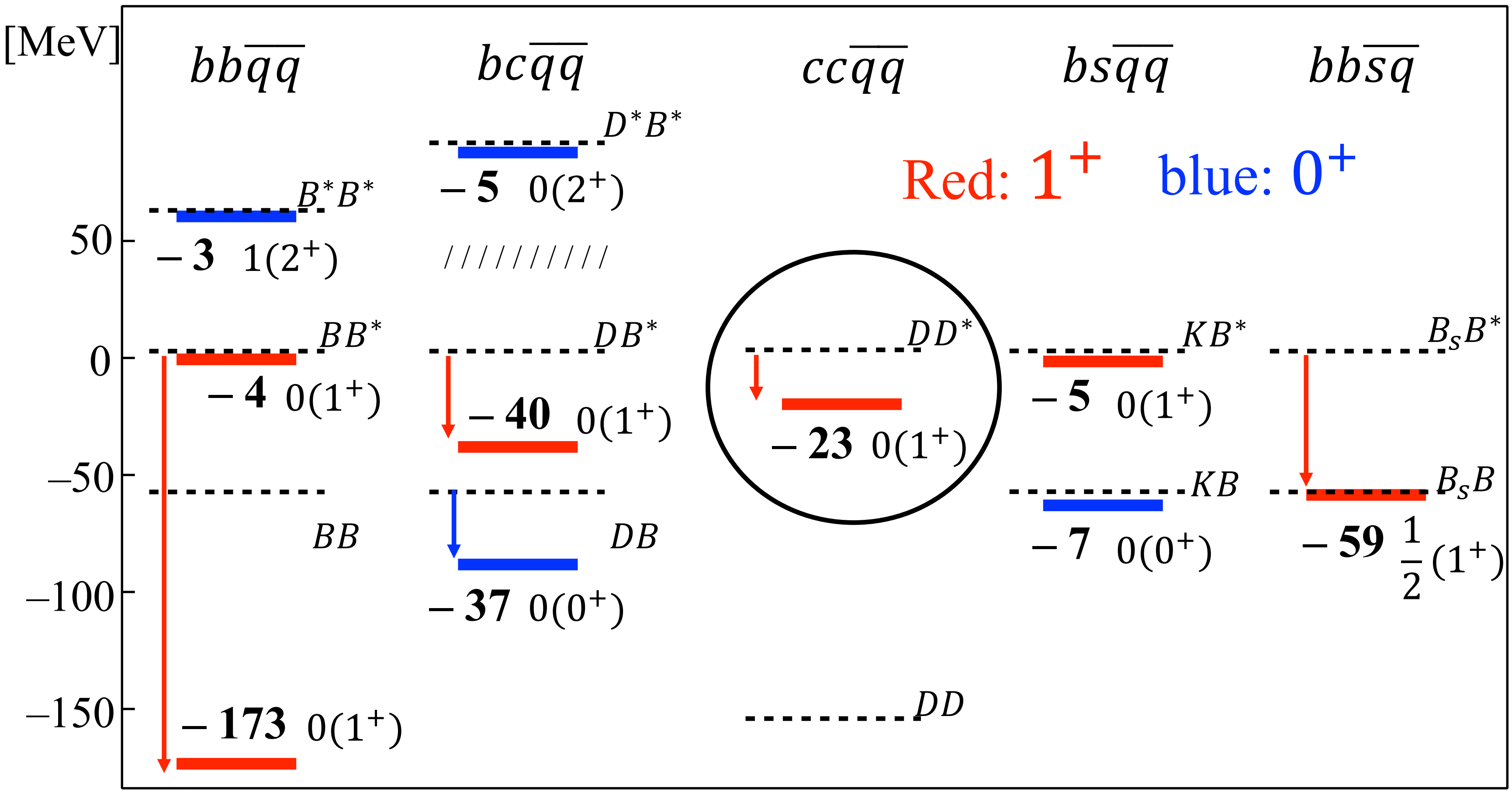
Diagonalize

Comparison with threshold energies important

=> Consistency check with meson masses ~ accuracy of the model/method

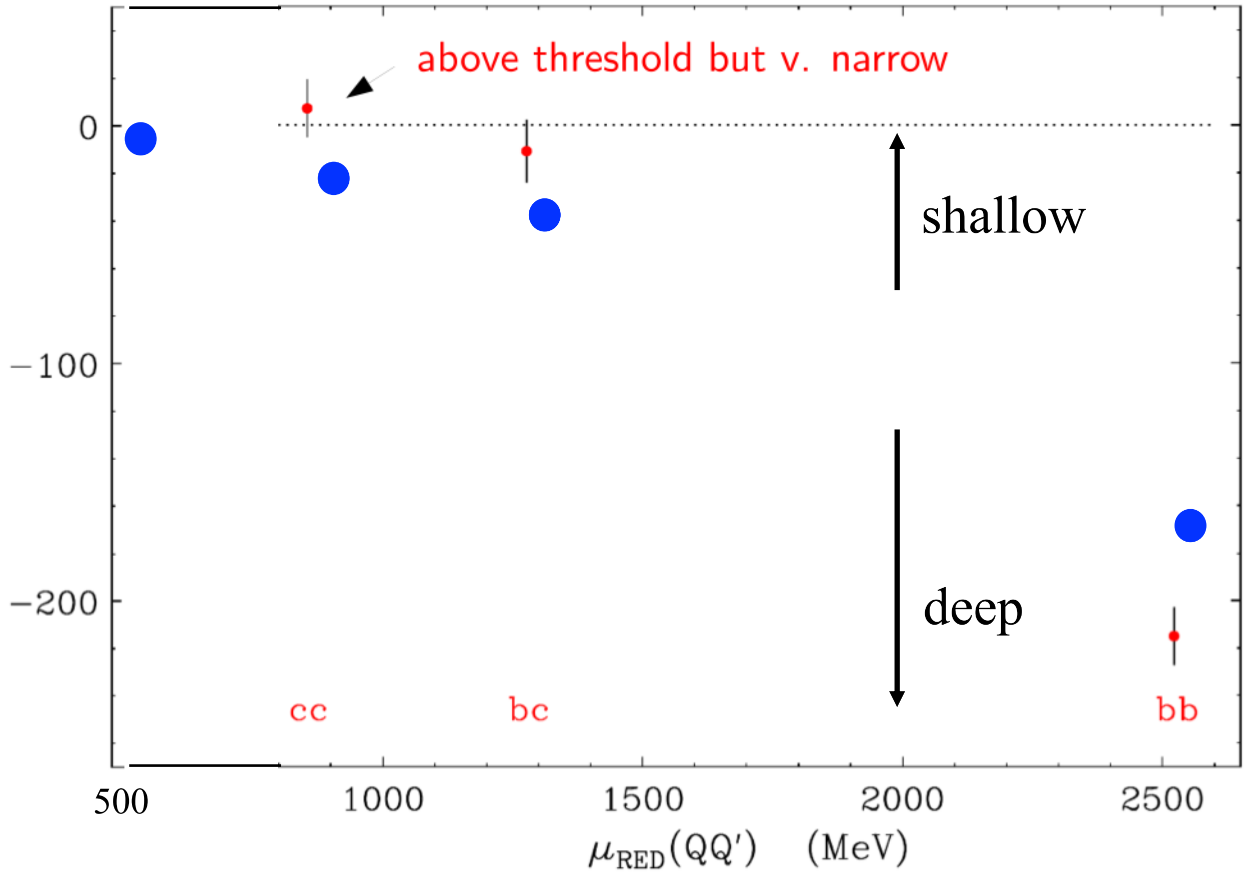
Parameters		Masses (MeV)	Cal	Exp
$m_{u,d}$ (GeV)	0.277	$\eta_b(0^-)$	9375	9399
m_s (GeV)	0.593	$\Upsilon(1^-)$	9433	9460
m_c (GeV)	1.826	$\eta_c(0^-)$	2984	2984
m_b (GeV)	5.195	$J/\psi(1^-)$	3102	3097
p	2/3	$B^-(0^-)$	5281	5279
κ	0.4222	$B^{*-}(1^-)$	5336	5325
κ'	1.7925	$B_s(0^-)$	5348	5367
λ (GeV$^{5/3}$)	0.3798	$B_s^*(1^-)$	5410	5415
Λ (GeV)	1.1313	$D^-(0^-)$	1870	1870
A (GeV^{B-1})	1.5296	$D^{*-}(1^-)$	2018	2010
B	0.3263			

Results — bound states



Arrows indicate the energy gain (binding energy) from the relevant thresholds

M. Karliner:
Proc. 8th Int. Conf. Quarks and Nuclear Physics (QNP2018)
JPS Conf. Proc. 26, 011005 (2019) https://doi.org/10.7566/JPSCP.26.011005
Blue dots are added by AH from our results



Comparison with lattice results

	$I(J^P)$	This work	[27]	[28]	[29]	[30]	[31]
$bb\bar{q}\bar{q}$	$0(1^+)$	-173	-189 ± 13	-143 ± 34	–	-186 ± 15	-128 ± 26
$bc\bar{q}\bar{q}$	$0(1^+)$	-40	–	–	13 ± 3	–	–
$cc\bar{q}\bar{q}$	$0(1^+)$	-23	–	-23 ± 11	–	–	–
$bs\bar{q}\bar{q}$	$0(1^+)$	-5	–	–	16 ± 2	–	–
$bb\bar{s}\bar{q}$	$\frac{1}{2}(1^+)$	-59	-98 ± 10	-87 ± 32	–	–	–
$bb\bar{q}\bar{q}$	$1(0^+)$	N	–	-5 ± 18	–	–	–
$bc\bar{q}\bar{q}$	$0(0^+)$	-37	–	–	17 ± 3	–	–
$cc\bar{q}\bar{q}$	$1(0^+)$	N	–	26 ± 11	–	–	–
$bs\bar{q}\bar{q}$	$0(0^+)$	-7	–	–	18 ± 2	–	–

[27] A. Francis, R.J. Hudspith, R. Lewis, K. Maltman, Phys. Rev. Lett. 118,(2017) 142001
$m_\pi = 164{,}299{,}415 MeV$

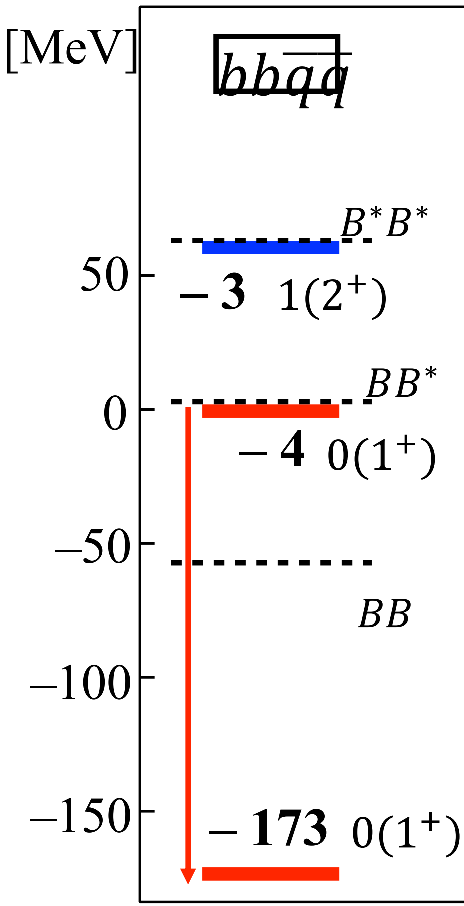
[28] P. Junnarkar, N. Mathur, M. Padmanath, Phys. Rev. D 99 (2019) 034507,
$m_\pi = 153 - 689 MeV$

[29] R. Hudspith, B. Colquhoun, A. Francis, R. Lewis, K. Maltman, Phys Rev D.102.114506 (2020).
$m_\pi = 164{,}299{,}415 MeV$

[30] P. Mohanta, S. Basak, Phys Rev D.102. 094516 (2020)

[31] L. Leskovec, S. Meinel, M. Pflaumer, M. Wagner, Phys. Rev. D 100 (1) (2019)

Results — bound states



There are two states of $J^P = 1^+$

One deep bound state

With “tightly bound” *bb* heavy-diquark)

Compact structure

Another is shallow

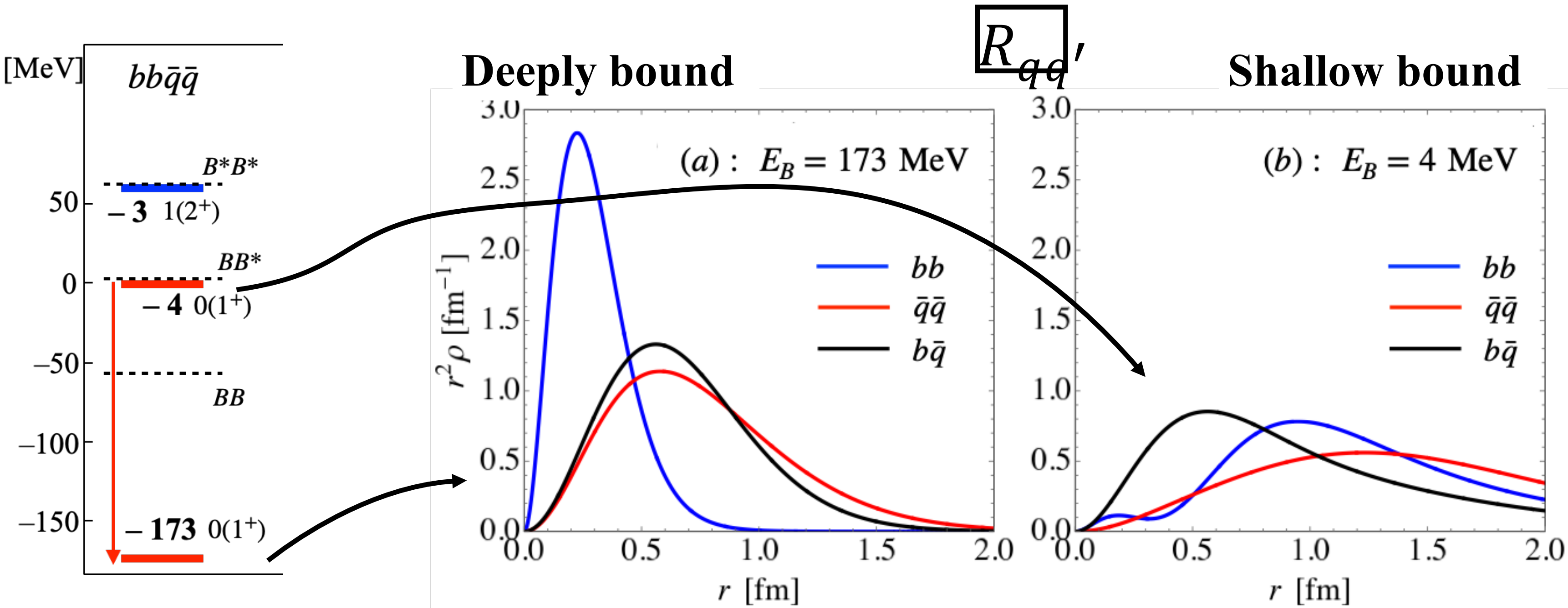
With “radially excited” *bb* heavy-diquark

Molecular-like

There are two states near thresholds BB^* and B^*B^*

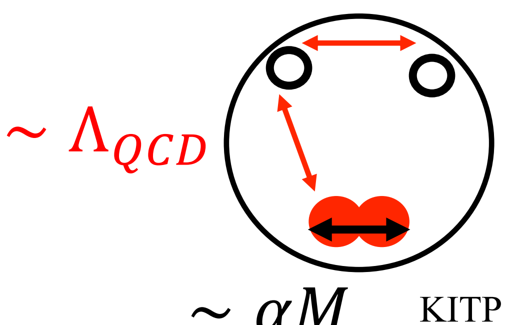
Arrows indicate the energy gain (binding energy) from the relevant thresholds

Results — bound states

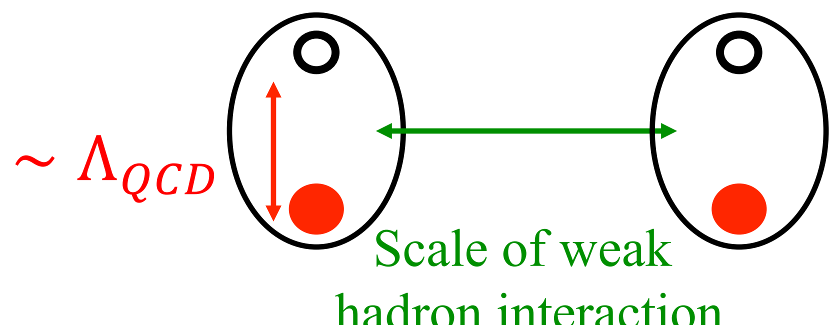


$$R_{qq'} \equiv \left(\int r^2 \rho_{qq'}(r)\, r^2 dr \Big/ \int \rho_{qq'}(r)\, r^2 dr \right)^{1/2} \qquad \rho_{qq'}(r_{qq'}) = \int d\hat{\mathbf{r}}_{qq'} d\boldsymbol{x}_1 d\boldsymbol{x}_2 \, |\Psi_{JM}(\boldsymbol{r}_{qq'}, \boldsymbol{x}_1, \boldsymbol{x}_2)|^2$$

Singly heavy baryon like



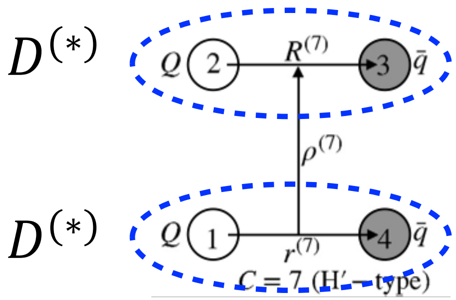
Molecular



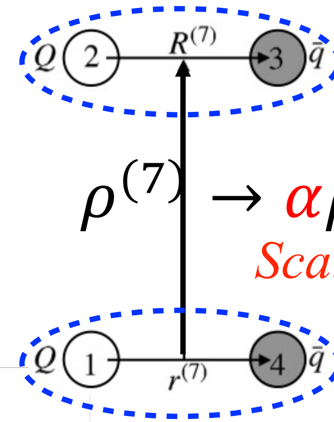
Results — Resonant states

Meng et al, PLB824 (2022) 136800

Scaling method



They can be scattering states



$\rho^{(7)} \rightarrow \alpha\rho^{(7)}$

Scaling factor

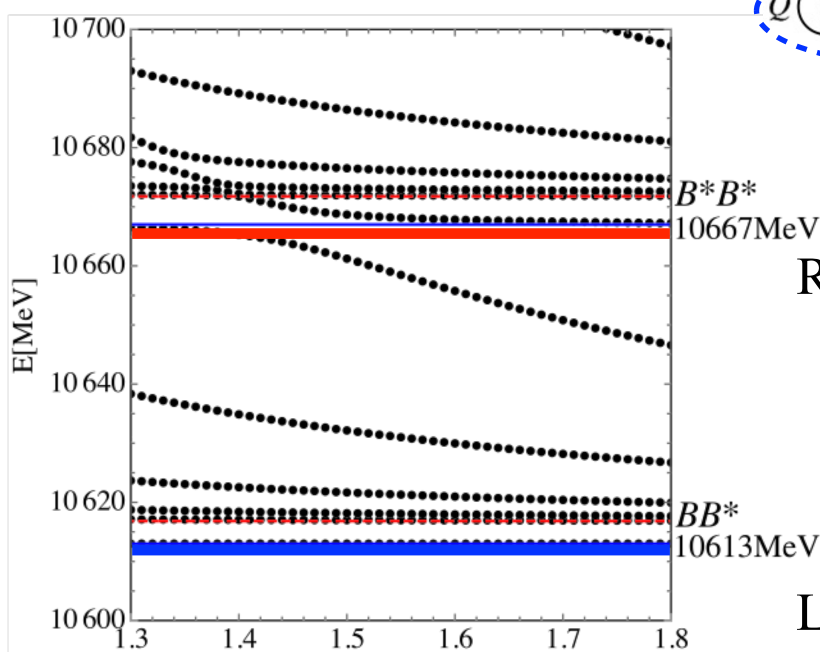
Resonances

Position:

Sequence of horizontal lines that repel each other.

Width:

Distance of repulsion

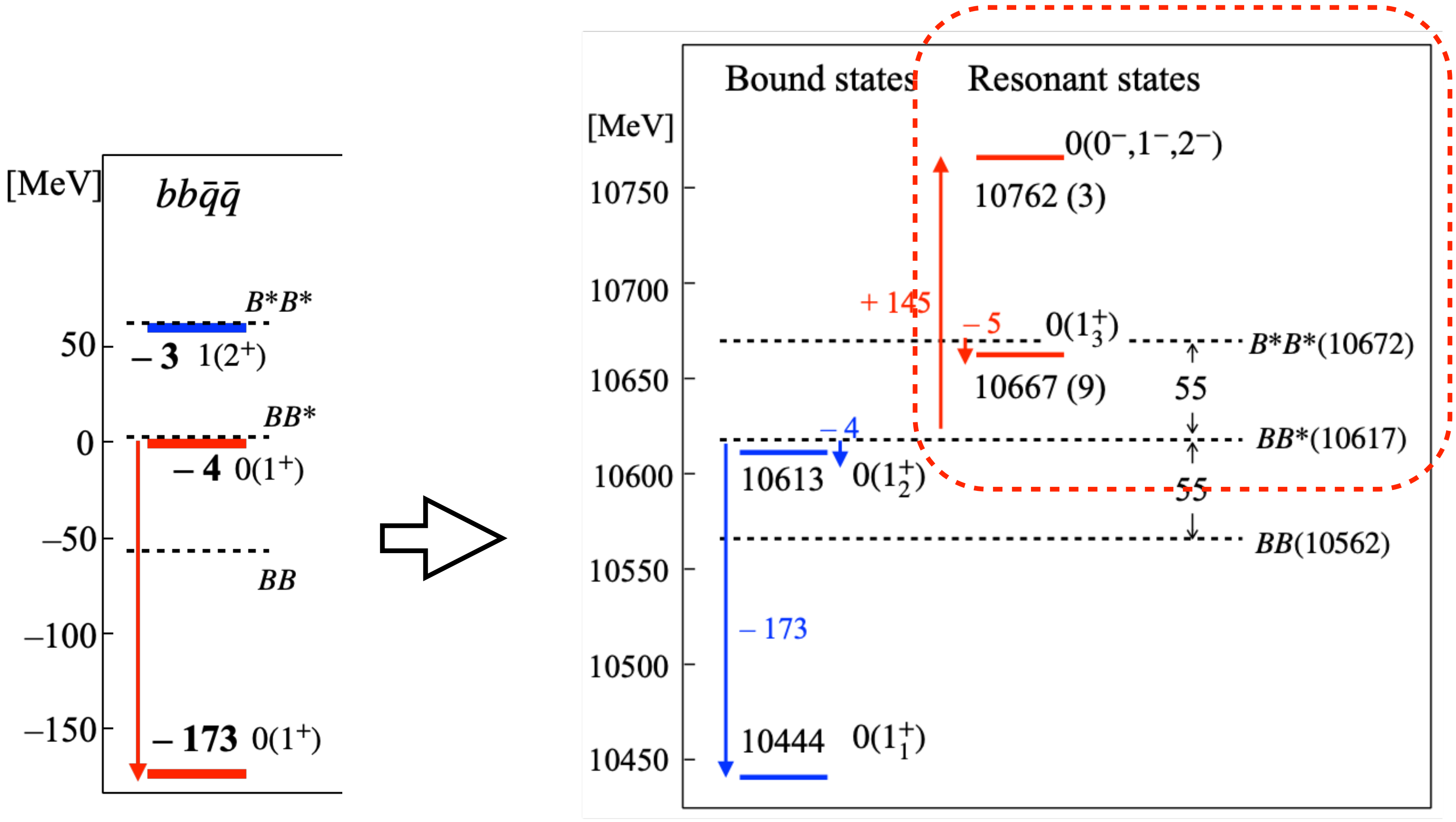


α

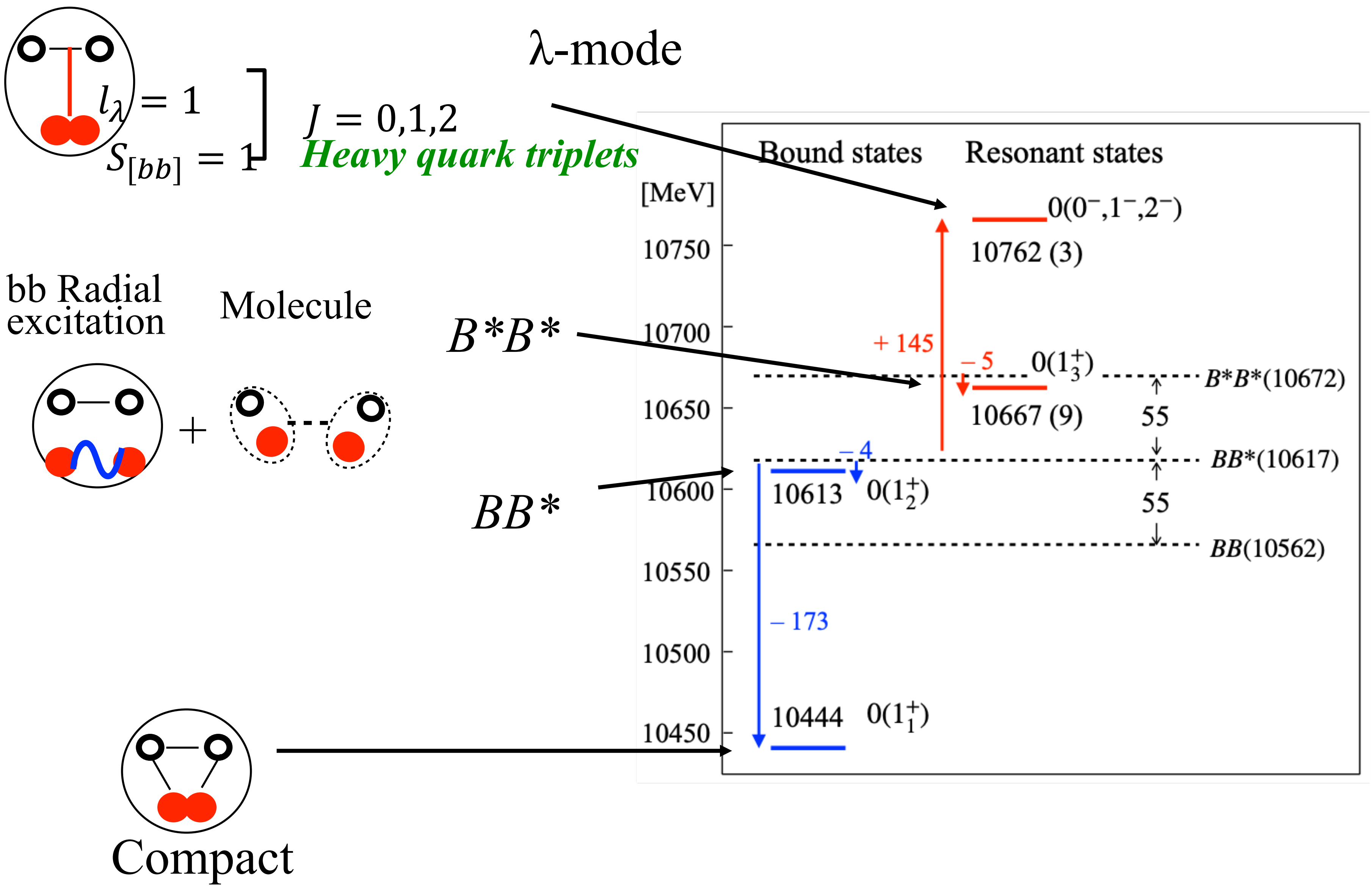
Resonant state

Loosely bound state

Results — Resonant states



Results — Resonant states



Summary for T_{QQ}

- Stable tetraquarks exist for $QQ\overline{qq}$ (Heavy + light)
- Various different configurations are formed

- The most stable one looks like a $\overline{Q}\overline{qq}(\sim Qqq)$
- Shallow ones are like a molecule
- No stable all heavy $QQ\overline{QQ}(> Q\overline{Q} + Q\overline{Q})$
- We have compared the results with lattice ones

- Resonances are also discussed

Future

- Decays, inclusion of pion exchange interaction

Roles of pions for baryons

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - gG\!\!\!/)q - m\bar{q}q + \mathcal{L}_{YM}$$

Quarks

$$+\bar{q}V\!\!\!/ q + g_A\bar{q}A\!\!\!/\gamma_5 q + \frac{1}{4}f_\pi^2 \mathrm{tr}\,\partial^\mu U\,\partial_\mu U^\dagger - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \cdots$$

Pion-quark **Pions**

Are there any places where pions (mesons) play manifestly?

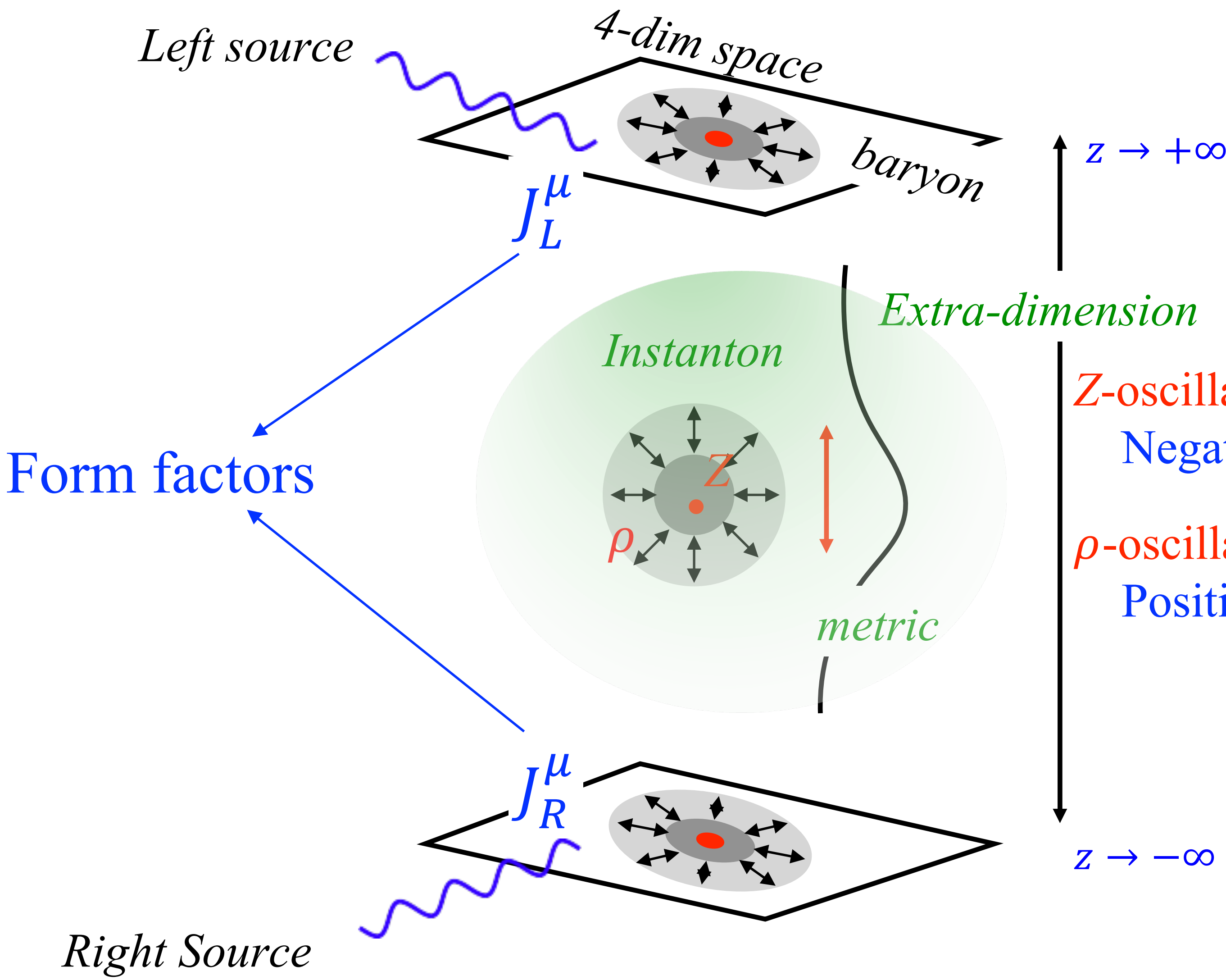
Holographic approach: the Sakai-Sugimoto model (D4/D8 brane)

T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113 (2005), 843; 114 (2005), 1083

- 10 dim. string theory => 5-dim. flavor gauge theory
- 1 extra (*z*) dimension for (infinite) tower of meson excited states
- Baryons emerge as instantons in the 5-dim. space
- Physical states are collective excitations of the instantons

- Implement SSB of chiral symmetry
- Non-linear meson dynamics included in baryon structure

Baryon in 5-dim projected to 4-dim



Z-oscillation:
Negative parity Resonance

ρ-oscillation:
Positive parity Resonance

Quick outline

$$S = S_{YM} + S_{CS}$$

$$S_{\rm YM} = -\kappa \int d^4x dz \, {\rm tr}\left[\frac{1}{2} h(z) \mathcal{F}_{\mu\nu}^2 + k(z) \mathcal{F}_{\mu z}^2\right], \qquad \kappa = \frac{\lambda N_c}{216\pi^3} = a\lambda N_c$$

$$S_{\rm CS} = \frac{N_c}{24\pi^2} \int_{M^4\times\mathbb{R}} \omega_5(\mathcal{A}), \qquad \omega_5(\mathcal{A}) = {\rm tr}\left(\mathcal{A}\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right)$$

$\mathcal{A} = \mathcal{A}_\mu dx^\mu + \mathcal{A}_z dz$ is the 5-dimensional $U(N_f)$ gauge field

$$U(2) \sim SU(2) \times U(1)$$

$\pi, \rho, \ldots$ $\qquad \omega$

Metric in 5th dimension dominates the dynamics

$$h(z) = (1+z^2)^{-1/3}, \qquad k(z) = 1 + z^2$$

A remark on the instanton

- Due to $h(z)$ and $k(z)$ the (BPST type of isospin $SU(2)$) instanton is not scale invariant, with the size $\rho = 0$.

$$A_M(x) = -if(\xi)\, g\partial_M g^{-1}$$

$$M = x_1, x_2, x_3, z$$

$$f(\xi) = \frac{\xi^2}{\xi^2+\rho^2}\,, \quad \xi = \sqrt{(\vec{x}-\vec{X})^2 + (z-Z)^2}$$

$$g(x) = \frac{(z-Z) - i(\vec{x}-\vec{X})\cdot\vec{\tau}}{\xi}\,,$$

- A coupling of the $U(1)$ part (~ omega meson) makes ρ finite.

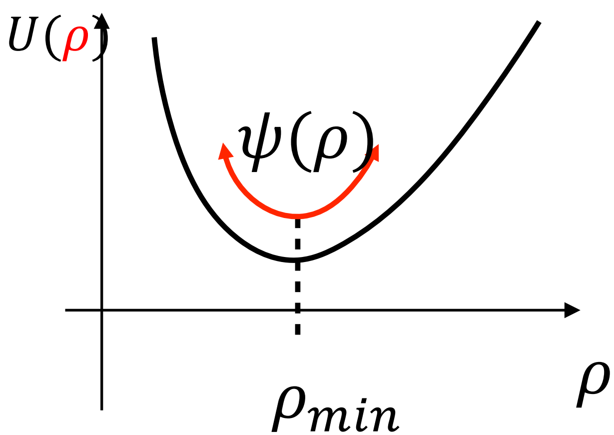
$$\widehat{A}_0 = \frac{1}{8\pi^2 a}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\rho^2+\xi^2)^2}\right]$$

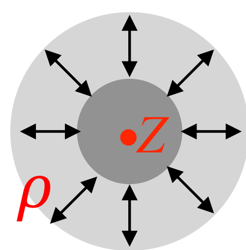
Collective Hamiltonian

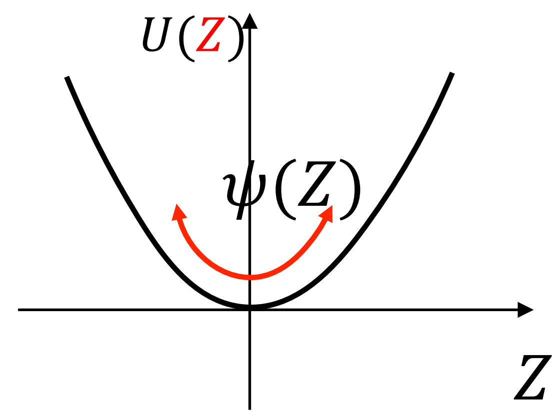
$$H = -\frac{1}{2M_0}\left(\vec{\partial}_X^2 + \partial_Z^2\right) - \frac{1}{4M_0}\partial_\theta^2 + U(\rho, Z)$$

$$U(\rho, Z) = M_0 + \frac{M_0}{6}\rho^2 + \frac{N_c^2}{5M_0}\frac{1}{\rho^2} + \frac{M_0}{3}Z^2$$

X and θ are zero modes







Static properties

Hata Sakai Sugimoto, Yamato, Prog. Theor. Phys. 117 (2007) 1157

(n_ρ, n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(l=1)$	940^+	1348^+	1348^-	1756^-	$1756^+, 1756^+$	$2164^-, 2164^-$	$2164^+, 2164^+$
$\Delta\,(l=3)$	1240^+	1648^+	1648^-	2056^-	$2056^+, 2056^+$	$2464^-, 2464^-$	$2464^+, 2464^+$

Hashimoto, Sakai and Sugimoto, Prog. Theor. Phys. 120 (2008) 1093

Nucleon

	our model	Skyrmion[14)]	experiment
$\langle r^2\rangle^{1/2}_{I=0}$	0.742 fm	0.59 fm	0.806 fm
$\langle r^2\rangle^{1/2}_{M,I=0}$	0.742 fm	0.92 fm	0.814 fm
$\langle r^2\rangle_{E,p}$	$(0.742\ \text{fm})^2$	∞	$(0.875\ \text{fm})^2$
$\langle r^2\rangle_{E,n}$	0	$-\infty$	$-0.116\ \text{fm}^2$
$\langle r^2\rangle_{M,p}$	$(0.742\ \text{fm})^2$	∞	$(0.855\ \text{fm})^2$
$\langle r^2\rangle_{M,n}$	$(0.742\ \text{fm})^2$	∞	$(0.873\ \text{fm})^2$
$\langle r^2\rangle^{1/2}_{A}$	0.537 fm	–	0.674 fm
μ_p	2.18	1.87	2.79
μ_n	−1.34	−1.31	−1.91
$\left\lvert \frac{\mu_p}{\mu_n} \right\rvert$	1.63	1.43	1.46
g_A	0.734	0.61	1.27
$g_{\pi NN}$	7.46	8.9	13.2
$g_{\rho NN}$	5.80	–	$4.2 \sim 6.5$

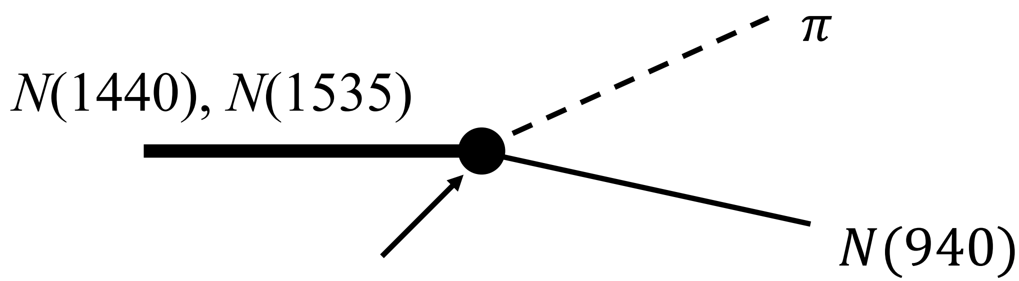
Resonances

	n, p	$N(1440)$	$N(1535)$
$\langle r^2\rangle_{E,p}$	$(0.742\ \text{fm})^2$	$(0.742\ \text{fm})^2$	$(0.699\ \text{fm})^2$
$\langle r^2\rangle_{E,n}$	0	0	0
$\langle r^2\rangle_{M,p}$	$(0.742\ \text{fm})^2$	$(0.742\ \text{fm})^2$	$(0.699\ \text{fm})^2$
$\langle r^2\rangle_{M,n}$	$(0.742\ \text{fm})^2$	$(0.742\ \text{fm})^2$	$(0.699\ \text{fm})^2$
$\langle r^2\rangle^{1/2}_{A}$	0.537 fm	0.537 fm	0.435 fm
μ_p	2.18	2.99	2.18
μ_n	−1.34	−2.15	−1.34
$\left\lvert \frac{\mu_p}{\mu_n} \right\rvert$	1.63	1.39	1.63
g_A	0.734	1.07	0.380
$g_{\pi BB}$	7.46	16.7	6.32
$g_{\rho BB}$	5.80	5.80	4.51

Transitions

Fujii, Hosaka, PRD104, 014022 (2021); Iwanaka et al, in preparation

$N(1440) \to N(940) + \pi$ $\qquad$ $N(1535) \to N(940) + \pi$



$$g_A^{NN^*} \sim \langle N^* | A_\mu | N \rangle$$

axial curent

Current couples to the external source:

$$\mathcal{A}_\alpha(x^\mu, z) = \mathcal{A}_\alpha^{\rm cl}(x^\mu, z) + \delta\mathcal{A}_\alpha(x^\mu, z) \longrightarrow S_{\rm YM} = -\kappa \int d^4x dz \ {\rm tr}\left[\frac{1}{2} h(z) \mathcal{F}_{\mu\nu}^2 + k(z) \mathcal{F}_{\mu z}^2\right]$$

$$S\Big|_{\mathcal{O}(\mathcal{A}_L, \mathcal{A}_R)} = -2 \int d^4x \ {\rm tr}\left(\mathcal{A}_{L\mu} \mathcal{J}_L^\mu + \mathcal{A}_{R\mu} \mathcal{J}_R^\mu\right)$$

$$\boxed{\mathcal{J}_{L\mu} = -\kappa \left(k(z)\, \mathcal{F}_{\mu z}^{\rm cl}\right)\Big|_{z=+\infty}, \quad \mathcal{J}_{R\mu} = +\kappa \left(k(z)\, \mathcal{F}_{\mu z}^{\rm cl}\right)\Big|_{z=-\infty}}$$

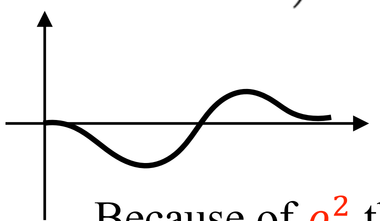
Results — One pion emission

N(1440)

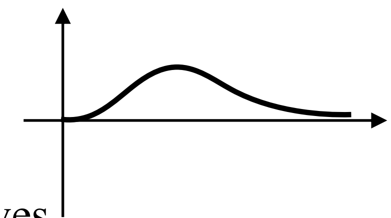
Fujii, Hosaka, PRD104, 014022 (2021)

$$g_A^{NN^*}(\vec{q}) = \frac{8\pi^2\kappa}{3}\langle R_{N^*}|\rho^2|R_N\rangle \sum_{n=1} \frac{g_{a_n}\langle \partial_Z \psi_{2n}(Z)\rangle}{\vec{q}^2+\lambda_{2n}}$$

N^*: $$R(\rho) = \left(\frac{2M_0}{\sqrt{6}}\rho^2 - 1 - 2\sqrt{1+N_c^2/5}\right)\rho^{-1+2\sqrt{1+N_c^2/5}}e^{-\frac{M_0}{\sqrt{6}}\rho^2}$$



N: $$R(\rho) = \rho^{-1+2\sqrt{1+N_c^2/5}}e^{-\frac{M_0}{\sqrt{6}}\rho^2}$$



Because of ρ^2 the matrix element survives

- This is the dominant operator that causes the transition
- The NR quark model misses.
- Can it be an evidence of collective dynamics of the solitonic baryons?

Decay width

Holographic: $\Gamma_{N^*(1440)\to N+\pi} = 64$ MeV vs Exp: $\Gamma_{N^*\to \pi N} \sim 90\text{–}140$ MeV

Model independent relation

$$g_A^{NN}/g_A^{NN^*} = \left(1+2\sqrt{1+\frac{N_c^2}{5}}\right)^{1/2} = 2.08$$

Results — One pion emission

N(1535)

Iwanaka, Fujii, Hosaka, in preparation

$$g_A^{NN^*}(\tau^a)_{I_3 I_3'} = 2\int d^3x\, \langle N, I_3' | J_A^{a0} | N^*, I_3 \rangle$$

$$= \int_{-\infty}^{\infty} dZ \psi_Z(Z) \psi_Z'(Z) \sum_{n=1}^{\infty} \frac{g_{a^n} \psi_{2n}(Z)}{\lambda_{2n}}$$

$$= \frac{e^{\sqrt{\frac{2}{3}} M_0}}{\left(\frac{2}{3}\right)^{1/4} \sqrt{\pi M_0}} \mathrm{erfc}\left(\left(\frac{2}{3}\right)^{1/4} \sqrt{M_0}\right)$$

$a = 3, I_z, I_z'$

Decay width

$\Gamma^{\text{Holographic}}_{N^* \to N\pi} = 44\ \text{MeV}$ vs $\Gamma^{\text{Exp}}_{N^* \to N\pi} = 42 - 68\ (\approx 55)\ \text{MeV}$

Summary

- **Holographic model** describes the nucleon and resonances well
- Not only static but also dynamic properties (decay)
- Some distinct features of solitonic approach which are physically of collective nature have been shown, different fromthe quark model (single particle)

• Quark model has predictive power also for exotics (multi-quarks)
• Dynamic processes may need meson (pion) dynamics
• How we can find links to QCD?