Universal Deformations

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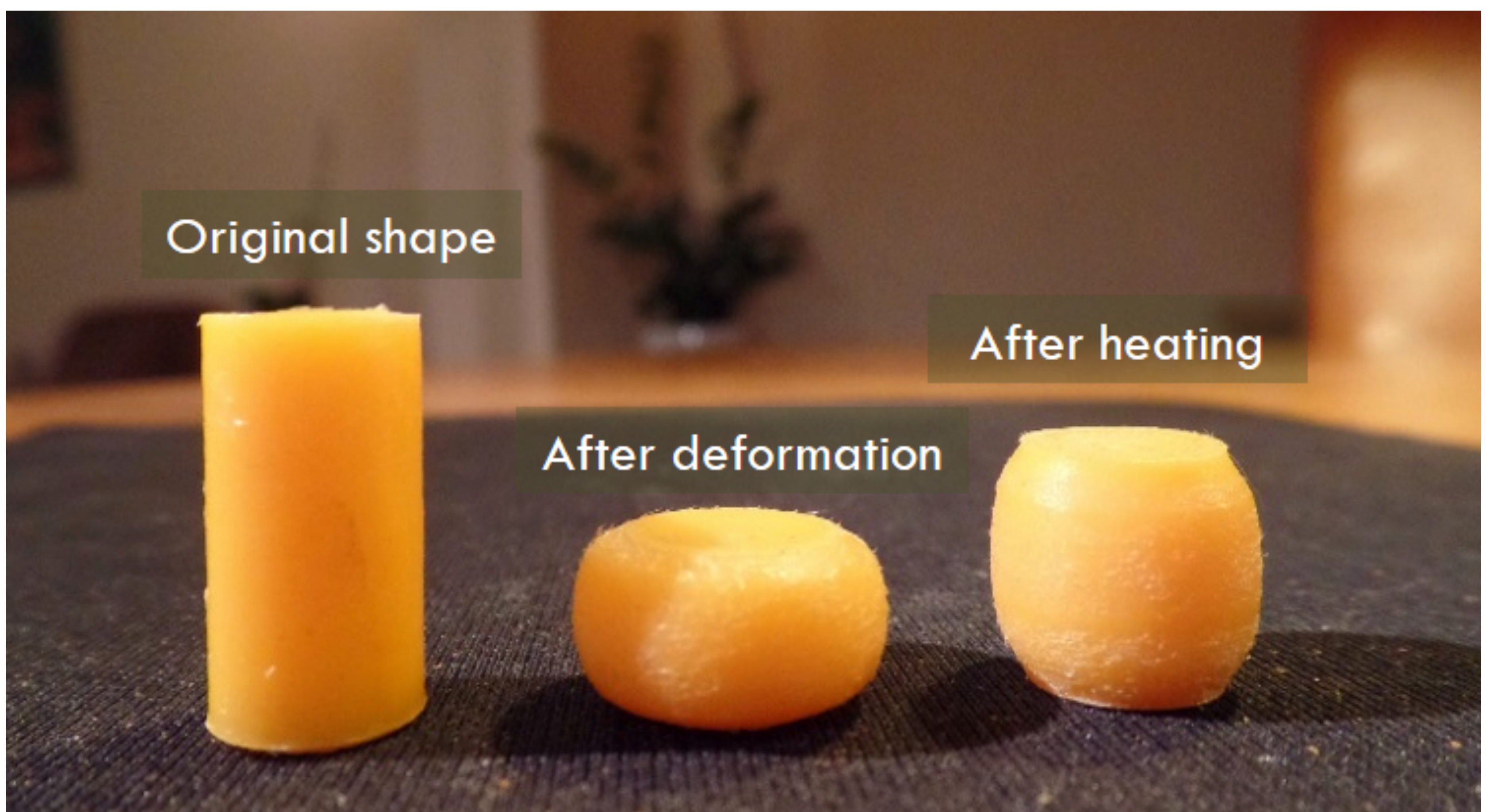
Deformations of QFTs

Much of QFT exploration is the study of deformations.

``Deformation'' = dialing parameter and asking what happens

- Dial chemical potential/density
- Dial magnetic field
- Dial mass parameters



viscoelasticity.info

Deformations of QFTs

$$S_{\text{new}} = S + \Lambda^{d-\Delta} \int d^d x \, \mathcal{O}(x)$$

If the scaling dimension $\Delta > d$, negligible at long distance, but if $\Delta < d$, the deformation is relevant, interesting effects!

Usually we can't determine dependence on Λ exactly.

- Perturbation theory tends to be the best we can do.

Punchlines

- There are non-SUSY interacting QFTs with exactly solvable relevant deformations

- To understand them, need a fancy perspective on meaning of 'symmetry'

- In 2d QFTs, dramatic implications for confinement

- Examples of QFTs that violate the EFT naturalness principle.

Symmetry

Textbooks continuous symmetry example:

$$S = \int d^4x \left(|\partial\phi|^2 + m^2 |\phi|^2 \right)$$

$\phi \to e^{i\alpha}\phi$ leaves S invariant, gives a j_μ such that $\partial^\mu j_\mu = 0$. Then we can construct ``symmetry generator" operators:

$$U_\alpha = e^{i\alpha \int_{\text{space}} j_0} = e^{i\alpha Q}$$

Very useful, but there is a different (deeper?) perspective.

Gaiotto, Kapustin, Seiberg, Willett, 2014

Symmetry and topology

$$U_\alpha = e^{i\alpha \int_{\text{space}} j_0} \quad \Rightarrow \quad U_\alpha(M_3) = \exp\left(i\alpha \int_{M_3} \star j \right)$$

$d \star j = 0 \Rightarrow U_\alpha$ only has topological dependence on M_3

If M_3 is deformed past a charged operator $\phi(x)$, then

linking number

$$U_\alpha(M_3)\phi(x) = e^{i\alpha \ell(M_3, x)} U_\alpha(\tilde{M}_3)\phi(x)$$

$$U_\alpha(M_3) U_\beta(M_3) = U_{\alpha+\beta}(M_3)$$

Existence of $U(1)$ symmetry $\Rightarrow$ existence of (d-1)-dimensional topological operators

Symmetry as topology

Gaiotto, Kapustin, Seiberg, Willett, 2014

New definition:

existence of (d-1)-dimensional **topological operators** $\Rightarrow$ Existence of a symmetry

Everything we do with normal symmetries can be rephrased in terms of manipulations of topological operators.

For standard symmetries, don't learn anything new from this fancy rewording.

shinesty

Symmetry as topology

Benefit of fancy topological language is generalizations!

existence of (d-1-p)-dimensional topological operators $\Rightarrow$ Existence of a 'p-form' symmetry

Topological operators need to satisfy some 'fusion rule' like

$$U_i(M)U_j(M) = \sum_k N_{ij}^k U_k(M)$$

and have some consistent topological action on charged **p-dimensional** operators.

Wild consequences, such as symmetry $\neq$ symmetry group

Chang, Lin, Shao, Yin 2018

QFT in 1+1d

Focus on 1+1d because it allows us to draw nice pictures.

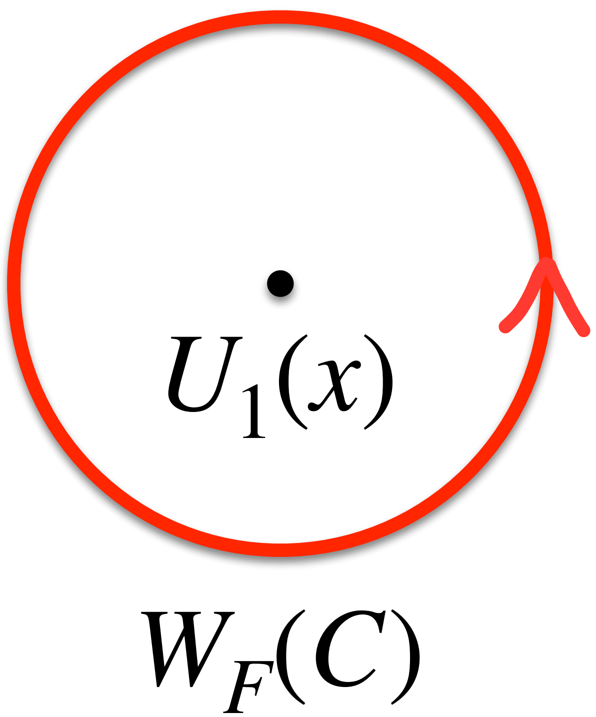
``1-form symmetry” is generated by 2-1-1 = 0-dimensional topological operators.

- So 2d QFTs with 1-form symmetries have **local** topological operators, which act on charged **line operators**
  - The charged objects are Wilson loops!
    - Schwinger model (2d QED) with fermions of charge N
    - 2d SU(N) YM theory
    - 2d adjoint QCD
    - …

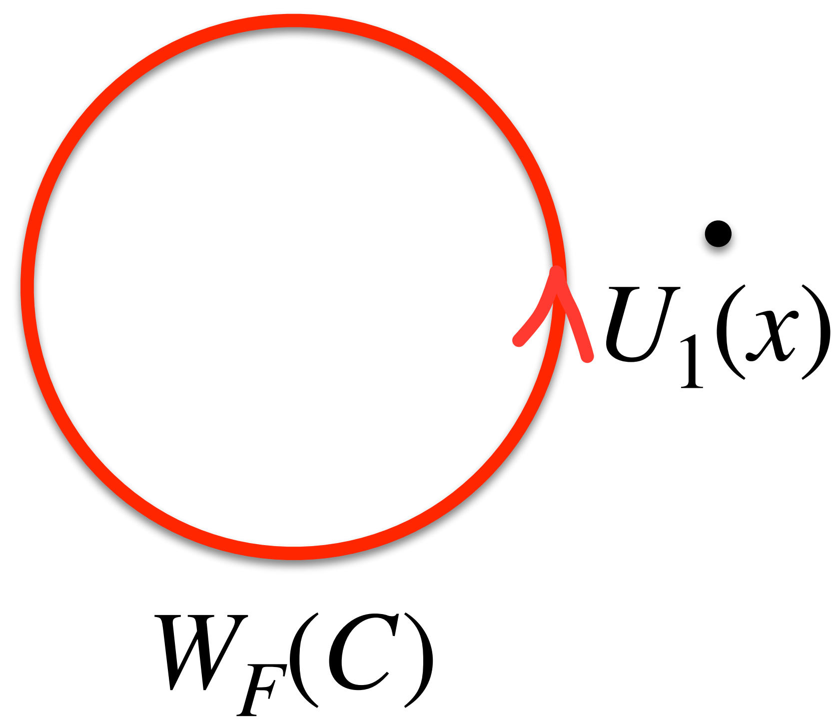
1-form symmetry = modern upgrade of 1980s “center symmetry”

$\mathbb{Z}_N$ 1-form symmetry in 1+1d

Suppose the 1-form symmetry has a $\mathbb{Z}_N$ group structure. Then



$$= e^{\frac{2\pi i}{N}} \times$$



$$U_1(x) \quad U_1(x') \quad = \quad U_1(x) \qquad U_1(x'')$$

$$U_i(x)U_j(x) = U_{i+j \text{ mod } N}(x)$$

Universal deformations

Consider any 2d theory with a $\mathbb{Z}_N$ 1-form symmetry.

- Means it has local topological operators (LTOs) $U_n(x)$
- Local operators can be added to the Lagrangian - defines a deformation of the theory!

$$S_{\text{new}} = S + \sum_n \Lambda_n^{2-\Delta} \int d^d x\, U_n + \text{h.c.}$$

What is Δ? Work near UV fixed point, then:

$$\langle U_n^\dagger(x) U_n(0) \rangle \longrightarrow \lambda^{-2\Delta} \langle U_n^\dagger(\lambda^{-1} x) U_n(0) \rangle = \lambda^{-2\Delta} \langle U_n^\dagger(x) U_n(0) \rangle$$

$$\Rightarrow \Delta = 0$$

This deformation is maximally relevant!

$\Delta = 0 \Rightarrow$ boring?

We've all heard of one famous dimension-0 deformation before:

$$S = \int d^4x \sqrt{-|g|} \left(\frac{1}{\kappa} R - \Lambda^4 + \mathcal{L}_{\mathrm{matter}} \right)$$

Cosmological constant term is a deformation by $\mathbf{1}$, and has $\Delta = 0$, but boring within QFT without gravity!

hcamag.com

LTO deformations have physical effects within QFT, like driving phase transitions.

ere.net

Universal deformations

Deformations by $\sum_n U_n(x)$ are `universal', in the sense that:

1. Effects are universal and exactly calculable.
2. Direct effect is on the vacuum energy of `universes'.

Universal deformations

Can always do a formal expansion in powers of a deformation in the path integral:

$$Z_{\text{new}} = \int d[\text{fields}] e^{-S_{\text{old}}} e^{-\Lambda^2 \int d^2 x U(x) + \text{h.c.}}$$

$$= Z_{\text{old}} \sum_{I,J=0}^{\infty} \int dx_i dy_j c_{I,J} \Lambda^{2I} \Lambda^{2J} \left\langle \prod_{i=1}^{I} \prod_{j=1}^{J} U_n(x_i) U_n^{\dagger}(y_j) \right\rangle_{\text{old}}$$

Normally useless, but here we know all the correlation functions!

Summing up, we get difference between Z_{old} and Z_{new} exactly.

The effect is on sectors called universes.

Universes

Expectation values of $\mathbb{Z}_n$ LTOs are tightly constrained:

$$\langle U_1(x) \rangle = e^{2\pi i k/N}$$

Hellerman et al, 2006;
Tanizaki, Unsal 2019;
Komargodski et al, 2020

k labels sectors of the QFT called 'universes'.

- Universe domain walls have *infinite* tension
  - $\langle U_1(x) \rangle$ is topological, so can't change smoothly
- Excitations can't take you from one 'universe' to another.
- **Physically, universe walls are just probe Wilson lines!**
  - Wilson loops ⇔ infinitely-heavy probe particles, useful to explore phase structure.

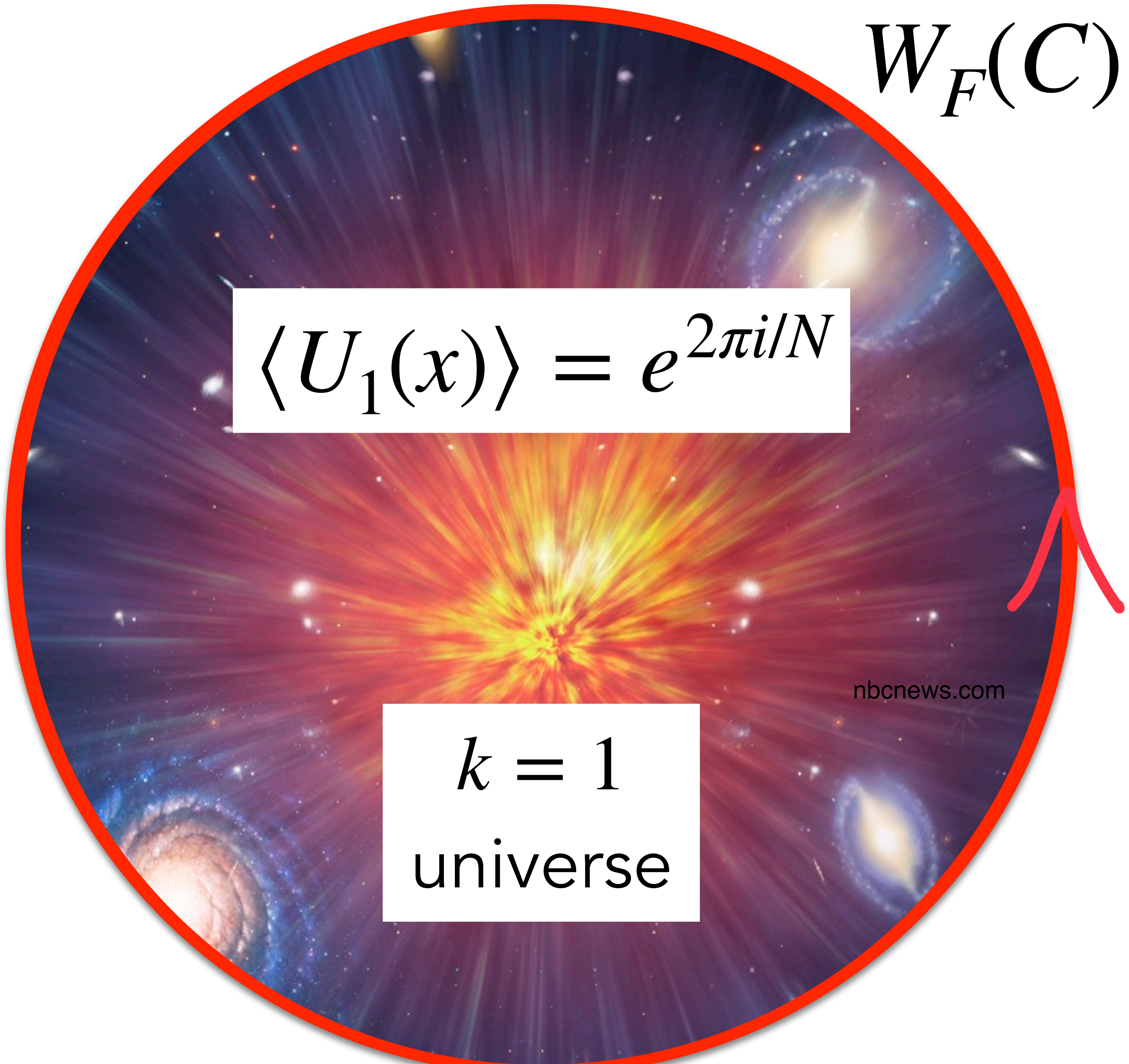
Universes

$$W_F(C) \sim e^{i\int_C a}$$

$$\langle U_1(x) \rangle = 1$$

$$k = 0$$

universe

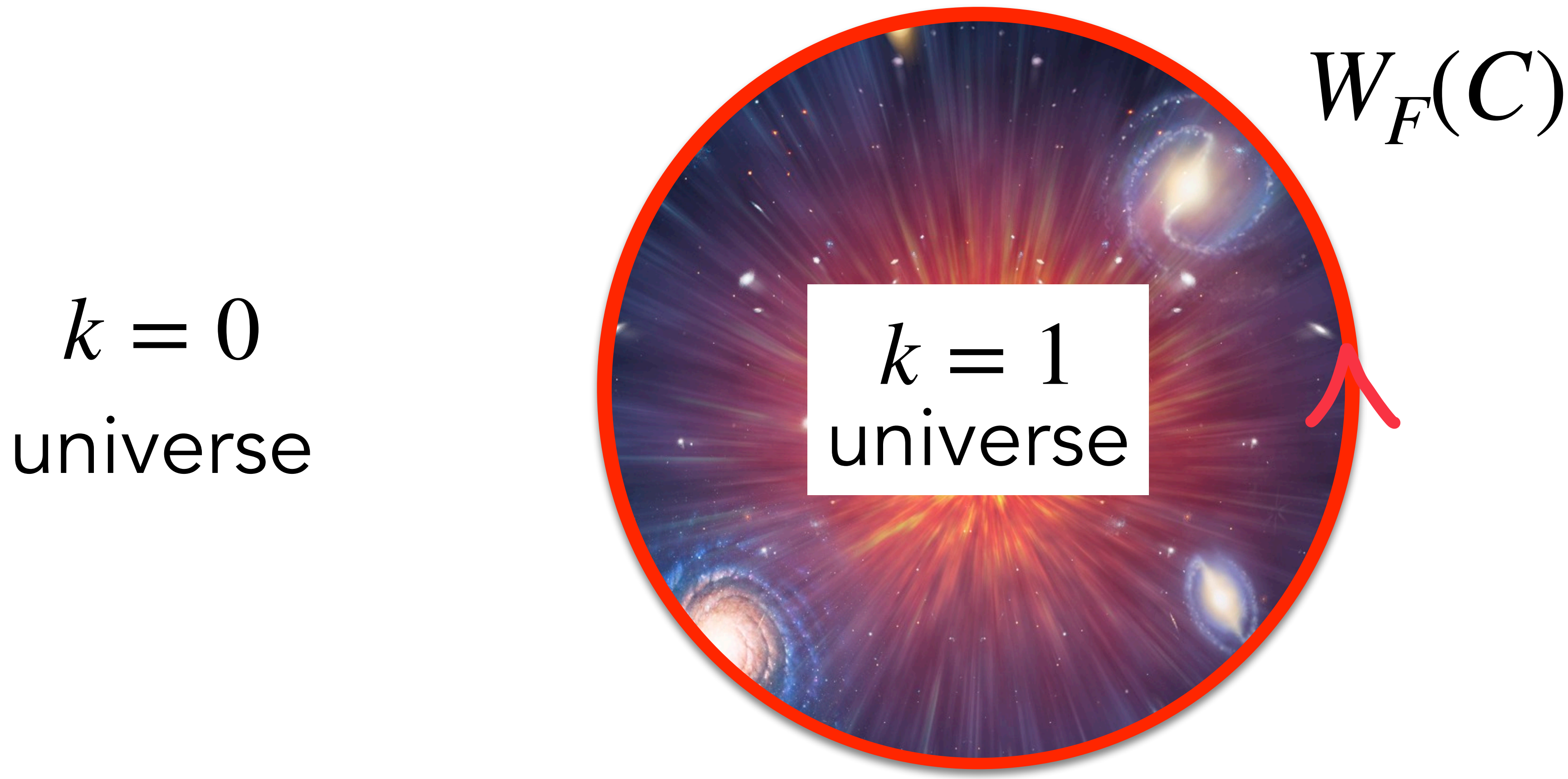


$$\langle U_1(x) \rangle = e^{2\pi i/N}$$

$$k = 1$$

universe

Universes and confinement



If vacuum energy density inside is bigger than outside, gain energy by shrinking C

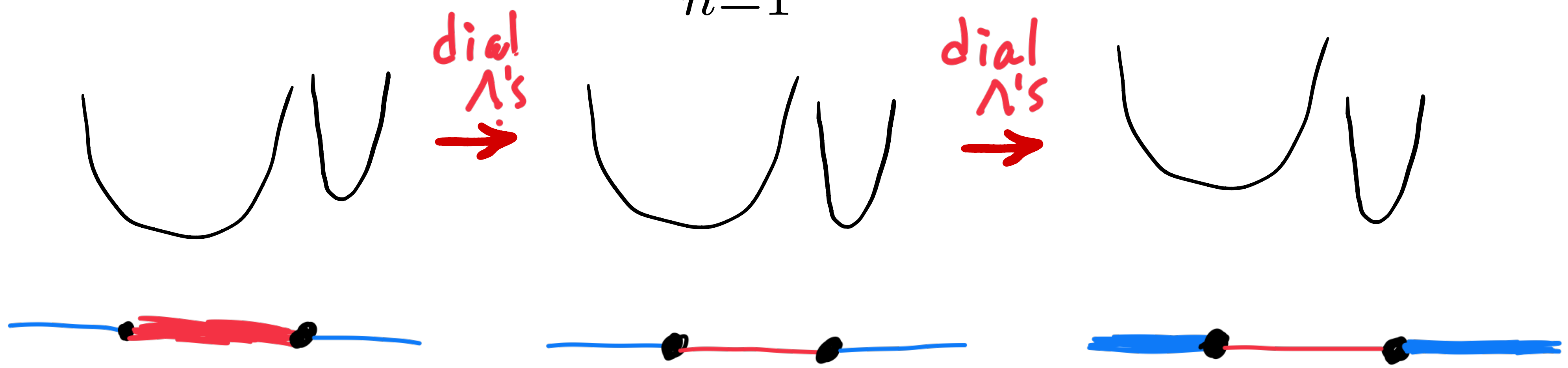
- Then $\langle W_F(C) \rangle \sim e^{-TA}$ - this is quark/charge confinement.

If vacuum energy densities inside = outside, only cost is from the edge, so $\langle W_F(C) \rangle \sim e^{-\mu L_C}$ - deconfinement!

Universal deformations

Within k-th universe, $\langle U_n \rangle = e^{2\pi i n k/N}$, so the effect of deformation is simply to shift all states by the same amount.

$$\mathcal{E}_{\text{new}} = \mathcal{E}_{k,\text{old}} + \sum_{n=1}^{N-1} \Lambda_n^2 \cos\left(2\pi k n/N\right)$$



Λ_n affects relative vacuum energy densities of universes.

- Makes their effects observable.
- Takes 2d QFTs through deconfinement phase transitions!

Concrete example

2d QED: U(1) gauge theory with charge N Dirac fermion.

$$S_\psi = \int d^2x \left(\frac{1}{4e^2} f_{\mu\nu} f^{\mu\nu} + \frac{i\theta}{2\pi} \epsilon^{\mu\nu} \partial_\mu a_\nu + \bar{\psi}(\not{\partial} - i N \not{a} - m_\psi)\psi \right)$$

$$\frac{1}{2\pi} \int_{M_2} da \in \mathbb{Z}$$

Charge N Schwinger model!

Famous playground for exploring confinement

- Do charge 1 probes feel linear potential?
- Analytic control in $m_\psi \ll e$ regime.

Concrete example

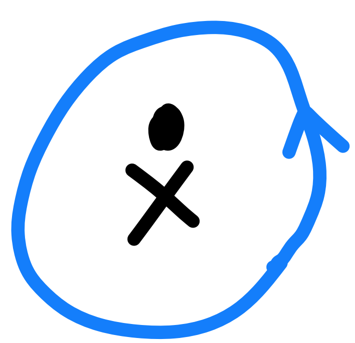
2d QED: U(1) gauge theory with charge N Dirac fermion.

$$S_\psi = \int d^2x \left(\frac{1}{4e^2} f_{\mu\nu} f^{\mu\nu} + \frac{i\theta}{2\pi} \epsilon^{\mu\nu} \partial_\mu a_\nu + \bar{\psi}(\partial\!\!\!/ - iN a\!\!\!/ - m_\psi)\psi \right)$$

Known to have a $\mathbb{Z}_N$ 1-form symmetry, so it has local topological operators!

Defined as abstract Gukov-Witten-type `disorder' operators.

- No simple description in terms of a_μ, ψ fields.

$U_1(x) =$  $e^{i\oint a} = e^{2\pi i/N}$

Concrete example: Schwinger model

In 2d, there is an equivalent bosonic theory:

$$S_\varphi = \int_M \bigg(\frac{e^2}{2} \|b\|^2 + \frac{1}{8\pi} \|d\varphi\|^2 + \frac{i}{2\pi} (N\varphi + 2\pi b + \theta) \wedge da - m\mu \cos\varphi \bigg)$$

$$e^{i\varphi} \sim \bar{\psi}_L \psi_R \, , \quad m = \frac{e^{\gamma_E}}{2\pi} m_\psi \, , \quad \mu \sim e$$

Slightly unconventional: no explicit $f_{\mu\nu} f^{\mu\nu}$ term.

- Integrating out b field gives $f_{\mu\nu} f^{\mu\nu}$ Maxwell term, but to write LTOs explicitly, it is useful to keep b in action.

Local topological operator

$$U_n(x) = \exp\left[i\frac{2\pi n}{N}\left(b + \frac{N}{2\pi}\varphi + \frac{\theta}{2\pi}\right)\right]$$

EoM for a is $d(N\varphi + 2\pi b + \theta) = 0$, so it is constant on-shell.

- Can verify $U_n(n)$ as given above satisfies the right fusion and commutation rules to generate the $\mathbb{Z}_N$ 1-form symmetry

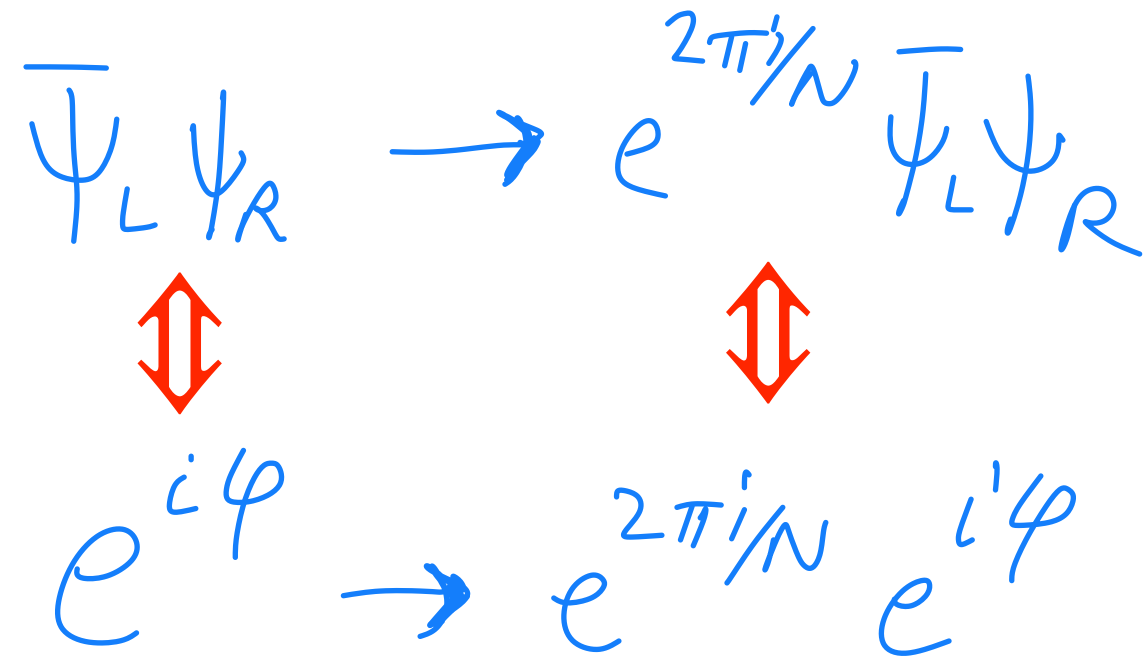
$$S_{\text{new}} = S_{\text{old}} + \sum_{n=1}^{N-1} \Lambda_n^2 \int d^2x \left(U_n(x) + \text{h.c.}\right)$$

$U_n(x)$ remains a topological operator when $\Lambda_n \neq 0$.

Chiral symmetry

$$U_n(x) = \exp\left[i\frac{2\pi n}{N}\left(b + \frac{N}{2\pi}\varphi + \frac{\theta}{2\pi}\right)\right]$$

If $m = 0$, then there is a $\mathbb{Z}_N$ chiral symmetry:



$U_1(x) \to e^{2\pi i/N} U_1(x)$ under chiral symmetry.

- Mixed 't Hooft anomaly for 1-form and 0-form symmetries.
- Adding $U_n(x)$ to the action breaks chiral symmetry!

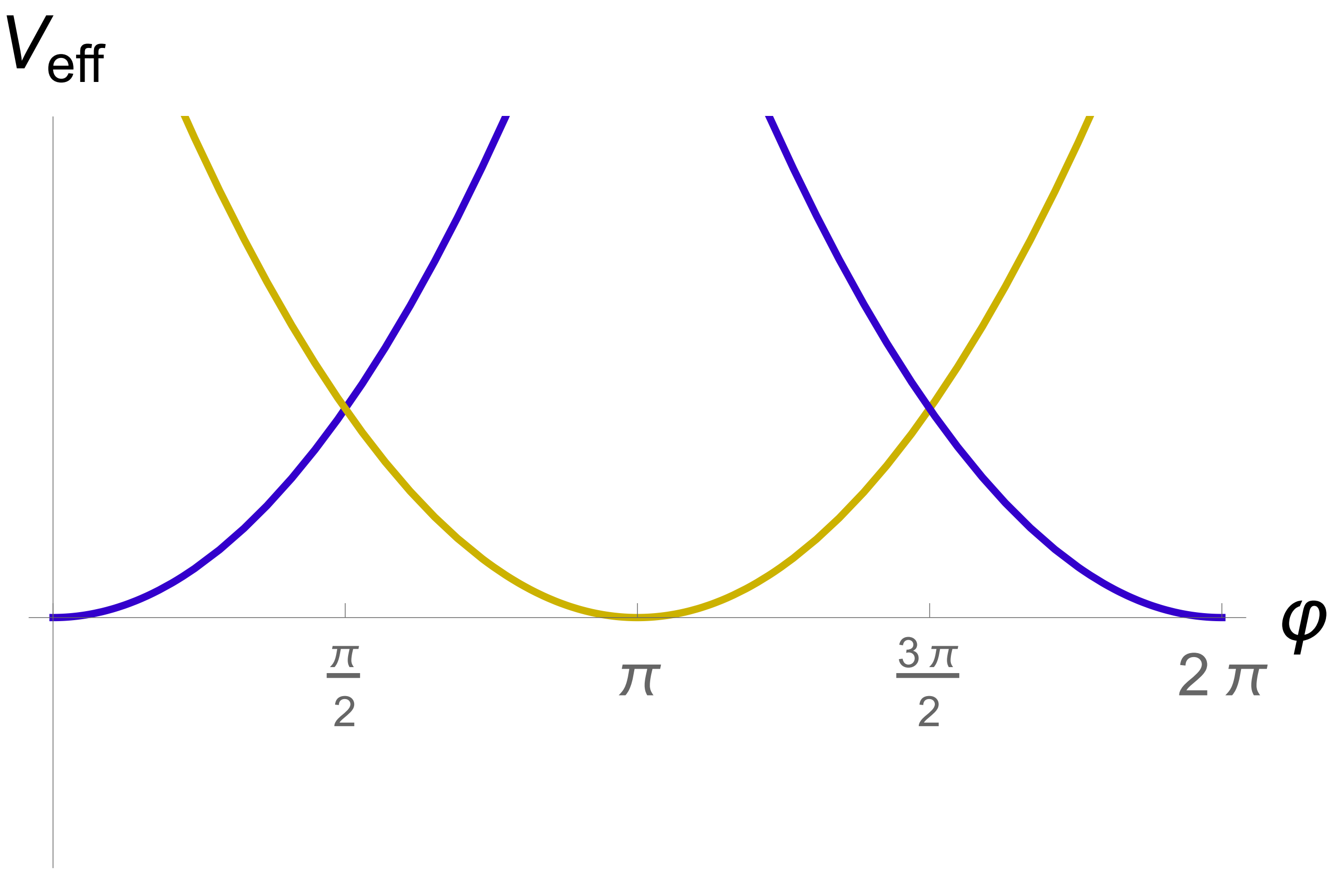
The complicated fate of confinement

- If $m = \Lambda_n = 0$, both 1-form $\mathbb{Z}_N$ symmetry and chiral symmetry are spontaneously broken.
  - deconfinement. ← widely known since 1970s!
- If $m = 0$ and $\Lambda_n \neq 0$, chiral symmetry is explicitly broken, and Schwinger model with $m = 0$ confines!
- If $m \neq 0$, in general we get confinement, but if we tune Λ_n as a function of m, can get deconfinement with $m \neq 0$

Charge-2 Schwinger model universes

- If N=2, then $\left(\mathbb{Z}_2\right)_{\text{chiral}} : \varphi \to \varphi + \pi$.
- To draw some pictures it's helpful to integrate out everyone except φ.
- Physics of confinement and chiral symmetry can be read off from the resulting potential V_{eff} for φ.

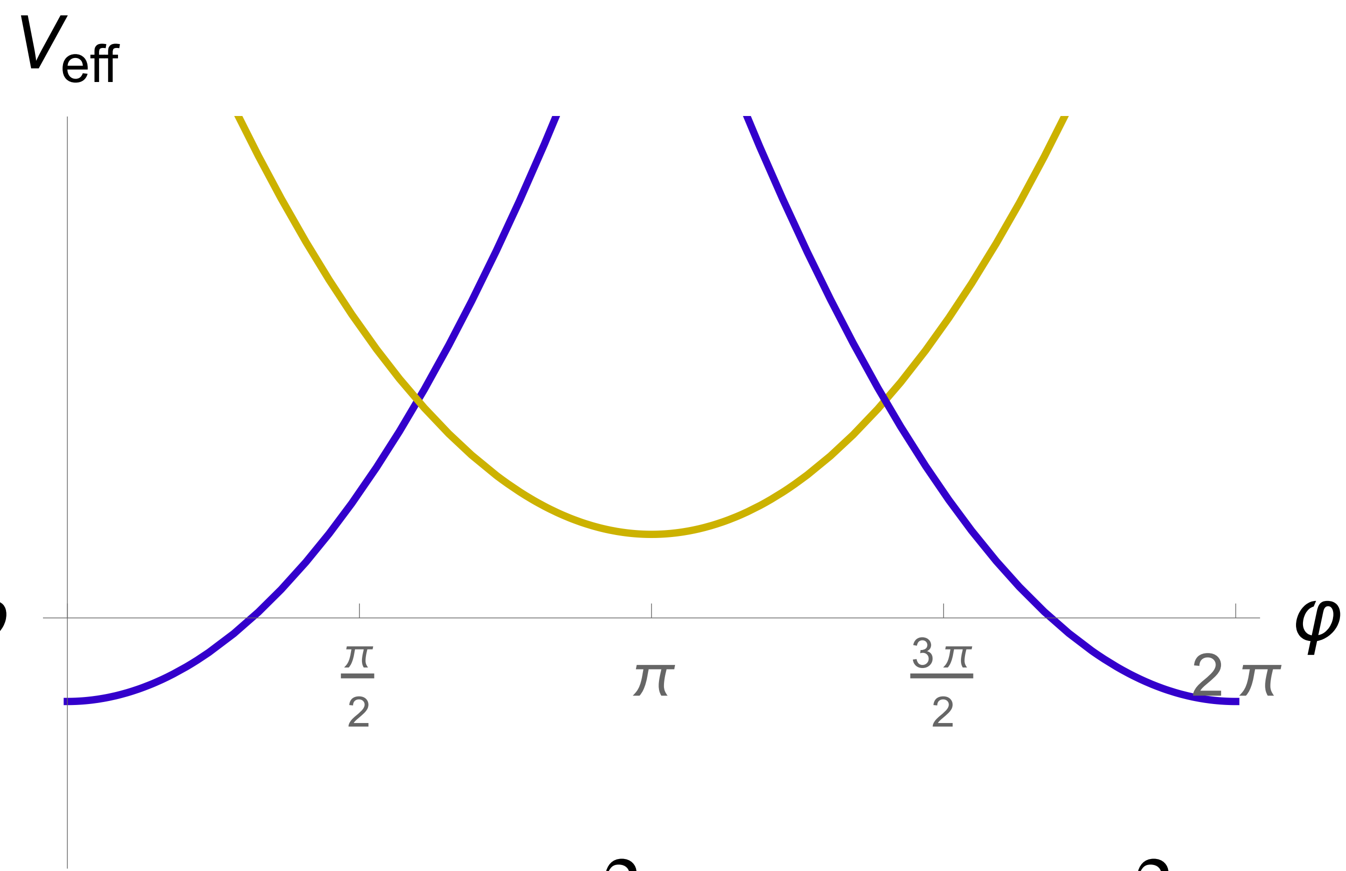
Charge-2 Schwinger model universes



$$m = \Lambda = 0$$

Spontaneously broken $\mathbb{Z}_2$ chiral symmetry

Spontaneously broken $\mathbb{Z}_2$ 1-form symmetry

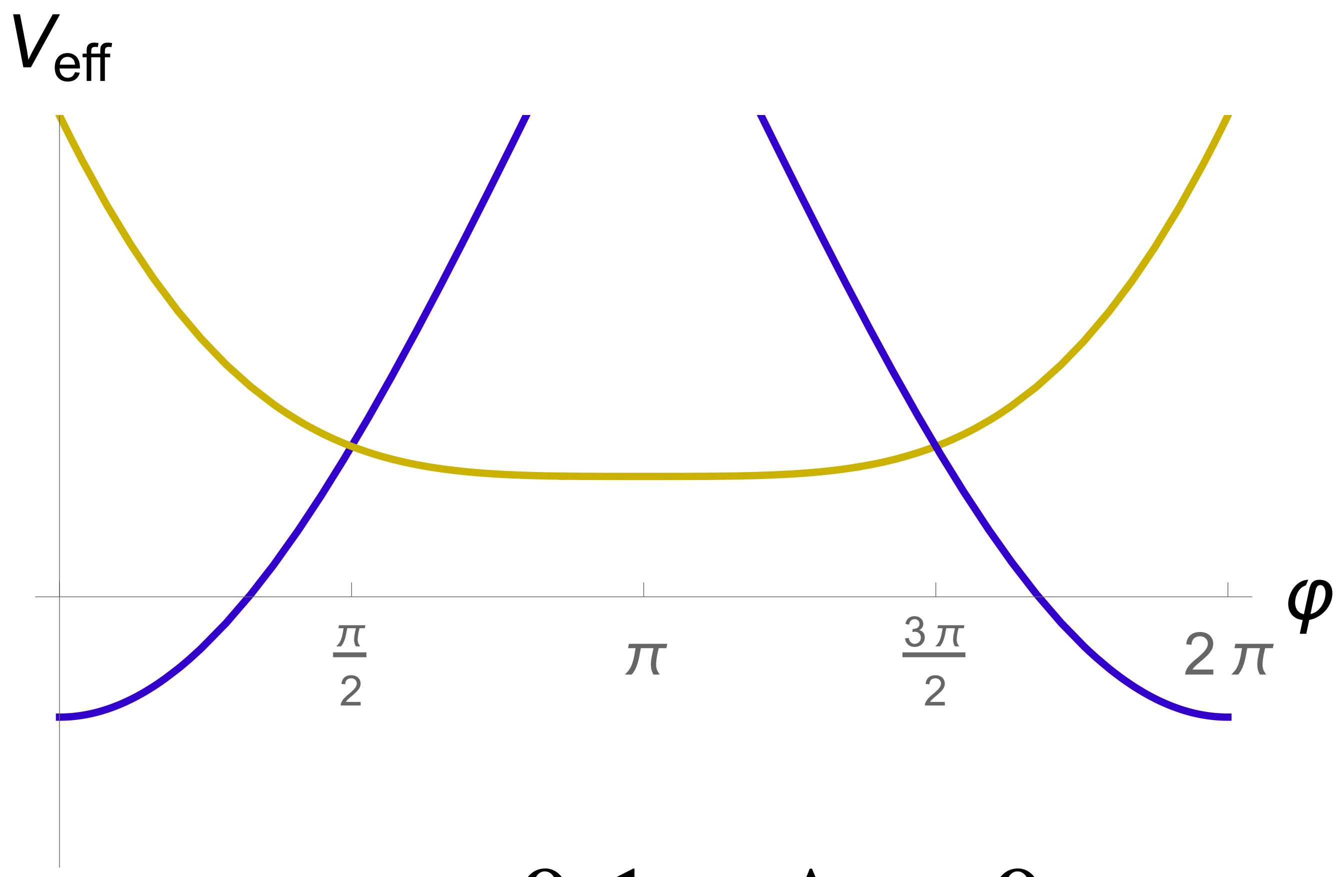


$$m = 0,\ \Lambda^2 = -0.05e^2$$

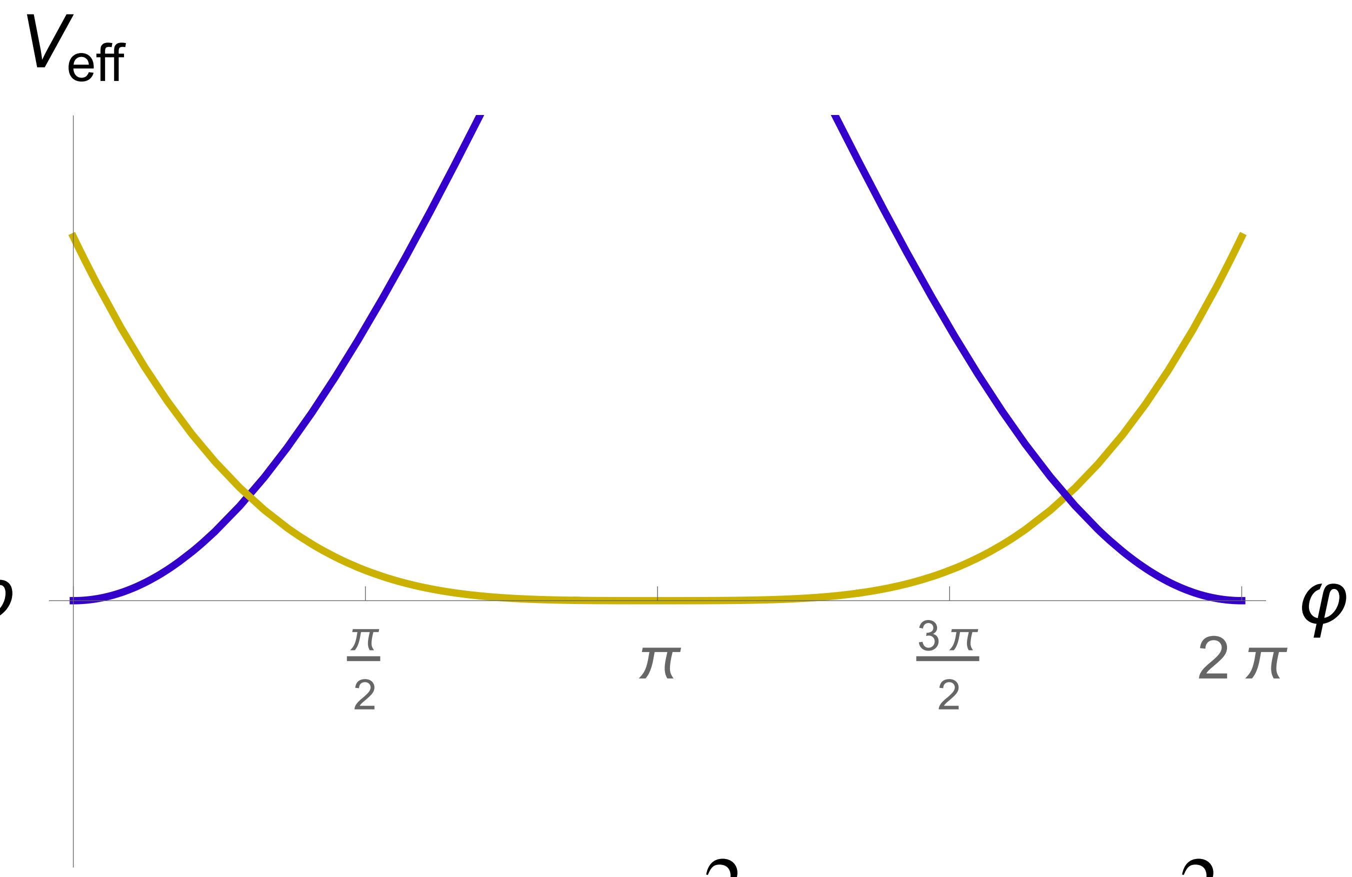
Explicitly broken $\mathbb{Z}_2$ chiral symmetry

Unbroken $\mathbb{Z}_2$ 1-form symmetry: confinement.

Massive Charge-2 Schwinger model



$m = 0.1e,\ \Lambda = 0$



$m = 0.1e,\ \Lambda^2 \simeq -0.1e^2$

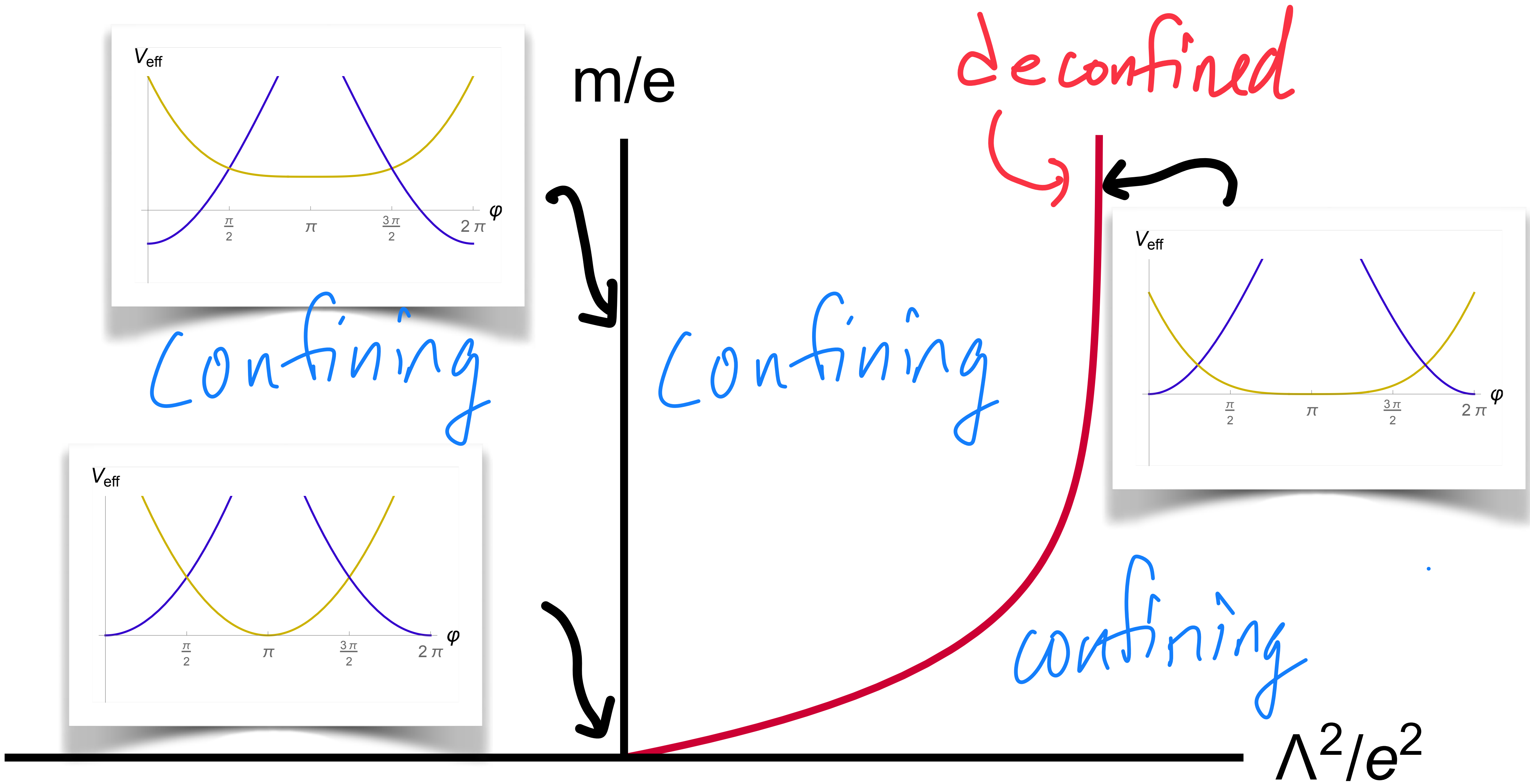
Explicitly broken $\mathbb{Z}_2$ chiral symmetry

Unbroken $\mathbb{Z}_2$ 1-form symmetry

Explicitly broken $\mathbb{Z}_2$ chiral symmetry

Spontaneously broken $\mathbb{Z}_2$ 1-form symmetry

Phase diagram for $N = 2$



Assuming C symmetry and $\theta = 0$

Consequences

- Counterexample to EFT naturalness principle
- Confinement in 1+1d
- (For others see our paper!)

Discuss one by one.

Naturalness Principle

Effective field theory naturalness principle:

- All operators not forbidden by a symmetry of IR theory will be generated by fluctuations, with scale set by scale of operators that break the symmetry in the UV.
- To avoid this, need to fine-tune UV parameters.

Huge fraction of modern particle physics research is dedicated to exploiting or fighting this principle.

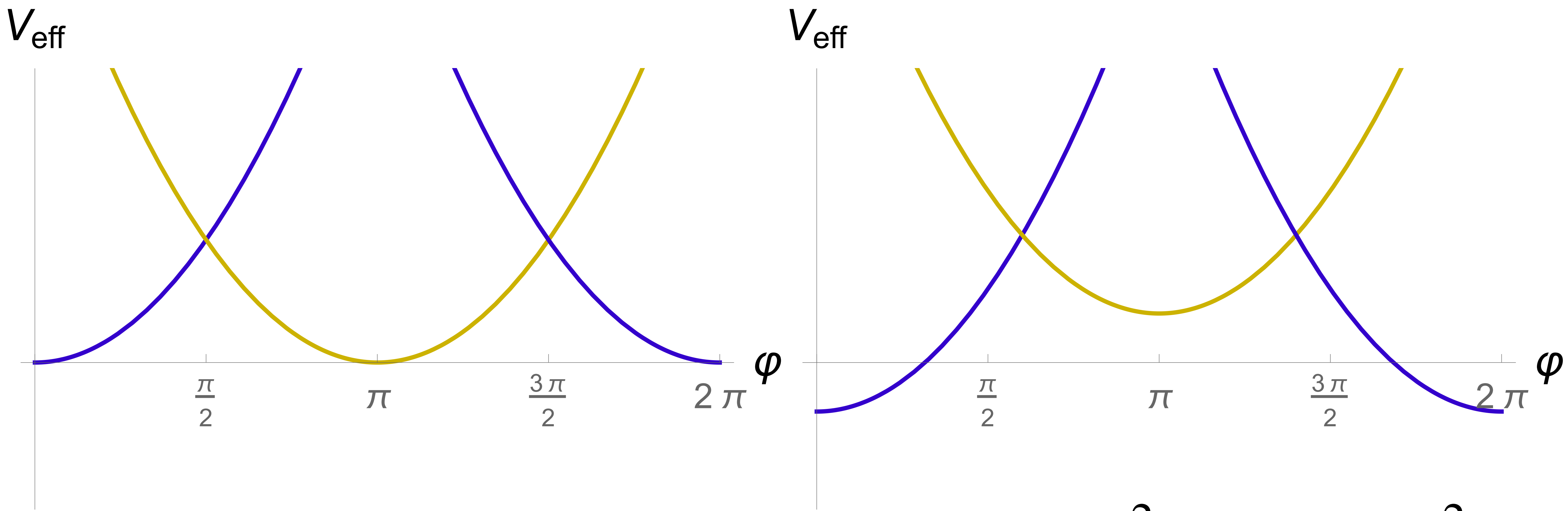
- Higgs boson mass
- Cosmological constant
- Strong CP

Unnaturalness

Our deformations give a counter-example to EFT naturalness.

- If 1-form and 0-form symmetries have 't Hooft anomaly, local topological operators are charged.
- When added to action, they are a relevant 0-form symmetry-breaking deformation.
- Effect of universal deformations is exactly calculable. Other symmetry breaking terms are not generated.
  - If they were, particle spectrum would be affected by deformations - and it isn't!
  - The **only** effect is to shift relative vacuum energies.

Massive Charge-2 Schwinger model



$m = \Lambda = 0$ $\qquad$ $m = 0,\ \Lambda^2 = -0.05e^2$

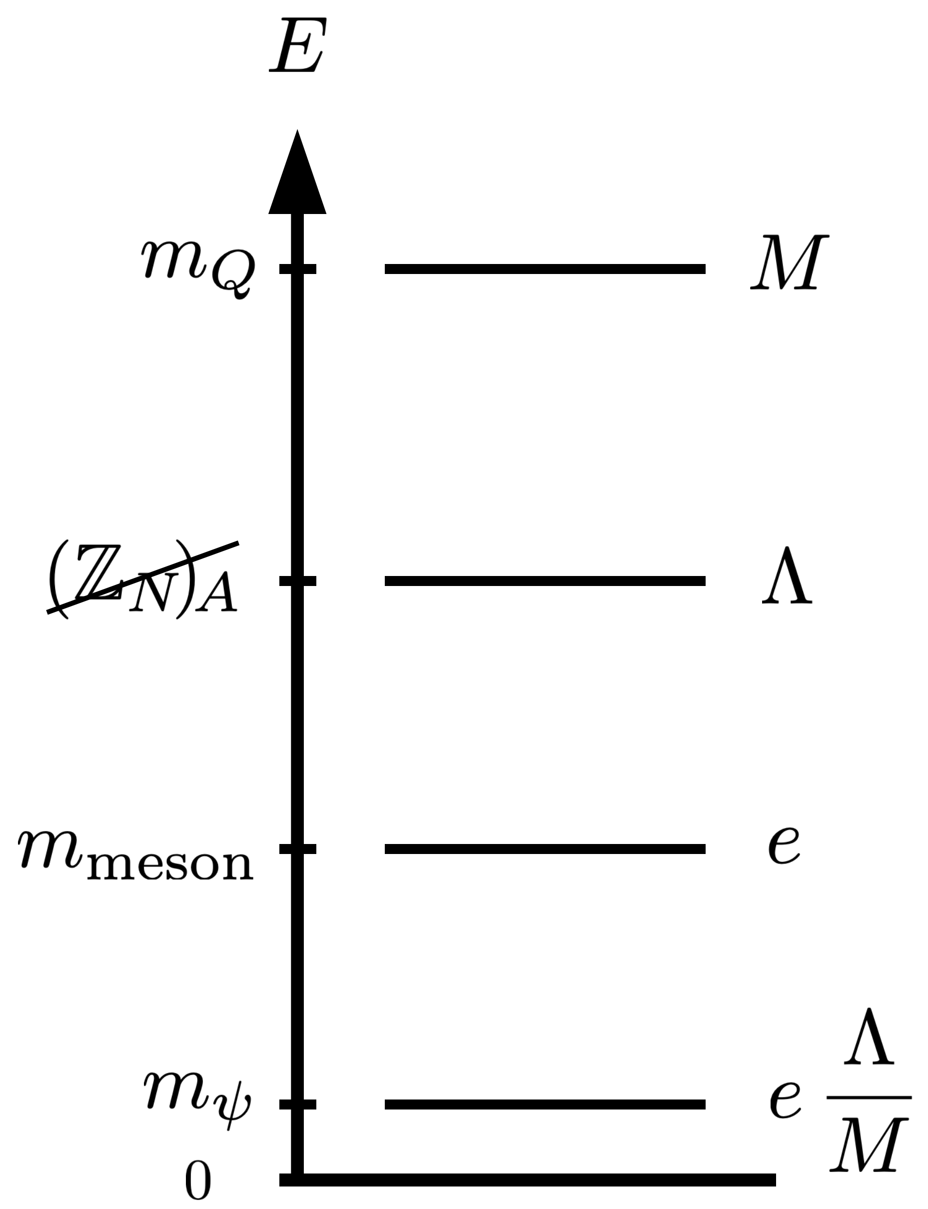
Particle spectrum comes from shape of potential curves —

and shape doesn't depend on the universal deformation

Turning on $\Lambda^2 \int d^2x\, U(x)$ does **not** induce mass term!

Breaking symmetries

If 1-form symmetry is explicitly broken at a high scale M, expect QFTs with unnaturally-small mass scales

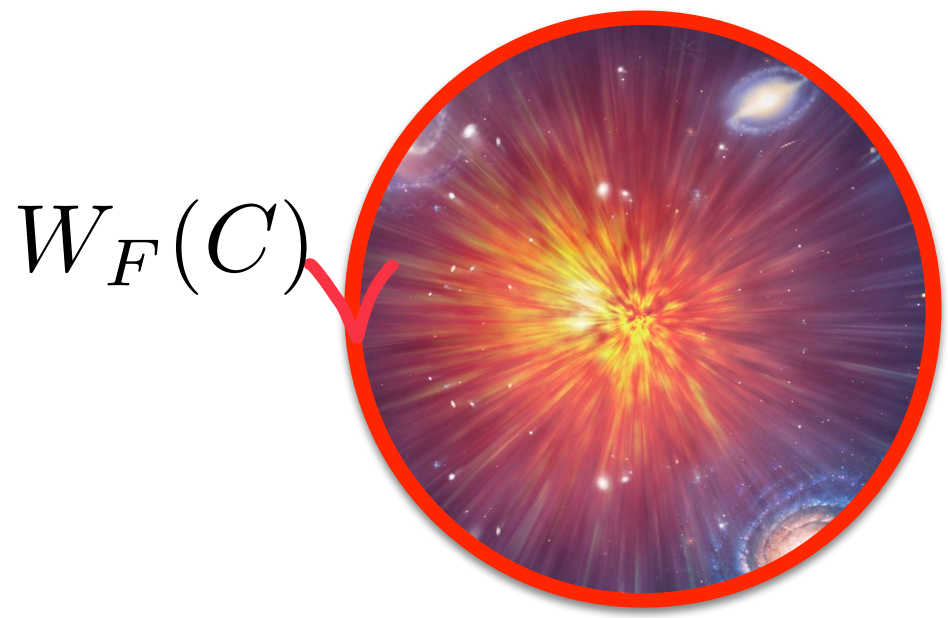


Would be great to understand this in detail!

Confinement in 1+1d

Confinement is a question about the behavior of large Wilson loops on $\mathbb{R}^2$: do they have area law, or not?

Behavior of $\langle W_F(C) \rangle$ determined by comparison of **vacuum energy densities** inside and outside a Wilson loop.



Vacuum energies are sensitive to universal deformations, so confinement is as well!

2d adjoint QCD

SU(N) YM + 1 adjoint massless Majorana fermion,

- Conventional symmetries: $\mathbb{Z}_N$ 1-form symmetry; $\mathbb{Z}_2$ chiral, $(-1)^F$, charge conjugation, parity 0-form symmetries

Does it confine fundamental test quarks? Many answers:

No
Gross, Klebanov, Matytsin, Smilga, 1990s;
…
Dempsey, Klebanov, Pufu 2021

Yes
AC, Jacobson, Unsal, Tanizaki 2019

No*
Komargodski, Ohmori, Roumpedakis, Seifnashri 2020

Yes and No
AC, Jacobson, Neuzil 2021

2d adjoint QCD

Odd N: (N-1)/2 + 2 relevant and (classically) marginal parameters consistent with all conventional symmetries.

$$\sum_{k=1}^{(N-1)/2} \Lambda_k^2 \left(U_k(x) + \text{h.c.}\right) + c_1 \text{tr}\ \psi_+\psi_+\psi_-\psi_- + c_2 \text{tr}\ \psi_+\psi_- \text{tr}\ \psi_+\psi_-$$

Confined at generic points in parameter space! But one can tune Λ_k to make it deconfine.

- Note that particle spectrum cares about c_1, c_2, m/e, but it does **not** care about Λ_k .
- Wilson loops care about all (N-1)/2 + 2 parameters: to check confinement, have to calculate Wilson loops VEVs.

2d adjoint QCD

Odd N: (N-1)/2 + 2 relevant and (classically) marginal parameters consistent with all conventional symmetries.

$$\sum_{k=1}^{(N-1)/2} \Lambda_k^2 \left(U_k(x) + \text{h.c.}\right) + c_1 \text{tr}\ \psi_+\psi_+\psi_-\psi_- + c_2 \text{tr}\ \psi_+\psi_- \text{tr}\ \psi_+\psi_-$$

Komargodski et al 2020

If $\Lambda_k = 0, c_2 = 0$, there are non-invertible topological lines.

- One line carries charge 1 under $\mathbb{Z}_N$ 1-form symmetry $\Rightarrow$ 't Hooft anomaly between exotic zero-form symmetry and 1-form symmetry $\Rightarrow$ deconfinement.

To forbid Λ_k from being generated, regulator must respect the exotic symmetry. Which regulators do this?

Lattice? DLCQ? Conformal truncation?

Generalizations

- There are 3+1d QFTs with local topological operators — they generate 3-form global symmetries.
    - 1-flavor SU(N) QCD with a restricted instanton sum has $\mathbb{Z}_p$ 3-form and 0-form chiral symmetry
    - Axion + SU(N) YM with a restricted instanton sum has $\mathbb{Z}_p$ 3-form and 0-form chiral symmetry
    - …

Seiberg, 2010 Tanizaki, Unsal, 2019 AC, Jacobson, 2020

Our analysis also applies to these 4d QFTs.

Conclusions

- There are interacting non-SUSY QFTs with **exactly solvable** relevant deformations!
  - All one needs is a (d-1)-form symmetry.

- In e.g. 2d QFTs, interesting implications for confinement.

- Examples of QFTs that violate EFT naturalness principle.

Next steps:

- Explore the new naturalness violation further.
- Understand *emergent* 1-form symmetries better.

Thanks for listening!