Office DEPOT

Semiclassics on $R^3 \times S^1$ & flux tubes
(confining strings "from flesh & blood")
(Misha S.)

Refs: flux tube on $R^3 \times S^1$
1501.06773 w/ Anber, Sulejmanpasic
1708.08821 w/ Shalchian
1909.10979 w/ Cox & Wong
2010.04330 w/ Bub & Wong
+ in progress w/ Cox

recently: general $R^3 \times S^1$ review/pedagogical intro 2111.10243
(... all work of Mithat Ü's ref'd there.)

$R^{1,2} \times S^1$ is remarkable setup

S^1 - small $LN\Lambda \ll 2\pi$ $SU(N)$

→ IR dynamics semiclassically calculable

pillars of calculability
- center symmetry (along S^1)
- abelianization $SU(N) \to U(1)^{N-1}$

→ weak coupling

- ∃ fractional I's ~ 1990's
+ Polyakov confinement : $R^3 \to R^3 \times S^1$

today's theory space

SU(N) w/ n_f adjoint Weyl $n_f \leq 5$

① dYM $n_f \geq 2$ $m_f \sim \frac{1}{NL}$

universality class of pure YM

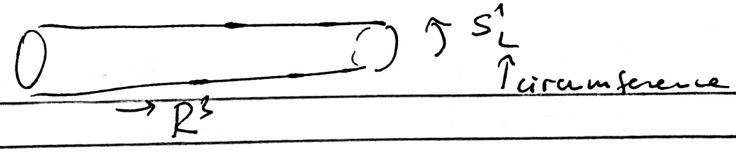
② SYM $n_f = 1$ $m_f = 0$

③ QCD(adj.) $n_f \geq 2$ $m_f = 0$

focus on $N=2$; ⇒ will answer $N>2$,
& show plots;
⇒ other gauge gps. SYM (in progress)

- focus on conf. strings, skip many detail
- weak coupling confinement, chiral SB, ~~parts~~
- some evidence for $\mathbb{R}^3 \times S^1 \to \mathbb{R}^4$
"adiabatic continuity"

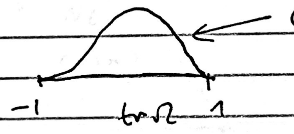
SU(2) & main points on dynamics ⟶



$\Omega = S^1$- Polyakov loop Fund. $= \frac{1}{2} P e^{i\oint_{S^1} A}$

$\mathrm{tr}_F \Omega \in [-1, 1]$

purely bosonic YM : $L = \beta = \frac{1}{T}$



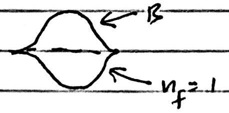
GPY : $T \gg \Lambda$

$\mathbb{Z}_2^{(0)} \longrightarrow$ center along S^1_L broken

here call: "0-form center"

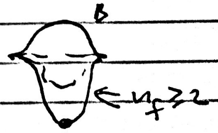
SYM w/ PERIODIC $\lambda^a \longleftarrow$ adjoint Weyl

Coulomb branch perturb. exact



so no potential

QCD (adj) w/ n_f periodic λ



center stability $\longrightarrow$ $\mathrm{tr}\Omega = 0$

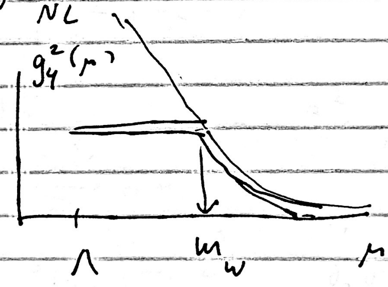
$\langle \lambda \rangle\Big|_{QCD(adj)} = i\sigma_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$

adjoint Higgs

$SU(2) \to U(1)$

$SU(N) \to U(1)^{N-1}$

$m_W = \frac{2\pi}{NL}$: scale of lightest W-boson mass



$m_W \gg \Lambda$

$\downarrow$

$\boxed{\Lambda N L \ll 2\pi}$

weak coupling

- L small & fixed
- $g_4^2(1/NL)$ fix, or Λ fix. $\left(\underline{\text{NOT Dim Red.}} : \underset{\text{fix}}{g_3^2} = \frac{g_4^2(1/L) \to 0}{L \to 0} \right)$

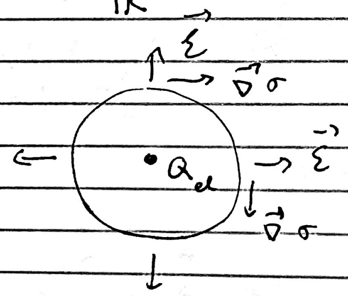
IR theory, perturbatively

$$\int_{\mathbb{R}^3} \frac{L}{g_4^2(1/L)} F_{\mu\nu}^2 + \dots$$

$\uparrow$ $\mathbb{R}^3$ abelian photon (Cartan)

dual photon

$$\int_{\mathbb{R}^3} \frac{g_4^2}{L} (\partial_\mu \sigma)^2 + \dots$$



$$\frac{g_4^2}{L} \partial_\mu \sigma \sim \epsilon_{\mu\nu\lambda} F_{\nu\lambda}$$

$$\partial_1 \sigma \sim E_2$$

$$\partial_2 \sigma \sim -E_1$$

$\oint d\sigma = Q_{el}$: σ has monodromy 2π 'round fundamental charges.

SU(2) $\sigma \simeq \sigma + 2\pi$

$(\vec{\sigma} \simeq \vec{\sigma} + 2\pi \vec{w}_k : G)$

$\uparrow$ weight lattice

K. Lee + P. Yi / Kraan, v. Baal

1990's

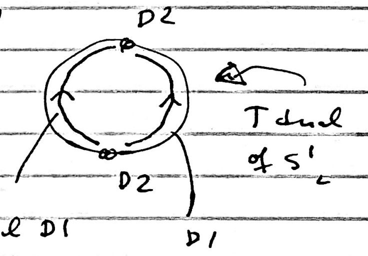
$SU(2) \to U(1)$ • M_1 & M_2 $\qquad (M_1 \ldots M_N \quad SU(N))$

• $A(SD)$; size $\sim \frac{1}{NL}$ $\qquad M_1 \ldots M_{r+1} \quad G$

	Q_m	Q_{top}
M_1	1	1/2
M_2	−1	1/2
BPST	0	1

$Z_2^{(0)}$

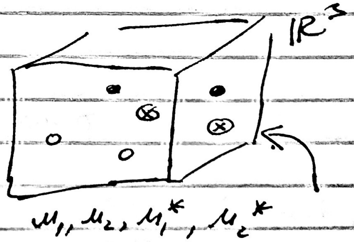
(interchanged by center Z_2)



action: $\frac{8\pi^2}{g_4^2 N} \equiv S_0$

$$M_{1,2} \sim e^{-S_0} e^{i\frac{\theta}{2}} m_W^3$$

+ Coulomb inter'n



$$M_{1,2} \sim m_W^3 e^{-S_0} e^{i\frac{\theta}{2}} e^{\pm i\sigma}$$

very much like Polyakov model

except (for $SU(2)$) 2 kinds of $\mathcal{M}$'s.
(N kinds $SU(N)$)

$\mathcal{M}_1 \quad e^{i\sigma} e^{-S_0} e^{i\theta/2} m_W^3$
$\mathcal{M}_2 \quad e^{-i\sigma} e^{-S_0} e^{i\theta/2} m_W^3$
$\mathbb{Z}_2^{(\sigma)} : \sigma \to -\sigma$

$\mathcal{M}_1^* \quad e^{-i\sigma} e^{-S_0} e^{-i\theta/2} m_W^3$
$\mathcal{M}_2^* \quad e^{i\sigma} e^{-S_0} e^{-i\theta/2} m_W^3$
$\mathbb{Z}_2^{(\sigma)} : \sigma \to -\sigma$

dilute gas:

$\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_1^* + \mathcal{M}_2^*$

$$\frac{g_4^2}{L}(\partial_\mu\sigma)^2 - c\, m_W^3 e^{-S_0}\left(\cos\sigma\left(e^{i\theta/2} + e^{-i\theta/2}\right)\right) + \ldots$$

dYM:

$$\frac{g_4^2}{L}\left[(\partial_\mu\sigma)^2 - m_\sigma^2 \cos\theta/2 \cos\sigma + \ldots\right]$$

$$m_\sigma^2 \sim m_W^2 e^{-\frac{8\pi^2}{g^2 N}}$$

nonpert mass gap.

likewise, ignoring fermions (bosonic $V(\sigma)$ only)

QCD(adj) $\frac{g_4^2}{L}\left[(\partial_\mu\sigma)^2 - \tilde{m}_\sigma^2 \cos 2\sigma\right]$

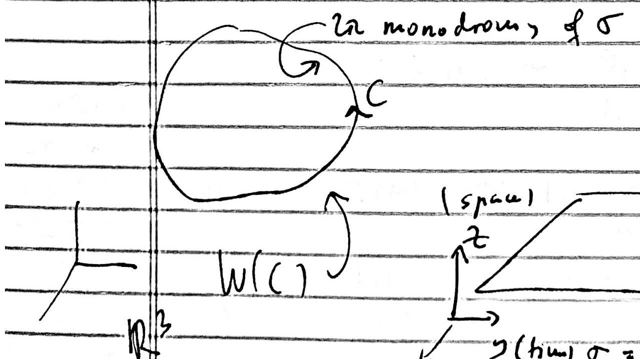
SYM: $\frac{g^4}{L}\left[(\partial_\mu\sigma)^2+(\partial_\mu\phi)^2+\tilde{m}_\sigma^2(\cosh 2\phi - \cos 2\sigma)\right]$

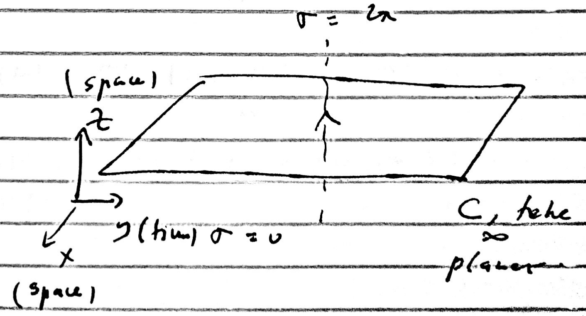
latter have discrete chiral sym:

- $\sigma \to \sigma + \pi$
- requires $\cos(2k\sigma)$

due to 2π shift of θ

Focus on flux tubes:



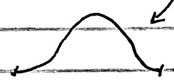


Ex. 11

$\underline{\underline{d\,YM}}$: $\frac{g_Y^2}{L}\left((\partial\sigma)^2 - m_\sigma^2(\theta)\cos\sigma + \dots\right)$

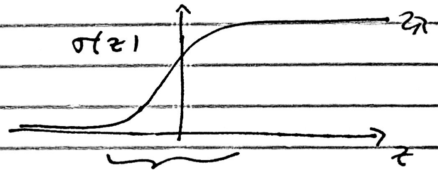
(vacua are equiv.) "DW"

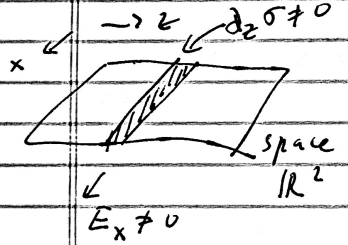
$-\cos\sigma$

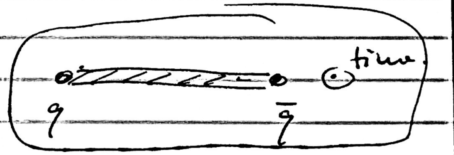


$\sigma = 0$ $\sigma = 2\pi$

= "DW" = string







el. flux in x ; flux = fund qu.

$$\text{DW Tension} \sim \frac{g_Y^2}{L} m_\sigma(\theta)$$

embarrassing typo in ref. 2111. !

↓

fix!

$\Rightarrow SU(2)$ $T(\theta) = T(0)\left(1 - \frac{\theta^2}{16}\right) \simeq T(0)(1 - 0.06\,\theta^2)$

(vs. lattice $SU(3)$ $(1 - 0.08\,\theta^2)$)

Del Debbio '06

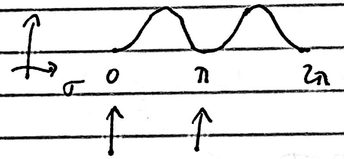
Ex 2. dYM: $\theta = \pi$ | in both

SYM/QCD adj.: | $(\partial\sigma)^2 - \tilde{m}_\sigma^2 \cos 2\sigma$

pot'l.

parity dYM: $\sigma \to \sigma + \pi$
chiral SYM/adj.

$V(\sigma)$

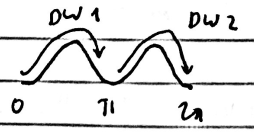


- two vacua $\langle e^{i\sigma} \rangle = \pm 1$

- $\cancel{P}$, $\mathbb{Z}^{\chi}_{4n_f} \to \mathbb{Z}^{\chi}_{2n_f}$; $\mathbb{Z}_2^{(0)}$ unbroken.

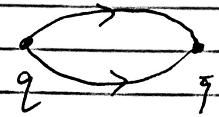
- DW carries ½ flux $\Delta\sigma = \pi$ NOT 2π!

- $\exists$ 2 distinct DW related by $\mathbb{Z}_2^{(0)}$ center.



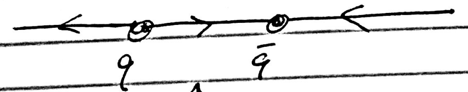
(BPS in SYM)

static pic:



each wall ½ flux

same tension ($\mathbb{Z}_2^{(0)}$!)



QH's deconfined on DW

(! old story. !)

→ here QFT

→ modern parity / $d\chi$ - center anomaly.

→ 2 degenerate DWs between $d\chi$/P-broken vacua

~ TQFT "lives" on DW

on slides G & Blood

here $\mathbb{Z}_2$ - 2d: $\frac{N}{2\pi} \int_{\text{worldvolume}} \phi^{(0)} \, d a^{(1)}$

$(\mathbb{Z}_N)$

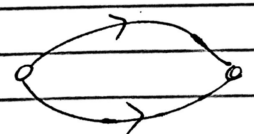
- two (N) states in Hilbert space

- mixed 0-form - 1 form anomaly

(dim red. of $SU(N)_1$ CS on 3d DW worldv.)

for flux tubes : suggests change for $\theta=\pi$
, see.

$\theta=\pi$

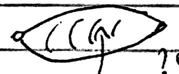
$d\chi$. $\Rightarrow$ 

$\hookrightarrow$ - goldstone + masslers
- breather mode
(theory dep... m_σ on dYM/SYM)
- massive modes (m_σ)

(but maybe lighter in QCD(adj))

show plots.

Conclusions : - $R^3 \times S^1$ remarkable "flesh & blood"
- suggests $\theta=\pi$ & $d\chi$ things different

on R^4  ?cartoon?

other vac inside

- $L \to \infty$?

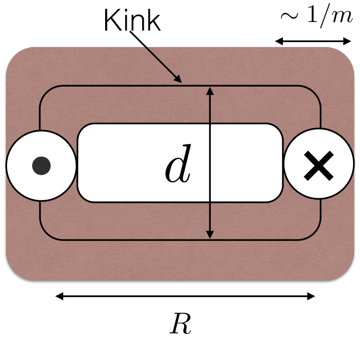
? - Ideas about TQFTs on DW -
- w/ or w/out mix anomaly/center/ all gauge groups SYM!

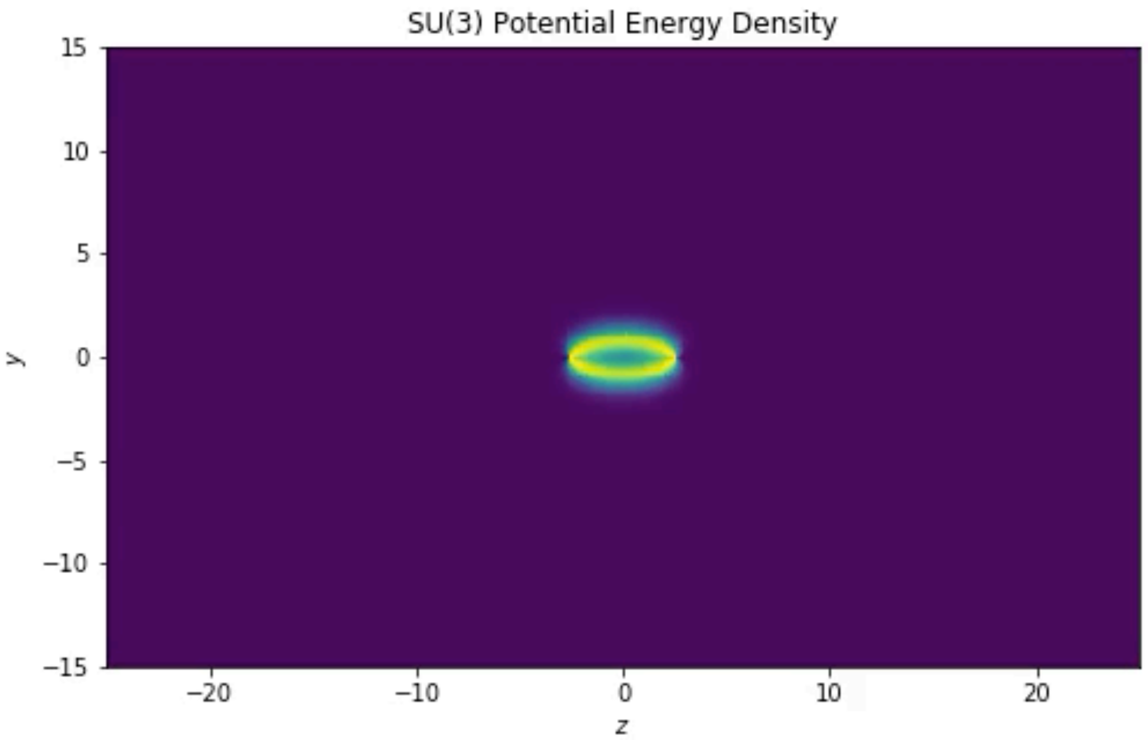
String separation

Using some naïve assumptions about domain wall repulsion and the double string geometry, can obtain a logarithmically growing string separation [Anber, Poppitz, & Sulejmanpašić (2015)].

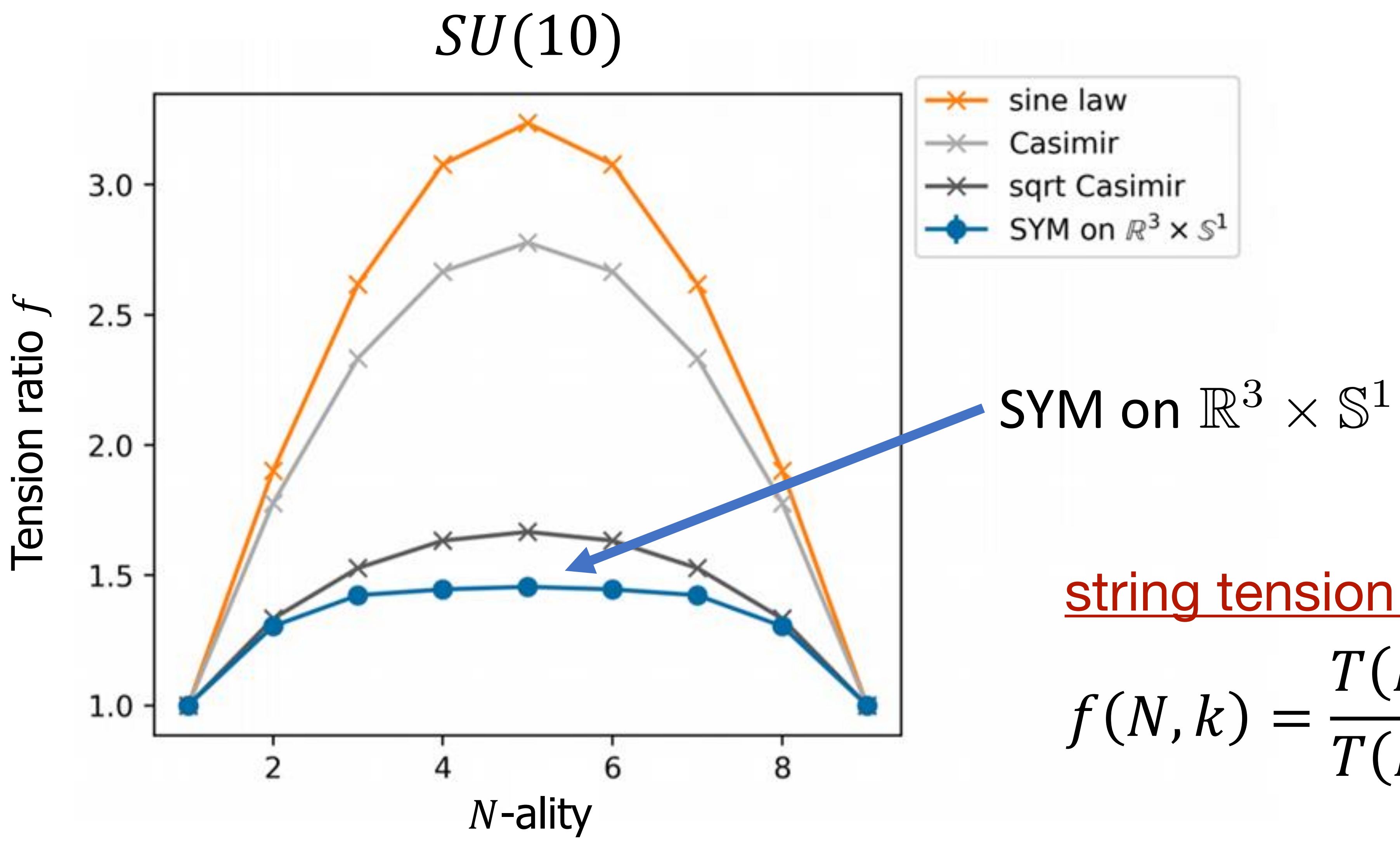
$$d \sim \frac{1}{m}\log(mR)$$

$$E \sim T(R+d) + TRe^{-md}$$





String tensions and N-ality dependence from 2010.04330 w/ Bub and Wong

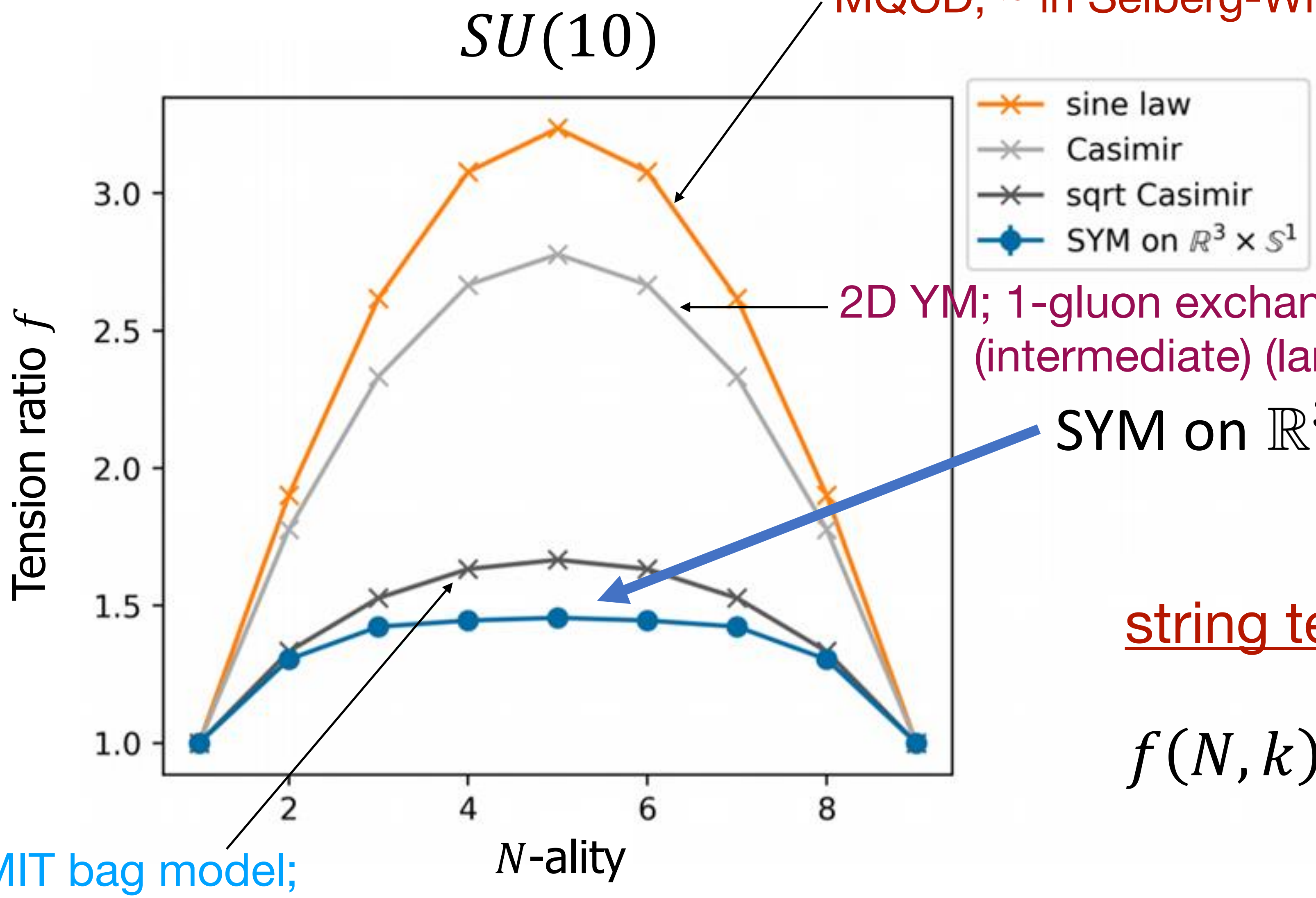


string tension ratio

$$f(N,k) = \frac{T(N,k)}{T(N,1)}$$

String tensions and N-ality dependence

MQCD; ~ in Seiberg-Witten; (small-m) 2D QCD(adj)



2D YM; 1-gluon exchange; ~ lattice strong-coupling;
(intermediate) (large-N) … 4D YM

SYM on $\mathbb{R}^3 \times \mathbb{S}^1$

MIT bag model;
approximate in deformed-YM $\theta \neq \pi$

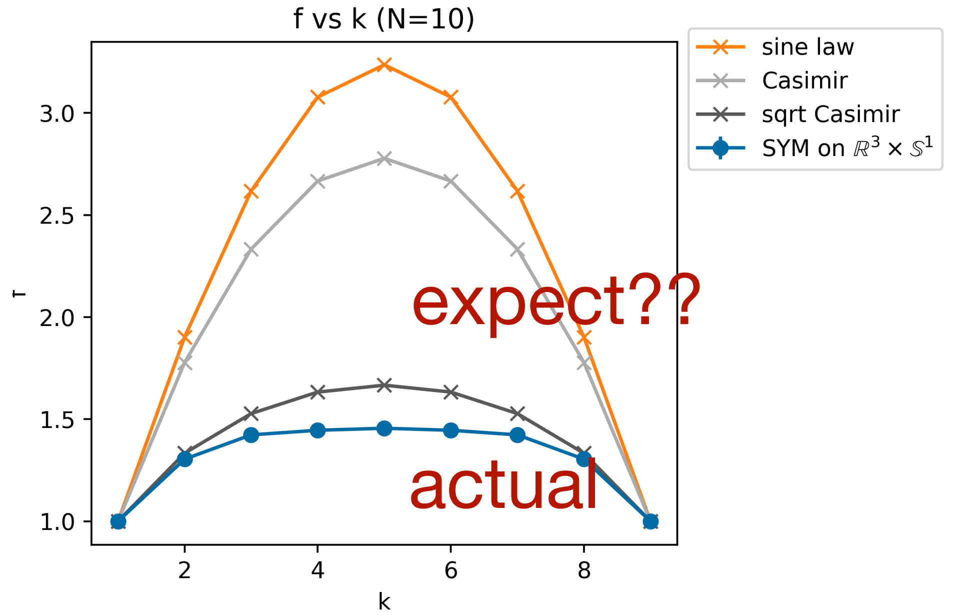
string tension ratio

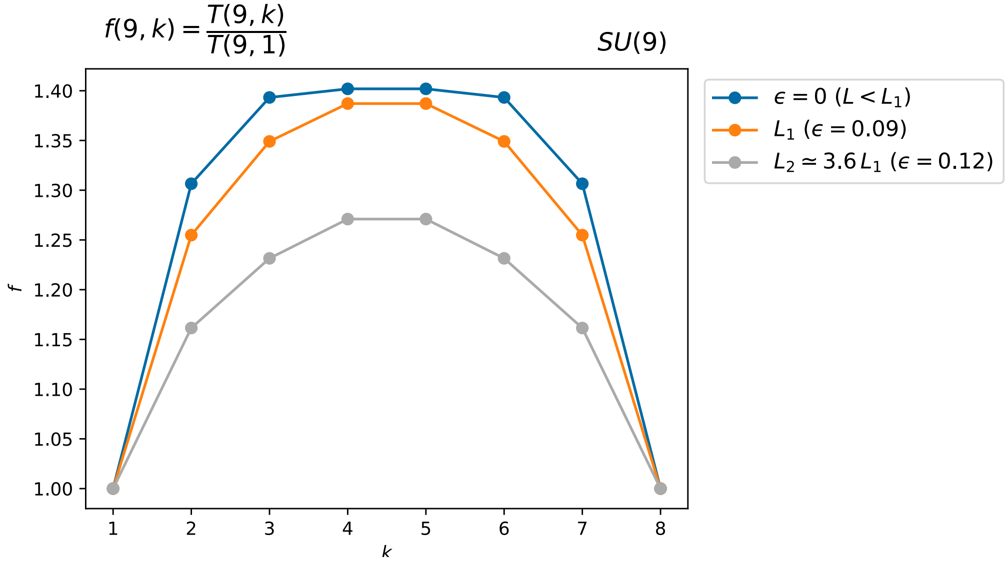
$$f(N,k) = \frac{T(N,k)}{T(N,1)}$$

String tensions and N-ality dependence

how do T(N,k) and f(N,k) behave as L increases?

$$T_{(N,k)} = .675\Lambda^2 \frac{\Lambda L N}{4\pi} \tilde{T}_{(N,k)}(\epsilon)$$





Thank you!

“*Color field,*” Mark Rothko (MoMA)

study the colour field