Higher form symmetries and large-N confinement in gauge/gravity duality

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KITP Confinement, Flux Tubes, and Large N
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Outline

Magical features of gravity.

Center symmetry in gauge theory/gravity.

Eguchi-Kawai mechanism reproduces magical features.

Miscellaneous lessons about Eguchi-Kawai and gravity.

Gravity as phase of matter

Given holography, can gravity be thought of as a phase of matter?

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Landau paradigm requires existence of symmetries and pattern of symmetry breaking:

liquid $\to$ ice

paramagnet $\to$ ferromagnet

Bose gas $\to$ superfluid

Universality in holography

Local Einstein-like AdS/CFT dualities require special CFTs:

- Large N
- Strong interactions
- CFT is almost always some sort of gauge theory
- Universality of free energy [Hartman, Keller, Stoica; Belin, de Boer, Kruthoff, Michel, ES, Shyani]
- Large dimensions for single-trace higher-spin operators [Heemskerk, Penedones, Polchinski, Sully]
- Correlators under bulk spacetime quotients obtained by method of images
- …

Symmetry principle must require or explain some of the above.

Black hole entropy

Black hole entropy in Einstein gravity:

$$S = \frac{A}{4G}$$

Quotient spacetime such that the horizon $\mathcal{M}^{d-1}$ transforms:

$$\mathcal{M}^{d-1} \to \mathcal{M}^{d-1}/\Gamma \qquad \Longrightarrow \qquad S \to S/|\Gamma|$$

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Not true for non-geometric quantum corrections:

$$S \to S + \log S$$

Correlation functions

Connected two-point function at finite size:

$$\langle O(x,\vec{y})O(0)\rangle_{S^1_L\times\mathcal{M}^{d-1}} = \sum_{n=-\infty}^{\infty}\langle O(x+nL,\vec{y})O(0)\rangle_{\mathbb{R}\times\mathcal{M}^{d-1}}\,,$$

Also true for n-point functions. Follows from method of images.

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Entanglement entropy in e.g. holographic CFT$_2$ on $S^1_\beta\times S^1_L$:

$$S_{EE} = \begin{cases} \frac{c}{3}\log\left(\frac{L}{\epsilon}\sin\frac{\pi\ell}{L}\right) & L<\beta \\ \frac{c}{3}\log\left(\frac{\beta}{\epsilon}\sinh\frac{\pi\ell}{\beta}\right) & L>\beta \end{cases}$$

Proposal

Center symmetry and general higher-form symmetries in QFT characterize the gravitational phase through their pattern of breaking.

And they imply strings/branes.

Center symmetry in gauge theory

$$A_\mu \to g A_\mu g^{-1} + g \partial_\mu g^{-1}\,, \qquad g : \mathcal{M}^{d-1} \times S^1_\beta \to SU(N)$$
$$g(x, \tau + \beta) = g(x, \tau) h \qquad \text{for} \qquad h \in \mathbb{Z}_N$$

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Polyakov loop transforms as

$$W(C) = \mathrm{Tr}_F P \exp\left(\oint A_\mu dx^\mu\right) \to h W(C)\,,$$

so $\langle W(C)\rangle$ is an order parameter for center symmetry.

Diagnoses deconfinement transition. It is a 1-form symmetry.

Center symmetry in gravity

In gauge-gravity duality, $\langle W(C) \rangle$ measured by classical string worldsheet with disk topology ending on C. Non-contractible cycle $\implies \langle W(C) \rangle = 0$.

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In gauge-gravity duality, $\langle W(C) \rangle$ measured by classical string worldsheet with disk topology ending on C. Non-contractible cycle $\implies \langle W(C) \rangle = 0$.

In smooth gravitational description, only one cycle can cap off in the bulk, so center preserved along all but one cycle.

e.g. bulk phases of holographic CFT_2 on $S^1_\beta \times S^1_L$:

$$ds^2 = \begin{cases} (1+r^2)d\tau^2 + \frac{dr^2}{1+r^2} + r^2 d\phi^2 \,, & S^1_\beta \times D_L \,, \quad \langle W_\beta \rangle = 0, \quad \langle W_L \rangle \neq 0 \\ (r^2 - r_+^2)d\tau^2 + \frac{dr^2}{r^2 - r_+^2} + r^2 d\phi^2 \,, & D_\beta \times S^1_L \,, \quad \langle W_\beta \rangle \neq 0, \quad \langle W_L \rangle = 0 \end{cases}$$

Hawking-Page transition = deconfinement transition. [Witten]

Eguchi-Kawai mechanism (large-N volume independence)

For large-N center-symmetric gauge theory on $\mathcal{M}^{d-n} \times \mathbb{T}^n$, if translation symmetry and center symmetry are preserved along an $S^1 \in \mathbb{T}^n$, then appropriate observables are independent of the size of the S^1 at leading order in N. [Eguchi, Kawai; Kovtun, Unsal, Yaffe]

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Helps large-N systems mimic large-volume thermodynamics.

Entropy and free energy

Assuming symmetry breaking pattern for holographic CFT_2, we find

$$\frac{\log Z}{\beta L} = \begin{cases} cL^{-2}\,, & \text{for } \beta > L \qquad (\text{used } \langle W_\beta \rangle = 0\,, \quad \langle W_L \rangle \neq 0) \\ c\beta^{-2}\,, & \text{for } \beta < L \qquad (\text{used } \langle W_\beta \rangle \neq 0\,, \quad \langle W_L \rangle = 0) \end{cases}$$

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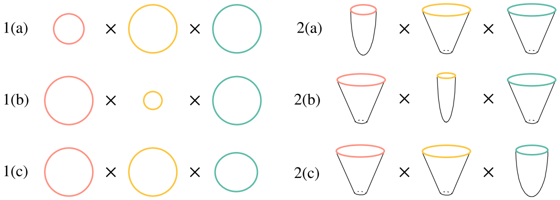
Argument extends to $SL(2,\mathbb{Z})$ family of black holes and higher dimensions:



Euclidean gravity a la Gibbons and Hawking

For spacetimes with a non-contractible S^1_β:

$$\log Z(\beta) \propto \beta \implies S(\beta) = (1 - \beta\partial_\beta) \log Z(\beta) = 0$$

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Quantum-mechanical dual is preservation or breaking of center: non-contractible $S^1_\beta \implies \langle W_\beta \rangle = 0 \implies \log Z(\beta) \propto \beta$.

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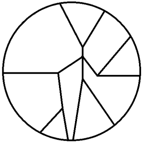
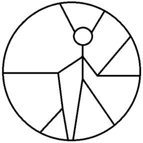
This is precisely what you get in BTZ or thermal AdS:

$$\langle O(t,\phi)O(0,0)\rangle_{\mathrm{BTZ}} = \sum_{n=-\infty}^{\infty} \left[\cosh\left(\frac{2\pi t}{\beta}\right) - \cosh\left(\frac{2\pi(\phi+nL)}{\beta}\right)\right]^{-2\Delta}$$

Extends to n-point functions:





Quotients of curved manifolds

Gravity manifests "topological volume independence": e.g. CFT_4 on $S^1 \times S^3/\mathbb{Z}_p$ independent of p in deconfined phase.

Phase structure between Eguchi-Hanson soliton and quotient black hole.

Introducing geometry through center symmetry

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Pattern of symmetry breaking also important, e.g. $\mathcal{N} = 4$ SYM at weak coupling.

Higher-form symmetries and strings/branes

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More generally a higher-form symmetry is relevant, e.g. 2-form symmetry of $\mathcal{N} = (2, 0)$ theory in 6d. Dual to M2-branes.

Gravity as phase of matter

Diagnose gravity as a phase of matter through pattern of higher-form symmetry breaking.

Requires strong coupling, large N.

Predicts universal large-N properties consistent with gravity.

Also implies strings/branes behind the scenes!

Helps explain the *gauge* in gauge-gravity duality.