Soliton Vortex Strings in Yang-Mills

A journey from vortex string to fundamental strings

M. Shifman

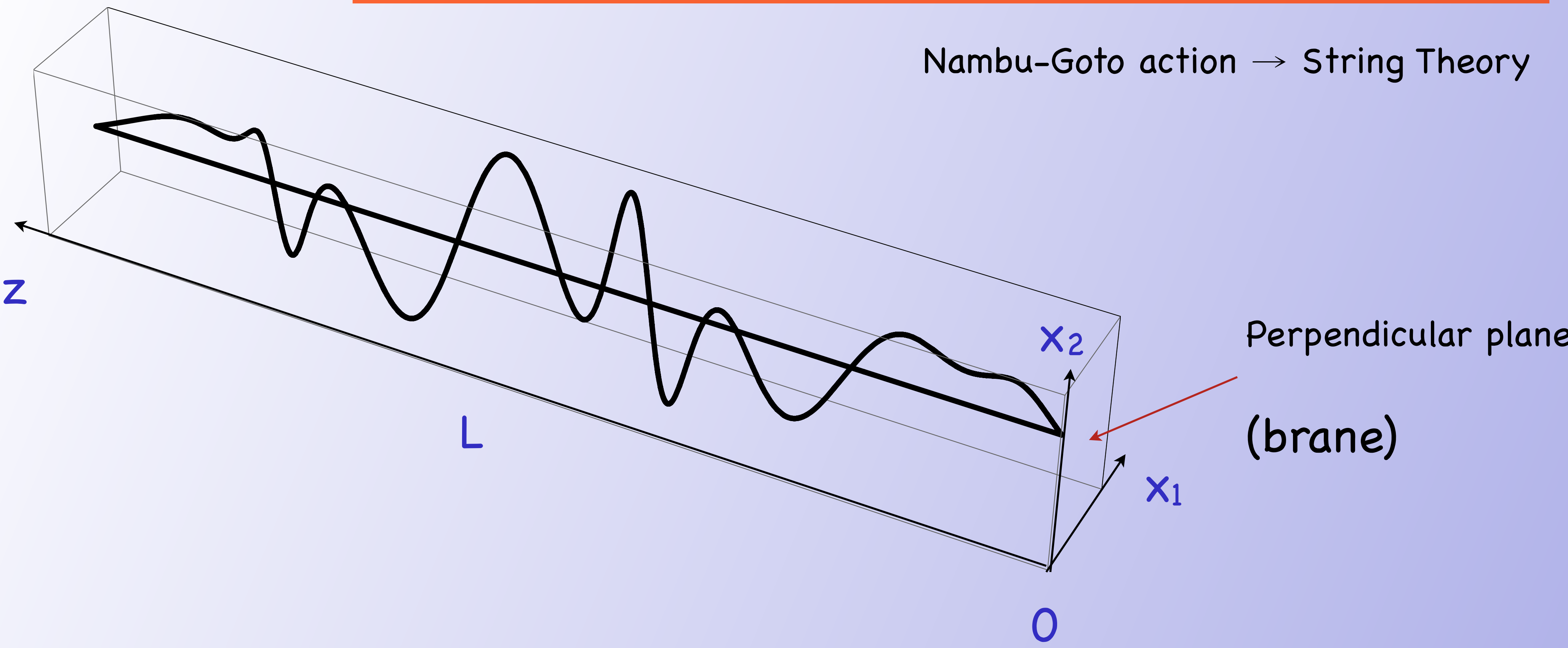
W.I. Fine Theoretical Physics Institute, University of Minnesota

(Mostly with A. Yung)

KITP Workshop on Flux Tubes (1/25/22)

Giant Swirl Phenomenon, Crater Lake National Park, Oregon

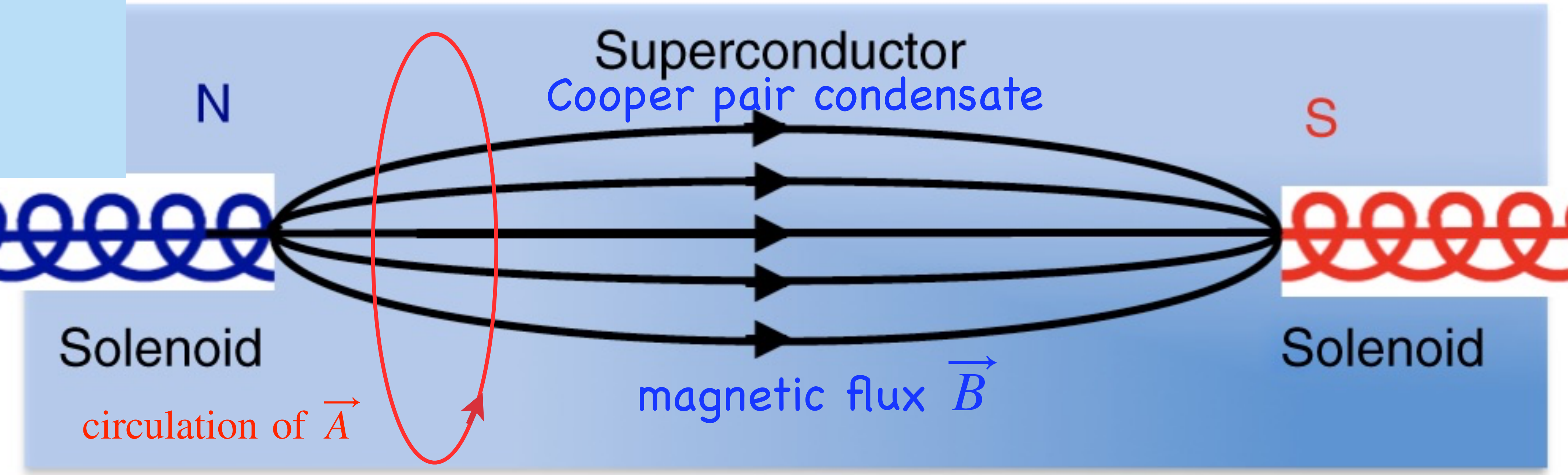
Fundamental ("classical") string theory



Infinitely **thin** string, **fully** defined by its coordinates

Critical in 10 dimensions : **Non-critical string → this program**

4D EFT: Expanding in derivatives, adding some terms, string bootstrap data,...



Abelian ☚

The most primitive Abrikosov (ANO) vortex (flux tube)

Superconductor of the 2nd kind

☞ Non-Abelian vortex strings

flavor index $A = 1{,}2$

$$\mathscr{L} = -\frac{1}{4g_2^2}\left(F^a_{\mu\nu}\right)^2 - \frac{1}{4g_1^2}\left(F_{\mu\nu}\right)^2 + \left(\mathscr{D}_\mu\phi^A\right)^*\left(\mathscr{D}_\mu\phi^A\right)$$

$SU(2)$ $\quad$ $U(1)$

$$-\frac{g_2^2}{2}\left(\phi_A^*\frac{\tau^a}{2}\phi^A\right)^2 - \frac{g_1^2}{8}\left[(\phi_A^*)(\phi^A) - 2v^2\right]^2$$

$$\mathscr{D}_\mu\phi = \partial_\mu\phi - \frac{i}{2}A_\mu\phi - \frac{i}{2}A^a_\mu\tau^a\phi$$

$$\Phi = \begin{pmatrix} \phi^{11} & \phi^{12} \\ \phi^{21} & \phi^{22} \end{pmatrix}$$

$$U(\Phi, \Phi^\dagger) = \frac{g_2^2}{2} \mathrm{Tr}\left(\Phi^\dagger \frac{\tau^a}{2} \Phi\right) \mathrm{Tr}\left(\Phi^\dagger \frac{\tau^a}{2} \Phi\right) + \frac{g_1^2}{8} \left[\mathrm{Tr}\left(\Phi^\dagger \Phi\right) - 2v^2\right]^2$$

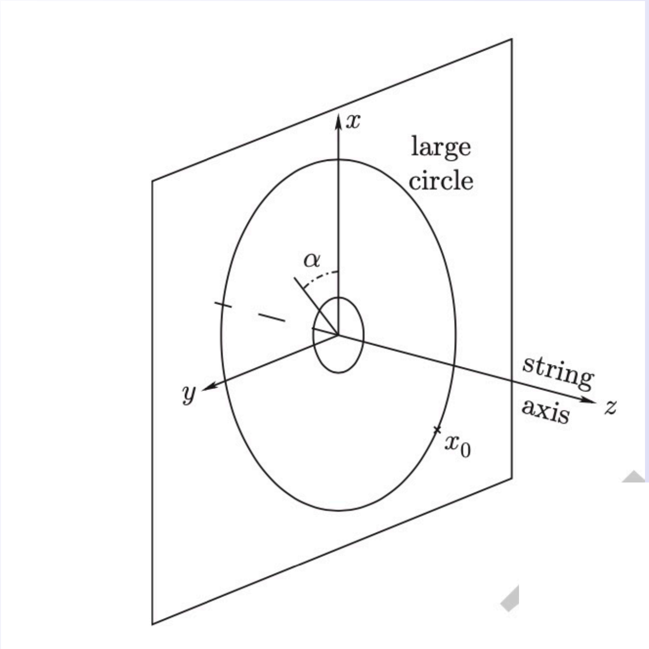
$$SU(2)_{\mathrm{color}} \times SU(2)_{\mathrm{flavor}} \longrightarrow SU(2)_{\mathrm{diag\ global}}$$

$$\Phi_{\mathrm{vac}} = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}$$

Color-flavor locking

Vortex Strings

$\pi_1(U(1)) = Z; \quad \Phi(x) = ve^{i\alpha(x)}, \quad A_i = -2\varepsilon_{ij}\frac{x_j}{r^2}, \quad i,j = 1,2, \quad |x|_{perp} \to \infty$

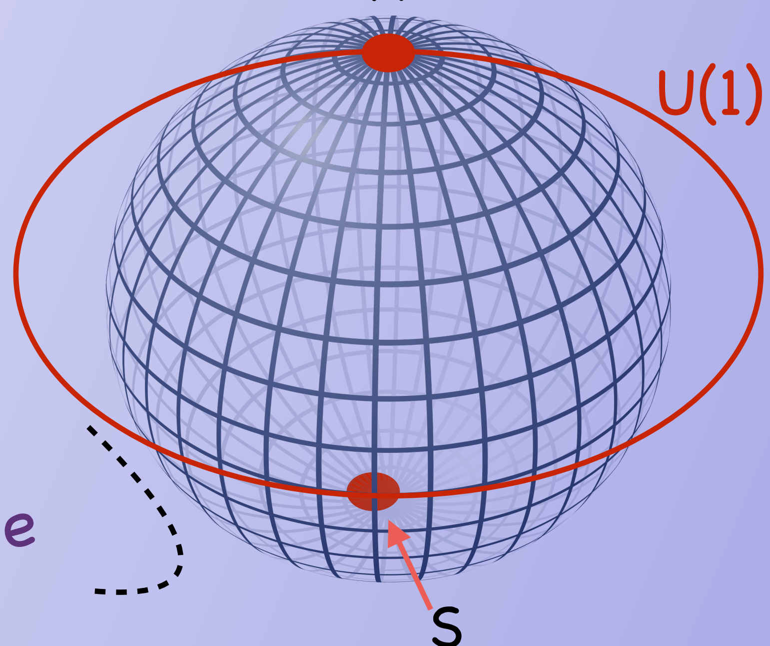


NAS solution breaks $SU(2)_{global}$ to SU(2)/U(1) = CP(1)

$T_{ANO} = 4\pi v^2$

$T_{NAS} = 2\pi v^2$

U(1) & SU(2) have common center Z_2



$\pi_1(S_2 \times U(1)/Z_2) = Z_2;$

$$\Phi(x) = \begin{pmatrix} ve^{i\alpha} & 0 \\ 0 & v \end{pmatrix}, \quad A_i = -\varepsilon_{ij}\frac{x_j}{r^2},$$

Move along the meridian from pole to pole then in U(1)

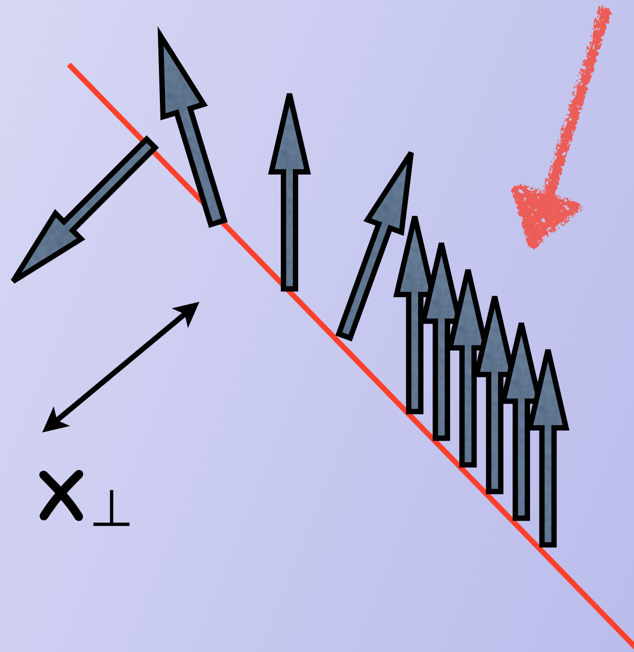
2003: Hanany, Tong
Auzzi, Yung, et al.
Yung + M.S.

CP(N-1) **N** =2 sigma model

$4D$ bulk, **N** =2 Yang-Mills
with N flavors & N colors

+ FI

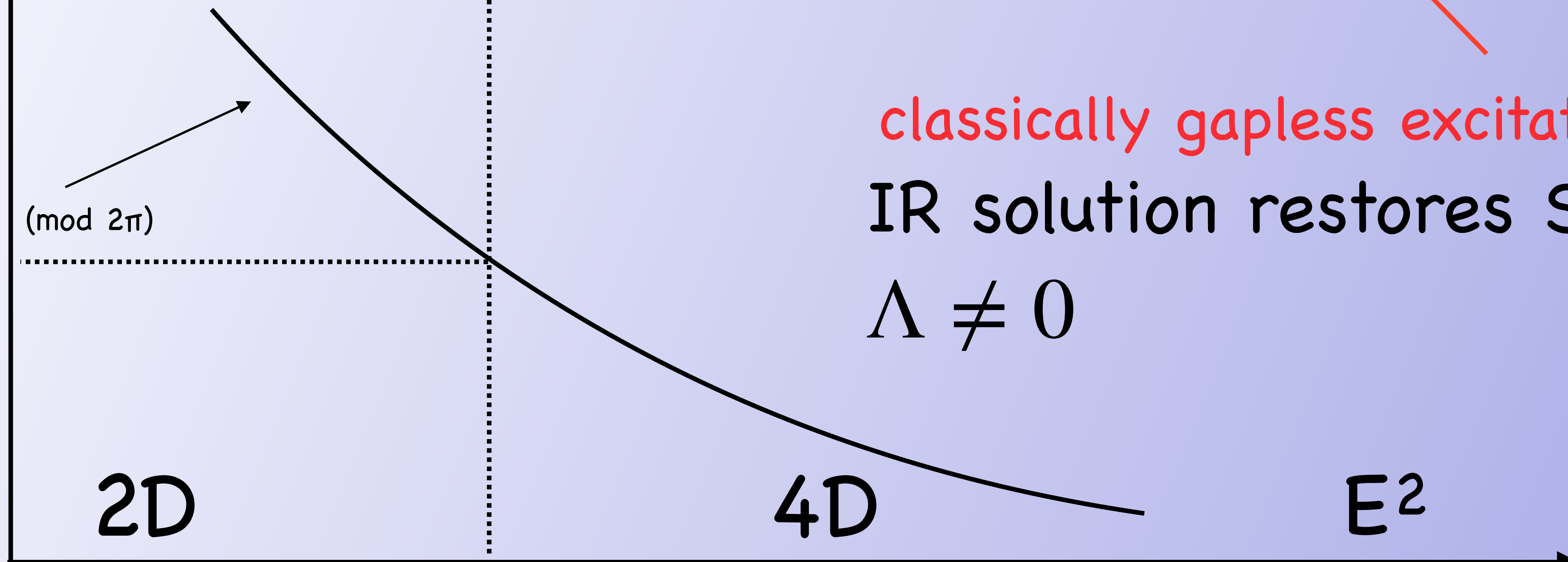
Non-Abelian internal d.o.f



classically gapless excitations

IR solution restores SU(2),

$\Lambda \neq 0$



In 2D $CP(N-1)$ model on the string we have

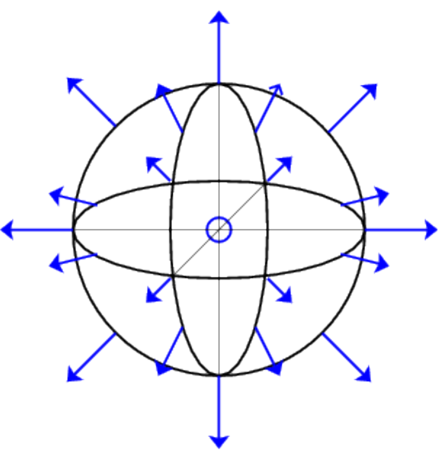
N vacua $= N$ Z_N strings

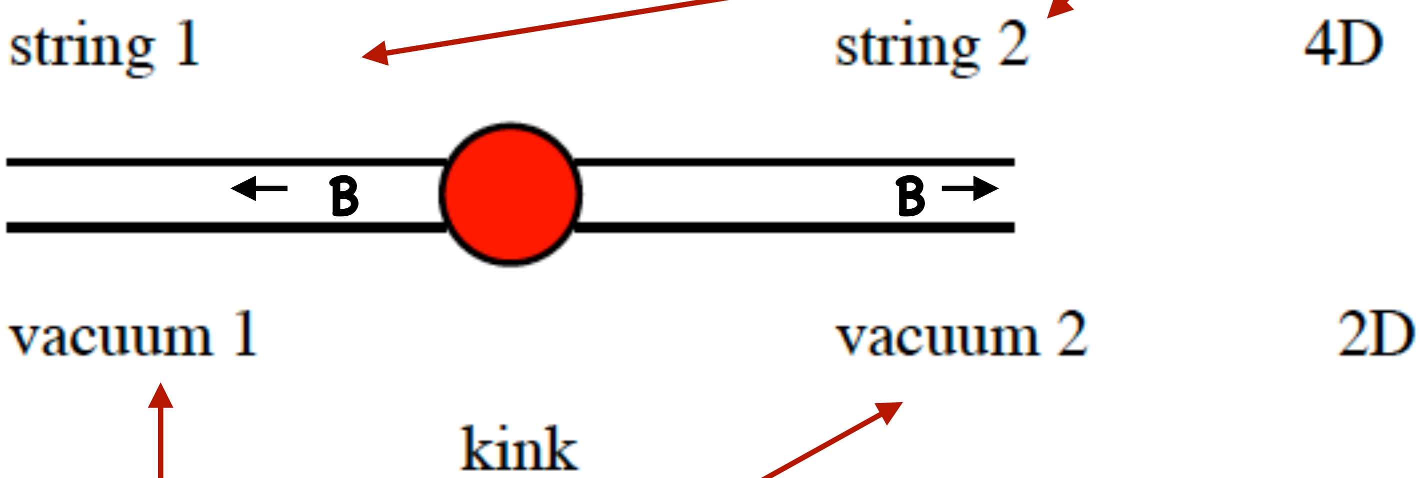
and kinks interpolating between these vacua

Kinks = confined monopoles

Yung + M.S.
Hanany, Tong

Degenerate because BPS saturated





Degenerate because SUSY vacua

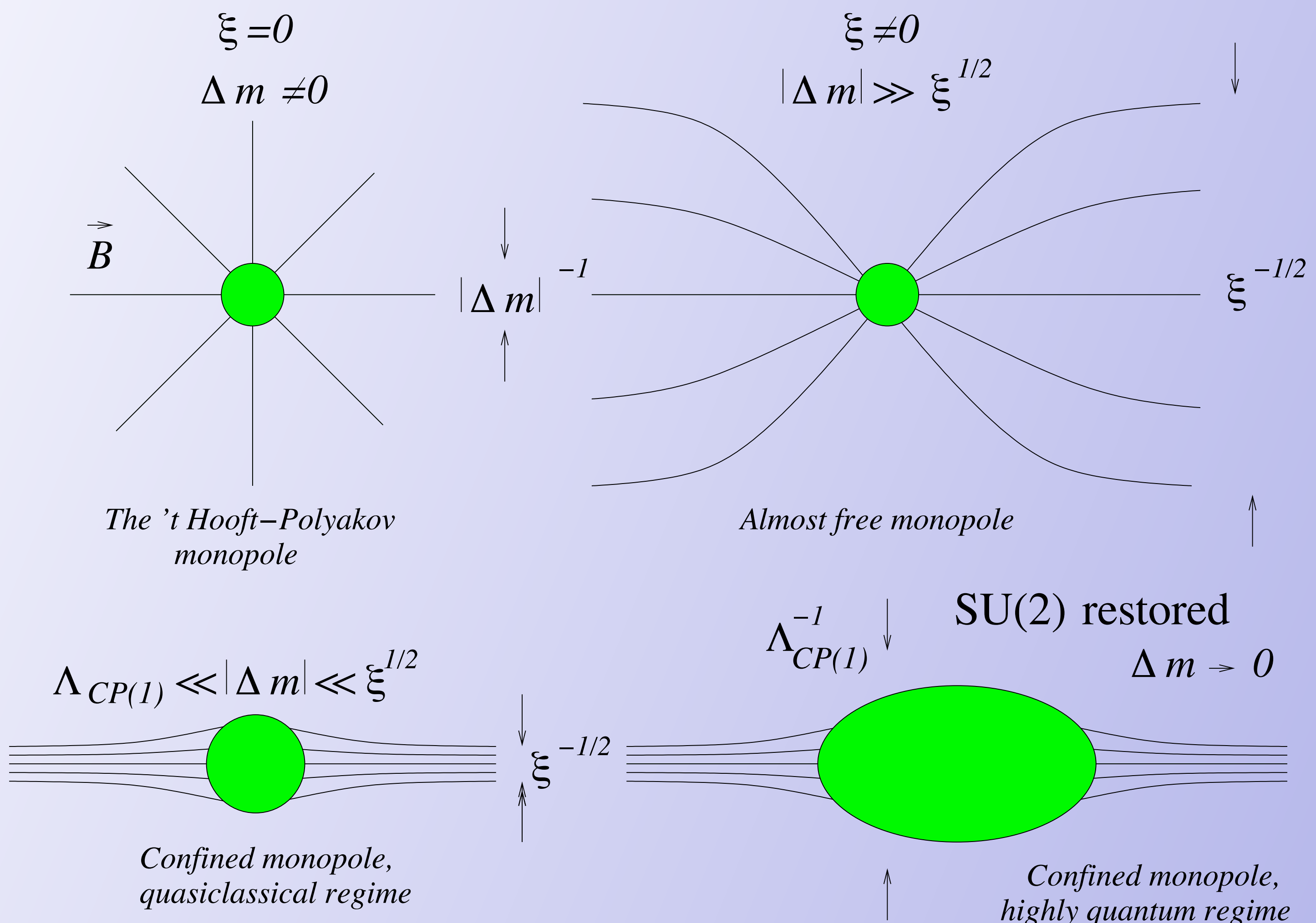
* Kinks are confined in 4D (attached to strings).
* Why?

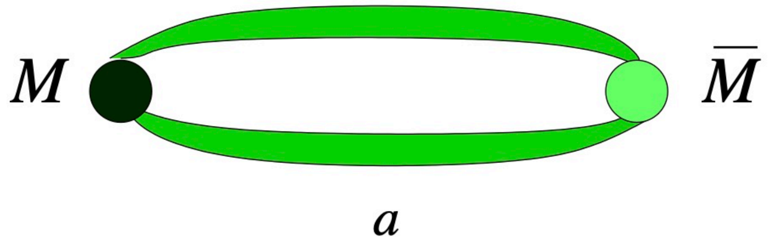
$$M_{2D\,k} = M_{4D\,,mon} = \left| \frac{\mu}{\pi} \left(\frac{1}{2} \ln \frac{\sqrt{\mu^2 + 4\Lambda^2} + \mu}{\sqrt{\mu^2 + 4\Lambda^2} - \mu} - \sqrt{1 + \frac{4\mu^2}{\Lambda^2}} \right) \right|$$

$$\mu \equiv 2m$$

4D ↔ 2D correspondence

$$\xi \equiv v^2$$





$\mathcal{N}=2$ theory confinement IF $(\bar{M}M)$
is in the adjoint of $\mathrm{SU}(N_f)$

Non-Abelian vortex strings are supported in 4D Yang-Mills theories with matter with $\mathcal{N}$=2,1, and 0 supersymmetry.

In QCD they are supported at hight density (chemical potential);

In condensed matter physics

$$\text{B phase He}^3 = SO(3)_{S+L}.$$

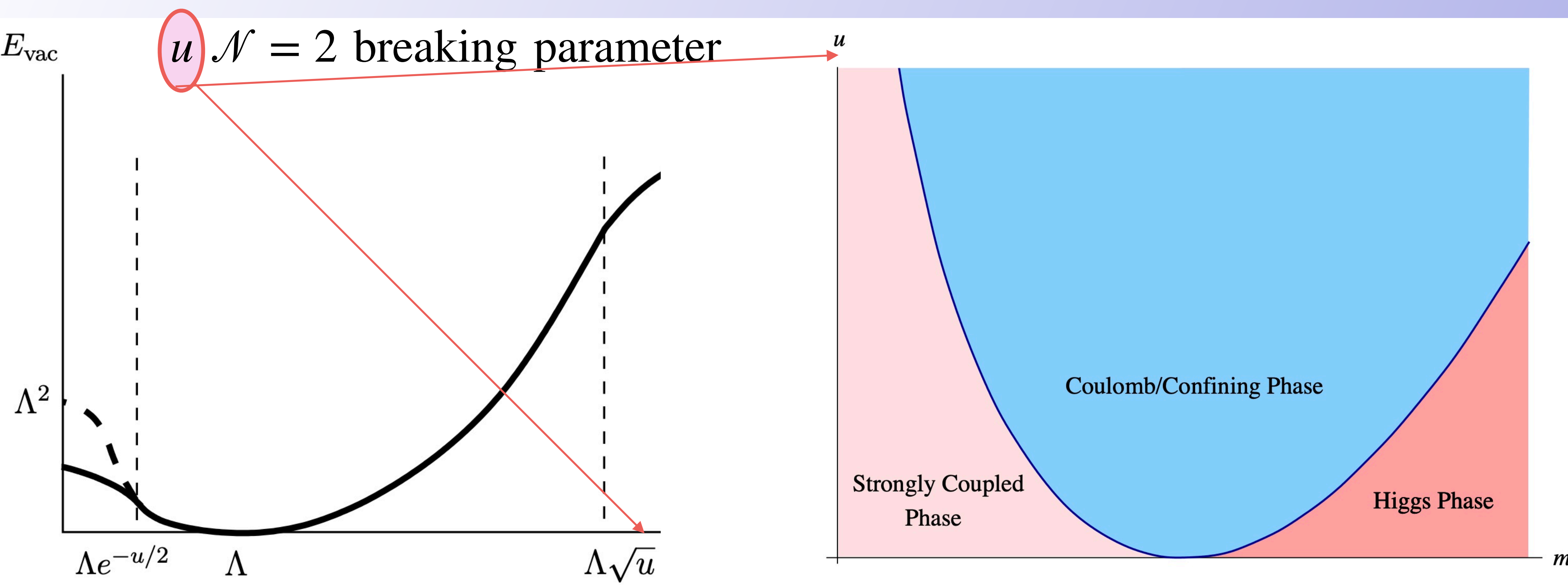
Free parameters:

$\mathcal{N}$

N, N_f, matter mass terms **(g^2 in conformal)**

A variety of interesting 2D models on the world sheet discovered, with various phase transitions, including SUSY breaking, e.g. :

(i) CP(N) non-minimal heterotic models
(ii) CP(1) minimal (0,2) model with N_f flavors,...



n energy density vs. m. The dashed line shows tl
rapolated into the strong coupling region.

Figure 8: The phase diagram of the twisted-mass deformed heterotic CP(N
Variable u denotes the amount of deformation, $u = \frac{8\pi}{N}|\omega|^2$. The phase transitic
are determined by Eq. (6.28).

$$G\left\{\partial^\mu\phi\partial_\mu\phi^\dagger + i\psi_L^\dagger \overleftrightarrow{D}_{RR}\psi_L \quad + i\sum_{k=1}^{N_f}(\psi_{Rk})^\dagger D_{LL}\psi_{Rk} \quad\right\}$$

$$\beta(g^2) = -\frac{g^4}{2\pi}\frac{1-\frac{N_f\, g^2}{4\pi}}{1-\frac{g^2}{4\pi}}$$ ← second order

$$= -\frac{g^4}{2\pi}\frac{1+\frac{N_f\gamma}{2}}{1-\frac{g^2}{4\pi}},$$ ← exact NSVZ; γ negative

If $\frac{g_*^2}{2\pi} = \frac{2}{N_f} \ll 1$ the Banks-Zachs conformal window!

With X. Cui

Ten-dimensional critical string as a soliton in four-dimensional super-Yang-Mills theory

4D bulk, $\mathcal{N}=2$ Yang-Mills

with N_f flavors and N colors;

U(N) gauge group

8 supercharges;

supports 1/2 BPS

"non-Abelian" vortices

For long time

$N_f=N$

$\Rightarrow$ AF CP(N-1)

UV incomplete

If N_f = 2N, then

bulk theory "CONFORMAL" (mod ξ)

n^A, ρ^A and x form **10D** target space $\mathbb{R}_4 \times Y_6$

$$S_1 = \int d^2\sigma \sqrt{h}\left\{h^{\alpha\beta}\left(\tilde{\nabla}_\alpha \bar{n}_P \nabla_\beta n^P + \nabla_\alpha \bar{\rho}_K \tilde{\nabla}_\beta \rho^K\right) + \frac{e^2}{2}\left(|n^P|^2 - |\rho^K|^2 - \beta\right)^2\right\} + \text{fermions},$$

$$\nabla_\alpha = \partial_\alpha - iA_\alpha, \qquad \tilde{\nabla}_\alpha = \partial_\alpha + iA_\alpha$$

Weighted CP(2,2) model

Non-compact Calabi-Yau, Ricci-flat!!!!!

Six-dimensional $\to$ 4+4-2=6

World-sheet theory → WCP(2,2): Verification:

Virasoro central charge (including ghosts) = 0

↓

ghost

$$c_{\rm Vir} = \frac{3}{2}\left(D + \frac{2}{3}c_{\rm WCP} - 10\right),$$

$$c({\rm WCP}(N,N)) = 3(N+N-1)$$

↓

N=2

- arXiv:1502.00683

At selfdual point (strong coupling) weighted CP (2, 2) admits deformation of complex structure → a single massless hypermultiplet in the bulk. We interpret it as a composite "baryon"   [Q(U(1))=2]

$$\ell^{-2} \to \xi \times \begin{cases} g^2, & g^2 \ll 1 \\ \infty, & g^2 \to 4\pi \\ 16\pi^2/g^2, & g^2 \gg 1 \end{cases}$$

β = 0 in the self dual point;

Target space develops conical singularity!

- $U(1)_B$ in the bulk is unconventional

Take $U(1)$ from $U(2)_{gauge}$;

- Define $U(1)_{flavor}$ as a $U(1)$ rotation of f_1 and f_2 in one direction & f_3 and f_4 in opposite;
- $U(1)_{gauge} \times U(1)_{flavor} \rightarrow U(1)_{diag} = U(1)_B$

Unbroken by $\langle f_1 \rangle$, $\langle f_2 \rangle$

On the world sheet

	n	ρ
$U(1)_g$	-1/2	1/2
$U(1)_f$	1/2	1/2
$U(1)_{diag}$	0	1

Deformed conifold par.

$w \sim n \times \rho$; $w^2 \sim b$; $Q_B(b) = 2$

IIA String on non-compact Calabi-Yau:

Must be normalizable

2) No spint-2 massless graviton ◀   4D → 6D

3) Spin-0 massless "baryon" ◀

3a) From supergravity, arXiv:1605.08433

3b) From 2D FT and 2D-4D correspondence, with Ievlev, 2006.12054

4) Massive states from little strings

Massive excitations

- **Non-critical $c = 1$ string** (with Liouville, e.g. Kutasov et al.)

Hadrons of N = 2 supersymmetric QCD in four dimensions from little string theory, 1805.10989

Spin-2 states

$$(M^G)^2_{j,l} = 8\pi T \left(l^2 + \frac{1}{4} \right)$$

Hadrons of N = 2 supersymmetric QCD in four dimensions from little string theory, 1805.10989

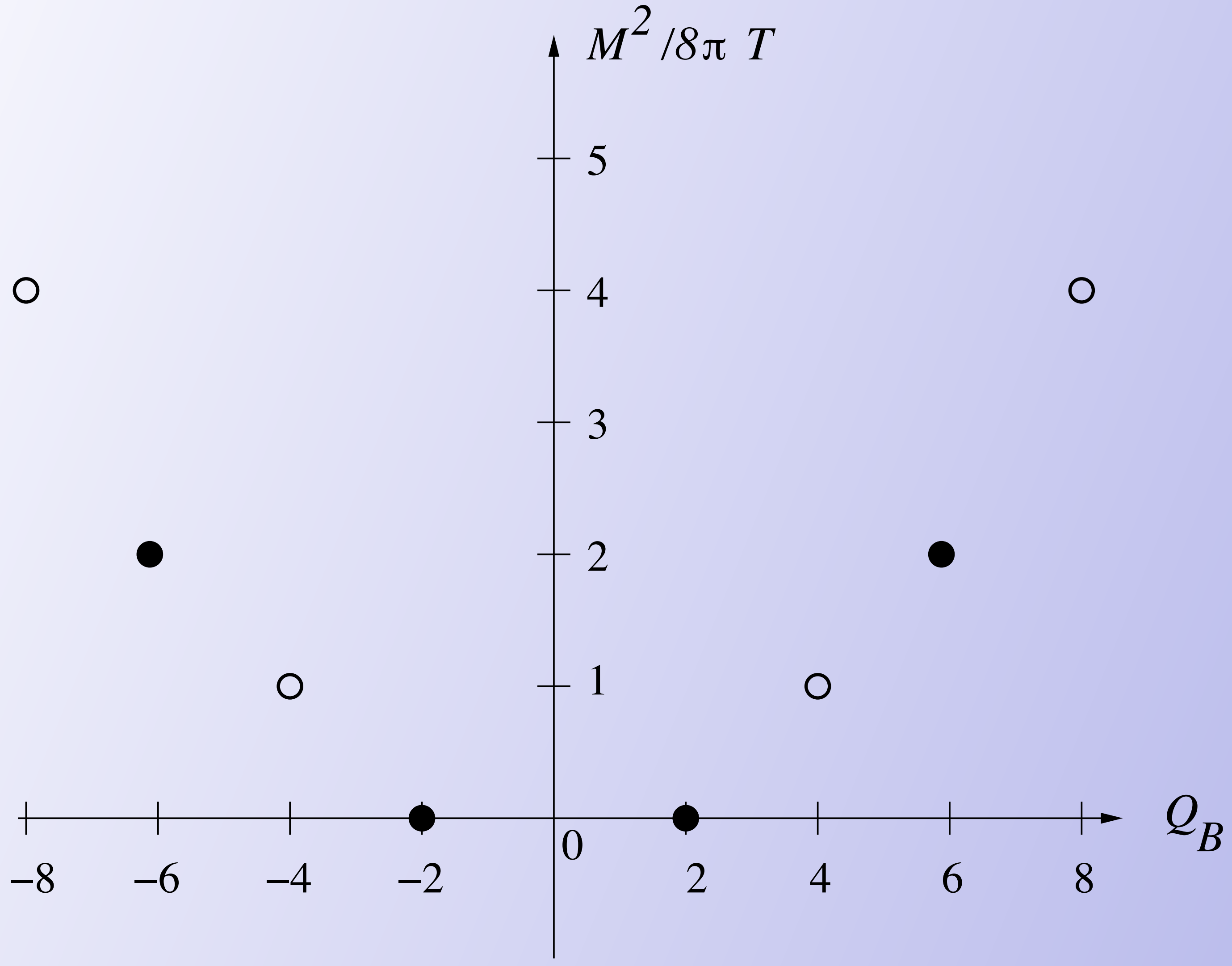


Figure 1: Spectrum of spin-0 and spin-2 states as a function of the baryonic charge. Closed and open circles denote spin-0 and spin-2 states, respectively.

From: 1805.10989

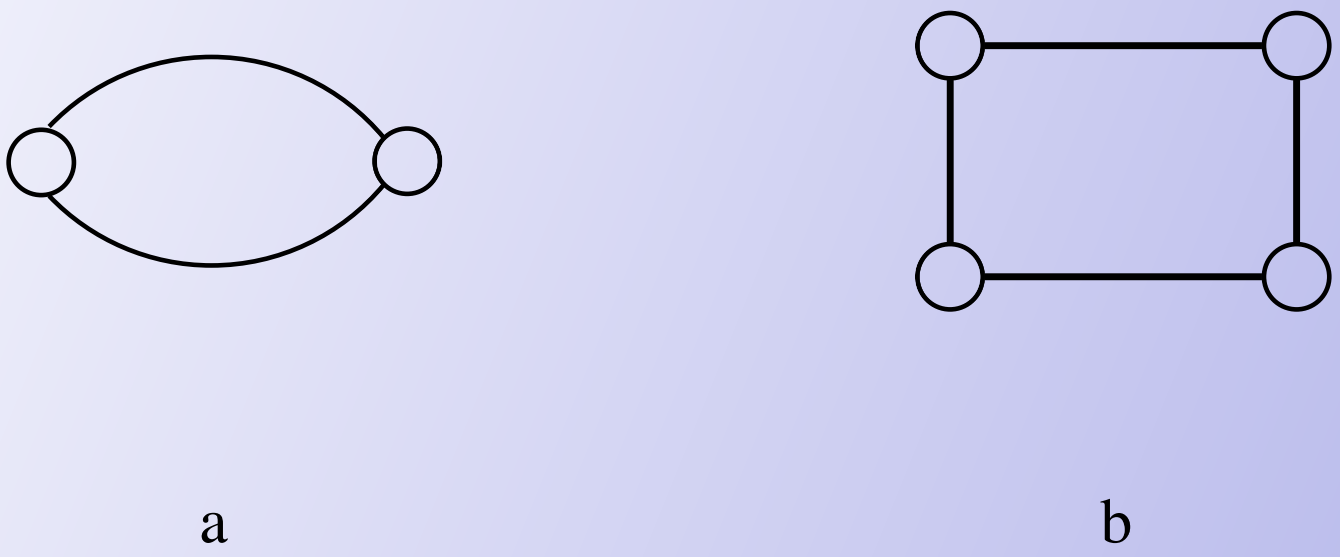


Figure 2: Examples of the monopole "necklace" baryons: a) Massless b-baryon with $Q_B = 2$; b) Spin-2 baryon with $Q_B = 4$. Open circles denote monopoles.

Conclusions:

☞ Non-Abelian vortex strings in $\mathcal{N} = 2$ YM with U(N) gauge group, judiciously chosen matter and FI parameter confine

☞ In 4D with U(2) gauge, FI term, and four quark flavors a critical string is supported. It lives on 6D non-compact CY manifold. Excitation spectrum (or a part of it) can be (and was found)..

Reverse Holography?