Microsoft Research

# Gapped and gapless spin liquid phases on the Kagome lattice from chiral three-spin interactions

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Michele Dolfi (ETH Zurich)

# Today's plan

Intuitive picture for emergent phases in a network of edge states

Numerical evidence for a gapped and a gapless spin liquid in a local SU(2)-invariant spin-1/2 system on the Kagome lattice

# A very short overview

## *Gapped topological SL*

- Two flavors: ***broken*** or ***unbroken*** time reversal (chiral - non-chiral)
- Long history for chiral spin liquids
  - Kalmeyer & Laughlin '89: triangular lattice Heisenberg AFM = Laughlin state?
  - Exact, but complicated parent Hamiltonians (Schroter, Thomale,Greiter/Yao)
  - Topological flat band models
- More recently interest in non-chiral topological spin liquids ($Z_2$ spin liquid)


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## *Gapless SL*

- Two flavors: gapless excitations at points in momentum space, or "Fermi surfaces"
    - Points in momentum space: *algebraic* spin liquid - Kitaev's honeycomb model
    - "Fermi surfaces": d-wave correlated Bose liquid, spinon Fermi surface



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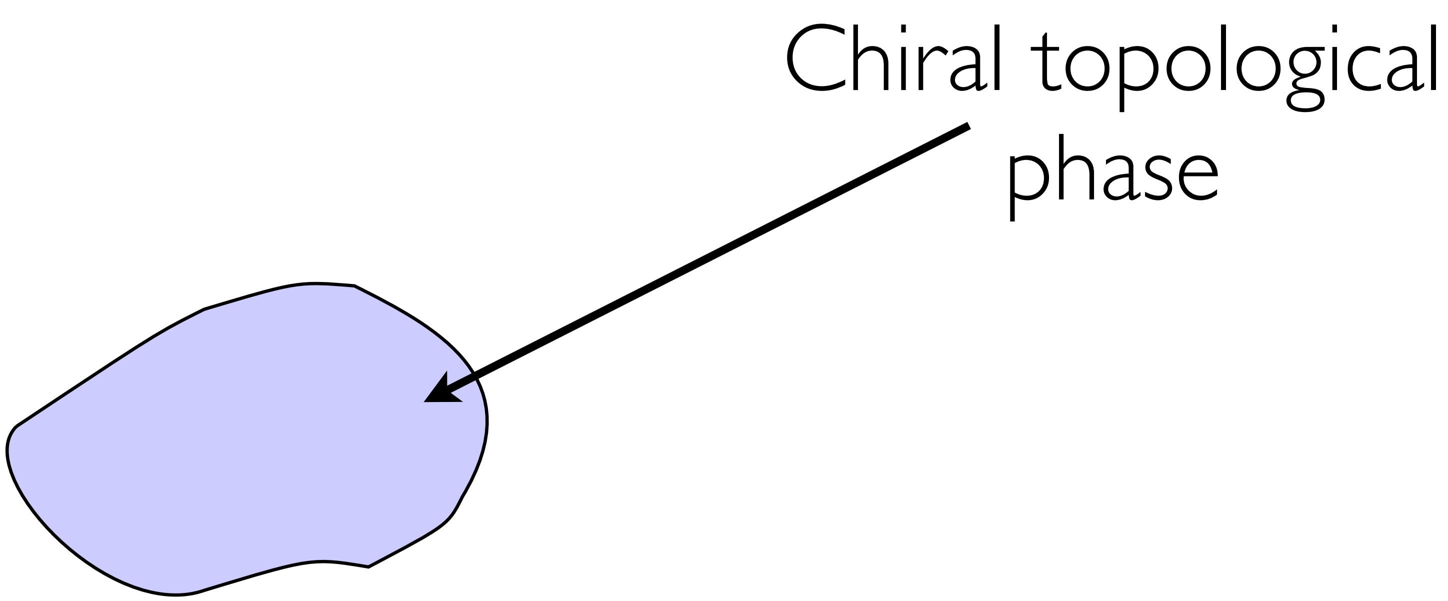
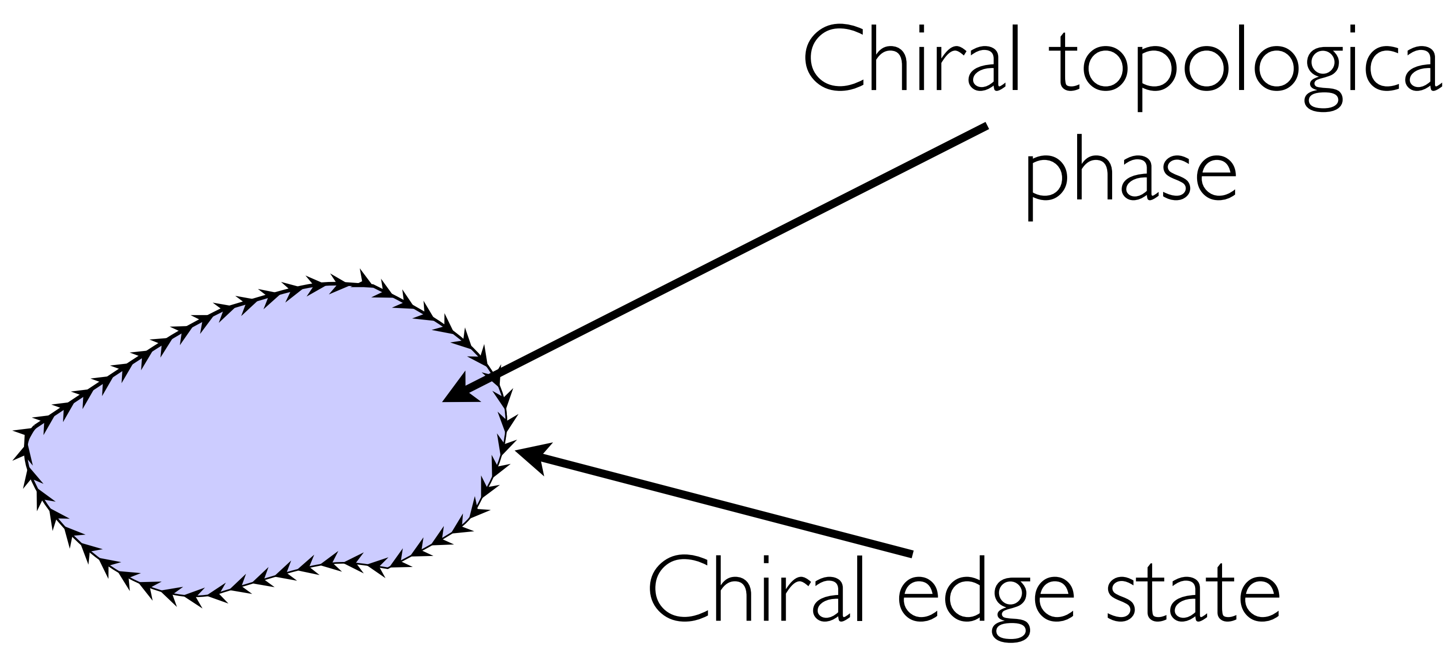
# Network of edge states



# Network of edge states



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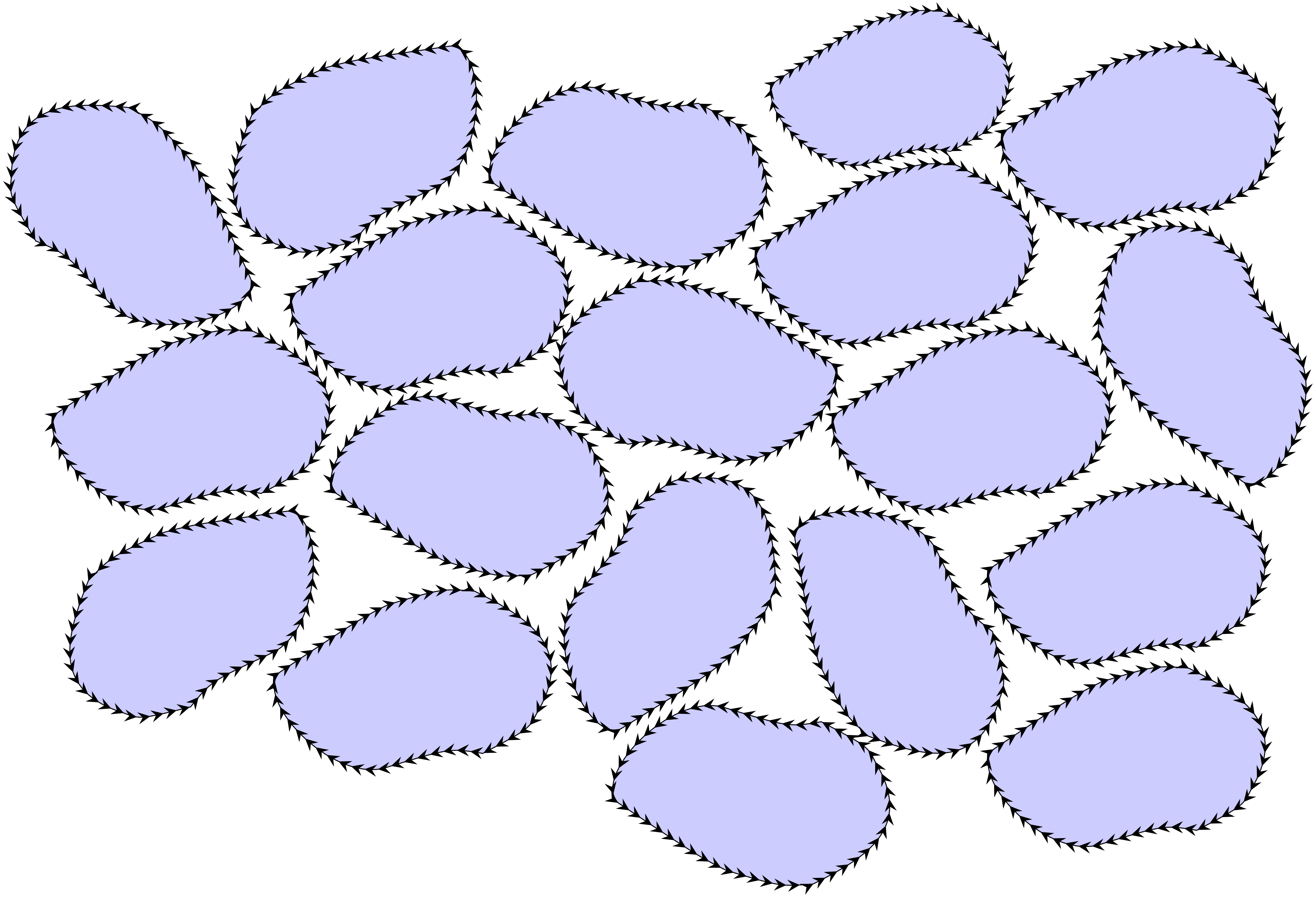

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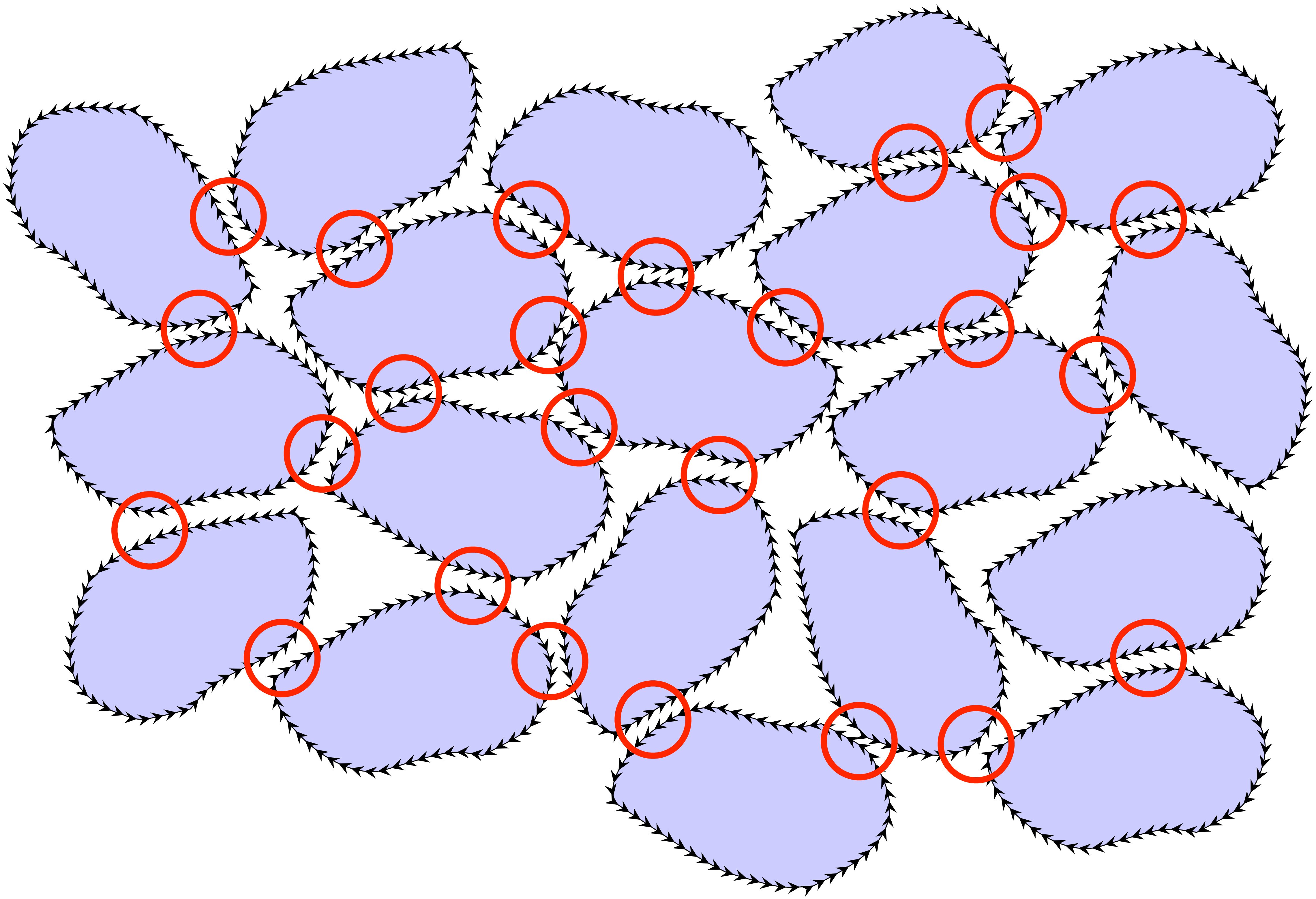


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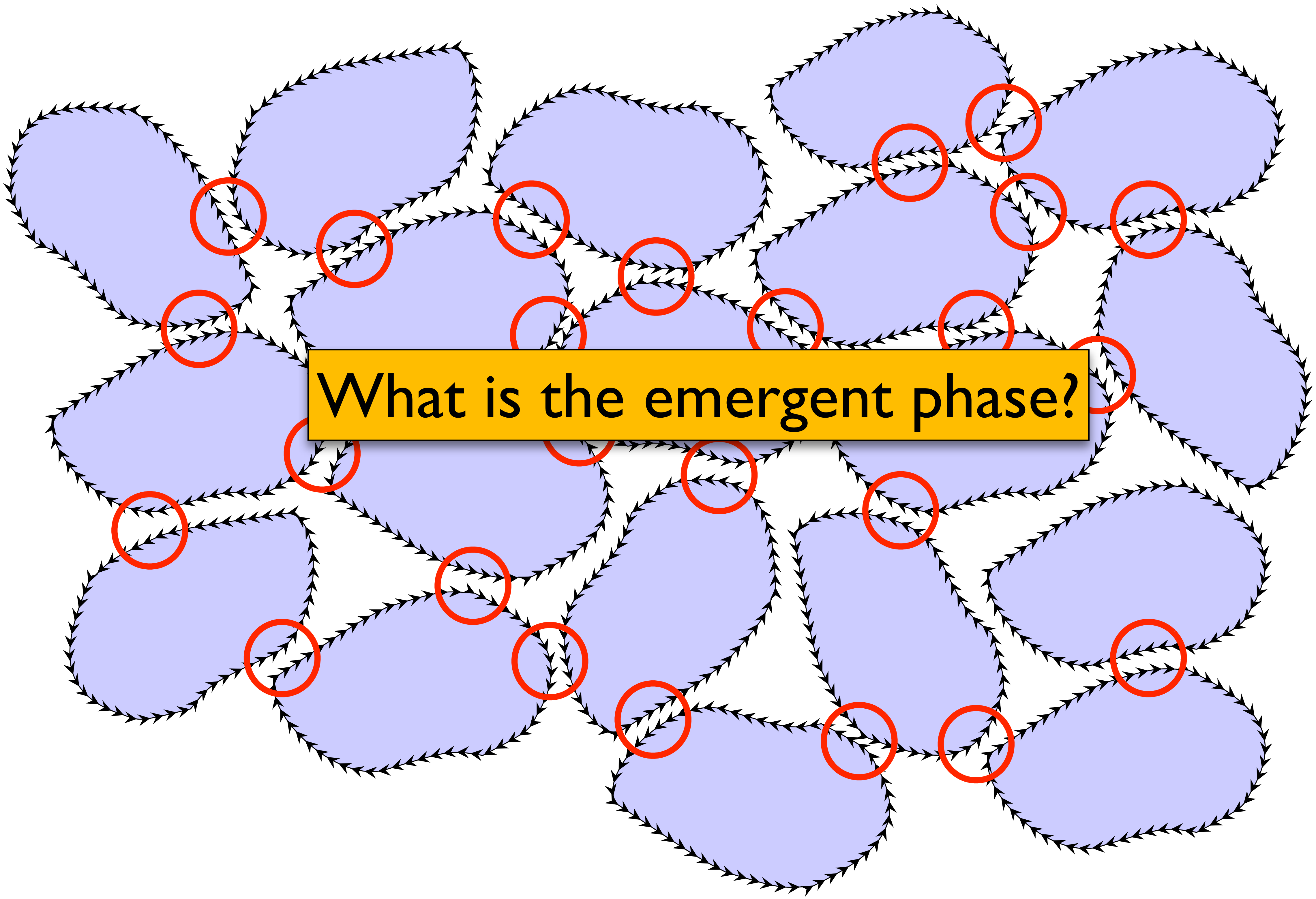
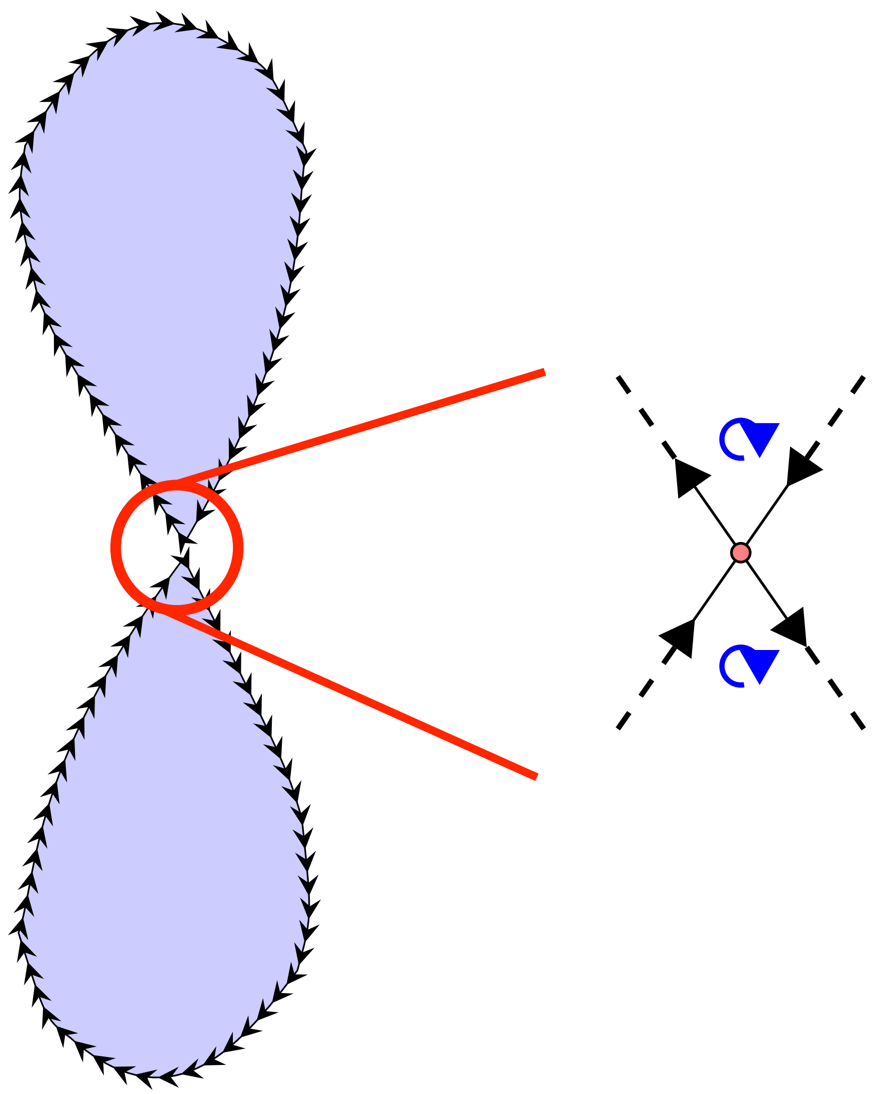


# Building block: two puddles

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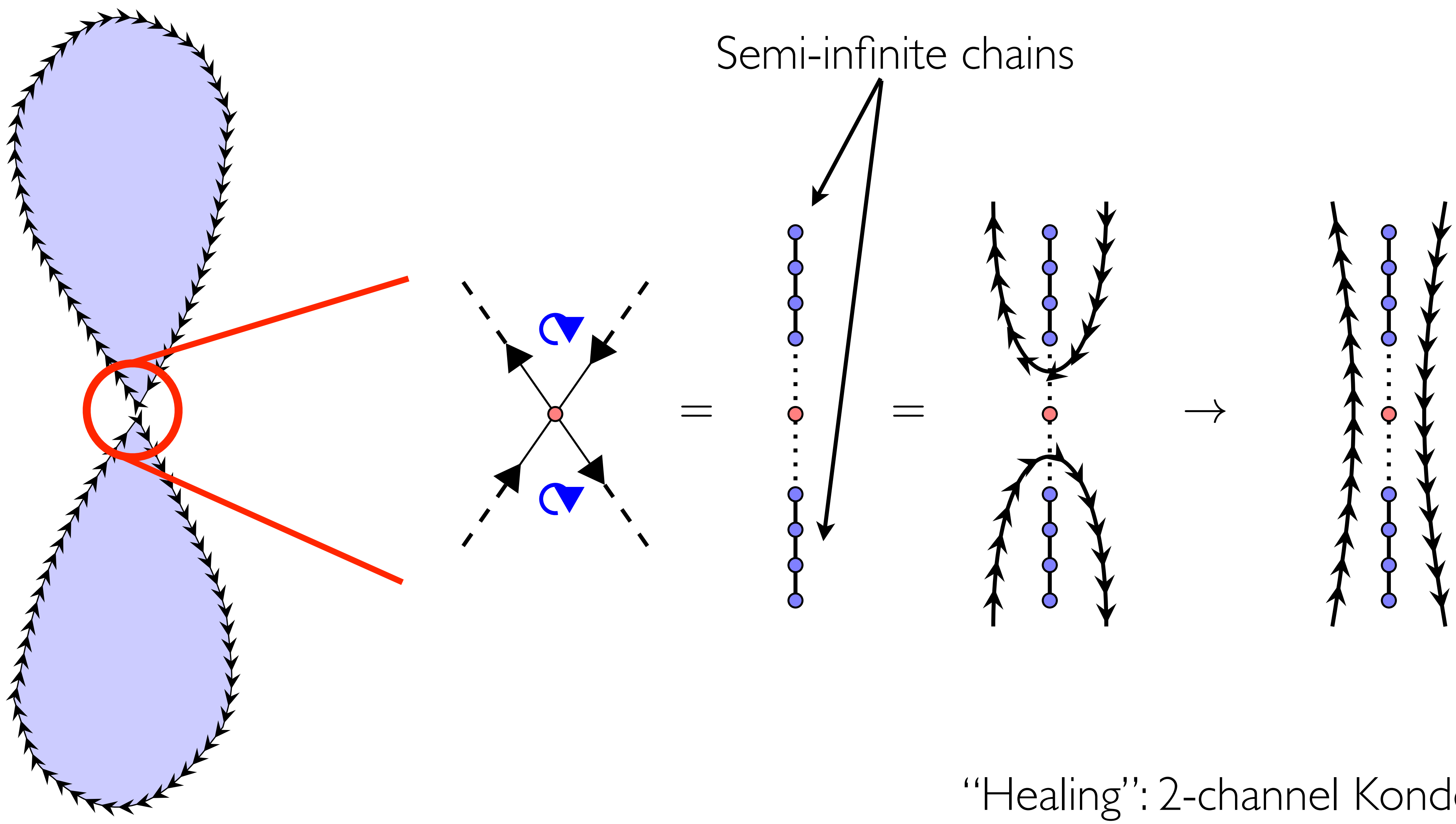
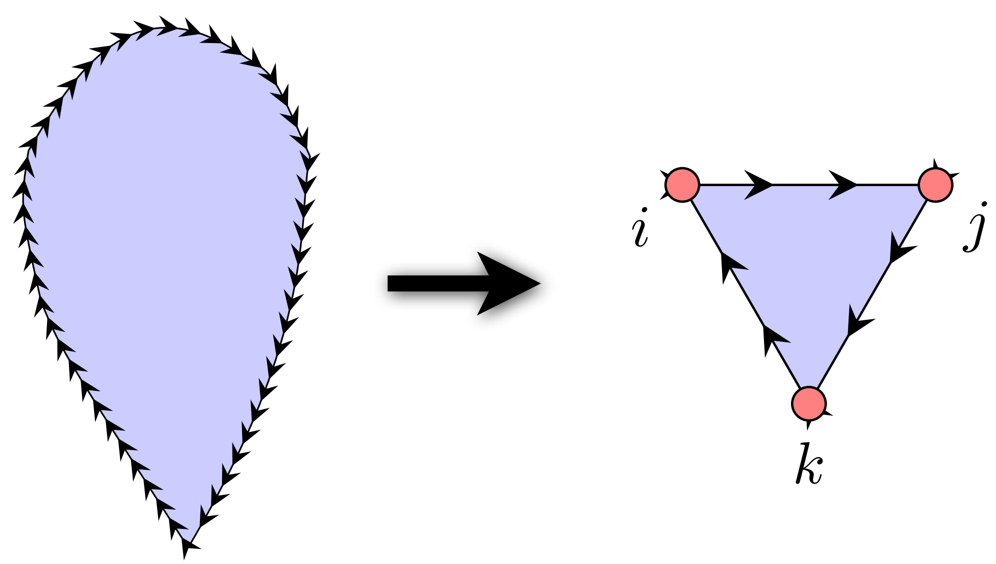



"Healing": 2-channel Kondo effect or resonant tunneling

# Puddles to triangles

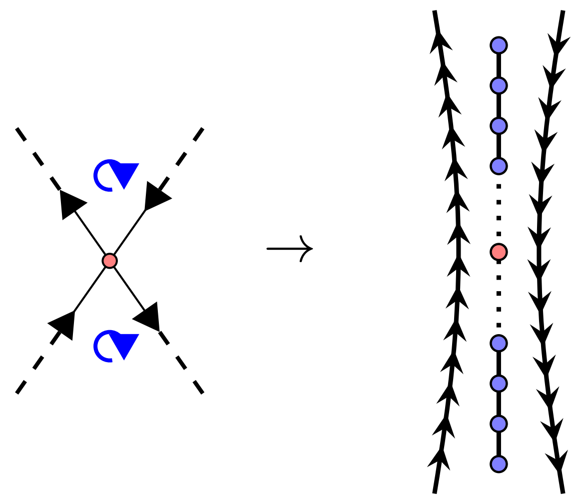


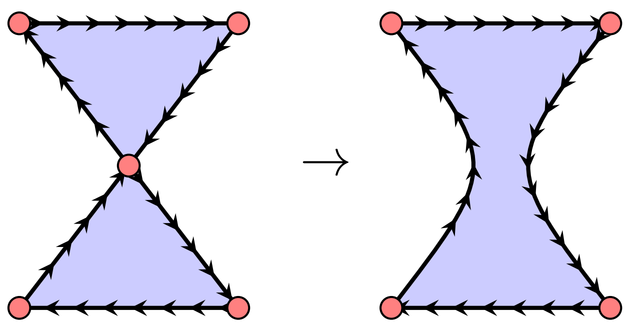

Break time reversal on each triangle!

Spin 1/2: $\chi_{ijk} = \frac{i}{2}\vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$

Majorana fermions: $\tilde{\chi}_{ijk} = i(\gamma_i\gamma_j + \gamma_j\gamma_k + \gamma_k\gamma_i)$

# Two triangles





Equal chirality



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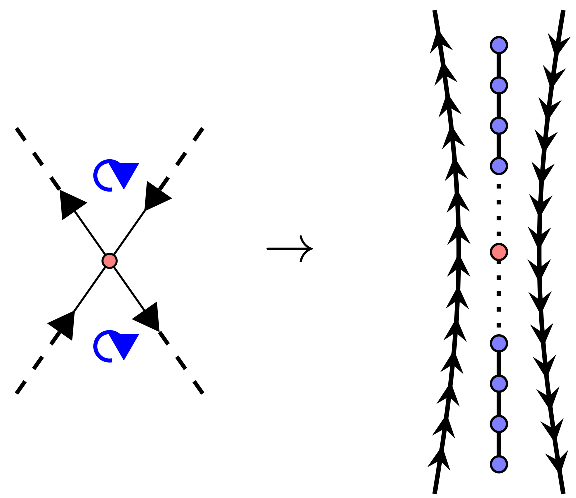
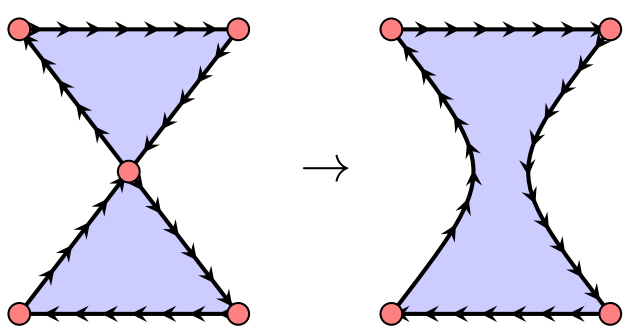
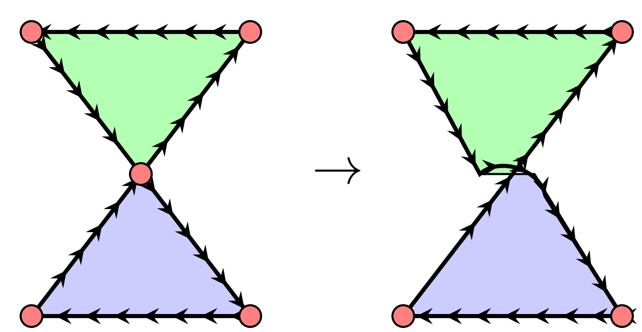
# Two triangles





Equal chirality



Different chirality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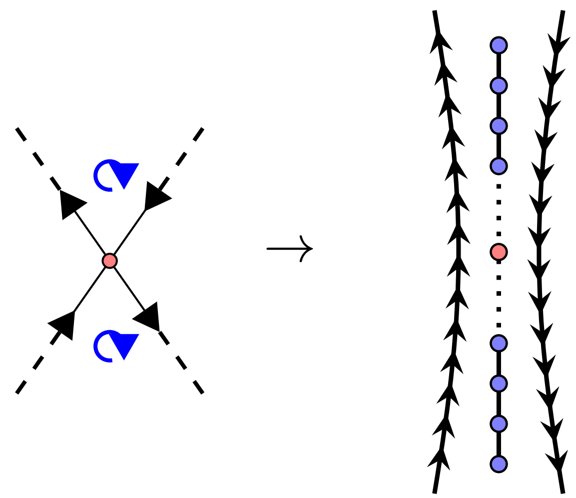
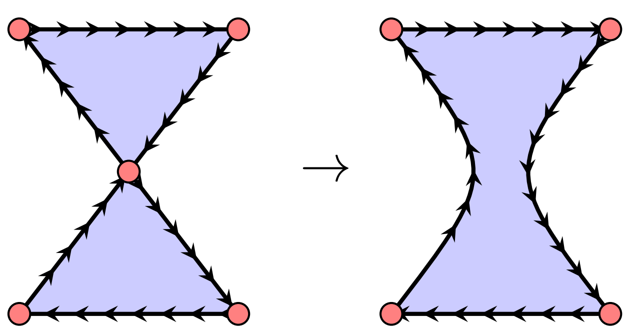
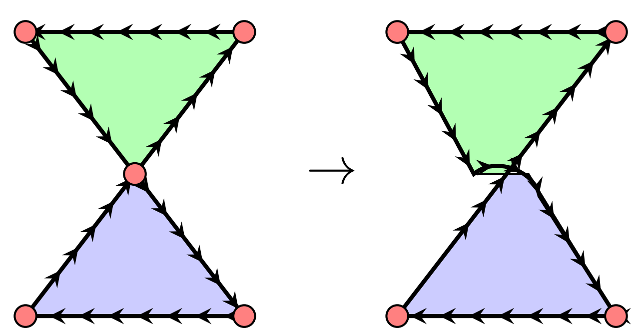
# Two triangles





Equal chirality



Different chirality

Both cases: ***one*** edge state remains



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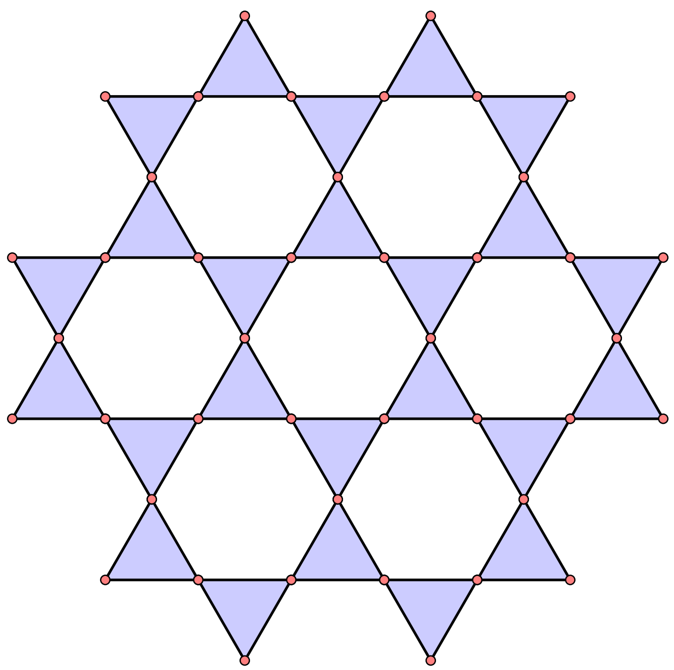
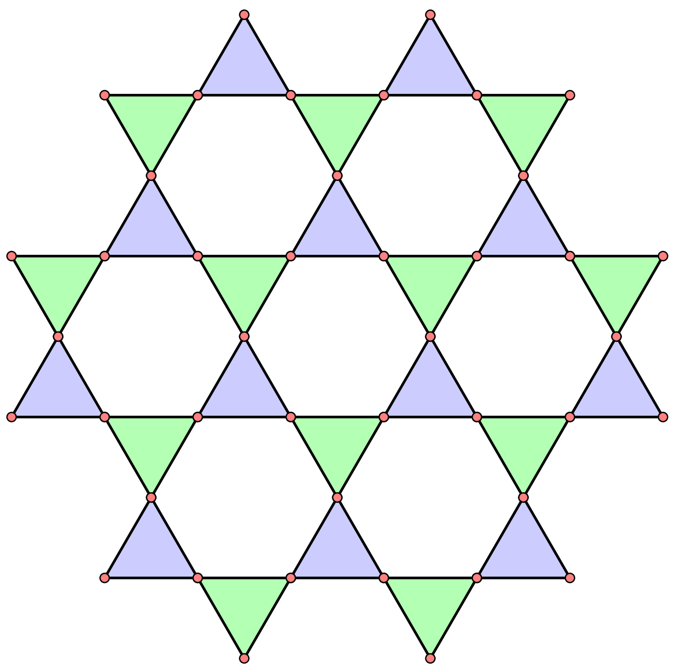
# Kagome lattice: two cases



Homogeneous



Staggered



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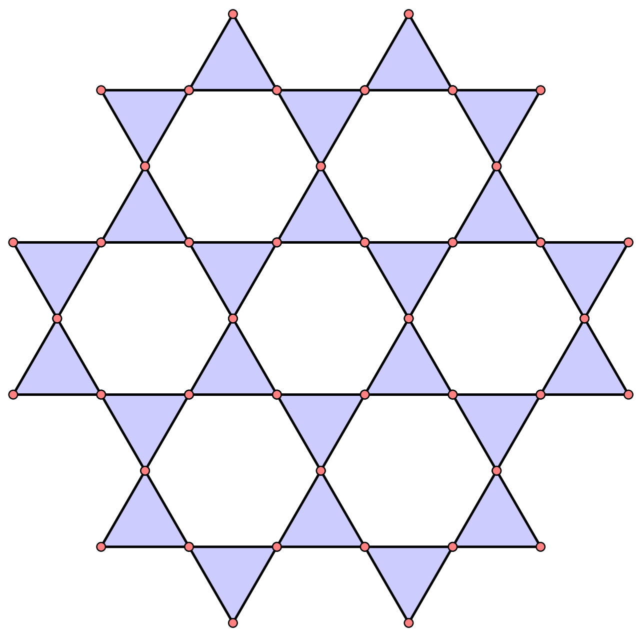
# Homogeneous phase



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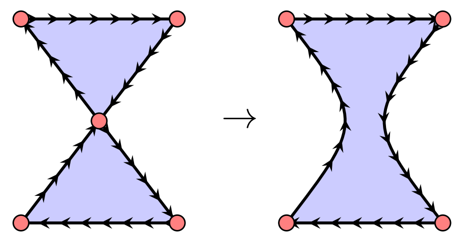

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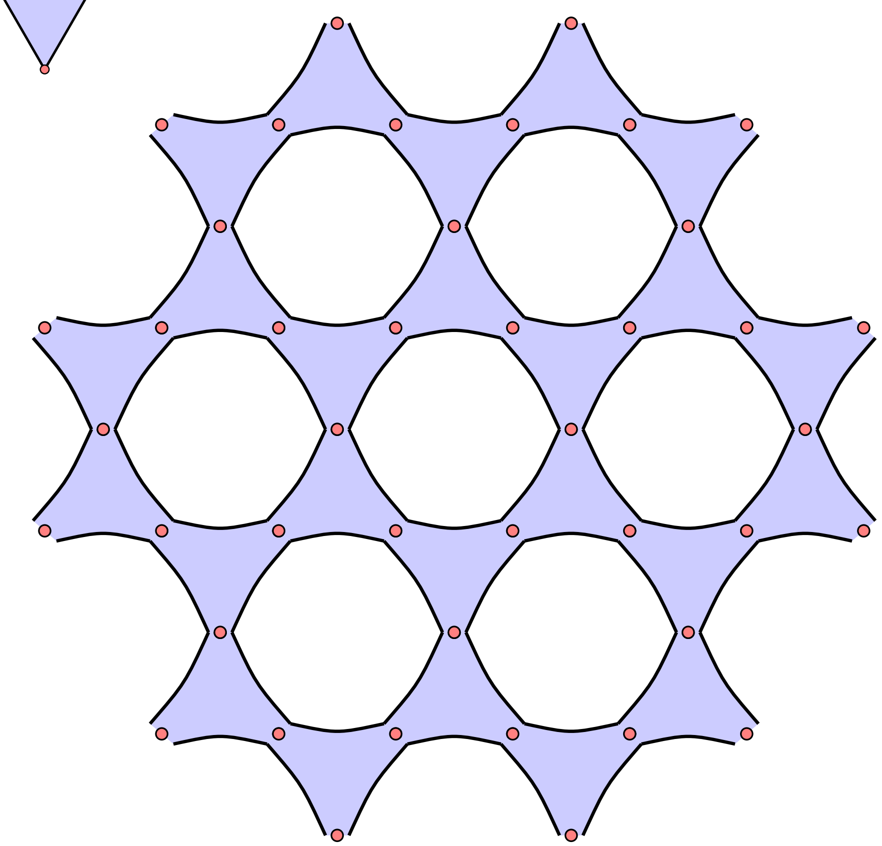



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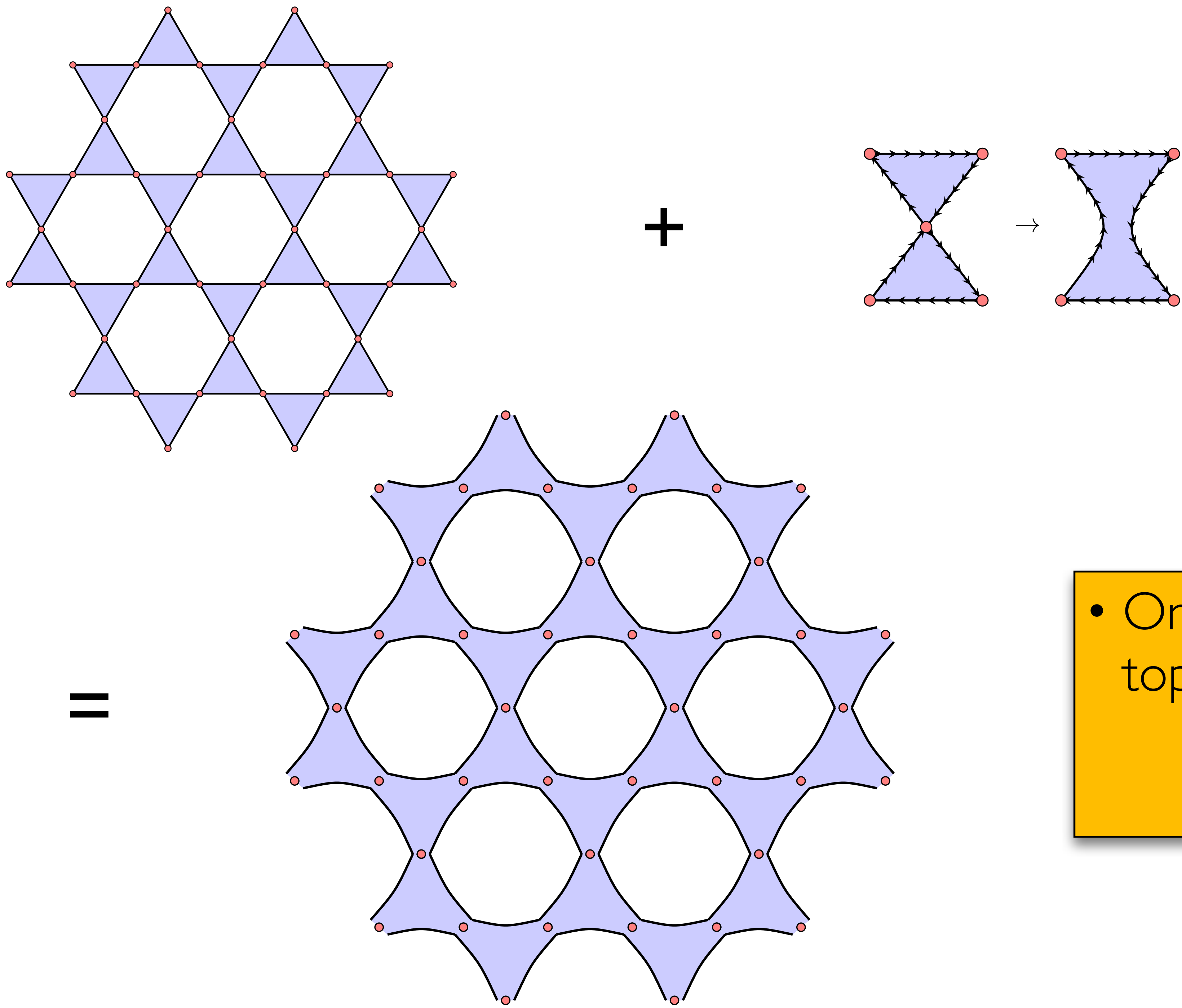
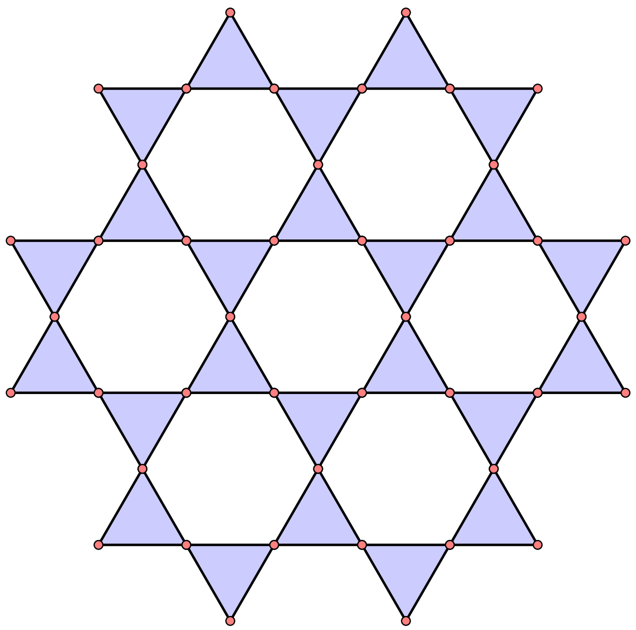



- One extended region of the topological phase

# Homogeneous phase



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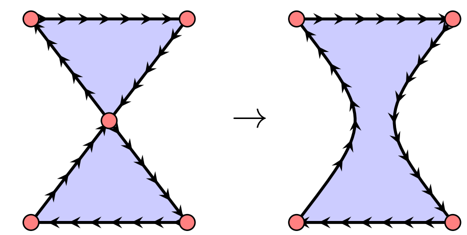

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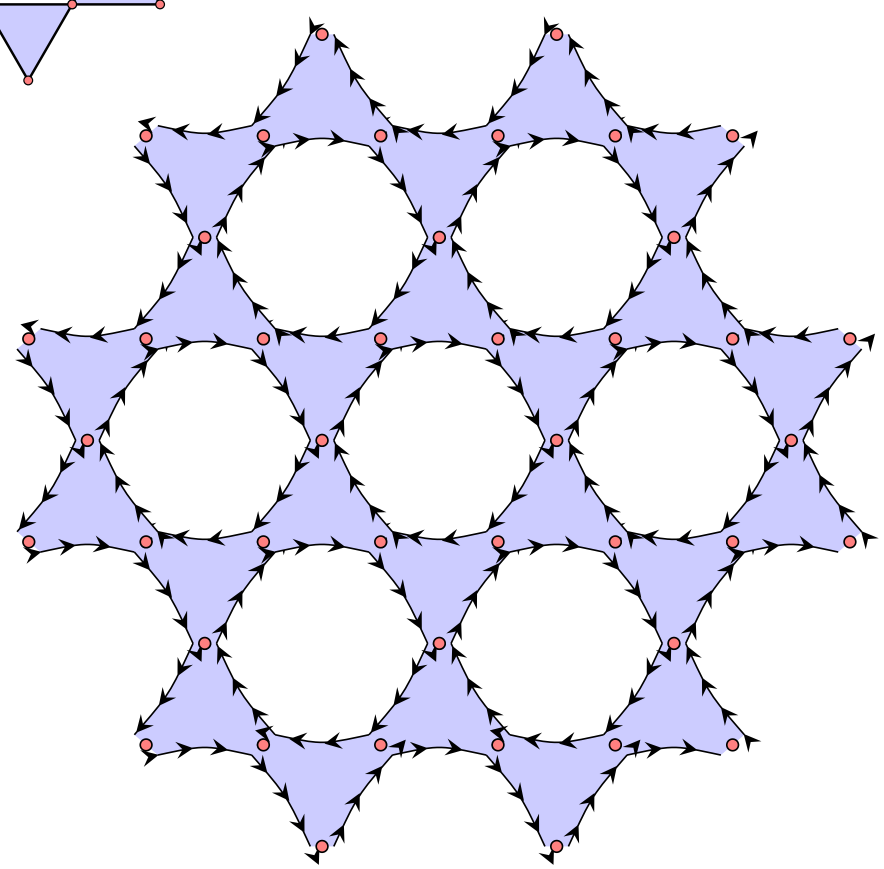
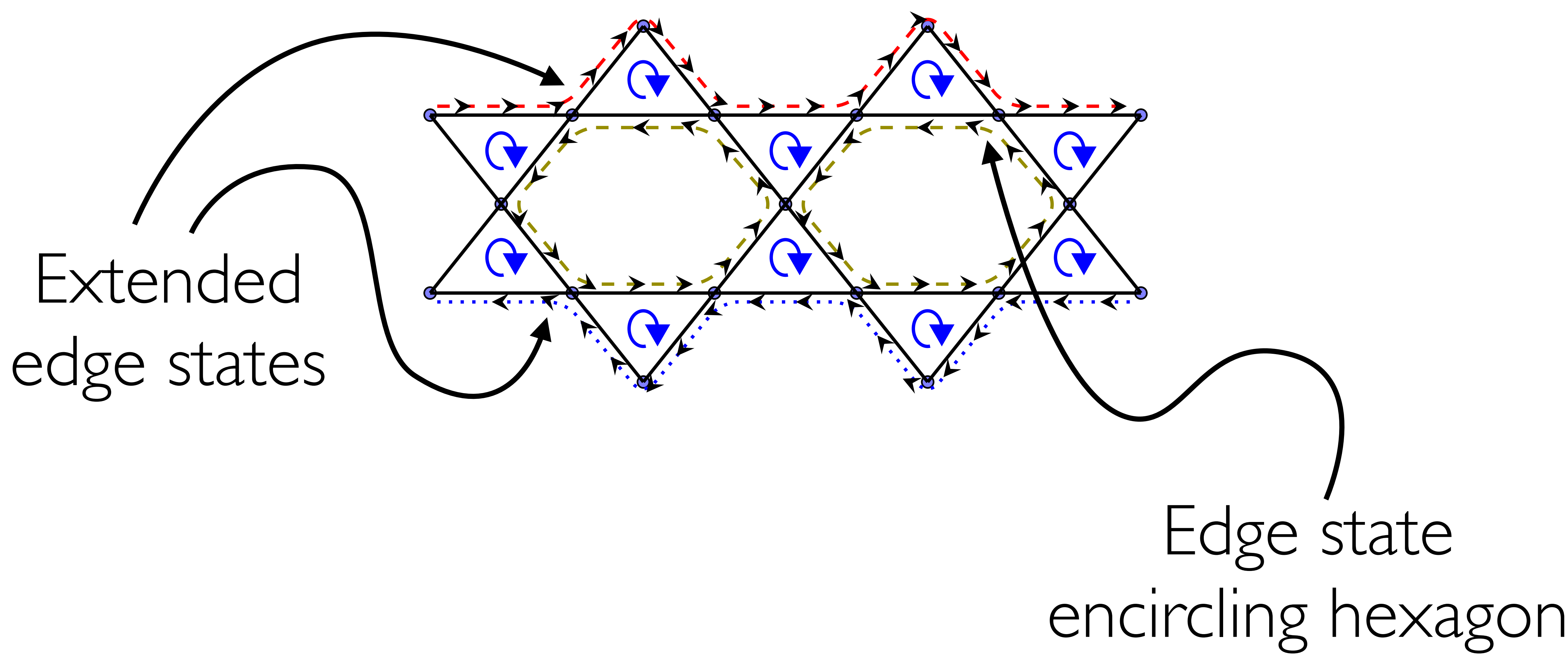


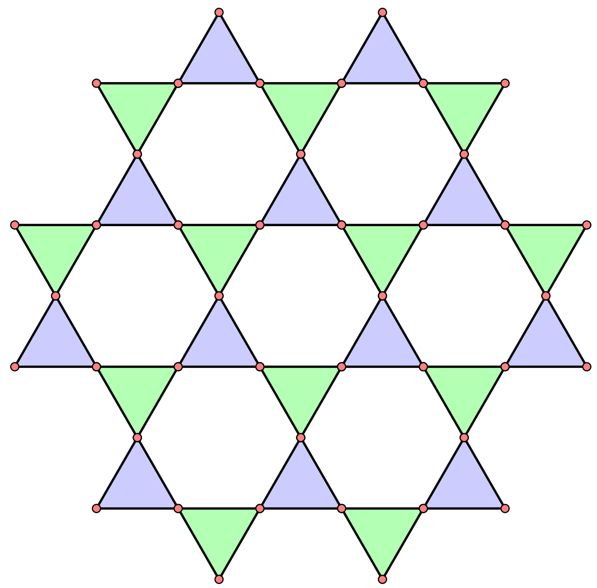
- One extended region of the topological phase
- One edge state encircling the whole system

# Homogeneous phase

2x*L* cluster



# Staggered phase



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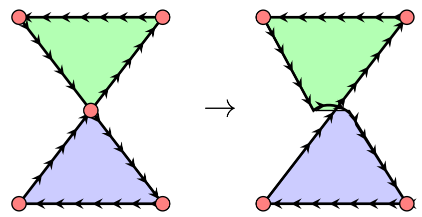

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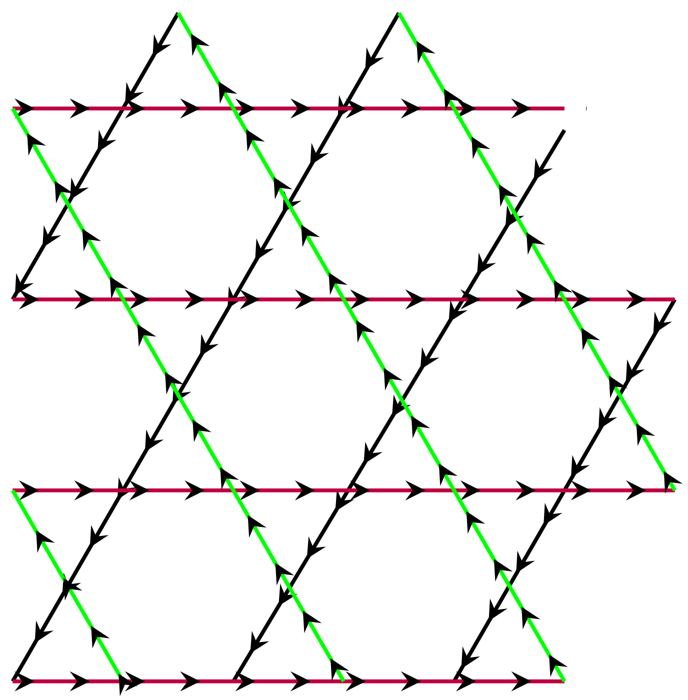

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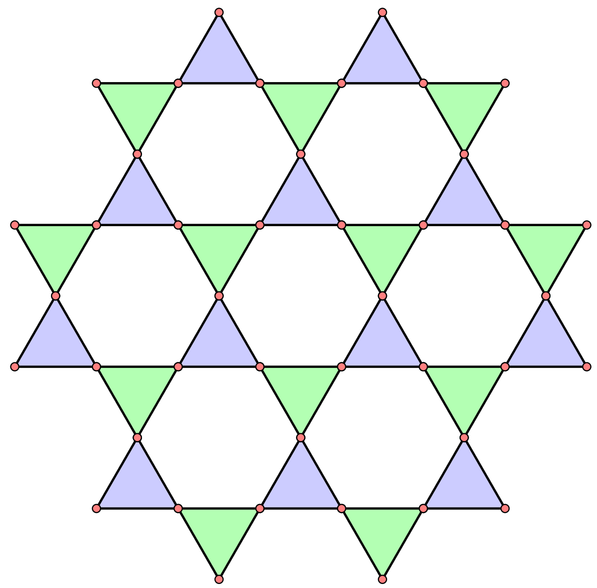

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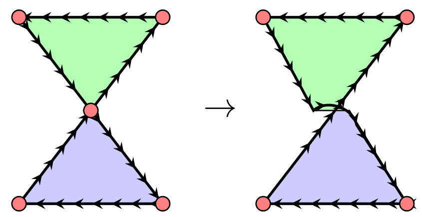

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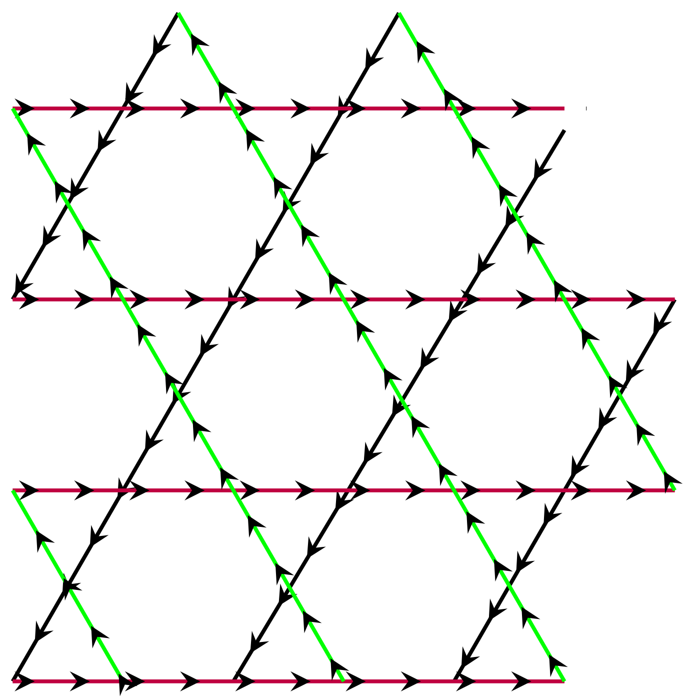

Uncoupled gapless edge states on the chains give rise to bulk gapless phase!



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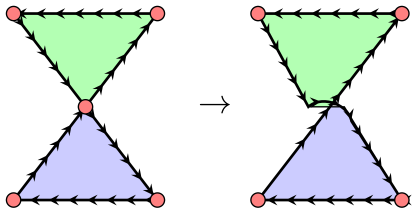

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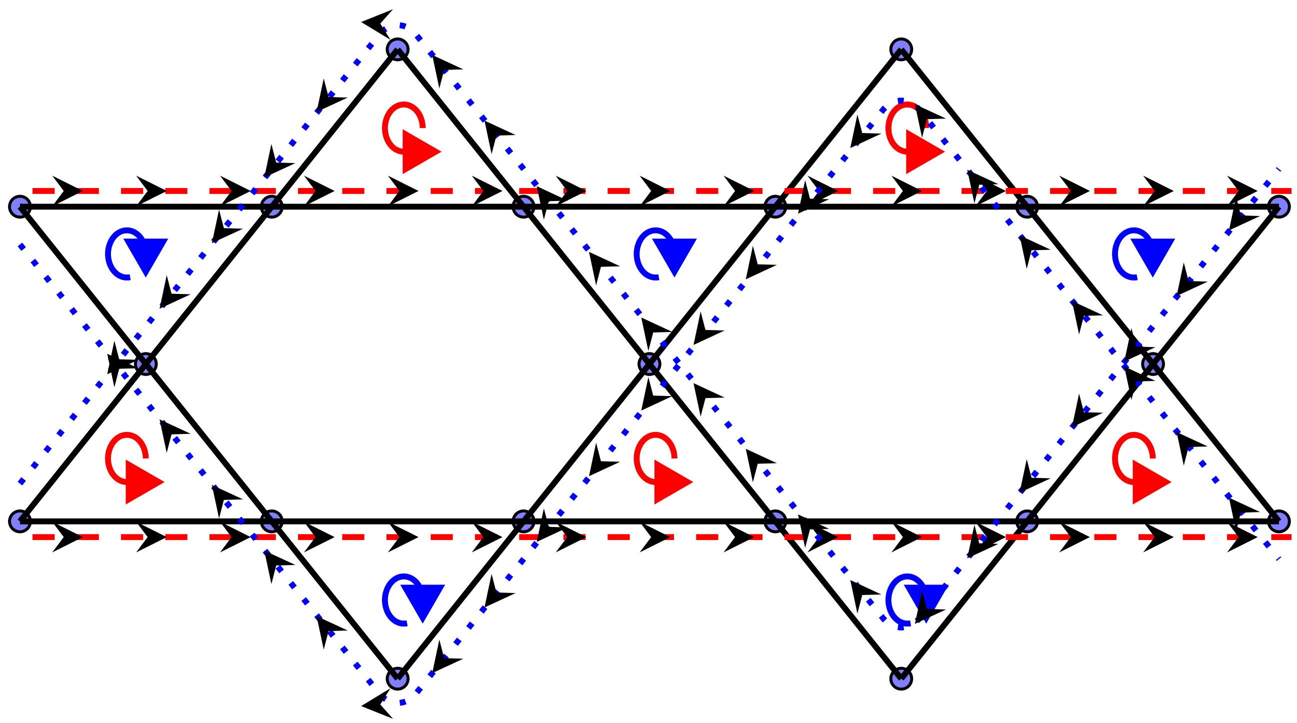
2xL cluster





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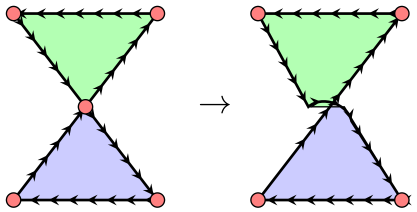
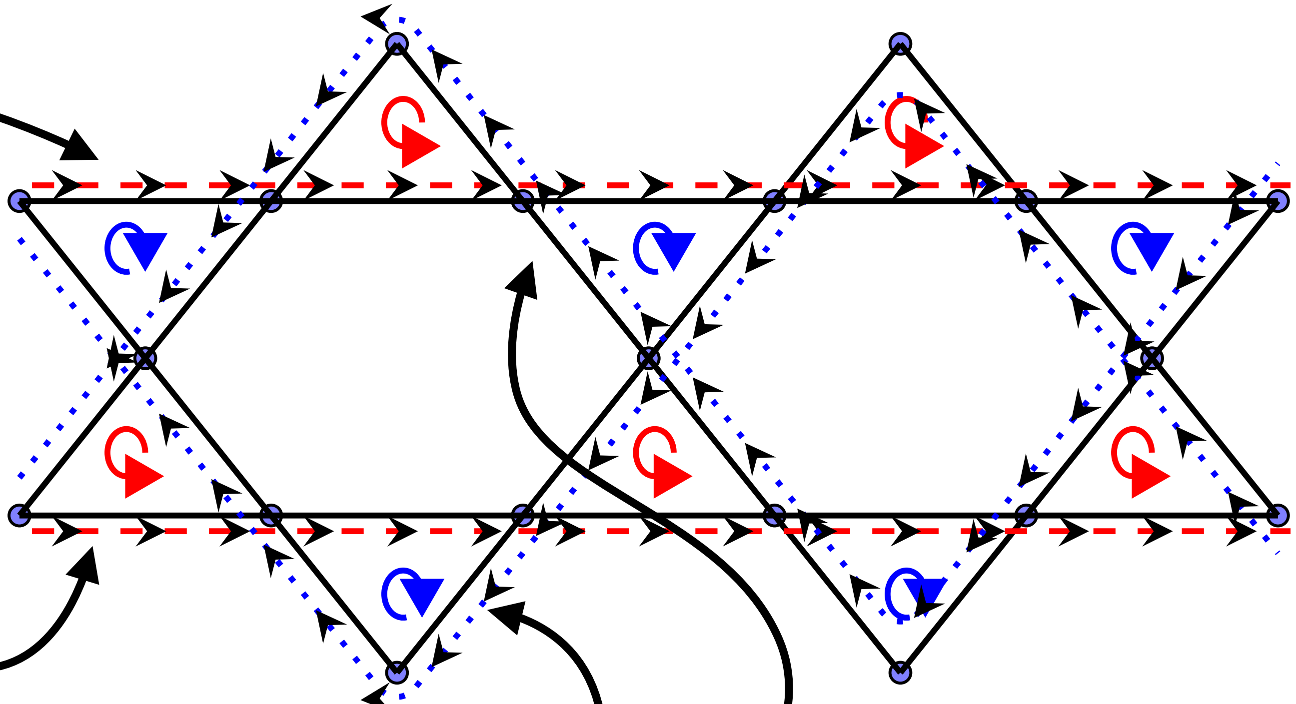


# Staggered phase

2x*L* cluster



Extended right-moving edge states

Extended left-moving edge states



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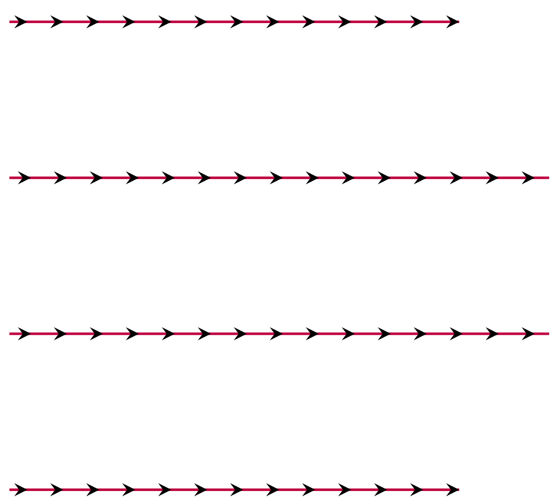
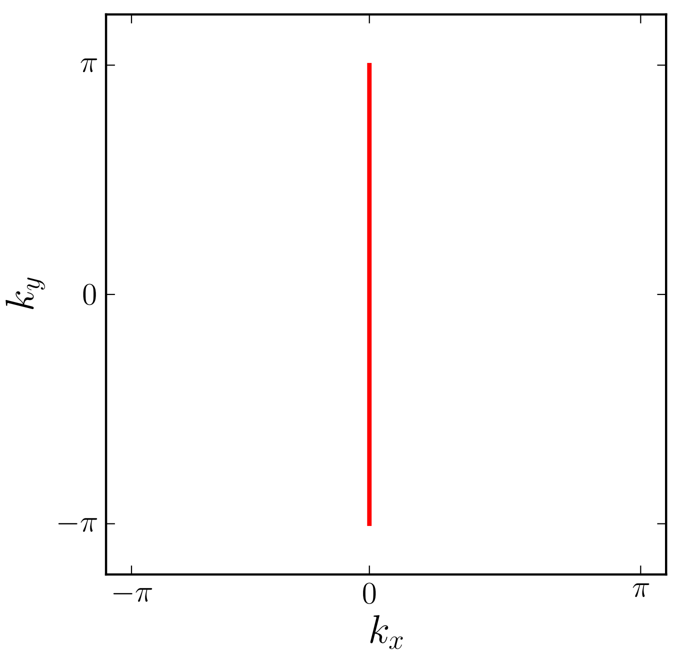
# Fermi surface

Real space



Momentum space



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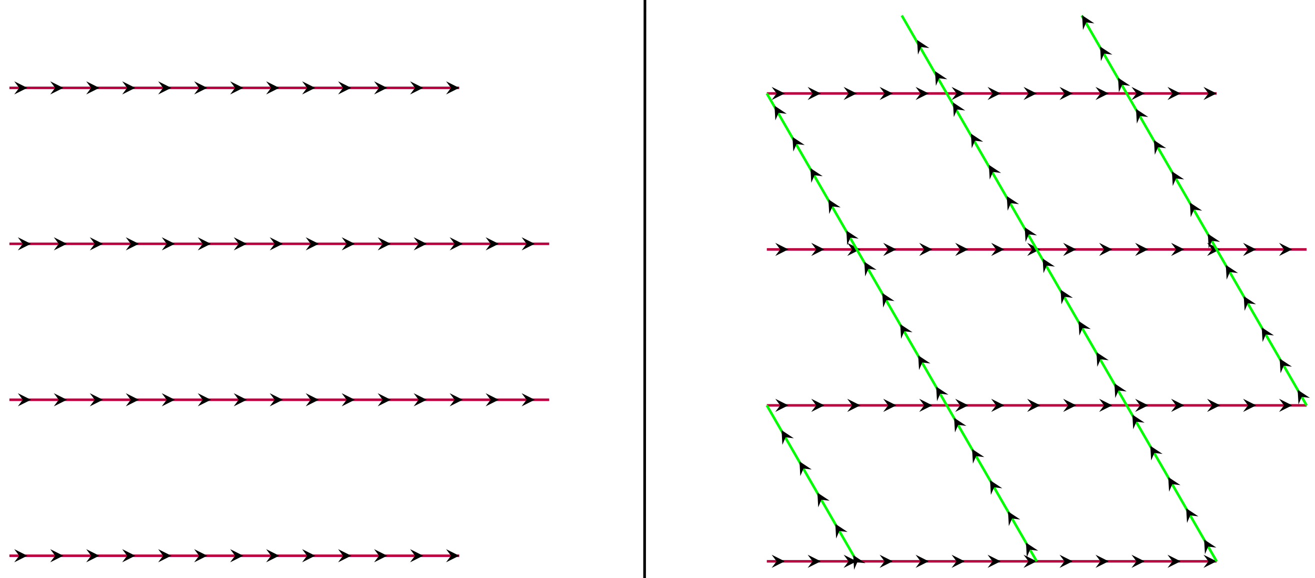
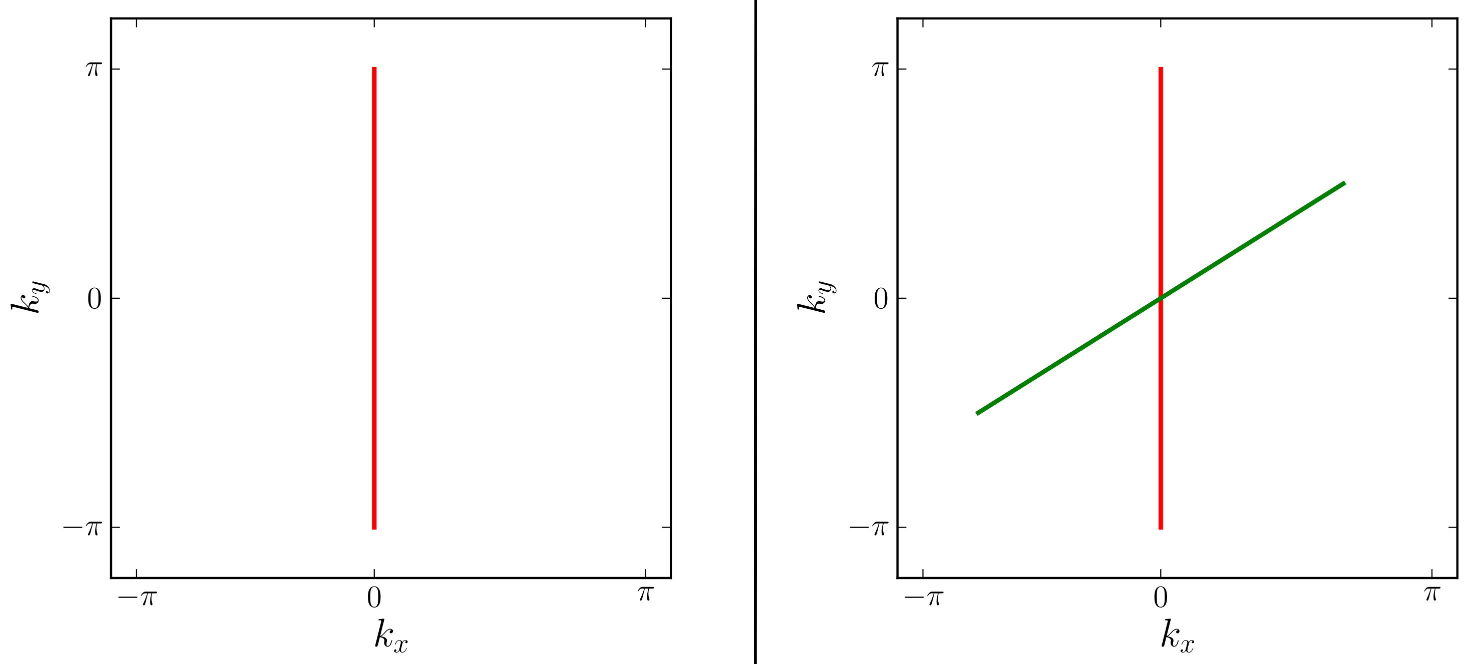
Real space



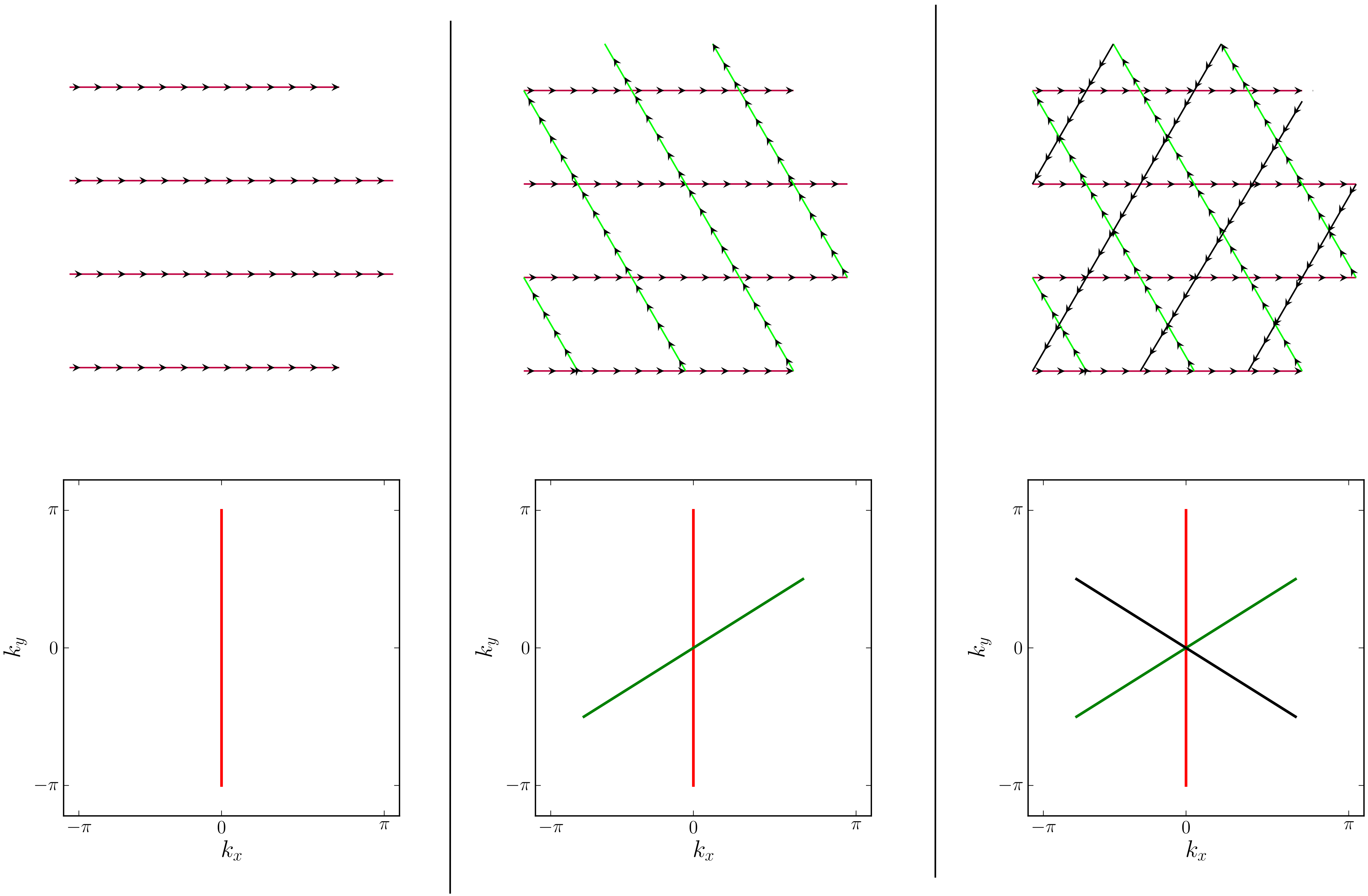
Momentum space



# Fermi surface

Real space

Momentum space



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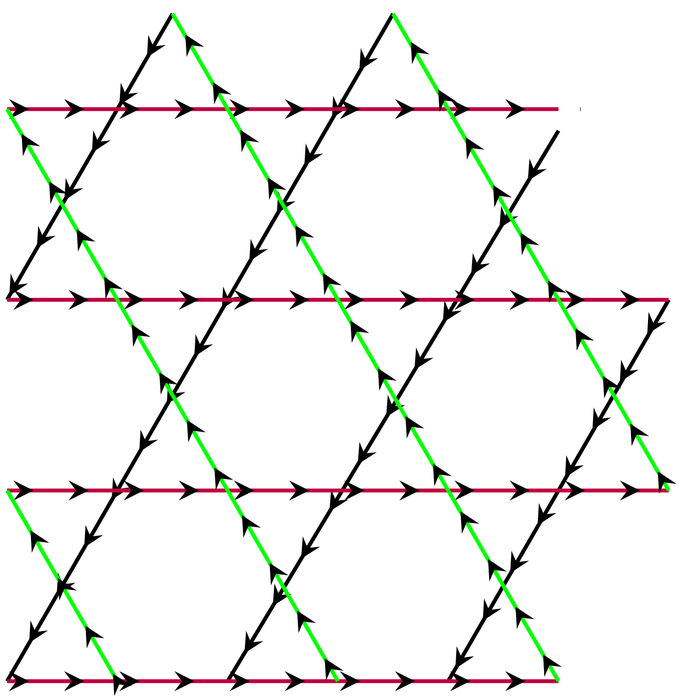
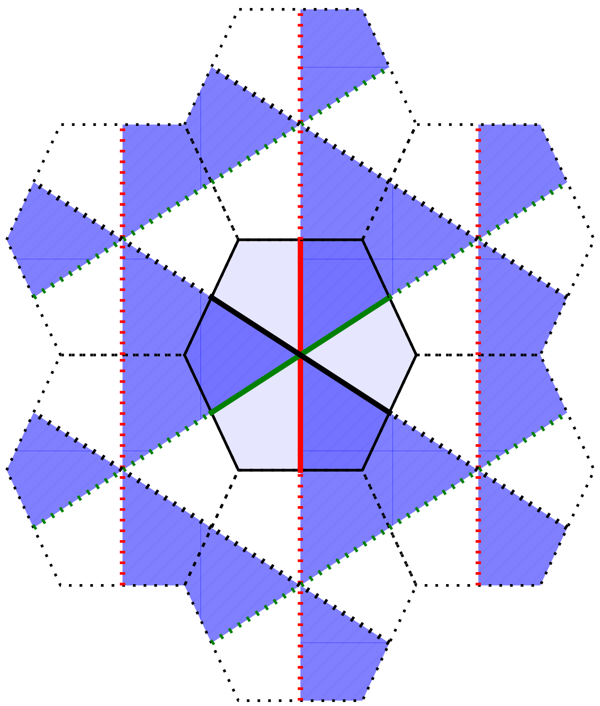
Real space



Momentum space



- Light blue: first BZ
- Dark blue: filled regions



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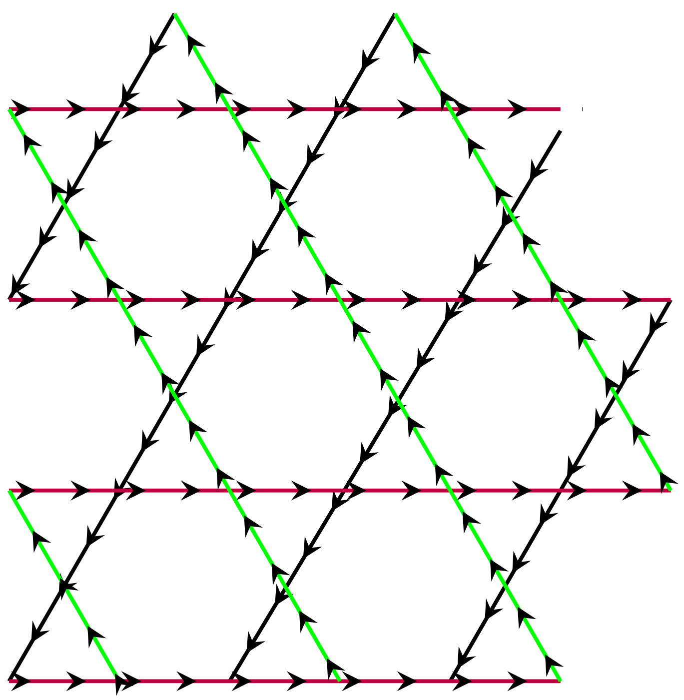
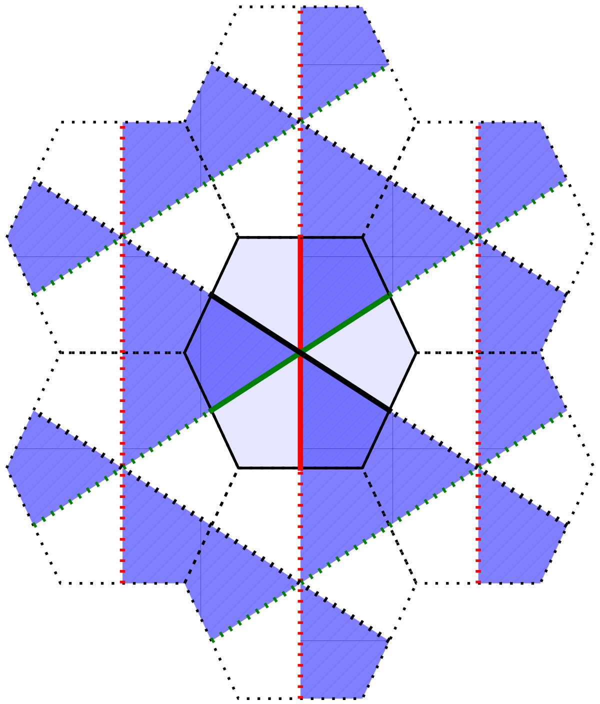
# Fermi surface

Real space



Momentum space



- Light blue: first BZ
- Dark blue: filled regions

- Each direction of edge states leads to a gapless line in the BZ
- Lines enclose triangles of filled states



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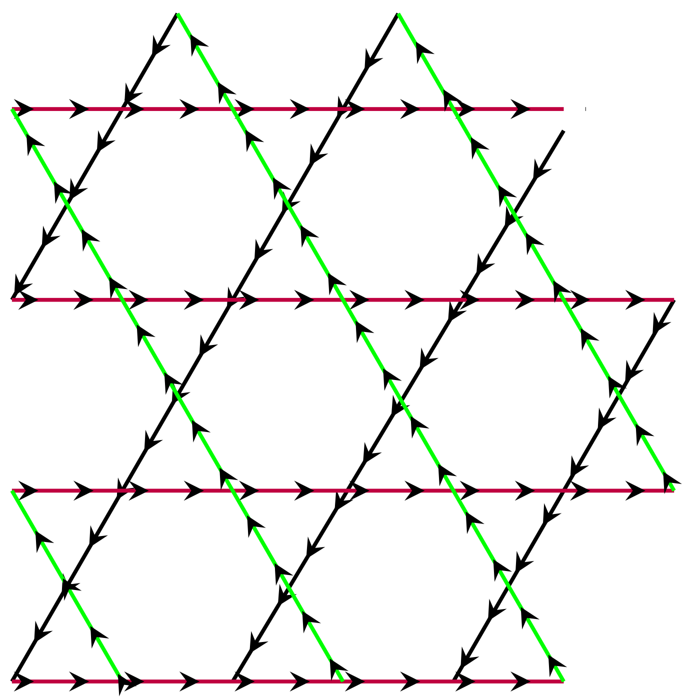
# Symmetries

Real space

Momentum space



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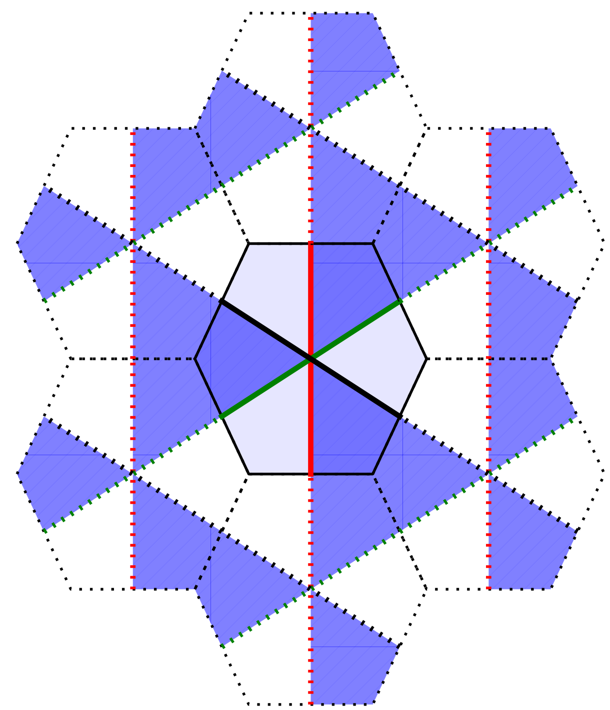

- Rotational symmetry: *6*-fold to *3*-fold
- Inversion symmetry $x \leftrightarrow -x$: broken
- Inversion symmetry $y \leftrightarrow -y$: not broken
- (Inversion $x \leftrightarrow -x$) x (time reversal): additional symmetry



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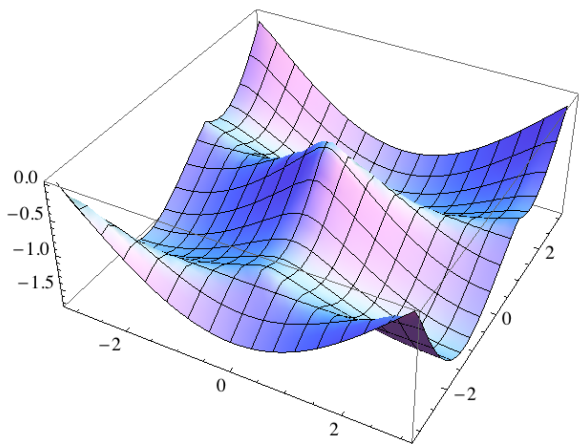
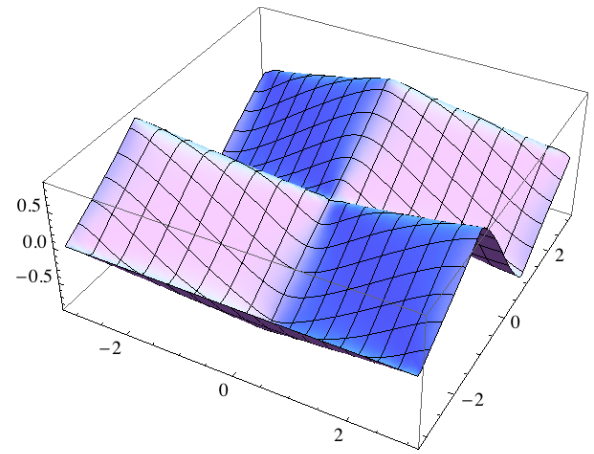
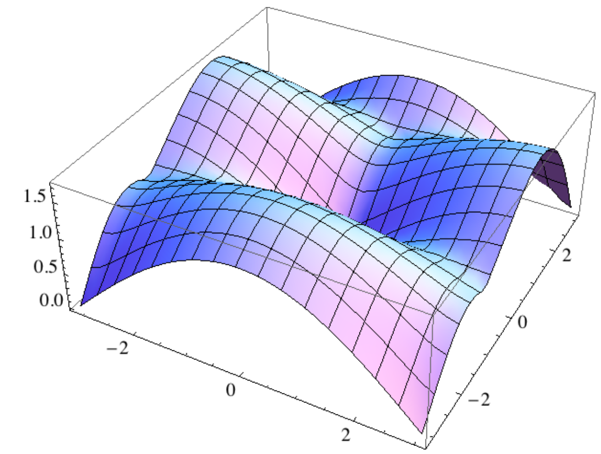
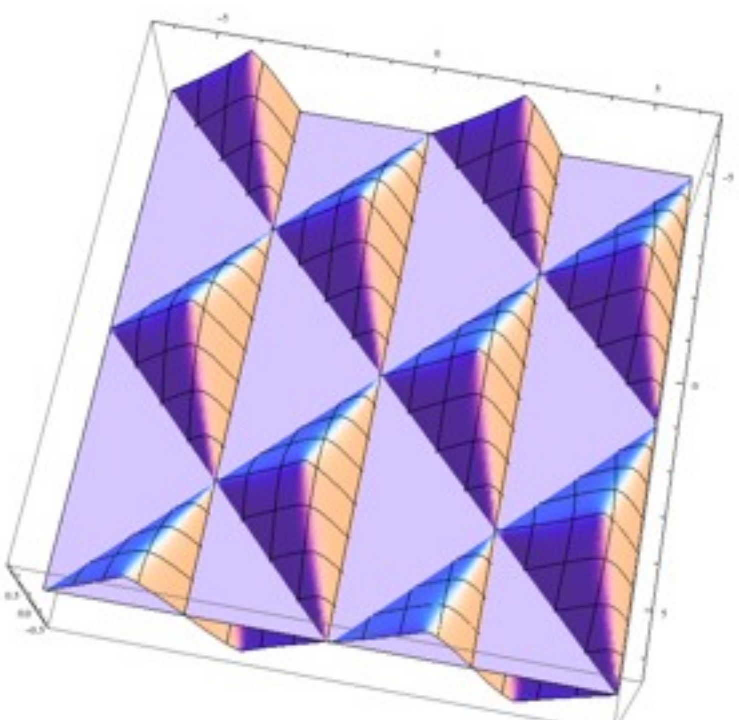
# Majorana fermions: staggered

$$\tilde{\chi}_{ijk} = i(\gamma_i\gamma_j + \gamma_j\gamma_k + \gamma_k\gamma_i) \qquad H = \sum_{\triangle} \tilde{\chi}_{ijk} - \sum_{\triangledown} \tilde{\chi}_{ijk}$$



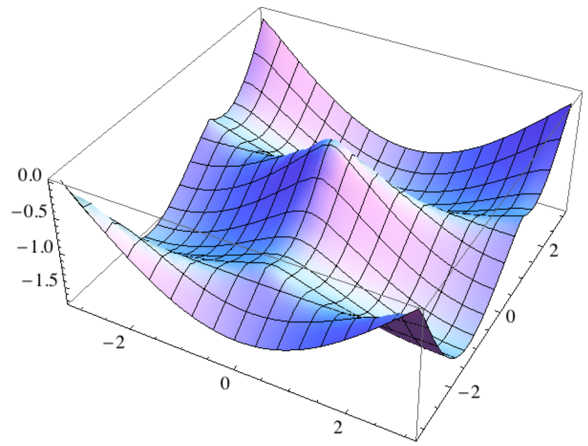


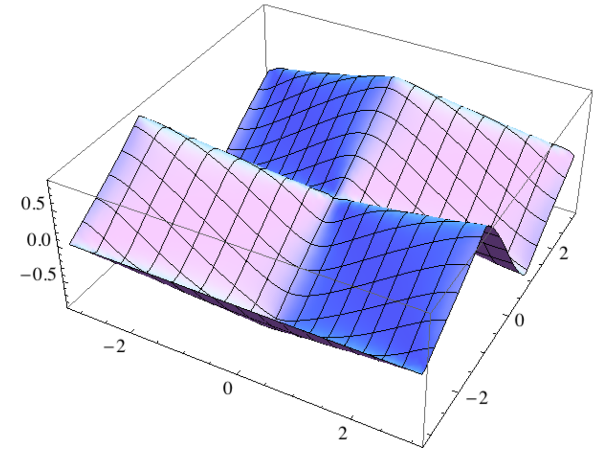


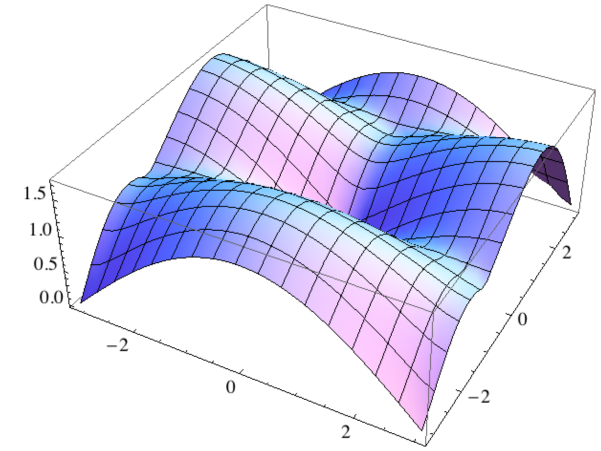


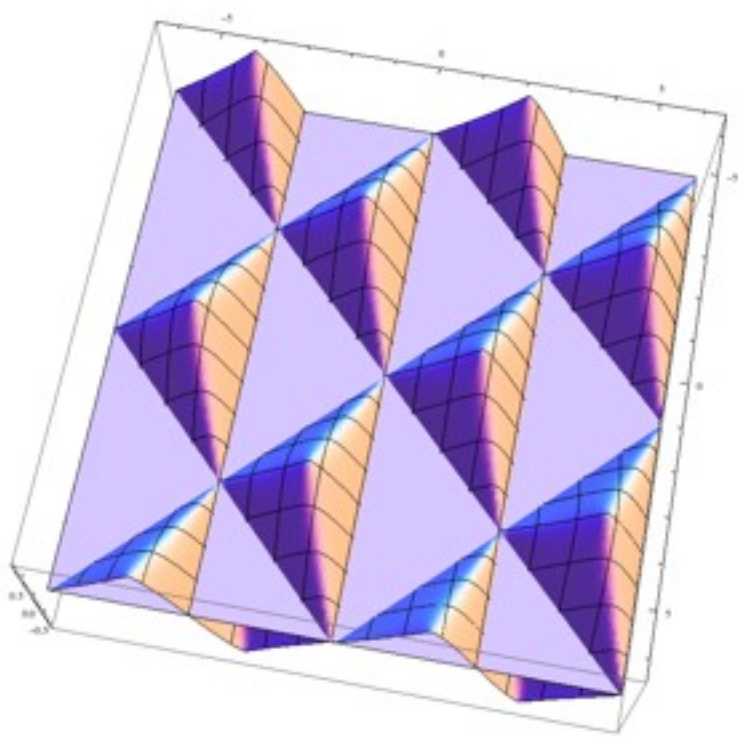
# Majorana fermions: staggered

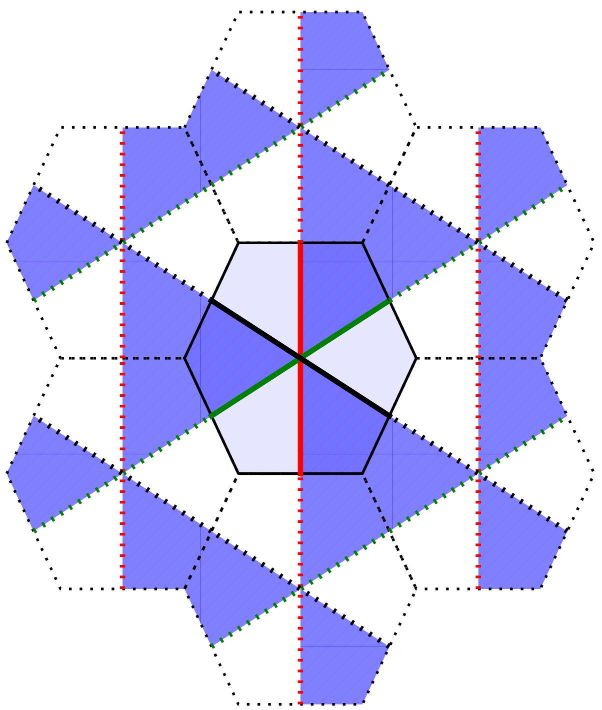
$$\tilde{\chi}_{ijk} = i(\gamma_i\gamma_j + \gamma_j\gamma_k + \gamma_k\gamma_i) \qquad H = \sum_{\triangle} \tilde{\chi}_{ijk} - \sum_{\bigtriangledown} \tilde{\chi}_{ijk}$$



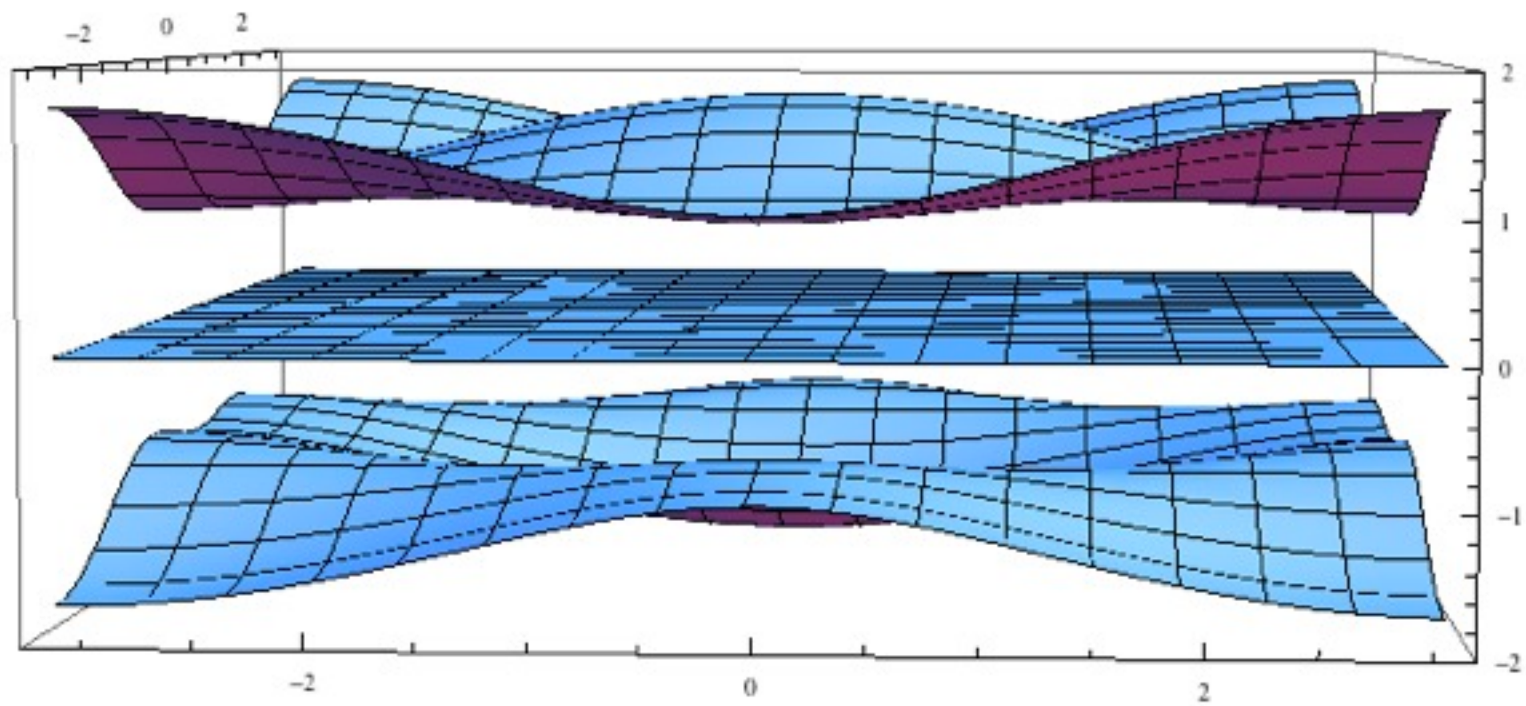





# Majorana fermions: homogeneous




- Gapped spectrum
- Chern number of the top and bottom bands: $C=\pm 1$
- Dispersionless band: localized zero-energy states on hexagons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# The same with spins?

$$\chi_{ijk} = \frac{i}{2}\vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) \qquad\qquad H = \sum_{\triangle} \chi_{ijk} \pm \sum_{\triangledown} \chi_{ijk}$$

- Can only be solved numerically: DMRG and ED
  - quasi-*1d* systems with DMRG: "thin torus" or "thin cylinder" limit
- Our numerical agenda:
  - ***Topological phase***
    - Spin gap
    - Degeneracy for torus vs cylinder
    - Edge state on strip vs cylinder
  - ***Gapless phase***
    - Spin gap
    - Central charge


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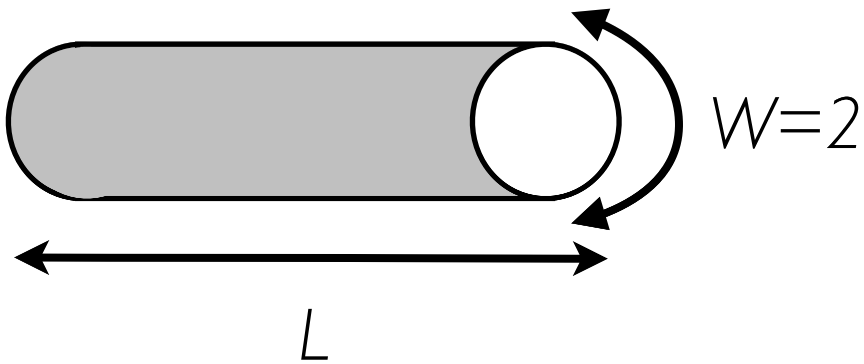
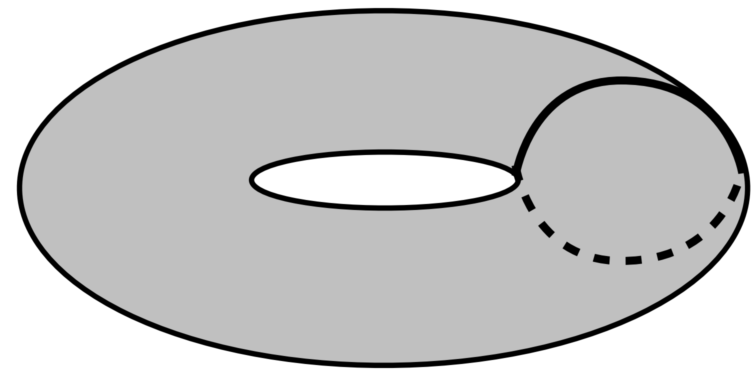
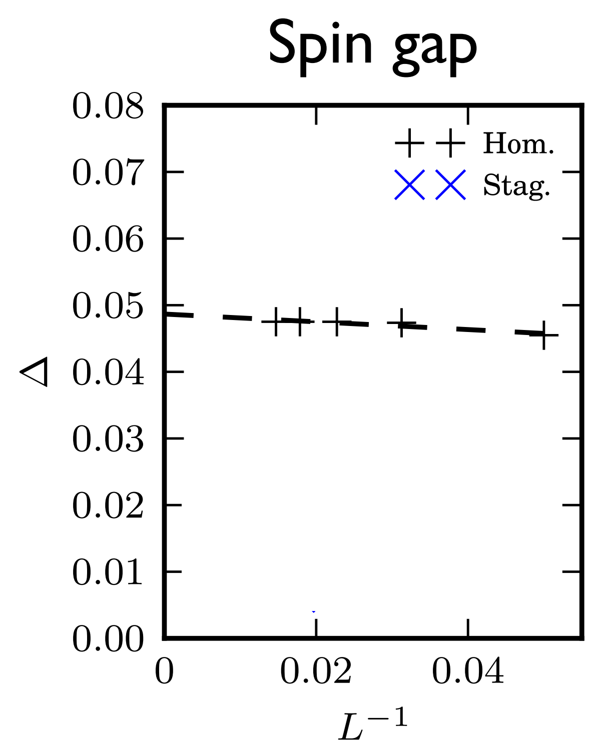
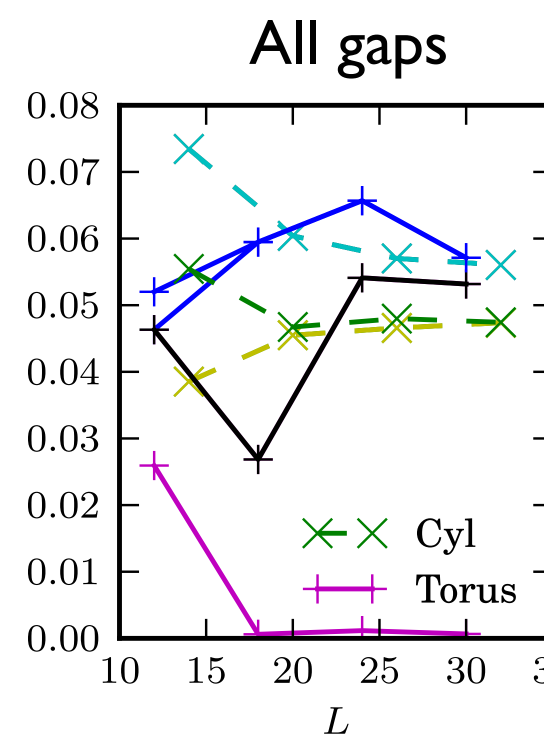
# Spin gap



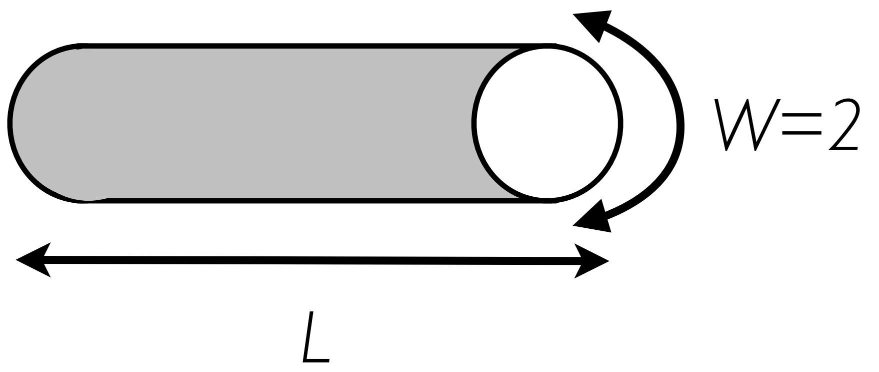



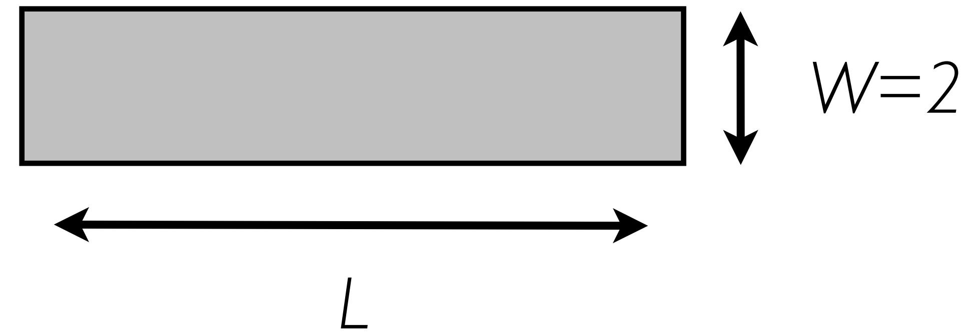
- *2xL* cylinders (DMRG & ED) and tori (ED)
  - DMRG with up to 2400 states
  - ED up to 32 sites (2x5x3+2)
- ***Finite gap (Δ~0.05)***
- ***Additional low-lying state on the torus***



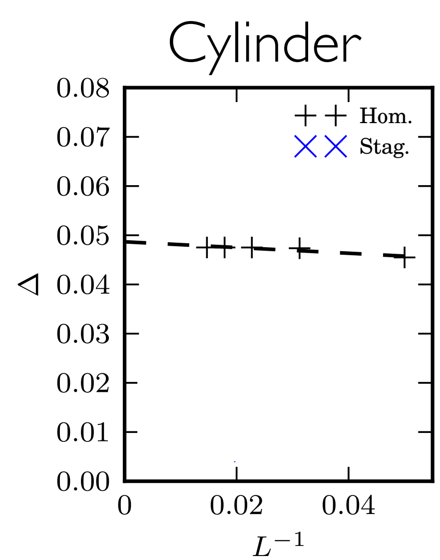



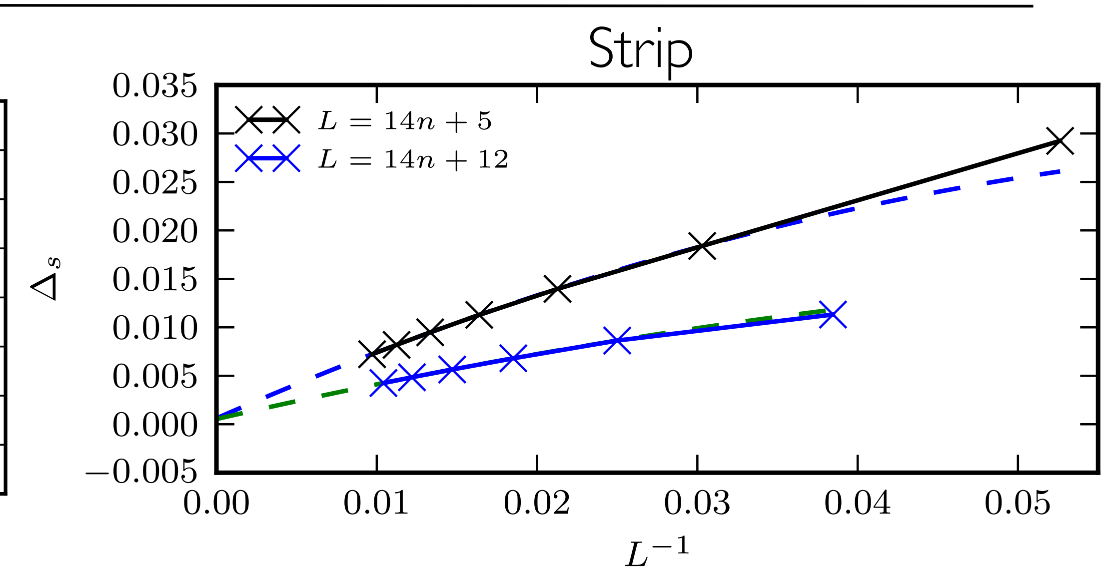
# Edge state



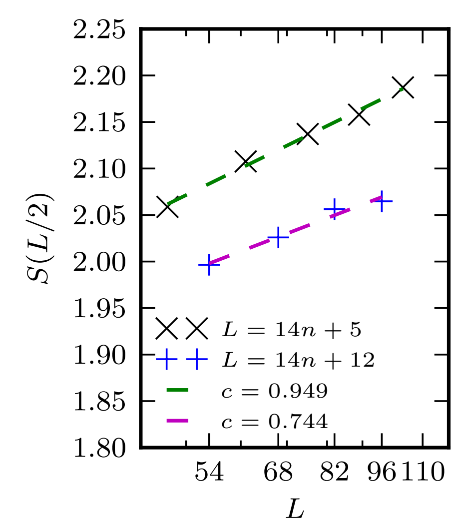



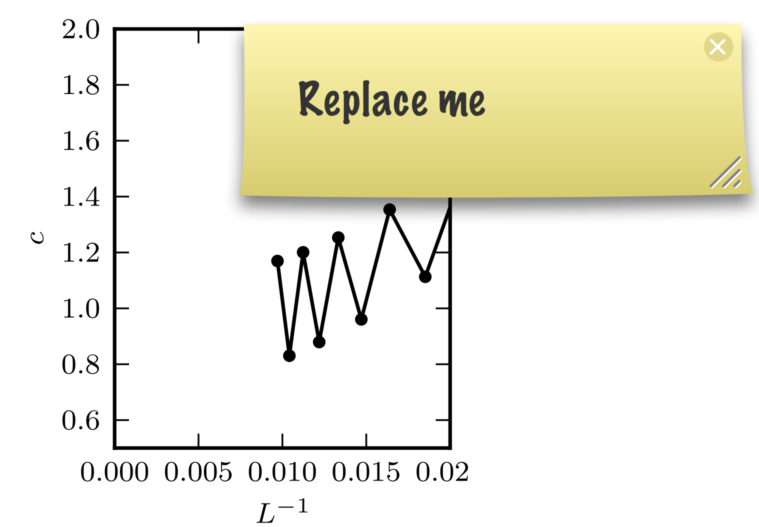

- Spin gap for a fully open system:
  $\Delta \sim L^{-1}$




- Entanglement entropy:
  *$S \sim \log(L)$*
- ***Fit consistent with central charge c=1***





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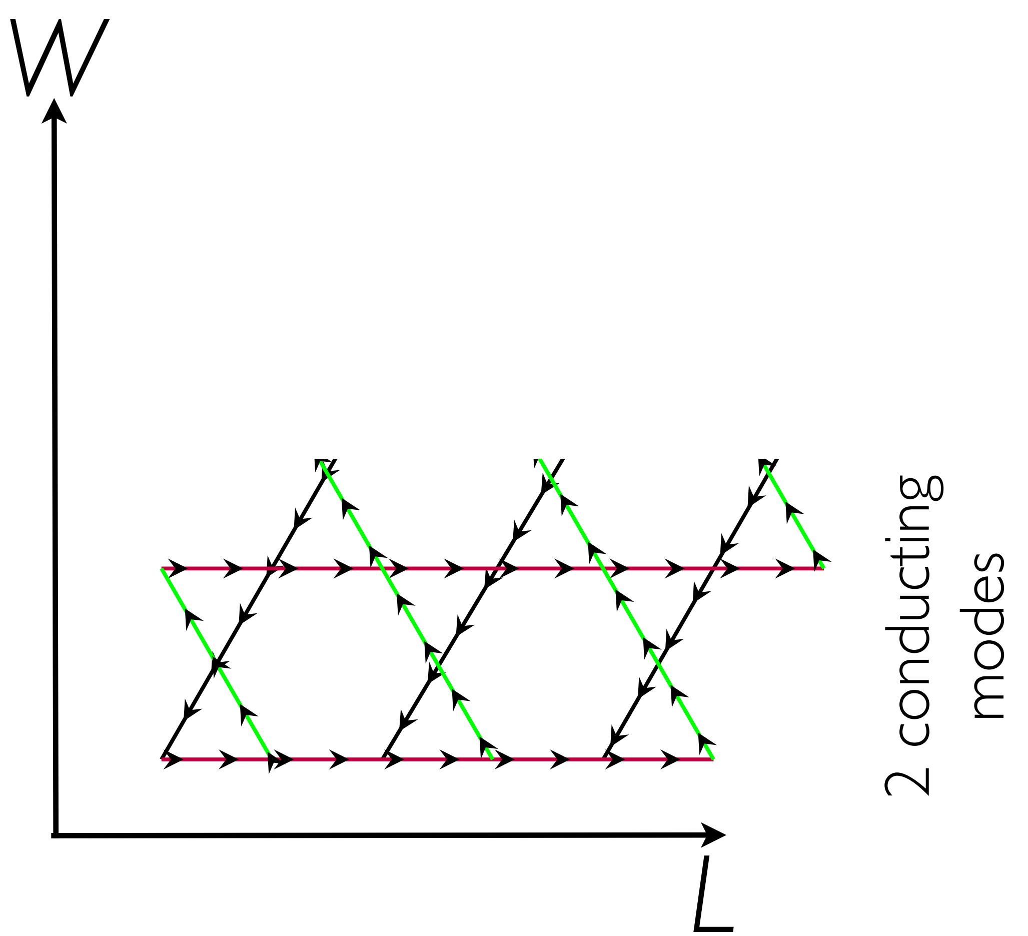
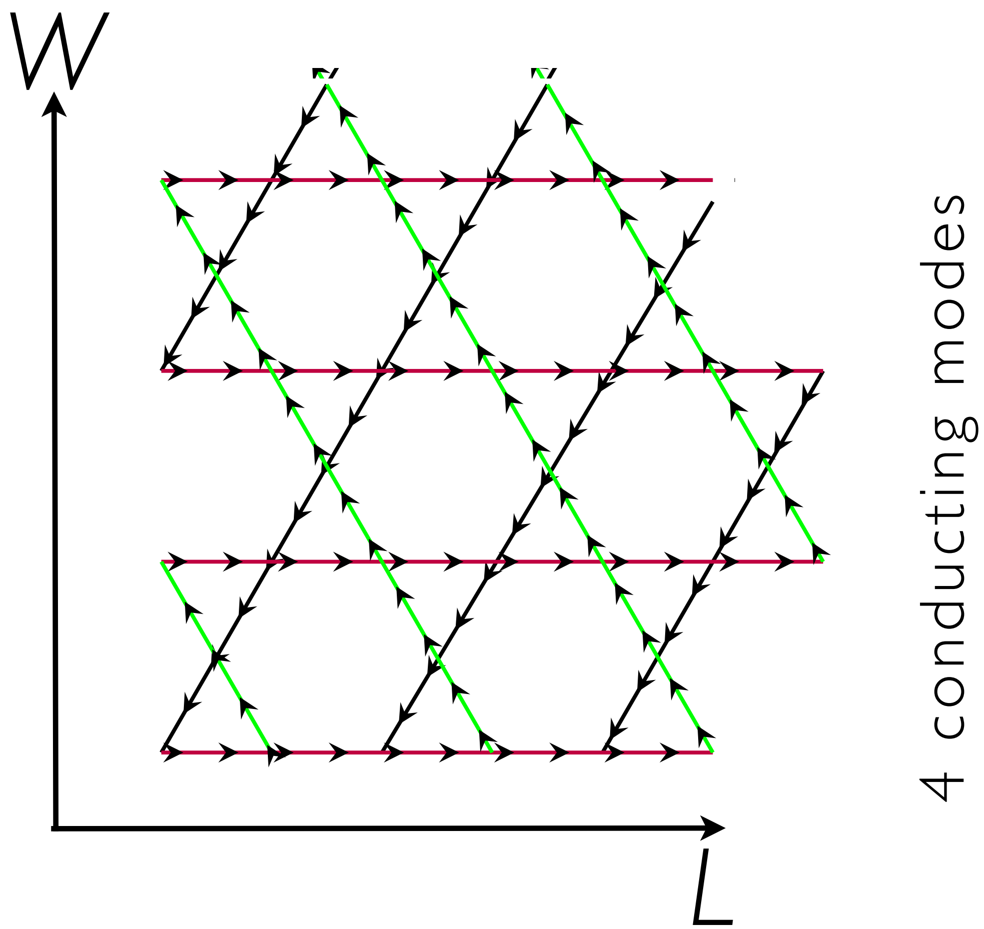
# The same with spins?

- Our numerical agenda:
  - *Topological phase: consistent with $\nu=1/2$ bosonic Laughlin state*
    - Spin gap
    - Degeneracy for torus vs cylinder
    - Edge state on strip vs cylinder
  - *Gapless phase*
    - Spin gap
    - Central charge

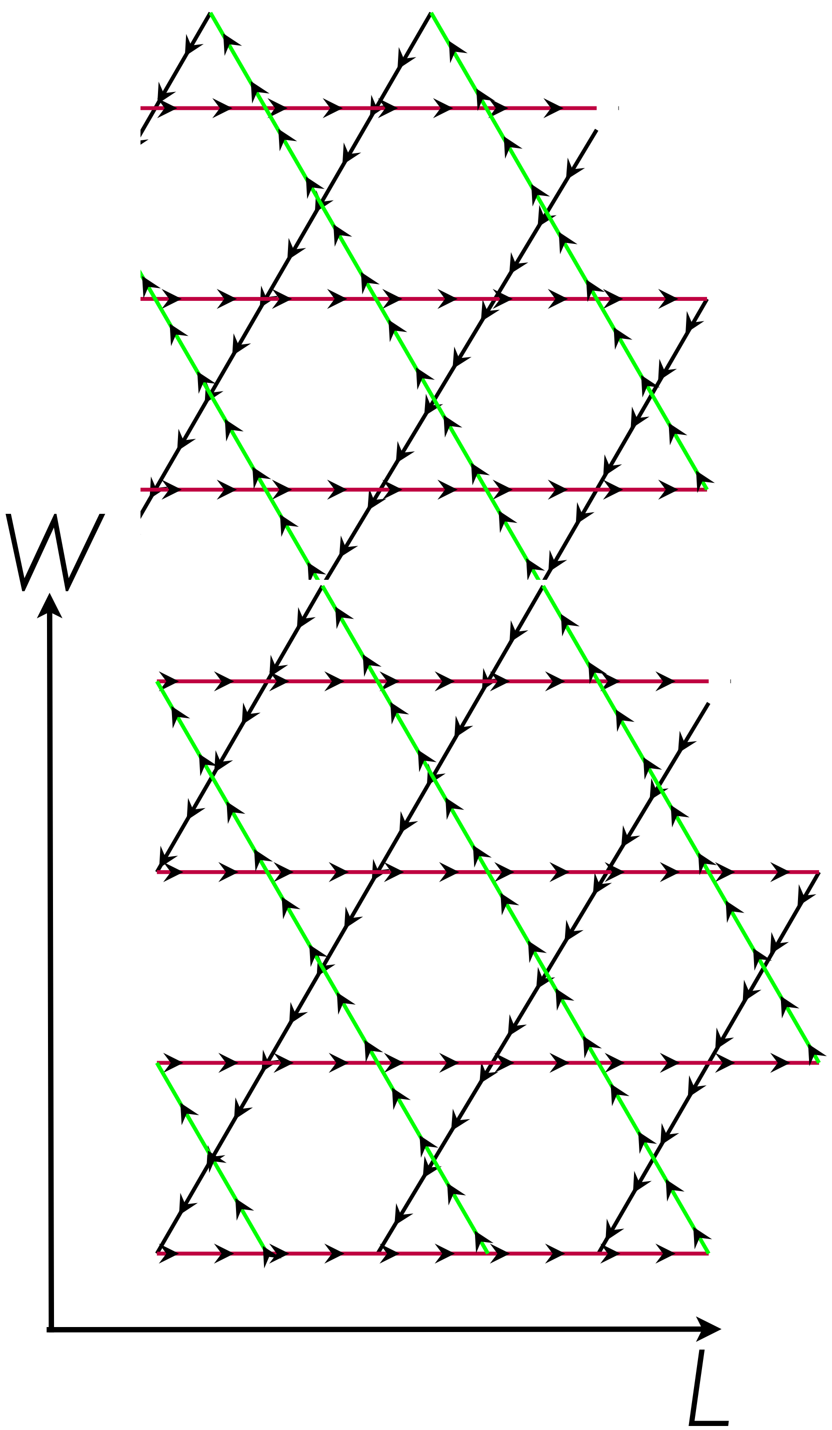
# Quasi-1d predecessors



# Quasi-1*d* predecessors

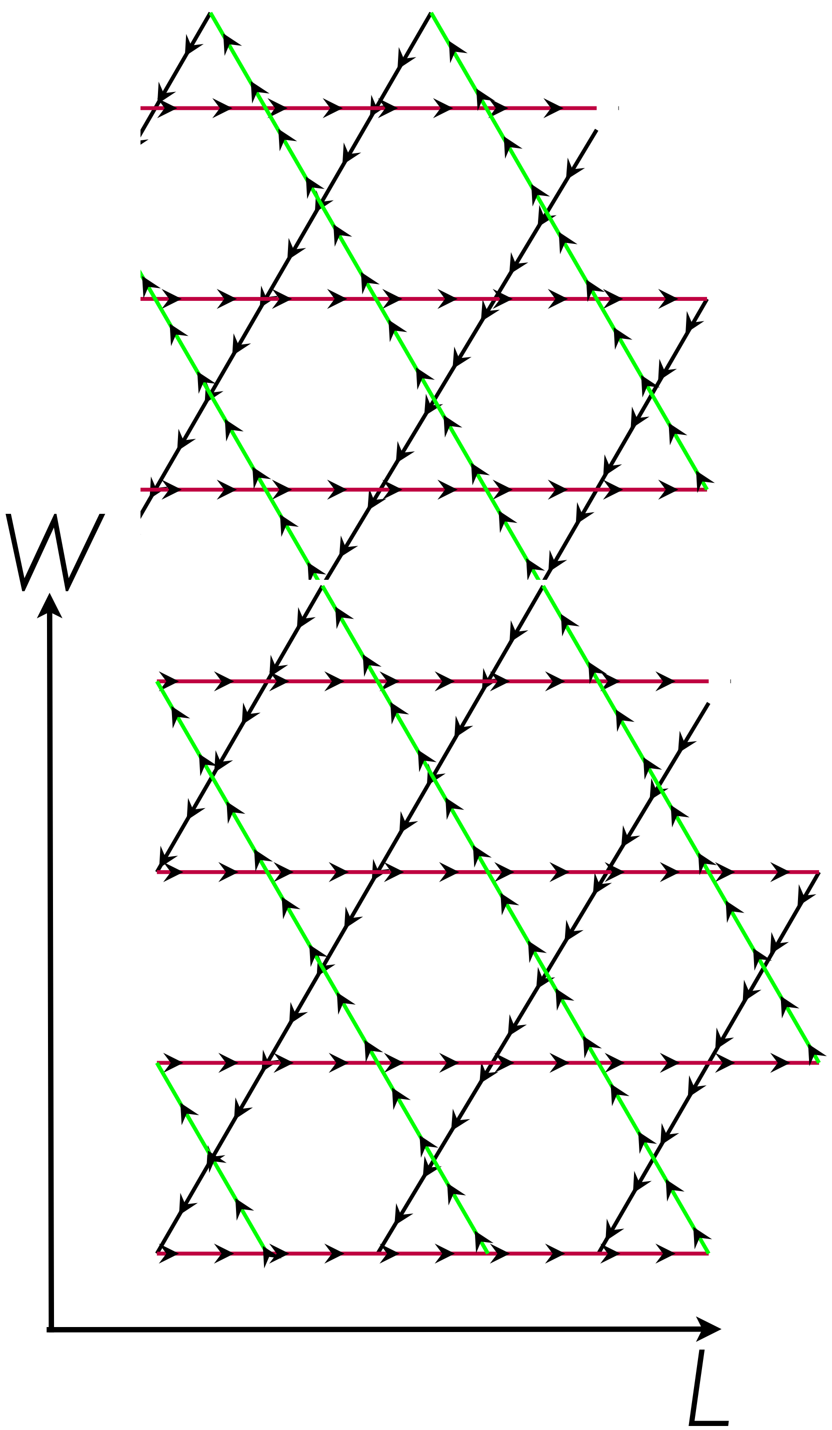


# Quasi-*1d* predecessors

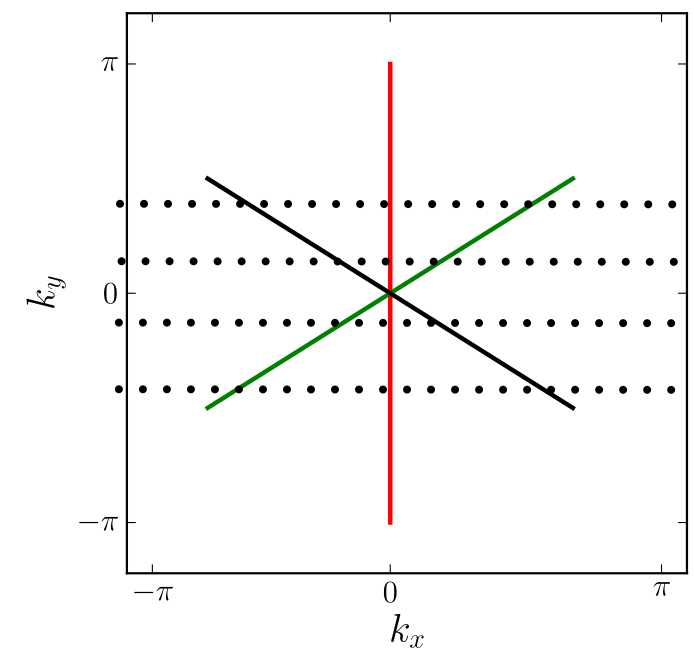



W conducting modes

# Quasi-1*d* predecessors



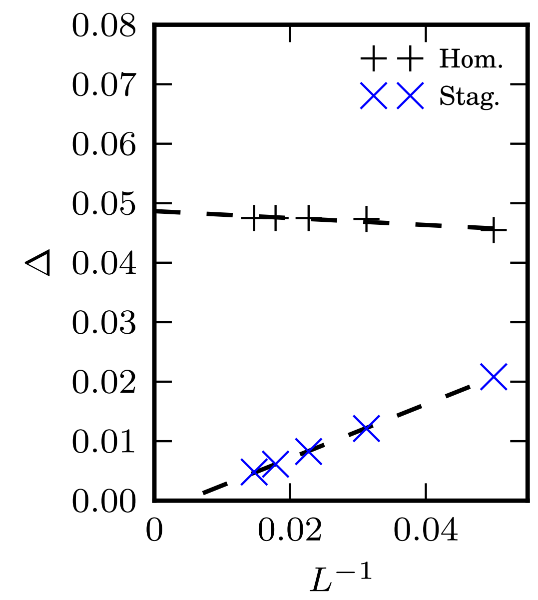

W conducting modes




W
~ # of ky points
~ number of gapless modes

# Staggered phase

- Gap vanishes as 1/L

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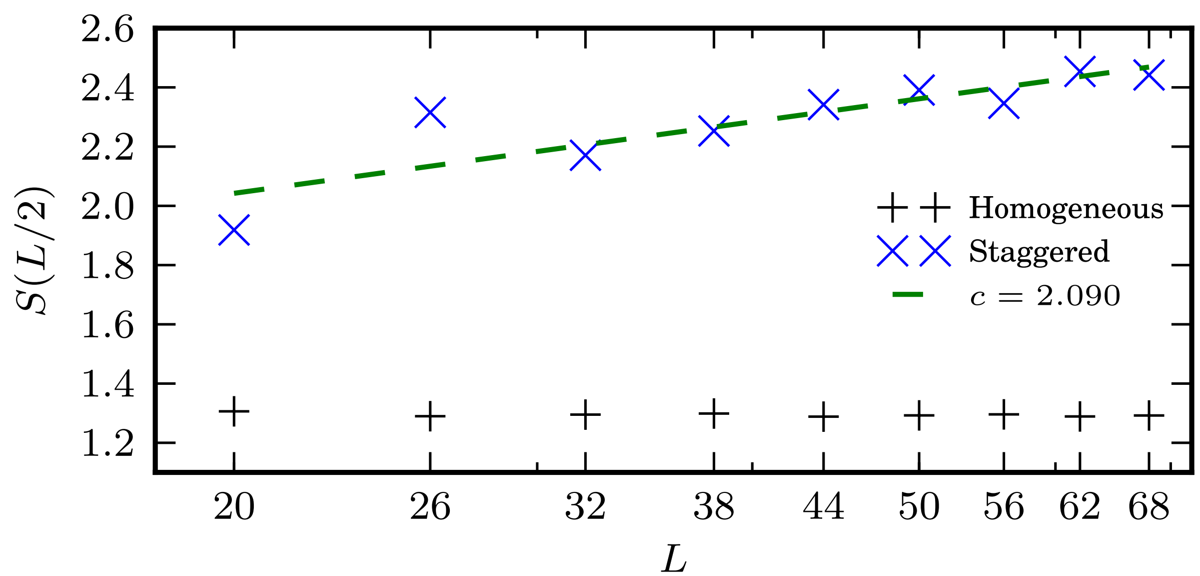
- Entanglement entropy at the center of the system
    - Gapped: $S \sim const$
    - Gapless: $S \sim log(L)$
- ***Fit yields expected c=2***



# The same with spins?

- Our numerical agenda:
  - *Topological phase: consistent with $\nu=1/2$ bosonic Laughlin state*
    - 👍 Spin gap
    - 👍 Degeneracy for torus vs cylinder
    - 👍 Edge state on strip vs cylinder
  - *Gapless phase*
    - 👍 Spin gap
    - 👍 Central charge for $W=2$

# Conclusions & Outlook

- We can construct a topological spin liquid and a gapless phase with an emergent surface of excitations from coupling the edge states encircling seeds of a topological phase.
- We can predict the shape of the Fermi surface from the edge state picture.
- Ongoing work:
  - Understand the effect of interactions on the gapless lines in the SU(2) spin liquid (stability of the gapless phase)
- Future work:
  - Other numerical approaches for gapless phase? VMC?
  - Possible relations to other models?

Thank you for your attention!