KITP Conference : Exotic Phases of Frustrated Magnets

(Oct 08 – Oct 12, 2012)

Identifying Topological Order by Entanglement Entropy in Physical Realistic Models

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HCJ, H. Yao, L. Balents, PRB 86, 024424

HCJ, Z. Wang, L. Balents, arXiv:1205.4289

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Oct. 09, 2012, KITP, UCSB

Outline

- Determining topological order by entanglement entropy

  (1) Introduction and Motivation

  (2) Cylinder construction: DMRG and Toric-Code model

- Topological spin liquid (SL) state in physical realistic models

  (1) Topological SL state of the S=1/2 Kagme Heisenberg model

  (2) Topological SL state of the S=1/2 Square J_1-J_2 Heisenberg model

- Summary and Conclusion

Conventional and Exotic States of Matters

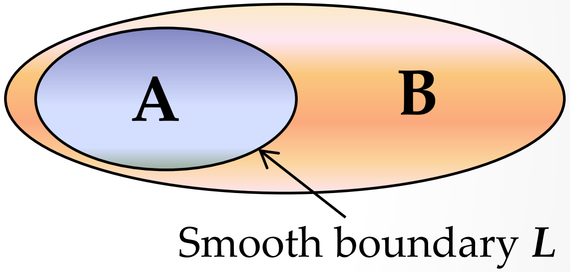
- **Topological order** (X. G. Wen, et al)

  Related to long range entanglement, and can be a new set of quantum numbers, such as **ground state degeneracy**, **quasiparticle fractional statistics**, **edge states**, **topological entanglement entropy**, etc.

- Von Neumann Entanglement Entropy

$$S_1(\rho_A) = -\mathrm{Tr}(\rho_A \ln \rho_A)$$

$$\rho_A = \mathrm{Tr}_B |\psi\rangle\langle\psi|$$



- Gapped phase

$$S(A) = \boxed{aL} - \boxed{\gamma}$$

Kitaev and Preskill Phys. Rev. Lett. 96, 110404 (2006)
Levin and Wen, Phys. Rev. Lett. 96, 110405 (2006)

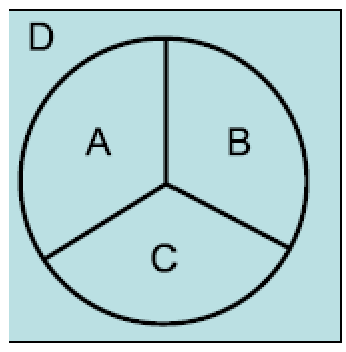
Boundary law term

Universal constant term
topological entanglement entropy (TEE) γ

(1) For topological trivial phase, γ=0;
(2) For topological ordered phase, γ=ln(*D*) (*D* the total quantum dimension)

Kitaev-Preskill and Levin-Wen construction

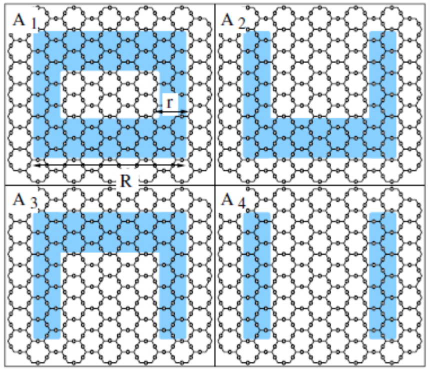
Kitaev-Preskill construction
(Microscopic Hamiltonian)



$$S_{\text{topo}} = -\gamma$$

$$S_{\text{topo}} \equiv S_A + S_B + S_C - S_{AB} - S_{BC} - S_{AC} + S_{ABC}$$

Levin-Wen construction
(Ground state wavefunction)



$$S_{\text{topo}} \equiv (S_1 - S_2) - (S_3 - S_4) = -\log(D^2) = -2\gamma$$

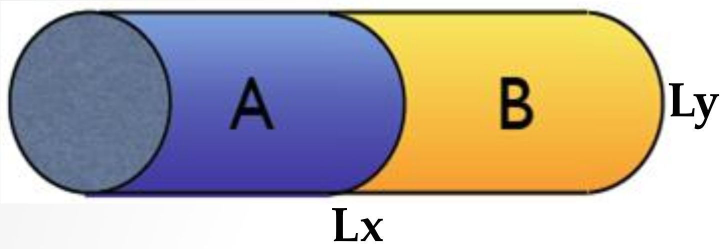
- The cancellation of the "area law" terms and corner contributions will allow us to extract the universal constant term, i.e., TEE γ.

  Kitaev and Preskill, PRL 2006; Levin and Wen, PRL 2006

- However, in practical applications, large finite-size effect due to sharp corners can be problematic.

  Zhang, Grover, Truner, Oshikawa, Vishwanath, PRB 2012

Cylinder construction



Von Neumann Entanglement Entropy
In infinite cylinder Lx= ∞

$$S(A) = aL_y - \gamma$$

However, there is an ambiguity in estimate of TEE, when topological degeneracy is present, e.g., the chiral spin liquid on the torus. The estimated TEE will be 0≤γ≤Ln(D) .

Zhang, Grover, Truner, Oshikawa, Vishwanath, PRB 2012

For cylinder construction, we show that in the long cylinder limit, i.e., Lx= ∞, DMRG naturally favors Minimal Entropy States (MES) with maximal value of TEE.

(a) For topological ordered state, γ=Ln(D),

(b) For topological trivial state, γ=0.

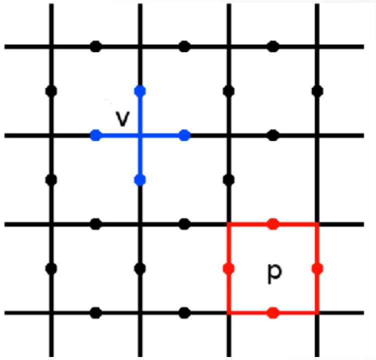
HCJ, Z. Wang, and L. Balents, arXiv:1205.4289

Cylinder construction: Toric code model

1. Pure toric-code model

$$H_{TC} = -J_x \sum_s A_s - J_z \sum_p B_p$$

$$A_v = \prod_{i \in v} \sigma_i^x, \; B_p = \prod_{i \in p} \sigma_i^z.$$



(1) Exactly solvable

(2) Z_2 topological ordered ground state

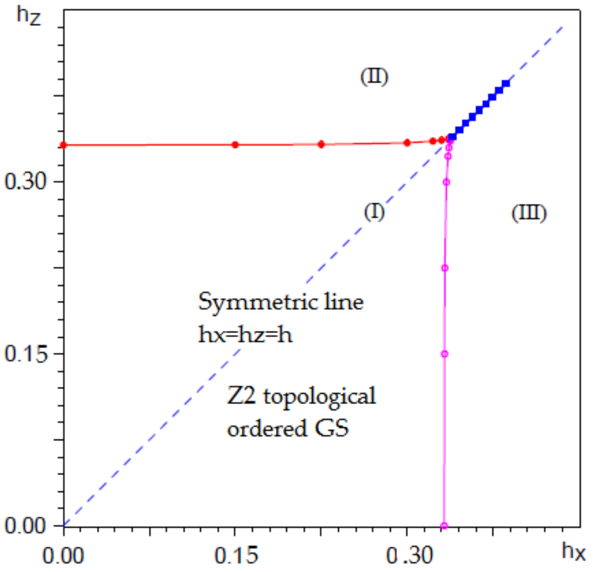
(3) Zero correlation length ξ=0

Kitaev 2003

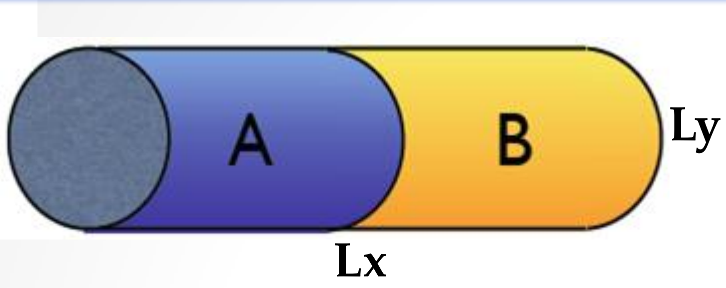
2. Toric-code model in magnetic fields

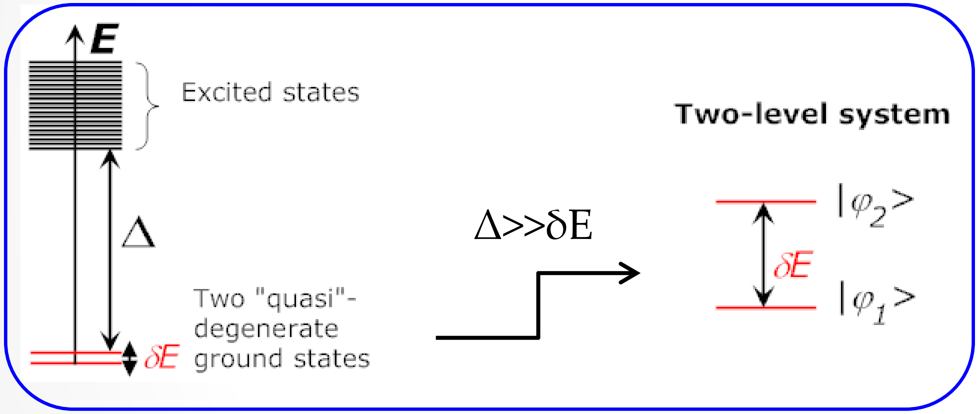
$$H_Q = H_{TC} - h_x \sum_b \sigma_b^x - h_z \sum_b \sigma_b^z$$

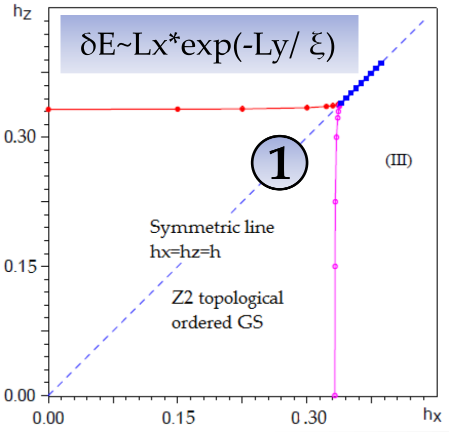
I.S. Tupitsyn et.al., PRB 2010

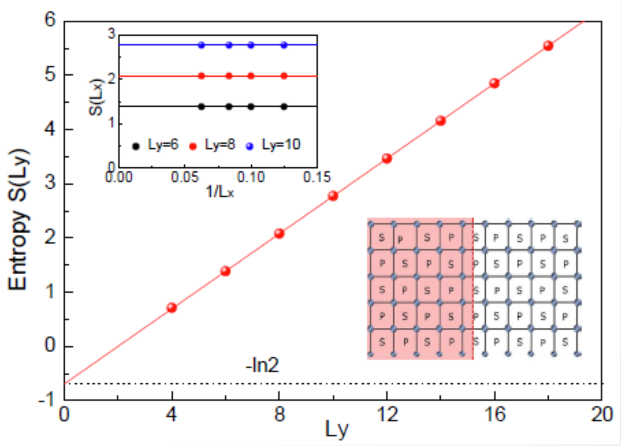


Cylinder construction: DMRG and Toric-Code model









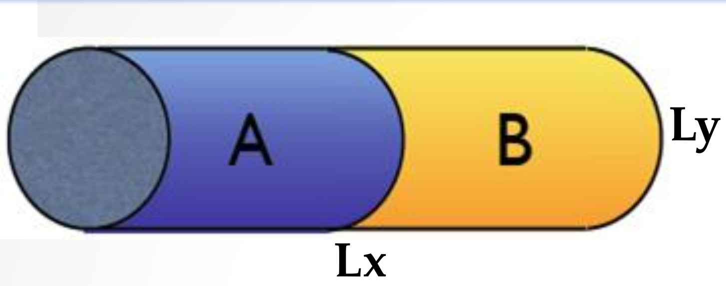
① Topological splitting is **δE ~ Lx*exp{-Ly/ ξ}.**

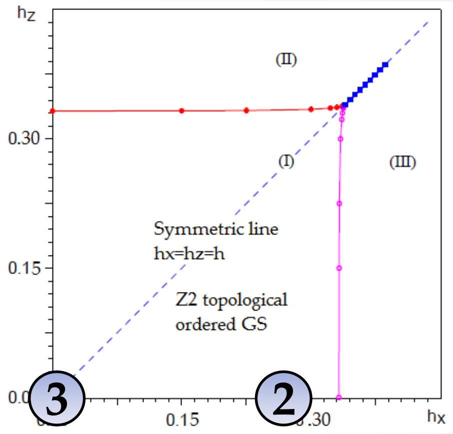
E.g., TC-model with magnetic field along the symmetric line (h>0), with $\xi_x = \xi_z = \xi > 0$.

δE=∞, when Lx =∞, i.e., the infinite cylinder, only |φ_1>, is obtained with maximal and ideal TEE γ=Ln(D). Such a state is Minimal Entropy State (MES).

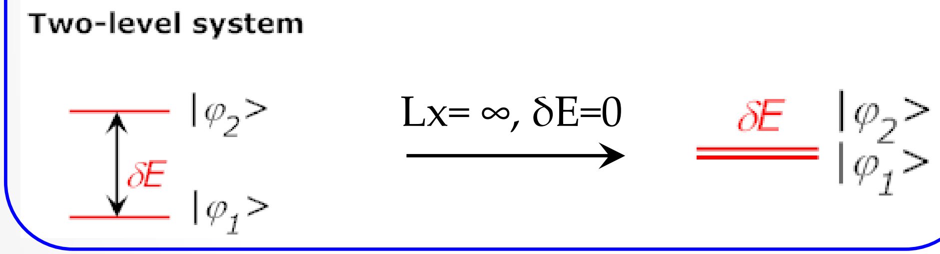
HCJ, Z. Wang, and L. Balents, arXiv:1205.4289

Cylinder construction: DMRG and Toric-Code model





② For hx≠0, hz=0, so $\xi_x>0$, while $\xi_z=0$.
δE~Ly*exp{-Lx/ξx}. So, **δE=0, when Lx =∞**



③ For pure TC model, hx=hz=0,
and $\xi_x=\xi_z=0$. Therefore,
δE=0, for any Lx and Ly

Therefore, for both cases, δE=0, when Lx =∞. Can we always get MES as well?

Answer is Yes!

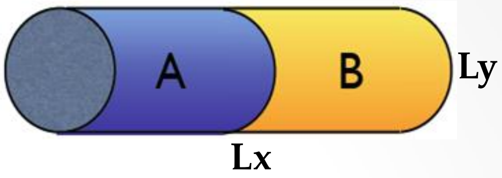
HCJ, Z. Wang, and L. Balents, arXiv:1205.4289

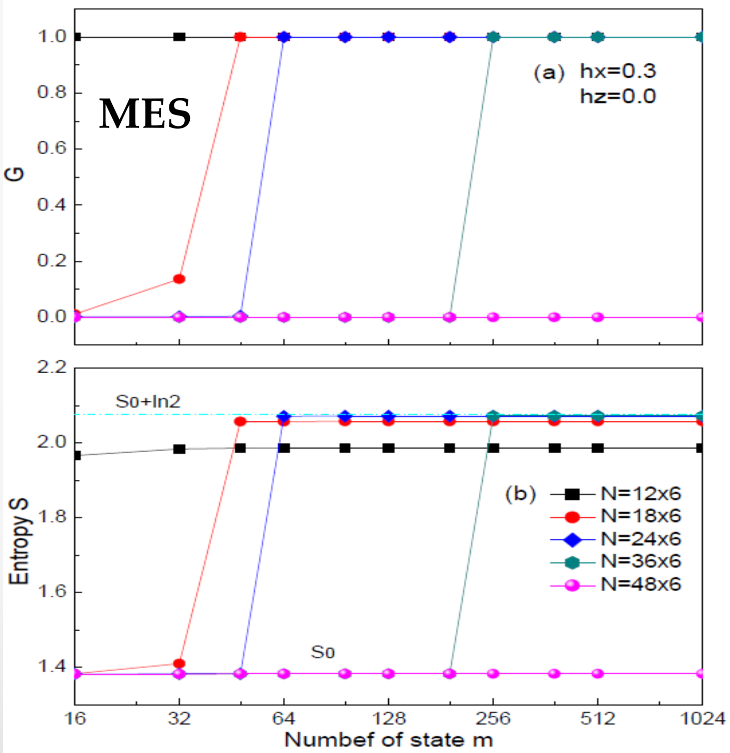
Cylinder construction: DMRG and Toric-Code model

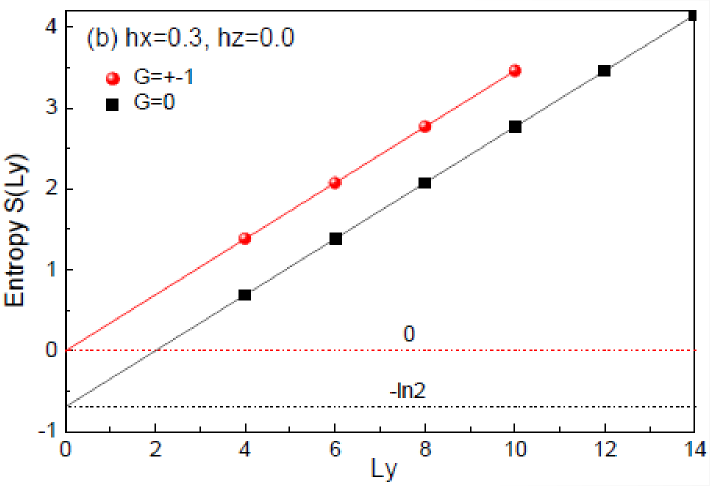
Toric-code model with magnetic field hx

$$H_Q = H_{TC} - h_x \sum_b \sigma_b^x$$

$$G \;=\; G_y = \prod_{x=1}^{L_x} \sigma_{x,y}^x \;=\pm 1$$







1. Global state |G=±1> gives γ=0
2. Minimal entangled state (MES) |G=0>, gives TEE γ=ln(D)

HCJ, Z. Wang, and L. Balents, arXiv:1205.4289

Cylinder construction: DMRG and Toric-Code model

Toric-code model with magnetic field hx

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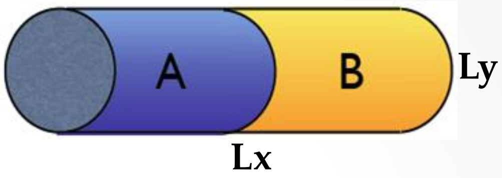
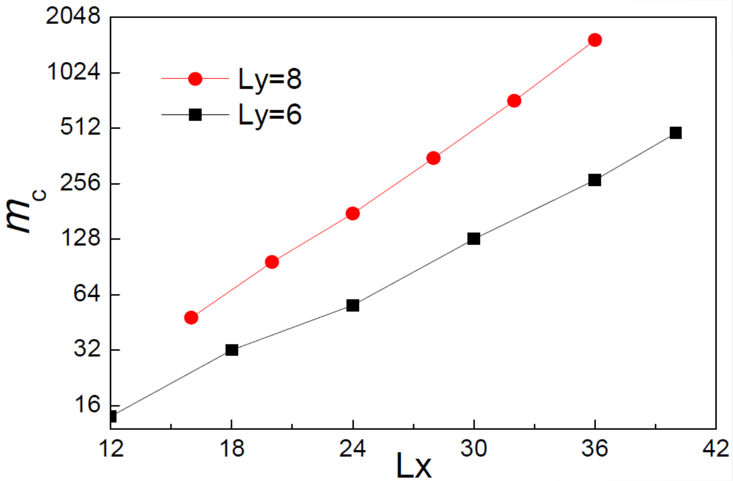
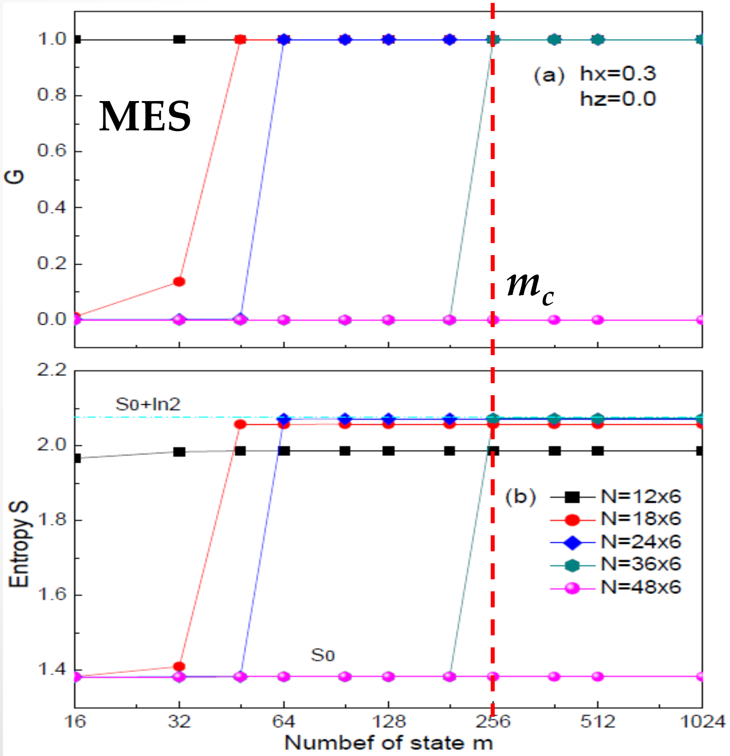






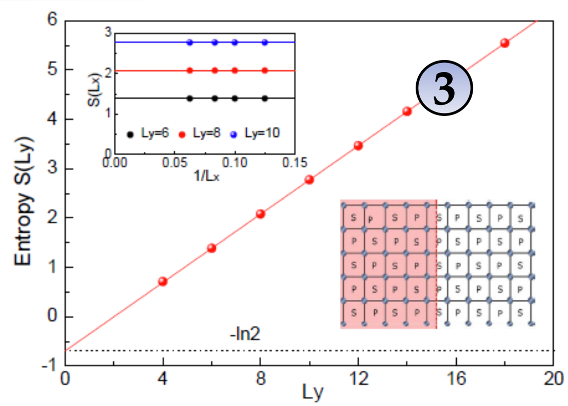
1. m_c grows exponentially fast with Lx
2. In the infinite cylinder limit, i.e., Lx =∞, MES is guaranteed with maximal and ideal value γ=ln(D)

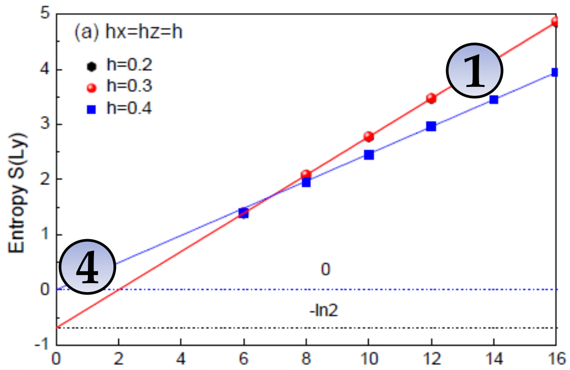
HCJ, Z. Wang, and L. Balents, arXiv:1205.4289

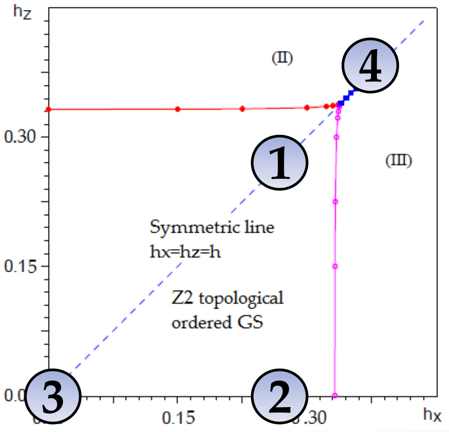
Cylinder construction: DMRG and Toric-Code model

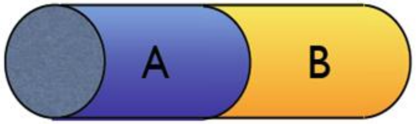
Toric-code model in magnetic fields

$$H_Q = H_{TC} - h_x \sum_b \sigma_b^x - h_z \sum_b \sigma_b^z$$









For cylinder construction, we show that

in the long cylinder limit, i.e., Lx= ∞, DMRG naturally favors MES with maximal TEE.

(a) For topological ordered state, γ=Ln(D),

(b) For topological trivial state, γ=0.

HCJ, Z. Wang, and L. Balents, arXiv:1205.4289

Outline

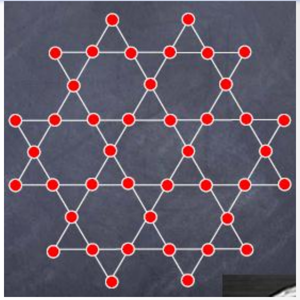
- Determining topological order by entanglement entropy
  - (1) Introduction and Motivation
  - (2) Kitaev-Preskill and Levin-Wen Construction
  - (3) Cylinder construction: DMRG and Toric-Code model
- Topological spin liquid (SL) state of physical realistic models
  - (1) Topological SL state of the S=1/2 Kagme Heisenberg model
  - (2) Topological SL state of the S=1/2 Square J_1-J_2 Heisenberg model
- Summary and Conclusion

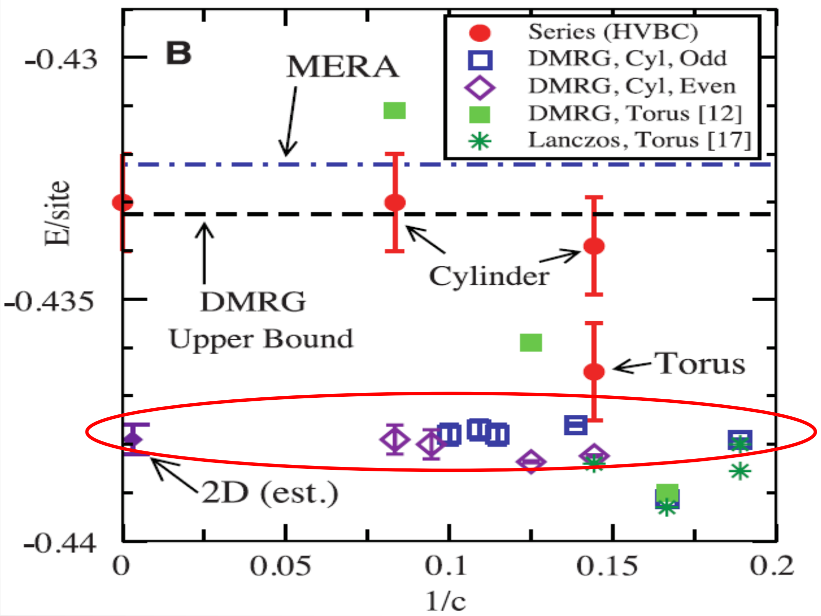
S=1/2 Kagome J_1-J_2 Heisenberg model

$$H = J_1 \sum_{<ij>} S_i \cdot S_j + J_2 \sum_{<<ij>>} S_i \cdot S_j$$

Quantum spin liquid GS

1) No magnetic order (ξ~1)
2) No VBS order (ξ~1)
3) Spin excitation is fully gapped





H.C. Jiang et al, PRL 2009

S. Yan et al., Science 2011

H. C. Jiang et al, arXiv:1205.4289

S. Depenbrock et al, arXiv:1205.4858

S. White, talk in March Meeting 2012

1. The $J_2 = 0$ point is near the edge of a substantial spin liquid phase centered near $J_2 = 0.05$-0.15.
2. For example, at $J_2 = 0.05$-0.15, spin singlet and triplet gaps are robust, and around 0.15.

See S. White's talk this morning for detail

S=1/2 Kagome J_1-J_2 Heisenberg model

$$H = J_1 \sum_{<ij>} S_i \cdot S_j + J_2 \sum_{<<ij>>} S_i \cdot S_j$$

Quantum spin liquid GS

1) No magnetic order (ξ~1)
2) No VBS order (ξ~1)
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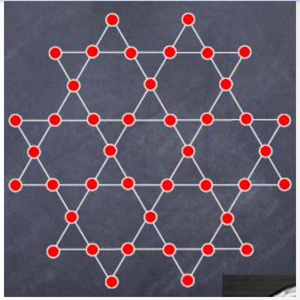
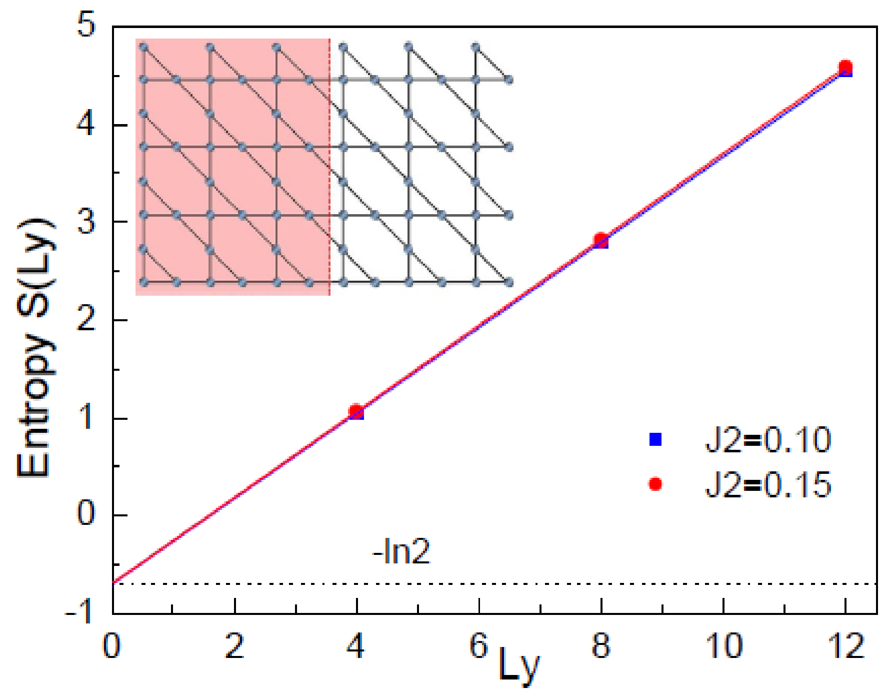




TEE is γ=ln(2)=0.693

(1) J2=0.10, γ=0.698(8)

(2) J2=0.15, γ=0.694(6)

Positive evidence for the topological spin liquid of Kagome Heisenberg model

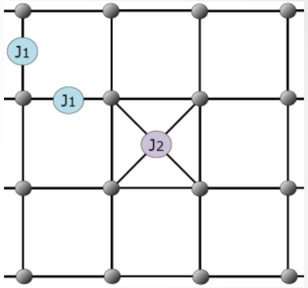
HCJ, Z. Wang, and L. Balents, arXiv:1205.4289

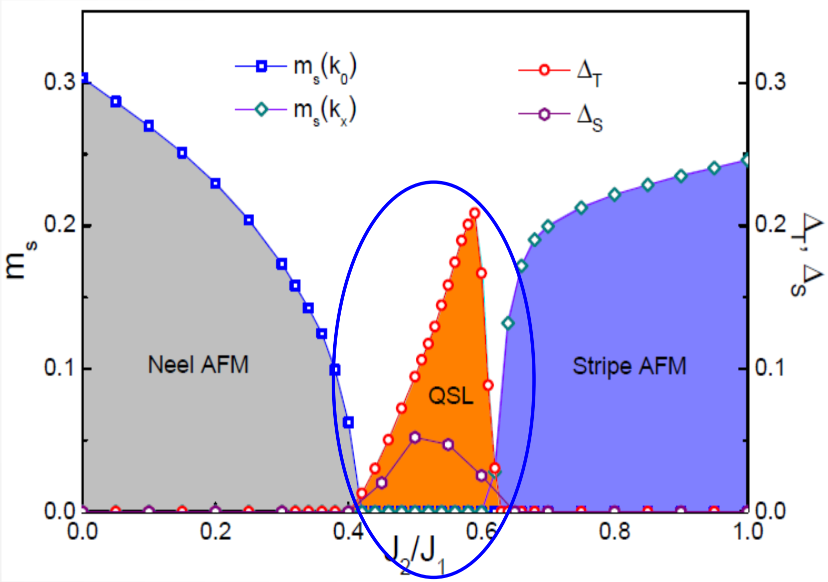
S=1/2 Square J_1-J_2 Heisenberg model

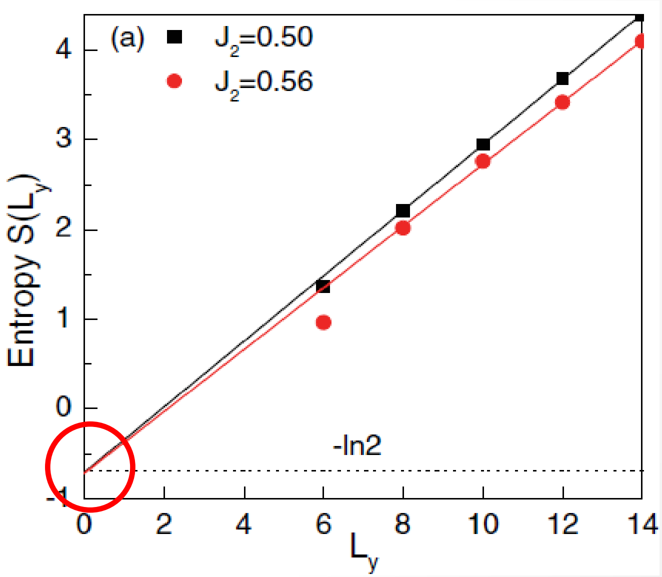
$$H = J_1 \sum_{<ij>} S_i \cdot S_j + J_2 \sum_{<<ij>>} S_i \cdot S_j$$

Quantum spin liquid GS
at $0.41<J_2/J_1<0.62$ region

1) No magnetic order (ξ~2-3)
2) No VBS order (ξ~4-5)
3) Spin excitation is fully gapped







HCJ, H. Yao, L. Balents, PRB 86, 024424

L. Wang, Z. C. Gu, F. Verstraete, X. G. Wen, arXiv.1112.3331

Topological Entanglement Entropy

(1) J_2=0.50, γ =0.70(2)

(2) J_2=0.56, γ =0.72(4)

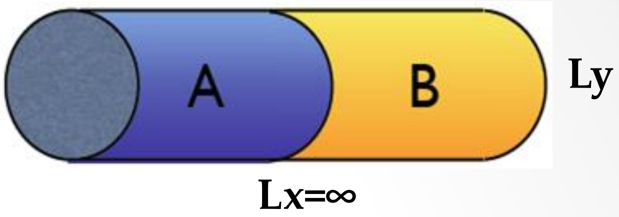
See H. C. Jiang's talk 9/11/2012, KITP

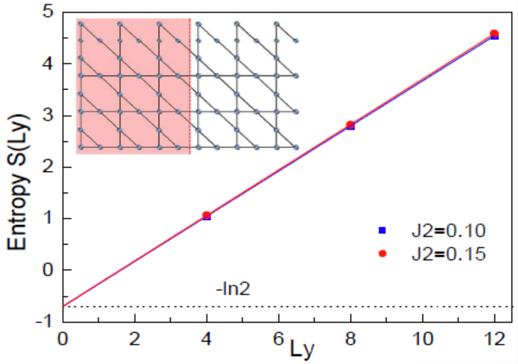
Summary and Conclusion

1. For cylinder construction, we show that in the long cylinder limit, i.e., Lx= ∞, DMRG naturally favors MES with maximal TEE.

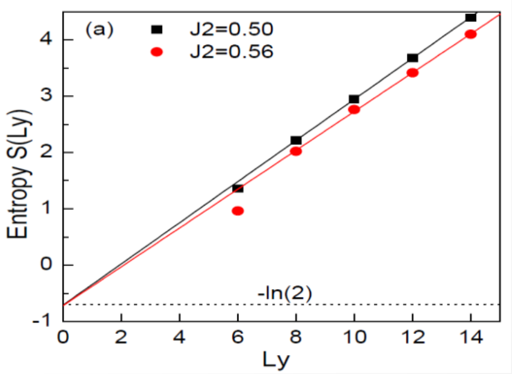
(a) For topological ordered state, γ=Ln(D),

(b) For topological trivial state, γ=0.





2, Give positive evidence to show that the ground state of S=1/2 Kagome Heisenberg model is topological QSL with TEE γ=ln(2)



3, Give positive evidence to show that the ground state of S=1/2 Square J_1-J_2 Heisenberg model is topological QSL with TEE γ=ln(2)