3d Symmetry Protected Topological Phases close to AF Neel order

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References:
arXiv:1209.4399
arXiv:1112.5303

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Outline:

(1) Introduction: Conventional and Exotic quantum ground states, definition for SPT phases;

(2) Review of 1d SPT with SO(3) symmetry, i.e. Haldane phase, field theory and lattice spin wave function using slave-fermion;

(3) 3d SPT of SU(2N) spin system, generalization of 1d Haldane phase, field theory and lattice spin wave function.

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Nontrivial/exotic quantum disordered phase:

Type 1: topological phase: fully gapped, topological ground state degeneracy, fractionalization, etc.
Example: fractional quantum Hall liquids.

Type 2: stable gapless spin/Bose liquid phase, power-law correlation,
Example: 1d spin chain, all the gapless spin liquids in 2d/3d materials

Type 3: symmetry protected topological phase.
3.1 bulk is gapped, nondegenerate,
3.2 with certain symmetry, boundary is either gapless or degenerate,
3.3 boundary cannot be realized as a low dimensional lattice system,
Example: 2d quantum spin Hall, 3d topological insulator
(with interaction)

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SPT has become very popular lately:

arXiv:1106.4772, Chen, Gu, Liu, Wen,
Cohomology classification of bosonic SPT.

arXiv:1201.2648, Gu, Wen,
SPT for fermions, super-cohomology group classification.

arXiv:1206.1604, Levin, Senthil,
Construction for 2d bosonic SPT with **U(1)** symmetry.

arXiv:1205.3156, Lu, Vishwanath,
2d SPT with **U(1)** symmetry, classification using CS theory.

arXiv:1209.3058, Vishwanath, Senthil,
Description of 3d SPT using Θ-term in EM response, + ……

More recent ones: Grover, Vishwanath, and Lu, Lee……

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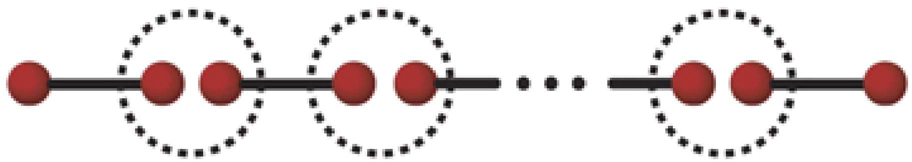
Will focus on **Bosonic spin** systems with **nonabelian** spin symmetry in this talk.

1d SPT with SO(3), Haldane phase

1d spin-1 chain,



Field theory description: O(3) NLSM + Θ-term, for $\pi_2[S^2] = Z$.
This field theory gives Z2 classification of 1d SO(3)-SPT

$$L = \frac{1}{g}(\partial_\mu \vec{n})^2 + \frac{i\Theta}{8\pi}\epsilon_{abc}\epsilon_{\mu\nu}n^a\partial_\mu n^b\partial_\nu n^c \qquad \Theta = 2\pi$$

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Wave function description: using slave-fermion as an example.

$$\vec{S}_j = \frac{1}{2}\sum_{A=1}^{2} f^\dagger_{j,A,\alpha}\vec{\sigma}_{\alpha\beta} f_{j,A,\beta}$$

Describing spin-1 operator using spin-1/2 slave-fermion with 2 colors, with the following gauge constraints:

$$\sum_{\alpha,A} f^\dagger_{i,A,\alpha} f_{i,A,\alpha} = 2$$

$$\sum_{\alpha,A,B} f^\dagger_{j,A,\alpha}\rho^{\mu}_{AB} f_{j,B,\alpha} = 0,$$

Fixed on-site particle number

Color-singlet on every site

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Haldane phase corresponds to the following slave fermion mean field state:

Haldane phase corresponds to the following slave fermion field theory:

$$\mathcal{L} = \bar{\psi}\gamma_\mu \partial_\mu \psi + m_4 \bar{\psi}\rho^z \psi$$

Coupling slave-fermions to Neel order parameter:

$$(-1)^j \vec{n}_j \cdot \sum_A f^\dagger_{j,A,\alpha} \vec{\sigma}_{\alpha\beta} f_{j,A,\beta} \sim \vec{n} \cdot \bar{\psi}\gamma_5 \vec{\sigma}\psi$$

Θ-term with Θ = 2π is generated after integrating out the slave fermions.

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Generalization to 3d

First thought: One state of the Haldane phase is the 1d AKLT state, then we just need 2d/3d AKLT state, whose edge states is a1d/2d spin-1/2 system, hence must be gapless ore degenerate.

However, the stability of Haldane phase does not need any translation symmetry; while **2d and 3d AKLT state relies on translation symmetry.**

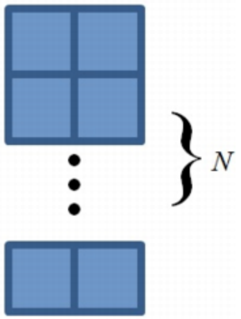
We want to look for 3d generalization of Haldane phase that does not need any translation symmetry.

Generalization to 3d

Basic idea: Haldane phase is described by a NLSM + Θ-term,
Because Neel order has manifold $\mathbf{S^2}$, and $\boldsymbol{\pi_2[S^2] = Z}$.

Then in 3d, in order to find analogue of Haldane phase, we should first look for spin systems whose Neel order has manifold M, that satisfies $\boldsymbol{\pi_4[M] = Z}$.

This is satisfied in SU(2N) spin system with self-conjugate rep: (actual symmetry is PSU(2N) = SU(2N)/Z_{2N})



$$\mathcal{M} = \frac{\mathrm{U}(2N)}{\mathrm{U}(N) \times \mathrm{U}(N)}, \quad \pi_4[\mathcal{M}] = \mathbb{Z}, \text{ for } N \geq 2$$

For $N = 1$, $M = S^2$, so M is a SU(2N) generalization of Neel order.

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Generalization to 3d

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For $N = 1$, $M = S^2$, so M is a SU(2N) generalization of Neel order.

Neel order parameter P, can be represented as

$$\mathcal{P} = V^\dagger \Omega V, \quad \Omega = \begin{pmatrix} \mathbf{1}_{N\times N}, & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N}, & -\mathbf{1}_{N\times N} \end{pmatrix}$$

When $N = 1$, $\mathcal{P} = \vec{n} \cdot \vec{\sigma}$

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Comparing 1d Haldane phase, and new 3d SPT

Haldane phase for spin-1

1, The field theory:
1+1d O(3) NLSM + Θ-term,
at Θ = 2π;

$$\mathcal{S} = \int dx\, d\tau\ \frac{1}{g}(\partial_\mu \vec{n})^2 + \frac{i\Theta}{8\pi}\epsilon_{\mu\nu}\epsilon_{abc}\ n^a \partial_\mu n^b \partial_\nu n^c$$

New 3d SPT for SU(2N) spin

1, The field theory:
3+1d NLSM + Θ-term,
at Θ = 2π;

$$\mathcal{S} = \int d^3x\, d\tau\ \frac{1}{g}\mathrm{tr}[\partial_\mu \mathcal{P} \partial_\mu \mathcal{P}] + \frac{i\Theta}{256\pi^2}\mathrm{tr}[\mathcal{P}\partial_\mu \mathcal{P}\partial_\nu \mathcal{P}\partial_\rho \mathcal{P}\partial_\lambda \mathcal{P}]\epsilon_{\mu\nu\rho\lambda}$$

Comparing 1d Haldane phase, and new 3d SPT

Haldane phase for spin-1

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1+1d O(3) NLSM + Θ-term,
at Θ = 2π;

$$\mathcal{S} = \int dx d\tau \ \frac{1}{g}\mathrm{tr}[\partial_\mu \mathcal{P} \partial_\mu \mathcal{P}]$$

$$+ \ \frac{\Theta}{16\pi}\epsilon_{\mu\nu}\mathrm{tr}[\mathcal{P}\partial_\mu \mathcal{P}\partial_\nu \mathcal{P}].$$

$$\mathcal{P} = \vec{n}\cdot\vec{\sigma}$$

New 3d SPT for SU(2N) spin

1, The field theory:
3+1d NLSM + Θ-term,
at Θ = 2π;

$$\mathcal{S} = \int d^3x d\tau \ \frac{1}{g}\mathrm{tr}[\partial_\mu \mathcal{P} \partial_\mu \mathcal{P}]$$

$$+ \ \frac{i\Theta}{256\pi^2}\mathrm{tr}[\mathcal{P}\partial_\mu \mathcal{P}\partial_\nu \mathcal{P}\partial_\rho \mathcal{P}\partial_\lambda \mathcal{P}]\epsilon_{\mu\nu\rho\lambda}$$

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Comparing 1d Haldane phase, and new 3d SPT

Haldane phase for spin-1

2, boundary theory is a single free spin-1/2

3, the edge state carries projective representation of SO(3), i.e. not invariant under the Z_2 center of SU(2).

New 3d SPT for SU(2N) spin

2, boundary theory (2+1d) must be either gapless or degenerate (for detailed argument, please read arXiv:1209.4399)

3, edge states cannot be constructed using reps of PSU(2N) group, i.e. reps of SU(2N) invariant under Z_{2N} center. (for detailed argument, please read arXiv:1209.4399)

Comparing 1d Haldane phase, and new 3d SPT

Haldane phase for spin-1

4, wave function construction: 2-color slave fermion, color-1 has a nontrivial topological band with edge states, color-2 has trivial band.

$$\mathcal{S} = \int dx d\tau \ \bar{\psi}\gamma_\mu \partial_\mu \psi + m_4 \bar{\psi}\rho^z \psi$$

5, NLSM + Θ-term at Θ = 2π can be generated after coupling slave fermions to Neel order, and integrate out the fermions.

New 3d SPT for SU(2N) spin

4, wave function construction: 2-color SU(2N) slave fermion, color-1 has a 3d topological insulator band structure, color-2 has a trivial band.

$$\mathcal{S} = \int d^3x d\tau \ \bar{\psi}\gamma_\mu \partial_\mu \psi + m_4 \bar{\psi}\rho^z \psi$$

5, NLSM + Θ-term at Θ = 2π can be generated after coupling slave fermions to Neel order, and integrate out the fermions.

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Summary:

We found a class of 3d SPT for spin systems with PSU(2N) = SU(2N)/Z_{2N} symmetry, which is a generalization of 1d Haldane phase of spin-1 chain with SO(3) = PSU(2) symmetry.

Properties:

1. Described by 3+1d NLSM + Θ-term at Θ = 2π.
2. Its 2+1d edge states is either gapless or degenerate.
3. Its edge states cannot be constructed using representations of SU(2N)/Z_{2N} symmetry group.
4. Lattice construction….

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Symm. group	$d=0$	$d=1$	$d=2$	$d=3$
Z_2^T	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_1$	$\mathbb{Z}_2$
$Z_2^T \times \text{trn}$	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^4$
Z_n	$\mathbb{Z}_n$	$\mathbb{Z}_1$	$\mathbb{Z}_n$	$\mathbb{Z}_1$
$Z_n \times \text{trn}$	$\mathbb{Z}_n$	$\mathbb{Z}_n$	$\mathbb{Z}_n^2$	$\mathbb{Z}_n^4$
$U(1)$	$\mathbb{Z}$	$\mathbb{Z}_1$	$\mathbb{Z}$	$\mathbb{Z}_1$
$U(1) \times \text{trn}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}^2$	$\mathbb{Z}^4$
$U(1) \rtimes Z_2^T$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$
$U(1) \rtimes Z_2^T \times \text{trn}$	$\mathbb{Z}$	$\mathbb{Z} \times \mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_2^3$	$\mathbb{Z} \times \mathbb{Z}_2^8$
$U(1) \times Z_2^T$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_1$	$\mathbb{Z}_2^3$
$U(1) \times Z_2^T \times \text{trn}$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^9$
$U(1) \rtimes Z_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_2$	$\mathbb{Z}_2$
$U(1) \times Z_2$	$\mathbb{Z} \times \mathbb{Z}_2$	$\mathbb{Z}_1$	$\mathbb{Z} \times \mathbb{Z}_2^2$	$\mathbb{Z}_1$
$Z_n \rtimes Z_2^T$	$\mathbb{Z}_n$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}^2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}^2$
$Z_n \times Z_2^T$	$\mathbb{Z}_{(2,n)}$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}^2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}^2$
$Z_n \rtimes Z_2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}$	$\mathbb{Z}_n \times \mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}^2$
$Z_m \times Z_n$	$\mathbb{Z}_m \times \mathbb{Z}_n$	$\mathbb{Z}_{(m,n)}$	$\mathbb{Z}_m \times \mathbb{Z}_n \times \mathbb{Z}_{(m,n)}$	$\mathbb{Z}_{(m,n)}^2$
$D_2 \times Z_2^T = D_{2h}$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^6$	$\mathbb{Z}_2^9$
$Z_m \times Z_n \times Z_2^T$	$\mathbb{Z}_{(2,m)} \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,m)} \times \mathbb{Z}_{(2,n)} \times \mathbb{Z}_{(m,n)}$	$\mathbb{Z}_{(2,m,n)}^2 \times \mathbb{Z}_{(2,m)}^2 \times \mathbb{Z}_{(2,n)}^2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,m,n)}^4 \times \mathbb{Z}_{(2,m)}^2 \times \mathbb{Z}_{(2,n)}^2$
$SU(2)$	$\mathbb{Z}_1$	$\mathbb{Z}_1$	$\mathbb{Z}$	$\mathbb{Z}_1$
$SO(3)$	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}_1$
$SO(3) \times \text{trn}$	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_2^2$	$\mathbb{Z}^3 \times \mathbb{Z}_2^3$
$SO(3) \times Z_2^T$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2$	$\mathbb{Z}_2^3$
$SO(3) \times Z_2^T \times \text{trn}$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^5$	$\mathbb{Z}_2^{12}$

arXiv:1106.4772, Chen, Gu, Liu, Wen,