Space of non-Fermi liquids

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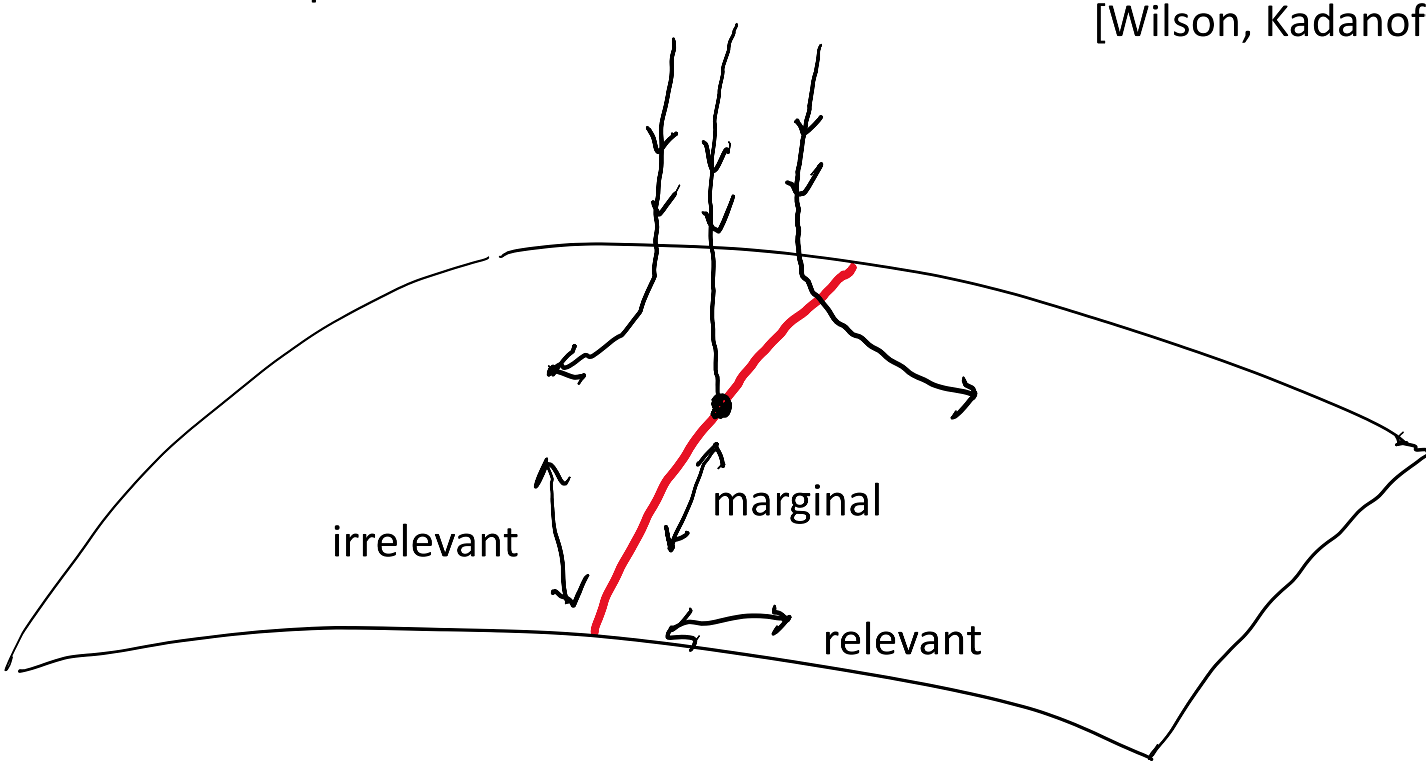
Mandate of low-energy theorists

To identify distinct phases of matter and uncover universal relations among low-energy observables

- -by identifying IR fixed points of RG flow
- -by expressing all low-energy observables in terms of marginal and relevant parameters

[Wilson, Kadanoff,..]



Complications for metals

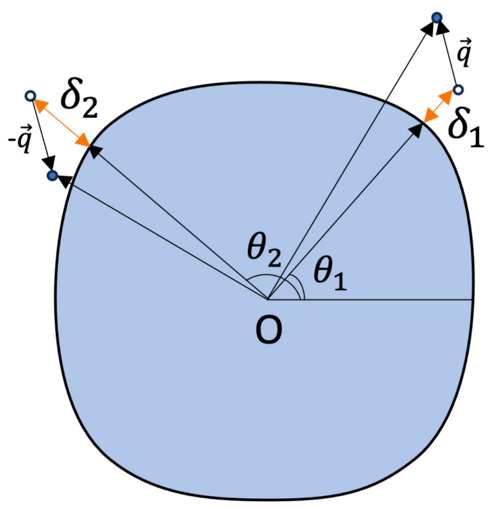
- The space of low-energy theories is an infinite-dimensional one spanned by Fermi velocity, Fermi momentum, couplings, which are functions of angles around FS

- The patch theory expands the full theory around a point on FS.
  - It does not capture observables that are non-local in k-space, e.g. Landau/pairing interactions for electrons with widely different k's
  - In the presence of strong inter-patch coupling, the patch theory breaks down. In this case, the low-energy physics is singularly controlled by k_F

UV(large-momentum)/IR(low-energy) mixing

Complications for metals

- There can not be a scale invariance due to Fermi momentum
  - In local theories*, one has to include scatterings with small but non-zero momentum transfer
  - Fermi momentum, which is a large momentum scale but a low-energy scale, grows incessantly under scale transformations



$$\Delta\theta \sim \frac{q}{K_F}$$

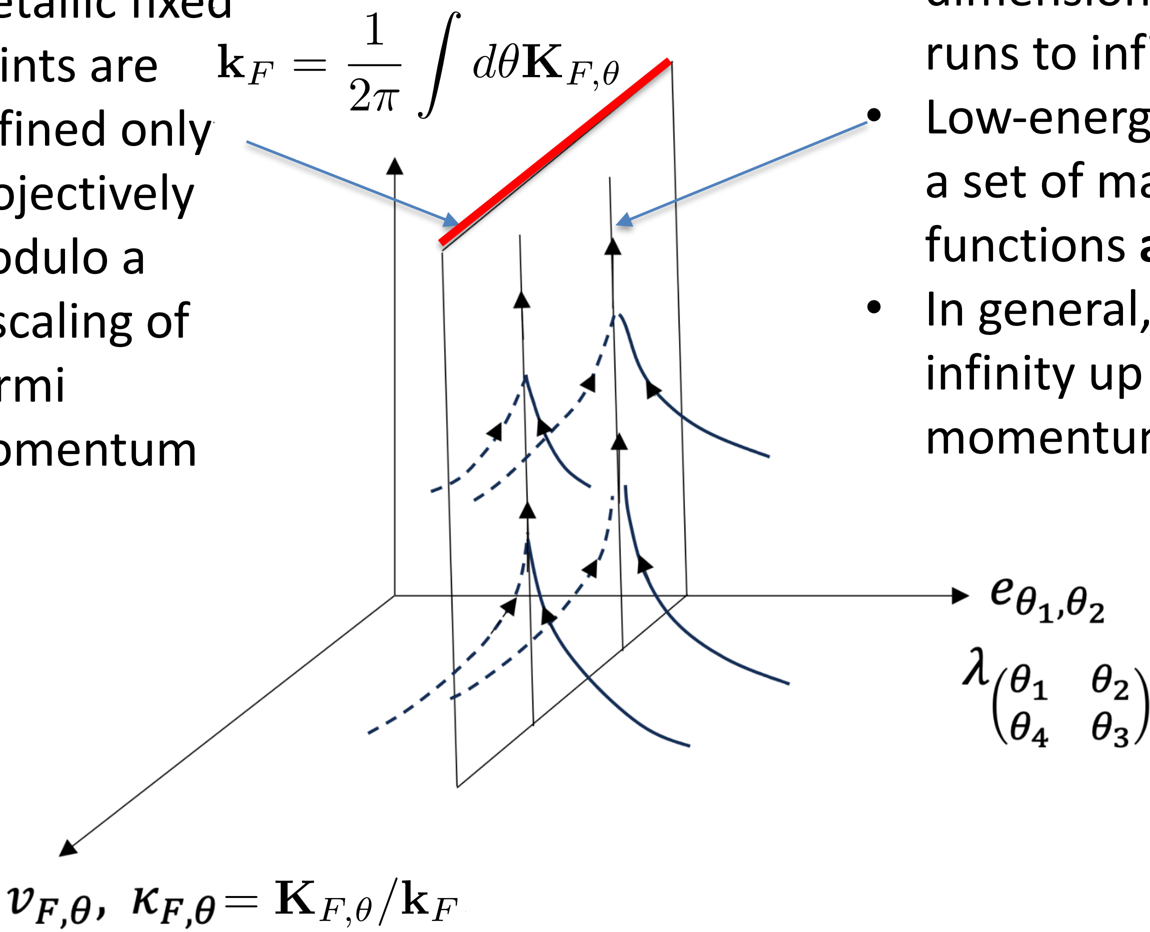
$$q \to q/b, \quad \theta \to \theta$$

$$K_F \to bK_F$$

*Landau's scale-invariant fixed-point theory for FL is non-local

Projective fixed point

Metallic fixed points are defined only projectively modulo a rescaling of Fermi momentum



- RG flow is attracted toward an one-dimensional manifold in which k_F runs to infinity
- Low-energy observables are fixed by a set of marginal/relevant coupling functions **and** Fermi momentum
- In general, one can not set k_F to infinity up front because large Fermi momentum limit can be singular

Fermi liquids

[Landau]

[Benfatto, Gallavotti] [Shankar] [Polchinski]

- Marginal parameters :
  - shape of FS, Fermi velocity, Landau function

$$\kappa_{F,\theta} = \mathbf{K}_{F,\theta}/\mathbf{k}_F,\ v_{F,\theta},\ \lambda_{\theta,\theta'} \qquad \left(\mathbf{k}_F = \frac{1}{2\pi}\int d\theta \mathbf{K}_{F,\theta}\right)$$

- The exact forward scattering fixes the universal profile of the near-forward scattering [H. Ma, SL (23)]
- Vanishing pairing interaction at the fixed point
- Emergent symmetry : loop U(1)

[Handane (93), Else, Thorgen, Senthil (22)]

$$\psi_j(\theta) \to e^{i\gamma(\theta)}\psi_j(\theta)$$

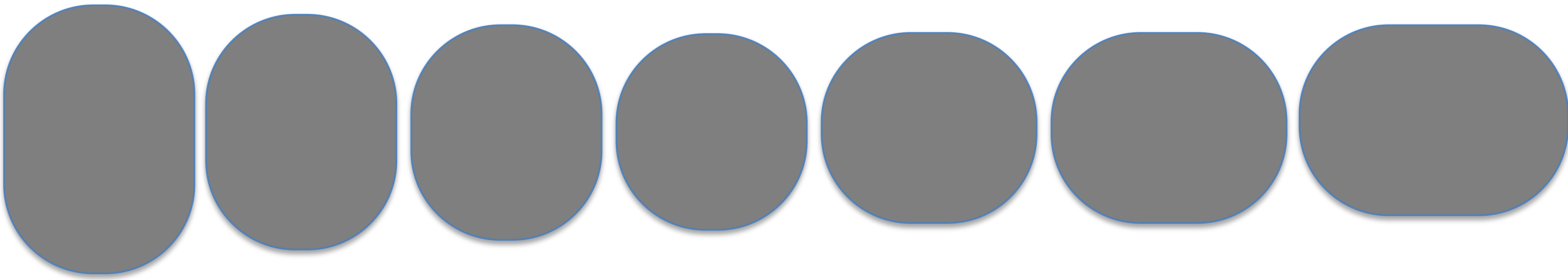
Charting the space of non-Fermi liquids

- One should identify IR fixed points in the space of enlarged space coupling functions
  - Minimal framework : field-theoretic functional RG
  - Antiferromagnetic quantum critical metal in the limit that the nesting angle is small

[A. Schlief, P. Lunts, SL (16)]   [F. Borges, A. Borissov, A. Singh, A. Schlief, SL (22)]

- The strong coupling problem is still prohibitive in NFL with hot Fermi surface in d=2

Ising-nematic quantum critical metal

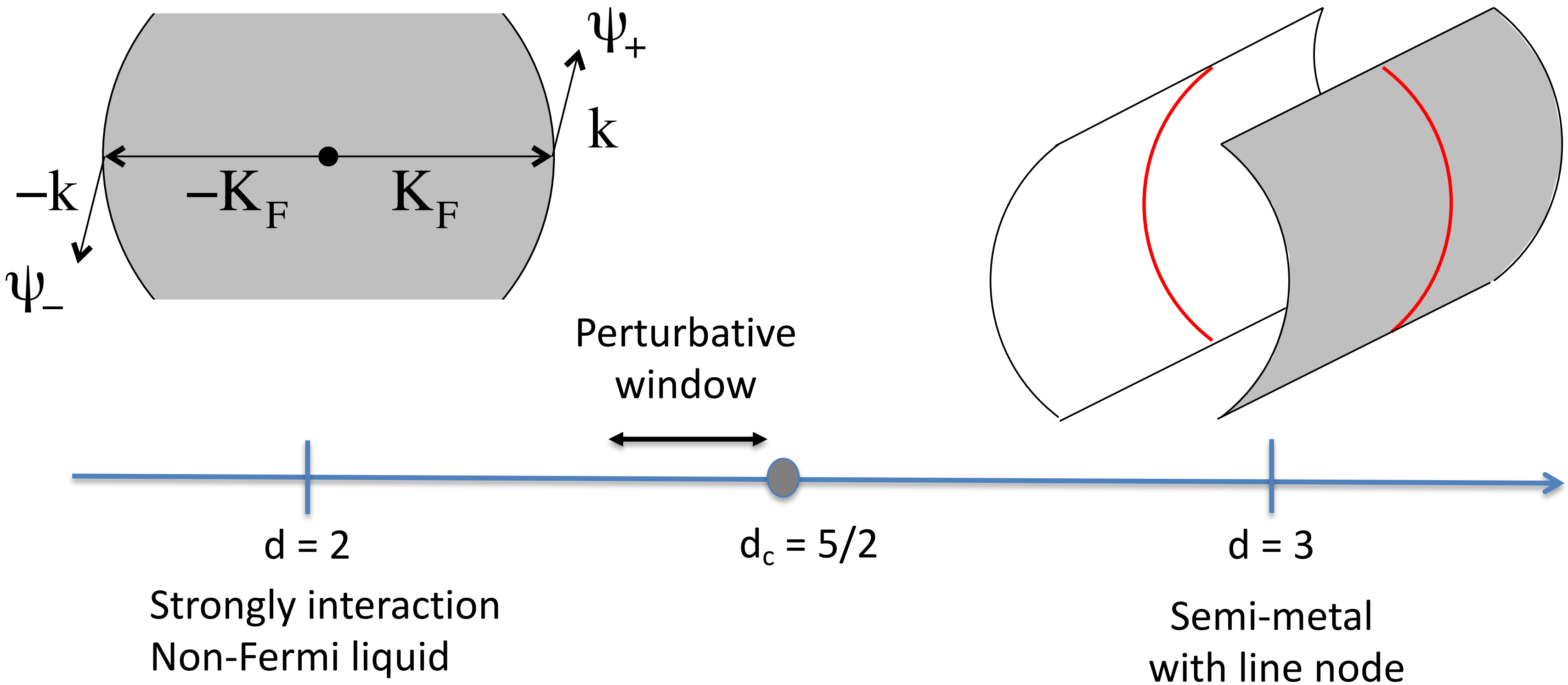


$$\phi(x,t)$$

Long-wavelength fluctuations of the nematic order parameter is described by a real gapless bosonic mode coupled to FS

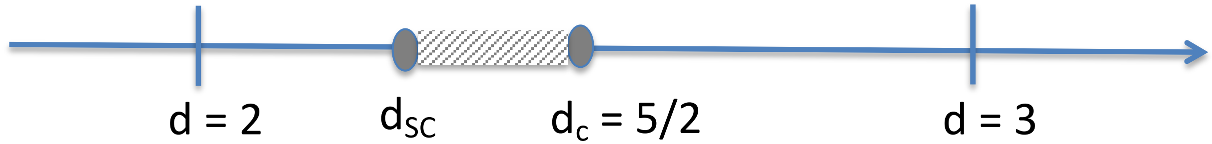
A dimensional regularization : an analytic continuation of the 2d metal to a semi-metal with line node in 3d

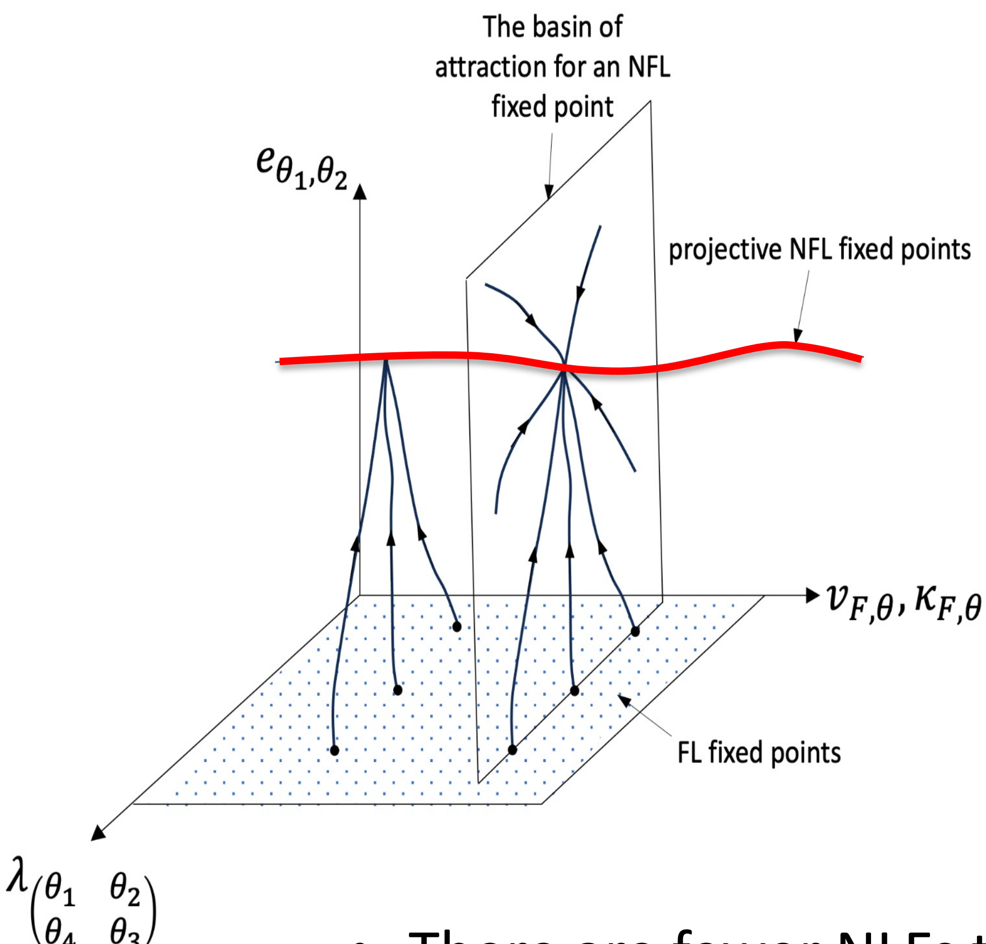
[D. Dalidovich, SL (2013)]



- Upper critical dimension : $d_c = 5/2$
- The regularization scheme breaks some global symmetry. The conclusion we draw does not depend on this artifact.

$d > d_{SC}$





- Stable projective NFL fixed points
- Only two marginal parameters for the shape of FS and Fermi velocity

$$\kappa_{F,\theta}, v_{F,\theta}$$

- Landau function and pairing interaction are non-zero, but fixed by the marginal parameters and Fermi momentum

- There are fewer NLFs than FLs
- SC fluctuations are intrinsic parts of NFLs

Universal pairing interaction (d > d_{SC})

Pairing interaction for Cooper pairs with center of mass momentum $\vec{q}$ and energy ω from angle $(\theta_1, \theta_1 + \pi)$ to $(\theta_2, \theta_2 + \pi)$

For $\vec{q} = 0$,

anomalous dimension of angle

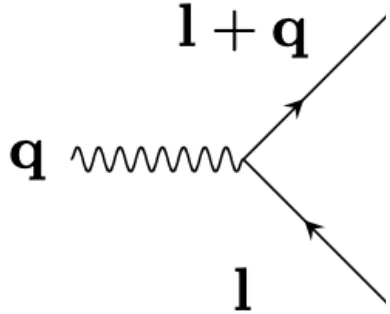
Universal exponent

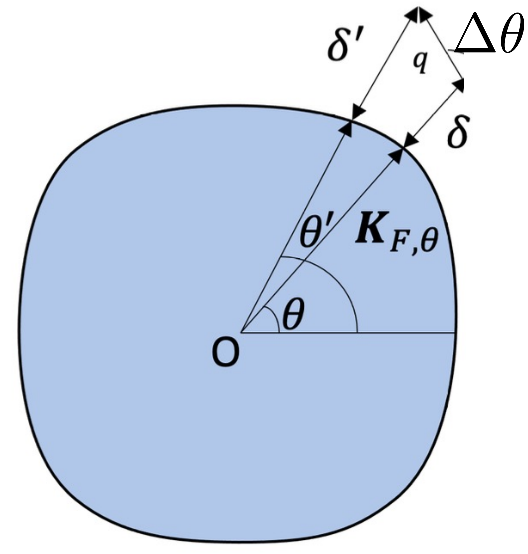
$$\Gamma^{(4)}_{\theta_1,\theta_2}(\vec{q} = 0, \omega) \sim \left| \left(\frac{\omega^{1/z}}{K_F} \right)^{1/2} \frac{1}{\theta_1 - \theta_2} \right|^{2\Delta}$$

$$z = \frac{3}{2(d-1)}$$

-Δ: scaling dimension of the pairing interaction

The anomalous dimension of angle





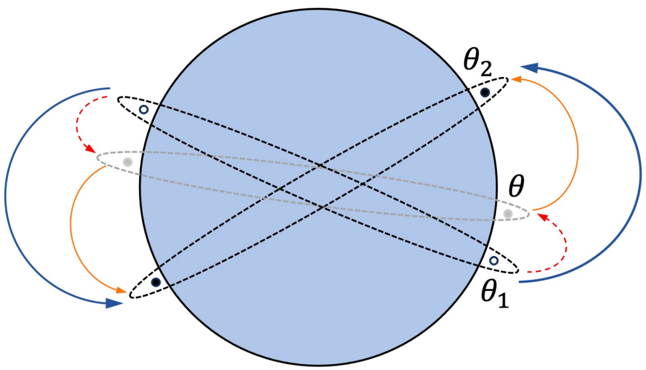
- The critical boson couple low-energy fermions within angular range $\Delta\theta \sim \sqrt{\frac{\omega^{1/z}}{K_F}}$

- With decreasing energy, FS is broken into smaller patches that lie outside the range of the critical boson

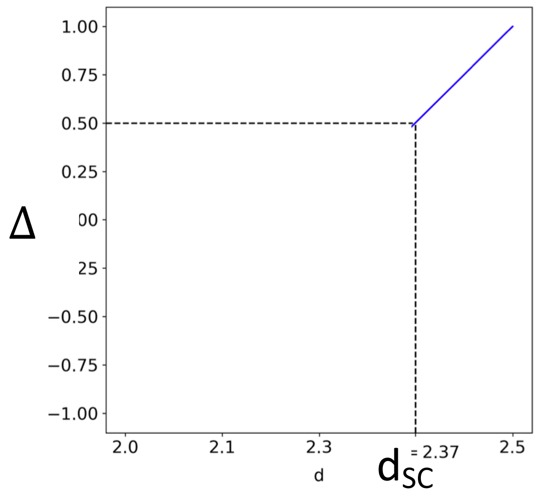
[Son (99), Metlitski, Mross, Sachdev, Senthil (15)]

Large-angle scattering assisted by small-angle scattering

- The singular small-angle scatterings give rise to an anomalous dimension of the short-range 4-fermion coupling that creates large-angle scatterings



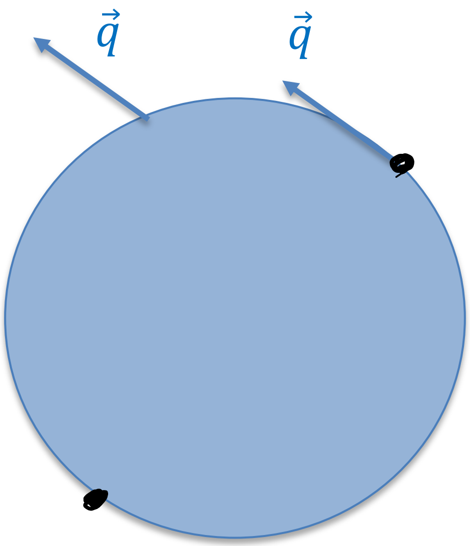
$$\Gamma^{(4)}_{\theta_1,\theta_2}(\vec{q}=0,\omega) \sim \left| \left(\frac{\omega^{1/z}}{K_F} \right)^{1/2} \frac{1}{\theta_1 - \theta_2} \right|^{2\Delta}$$



Universal pairing interaction for $\vec{q} \neq 0$

Crossover function

$$\Gamma^{(4)}_{\theta_1,\theta_2}(\vec{q},\omega) \sim \left| \left(\frac{\omega^{1/z}}{K_F}\right)^{1/2} \frac{1}{\theta_1 - \theta_2} \right|^{2\Delta} \mathcal{V}_{\theta_1}(\vec{q})\mathcal{V}_{\theta_2}(\vec{q})$$



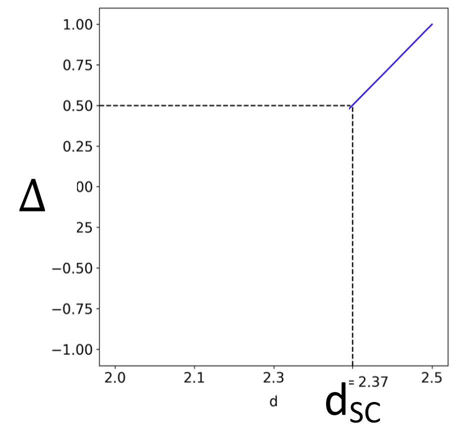
For large q,

$$\mathcal{V}_{\theta}(\vec{q}) \sim \begin{cases} \left(\frac{\omega^{1/z}}{q}\right)^{\frac{\eta_d}{2}}, & \vec{q} \text{ not parallel to FS at } \theta \\ \left(\frac{\omega^{1/z} K_F}{q^2}\right)^{\frac{\eta_d}{2}}, & \vec{q} \text{ almost parallel to FS at } \theta \end{cases}$$

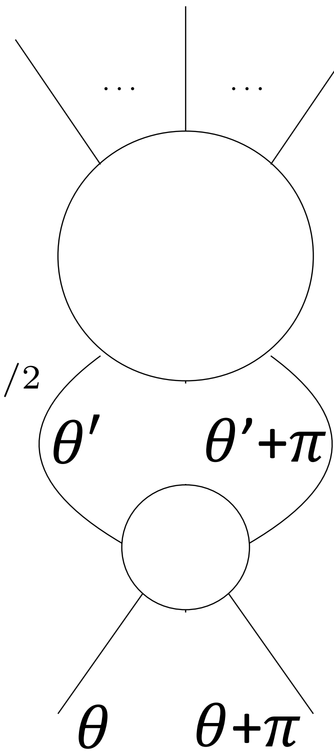
No unique dynamical critical exponent that sets the relative scaling between q and ω

UV/IR mixing

$$\Gamma^{(4)}_{\theta_1,\theta_2}(\vec{q}=0,\omega) \sim \left| \left(\frac{\omega^{1/z}}{K_F} \right)^{1/2} \frac{1}{\theta_1 - \theta_2} \right|^{2\Delta}$$

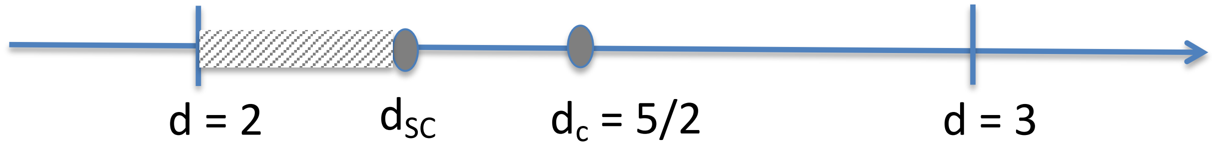


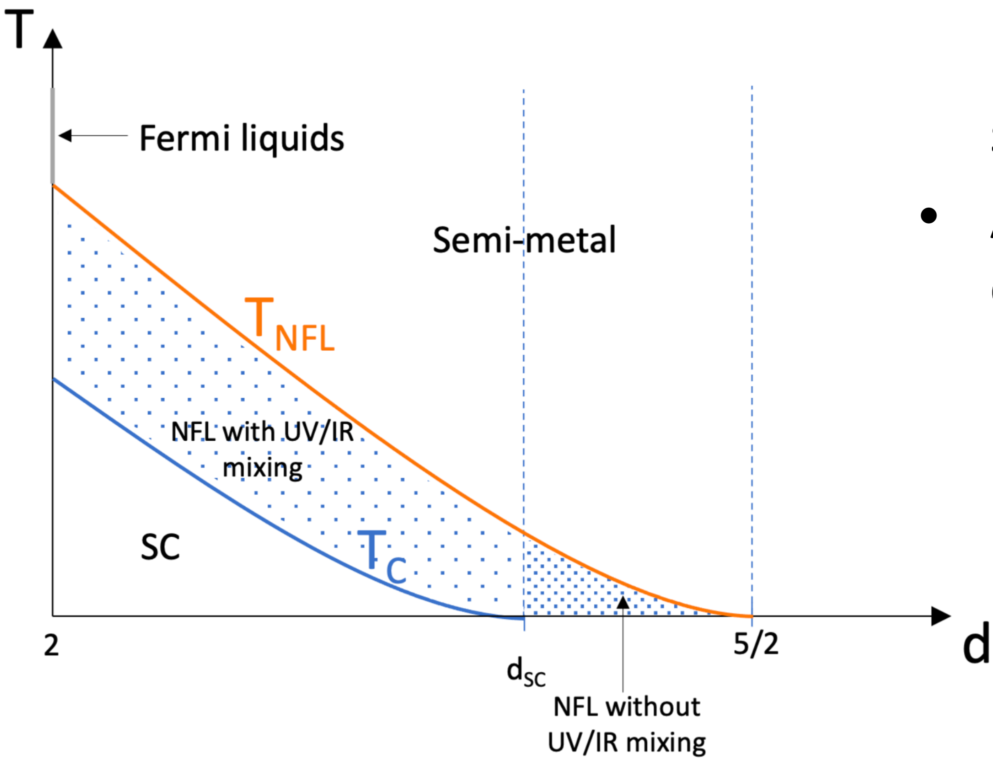
$$\int d\theta' \left(\frac{K_F}{\omega^{1/z}} \right)^{1/2}$$



- Naively, the inter-patch coupling are \`irrelevant' as far as $\Delta > 0$
- However, they become marginal for $\Delta = 1/2$ because the growth of the number of patches compensates the decay of the interaction strength
  - The large-angle scatterings generate singular quantum corrections in the pairing channel

$d < d_{SC}$





- The non-Fermi liquids become unstable against superconductivity for $d<d_{sc}$
- A window of energy scale controlled by NFL quasi-fixed point with strong UV/IR mixing
  - In d=2, LU(1) is expected to be broken down to the subgroup that only includes odd-angular momentum channel

$$\psi_j(\theta) \rightarrow e^{i\gamma(\theta)}\psi_j(\theta)$$

$$\gamma(\theta+\pi) = -\gamma(\theta)$$

Summary

- Due to Fermi momentum, fixed points of metals are defined only projectively
  - The absence of a unique dynamical critical exponent
  - Mismatch between scaling dimension and relevancy of couplings
- UV/IR mixing can lower the emergent symmetry of certain non-Fermi liquids realized above superconducting transition temperatures from that of Fermi liquids