Excitonic quantum criticality

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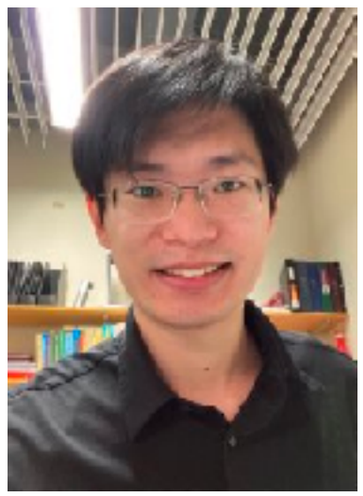

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ZDS, 2311.12922
ZDS, Goldman, Dong, Senthil, 2402.12436

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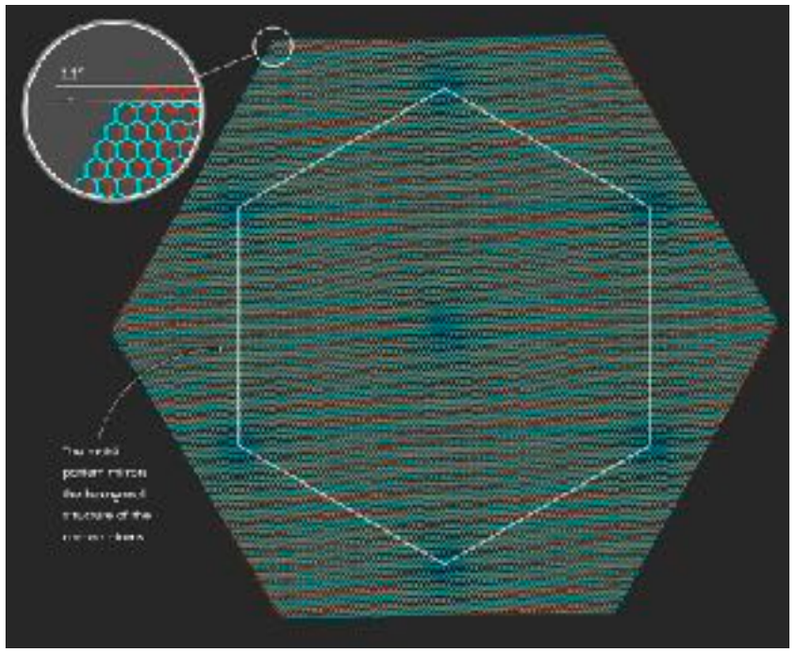
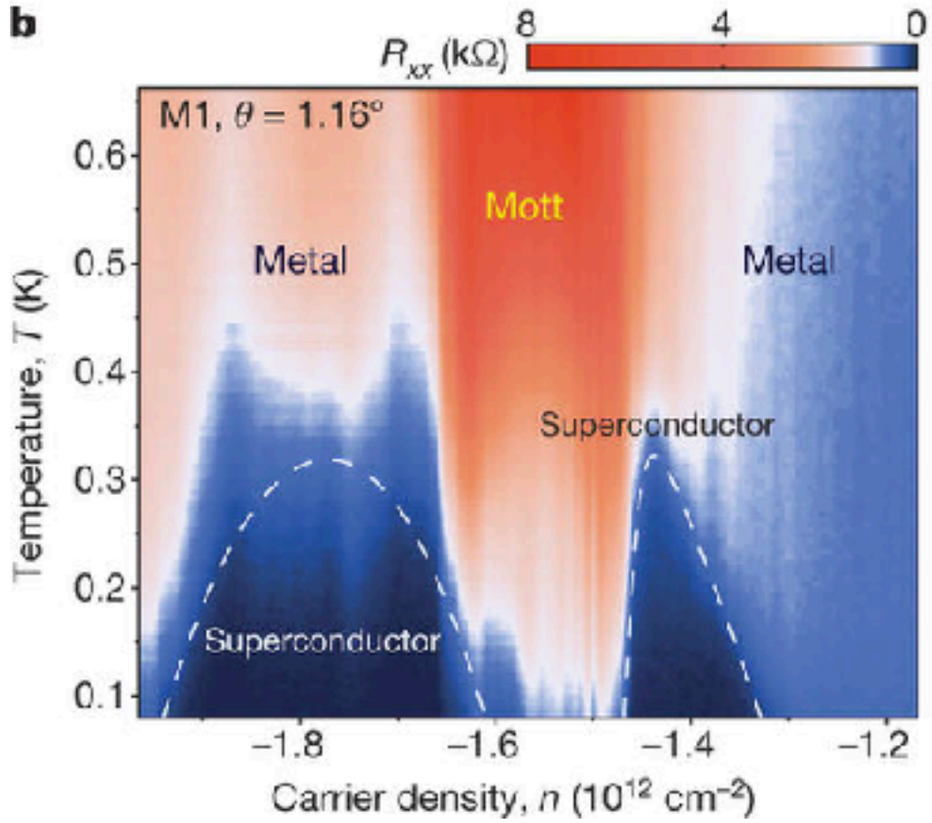
Gift from 2D materials

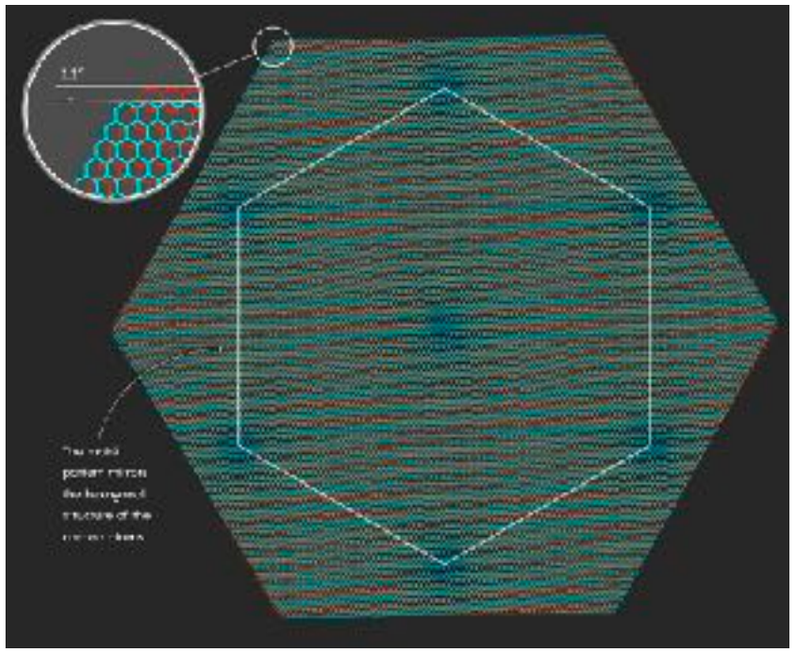


Credit: 5W Infographics



[Cao et al. 2018, Yankowitz et al. 2019, Lu et al. 2019]…

Gift from 2D materials



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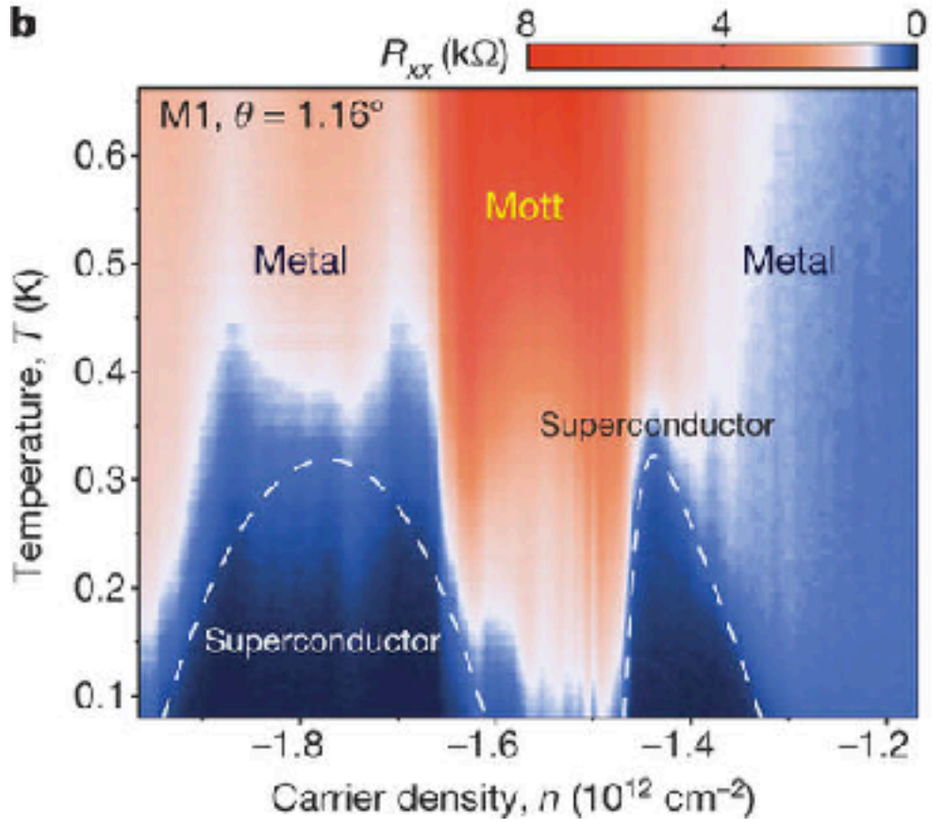

[Cao et al. 2018, Yankowitz et al. 2019, Lu et al. 2019]…

- Strong tunability: twist angle, carrier density, displacement field etc.
- Ideal for precision study of novel phases and transitions

Exotic phases really exist

- Fractional quantum anomalous Hall effect (FQAH)

  TMD [Cai … Xu, 2023], [H. Park … Xu, 2023], [Zeng … Mak & Shan, 2023]

  Pentalayer graphene [Lu … Ju, 2023]

- Fractional quantum spin Hall effect in tMoTe2 [Kang…Shan & Mak, 2024]
- … rapidly growing list

Much of recent development centered around exotic insulators

But metals are by far the more common fate at generic filling!

Metal ≠ boring

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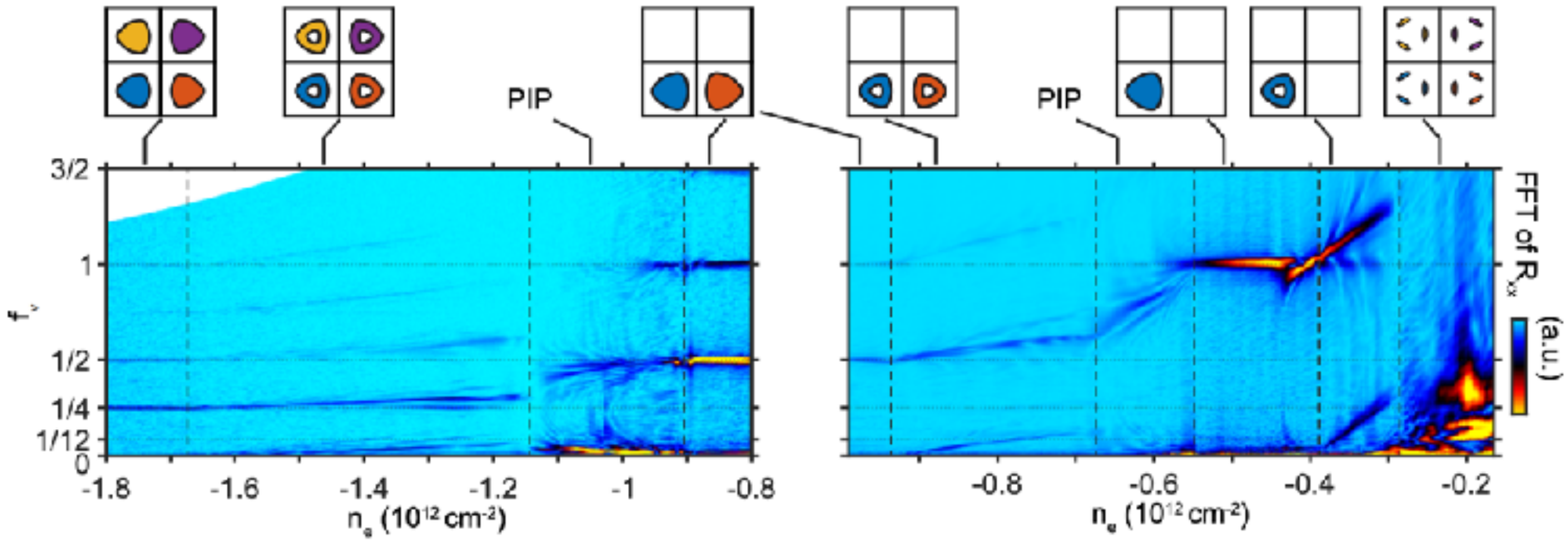
Huge variety of symmetry-breaking Fermi-liquid metals seen!

c.f. talks by Chenhao and Stevan on Wednesday



Rhombohedral trilayer graphene

Quantum oscillation frequency vs. filling

[Zhou…Andrea Young, 2021]

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Huge variety of symmetry-breaking Fermi-liquid metals seen!

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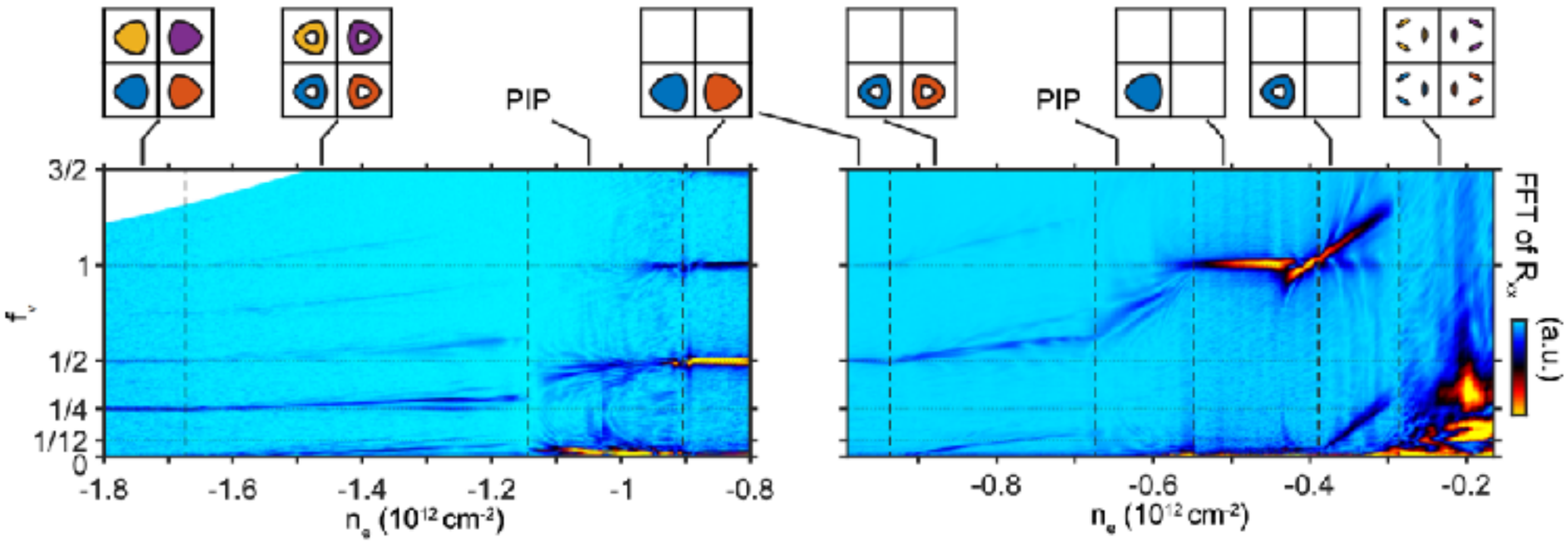
— So far, phase transitions appear to be first order

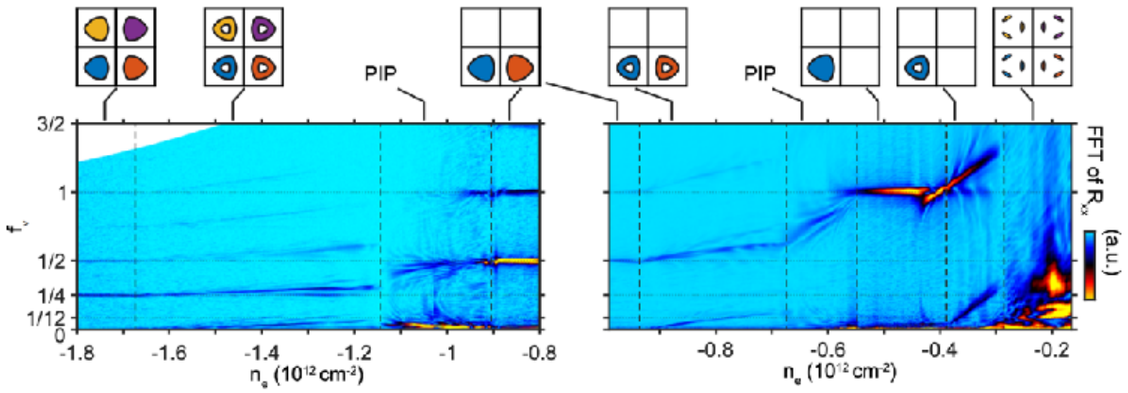


Rhombohedral trilayer graphene

Quantum oscillation frequency vs. filling

[Zhou…Andrea Young, 2021]

In search of metallic quantum criticality



Rhombohedral trilayer graphene

Quantum oscillation frequency vs. filling

[Zhou…Andrea Young, 2021]

Guiding theoretical question:

Can we find a clean 2D NFL at a flavor-SSB transition where

(1) Fluctuations do not destabilize the continuous transition?

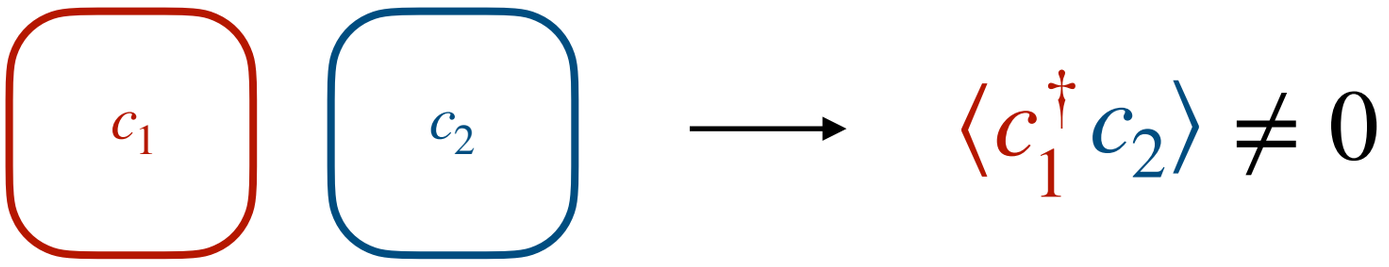
— Challenge: possible fluctuation-induced 1st order transitions [Chubukov et al. 2004], [Maslov et al. 2009] …

(2) Critical properties (e.g. exponents) are accessible in transport?

— Challenge: hard to access in single-flavor metallic QCPs due to non-perturbative constraints [Maslov et al. 2011, Shi et al. 2022, Guo et al. 2022, Gindikin and Chubukov 2024] …

Our answer:

Excitonic quantum criticality!



Two flavors of fermions forming FS

SSB of relative $U(1)$ symmetry

Many names for the same phenomenon

Flavor = spin: XY ferromagnet

Flavor = valley: Intervalley coherence (IVC)

Flavor = layer: Interlayer exciton

Main results

General theoretical development:

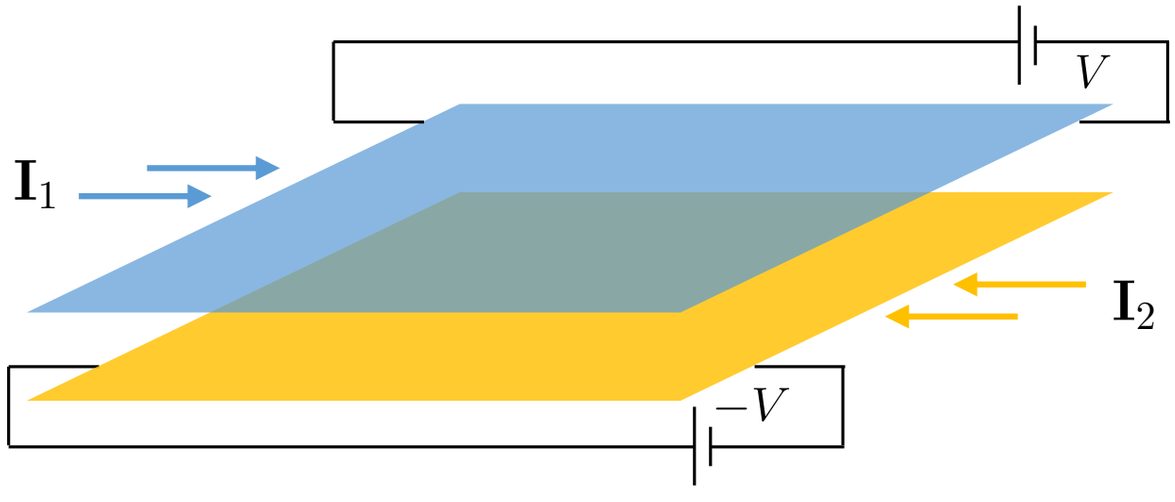
Refine and extend a controlled expansion for metallic QCPs

Use controlled expansion to answer guiding questions:

(1) Continuous transition survives fluctuations

(2) Counterflow probes the dynamical critical exponent

$$\rho_{\text{counterflow}}(\omega, T = 0) \sim \omega^{2/z}$$



$$V = \rho_{\text{counterflow}}\,(I_1 - I_2)$$

Roadmap

Part I: Hertz-Millis-Moriya framework for non-Fermi liquids

Part II: Strong coupling and a controlled expansion

Part III: Possibility of a continuous transition

Part IV: Counterflow transport

Part I

Hertz-Millis-Moriya framework For non-Fermi liquids

What is Hertz-Millis-Moriya?

IR description of Landau ordering transitions in a metallic environment

The minimal model of NFL: FS coupled to Landau order parameter

[Hertz, PRB 1976], [Moriya 1985], [Millis, PRB 1993] …

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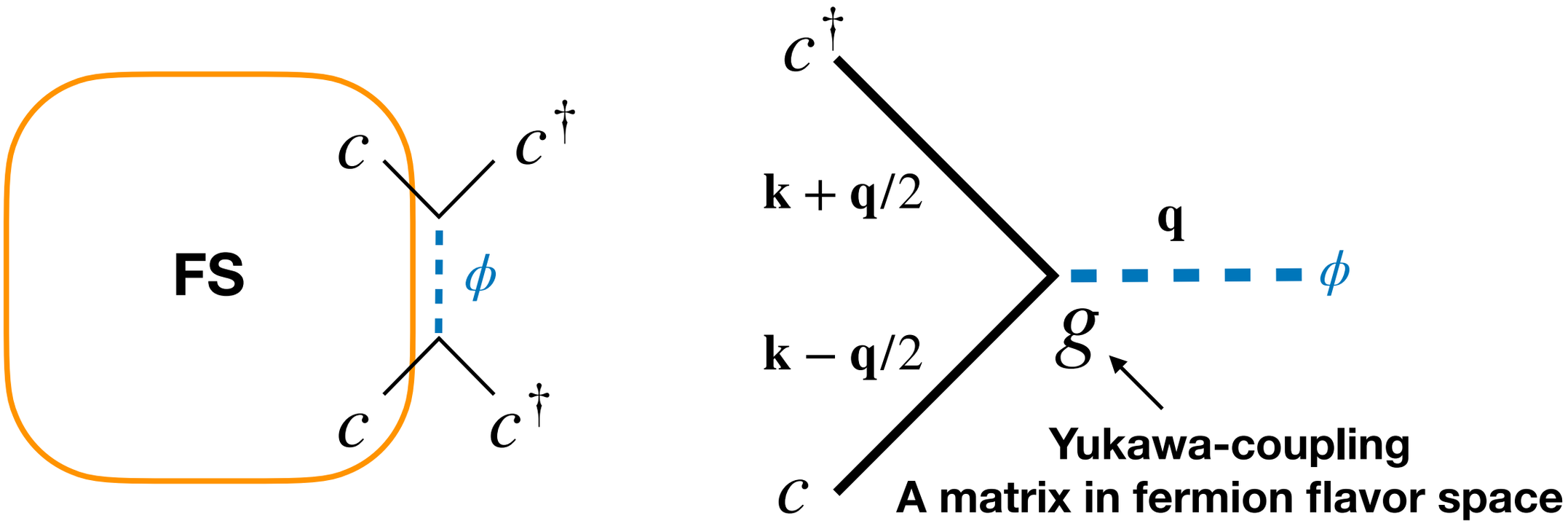
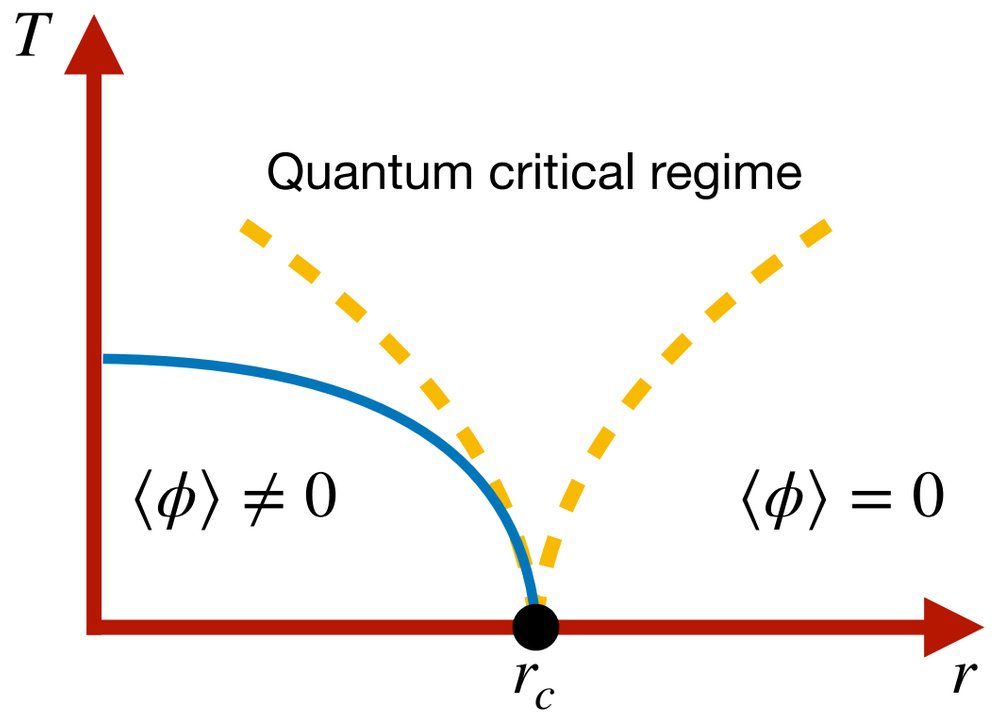


Landau action for boson:

(Standard choice z = 3)

$$S_{\text{Landau}}[\phi] = \frac{1}{2}\int \phi^*(\vec{q},\Omega)\left[\Omega^2 + |\vec{q}|^{z-1} + r\right]\phi(\vec{q},\Omega) + \ldots$$

Mean-field phase diagram



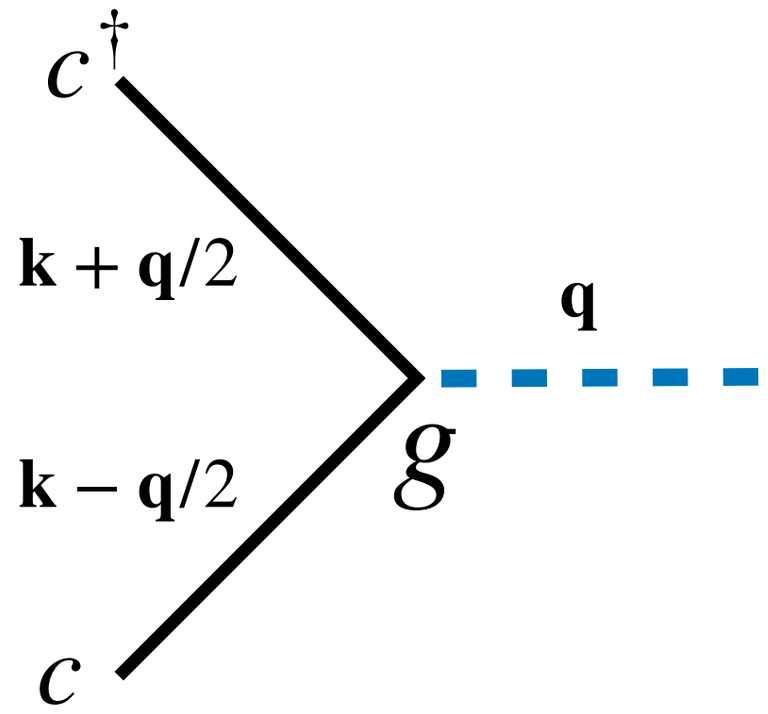
$$S_{\text{Landau}}[\phi] = \frac{1}{2}\int \phi^*(\vec{q},\Omega)\left[\Omega^2 + |\vec{q}|^{z-1} + r\right]\phi(\vec{q},\Omega) + \ldots$$

$r > r_c$ => integrate out gapped bosons => symmetric FL

$r < r_c$ => boson condenses => symmetry-broken FL

Enhanced fluctuations at $r = r_c$

Singular IR fluctuations



Yukawa coupling g strongly relevant under RG flow

=> “decoupled FS + boson” is not a good perturbative starting point

Need for alternative expansions to control the theory in the IR

Part II

Strong coupling and a controlled expansion

Try ordinary large N?

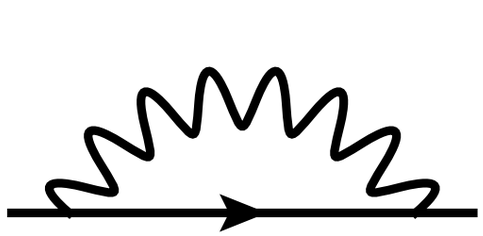
$$S = S_{\mathrm{FL}}[c_i] + S_{\mathrm{Landau}}[\phi] + S_{\mathrm{int}}[c_i, \phi]$$

$$S_{\mathrm{FL}}[c_i] = \int_{\vec{k},\,\omega_n} \sum_{i=1}^{N} c_i^{\dagger}(\vec{k},\omega_n)\,[i\omega_n - \epsilon(\vec{k})]\,c_i(\vec{k},\omega_n)$$

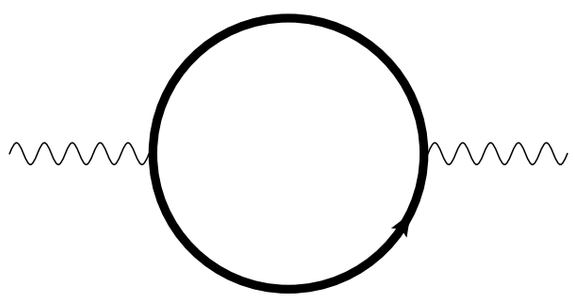
$$S_{\mathrm{Landau}}[\phi] = \frac{1}{2}\int \phi^*(\vec{q},\Omega)\left[\Omega^2 + |\,\vec{q}\,|^2 + r\right]\phi(\vec{q},\Omega)$$

$$S_{\mathrm{int}}[c_i, \phi] = \frac{g}{\sqrt{N}}\int \phi \sum_{i=1}^{N} c_i^{\dagger} c_i + h\,.\,c\,.$$

Compute the leading fermion and boson self energy:



$$\Sigma(\omega) \sim N^{-1}\omega^{2/3}$$



$$\Pi(\Omega, q) \sim |\Omega|/q$$

Large N is not enough

Ordinary large N expansion is difficult to control [Lee 2009]

Essential reason (c.f. Zhen Bi's talk)

$$\Sigma(\omega) \sim N^{-1}\omega^{2/3} \quad \longrightarrow \quad G_c(\omega, |\vec{k}| = k_F) \sim N\omega^{-2/3}$$

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Appearance of N in the numerator

=> Proliferation of higher loop diagrams

=> At least as complicated as planar $SU(N)$ gauge theory

(which we are not trying to solve)

An improved double expansion

N fermion flavors & $z = 2 + \epsilon$ with ϵN fixed [Mross et al. 2010]

$$S = S_{\mathrm{FL}}[c_i] + S_{\mathrm{Landau}}[\phi] + S_{\mathrm{int}}[c_i, \phi]$$

$$S_{\mathrm{FL}}[c_i] = \int_{\vec{k},\,\omega_n} \sum_{i=1}^{N} c_i^{\dagger}(\vec{k},\omega_n)\,[i\omega_n - \epsilon(\vec{k})]\,c_i(\vec{k},\omega_n)$$

$$S_{\mathrm{Landau}}[\phi] = \frac{1}{2}\int \phi^*(\vec{q},\Omega)\left[\Omega^2 + |\,\vec{q}\,|^{1+\epsilon} + r\right]\phi(\vec{q},\Omega)$$

$$S_{\mathrm{int}}[c_i, \phi] = \frac{g}{\sqrt{N}}\int \phi \sum_{i=1}^{N} c_i^{\dagger} c_i + h\,.\,c\,.$$

What saves us? Fermion self energy $\Sigma(\omega) \sim (N\epsilon)^{-1}\omega^{\frac{2}{2+\epsilon}}$

*No more large N enhancement if ϵN is held fixed**. Now let's use it!*

***Potential 4-loop divergence in [Ye et al. 2021]. Neglect for now. No effect on our three-loop results*

Part III

Possibility of a continuous transition

Why worry about continuity?

Gapless virtual fermions give singular corrections to boson action

=> Can trigger a fluctuation-induced first order transition [Halperin, Lubensky, Ma, 1974]

Borrow old results: negative boson self energy indeed generated in flavor-SSB transitions (see [Belitz et al. 2005] for a review)

$$\Pi_{\text{exciton}}(\vec{q}, \Omega = 0) = \xi(r)^{-2} - c\,\xi(r)^{-1}\,|\,\vec{q}\,|$$

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Does the singular negative term survive right at the QCP?

Strategy: direct calculation using the double expansion

Continuity regained

Result: calculate to leading two orders in $\epsilon \sim 1/N$

$$
\begin{aligned}
D^{-1}(\vec{q}, \Omega = 0) &= |\vec{q}|^{1+\epsilon} - b\epsilon\, |\vec{q}|^{\frac{2+\epsilon}{2}} + \mathcal{O}(\epsilon^2) \\
&= |\vec{q}| + \epsilon\, |\vec{q}| (\log |\vec{q}| - b) + \mathcal{O}(\epsilon^2)
\end{aligned}
$$

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$$= |\vec{q}| + \epsilon |\vec{q}| (\log |\vec{q}| - b) + \mathcal{O}(\epsilon^2)$$

Singular negative correction subleading relative to $\epsilon \log |\vec{q}|$

=> continuous transition survives fluctuations (at least for $\epsilon \ll 1$)

Caution: path to $\epsilon = N = 1$ need not be smooth. Need numerics/expts!

=> Possible instability as $\epsilon \to 1$: suggested by the pioneering works in [Chubukov et al. 2004], [Rech et al. 2006], [Maslov and Chubukov, 2009]

Part IV

Counterflow transport

Zeroth order question

Why do we need counterflow?

Why can’t we see critical fluctuations in ordinary transport?

Gift (curse?) from anomalies

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Momentum drag: lore cannot be literally correct

Current carries momentum => conservation law forbids dissipation

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Momentum drag: lore cannot be literally correct

Current carries momentum => conservation law forbids dissipation

Modern update [Else et al. 2021, ZDS et al. 2022, Guo et al. 2022 …]

Critical fluctuations give no singular correction to Drude peak

— Non-perturbative arguments using symmetries and anomalies

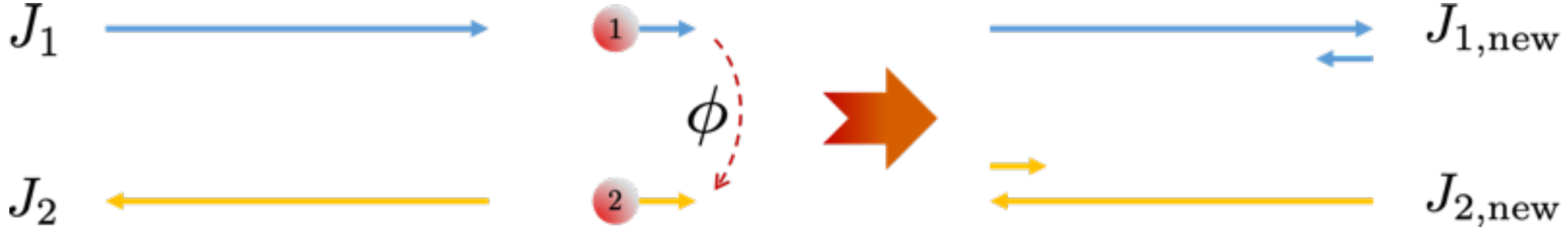
— Bolstered by perturbative checks [Maslov et al. 2011, ZDS et al. 2022, Guo et al. 2022, Gindikin and Chubukov 2024]…

Escape route: counterflow

Multi-component Hertz-Millis opens up new transport probes

— Can couple different fermion flavors to separate gauge fields

— Counterflow in bilayers: equal and opposite electric fields



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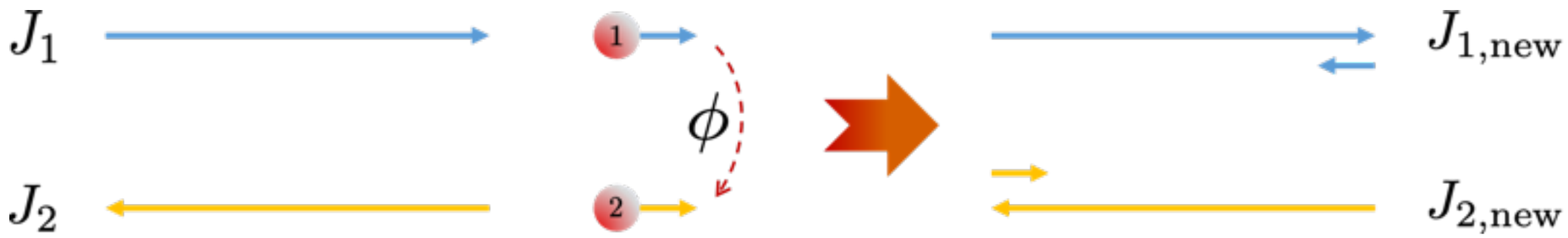
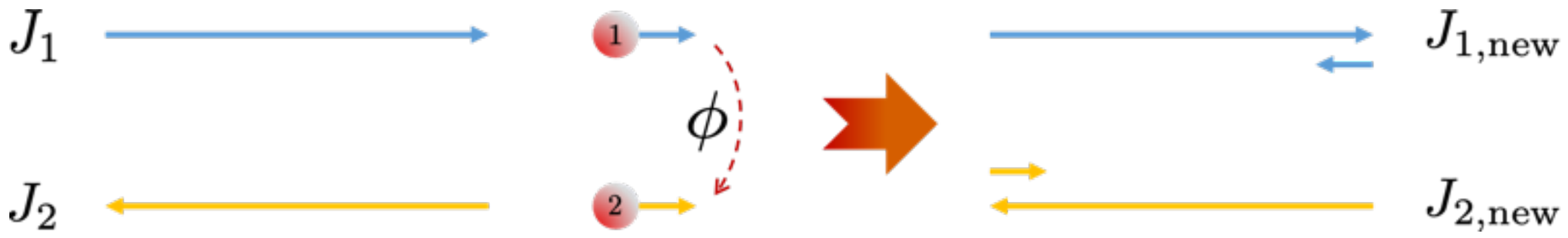


Intuition

— counterflow current $J_1 - J_2$ carries no momentum

— Critical scattering induced by ϕ efficiently degrades $J_1 - J_2$

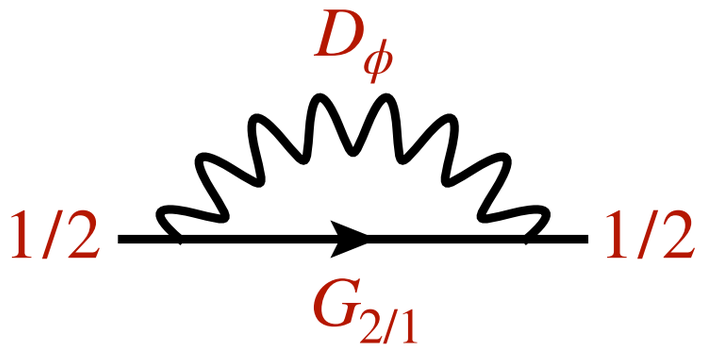
Estimating the scattering rate



No momentum drag => natural guess:

single fermion scattering rate ∝ transport scattering rate

$$\rho_{\mathrm{counterflow}}(\omega) \sim \Sigma_{1/2}(\omega)$$



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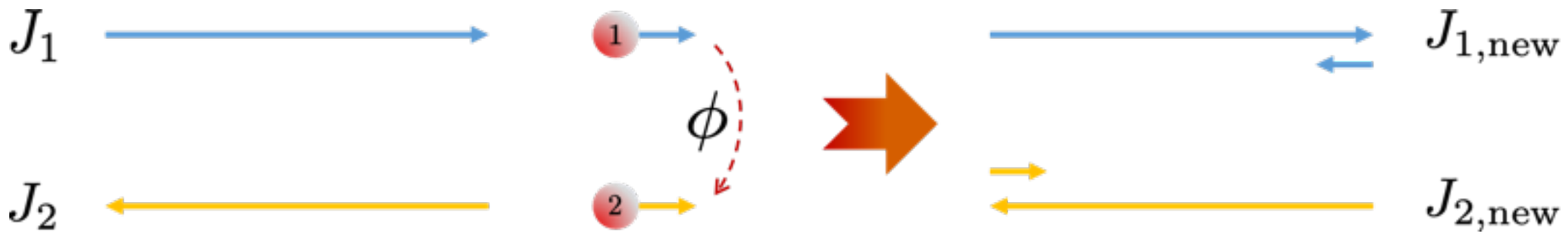

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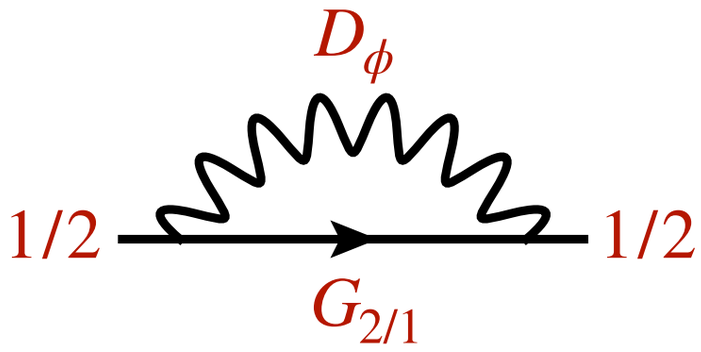

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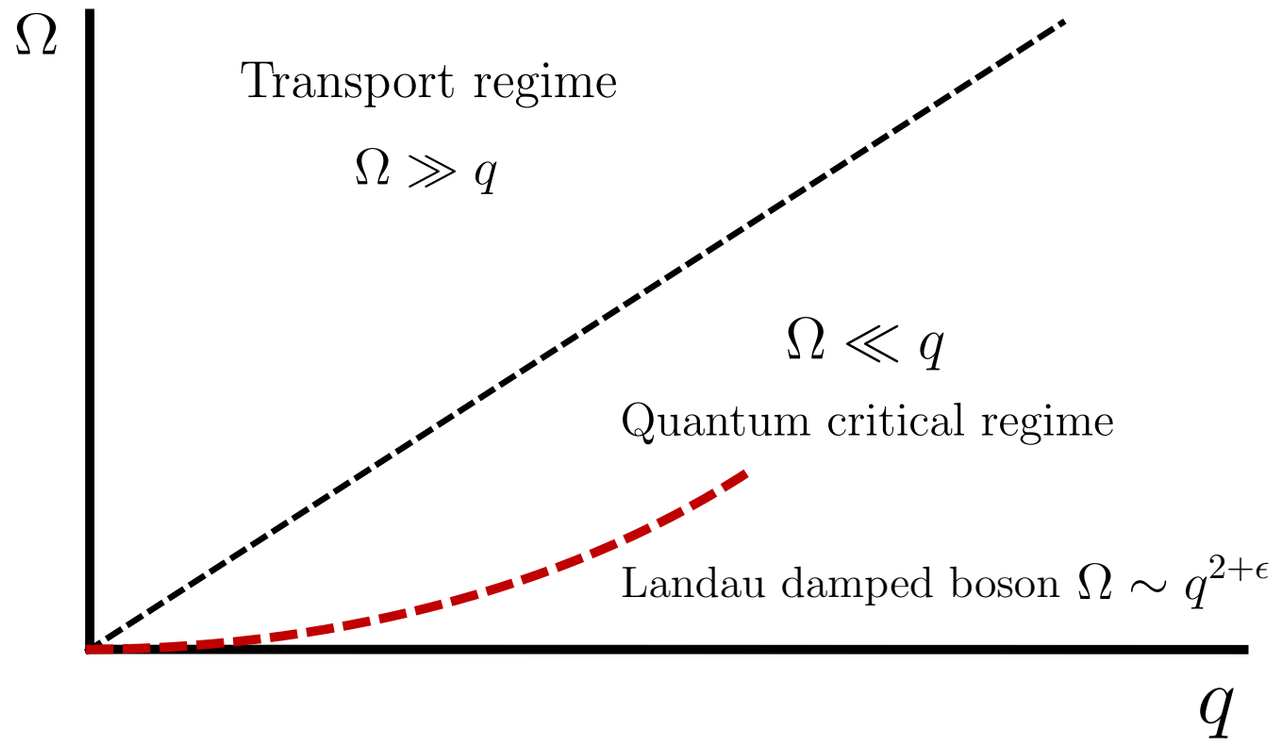
Correct scaling, but real story more subtle due to vertex corrections

Turn to the double expansion!

Transport in the double expansion

A surprise

N-scaling of diagrams depends on the kinetic regime!



Quantum critical regime: finitely many diagrams at each order in 1/N

Transport regime: infinitely many diagrams at each order in 1/N

Double expansion is not so controlled after all?

Summary and Outlook

Take-home messages

Excitonic quantum phase transitions can be continuous

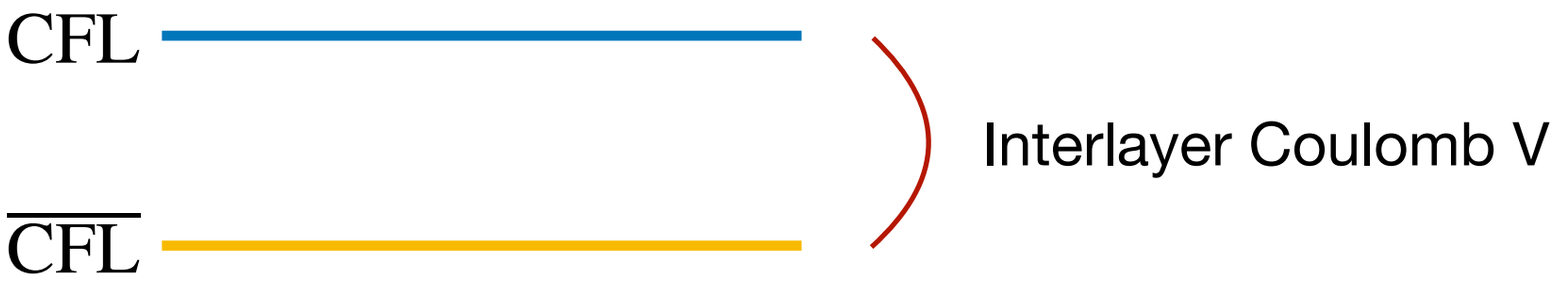
- Established within a particular large N expansion
- Dangerous path to small N: need input from numerics/ experiments!

Counterflow transport probes critical properties at the QCP

- Developed technique to compute in a large N expansion
- Directly encodes the dynamical critical exponent

A generalization

Excitonic criticality: from electrons to composite fermions



c.f. Chaoming's talk

Start with conjugate composite Fermi liquid [Myerson-Jain et al. 2023])

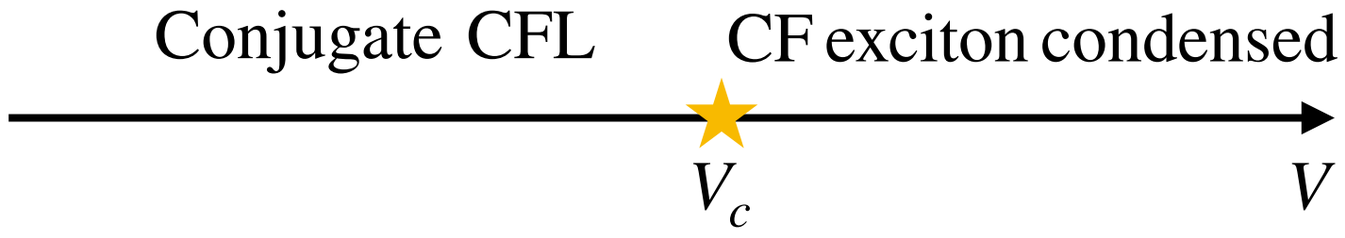
"Condense" composite fermion exciton to Higgs one gauge field

Remaining gauge field couples to a neutral Fermi surface

A gapless candidate for the FQSH effect found in [Kang et al. 2024]

Excitonic QCP of CFs

Previous works [Y. Zhang 2018], [Y. Zhang 2024] considered this phase but not the transition theory



Beyond-Landau, but essence = FS + gapless bosons + gauge fields

Generalized double expansion remains controlled for this theory!

$$\rho^{xx}_{\text{counterflow}}(\omega) \sim \omega^{2/z}$$

Counterflow continues to probe the dynamical critical exponent

Can be different from the exponent at the excitonic QCP of electrons

Future directions

1. Multi-layer transport as a general probe of clean non-Fermi liquid physics (layers need not be identical)

2. Breaking flavor-exchange symmetry: generically mismatched FS

- Some resemblance to AFM criticality but important differences
- WIP: weak mismatch => solve using method of [Schlief, Lunts, Lee 2017]

3. Incorporating spatial disorder into the controlled expansion?

$$\langle c_1^\dagger c_2 \rangle \neq 0$$

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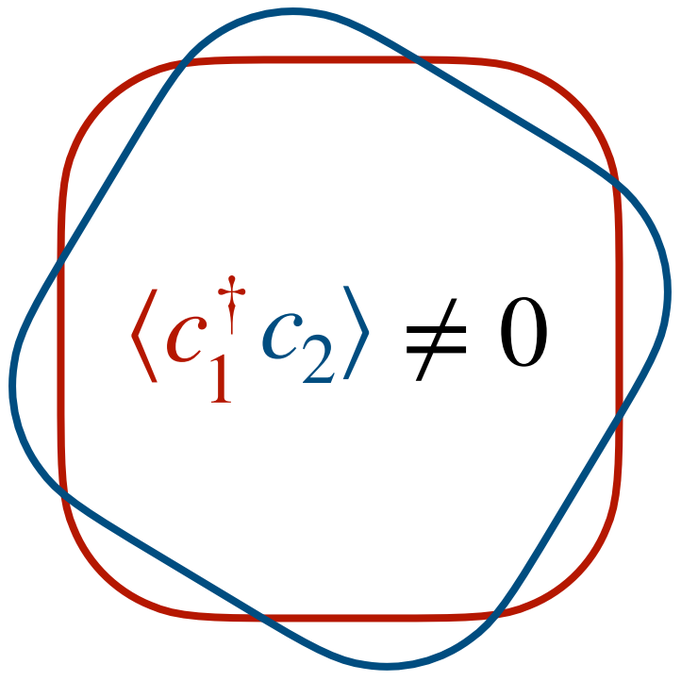
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