Issues in Strongly Interacting Fermi Gases

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Collaborator :

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Supported by NASA & NSF

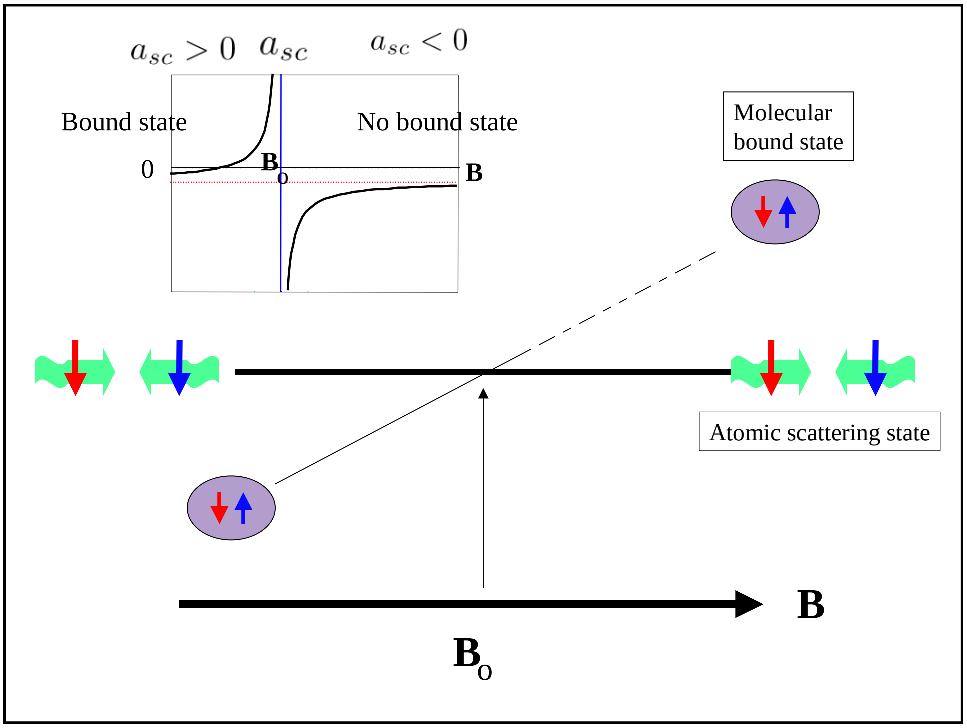
What have we here?

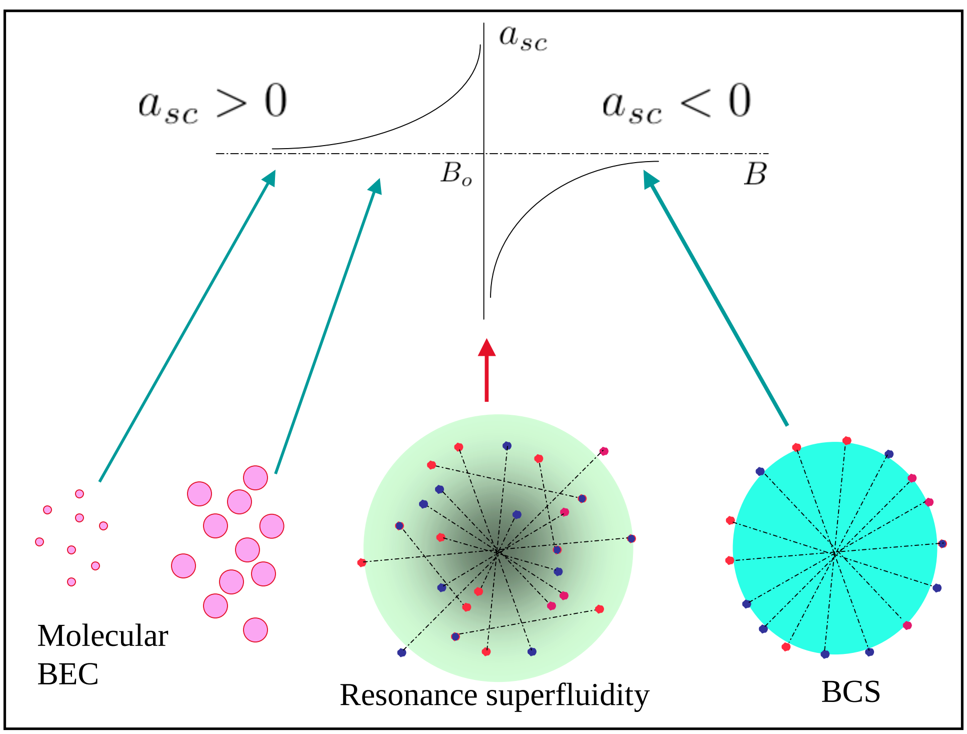
Background

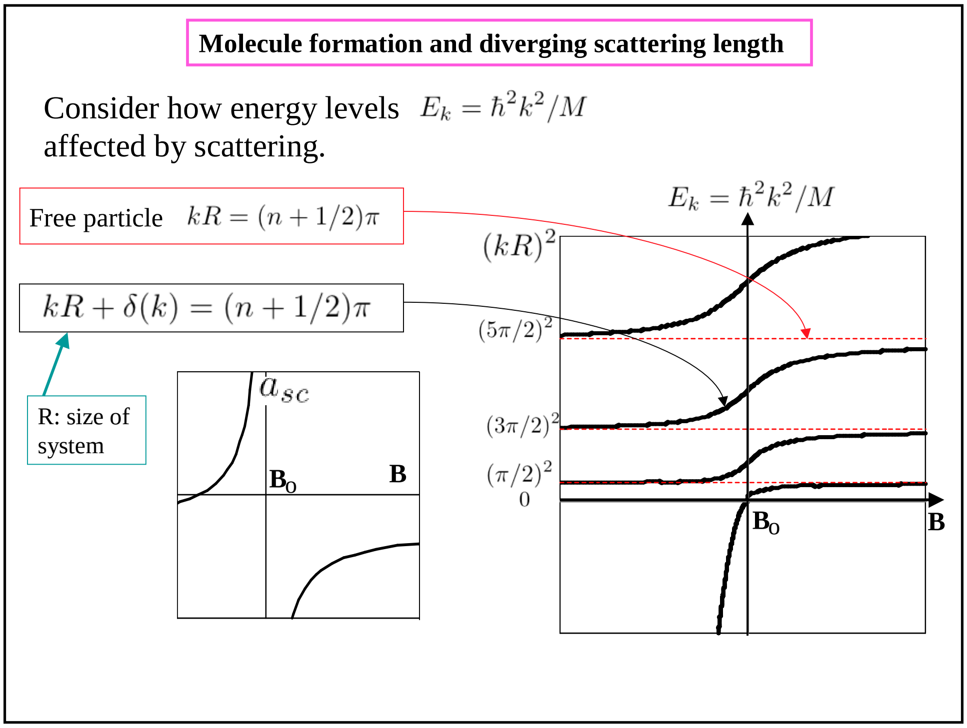
Key Experimental Facts and Isues

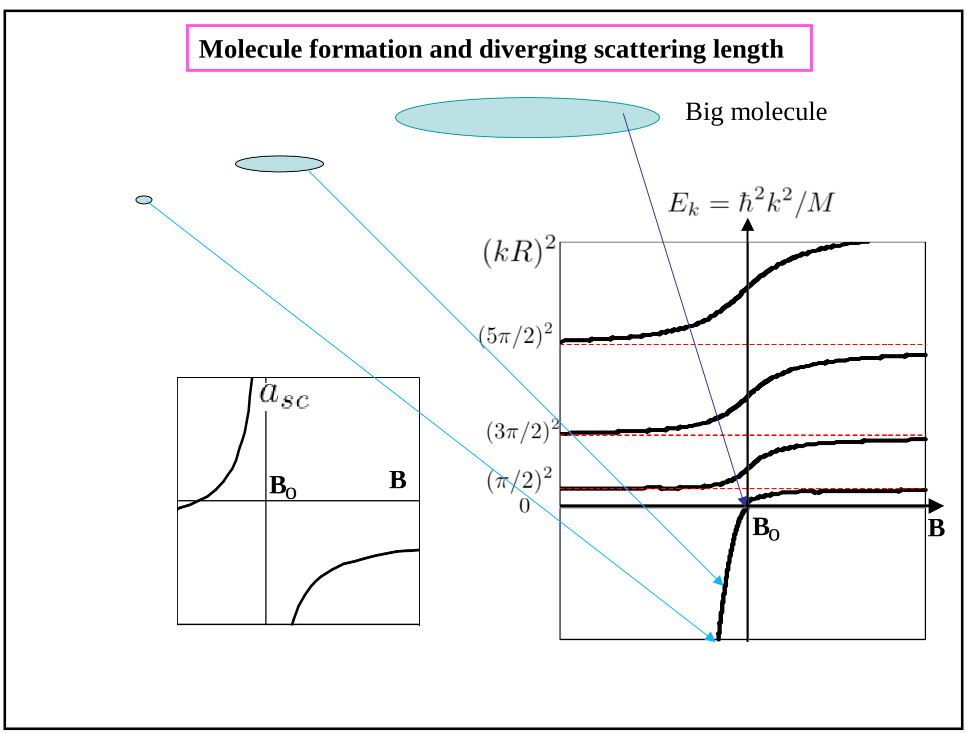
Current Situation in Theory

Resolutions of Key Issues


$a_{sc} > 0$
a_{sc}
$a_{sc} < 0$
Bound state
No bound state
0
B_o
B
Molecular bound state
Atomic scattering state
B
B_o

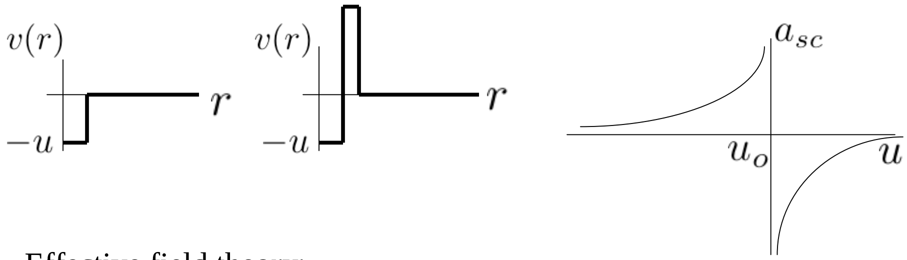

a_{sc}
$a_{sc} > 0$
$a_{sc} < 0$
B_o
B
Molecular BEC
Resonance superfluidity
BCS


Molecule formation and diverging scattering length
Consider how energy levels $E_k = \hbar^2 k^2/M$
affected by scattering.
Free particle $kR = (n+1/2)\pi$
$kR + \delta(k) = (n+1/2)\pi$
R: size of system
a_{sc}
B_O
B
$E_k = \hbar^2 k^2/M$
$(kR)^2$
$(5\pi/2)^2$
$(3\pi/2)^2$
$(\pi/2)^2$
0
B_O
B


Molecule formation and diverging scattering length
Big molecule
$E_k = \hbar^2 k^2/M$
$(kR)^2$
$(5\pi/2)^2$
$(3\pi/2)^2$
$(\pi/2)^2$
0
B_O
B
a_{sc}
B_O
B

Single Channel model

$$H = \int \left[\frac{\hbar^2}{2M} \nabla \psi_\sigma^\dagger \nabla \psi_\sigma + v(\mathbf{r}-\mathbf{r}') \psi_\uparrow^\dagger(\mathbf{r}) \psi_\downarrow^\dagger(\mathbf{r}') \psi_\downarrow(\mathbf{r}') \psi_\uparrow(\mathbf{r}) \right]$$



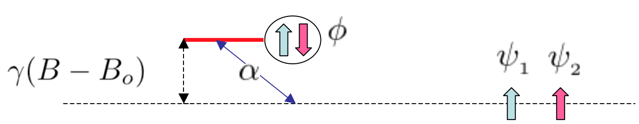
Effective field theory:

$$H = \int \left[\sum_{\mu=1,2} \frac{\hbar^2}{2M} \nabla \psi_\mu^\dagger \nabla \psi_\mu + \frac{g}{2} \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow + ... \right]$$

$$g = 4\pi\hbar^2 a_s / M$$

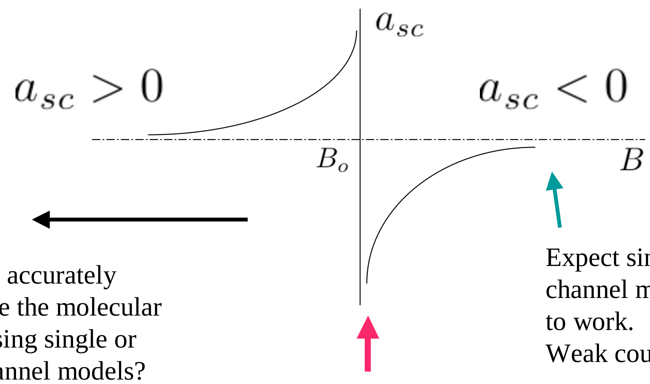
Resonance model : (two channel models)

$$H = \int \left[\frac{\hbar^2}{2M} \nabla \psi_\sigma^\dagger \nabla \psi_\sigma + \frac{\hbar^2}{4M} \nabla \phi^\dagger \nabla \phi + \mu_{co}(B - B_o) \phi^\dagger \phi \right]$$
$$+ \int \left[\alpha \left(\phi^\dagger \psi_1 \psi_2 + h.c. \right) + \frac{g_{bg}}{2} \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1 \right]$$



Chiafalo et.al. PRL 87, 120406 (2001) Ohashi and Griffin et.al. PRL (2002)

$$a = a_{bg} \left(1 - \frac{|\alpha|^2}{\mu_{co}(B - B_o)} \right) \qquad g_{bg} = 4\pi\hbar^2 a_{bg} / M$$



How to accurately describe the molecular BEC using single or two channel models?

Can single channel models capture the right physics?

Or must we use 2 channel models?

Expect single channel models to work. Weak coupling BCS

BCS-BEC Crossover theory:

Ground state remain pair condensate form as one crosses from the fermion to the molecular side.

$$|G\rangle = O^{\dagger N/2}|\text{vac}\rangle$$

$$O^{\dagger} = \int f(\mathbf{r}_1 - \mathbf{r}_2)\psi_{\uparrow}^{\dagger}(\mathbf{r}_1)\psi_{\downarrow}^{\dagger}(\mathbf{r}_2)$$

Advantage: |G> reduces to the correct ground state in the BCS and BEC limit

D.M. Eagles, Phys. Rev. {\bf 186}, 456 (1969);
A.J. Leggett, J. Phys. (Paris), Colloq. {\bf 41}, 7 (1980);
P. Nozieres and S. Schmitt-Rink, J. Low Temp. Phys. {\bf 59}, 195 (1985).
C. Sá de Melo, M. Randeria, and J. Engelbrecht, Phys. Rev. Lett. {\bf 71}, 3202 (1993)

The simplest crossover theory:
Single channel model with contact potential => universal behavior

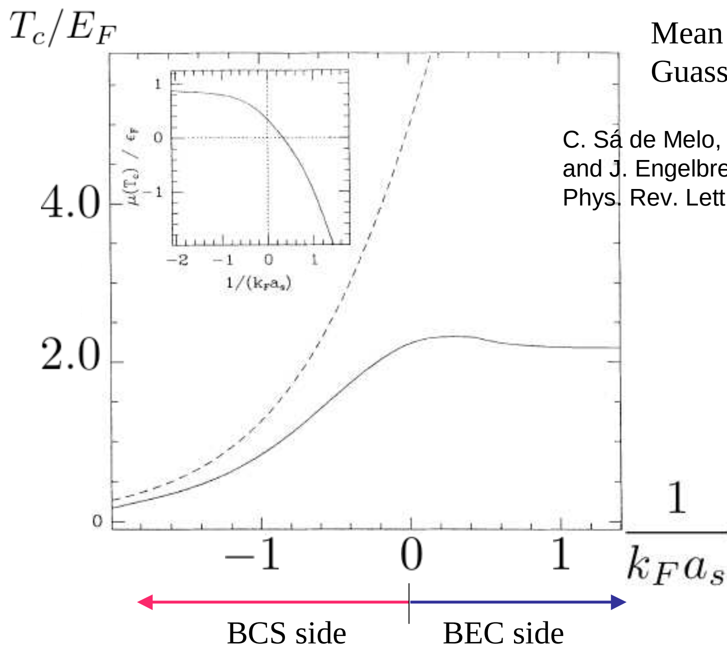
$$H = \int \left[\sum_{\mu=1,2} \frac{\hbar^2}{2M} \nabla\psi^\dagger_\mu \nabla\psi_\mu + \frac{g}{2}\psi^\dagger_\uparrow\psi^\dagger_\downarrow\psi_\downarrow\psi_\uparrow + ... \right]$$

T=0:

$$\frac{1}{g} = \frac{1}{\Omega}\sum_{\mathbf{k}} \left(\frac{1}{2E_{\mathbf{k}}} - \frac{1}{2\epsilon_{\mathbf{k}}} \right)$$ Gap equation

$$n = \frac{1}{2\Omega}\sum_{\mathbf{k}} \left(1 - \frac{\epsilon_{\mathbf{k}} - \mu}{E_{\mathbf{k}}} \right).$$ Number constraint

$$E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + \Delta^2}$$



Mean field +
Guassian Fluctuations

C. Sá de Melo, M. Randeria,
and J. Engelbrecht,
Phys. Rev. Lett. 71, 3202 (1993)

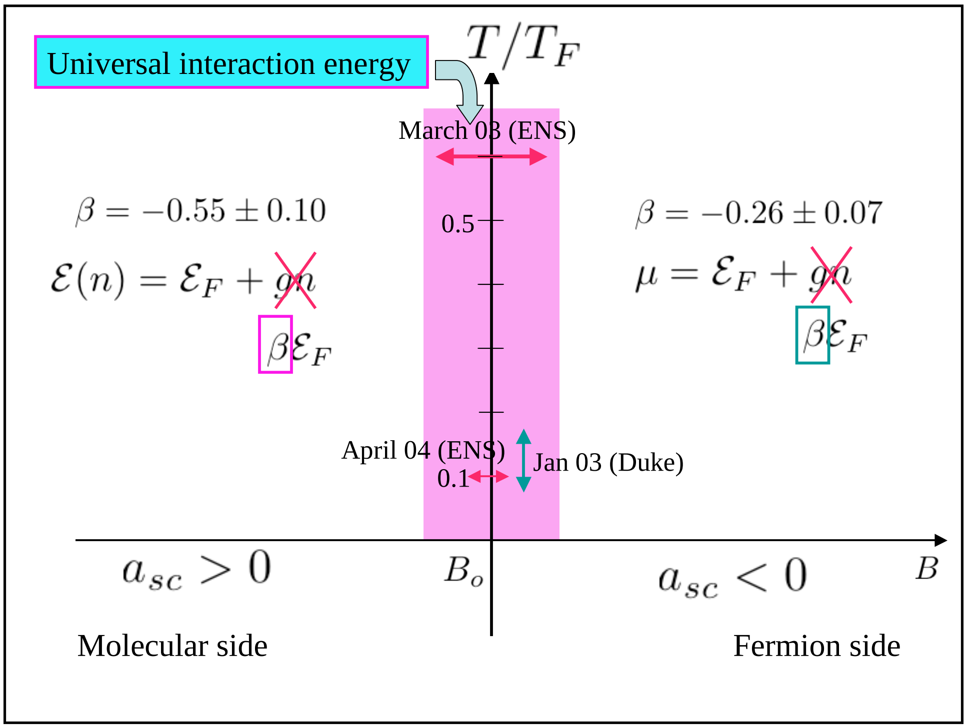
Situation of Experiments

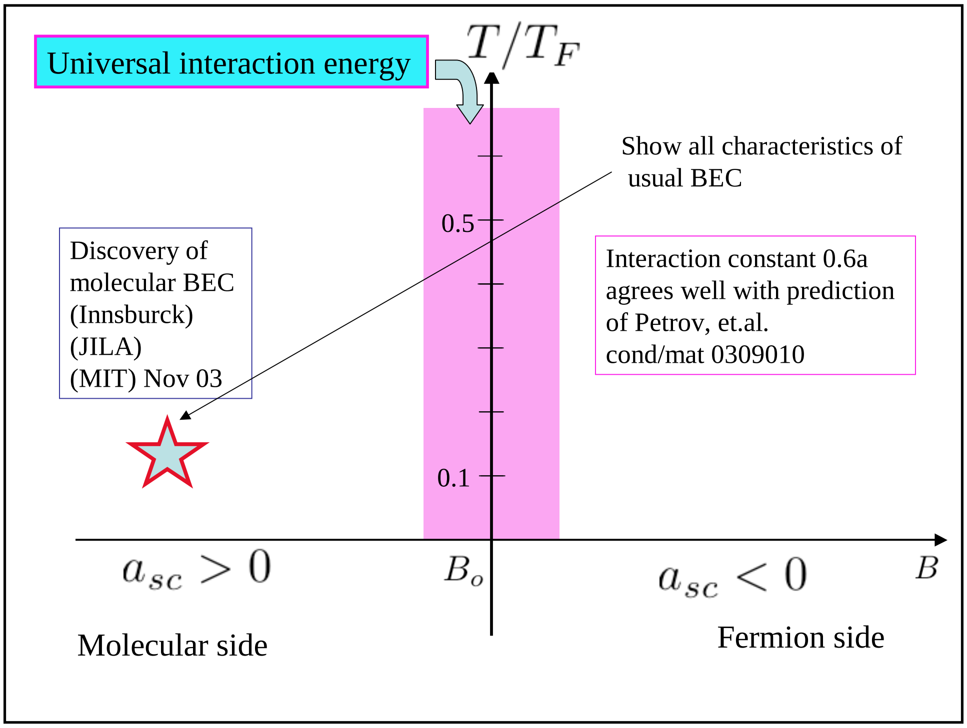
Universal interaction energy

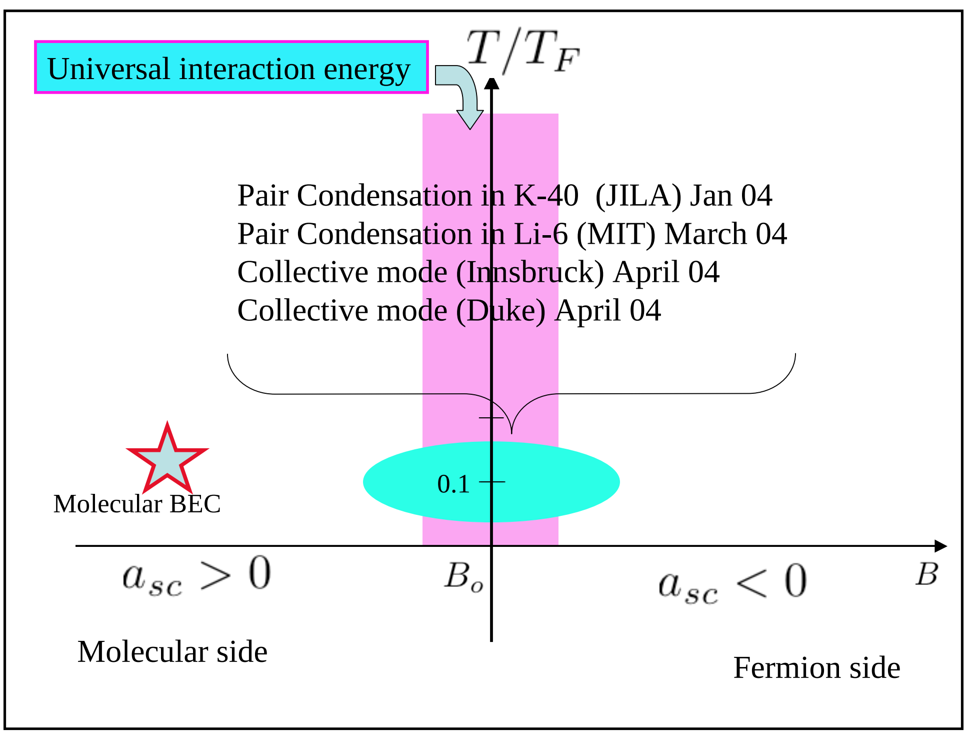
Molecular condensate

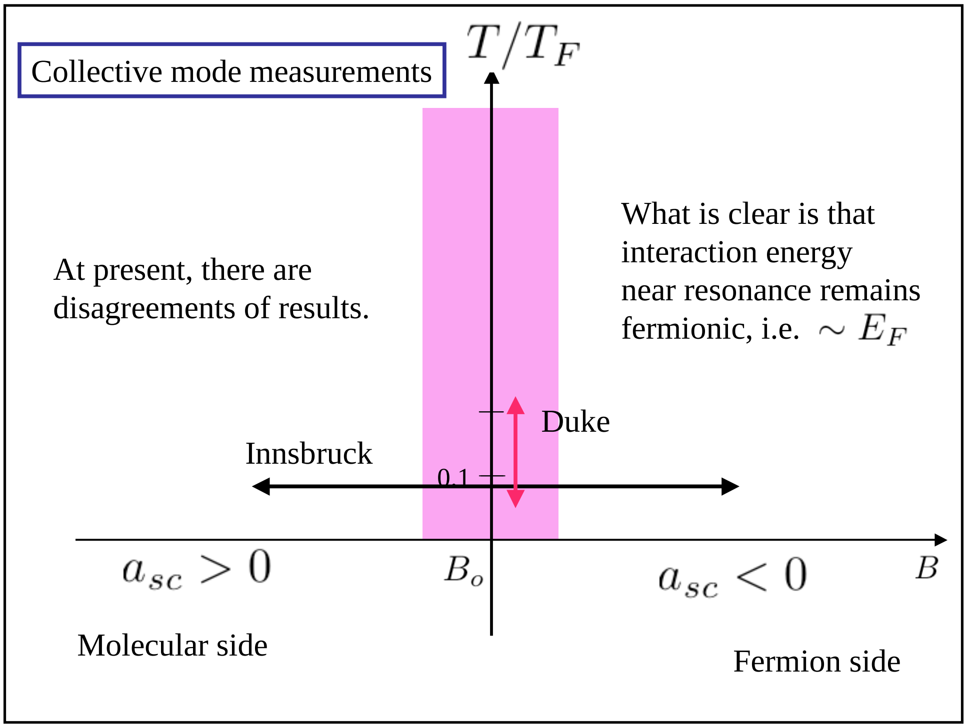
Collective modes

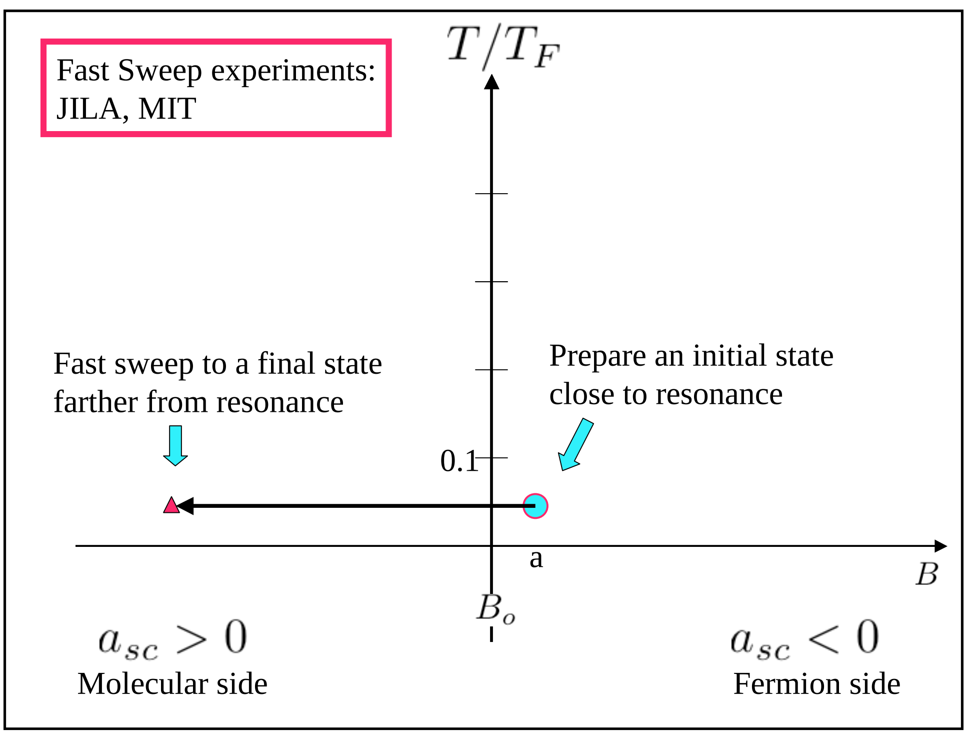
Fast Sweep Experiments and Condensation of Fermion pairs

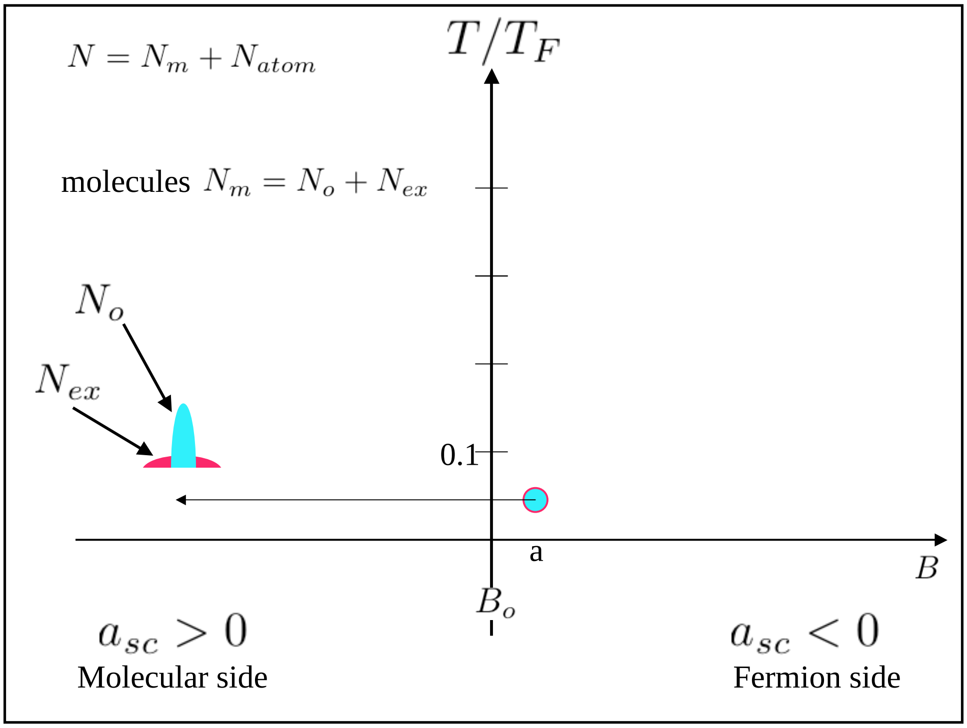


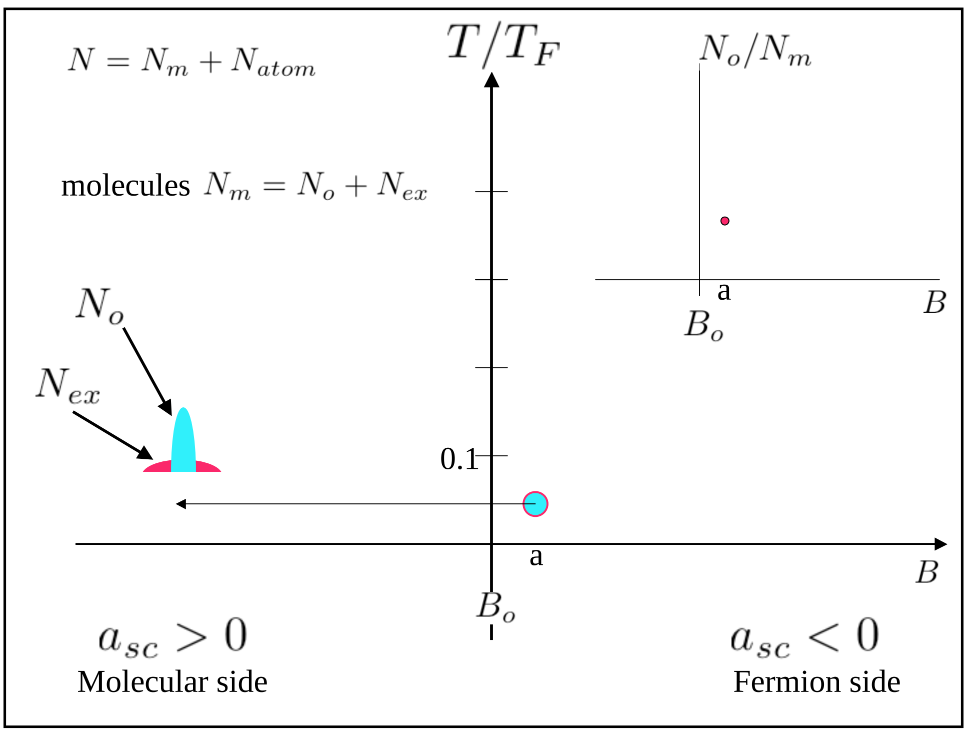

Universal interaction energy
T/T_F
Show all characteristics of usual BEC
0.5
Discovery of molecular BEC (Innsburck) (JILA) (MIT) Nov 03
Interaction constant 0.6a agrees well with prediction of Petrov, et.al. cond/mat 0309010
0.1
$a_{sc} > 0$
B_o
$a_{sc} < 0$
B
Molecular side
Fermion side

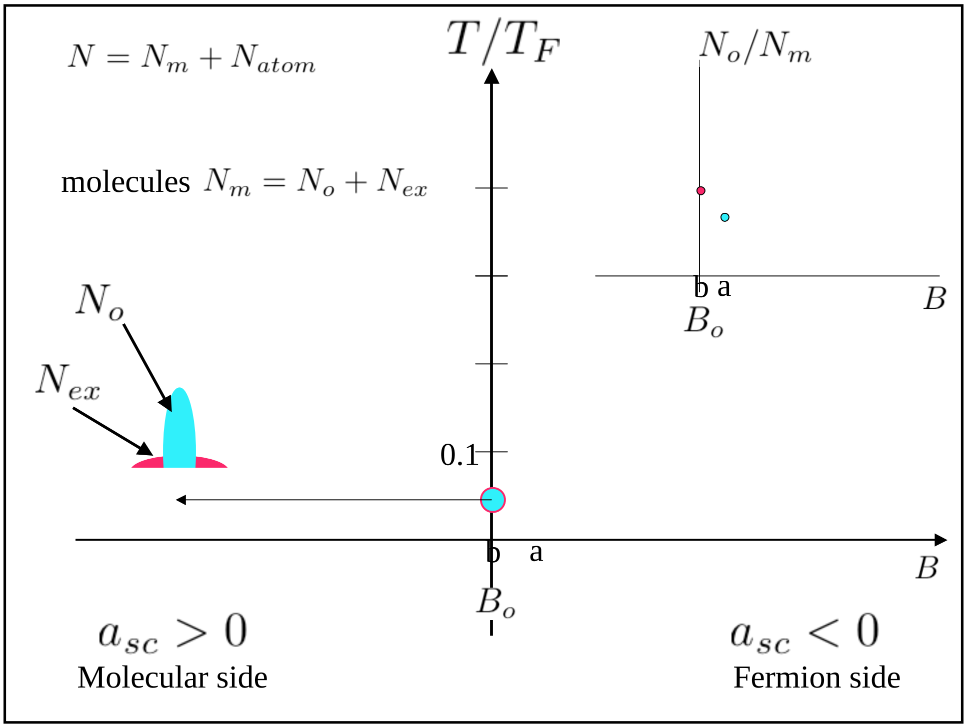

Universal interaction energy
T/T_F
Pair Condensation in K-40 (JILA) Jan 04
Pair Condensation in Li-6 (MIT) March 04
Collective mode (Innsbruck) April 04
Collective mode (Duke) April 04
Molecular BEC
0.1
$a_{sc} > 0$
B_o
$a_{sc} < 0$
B
Molecular side
Fermion side

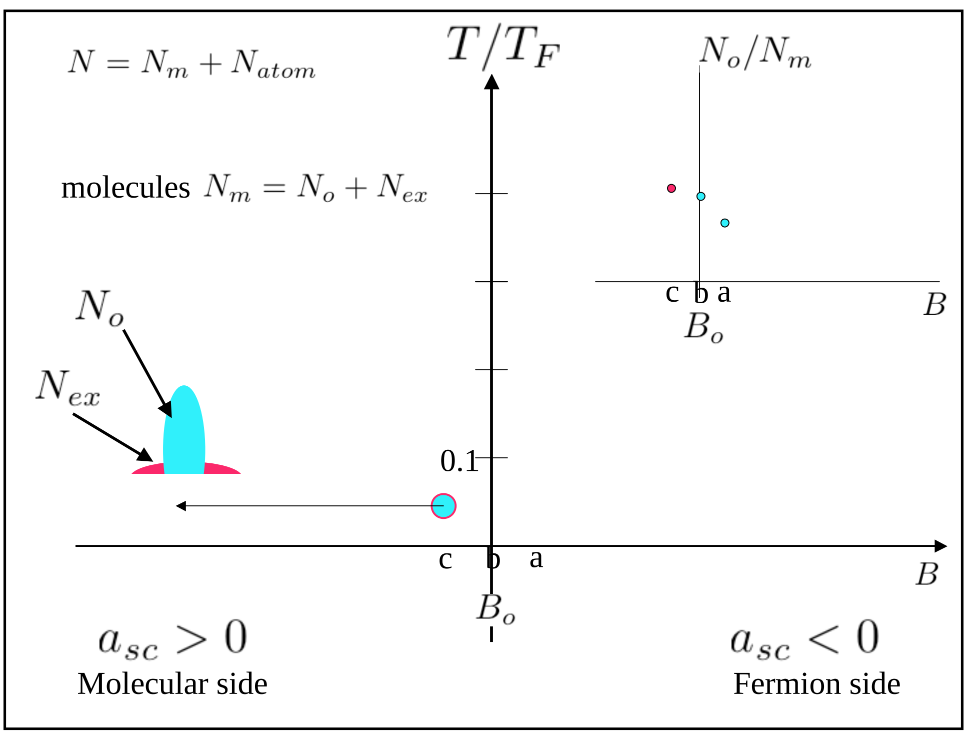

Collective mode measurements
T/T_F
At present, there are
disagreements of results.
What is clear is that
interaction energy
near resonance remains
fermionic, i.e. $\sim E_F$
Duke
Innsbruck
0.1
$a_{sc} > 0$
B_o
$a_{sc} < 0$
B
Molecular side
Fermion side


Fast Sweep experiments:
JILA, MIT
T/T_F
Fast sweep to a final state
farther from resonance
Prepare an initial state
close to resonance
0.1
a
B
B_o
$a_{sc} > 0$
Molecular side
$a_{sc} < 0$
Fermion side


T/T_F
$N = N_m + N_{atom}$
molecules $N_m = N_o + N_{ex}$
N_o
N_{ex}
0.1
a
B
B_o
$a_{sc} > 0$
Molecular side
$a_{sc} < 0$
Fermion side


T/T_F
$N = N_m + N_{atom}$
N_o/N_m
molecules $N_m = N_o + N_{ex}$
a
B
B_o
N_o
N_{ex}
0.1
a
B
B_o
$a_{sc} > 0$
Molecular side
$a_{sc} < 0$
Fermion side


$N = N_m + N_{atom}$
molecules $N_m = N_o + N_{ex}$
N_o
N_{ex}
T/T_F
0.1
N_o/N_m
b a
B_o
B
b a
B_o
B
$a_{sc} > 0$
Molecular side
$a_{sc} < 0$
Fermion side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molecules $N_m = N_o + N_{ex}$
N_o
N_{ex}
T/T_F
0.1
N_o/N_m
c b a
B_o
B
c b a
B_o
B
$a_{sc} > 0$
Molecular side
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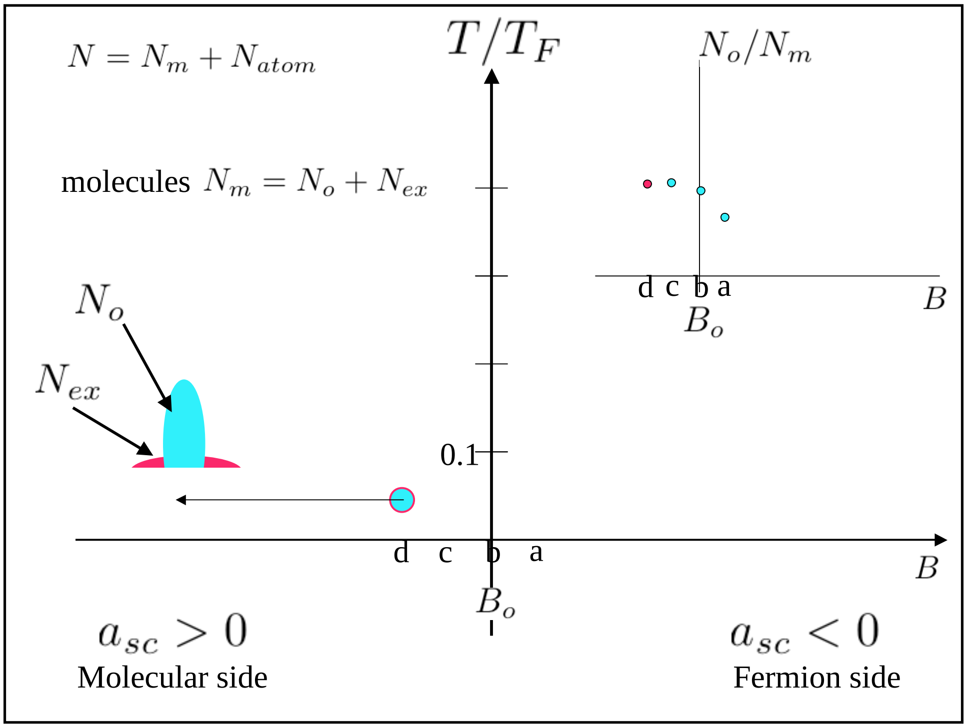

$N = N_m + N_{atom}$
T/T_F
N_o/N_m
molecules $N_m = N_o + N_{ex}$
N_o
N_{ex}
0.1
d c b a
B
B_o
d c b a
B
B_o
$a_{sc} > 0$
Molecular side
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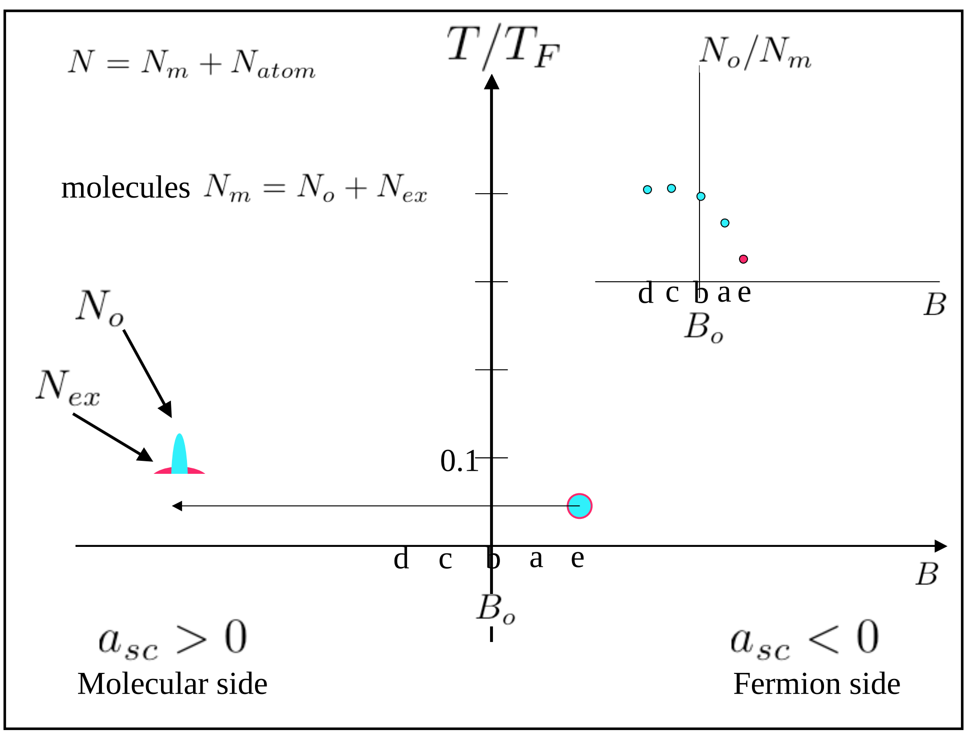

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T/T_F
N_o/N_m
molecules $N_m = N_o + N_{ex}$
N_o
N_{ex}
0.1
d c b a e
B
B_o
d c b a e
B
B_o
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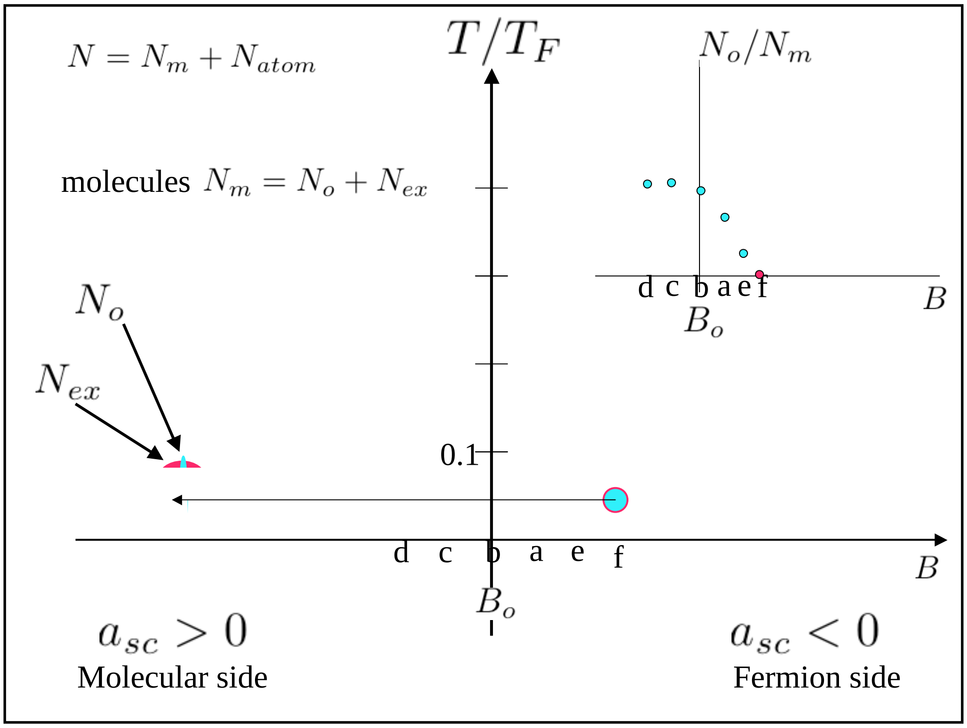
Fermion side



$N = N_m + N_{atom}$
T/T_F
N_o/N_m
molecules $N_m = N_o + N_{ex}$
N_o
N_{ex}
d c b a e f
B
B_o
0.1
d c b a e f
B
B_o
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Molecular side
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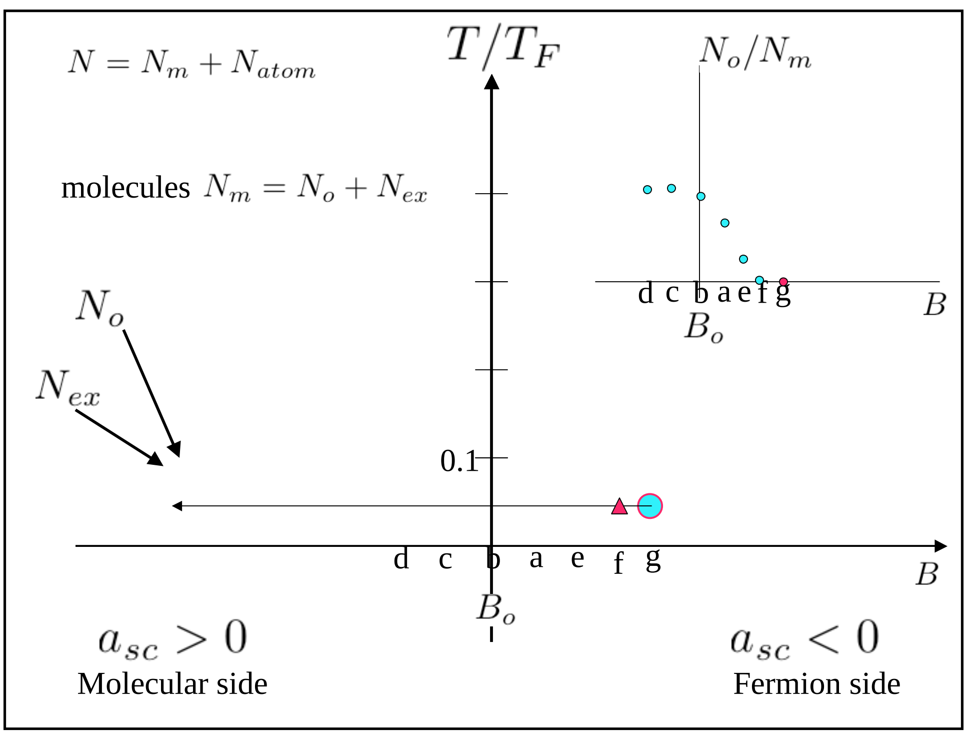
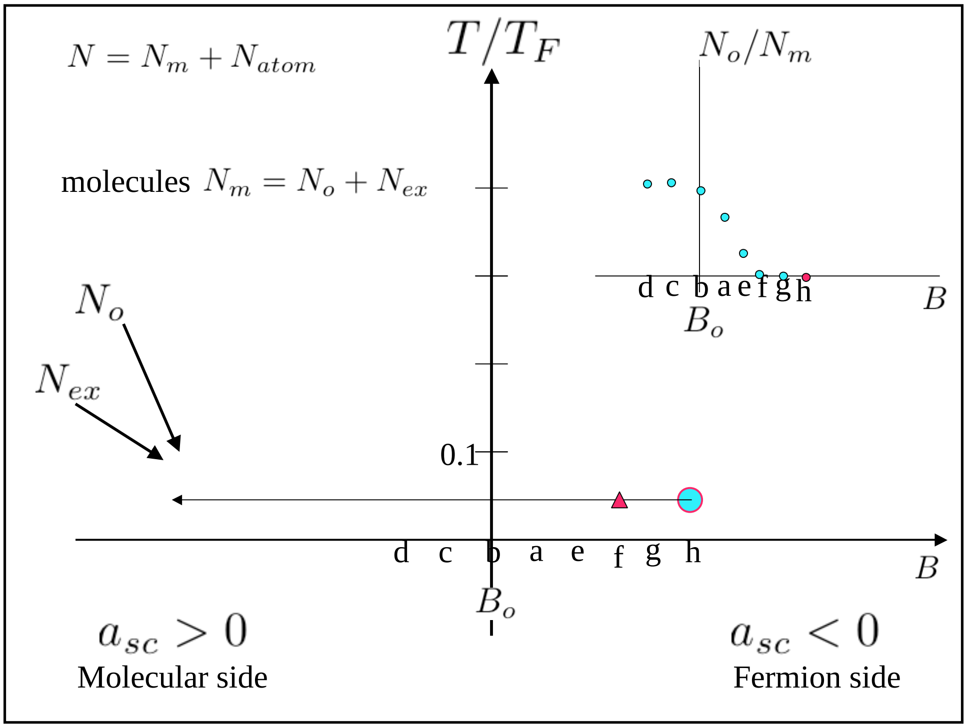
Fermion side



$N = N_m + N_{atom}$
T/T_F
N_o/N_m
molecules $N_m = N_o + N_{ex}$
N_o
N_{ex}
d c b a e f g
B
B_o
0.1
d c b a e f g
B
B_o
$a_{sc} > 0$
Molecular side
$a_{sc} < 0$
Fermion side


$N = N_m + N_{atom}$
T/T_F
N_o/N_m
molecules $N_m = N_o + N_{ex}$
N_o
N_{ex}
d c b a e f g h
B_o
B
0.1
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B
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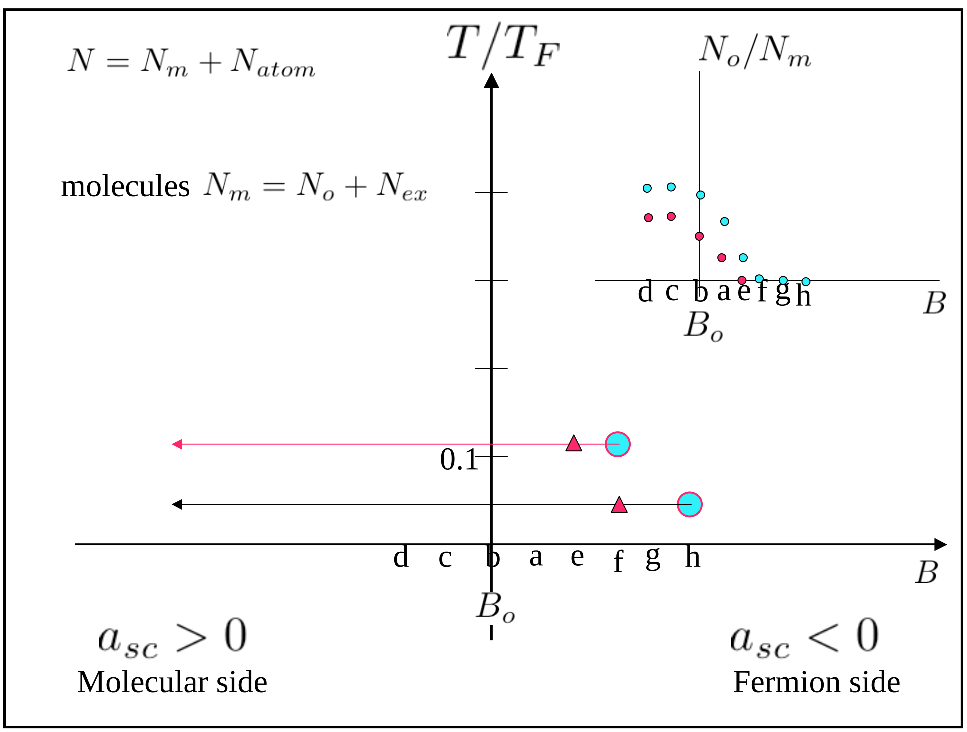
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T/T_F
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B_o
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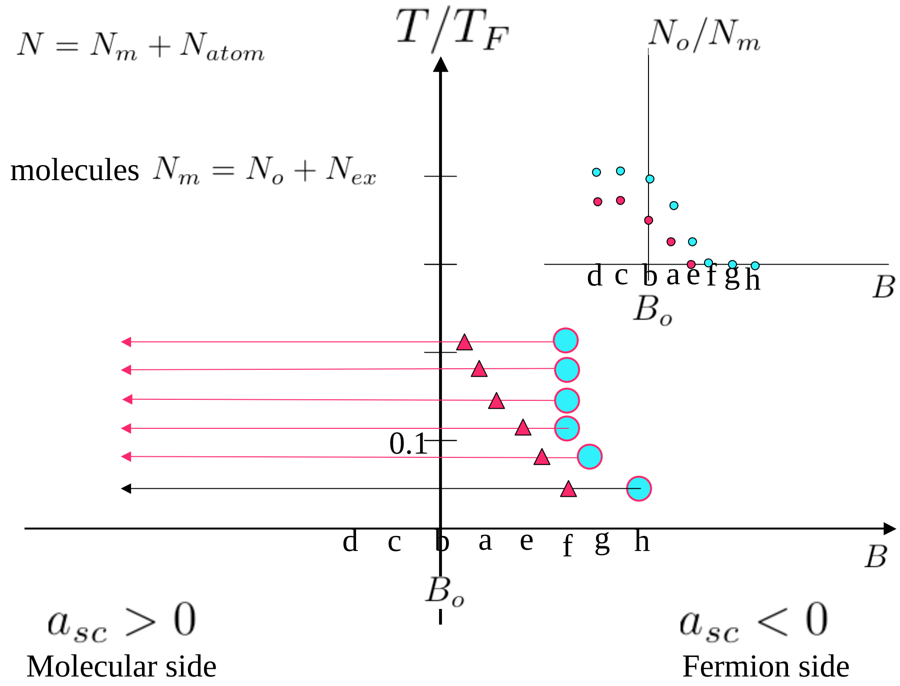
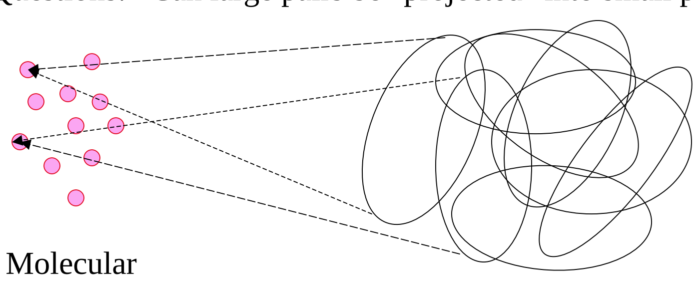


Definitely a clear boundary separating different behavior

Evidence for phase transition or artifact of fast sweep ?

Questions: Can large pairs be “projected” into small pairs?



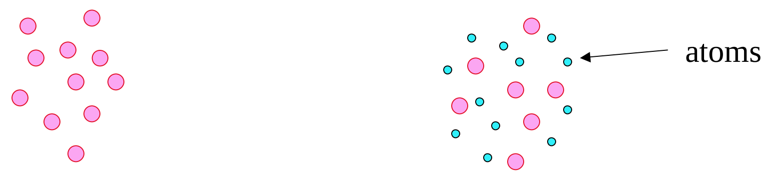
Universality => pair size $\sim n^{-1/3}$

How can one explain N_o/N_m independent of sweep rate?

Definitely a clear boundary separating different behavior

Evidence for phase transition or artifact of fast sweep ?

Questions: A condensate of molecules mixed with atoms?



Molecular
BEC

Atom-molecule mixture near resonance?
Molecules stabilized by Fermi sea.

How can one explain N_o/N_m independent of sweep rate?

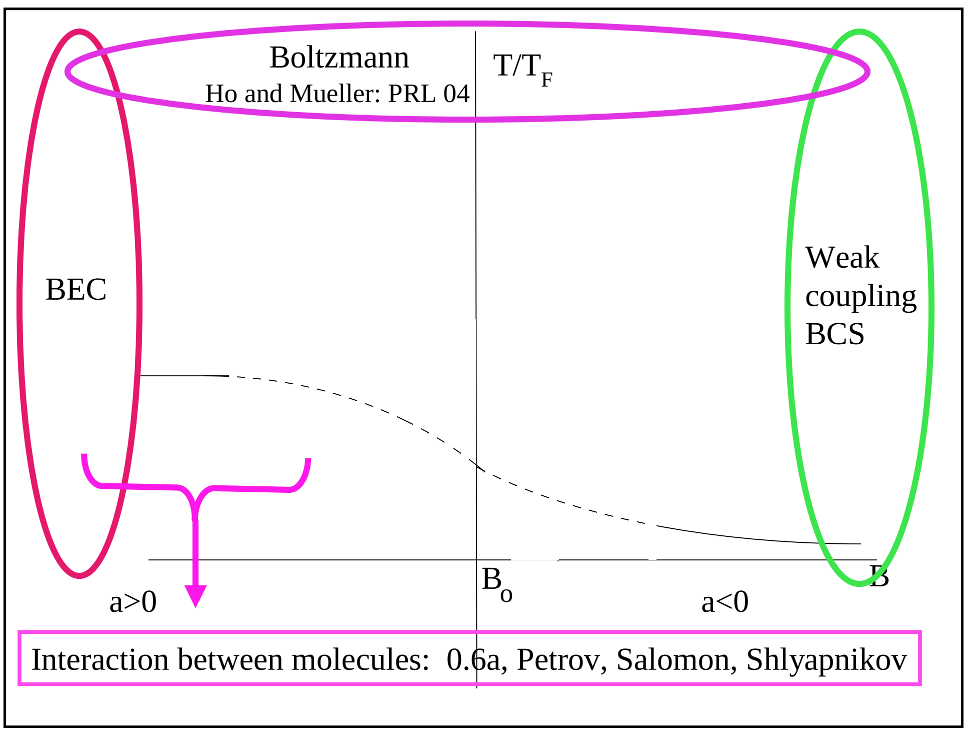
Key Questions:

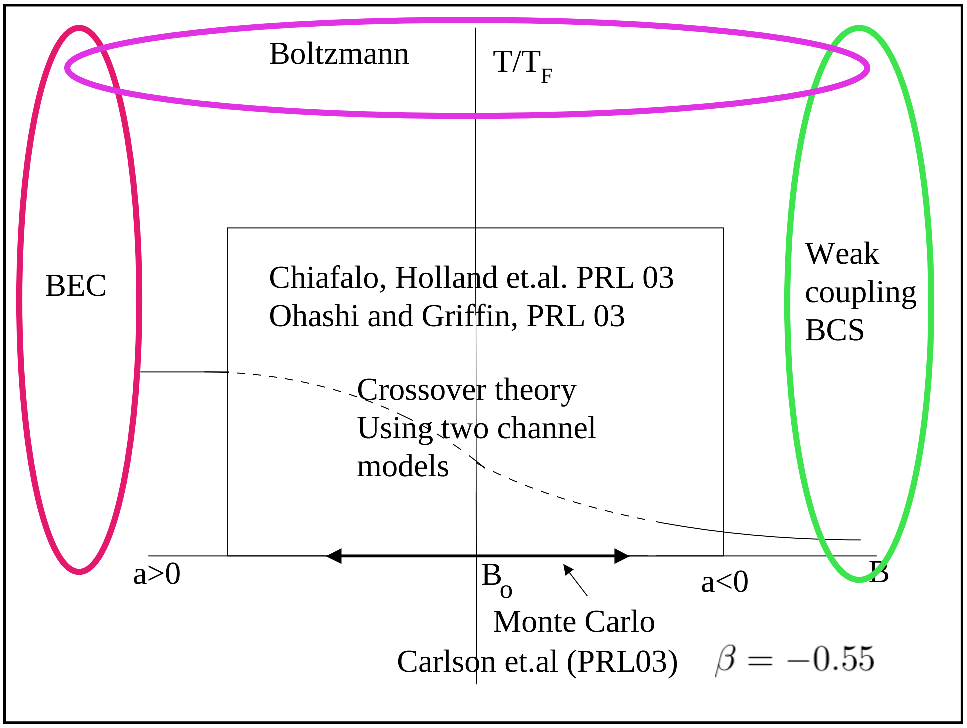
Is a clear phase transition observed?

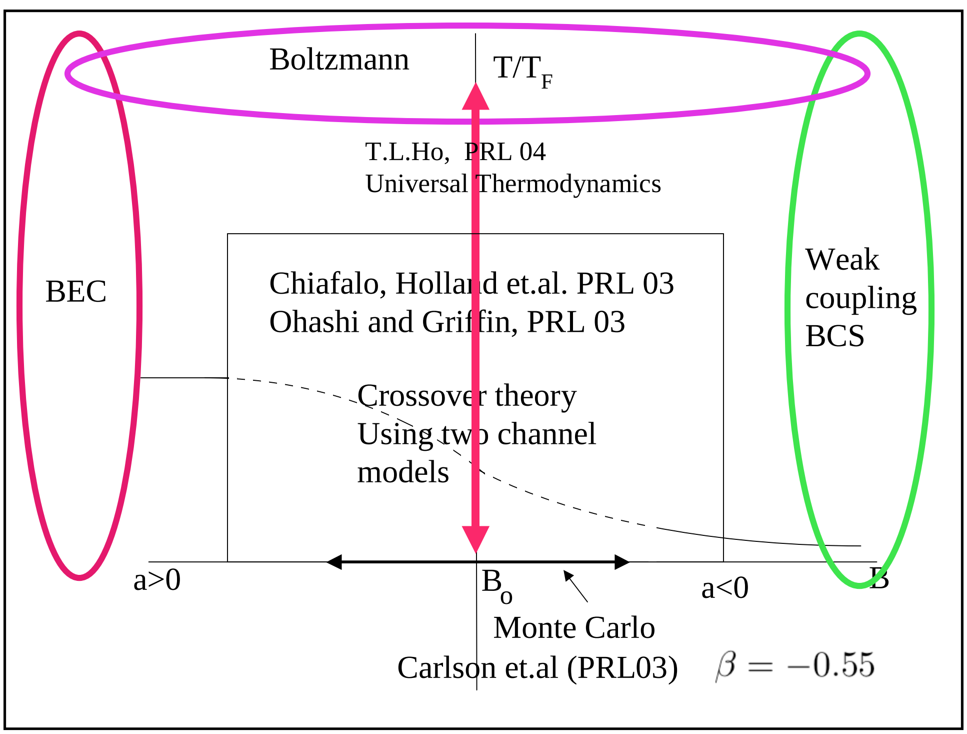
Do we have a Fermion pair condensate?

If it is a pair condensate, large pairs
or small pairs?

Summary of Theoretical Situation




Boltzmann
T/T_F
BEC
Chiafalo, Holland et.al. PRL 03
Ohashi and Griffin, PRL 03
Crossover theory
Using two channel
models
Weak
coupling
BCS
a>0
B_o
a<0
B
Monte Carlo
Carlson et.al (PRL03)
$\beta = -0.55$


Boltzmann
T/T_F
T.L.Ho, PRL 04
Universal Thermodynamics
BEC
Chiafalo, Holland et.al. PRL 03
Ohashi and Griffin, PRL 03
Crossover theory
Using two channel
models
Weak
coupling
BCS
a>0
B_o
a<0
B
Monte Carlo
Carlson et.al (PRL03)
$\beta = -0.55$

Theoretical Situation :

Molecular side : 0.6a agree well with expt.

BCS side : weak coupling BCS deep inside BCS limit

Resonance regime:

1. Universal energy: High temperature well under control,
   (no well controlled theory at low temperatures)
   Universal thermodynamics
2. Many calculations using resonance model
   (Mostly focuses on Tc. Also results finite T results).
3. Very little calculation on correlation functions except for single channel models. Need some work in order to compare with experiments
4. Proposal of molecular BEC invading to fermion side
5. Two vs one channel equivalence?

Answers to Key Questions:

What kind of ground state do we have ?

Are two channel models truly different from single channel models?

Answers to Key Questions:

What kind of ground state do we have ?

Condensate with large pairs

Are two channel models truly different from single channel models?

Depending on the width of the resonance.

Universality

Condition under which universality emerges

Relation between two-channel and single channel systems.

Universality emerges when there are no other length (or energy) scales except for interparticle spacing (or Fermi energy).

Hence, single channel models with contact potential

$$H = \int \left[\sum_{\mu=1,2} \frac{\hbar^2}{2M} \nabla\psi_\mu^\dagger \nabla\psi_\mu + \frac{g}{2}\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow + ... \right]$$

must exhibit universality at resonance $|g| \rightarrow \infty$.

However, all resonances have an intrinsic width $\mathcal{E}^* = \hbar^2/(2Mr^{*2})$ in energy space, hence intrinsic length scale r^* .

Expect universality emerges when

$k_F r^* << 1$ or $\mathcal{E}^*/E_F >> 1$

Extraction of r^* :

Recall that

$$a_s = a_{bf}\left(1 - \frac{\mu_{co}W}{\mu_{co}(B-B_o)}\right)$$

Near resonance,

$$a_s = -\frac{a_{bf}\mu_{co}W}{\mu_{co}(B-B_o)}$$

$$a_{bf}\mu_{co}W = \frac{\hbar^2}{2Mr^*}$$

Simple derivation of condition for universality

Starting with the two-channel model

$$H - \mu N = \sum_{\mathbf{k},\sigma} (\epsilon_{\mathbf{k}} - \mu) a^{\dagger}_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma} + \sum_{\mathbf{k}} (\epsilon_{\mathbf{k}}/2 - 2\mu + \overline{\nu}) b^{\dagger}_{\mathbf{k}} b_{\mathbf{k}}$$
$$+ \frac{\alpha}{\sqrt{\Omega}} \sum_{\mathbf{k},\mathbf{q}} (b^{\dagger}_{\mathbf{q}} a_{\mathbf{k}+\mathbf{q}/2,\uparrow} a_{-\mathbf{k}+\mathbf{q}/2,\downarrow} + h.c.)$$

$$\overline{\nu} = \nu + \frac{\alpha^2}{\Omega} \sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}} \qquad a_s = \frac{M}{4\pi\hbar^2} \frac{\alpha^2}{\nu}$$

Crossover theory at T=0:

$$\sqrt{n_m} = \langle b_{\mathbf{q}=\mathbf{0}} \rangle \neq 0 \qquad \langle a_{\mathbf{k}\uparrow} a_{-\mathbf{k}\downarrow} \rangle \neq 0 \qquad \Delta = \alpha \sqrt{n_m}$$

$$\frac{\nu - 2\mu}{\alpha^2} = \frac{1}{\Omega} \sum_{\mathbf{k}} \left(\frac{1}{2E_{\mathbf{k}}} - \frac{1}{2\epsilon_{\mathbf{k}}} \right) \qquad E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + \Delta^2}$$

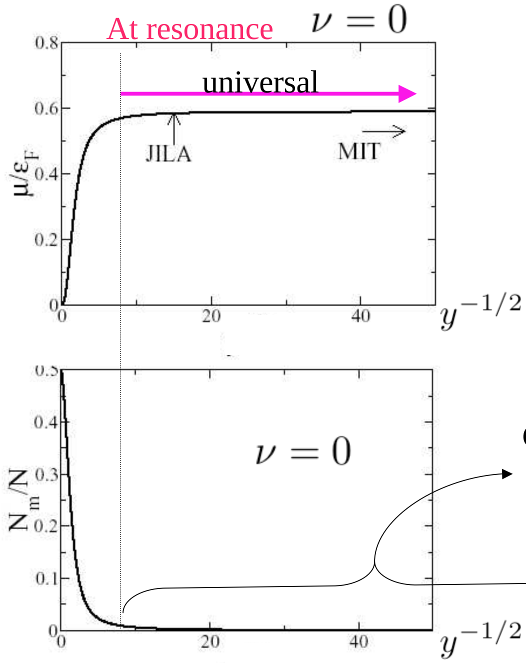
$$\Delta = \alpha \sqrt{n_m}$$

$$n = \frac{2\Delta^2}{\alpha^2} + \frac{1}{2\Omega} \sum_{\mathbf{k}} \left(1 - \frac{\epsilon_{\mathbf{k}} - \mu}{E_{\mathbf{k}}} \right)$$

Reduction to contact potential problem as $k_F r^* = k_F/\alpha^2 \to 0$

$$\frac{1}{g} = \frac{1}{\Omega} \sum_{\mathbf{k}} \left(\frac{1}{2E_{\mathbf{k}}} - \frac{1}{2\epsilon_{\mathbf{k}}} \right)$$

$$n = \frac{1}{2\Omega} \sum_{\mathbf{k}} \left(1 - \frac{\epsilon_{\mathbf{k}} - \mu}{E_{\mathbf{k}}} \right).$$



$$y = k_F r^*$$

$$a_{bf}\mu_{co}W = \frac{\hbar^2}{2Mr^*}$$

$$a_s = a_{bf}\left(1 - \frac{\mu_{co}W}{\mu_{co}(B - B_o)}\right)$$

Closed channel component becomes insignificant in universal regime at resonance

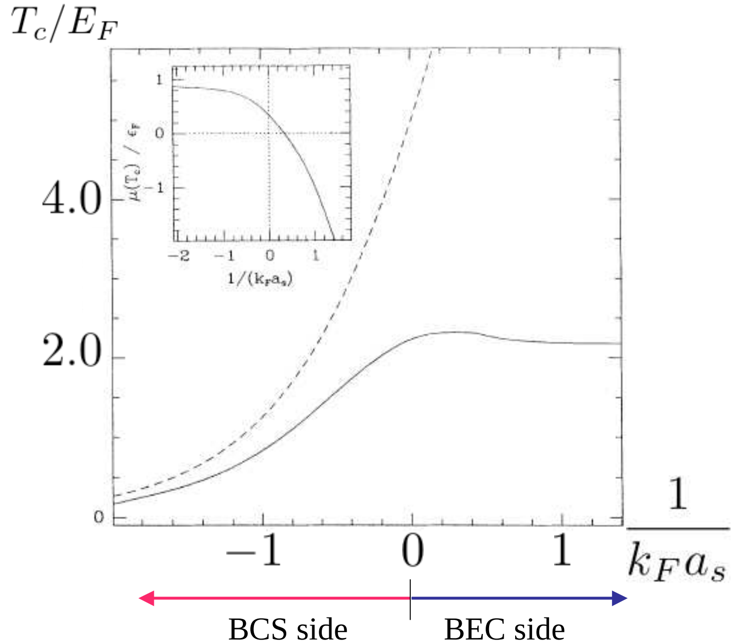
Diener and Ho
Cond-mat/0405174

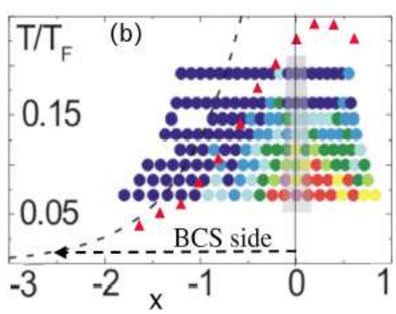
$$D^\dagger_{\mathbf{q}}(x) = \sum_{\mathbf{k},\alpha\beta} f_{\mathbf{k},\alpha\beta}(x) a^\dagger_{\mathbf{k}+\mathbf{q}/2,\alpha} a^\dagger_{-\mathbf{k}+\mathbf{q}/2,\beta}/2$$

$$|x\rangle = \mathcal{N} D^{\dagger N}_{\mathbf{q}=\mathbf{0}}(x)|\text{vac}\rangle$$

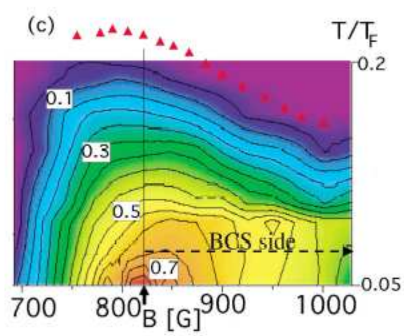
$$N_0 = \langle D^\dagger_{\mathbf{0}}(x) D_{\mathbf{0}}(x)\rangle_{x_o} = |\langle D_{\mathbf{0}}(x)\rangle_{x_o}|^2$$

$$N_{ex} = \sum_{\mathbf{q}\neq 0} \langle D^\dagger_{\mathbf{q}}(x) D_{\mathbf{q}}(x)\rangle_{x_o}$$





Diener and Ho
Cond-mat/0404517



Directly measuring the pair wavefunction using rf spectroscopy

Exciting a $a_\uparrow$ fermion to a different state c

Spin -5/2 Spin -7/2

$$\mathcal{R} = \frac{2\pi}{\hbar} \sum_f \left| \langle f | c^\dagger_{\mathbf{k}} a_{\mathbf{k},\uparrow} | G \rangle \right|^2 \delta(\mathcal{E}_f - \mathcal{E}_i - \hbar\omega)$$

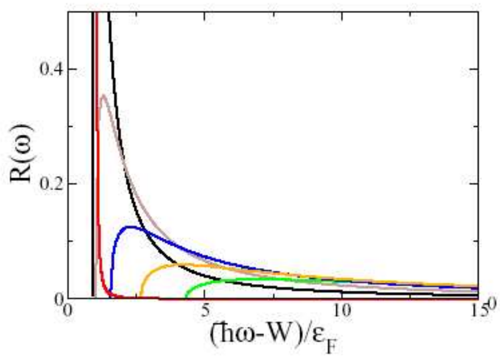
$$\mathcal{R}(\omega) = \frac{2\pi\lambda^2}{\hbar} \frac{1}{\Omega} \sum_{\mathbf{k}} v^2_{\mathbf{k}} \delta(E_{\mathbf{k}} + W + \epsilon_{\mathbf{k}} - \hbar\omega)$$

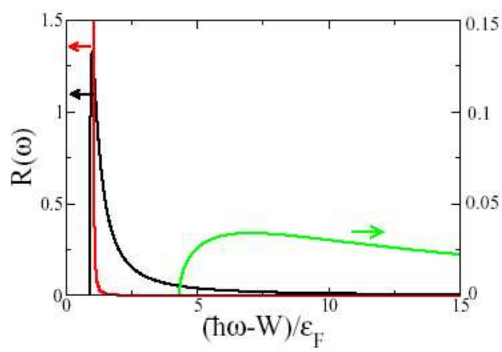
$$v^2_{\mathbf{k}} = [1 - (\epsilon_{\mathbf{k}} - \mu)/E_{\mathbf{k}})]/2$$

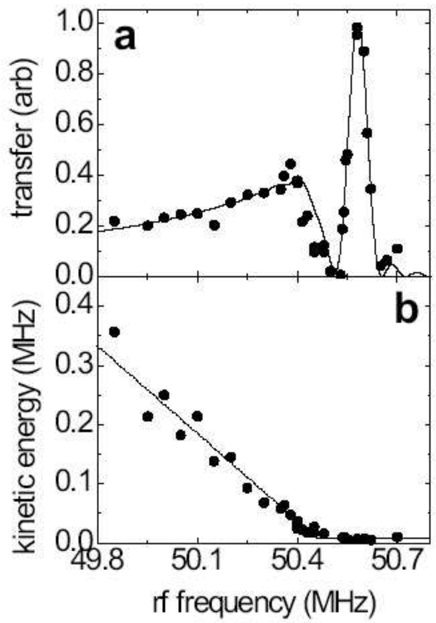
$$\mathcal{R}(\omega) = \frac{2\pi\lambda^2}{\hbar} \frac{1}{\Omega} D(\epsilon^*) \left| \frac{v^2_{\mathbf{k}}}{1 + \partial E_{\mathbf{k}} / \partial \epsilon_{\mathbf{k}}} \right|_{\epsilon^*}$$

$$\mathcal{R}(\omega) = \frac{2\pi\lambda^2}{\hbar} D(\epsilon^*) \left| v_{\mathbf{k}} / u_{\mathbf{k}} \right|^2_{\epsilon^*} ,$$

Black : x = 0, red : x=-2 , brown : 0.5 , blue 1.0,
Orange : 1.5 , green = 2.0







Black : x = 0, red : x=-2 , brown : 0.5 , blue 1.0,
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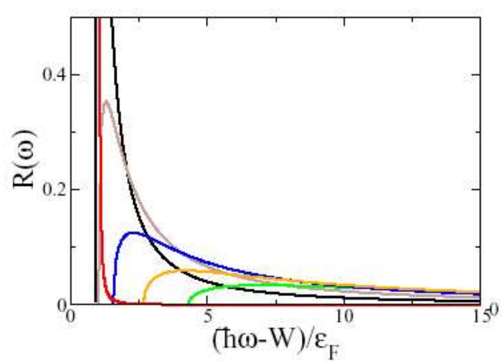
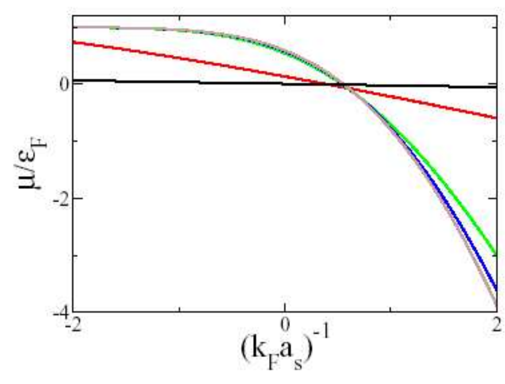


$$y \equiv k_F r^* << 1$$

$$\frac{2M\eta^2}{\hbar^2 \epsilon_F} >> 1$$



$$\mathcal{R}(\omega) = \frac{2\pi\lambda^2}{\hbar}\frac{1}{\Omega}\sum_{\mathbf{k}} v_{\mathbf{k}}^2 \delta(E_{\mathbf{k}} + W + \epsilon_{\mathbf{k}} - \hbar\omega)$$

$$v_{\mathbf{k}}^2 = [1 - (\epsilon_{\mathbf{k}} - \mu)/E_{\mathbf{k}})]/2$$

$$\mathcal{R}(\omega) = \frac{2\pi\lambda^2}{\hbar}\frac{1}{\Omega} D(\epsilon^*) \left| \frac{v_{\mathbf{k}}^2}{1 + \partial E_{\mathbf{k}}/\partial \epsilon_{\mathbf{k}}} \right|_{\epsilon^*}$$

$$\mathcal{R}(\omega) = \frac{2\pi\lambda^2}{\hbar} D(\epsilon^*) \left| v_{\mathbf{k}}/u_{\mathbf{k}} \right|_{\epsilon^*}^2 ,$$

$$D_{\mathbf{q}}^\dagger(x) = \sum_{\mathbf{k},\alpha\beta} f_{\mathbf{k},\alpha\beta}(x) a_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger a^\dagger_{-\mathbf{k}+\mathbf{q}/2,\beta}/2$$

$$f_{\alpha\beta}(\mathbf{r}; x) = \Omega^{-1/2} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} f_{\mathbf{k},\alpha\beta}(x)$$

$$|x\rangle = \mathcal{N} D_{\mathbf{q}=\mathbf{0}}^{\dagger N}(x) |\text{vac}\rangle$$

$$|\Psi(x)\rangle = \mathcal{N} \prod_{\mathbf{k},\alpha\beta} \left(u_{\mathbf{k}}(x) + v_{\mathbf{k},\alpha\beta}(x) a_{\mathbf{k},\alpha}^\dagger a^\dagger_{-\mathbf{k},\beta} \right) |\text{vac}\rangle$$

$$N_0 = \langle D_{\mathbf{0}}^{\dagger}(x) D_{\mathbf{0}}(x) \rangle_{x_o}$$

$$N_{ex} = \sum_{\mathbf{q} \neq 0} \langle D_{\mathbf{q}}^{\dagger}(x) D_{\mathbf{q}}(x) \rangle_{x_o}$$

$$a_s = \frac{\eta}{\nu} \qquad a_s = a_{bg}\left(1 - \frac{W}{B - B_o}\right)$$

$$a_{sc} = a_{bg}\left(1 - \frac{\mu_{co} W}{\mu_{co}(B - B_o)}\right)$$

$$a_{sc} = -\frac{a_{bg}\mu_{co} W}{\mu_{co}(B - B_o)}$$

$$\frac{a_s}{r^*} = \frac{\hbar^2/(2Mr^{*2})}{\nu} \qquad \eta = \frac{\hbar^2}{2Mr^*}$$