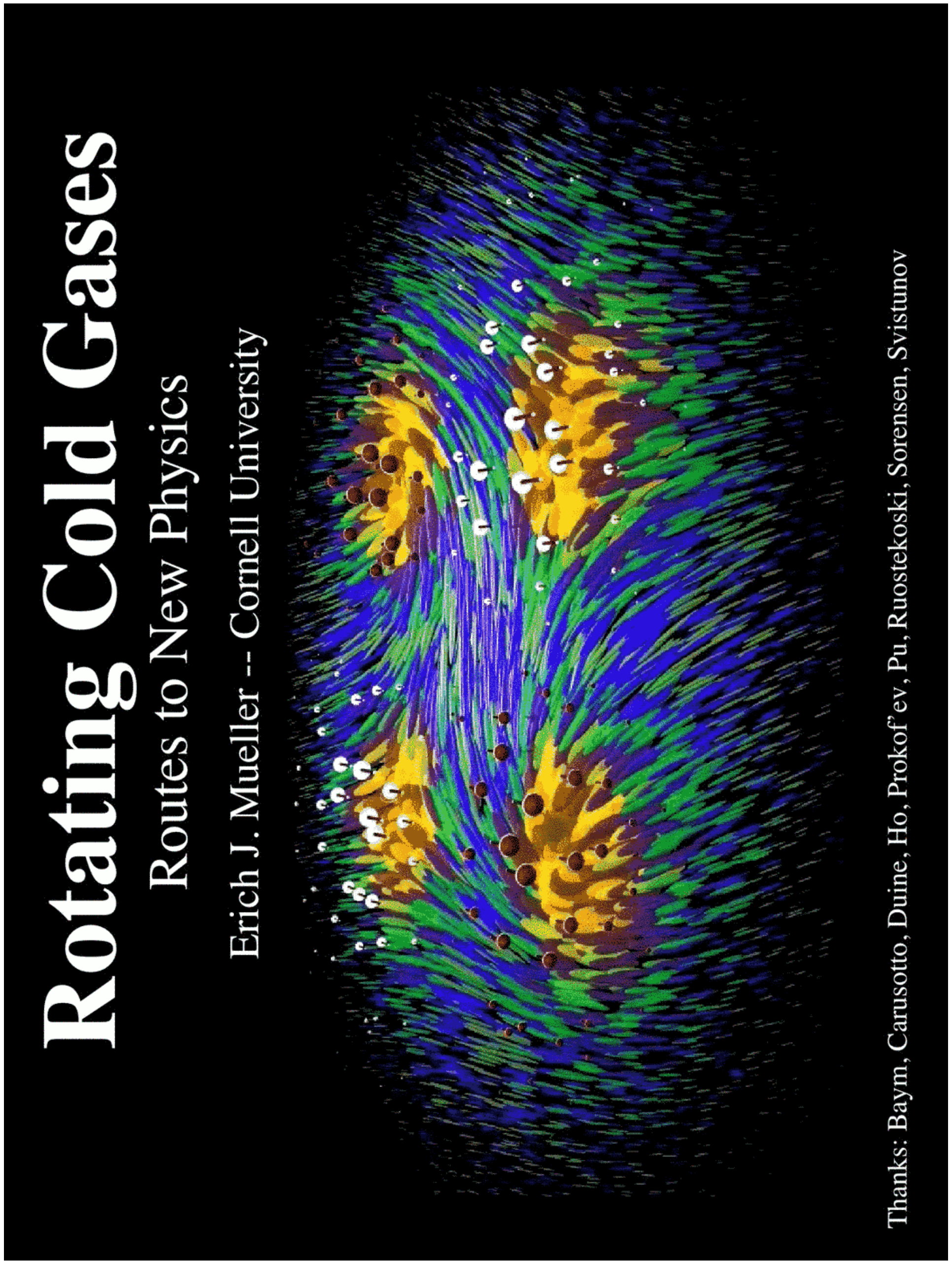
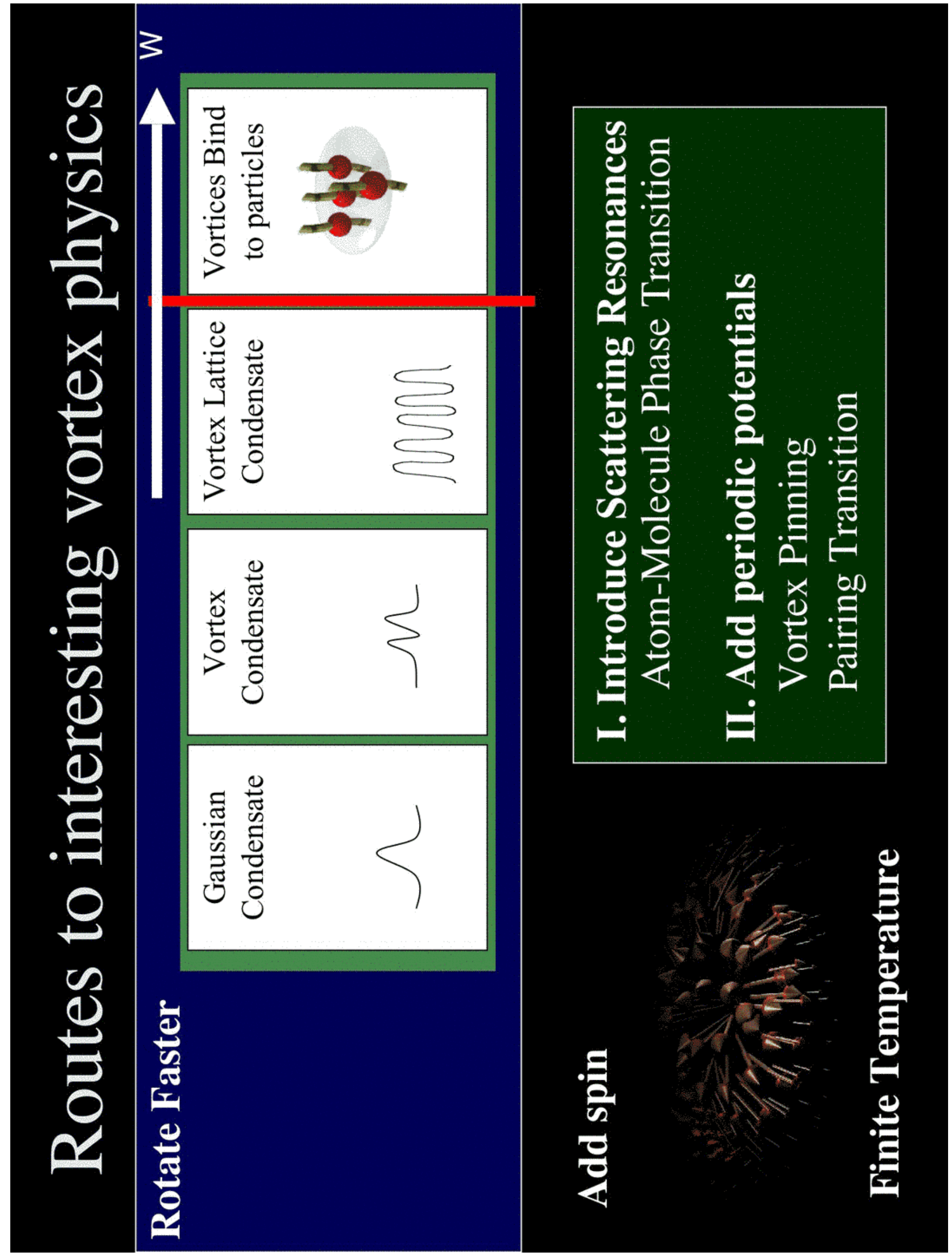
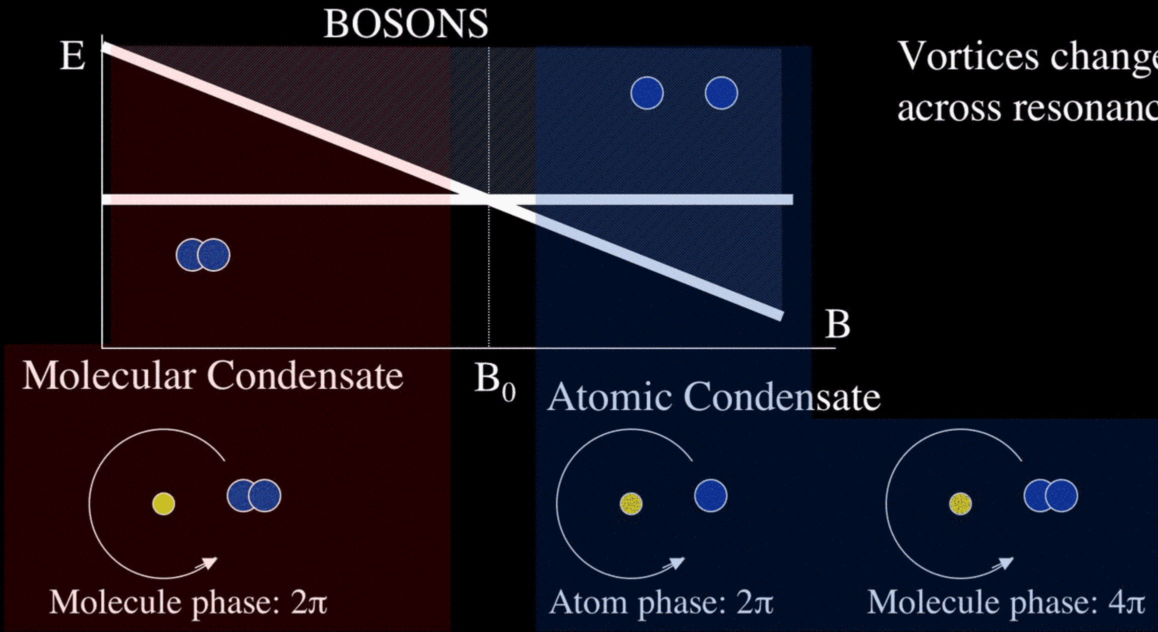

Rotating Cold Gases
Routes to New Physics
Erich J. Mueller -- Cornell University
Thanks: Baym, Carusotto, Duine, Ho, Prokof'ev, Pu, Ruostekoski, Sorensen, Svistunov


Routes to interesting vortex physics
Rotate Faster
W
Gaussian Condensate
Vortex Condensate
Vortex Lattice Condensate
Vortices Bind to particles
Add spin
Finite Temperature
I. Introduce Scattering Resonances
Atom-Molecule Phase Transition
II. Add periodic potentials
Vortex Pinning
Pairing Transition

I. Vortex Physics near resonance

M.W.J. Romans, R.A. Duine, S. Sachdev, H.T.C. Stoof, Quantum phase transition in an atomic Bose gas with a Feshbach resonance. **cond-mat/0312446**

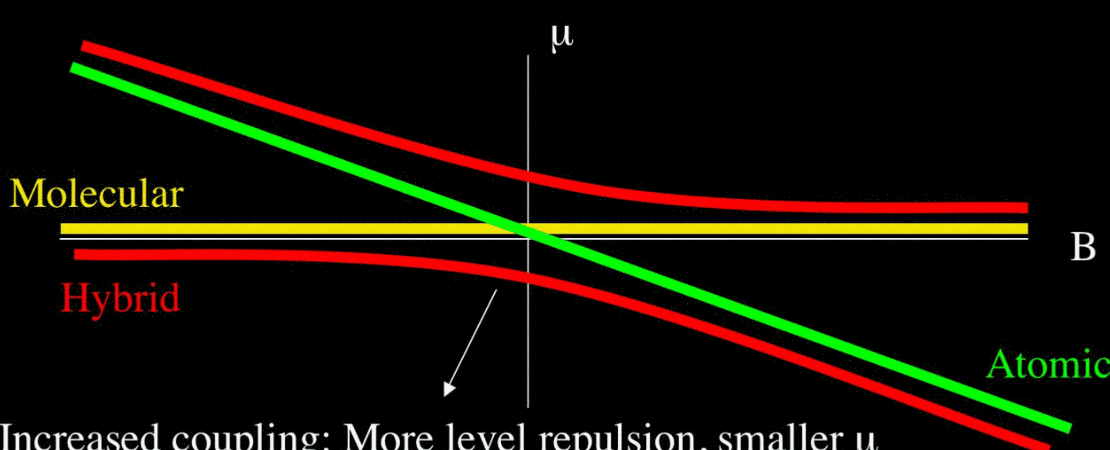
L. Radzihovsky, J. Park, P. B. Weichman, Superfluid transitions in bosonic atom-molecule mixtures near Feshbach resonance. **cond-mat/0312237**



Vortices change across resonance

Problem: Mechanical Instability

Eddy Timmermans, Paolo Tommasini, Robin Côté, Mahir Hussein, and Arthur Kerman
Rarified Liquid Properties of Hybrid Atomic-Molecular Bose-Einstein Condensates
Phys. Rev. Lett. 83, 2691 (1999)

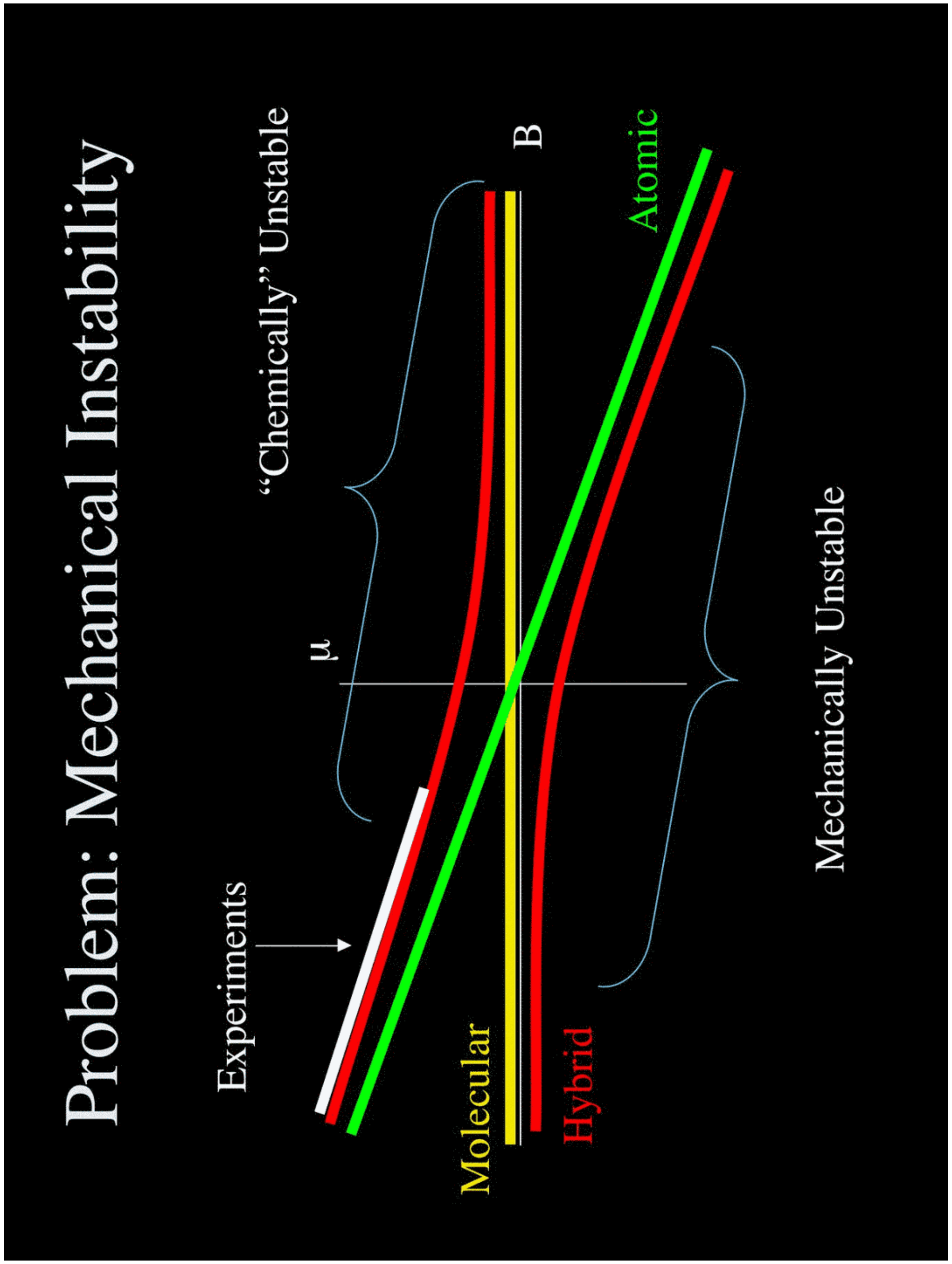


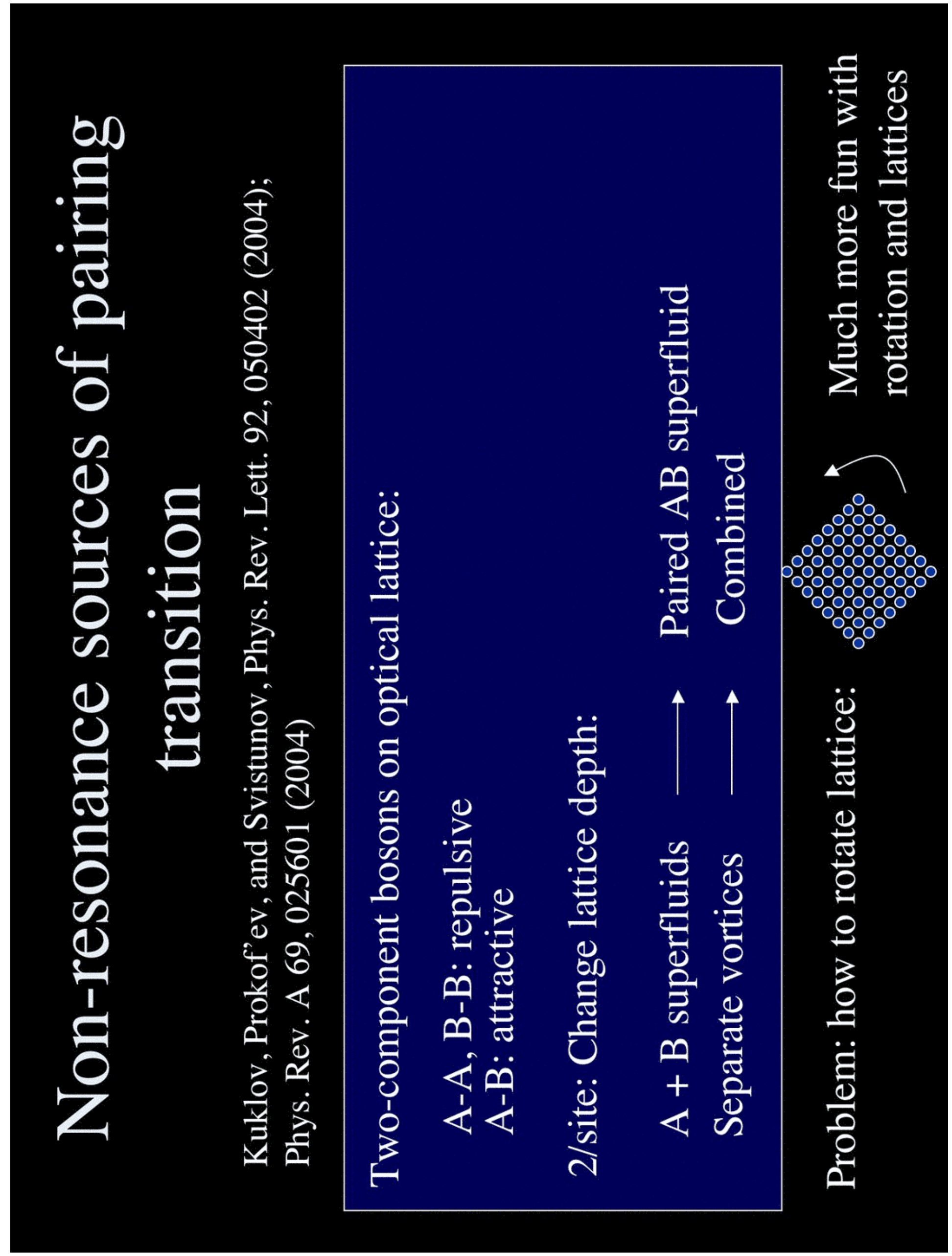
Increased coupling: More level repulsion, smaller μ

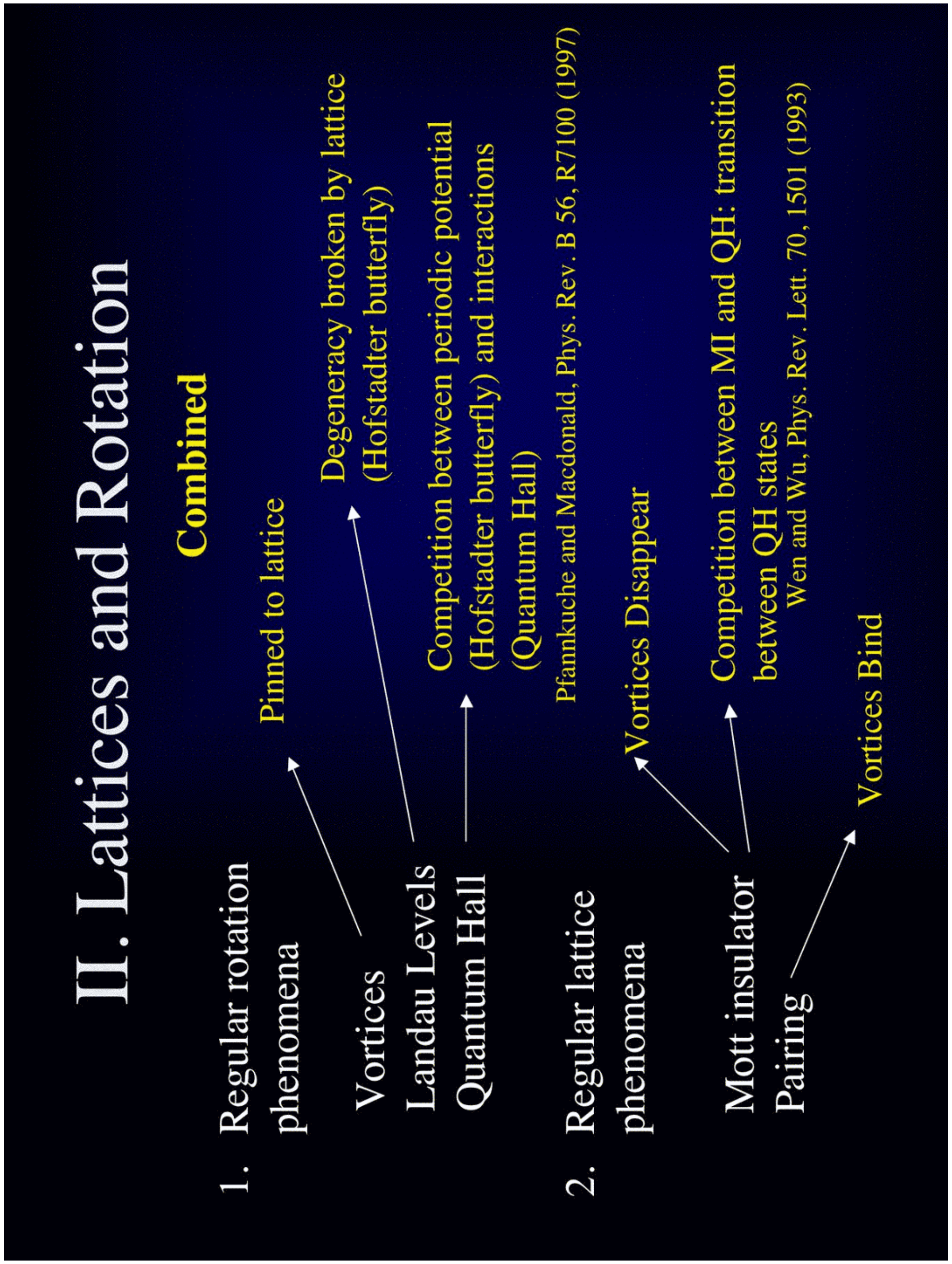
Coupling: nonlinear -- grows with N

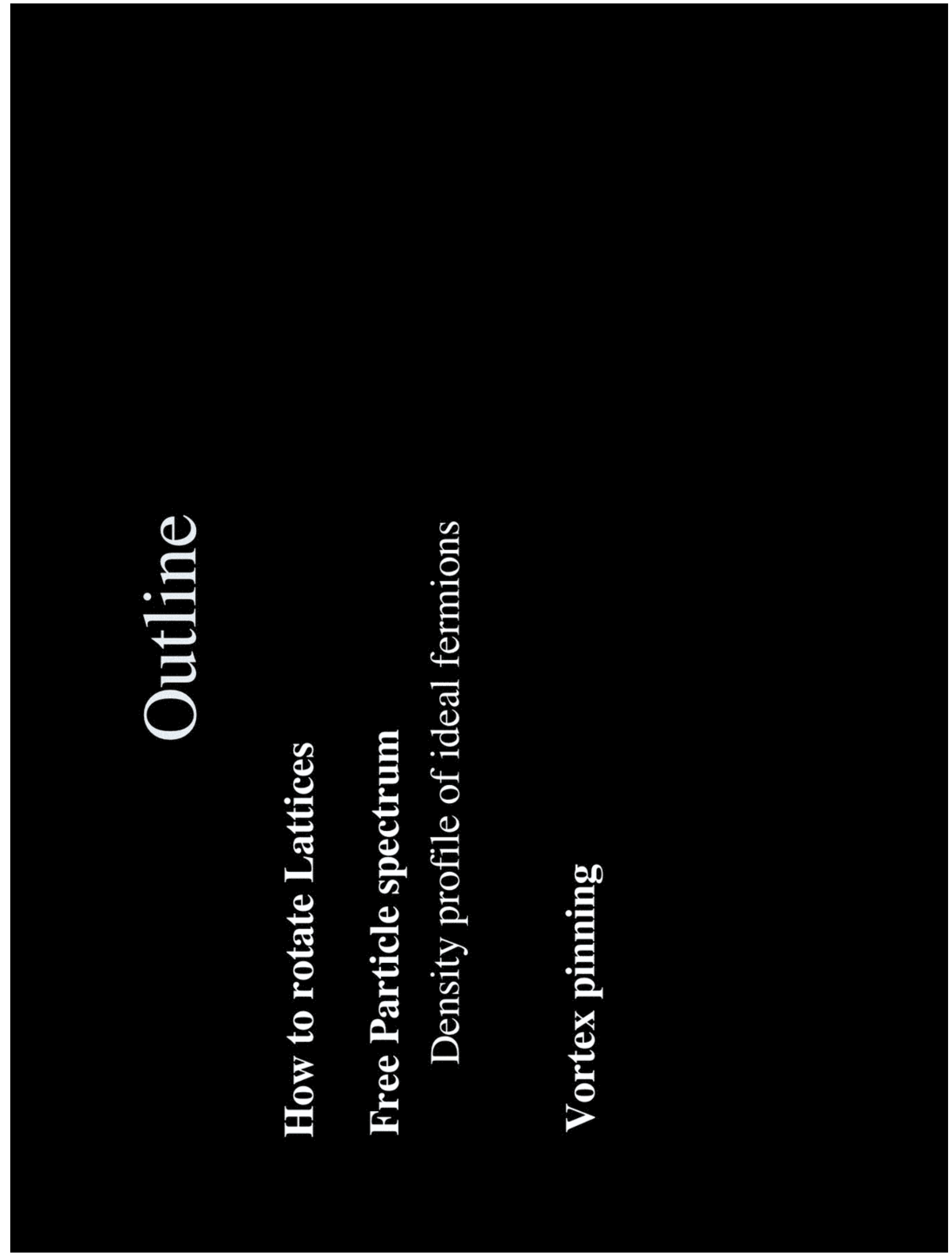
$$\frac{\partial \mu}{\partial N} < 0$$

Confirmed by exact diagonalization in Lowest Landau Level


Problem: Mechanical Instability
"Chemically" Unstable
B
Atomic
μ
Experiments
Molecular
Hybrid
Mechanically Unstable


Non-resonance sources of pairing transition
Kuklov, Prokof'ev, and Svistunov, Phys. Rev. Lett. 92, 050402 (2004);
Phys. Rev. A 69, 025601 (2004)
Two-component bosons on optical lattice:
A-A, B-B: repulsive
A-B: attractive
2/site: Change lattice depth:
A + B superfluids → Paired AB superfluid
Separate vortices → Combined
Problem: how to rotate lattice:
Much more fun with rotation and lattices


II. Lattices and Rotation
Combined
1. Regular rotation phenomena
Vortices
Landau Levels
Quantum Hall
Pinned to lattice
Degeneracy broken by lattice (Hofstadter butterfly)
Competition between periodic potential (Hofstadter butterfly) and interactions (Quantum Hall)
Pfannkuche and Macdonald, Phys. Rev. B 56, R7100 (1997)
2. Regular lattice phenomena
Mott insulator
Pairing
Vortices Disappear
Competition between MI and QH: transition between QH states
Wen and Wu, Phys. Rev. Lett. 70, 1501 (1993)
Vortices Bind


Outline
How to rotate Lattices
Free Particle spectrum
Density profile of ideal fermions
Vortex pinning

How to “rotate lattice”

Ideas:

1. Mirrors physically rotate lattice, holographic gratings, micro-lens arrays…
2. Lasers imprint phases on hopping atoms
   D. Jaksch and P. Zoller, New J. Phys. 5, 56 (2003);
   E. Mueller, cond-mat/0404306
3. Periodically Modulate hopping and quadrupolar potential
   A. S. Sorensen, E. Demler, and M. D. Lukin, cond-mat/0405079

Thinking about rotation:

Formal picture
Rotation -> Phase on hopping matrix element

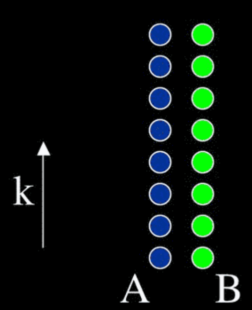
Intuitive picture
Coriolis force: move radially -- accelerate azimuthally

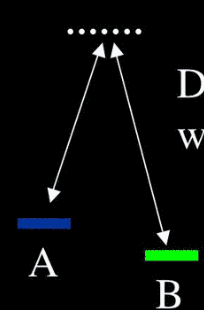
Electromagnetic analogy
Rotation ⟷ Magnetic Field

$$\frac{p^2}{2m} - \Omega\cdot r\times p = \frac{1}{2m}(p - m\Omega\times r)^2 - \frac{m}{2}(\Omega\times r)^2$$

Producing Phases

Hop A-B requires internal state change





Drive transition
with Raman lasers

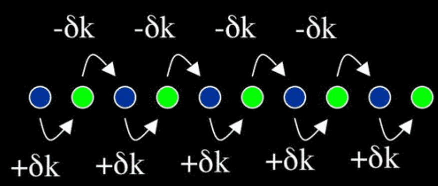
Phase of hopping ⟷ Phase of electromagnetic field

$$\phi = \mathbf{k}_{\mathrm{recoil}}\cdot\mathbf{r}$$

Alternate Picture:
Get momentum kick when you hop

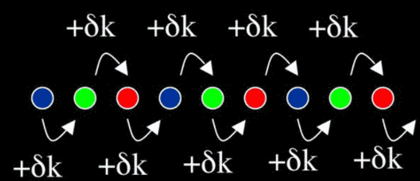
Scaling Up

Want: momentum kick up whenever we hop right



Need to break reflection symmetry

Solution: three states:



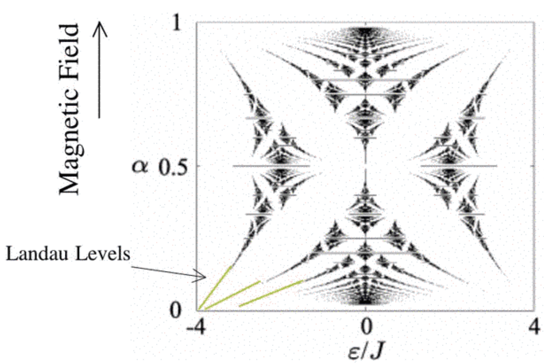
Separate lasers drive:

A-B, B-C, C-A

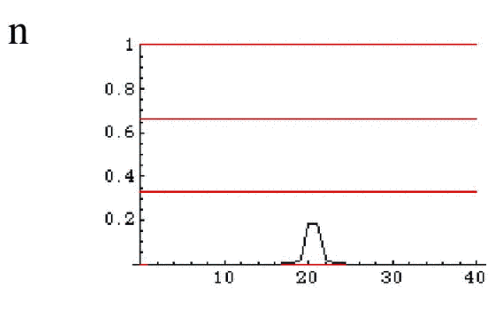
Select recoils so momentum kick always in same direction

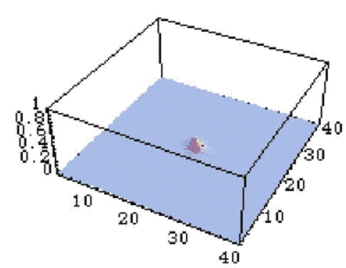
Next: Simple applications --
non-interacting fermions, vortex pinning

Free Particle Spectrum

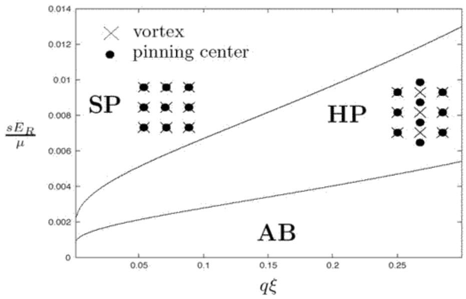


Free Fermions:
Gaps imply plateaus in density
in harmonic trap



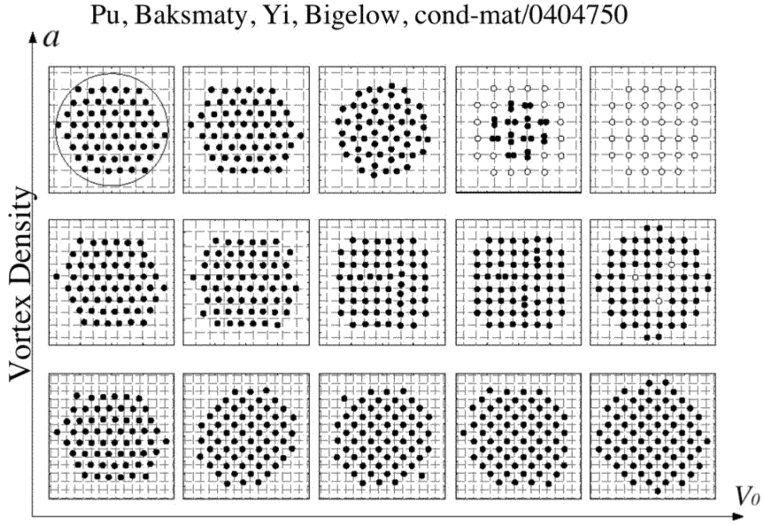


Vortex Pinning on Lattices

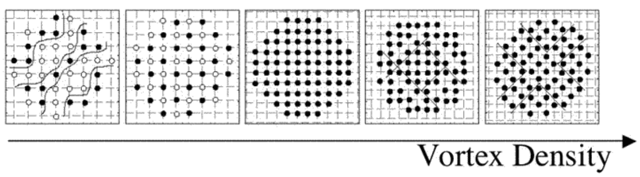


Reijnders and Duine, cond-mat/0401583

Pu, Baksmaty, Yi, Bigelow, cond-mat/0404750

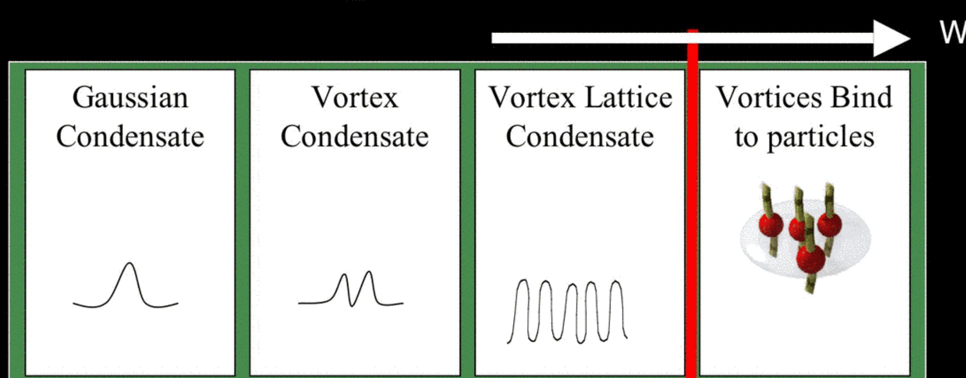


Pu, Baksmaty, Yi, Bigelow, cond-mat/0404750



Strong pinning regime

III. Spin/Mixtures



Textures

E. J. Mueller, Phys. Rev. A 69, 033606 (2004)
I. Carusotto and E. J. Mueller, J. Phys. B: At. Mol. Opt. Phys. 37, S115 (2004)
Reijnders, Lankvelt, Schoutens and Read, Phys. Rev. A 79, 023612 (2004)

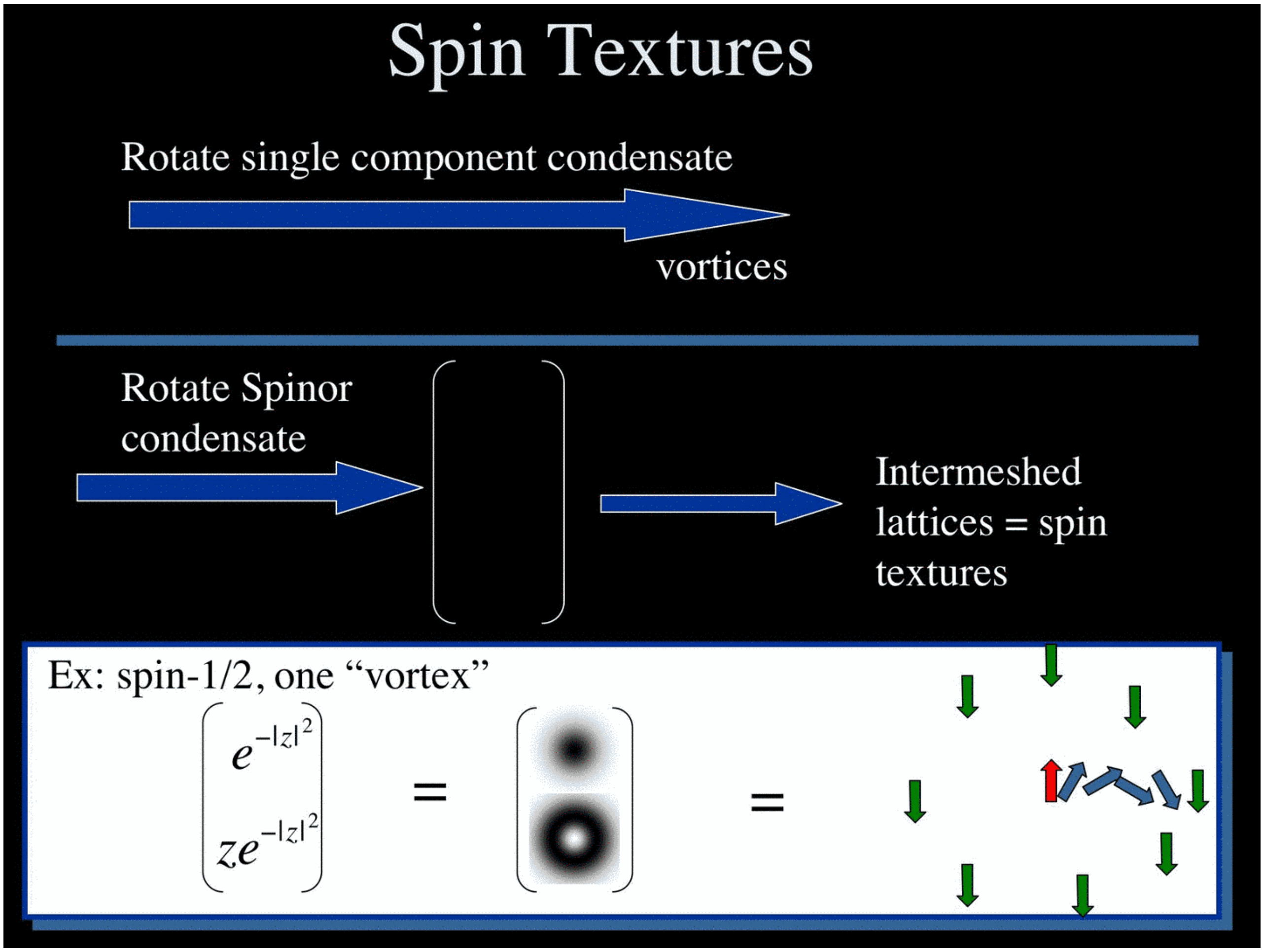
E. J. Mueller and T.-L. Ho, Phys. Rev. Lett. **88**, 180403 (2002)
Kasamatsu, Tsubota, Ueda, Phys. Rev. Lett. 91, 150406 (2003)

`Quantum Hall' States featuring

a) Correlations between spin and angular momentum

b) Fermionization and Spin-Charge separation

T.-L. Ho and E. J. Mueller, Phys. Rev. Lett. **89**, 050401 (2002)
J.W. Reijnders, F.J.M. van Lankvelt, K. Schoutens, and N. Read, Phys. Rev. Lett. **89**, 120401 (2002)
Paredes, Zoller, and Cirac, Phys. Rev. A, 033609 (2002)



Hamiltonian -- Spin 1

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{1}{2} m\omega^2 r_i^2 + \sum_{i<j} V_{\sigma_i\sigma_j}(r_i - r_j)$$

Short-range interaction, transforms as scalar under spin rotations: generically of form

$$V_{\sigma_i\sigma_j}(r_i - r_j) = (c_0 + c_2 \mathbf{S_i} \cdot \mathbf{S_j})\delta(r_i - r_j)$$

Density interaction → c_0; Spin interaction → $c_2 \mathbf{S_i} \cdot \mathbf{S_j}$

Tin-Lun Ho, PRL 81, 742 (1998)

Experiment:

^{87}Rb -- $|c_2| << c_0$, $c_2 < 0$
Ferromagnet: Wants local spin order → $\psi = \overleftrightarrow{R} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

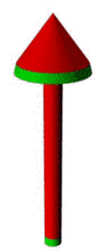
Uniform Condensate

^{23}Na -- $|c_2| << c_0$, $c_2 > 0$
Antiferromagnet: Dislikes local spin order → $\psi = \overleftrightarrow{R} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Symmetry Properties of Order Parameter

Ferromagnet

$$\psi = \overleftrightarrow{R} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$



Vector order parameter **<S>**

Antiferromagnet

$$\psi = \overleftrightarrow{R} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

No local spin **<S>=0**

Order Parameter:
Spin Fluctuations

$$\left\langle S_\mu S_\nu \right\rangle = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{matrix} x \\ y \\ z \end{matrix}$$

$x \quad y \quad z$

Nematic (headless arrow) order parameter →

Also see Fei Zhou: **cond-mat/0108473**,
Order Parameter: $\mathbf{S_2 x S_1 / Z_2}$

Vorticity

Scalar: $v = \frac{\hbar}{m}\nabla\phi$ $\qquad \nabla \times v = 0$

Spinor: $v = \frac{j}{n} = \frac{1}{n}\frac{\hbar}{m}\sum_\nu n_\nu \nabla\phi_\nu$ $\qquad \nabla \times v \neq 0$

Vector Order: (^{3}He-A, Ferromagnetic Spin 1 BEC…)

$$\nabla \times v = \epsilon_{abc}\frac{\hbar}{m} s_a \nabla s_b \times s_c$$

N. D. Mermin, and T.-L. Ho, Phys. Rev. Lett. 36, 594 (1976).

General Order:

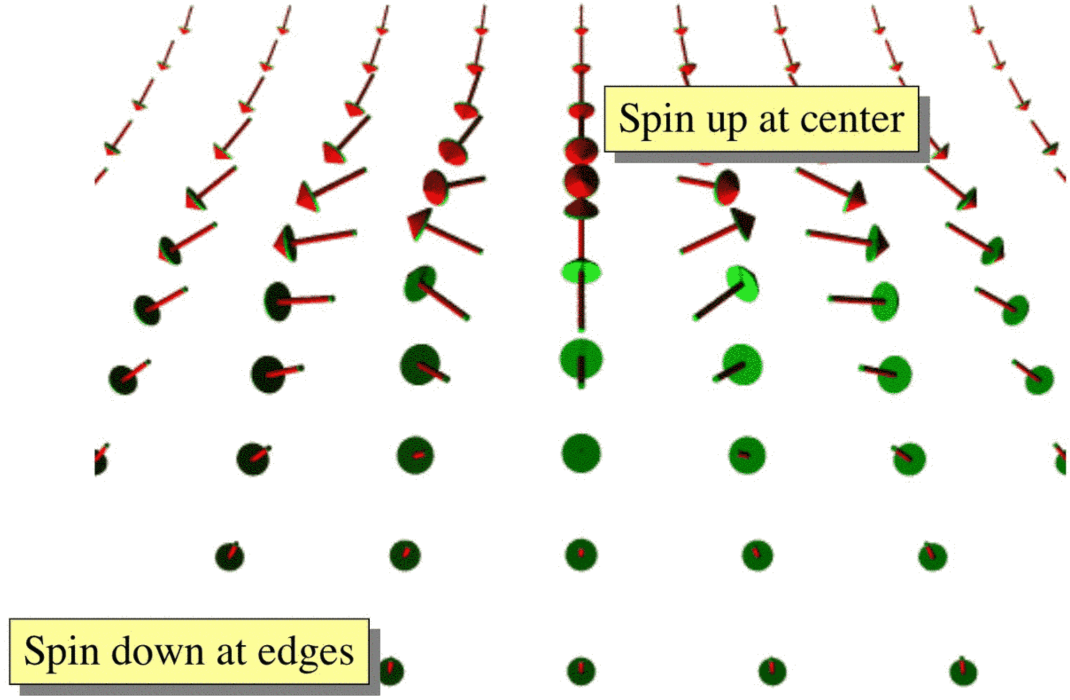
$$\nabla \times \mathbf{v} = i\frac{\hbar}{m} Q_{ab}(\nabla Q_{bc}) \times (\nabla Q_{ca})$$

$$Q_{ab} = \frac{\psi_a^* \psi_b}{\sum_c \psi_c^* \psi_c}$$

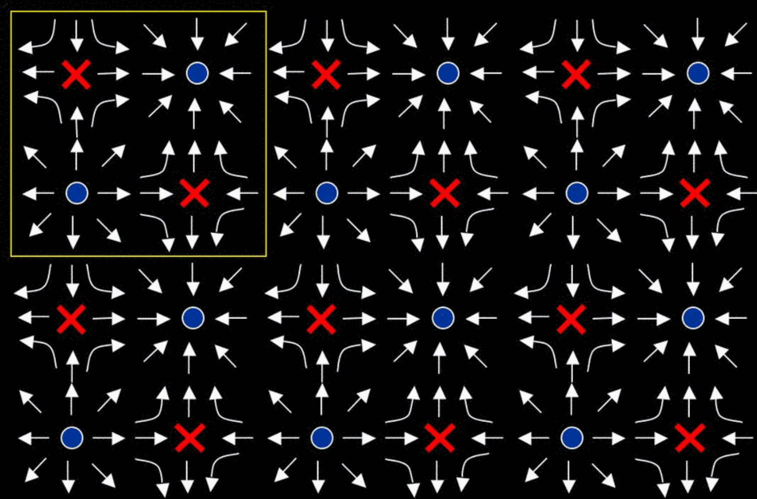
Independent of overall phase/density

Ferromagnetic Texture

Skyrmion/Meron/Coreless Vortex:



Lattice



● Out of board

✕ In to board

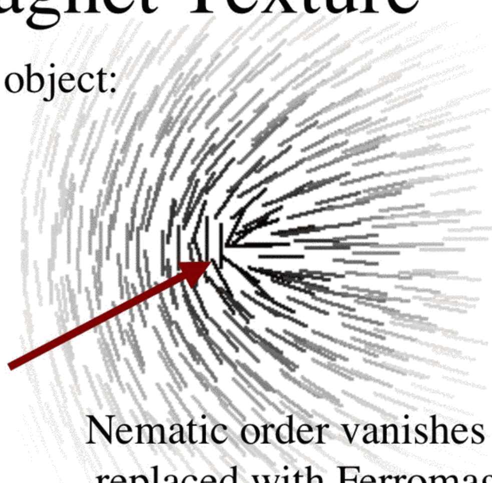
(^{87}Rb parameters)

Antiferromagnet Texture

Angular momentum carrying object:

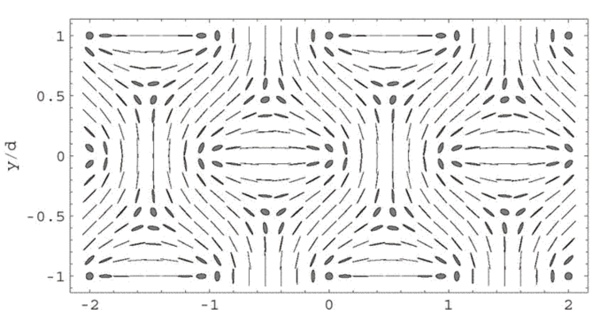
p-disclination or 1/2 vortex

$$\begin{pmatrix} ze^{-|z|^2} \\ 0 \\ e^{-|z|^2} \end{pmatrix}$$

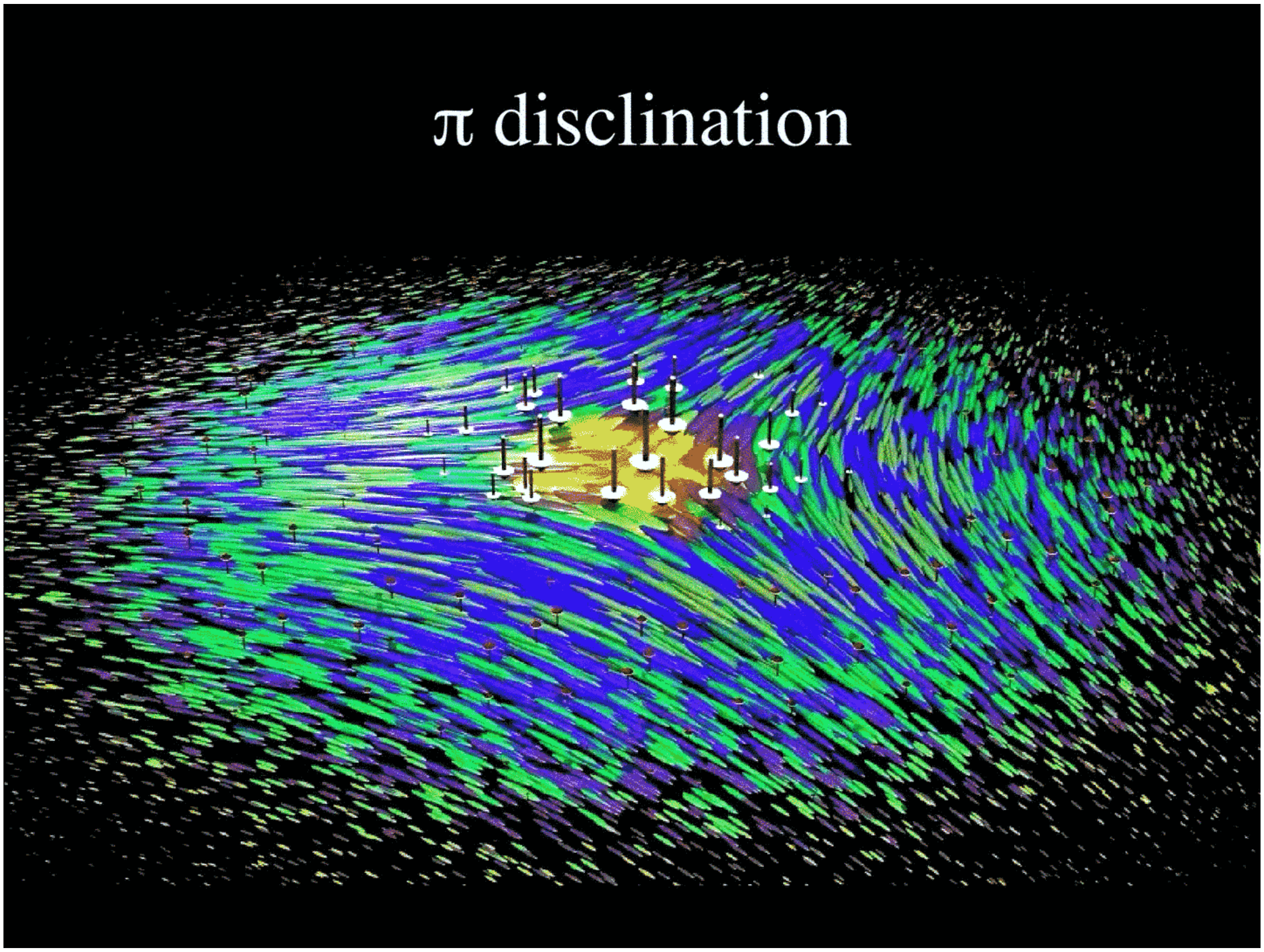


Topological Singularity

Nematic order vanishes at core: replaced with Ferromagnetic



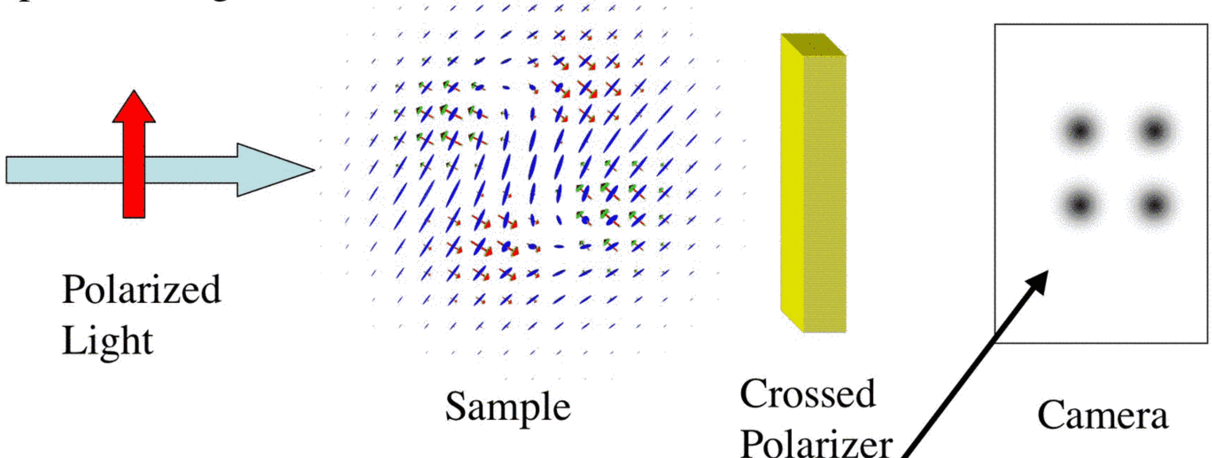
Lattice



Observing Spin Textures

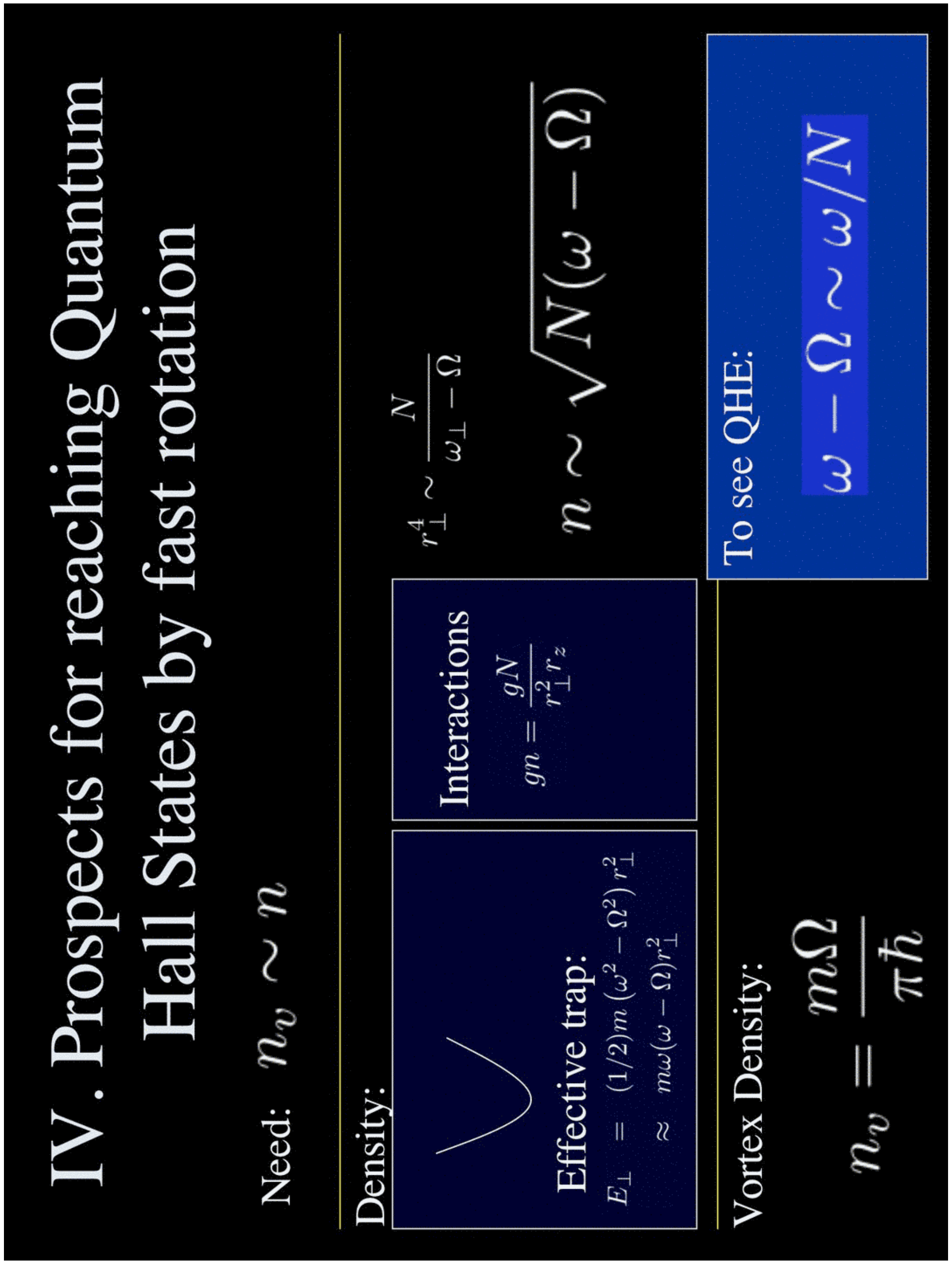
I. Carusotto and E. J. Mueller, J. Phys. B: At. Mol. Opt. Phys. 37, S115 (2004)

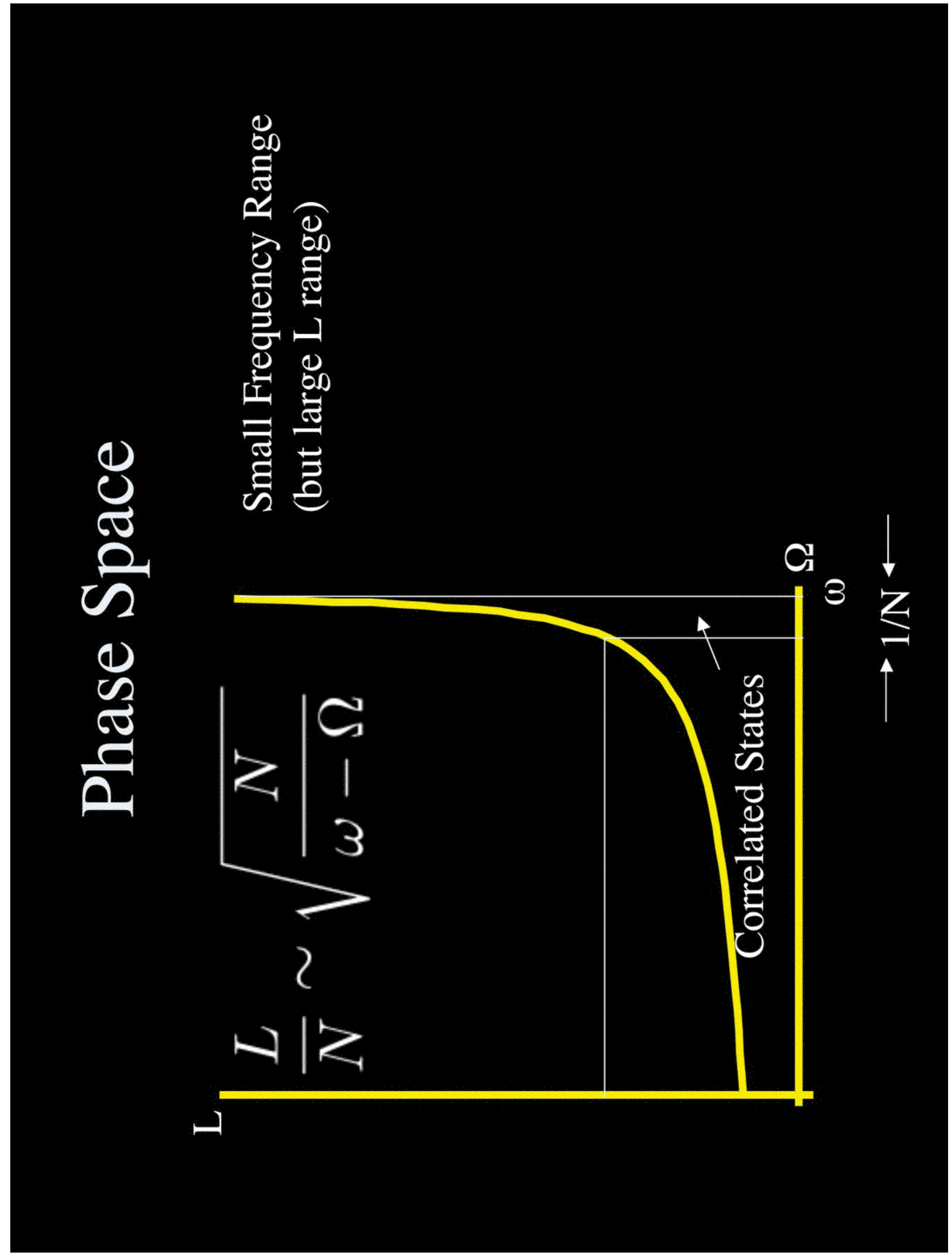
Correct Detuning: Ferromagnetic regions are *optically active*: polarized light is rotated.



See bright spots at locations of cores

Other detunings allow probing of nematicity

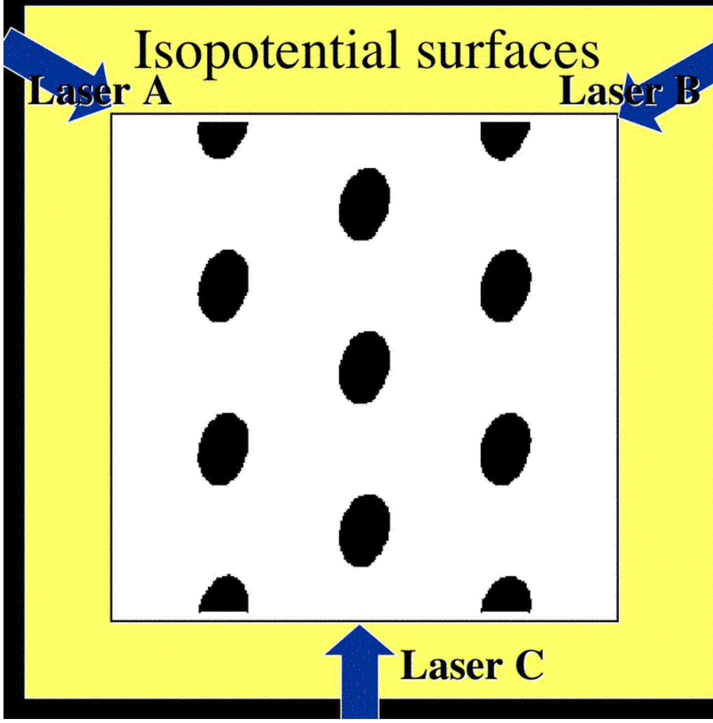

IV. Prospects for reaching Quantum Hall States by fast rotation
Need: $n_v \sim n$
Density:
Effective trap:
$E_\perp = (1/2)m(\omega^2-\Omega^2)r_\perp^2 \approx m\omega(\omega-\Omega)r_\perp^2$
Interactions
$gn = \frac{gN}{r_\perp^2 r_z}$
$r_\perp^4 \sim \frac{N}{\omega_\perp-\Omega}$
$n \sim \sqrt{N(\omega-\Omega)}$
Vortex Density:
$n_v = \frac{m\Omega}{\pi\hbar}$
To see QHE:
$\omega-\Omega \sim \omega/N$

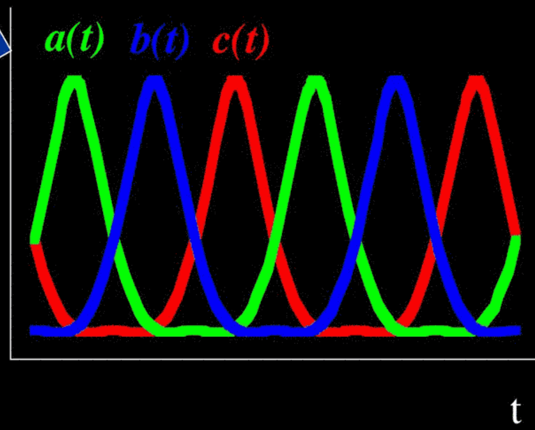

Phase Space
$\frac{L}{N} \sim \sqrt{\frac{N}{\omega-\Omega}}$
Small Frequency Range
(but large L range)
L
Correlated States
Ω
ω
1/N

Clusters in optical lattice

Modulate intensity of lasers:

$$I = \left| a(t)e^{i\vec{k}_A\cdot\vec{r}} + b(t)e^{i\vec{k}_B\cdot\vec{r}} + c(t)e^{i\vec{k}_C\cdot\vec{r}} \right|^2$$





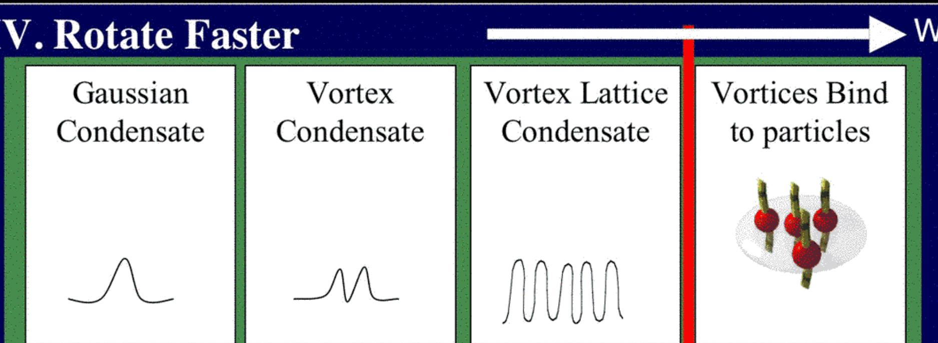
(harmonic in limit of vanishing anisotropy)

Typical frequency: 100 kHz

Summary

Many directions for exciting “vortex” physics:

IV. Rotate Faster

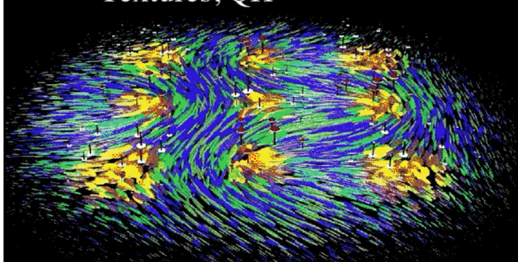


Small Ω range, Large L range

Clusters

III. Add spin

Textures, QH



I. Introduce Scattering Resonances

Atom-Molecule Phase Transition
Mechanical Instabilities

II. Add periodic potentials

Vortex Pinning
Pairing Transition
More Phase Transitions
Many tricks for “rotating”