Ground-state properties of artificial bosonic atoms, Bose interaction blockade and the single-atom pipette

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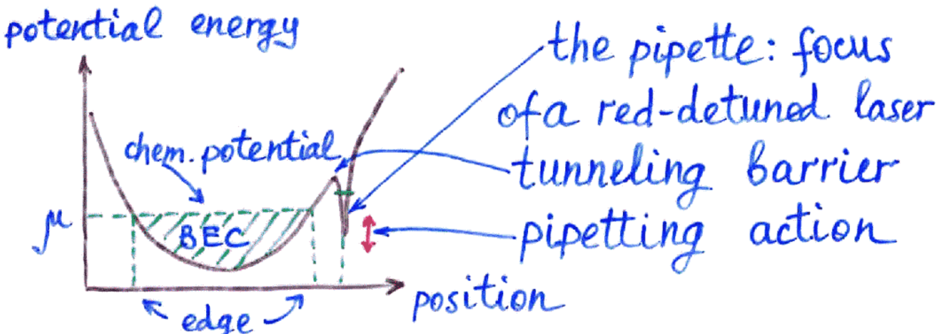
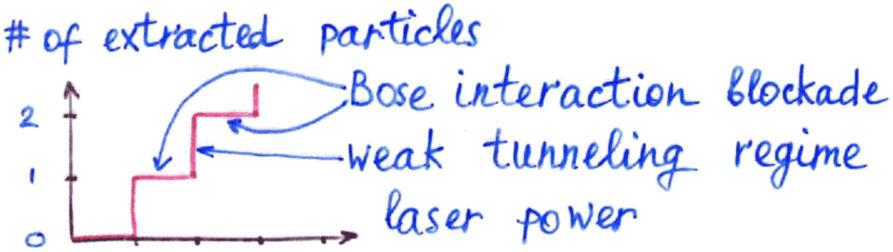
- How to make a device to precisely manipulate single neutral particles
- How to approach the physics involved

The Principle

System: Bose-Einstein condensate of alkali gases

Physics: particle discreteness combined with interparticle repulsion





Basic Questions

✓• Binding properties of isolated pipette:
pipette → artificial nucleous } artificial
bound particles → bosons } Bose-atom

Specifics:

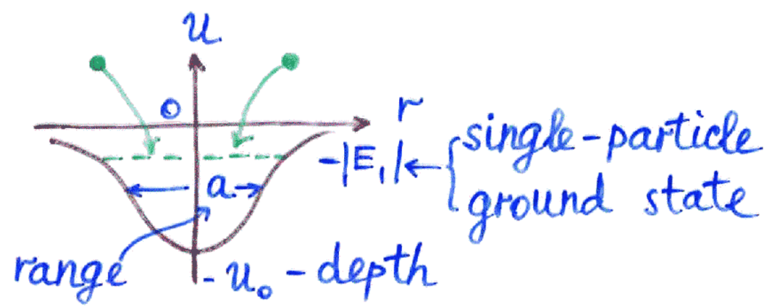
- There is no Pauli principle
- Interactions play major role
- All the interactions have short range

• Tunneling between the pipette and BEC

• Experimental feasibility

- The pipette trap
- The role of the BEC temperature
- Adiabaticity – tunneling times
- Shaking the pipette

Ground-state properties of artificial bosonic atoms



"Nuclear" attraction enhanced by the Bose statistics competes with interparticle repulsion

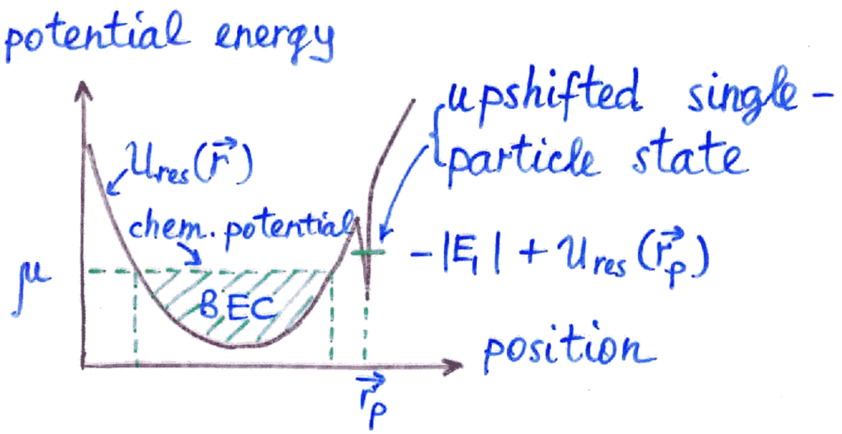
Toy model (Diener et al, 2002)

$$E_n = \underbrace{-|E_1|\, n}_{\text{BEC}} + \underbrace{\overset{\circ}{V} \cdot \frac{n(n-1)}{2}}_{\text{Hubbard-like penalty}}$$

interparticle repulsion

# of pair interactions

The pipette next to the BEC



Particle extraction takes place if

$$-|E_1| + U_{res}(\vec{r}_p) \le \mu$$

Relative to the BEC chemical potential:

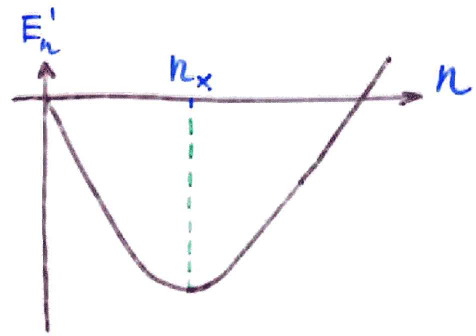
$$E'_n = E_n - \mu n = [-|E_1| + U_{res}(\vec{r}_p) - \mu]n + \frac{\nu n(n-1)}{2} -$$

looks like an energy of a metal grain of charge Q and capacitance C biased by a voltage V:

$$E(Q) = \frac{Q^2}{2C} - QV$$ – Coulomb blockade context

Borrowing Likharev et al, 1985 argument:

$$E'_n = [-|E_1| + U_{res}(\vec{r}_p) - \mu]n + \frac{\nu n(n-1)}{2}$$



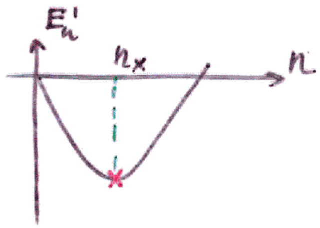
$$n_x = \frac{1}{2} + \frac{1}{\nu}[|E_1| + \mu - U_{res}(\vec{r}_p)]$$ – the parameter which can be tuned:

$\nu \to$ Feschbach resonance

$E_1 \to$ laser power; also affects tunneling barrier

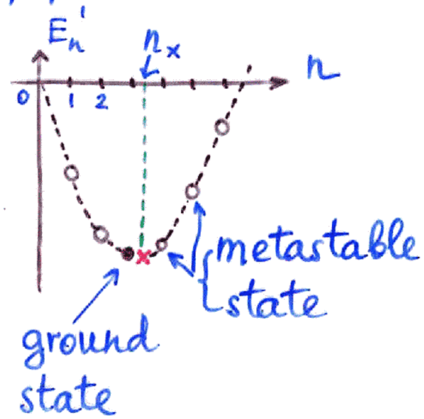
$U_{res}(\vec{r}_p) \to$ pipette location; also affects tunneling barrier

Strong tunneling → particle discreteness irrelevant → n in E_n' continuous → average pipette population



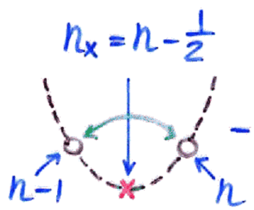
$\langle n \rangle = n_x$

Infinitesimally weak tunneling → particle discreteness relevant → pipette population (almost nearly) quantized



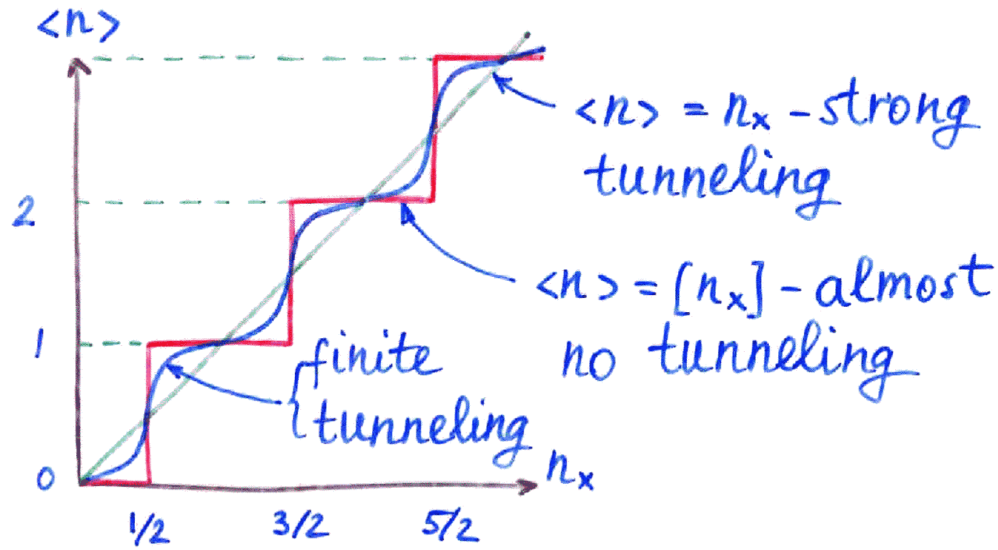
$\langle n \rangle = [n_x]$ – integer closest to n_x

Half-integer n_x:



– degeneracy → tunneling takes place

n_x – dependence of the pipette population $\langle n \rangle$



Do we just need to develop a theory of finite tunneling?

Not yet, as the toy model is partly misleading!

Toy model again

$E_n = -|E_1| n + \nu \frac{n(n-1)}{2}$ – n particles

Bose-condensed in the single-particle ground state mechanically penalized for their repulsions. The spatial extent of this state is the same as that of the single-particle state.

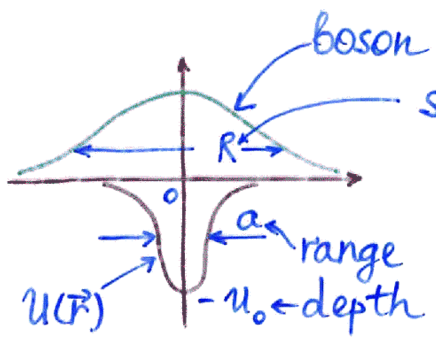
The toy model overlooks "swelling" of the ground state due to interparticle repulsion.

Range of applicability:

$\nu(n-1) \ll |E_1|$ – the conclusion that E_n has a minimum is beyond the range of applicability of the toy model.

But the arguments relied on the existence of the minimum!

Statement of the problem



dimensionality

If $R \gg a \rightarrow U(\vec{r}) \rightarrow -U_0 a^d \delta(\vec{r})$ – model-independent calculation

Dimensionless parameters

$\xi \simeq \frac{m U_0 a^2}{\hbar^2}$ – strength of "nuclear" attraction

$Z \simeq \frac{U_0}{\nu}$ – reduced "nuclear charge"

$v = \xi / Z$ – interparticle repulsions alone

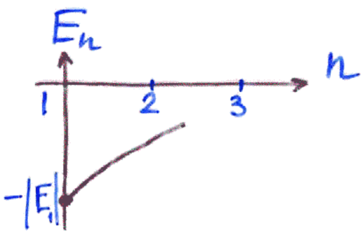
Typically $Z \gg 1$, $v \ll 1$ but $Z \simeq v \simeq 1$ is not impossible (Feschbach resonance and/or tight trap)

Methods

- Exact solution (in 1d only)
- Hartree-Fock → exact minimization (1d); → variational solution (general d)

Results

- Sufficiently strong interparticle repulsions ($Z \lesssim 1$)



Only single-boson atom is stable

- Otherwise ($Z \gtrsim 1$) the results are very sensitive to space dimension

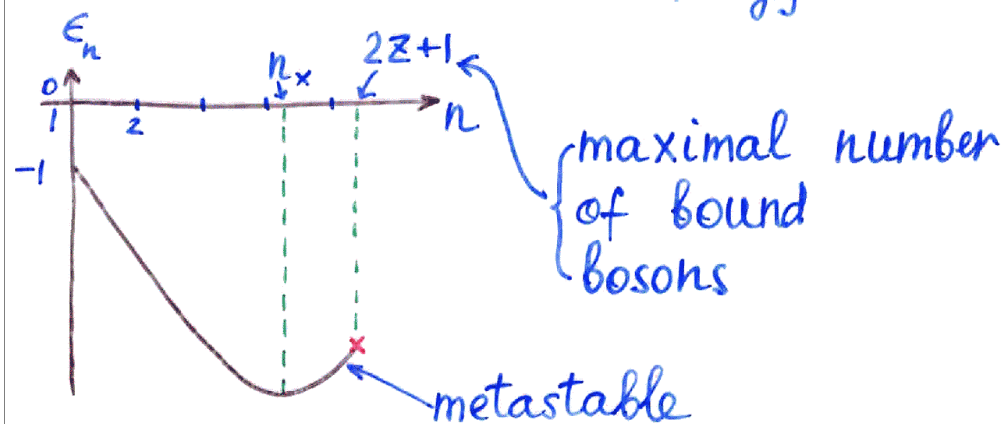
Results, continued

"Atomic" units are used below:

d=1, shallow "nuclear" well ($\xi \ll 1$)

$$\rho_n = \left(1 - \frac{n-1}{2Z}\right)^{-1}$$ – size of the ground state

$$\epsilon_n = -n\left[1 - \frac{n-1}{2Z} + \frac{(n-1)^2}{12Z^2}\right]$$ – ground-state energy

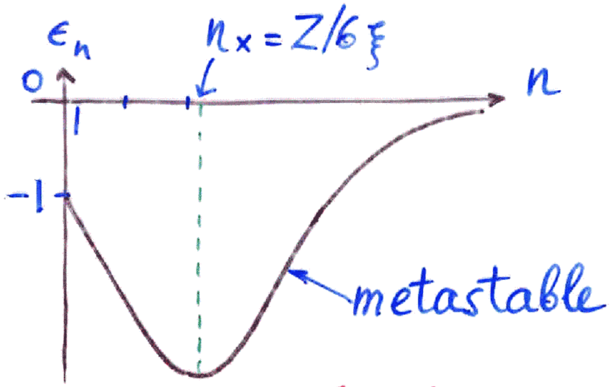


Only a limited number of particles can be bound in one dimension

$d=2$, shallow "nuclear" well ($\xi \ll 1$)

$$\rho_n = \exp\left[\frac{\sqrt{1+24\,\xi^2 (n-1)/Z}-1}{4\xi}\right] \text{ – ground-state size}$$

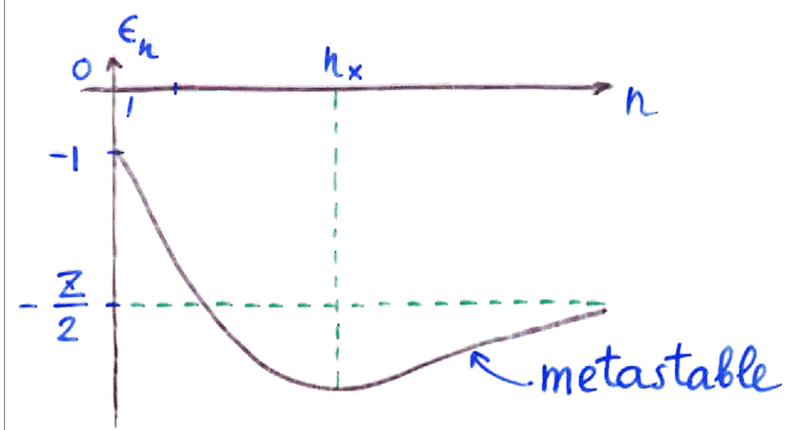
$$\epsilon_n = -n\sqrt{1+24\,\xi^2 (n-1)/Z}\;\exp\left[\frac{1-\sqrt{1+24\,\xi^2(n-1)/Z}}{2\xi}\right] \text{ – ground-state energy}$$



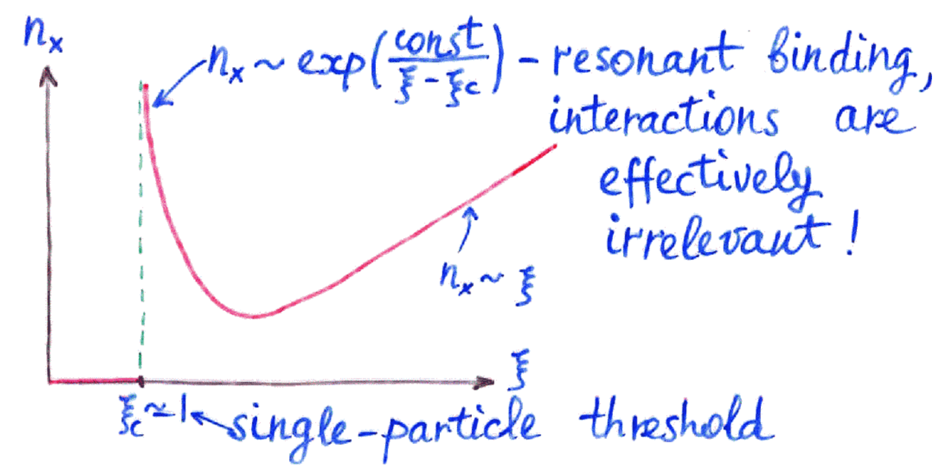
Arbitrary number of bosons can be bound in two dimensions

$d=3$, arbitrary "nuclear" well

$\rho_n \simeq 1 + 2(n-1)/Z$ – ground-state size



Arbitrary number of bosons can be bound in three dimensions



Conclusions

Our work ...

- Proposes a setup for single-particle manipulation
- Analyzes pertinent physics
- Shows experimental feasibility of single-particle manipulation