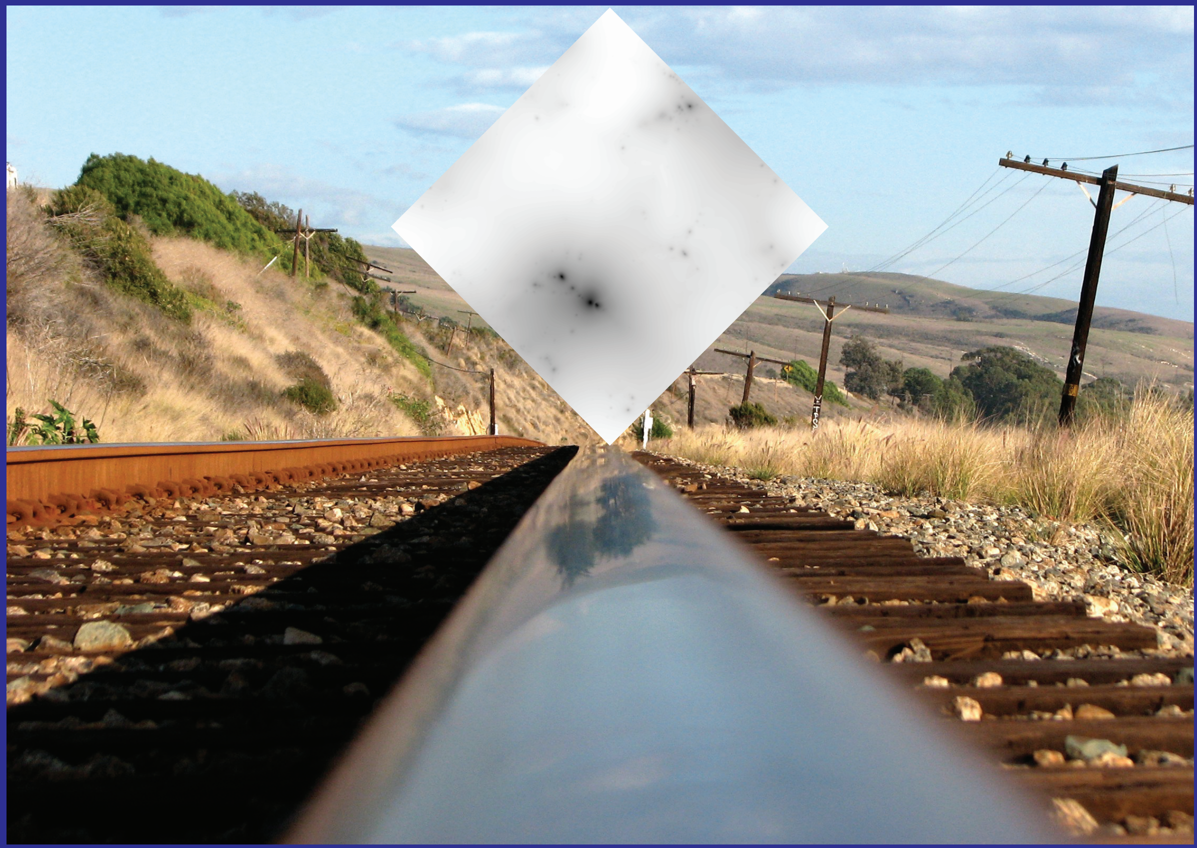
Testing Gravity with Galaxy Clusters



Wayne Hu

KITP

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Outline

- Falsifying ΛCDM and Smooth Dark Energy
- In favor of Modified Gravity?

- Collaborators:
  - Simone Ferraro
  - Dragan Huterer
  - Yin Li
  - Marcos Lima
  - Hiro Oyaizu
  - Michael Mortonson
  - Fabian Schmidt

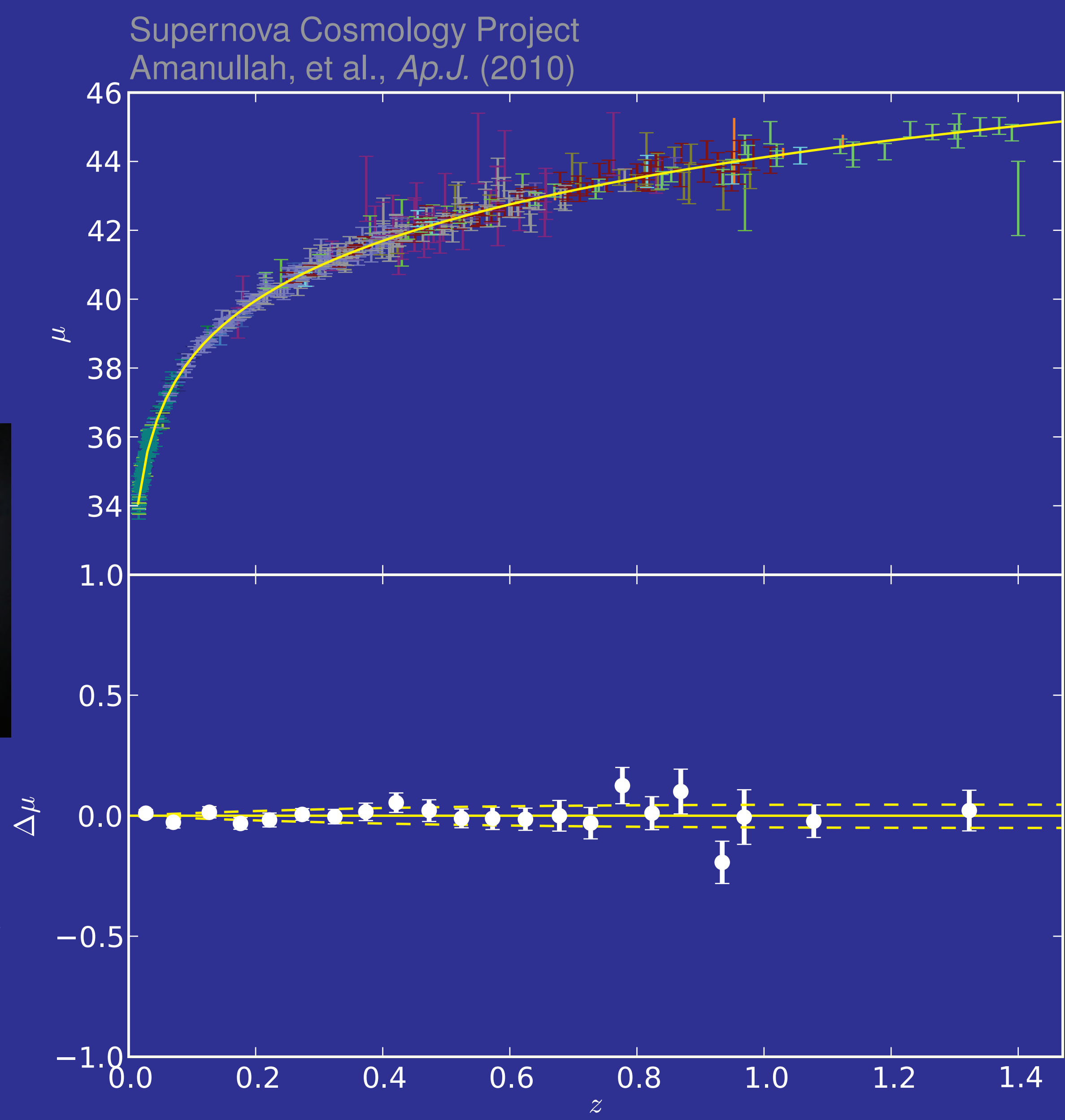
Falsifiability of Smooth Dark Energy

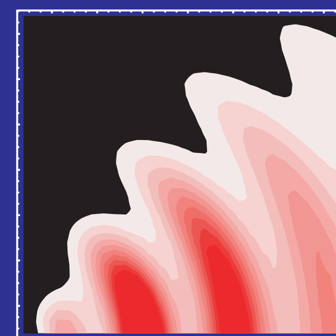
- With the smoothness assumption, dark energy only affects gravitational growth of structure through changing the expansion rate
- Hence geometric measurements of the expansion rate predict the growth of structure
  - Hubble Constant
  - Supernovae
  - Baryon Acoustic Oscillations
- Growth of structure measurements can therefore falsify the whole smooth dark energy paradigm
  - Cluster Abundance
  - Weak Lensing
  - Velocity Field (Redshift Space Distortion)

Falsifying ΛCDM

- Geometric measures of distance redshift from SN, CMB, BAO

Standard(izable)
Candle
Supernovae
Luminosity v Flux

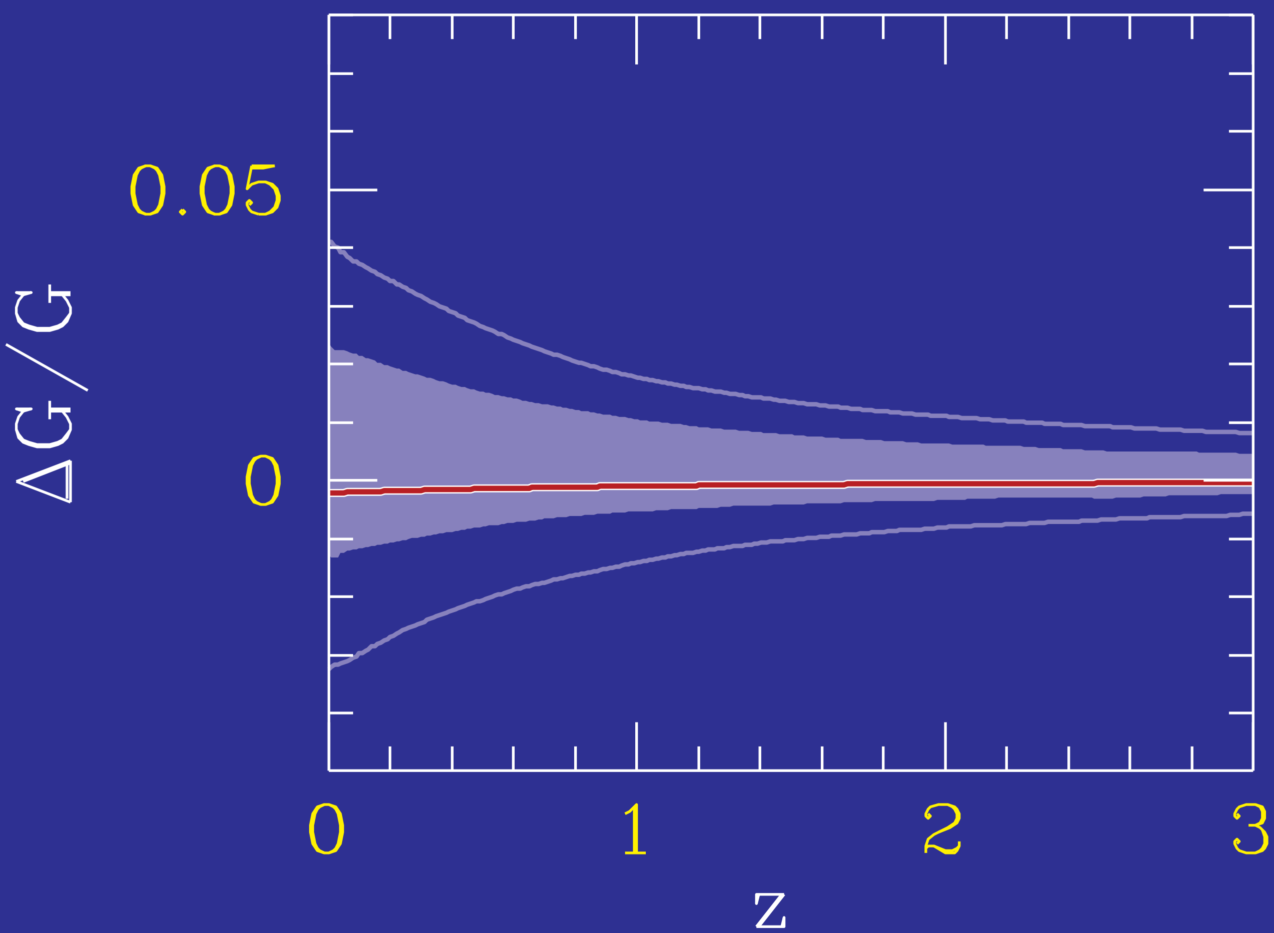




Standard Ruler
Sound Horizon
v CMB, BAO angular
and redshift separation

Falsifying ΛCDM

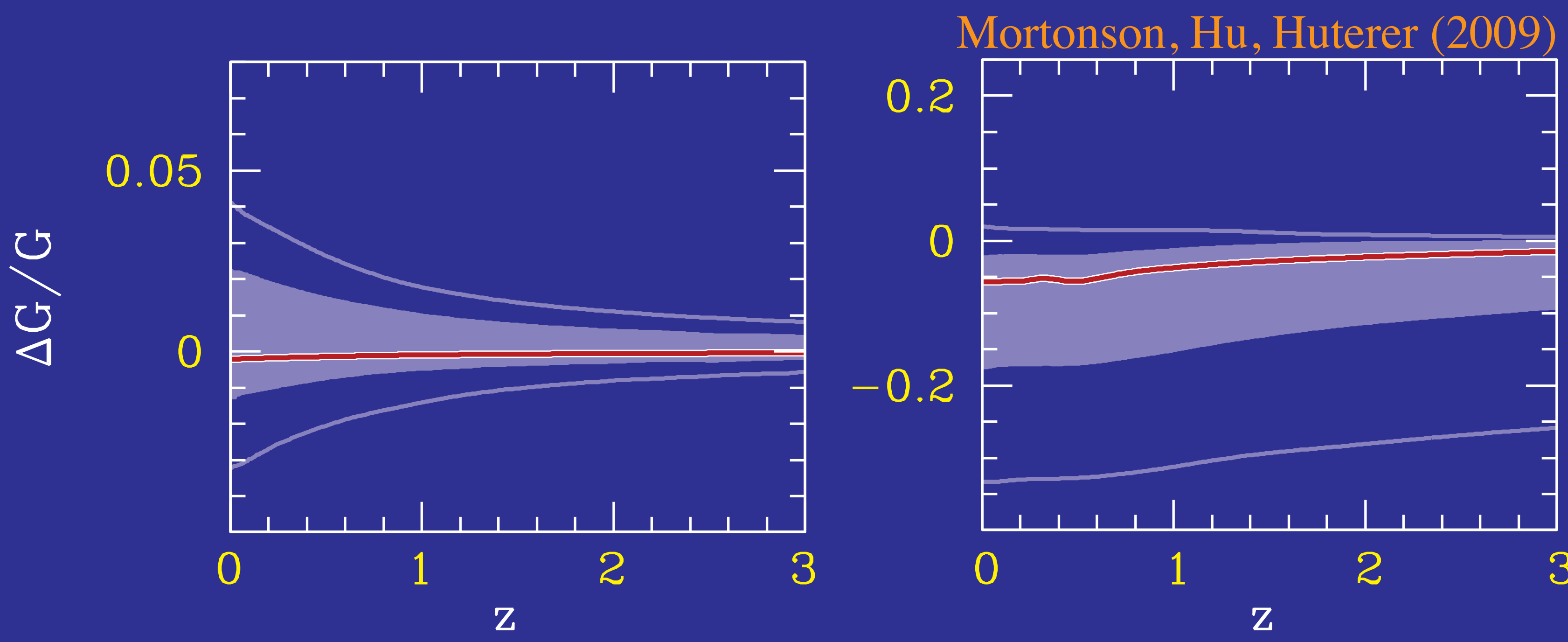
- Λ slows growth of structure in highly predictive way



Cosmological Constant

Falsifying Quintessence

- Dark energy slows growth of structure in highly predictive way



Cosmological Constant

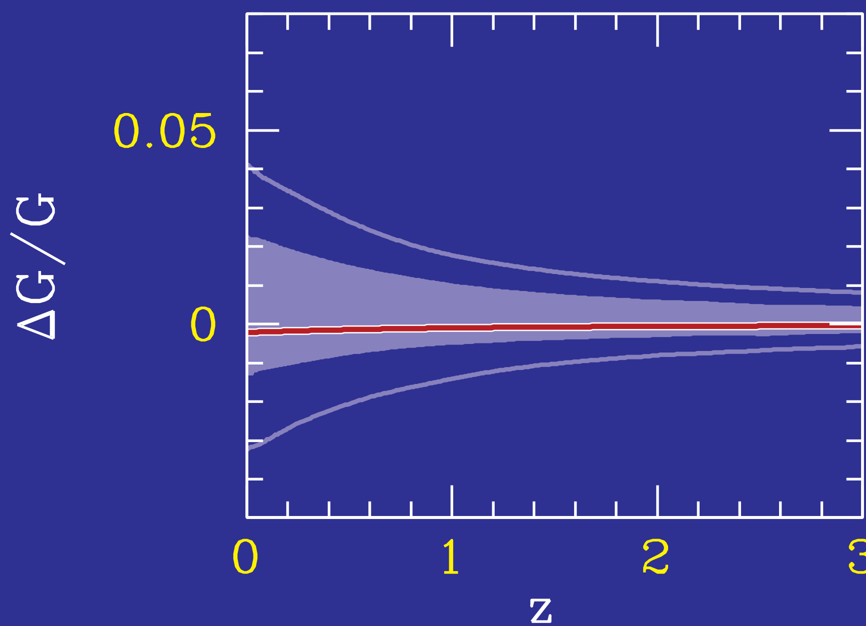
Quintessence

- Deviation significantly >2% rules out Λ with or without curvature
- Excess >2% rules out quintessence with or without curvature and early dark energy [as does >2% excess in H_0]

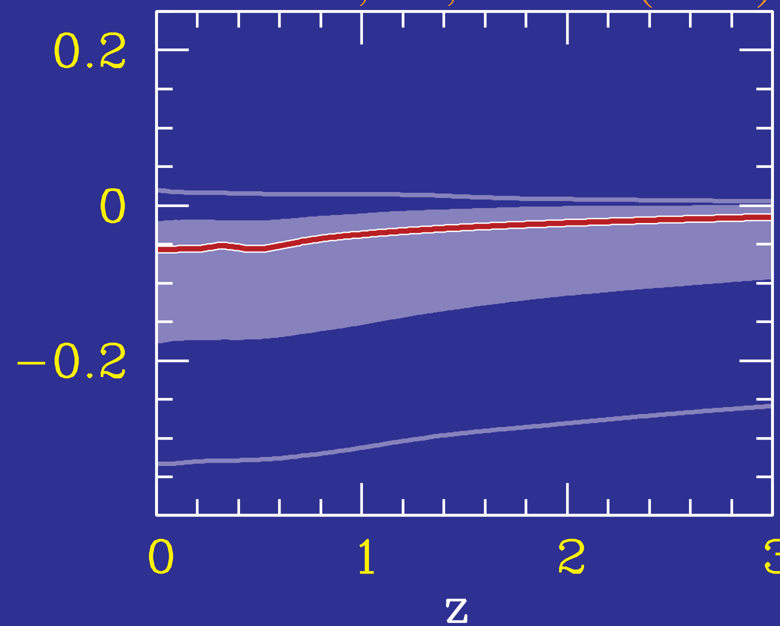
Dynamical Tests of Acceleration

- Dark energy slows growth of structure in highly predictive way

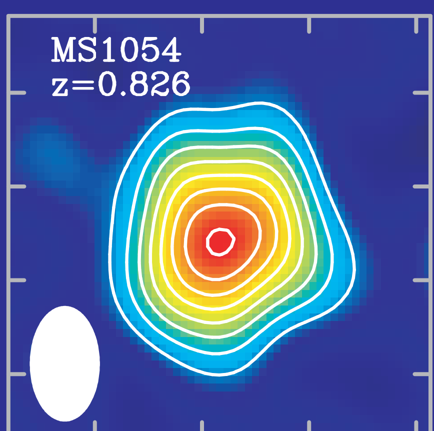
Mortonson, Hu, Huterer (2009)

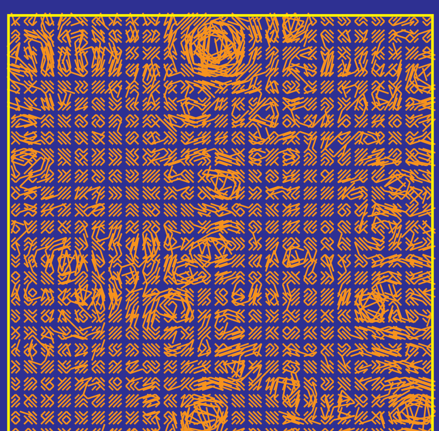


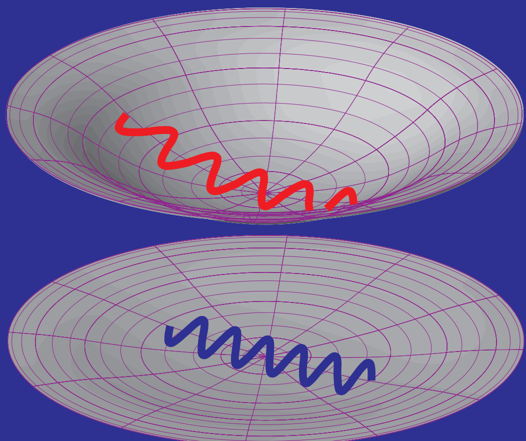
Cosmological Constant

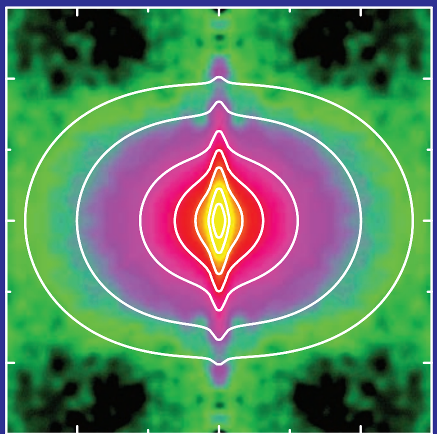


Quintessence



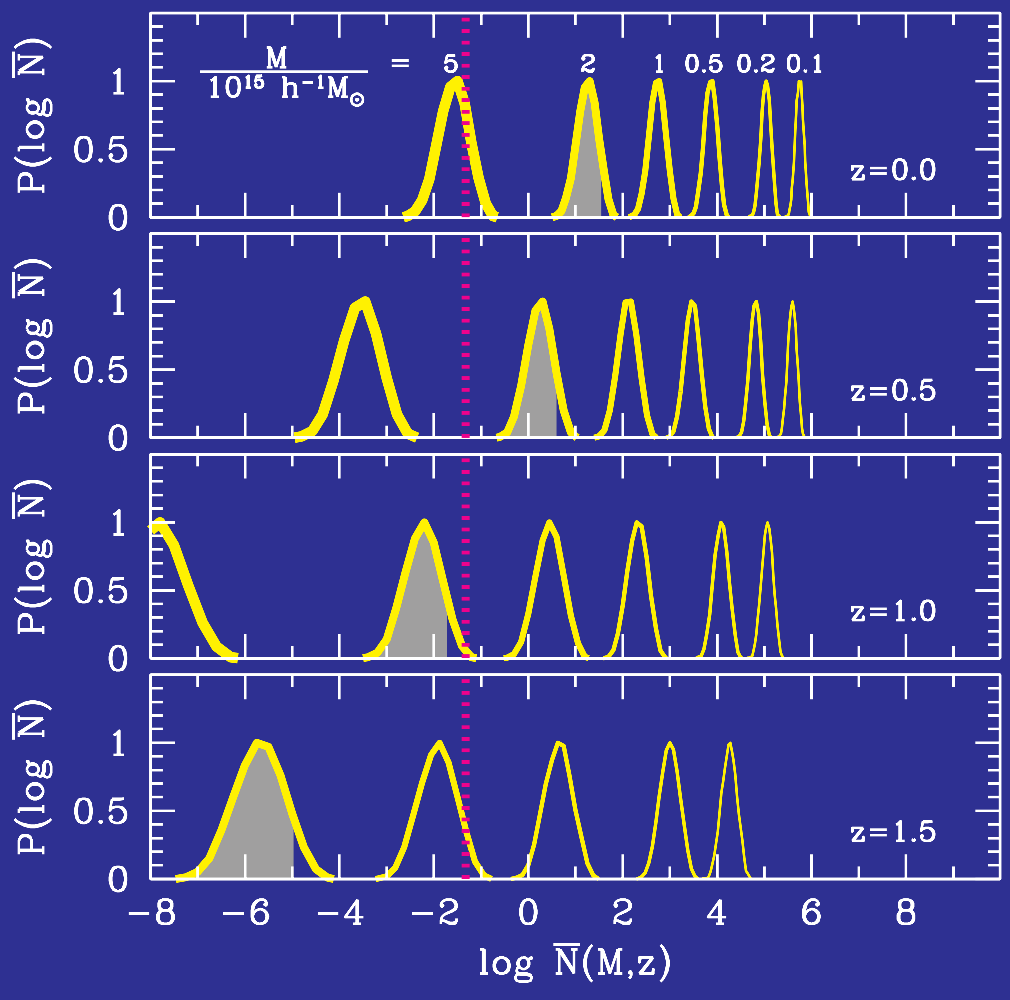






Elephantine Predictions

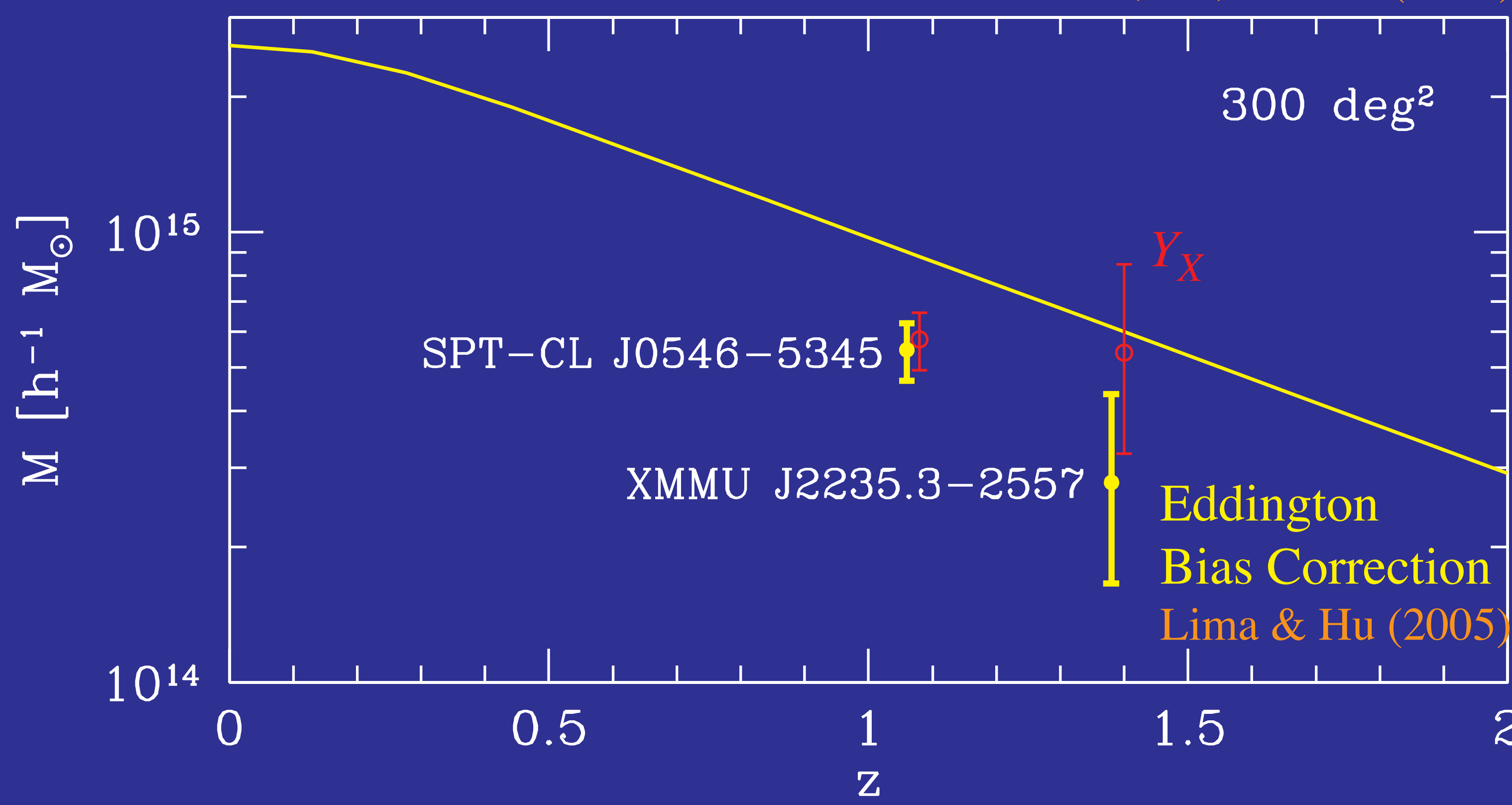
- Geometric constraints on the cosmological parameters of ΛCDM
- Convert to distributions for the predicted average number of clusters above a given mass and redshift



ΛCDM Falsified?

- 95% of ΛCDM parameter space predicts less than 1 cluster in 95% of samples of the survey area above the $M(z)$ curve
- No currently known high mass, high redshift cluster violates this bound

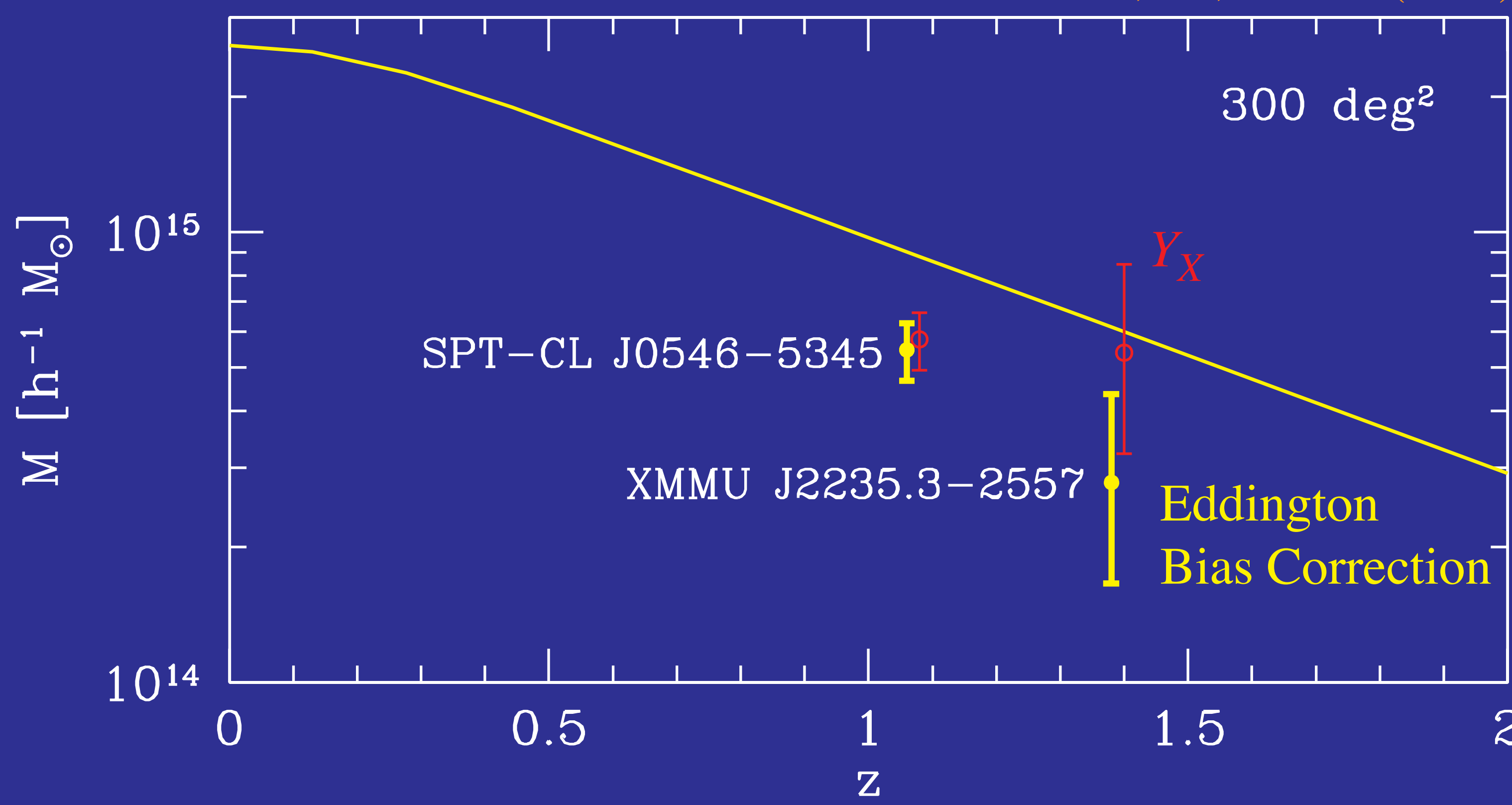
Mortonson, Hu, Huterer (2010)



ΛCDM Falsified?

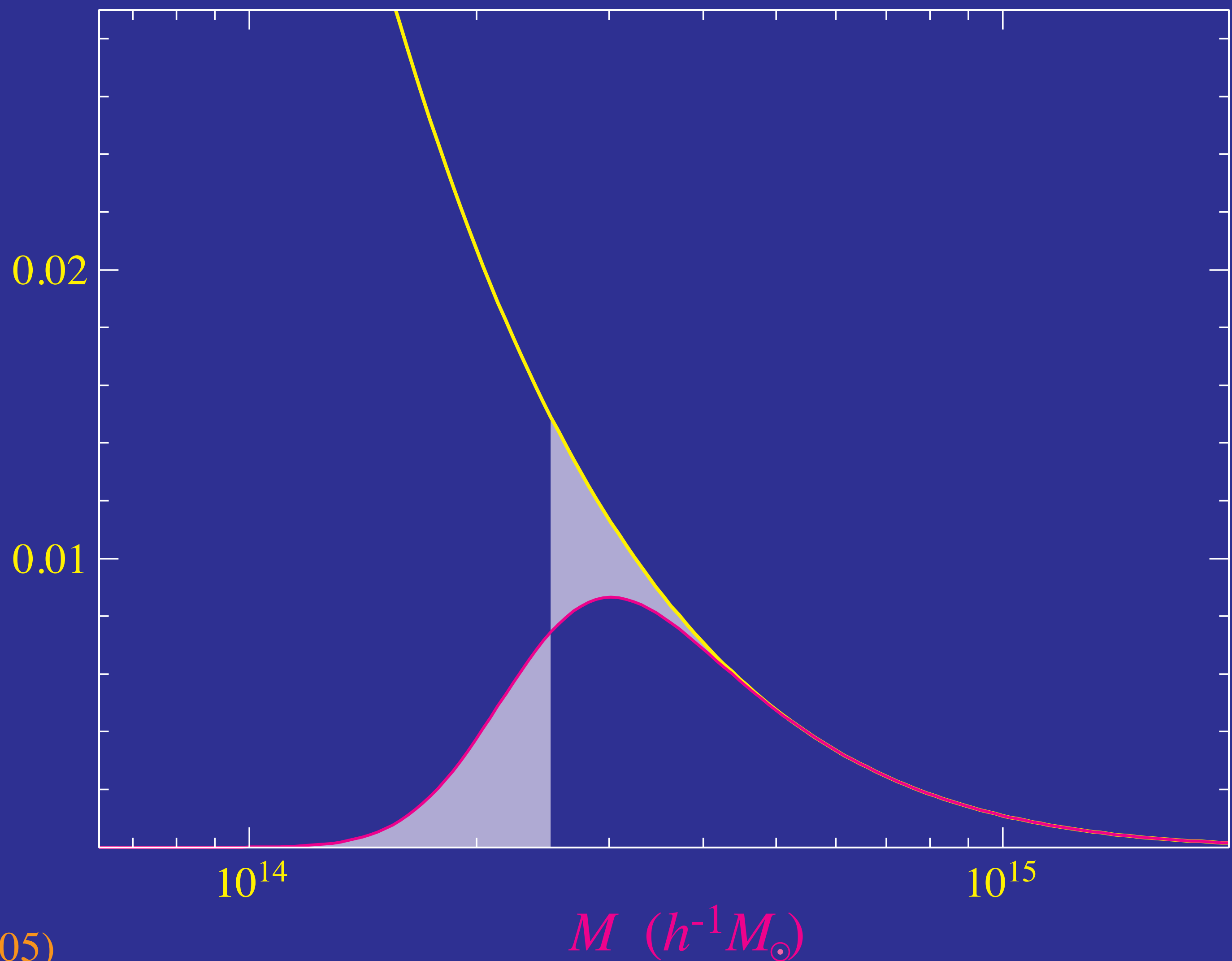
- 95% of ΛCDM parameter space predicts less than 1 cluster in 95% of samples of the survey area above the $M(z)$ curve
- Convenient fitting formulae for future elephants:
  http://background.uchicago.edu/abundance

Mortonson, Hu, Huterer (2010)



Number Bias

- For $>M_{\rm obs}$, scatter and steep mass function gives excess over $>M$
- Equate the number $>M_{\rm obs}$ to $>M_{\rm eff}$
- Not the same as best estimate of true mass given model!



Lima & Hu (2005)

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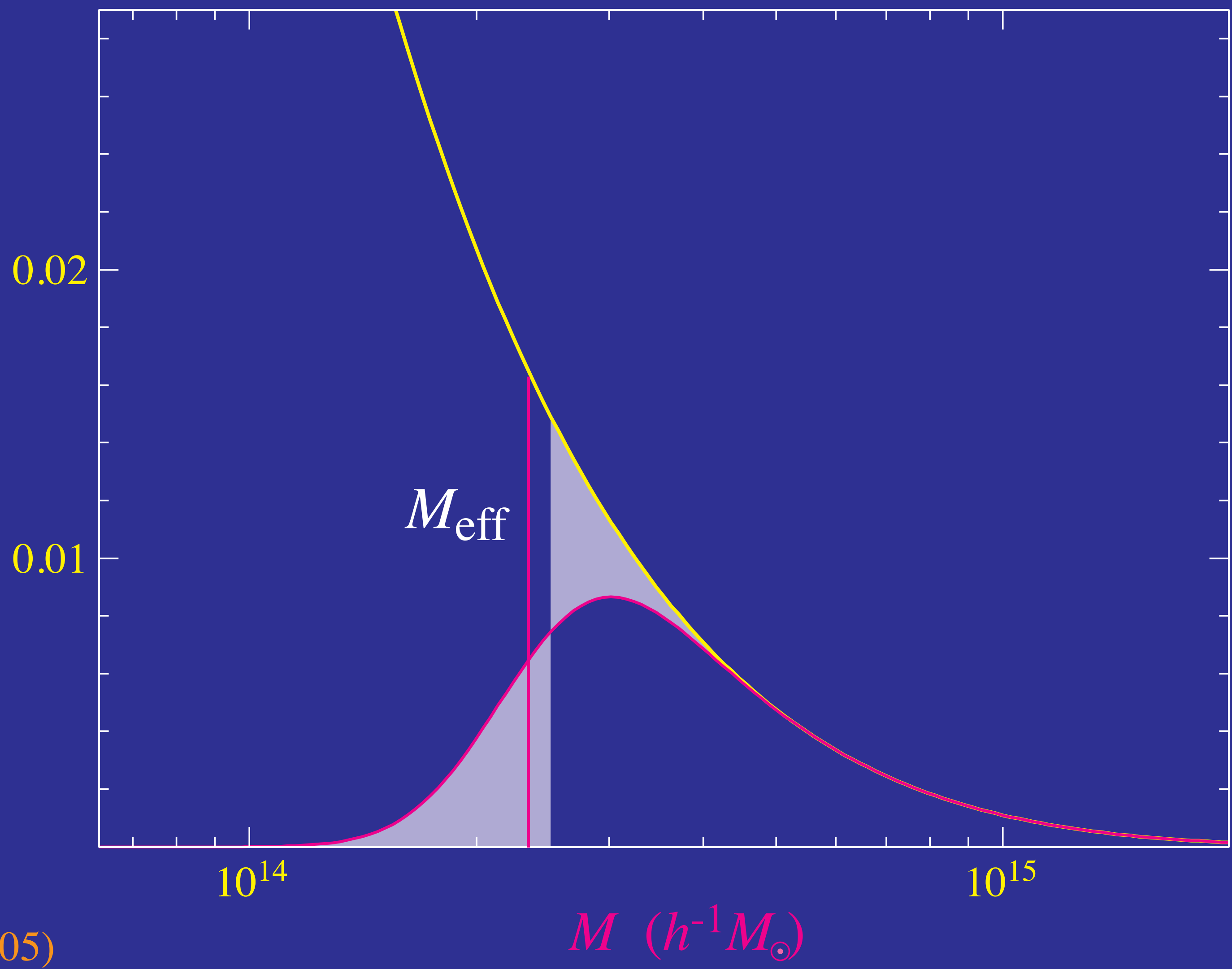
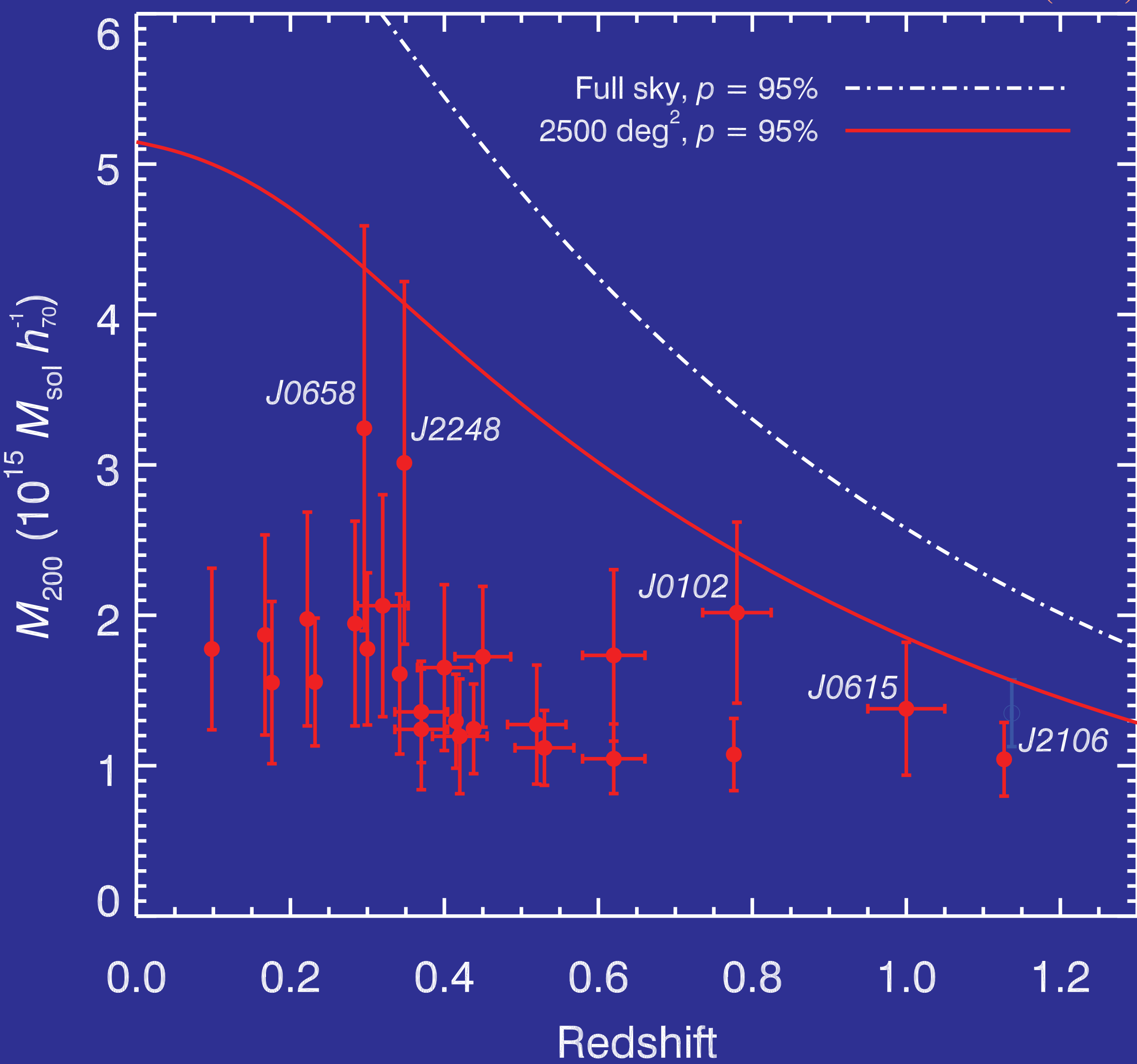


Lima & Hu (2005)

Pink Elephant Parade

- SPT catalogue on 2500 sq degrees

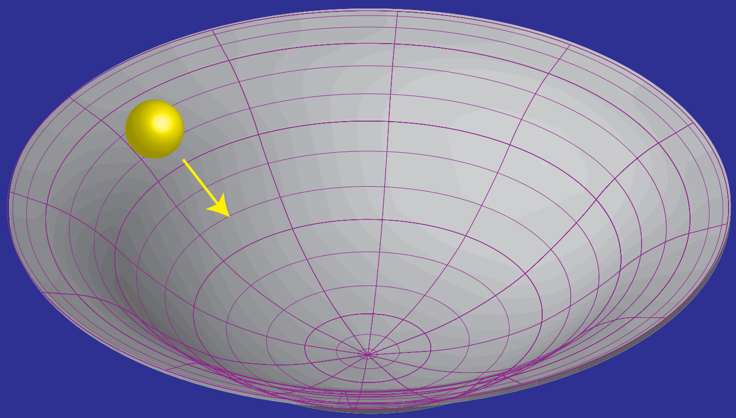
Williamson et al (2011)



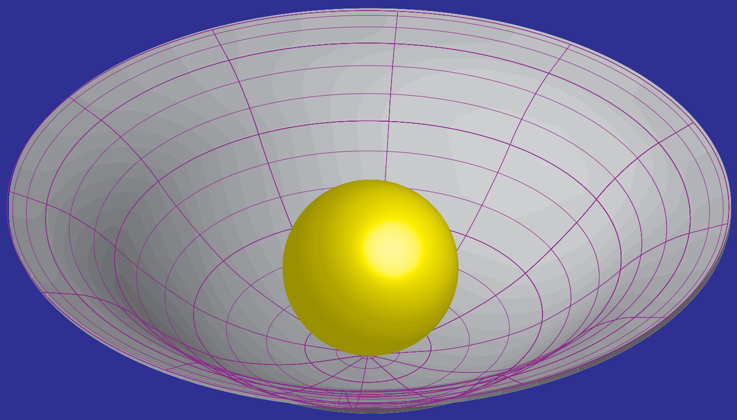
Falsify in Favor of What?

Mercury or Pluto?

- General relativity says Gravity = Geometry



- And Geometry = Matter-Energy



- Could the missing energy required by acceleration be an incomplete description of how matter determines geometry?

Modified Gravity = Dark Energy?

- Solar system tests of gravity are informed by our knowledge of the local stress energy content
- With no other constraint on the stress energy of dark energy other than conservation, modified gravity is formally equivalent to dark energy

$$F(g_{\mu\nu}) + G_{\mu\nu} = 8\pi G T^{\rm M}_{\mu\nu} \qquad - F(g_{\mu\nu}) = 8\pi G T^{\rm DE}_{\mu\nu}$$

$$G_{\mu\nu} = 8\pi G [T^{\rm M}_{\mu\nu} + T^{\rm DE}_{\mu\nu}]$$

and the Bianchi identity guarantees $\nabla^\mu T^{\rm DE}_{\mu\nu} = 0$

- Distinguishing between dark energy and modified gravity requires closure relations that relate components of stress energy tensor
- For matter components, closure relations take the form of equations of state relating density, pressure and anisotropic stress

Modified Gravity $\neq$ “Smooth DE”

- Scalar field dark energy has $\delta p = \delta\rho$ (in constant field gauge) – relativistic sound speed, no anisotropic stress
- Jeans stability implies that its energy density is spatially smooth compared with the matter below the sound horizon

$$
\begin{aligned}
ds^2 &= -(1+2\Psi)dt^2 + a^2(1+2\Phi)dx^2 \\
\nabla^2(\Phi - \Psi) &\propto \text{matter density fluctuation}
\end{aligned}
$$

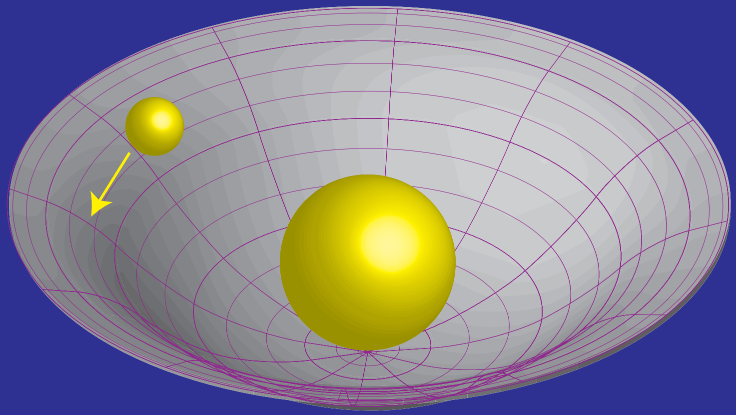
- Anisotropic stress changes the amount of space curvature per unit dynamical mass

$$
\nabla^2(\Phi + \Psi) \propto \text{anisotropic stress}
$$

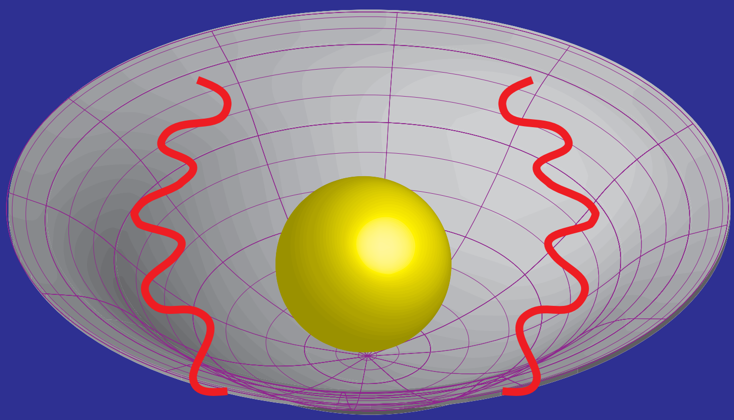
but its absence in a smooth dark energy model makes $g = (\Phi + \Psi)/(\Phi - \Psi) = 0$ for non-relativistic matter

Dynamical vs Lensing Mass

- Newtonian potential: $\Psi=\delta g_{00}/2g_{00}$ which non-relativistic particles feel



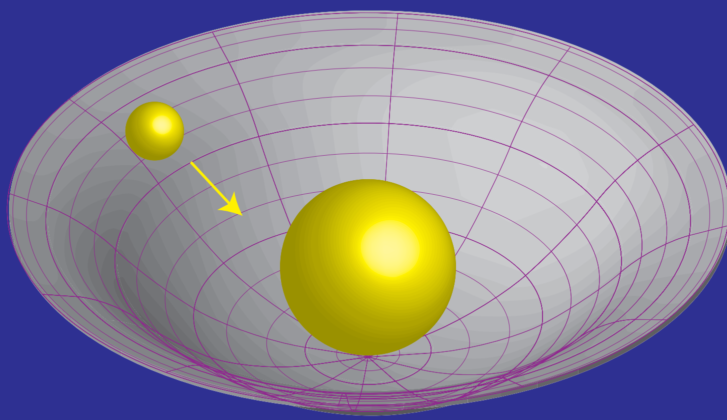
- Space curvature: $\Phi=\delta g_{ii}/2g_{ii}$ which also deflects photons



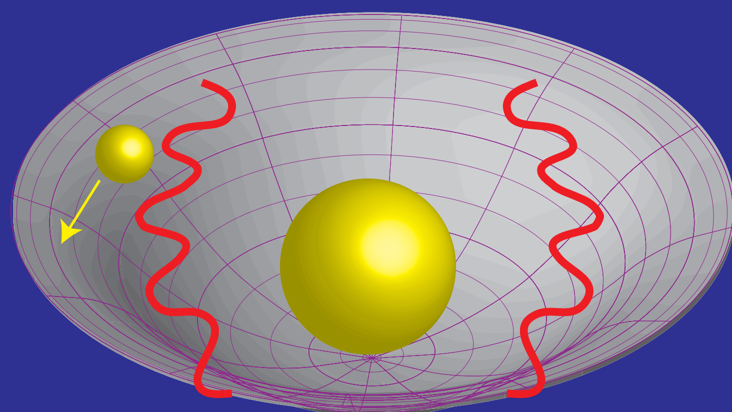
- Most of the incisive tests of gravity reduce to testing the space curvature per unit dynamical mass

Growth of Structure

- Alteration in how density sources Newtonian potential Ψ



- Changes the growth of structure and hence the masses of dark matter halos or the abundance at fixed mass



- Requires solution of the dynamical structure formation problem in the context of a model

Modified Action $f(R)$ Model

- R: Ricci scalar or "curvature"
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_{\rm m} \right]$$

- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2 f/dR^2$: Compton wavelength of f_R squared, inverse mass squared
- B: Compton wavelength of f_R squared in units of the Hubble length

$$B \equiv \frac{f_{RR}}{1+f_R} R' \frac{H}{H'}$$

- $' \equiv d/d\ln a$: scale factor as time coordinate

DGP Braneworld Acceleration

- Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)

$$S = \int d^5x \sqrt{-g} \left[\frac{{}^{(5)}R}{2\kappa^2} + \delta(\chi) \left(\frac{{}^{(4)}R}{2\mu^2} + \mathcal{L}_m \right) \right]$$

  with crossover scale $r_c = \kappa^2 / 2\mu^2$
- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001)
- Matter still minimally coupled and conserved
- Exhibits the 3 regimes of modified gravity
- Weyl tensor anisotropy dominated conserved curvature regime $r > r_c$ (Sawicki, Song, Hu 2006; Cardoso et al 2007)
- Brane bending scalar tensor regime $r_* < r < r_c$ (Lue, Soccimarro, Starkman 2004; Koyama & Maartens 2006)
- Strong coupling General Relativistic regime $r < r_* = (r_c^2 r_g)^{1/3}$ where $r_g = 2GM$ (Dvali 2006)

Three Regimes

- Fully worked $f(R)$ and DGP examples show 3 regimes
- Superhorizon regime: ζ =const., $g(a)$
- Linear regime - closure condition - analogue of "smooth" dark energy density:

$$\begin{aligned} \nabla^2(\Phi - \Psi)/2 &= -4\pi G a^2 \Delta\rho \\ g(a, \mathbf{x}) &\leftrightarrow g(a, k) \end{aligned}$$

G can be promoted to $G(a)$ but conformal invariance relates fluctuations to field fluctuation that is small

- Non-linear regime:

$$\begin{aligned} \nabla^2(\Phi - \Psi)/2 &= -4\pi G a^2 \Delta\rho \\ \nabla^2\Psi &= 4\pi G a^2 \Delta\rho - \frac{1}{2}\nabla^2\phi \end{aligned}$$

Nonlinear Interaction

Non-linearity in the field equation

$$\nabla^2 \phi = g_{\rm lin}(a) a^2 \left(8\pi G \Delta\rho - N[\phi]\right)$$

recovers linear theory if $N[\phi] \to 0$

- For $f(R)$, $\phi = f_R$ and

$$N[\phi] = \delta R(\phi)$$

a non-linear function of the field

Linked to gravitational potential

- For DGP, ϕ is the brane-bending mode and

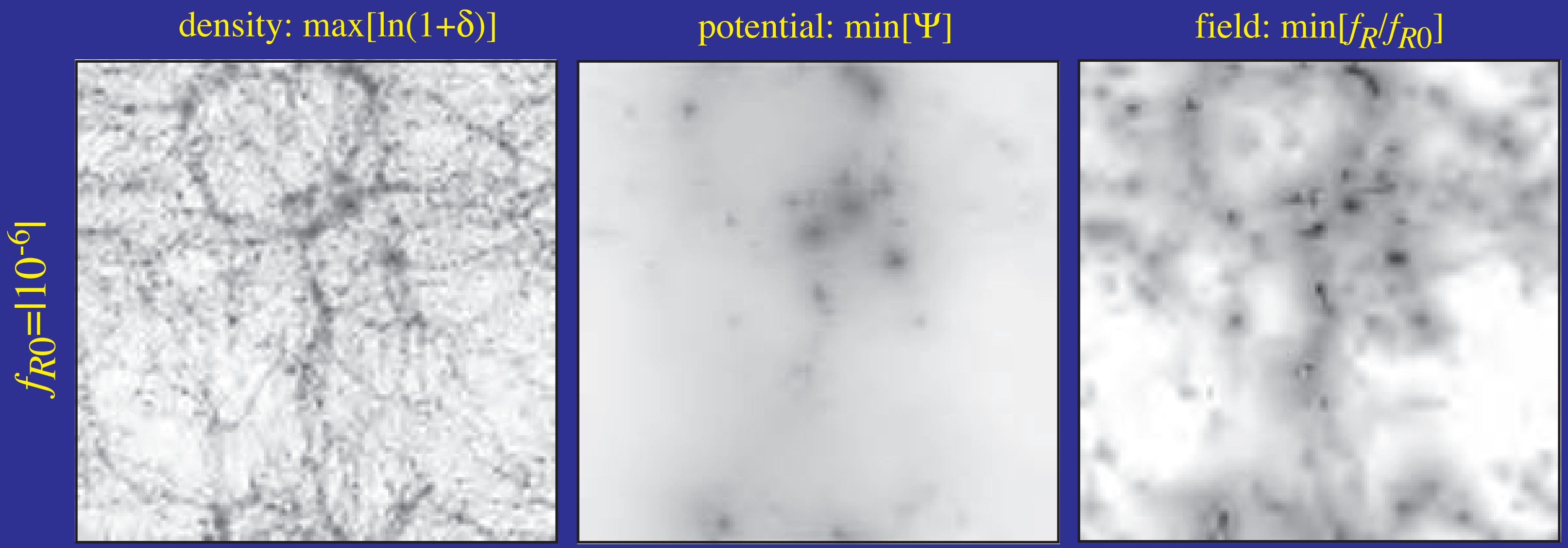
$$N[\phi] = \frac{r_c^2}{a^4}\left[(\nabla^2\phi)^2 - (\nabla_i \nabla_j \phi)^2\right]$$

a non-linear function of second derivatives of the field

Linked to density fluctuation

Environment Dependent Force

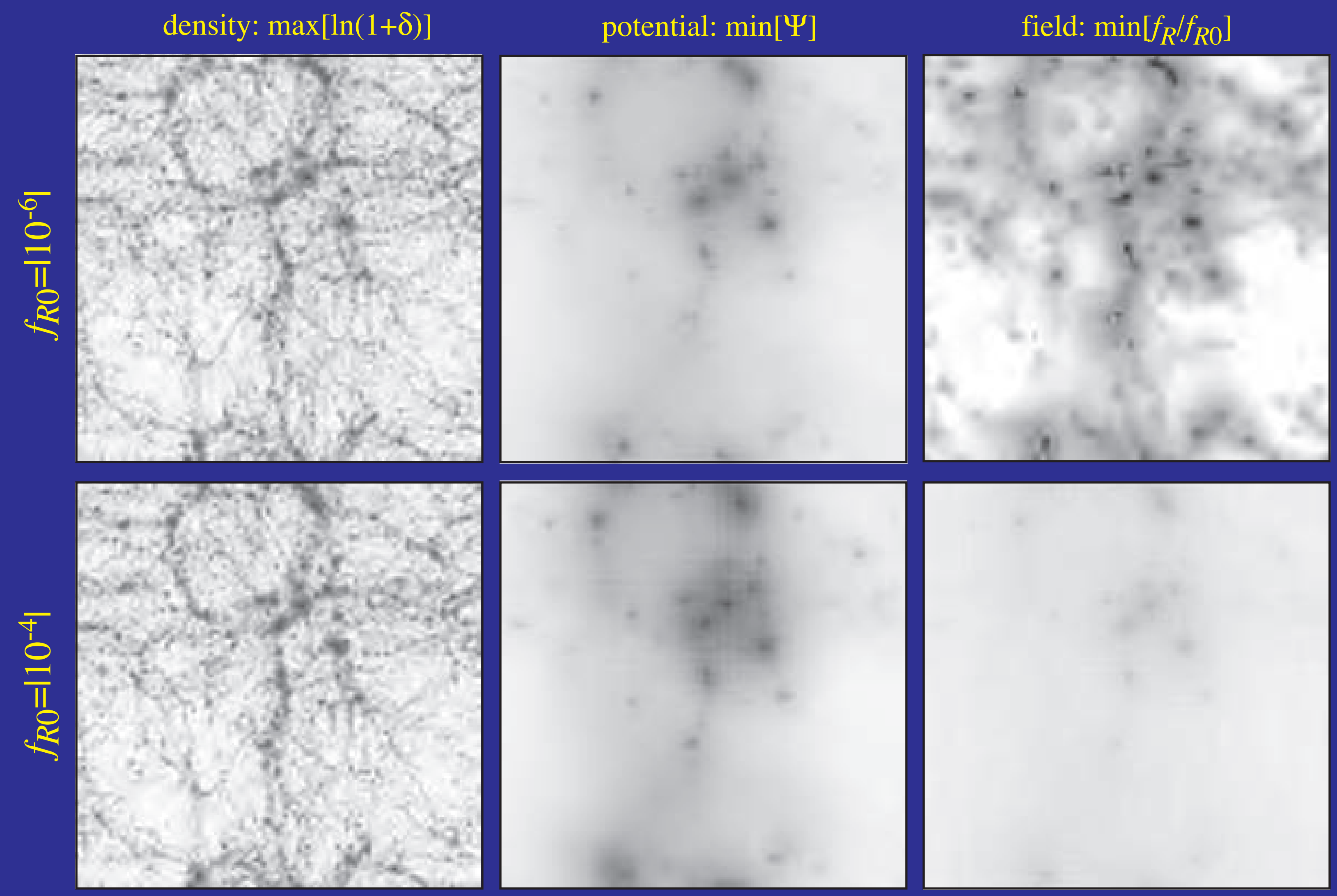
- Chameleon suppresses extra force (scalar field) in high density, deep potential regions



Oyaizu, Lima, Hu (2008)

Environment Dependent Force

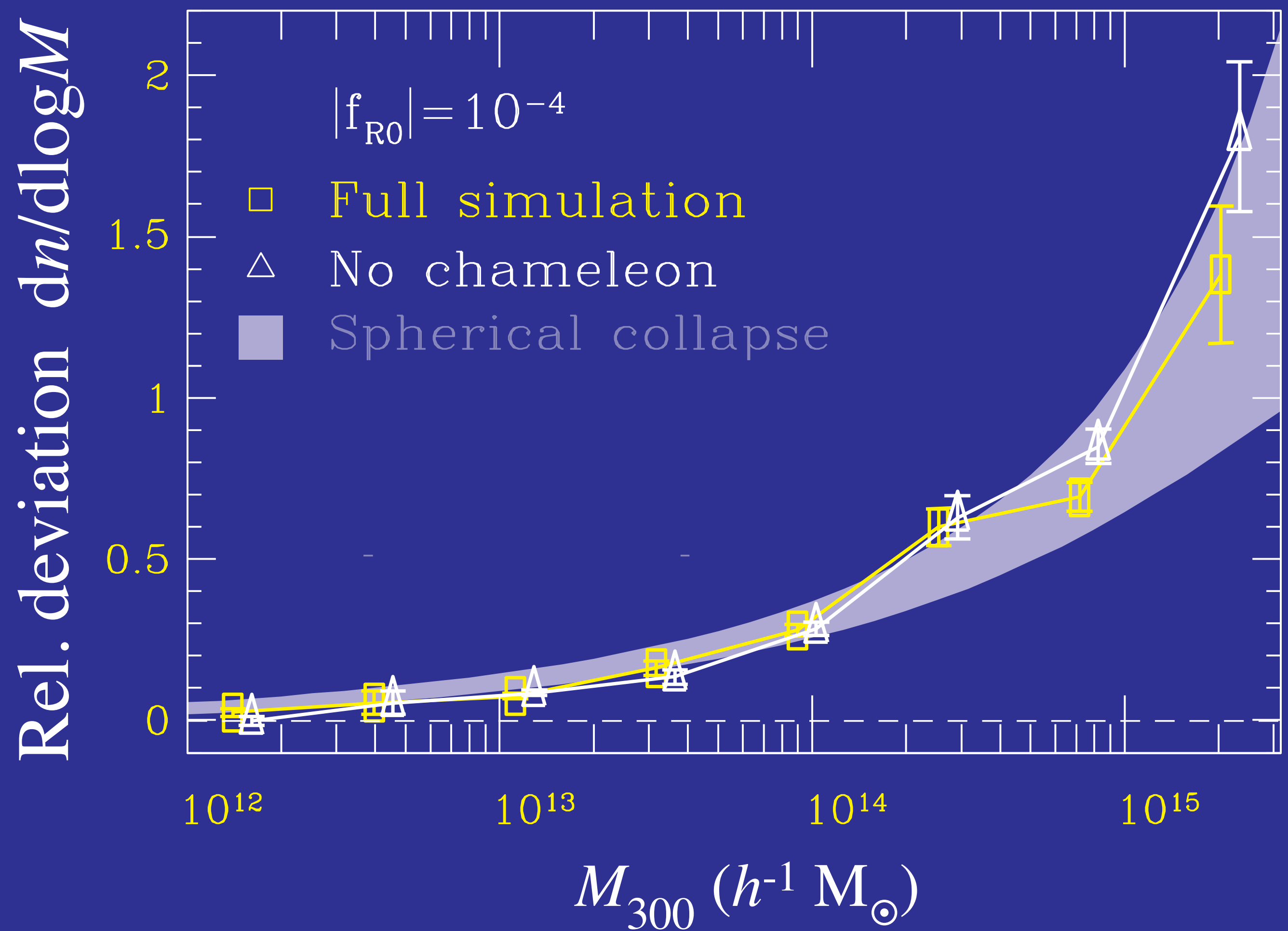
- For large background field, gradients in the scalar prevent the chameleon from appearing



Oyaizu, Lima, Hu (2008)

Cluster Abundance

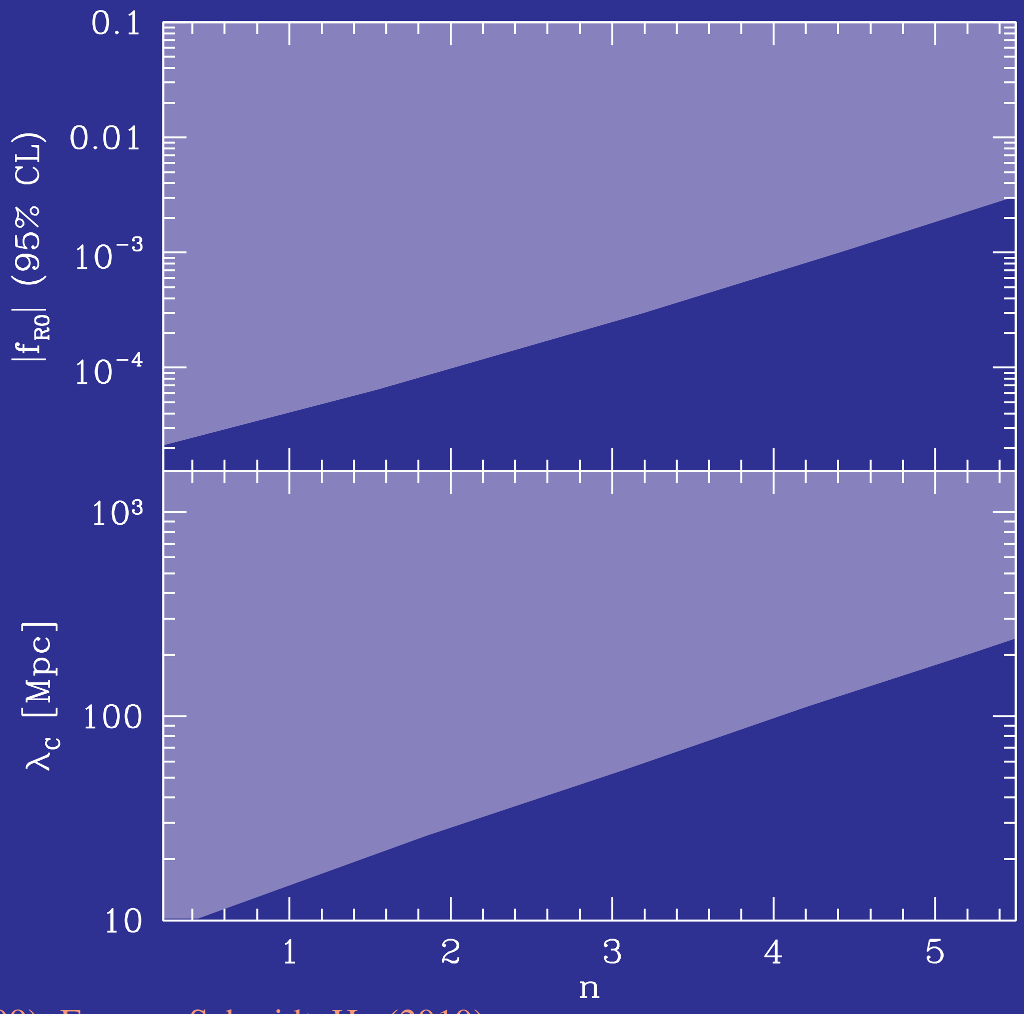
- Enhanced abundance of rare dark matter halos (clusters) with extra force



Lima, Schmidt, Oyaizu, Hu (2008)

Cluster *f*(*R*) Constraints

- Clusters provide best current cosmological constraints on *f*(*R*) models
- Spherical collapse rescaling to place constraints on full range of inverse power law models of index *n*



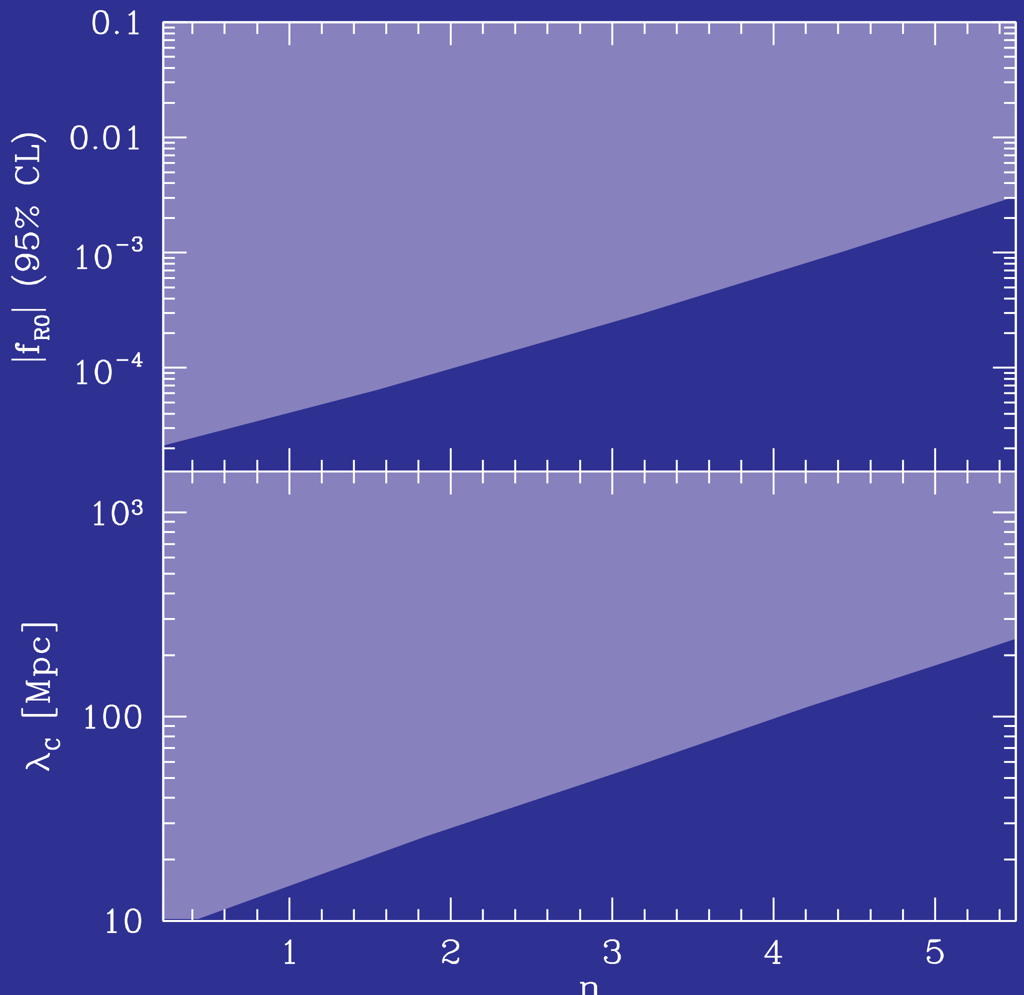
field amplitude

range of force

Schmidt, Vikhlinin, Hu (2009); Ferraro, Schmidt, Hu (2010)

Cluster $f(R)$ Constraints

- Approaching competitiveness with solar system + Galaxy constraints of few 10^{-6} at low n
- Vastly different scale



field amplitude

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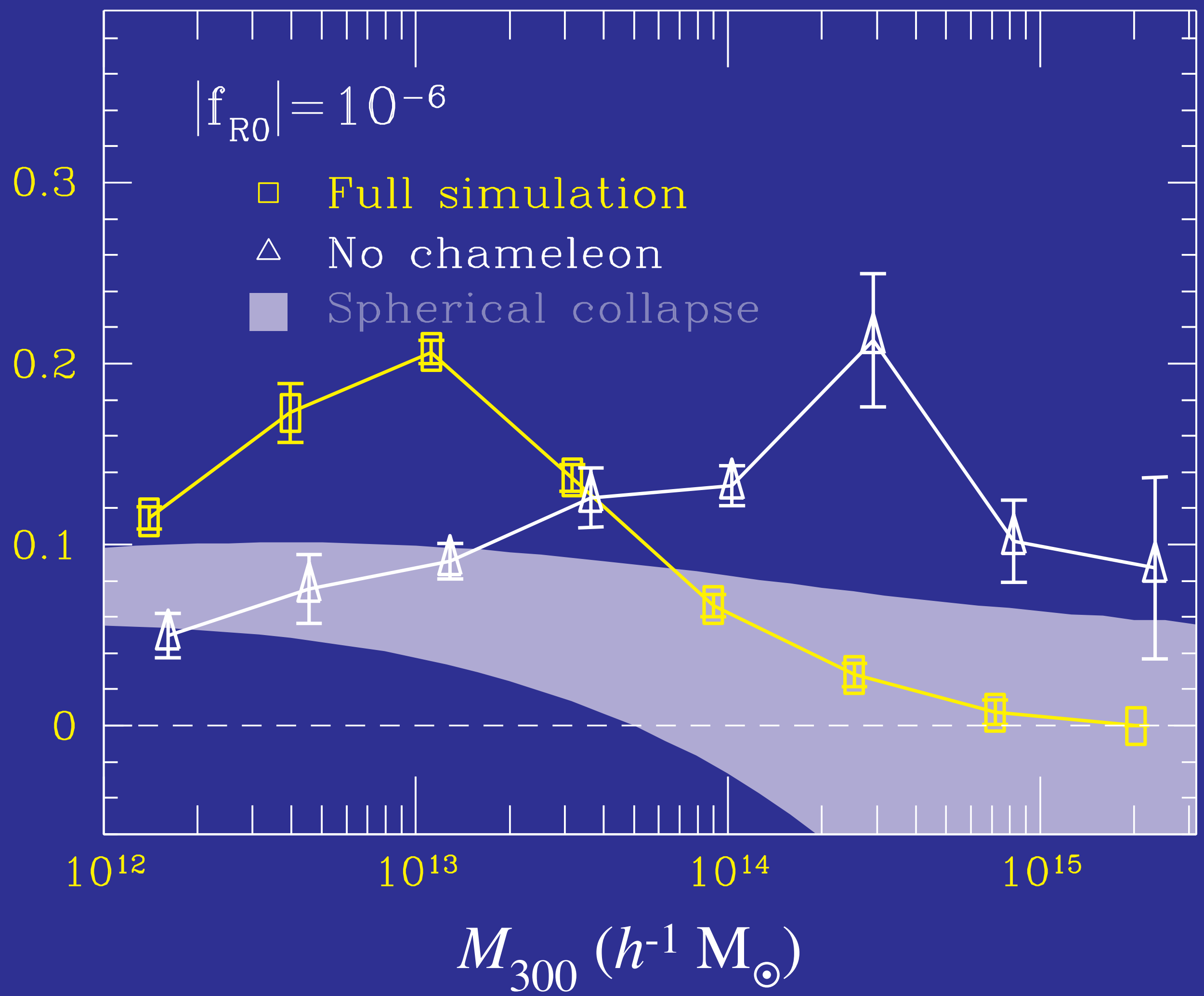
Chameleon Mass Function

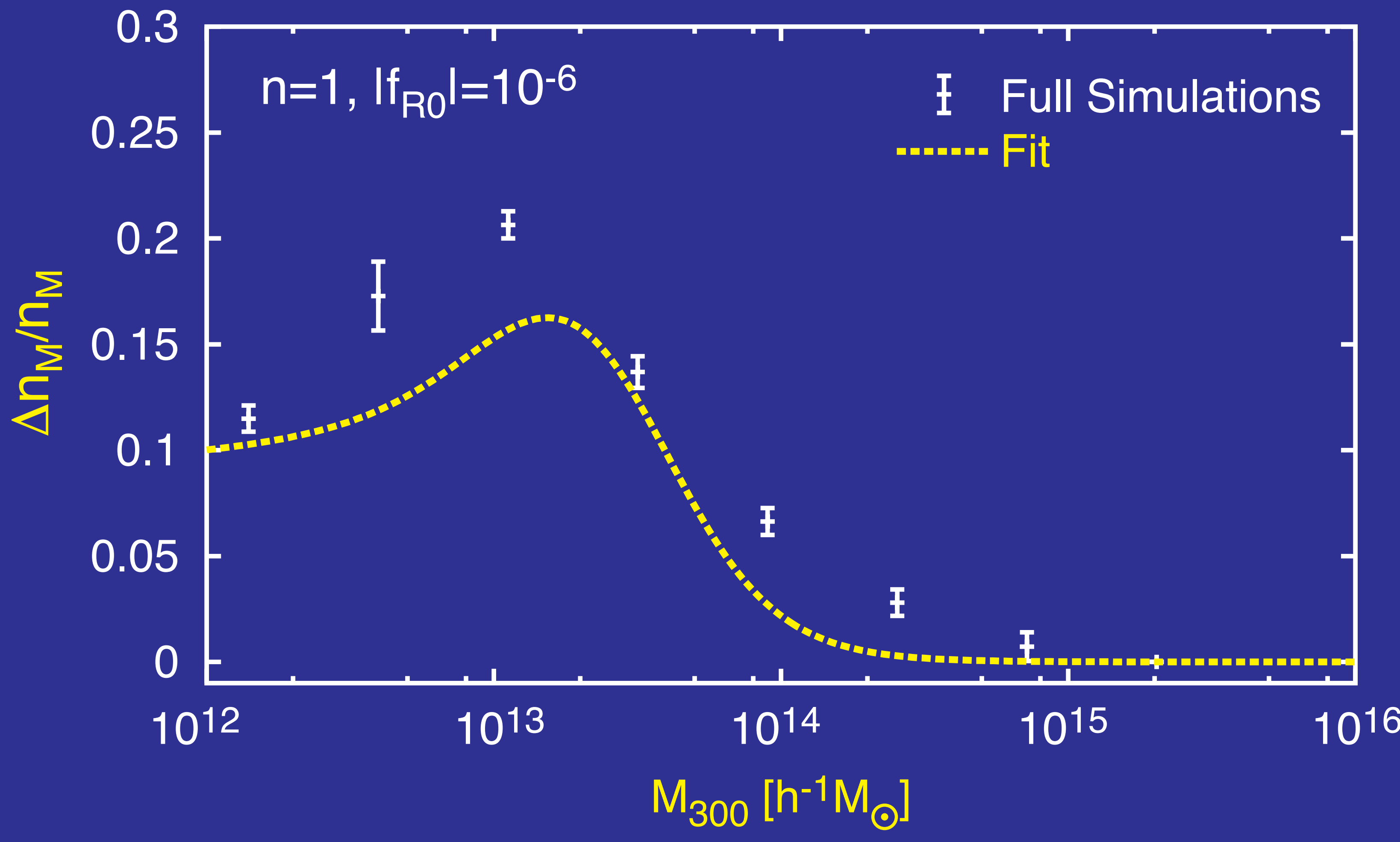
- Chameleon effect suppresses the enhancement at high masses
- Pile up of abundance at intermediate group scale



Lima, Schmidt, Oyaizu, Hu (2008)

Chameleon Mass Function

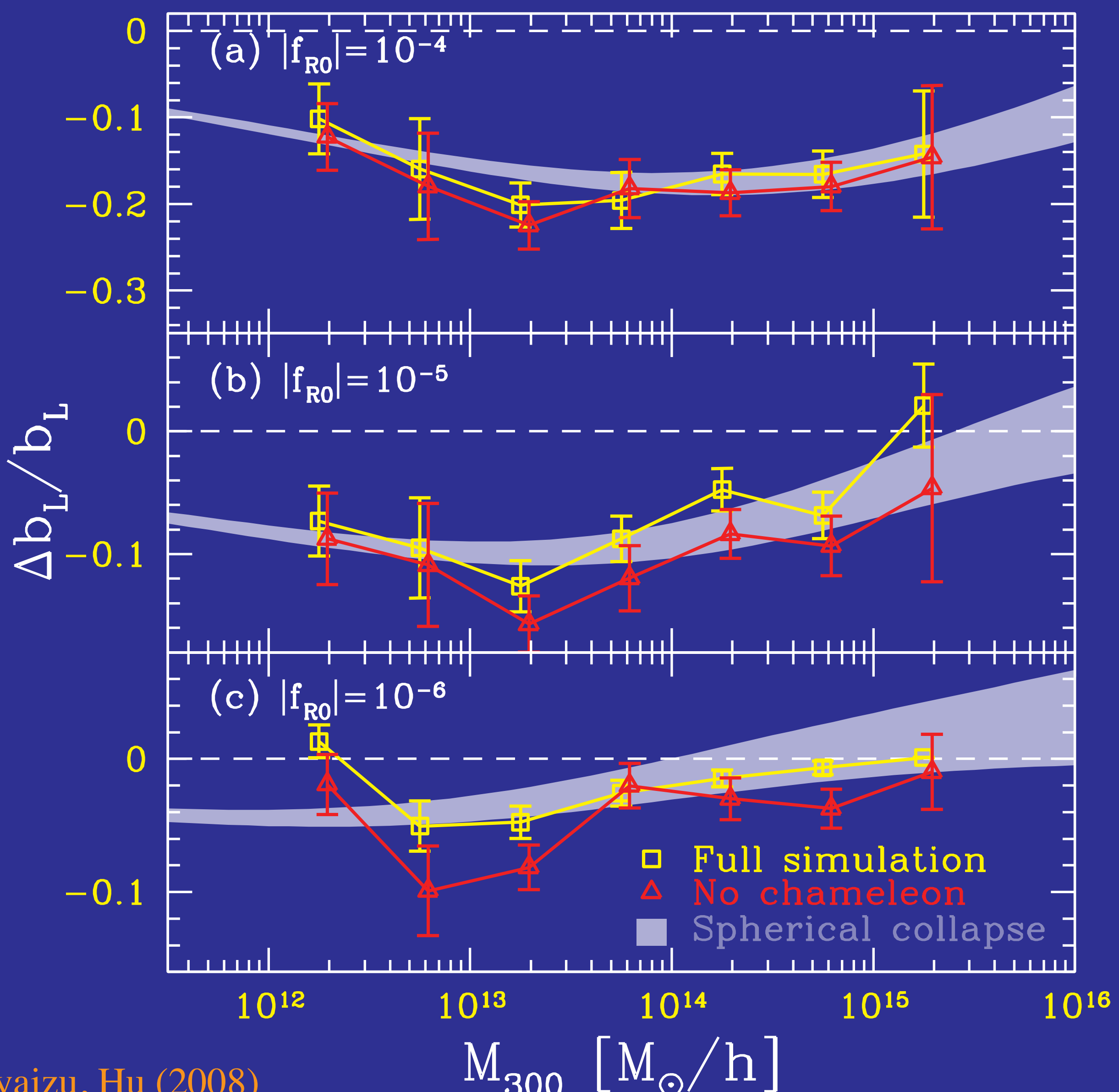
- Simple single parameter extention covers variety of models
- Basis of a halo model based post Friedmann parameterization of chameleon



Li & Hu (2011)

Halo Bias

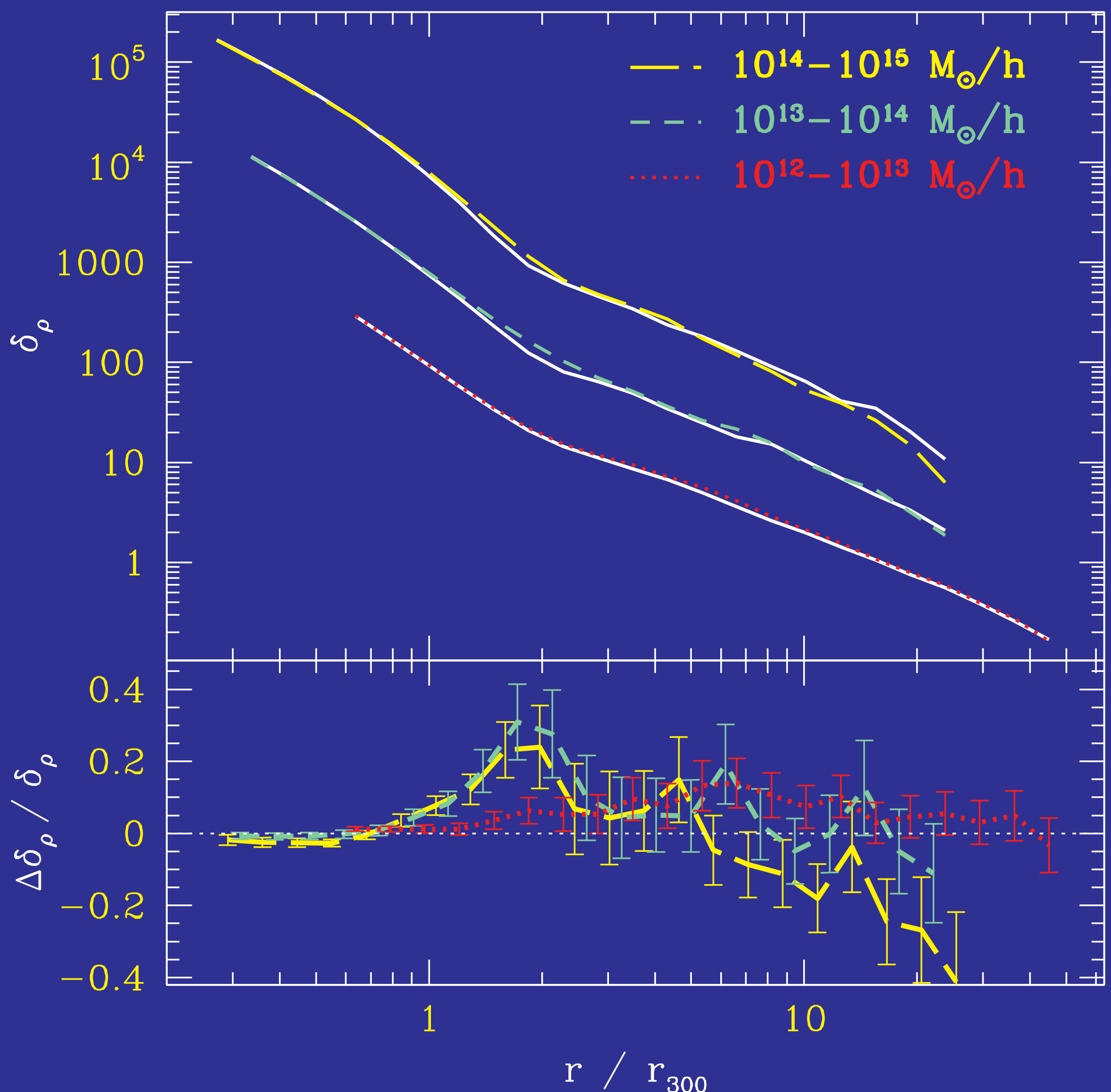
- Halos at a fixed mass less rare and less highly biased



Schmidt, Lima, Oyaizu, Hu (2008)

Halo Mass Correlation

- Enhanced forces vs lower bias



Schmidt, Lima, Oyaizu, Hu (2008)

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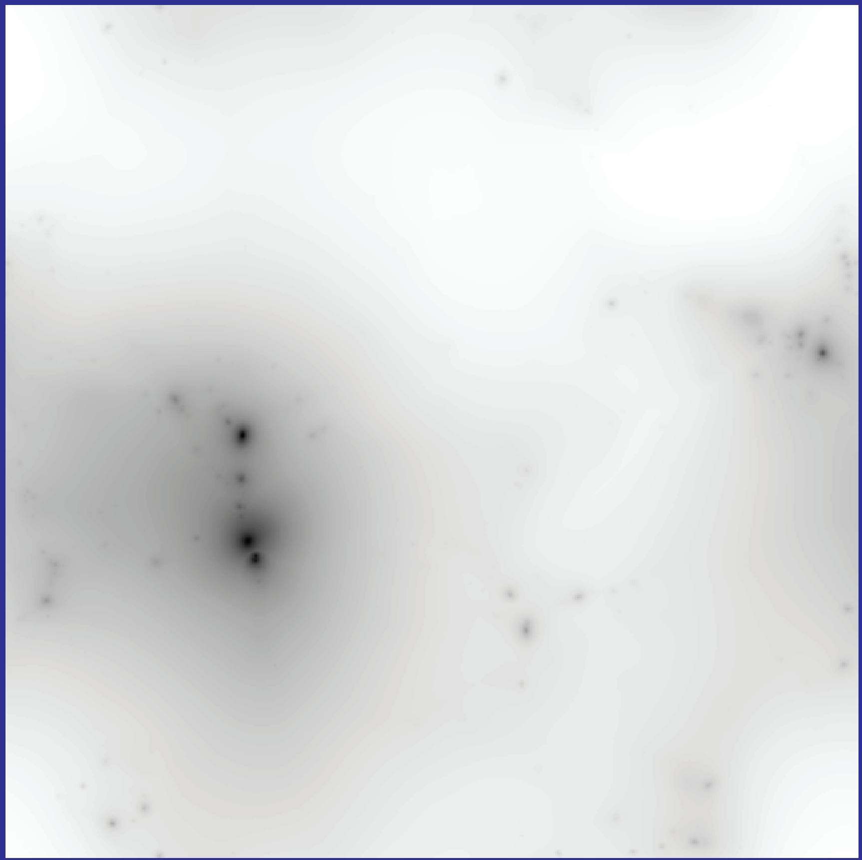
a non-linear function of second derivatives of the field

Linked to density fluctuation

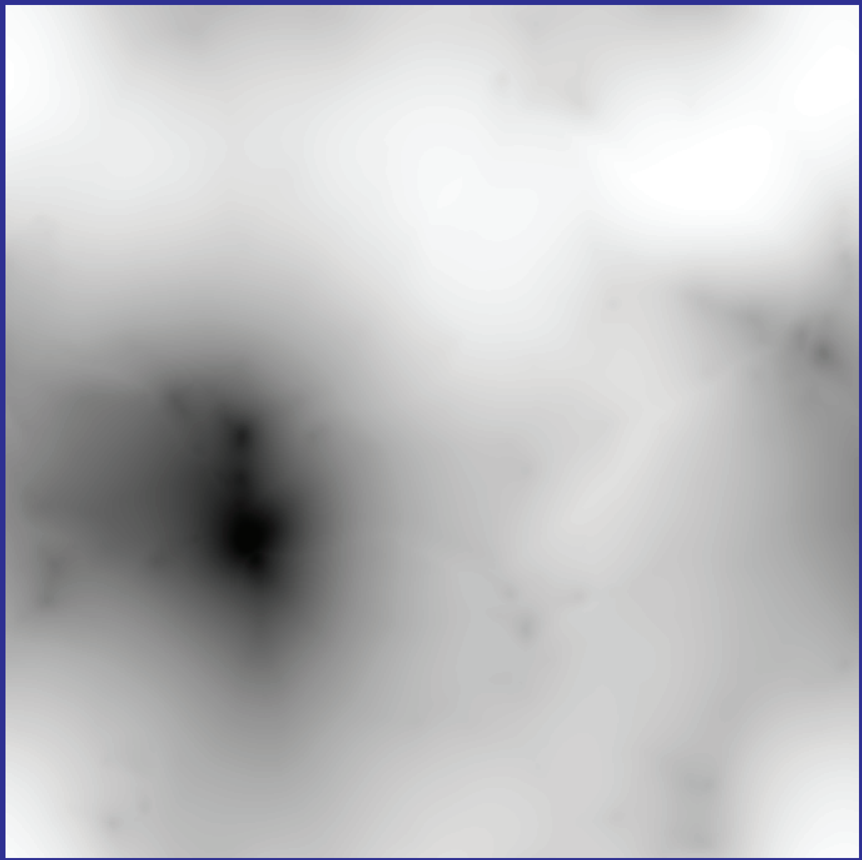
DGP N-Body

- DGP nonlinear derivative interaction solved by relaxation revealing the Vainshtein mechanism

Newtonian Potential



Brane Bending Mode



Schmidt (2009); Chan & Scoccimarro (2009) (cf. Khoury & Wyman 2009)

Summary

- Given current geometric data, ΛCDM and quintessence $(w > -1)$ are highly predictive and falsifiable
- Linear growth at all z cannot exceed fiducial $>$ few percent
- With Gaussian fluctuations, exponential sensitivity of cluster abundance exploits this test: e.g. high M, high z pink elephants
- No currently known single cluster falsifies ΛCDM
- Places currently the strongest cosmological constraints on modified gravity models with enhanced forces but ΛCDM expansion history, e.g. $f(R)$
- Future tests which complement the solar system constraints will need to move down the mass function
- Parameterized approaches should take into account that the force modifications depend on local environment, potential or density