Avalanche dynamics on an inclined plane

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Particle-Laden Flows in Nature

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Granular Flows and Avalanches

Statistical Mechanics Approach

- Based on grain-scale theories of grain interaction and instability of avalanches.
- Focus on statistical distributions of avalanche sizes and pattern formation
- Most developed for highly intermittent flows
- Now mostly used for problems besides granular flow

Fluid Mechanics Approach

- Based on approximations to rheology and conservation laws
- Rapid progress since seminal work of Pouliquen (1999)
- Weak connection to underlying particle mechanics, esp. for dense flows
- Most developed for steady and close-to-steady flows

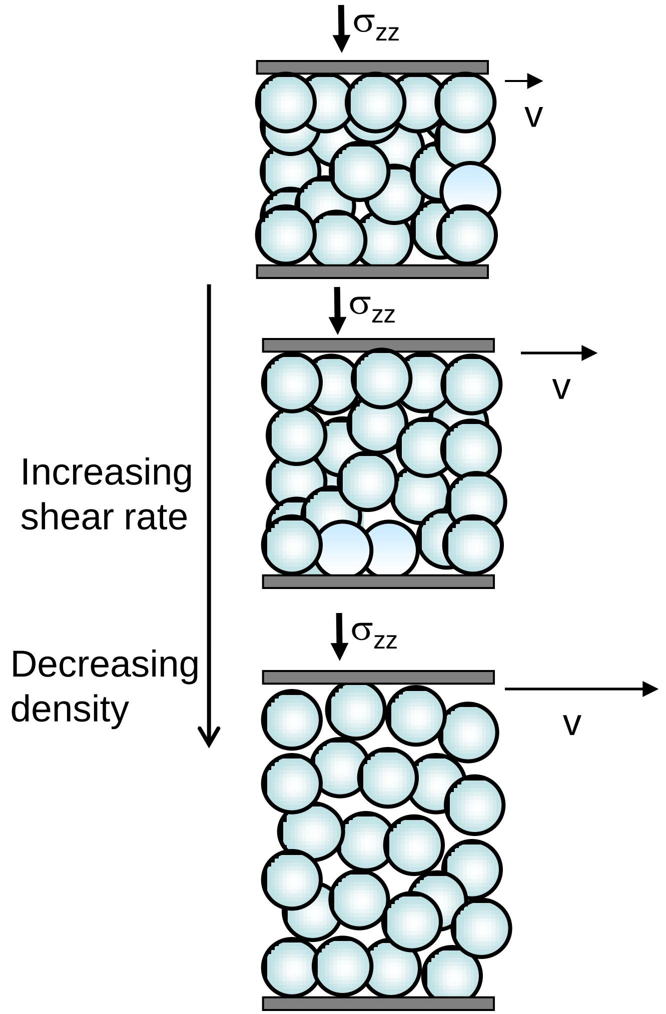
Dense Granular Flows

- Quasistatic Flow: Rate independent stress-strain constitutive relations (Critical State Soil Mechanics)

- Dense Granular Flow: dynamic contact network with multi-particle interactions

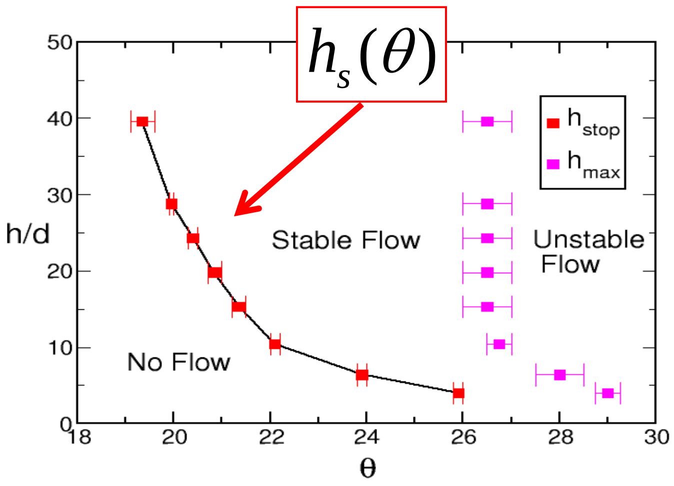
- Collisional Flow: Constitutive relations based on collision statistics (Kinetic Theory)

- Fluid-dominated flows
  - Wet dense granular flows
  - Turbidity currents



Rheology of Dense Granular Flows

- Well-established phenomenology for dry dense granular flows
  - Campbell, Pouliquen, Silbert et al.



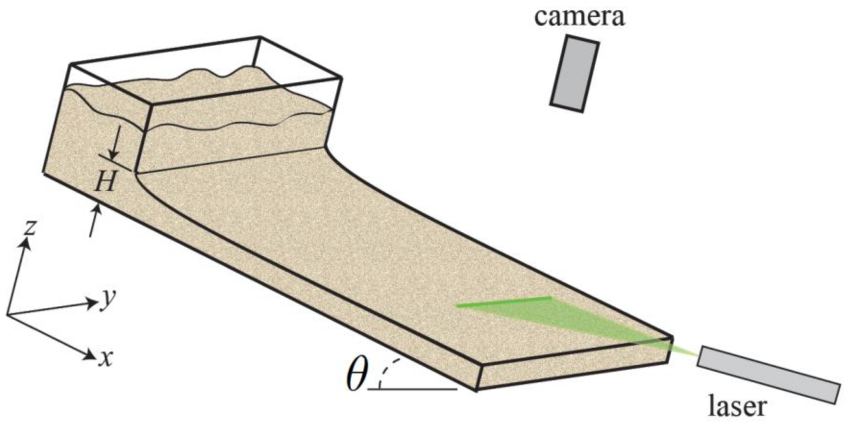
- Pouliquen flow rule on inclined plane

$$\frac{u}{\sqrt{gh}} \equiv Fr = \beta \frac{h}{h_s(\theta)} - \gamma$$

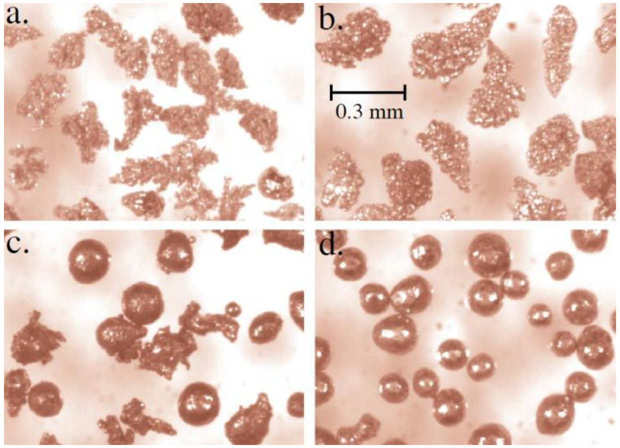
- Rheology is established for steady-state, near steady-state conditions
  - Usually for spherical grains

Can steady-state rheology be used to understand intermittent avalanche regime?

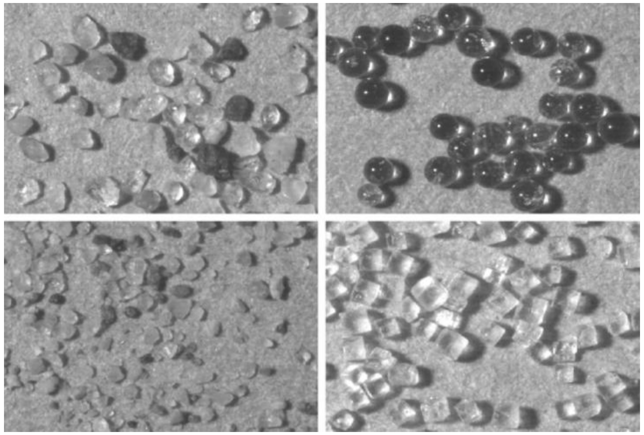
Experimental Approach (Börsönyi, Ecke)



Copper



Sand **Glass Beads**



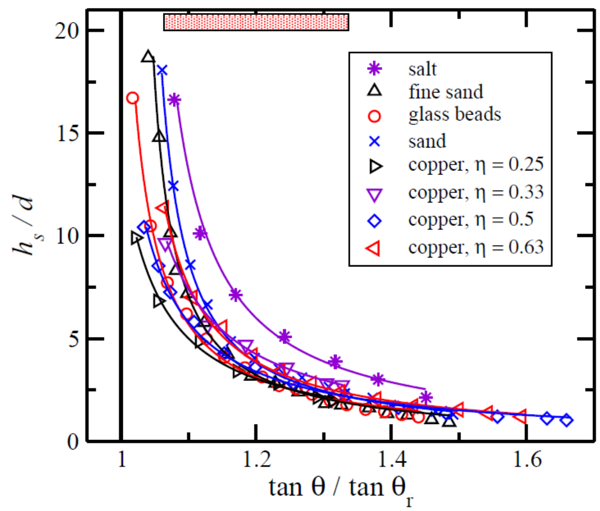
Salt

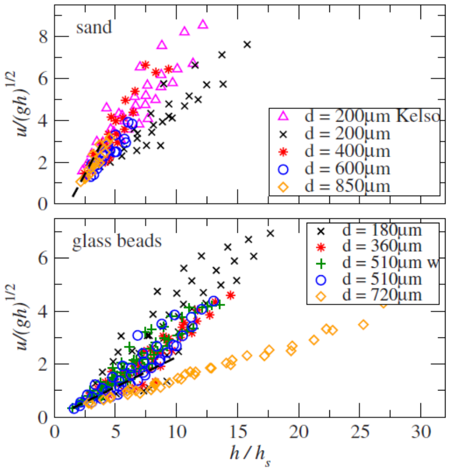
Overall Flow Character

- Qualitatively simple "phase diagram" for all materials
- Critical height as function of θ can be modeled as

$$\frac{h_s}{d} = \frac{a_1}{\tan\theta - \tan\theta_1}$$

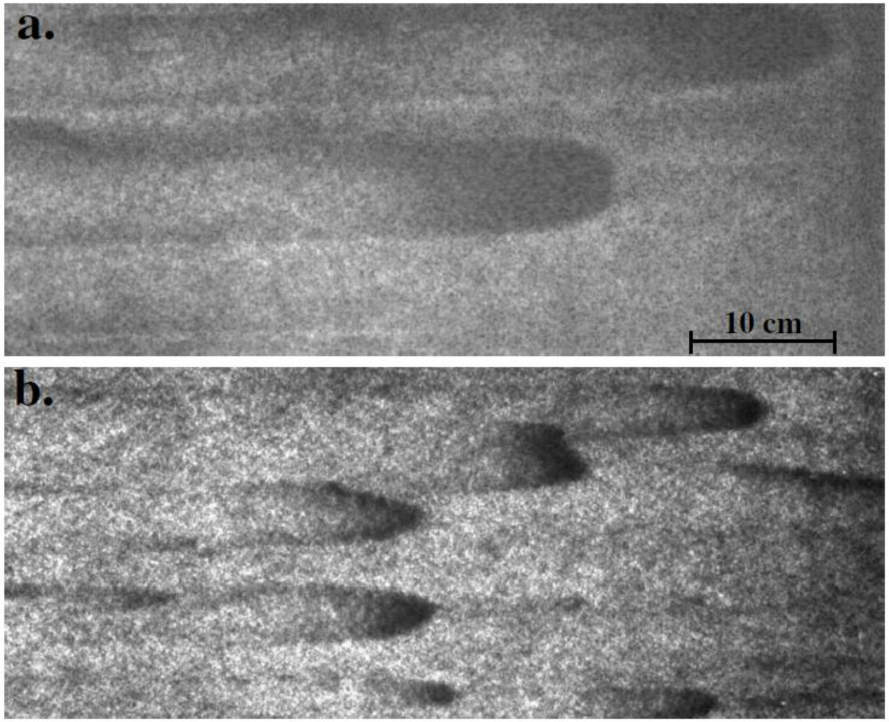
- Pouliquen flow rule (or modified Jenkins form) satisfied for sand, glass beads, less robust for copper particles
  - β for sand larger than for glass beads





Avalanches

Sand Avalanches



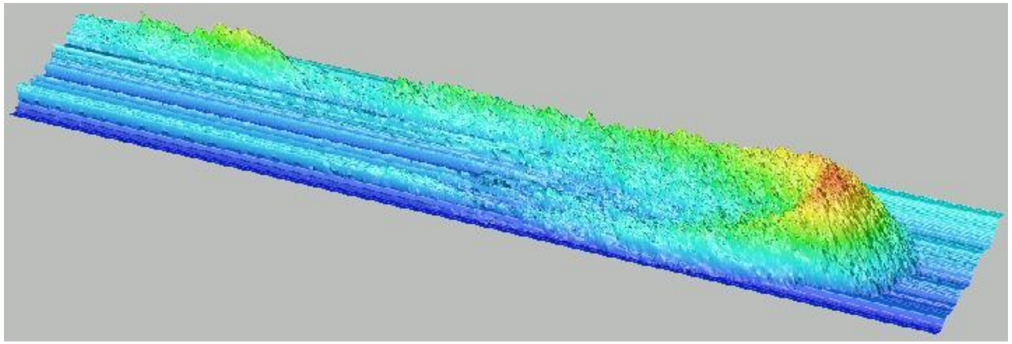
***Θ* = 33.6º**

***Θ* = 38.1º**

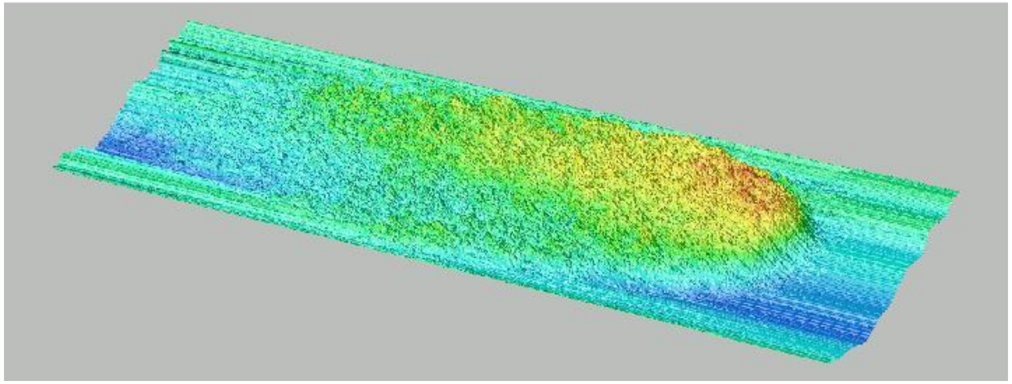
- Particles are added at top of incline
- Avalanches return slope to its critical value
- Avalanches structure and velocity are approximately constant

Weak and Strong Avalanches

Sand Avalanches



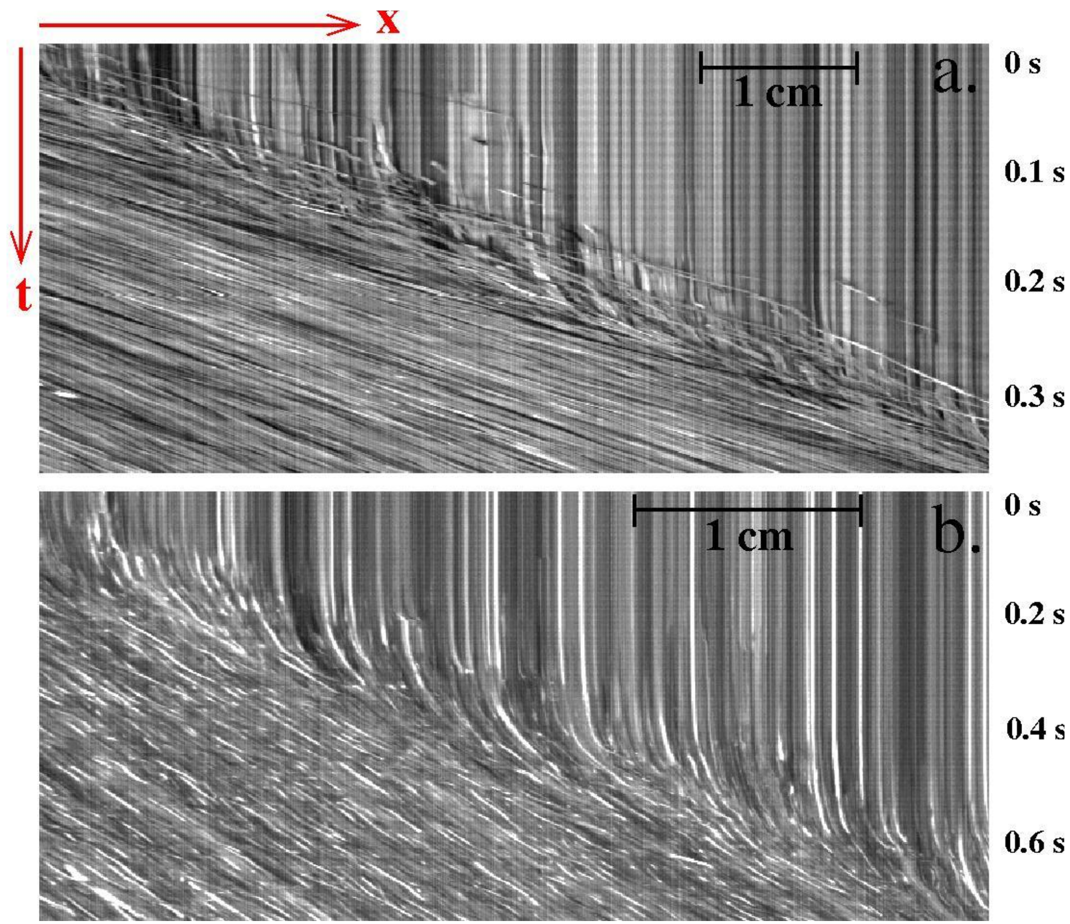
Glass Bead Avalanches



- Differing character of avalanches seen
  - Sand avalanches are larger and faster than glass bead avalanches, have a much more dramatic forward profile

Avalanche Structure

- For sand avalanches, front arrives suddenly, with particle velocity at front (at least at surface) exceeding front velocity
- For glass bead avalanches, particles are gradually accelerated as front arrives



Depth-Averaged Theory

Pouliquen flow rule

$$\frac{u(h,\theta)}{\sqrt{gh}} = \beta \frac{h}{h_s(\theta)} - \gamma$$

Conservation of mass

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0$$

Conservation of momentum

$$\frac{\partial (hu)}{\partial t} + \alpha \frac{\partial (hu^2)}{\partial x} = \left(\tan\theta - \mu(u,h) - K \frac{\partial h}{\partial x} \right) gh\cos\theta$$

Velocity profile (α)

Base friction ($\mu(u,h)$)

Normal stress difference (K)

$$\alpha = 5/4 \qquad \tan\theta = \mu(u(h,\theta),h) \qquad K \approx 1$$

Solution Structure

- Second order hyperbolic (wave) equation with characteristic velocities

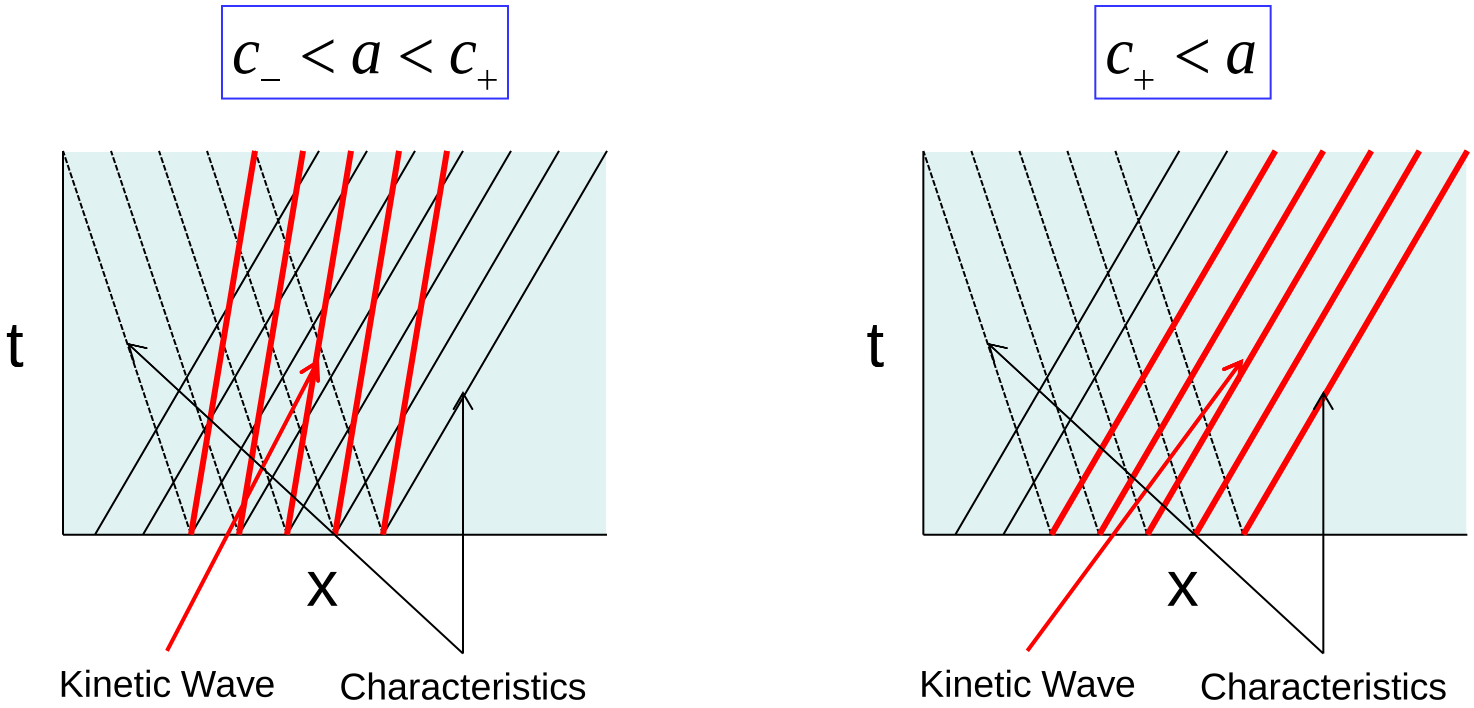
$$c_{\pm} = u\left(\alpha \pm \sqrt{\alpha(\alpha - 1) + K / \left(Fr^2 \cos\theta\right)} \right) \quad Fr = u / \sqrt{gh}$$

- But, for Fr << 1, equations of motion can be directly simplified to give kinematic waves

$$\frac{\partial h}{\partial t} + a(h)\frac{\partial h}{\partial x} = N\left(h, \frac{\partial h}{\partial x} \right) \quad a(h) = \sqrt{gh}\left(\frac{5}{2}\beta\frac{h}{h_s} - \frac{3}{2}\gamma \right)$$

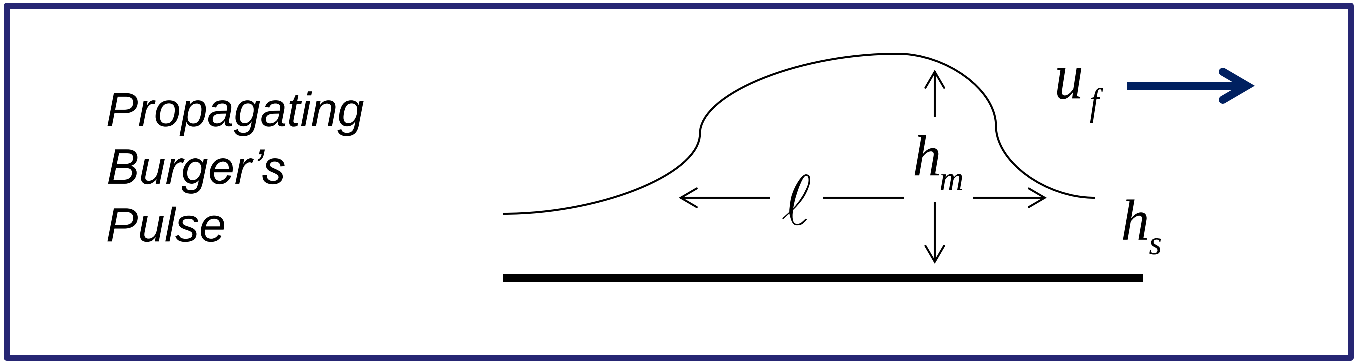
- Note that it is not automatic that $a < c_+$

Wave Hierarchy



- Kinematic wave cannot move faster than characteristic (maximum velocity of information transport). When $a \geq c_+$, the kinematic wave merges with the forward shock

Weak Avalanche

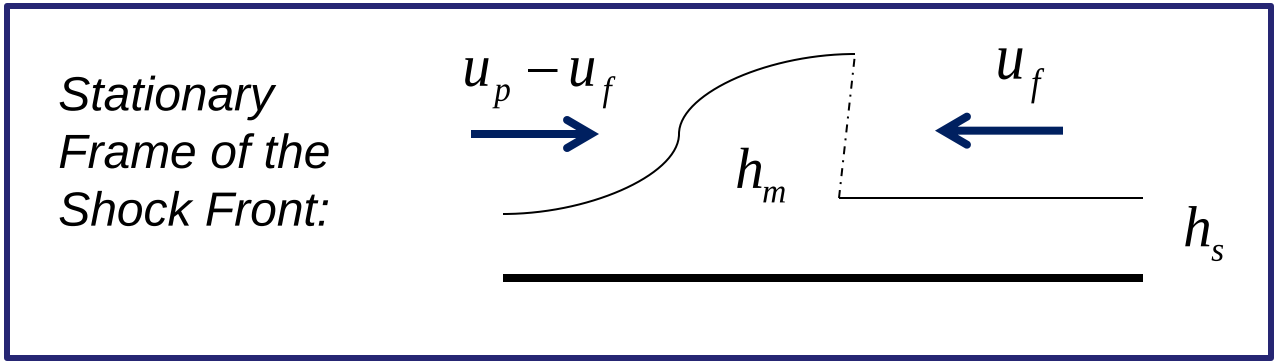


- Kinematic waves have a first-order wave, with a diffusive term on the right hand-side (like Burger's equation)
- Suggests that avalanche should broaden with time—not observed
  - May be too slow to observe in course of experiment
- For glass beads, pure first order theory predicts

$$u_f \approx 0.6a(h_m) \quad \ell \approx 6h_s$$

- Acceptable (but not impeccable) agreement

Strong Avalanche: Shock Solution



- For the shock solution, there will be a jump criterion connecting particle and front velocities with the height of the shock

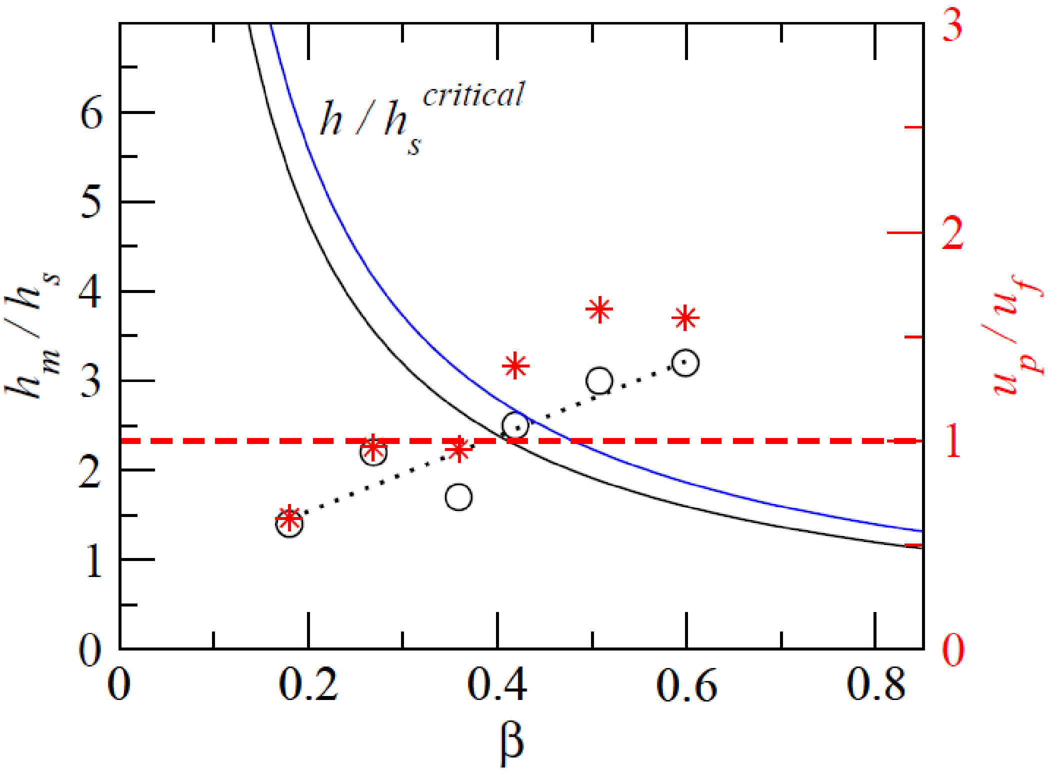
$$(u_p - u_f)h_m > u_f h_s$$

- Equivalently

$$\left(\frac{u_p}{u_f} - 1\right) > \left(\frac{h_m}{h_s}\right)^{-1}$$

- So that we must have $u_p > u_f$ at the shock!

Results for Various Particles

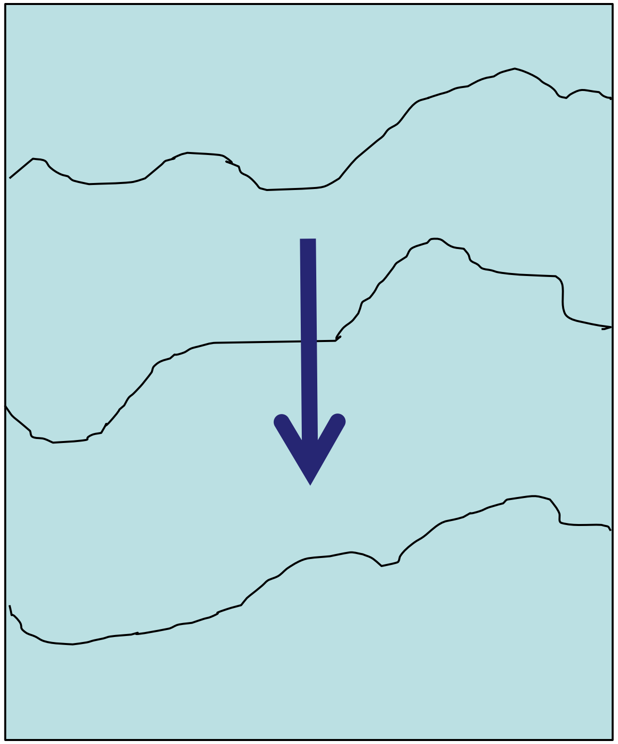


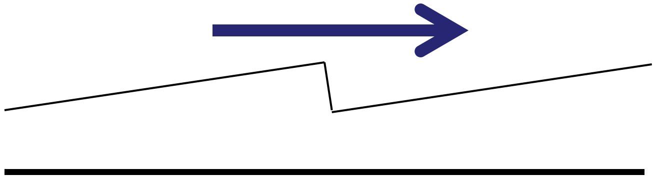
- Note strong correlation between super-critical vs. sub-critical avalanche height (corresponding to which side of the blue or black curves the points occupy) and the particle to front velocity ratio (shown on right)

Instabilities

- This is analogous to result for instabilities in steady flow, analyzed by Forterre and Pouliquen
- Glass beads
  - Flows near critical height were stable
  - Flows away from critical height were unstable
- Sand: the reverse
- Roll waves vs. flood waves
- Criterion for stability of flows:

$$a < c_+$$





Reservations

- Both strong and weak avalanches are propagating into static materials; for both types of avalanches the zone behind the avalanche front is settling back into a static state.
  - No modeling of zone of “passive Rankine failure” ahead of front
- Have not addressed lateral structure of avalanches
  - Could be done with straightforward extension of depth-averaged equations
- In practice, α should vary with height
  - linear velocity profile seen near threshold
  - Bagnold velocity profile seen for deeper flows

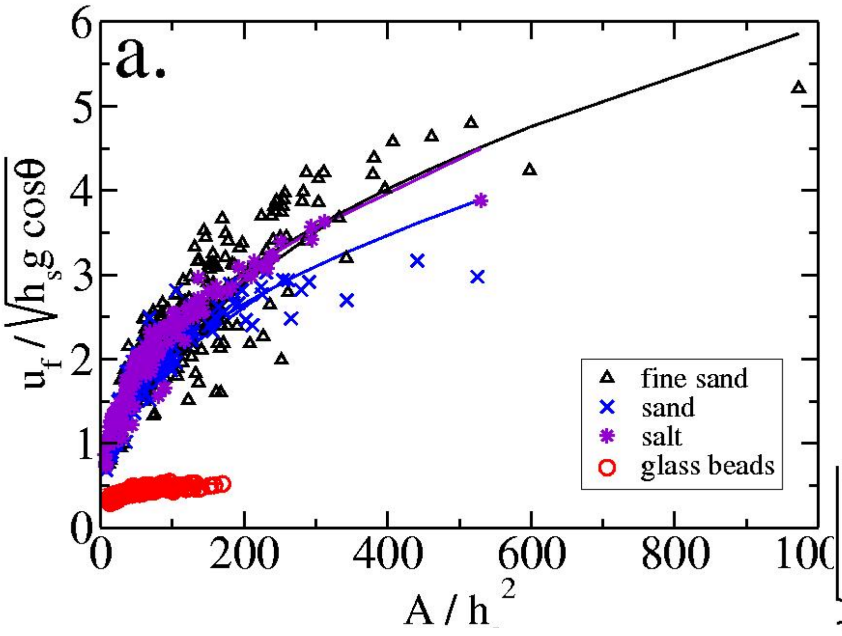
Outlook

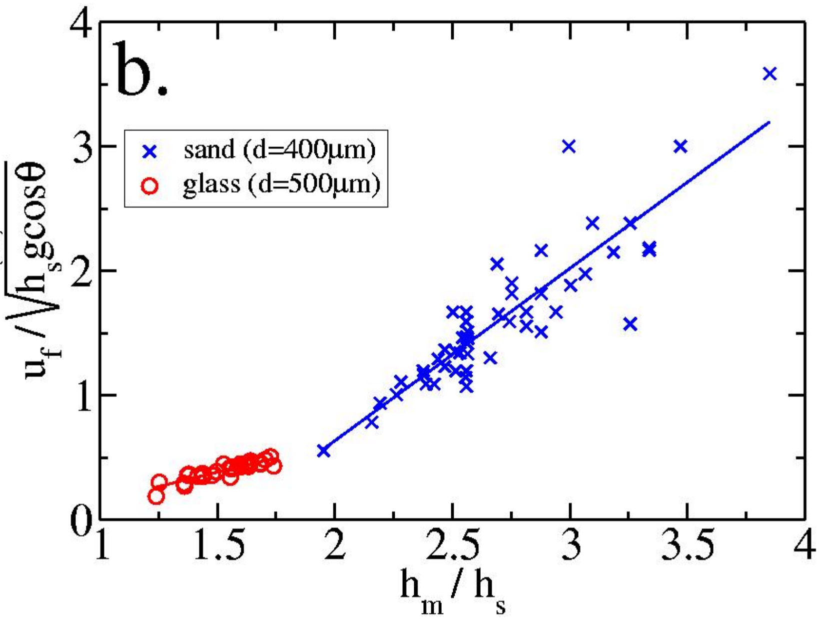
Can steady-state rheology be used to understand intermittent avalanche regime?
Yes! But statistical mechanics may still be needed to underpin fundamental rheology!

- Semi-quantitative theory accounts well for transition from weak to strong avalanches
  - Notwithstanding granular complexities, simple depth-averaged fluid mechanical approach is quite successful
- Alas, dry granular flows are limited in their geophysical importance
- “Wet granular flows” (Debris flows)—more complex rheology (although note Marseille group proposal)
- Turbidity currents—simple conceptually (Parker model and its descendants) but large phase space, mathematically more complex

Backup

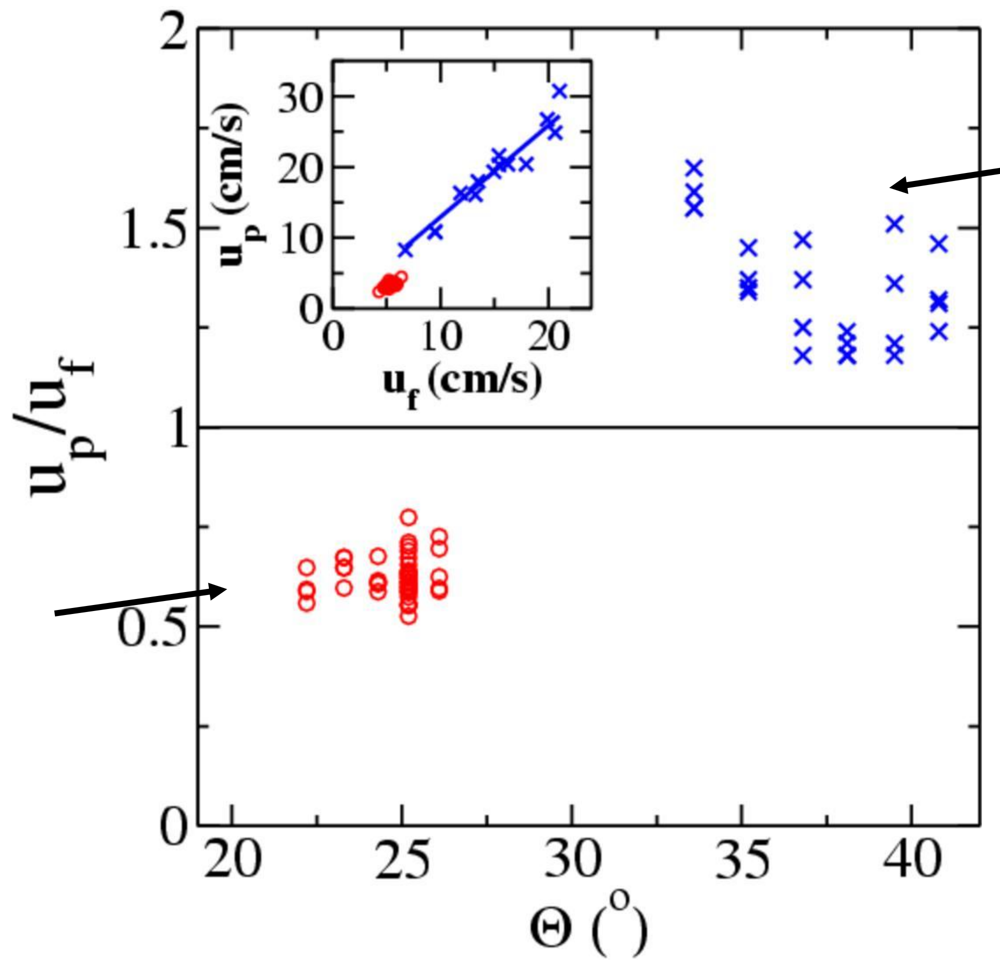
Avalanche Size and Speed





Front and Particle Velocities vs. Angle

Glass beads



Sand