Coulomb Effects in Granular Materials

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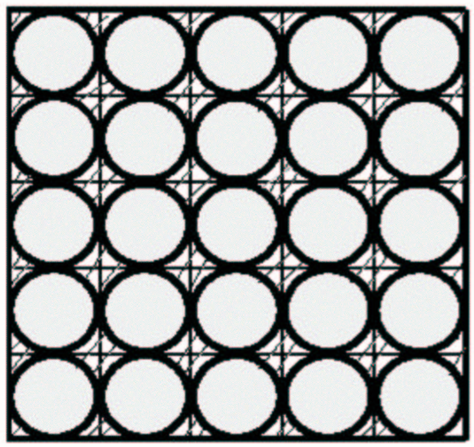
Typical structure of a granular metal:



d=50-200A

Coupling between the grains can vary: possibility of both macroscopically metallic and insulating states.

1

Experimental puzzles:

1. Strong coupling between the grains

$$\sigma = \sigma_0 + \alpha \ln T$$

2. Weak coupling between the grains

$$\sigma = a \exp(-b / \sqrt{T})$$

The dimensionality of the array does not seem to play any important role for both 1) and 2)!

Metal-Insulator transition?

If so, what is the reason for such a behavior?

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Some experimental curves (after A. Gerber *et al*, PRL (1997))

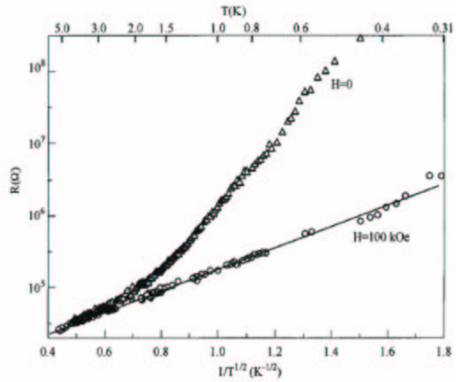

FIG. 1. Resistance of sample 1 measured at zero (triangles) and 100 kOe field (circles) as a function of the inverse square root of the temperature. Sample 1 room temperature resistance is $2 \times 10^3\ \Omega$.

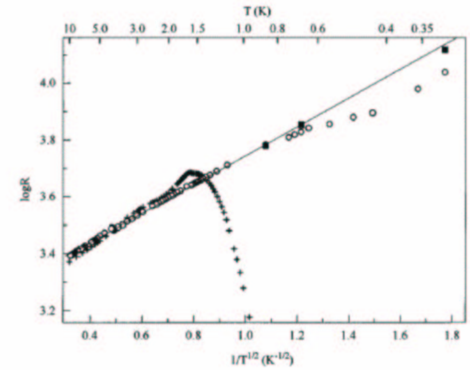

FIG. 2. Resistance of sample 2 measured at zero (crosses) and 100 kOe field (open circles) as a function of the inverse square root of the temperature. Open circles indicate resistance measured with a constant dc current $I = 10^{-6}$ A. Solid squares are zero bias resistances approximated from *I-V* measurements. Sample 2 room temperature resistance is $800\ \Omega$.

The weak coupling limit (insulator)

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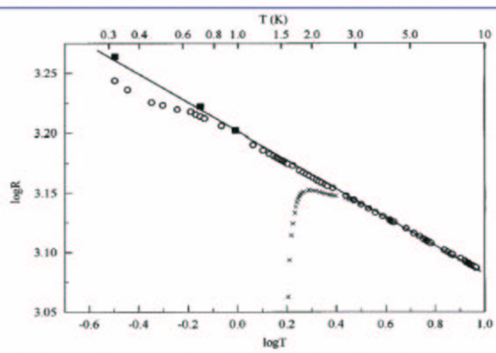

FIG. 3. Resistance of sample 3 as a function of temperature on a log-log scale, as measured at (zero) (×) and 100 kOe field (open circles). Open circles indicate resistance measured with a constant dc current $I = 10^{-5}$ A. Solid squares are zero bias resistances approximated from *I-V* measurements. Sample 3 room temperature resistance is $500\ \Omega$.

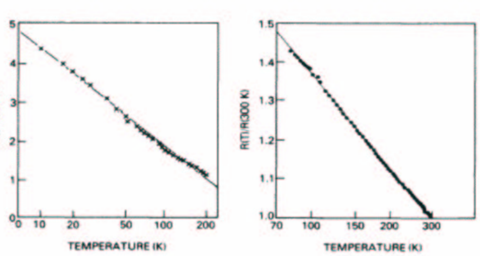

FIG. 5. The resistance normalized to the room-temperature value vs $\ln T$ for two different superconducting samples. The graph on the left is for a sample with $R_\square(300\text{ K}) = 2000\ \Omega/\square$, while that on the right is for $R_\square(300\text{ K}) = 100\ \Omega/\square$. The lines are guides to the eye.

$$R = AT^{-\alpha}$$

or

$$R = A(1 - \alpha \ln T)$$

$$\alpha = 0.117$$

R.W. Simon *et al*, Phys. Rev. B (1987)

Strong coupling limit. Metal?

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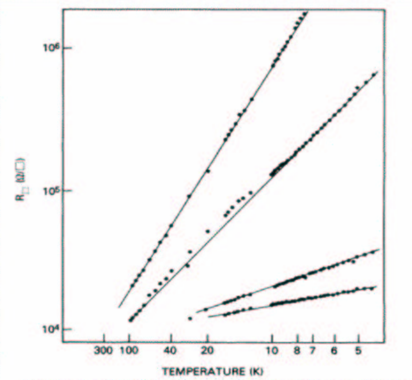

FIG. 4. The logarithm of the sheet resistance as a function of $1/\sqrt{T}$ for a variety of the nonsuperconducting samples. The lines are guides to the eye.

High resistivity sample again.

Both 1) and 2) are typical for many experiments!

Why?

It is always so!

How to describe the granular metals?

Coulomb interaction plays a crucial role!

$E_c = e^2/2C$

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E_c -charging energy

Other energies in the system:

$\delta = (\nu V)^{-1}$ -mean level spacing in a grain

$g\delta$ -tunneling energy between the grains

g-tunneling conductance

The Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_t + \hat{H}_c$$

$$\hat{H}_0 = \int \psi^+(\mathbf{r}) \left(-\frac{\nabla^2}{2m} + U(\mathbf{r}) \right) \psi(\mathbf{r})\, d\mathbf{r}$$

t_{ij}-tunneling amplitude from grain to grain, C_{ij} - capacitance matrix

$$\hat{H}_t = \sum_{\mathbf{i},\mathbf{j},\alpha,\alpha'} t_{\mathbf{ij}} \hat{\psi}^+_{\alpha\mathbf{i}} \hat{\psi}_{\alpha'\mathbf{j}}$$

$$\hat{H}_c = \frac{e^2}{2} \sum_{\mathbf{ij}} \hat{N}_{\mathbf{i}} C^{-1}_{\mathbf{ij}} \hat{N}_{\mathbf{j}}$$

$$\hat{N}_{\mathbf{i}} = \int \hat{\psi}^+(\mathbf{r}_{\mathbf{i}})\, \hat{\psi}(\mathbf{r}_{\mathbf{i}})\, d\mathbf{r}_{\mathbf{i}} - \bar{N}$$

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Methods of calculation:

1. Bosonization
2. Perturbation theory in the limit $g \geq 1$ (strong coupling between the grains).

Bosonization

Study of Coulomb interaction via bosonization is well known in superconductors where $\Delta_i(\tau) = \exp(2\phi_i(\tau))$

$\phi(\tau)$ is the phase.

One can reduce the electron Hamiltonian to an effective phase Hamiltonian H_{eff} (Efetov (1980))

$$H_{eff} = \sum_{ij} [B_{ij}\rho_i\rho_j - J_{ij}\cos(2(\phi_i - \phi_j))]$$

Where $\rho_i = -i\partial / \partial\phi_i$ (eigenvalues are integers)

How can one "bosonize" a normal metal?

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Scheme of the bosonization:

1. Hubbard-Stratonovich transformation

$$\exp\left(-\frac{e^2}{2}\sum_{\mathbf{ij}} \hat{N}_{\mathbf{i}} C_{\mathbf{ij}}^{-1} \hat{N}_{\mathbf{j}}\right) = \int \exp\left(-i\sum_i \int \left(\psi^*(\mathbf{r_i},\tau)\,\psi(\mathbf{r_i},\tau)\,d\mathbf{r_i} - \bar{N}\right) V_{\mathbf{i}}(\tau)\,d\tau\right) \times \exp\left(-\frac{1}{2e^2}\sum_{\mathbf{ij}} \int d\tau V_{\mathbf{i}}(\tau) C_{\mathbf{ij}} V_{\mathbf{j}}(\tau)\right) DV$$

2. Gauge transformation

$$\psi(\mathbf{r_i},\tau) \rightarrow e^{-i\varphi_{\mathbf{i}}(\tau)}\psi_{\mathbf{i}}(\mathbf{r_i},\tau), \qquad \dot{\varphi}_{\mathbf{i}}(\tau) = V_{\mathbf{i}}(\tau)$$

However, one should satisfy the fermionic boundary condition:

$$\psi(\mathbf{r_i},\tau) = -\psi(\mathbf{r_i},\tau+\beta), \quad \beta = 1/T$$

This is possible in the limit $T \geq \delta$ ⟹ Integration over the phases $\tilde{\phi}_i(\tau)$

$$\tilde{\phi}_{\mathbf{i}}(\tau) = \phi_{\mathbf{i}}(\tau) + 2\pi T k_{\mathbf{i}}\tau,$$

k_i are integers (winding numbers)

where $-\infty < \phi_{\mathbf{i}}(\tau) < \infty$, $\phi_{\mathbf{i}}(0) = \phi_{\mathbf{i}}(\beta)$.

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Final action: $S = S_c + S_t,$

S_c is the charging energy

$$S_c = \frac{1}{2e^2} \sum_{ij} \int_0^\beta d\tau C_{ij} \frac{d\tilde{\phi}_i(\tau)}{d\tau} \frac{d\tilde{\phi}_j(\tau)}{d\tau}$$

S_t describes the tunneling between the grains

$$S_t = \pi g \sum_{|i-j|=a} \int_0^\beta d\tau d\tau' \alpha(\tau - \tau') \sin^2\left(\frac{\tilde{\phi}_{ij}(\tau) - \tilde{\phi}_{ij}(\tau')}{2}\right)$$

where $\alpha(\tau) = T^2 \left(\mathrm{Re}\left(\sin(\pi T \tau + i\delta)\right)^{-1}\right)^2$ $\quad g = 2\pi\nu_0^2 t_{ij}^2$

This form is analogous to the Ambegaokar, Eckern and Schon (1982) action written to describe quantum dissipation.

However, it is applicable only for $T \geq \max(\delta, g\delta)$

↓

No quantum dissipation and no dephasing at T=0!

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The phase functional can be studied without difficulties in the limits $g \geq 1$ and $g \leq 1$

$g \geq 1$ One can use renormalization group integrating over fast variations of ϕ

RG Equation: $\frac{\partial g(\xi)}{\partial \xi} = -\frac{1}{2\pi d}$ $\quad \sigma = e^2 g(T) a^{2-d}$

Result: $g(T) = g - \frac{1}{2\pi d} \ln \frac{g E_c}{T}$ $\quad$ Valid as long as $g \geq 1$

$g \leq 1$ Expansion in g. $\quad B \propto E_c$ -is the energy of the lowest excitation

Result: $\sigma = 2\sigma_0 \exp(-B/T)$

At $T \geq E_c$ one has $\sigma = \sigma_0$

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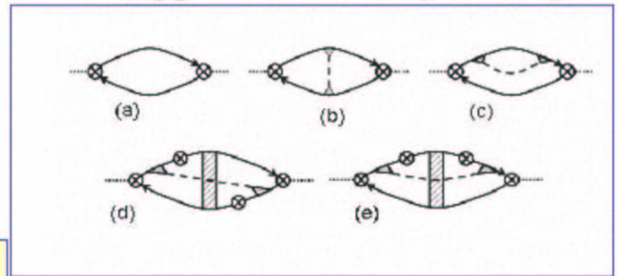
Perturbation theory in the limit $g \geq 1$ (applicable for any low T):

Relevant diagrams:

a) Classical conductivity

b-c) The "bosonic" contribution

d-e) Altshuler-Aronov contribution



Result:

$$\sigma = \sigma_0 + \delta\sigma_1 + \delta\sigma_2$$

$$\frac{\delta\sigma_1}{\sigma_0} = -\frac{1}{2\pi d g_T} \ln\left[\frac{g_T E_C}{\max(T, g_T\delta)}\right]$$

$$\frac{\delta\sigma_2}{\sigma_0} = \begin{cases} \frac{\alpha}{12\pi^2 g_T}\sqrt{\frac{T}{g_T\delta}} & D=3, \\ -\frac{1}{4\pi^2 g_T}\ln\frac{g_T\delta}{T} & D=2, \\ -\frac{\beta}{4\pi}\sqrt{\frac{\delta}{T g_T}} & D=1. \end{cases}$$

Altshuler-Aronov corrections, the same as in a homogeneous metal.

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Both $\delta\sigma_1$ and $\delta\sigma_2$ can be important (weak localization corrections are assumed to be killed by a magnetic field, which is of an experimental relevance).

Density of states

$$\frac{\delta\nu_2}{\nu_0} = -\frac{1}{16 g_T \pi^2}\begin{cases} 2\ln^2\frac{g_T E_C}{T} & T \gg g_T\delta, \\ \ln\frac{g_T\delta}{T}\ln\frac{g E_0^4}{T\delta^3} + 2\ln^2\frac{E_0}{\delta} & T \ll g_T\delta. \end{cases} \quad (1($$

2D

$$\frac{\delta\nu_3}{\nu_0} = -\frac{A}{2\pi g_T}\ln\left[\frac{E_C g_T}{\max(T, g_T\delta)}\right]$$

3D

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Are underdoped or optimally doped high- T_c cuprates granular?

VOLUME 75, NUMBER 25 PHYSICAL REVIEW LETTERS 18 DECEMBER 1995

Logarithmic Divergence of both In-Plane and Out-of-Plane Normal-State Resistivities of Superconducting $La_{2-x}Sr_xCuO_4$ in the Zero-Temperature Limit

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(Received 18 August 1995)

The low-temperature normal-state resistivities of underdoped $La_{2-x}Sr_xCuO_4$ crystals with T_c of 20 and 35 K were studied by suppressing the superconductivity with pulsed magnetic fields of 61 T. Both in-plane resistivity ρ_{ab} and out-of-plane resistivity ρ_c are found to diverge logarithmically as $T/T_c \to 0$. Logarithmic divergence is accompanied by a nearly constant anisotropy ratio, ρ_c/ρ_{ab}, suggesting an unusual three-dimensional insulator.

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Metal-to-Insulator Crossover in the Low-Temperature Normal State of $Bi_2Sr_{2-x}La_xCuO_{6+\delta}$

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(Received 18 February 2000)

We measure the normal-state in-plane resistivity of $Bi_2Sr_{2-x}La_xCuO_{6+\delta}$ single crystals at low temperatures by suppressing superconductivity with 60 T pulsed magnetic fields. With decreasing hole doping, we observe a crossover from a metallic to an insulating behavior in the low-temperature normal state. This crossover is estimated to occur near 1/8 doping, well inside the *underdoped* regime, and not at optimum doping as reported for other cuprates. The insulating regime is marked by a logarithmic temperature dependence of the resistivity over two decades of temperature, suggesting that a peculiar charge localization is common to the cuprates.

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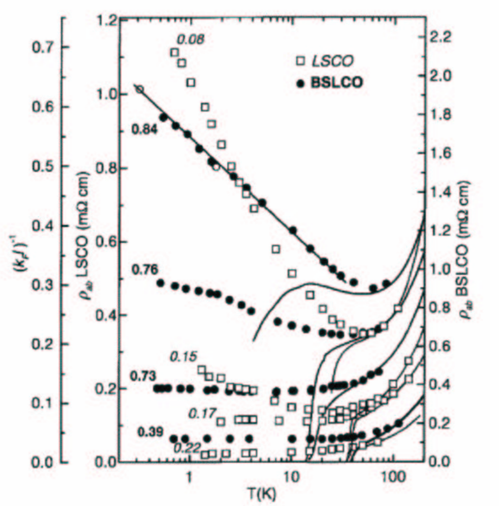

FIG. 3. Logarithmic plot of $\rho_{ab}(T)$ for BSLCO crystals in 0 T (solid lines) and in 60 T (filled circles), labeled by La concentration, x. The straight line emphasizes the $\log(1/T)$ behavior in the $x = 0.84$ sample and open circles are 30 T data from the long-pulse magnet. LSCO data in 0 T (dashed lines) and in 60 T (open squares), labeled by Sr concentration, y, are from Ref. [3]. All data are vertically scaled to directly compare the resistivity per CuO_2 layer in units of $(k_F l)^{-1}$.

$$\sigma = b + \frac{e^2}{\pi d\hbar}\ln T$$

Results of the fitting

For $La_{2-y}Sr_yCuO_4$:

d=2.3 at y=0.08 and
d=1.08 at y=0.15

For $Bi_2Sr_{2-x}La_xCuO_{6+\delta}$:

d=4.5 at x=0.84 and
d=3.57 at x=0.76

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Conclusions:

1. Bosonization and diagrammatic schemes are developed to study granular metals.

2. A crossover from the logarithmic to the exponential dependence of the conductivity on temperatures is found at not very low temperatures.

3. At low temperatures the conductivity by Altshuler-Aronov formulae but logarithmic corrections can be important in any dimensionality.

4. Logarithmic dependence of the conductivity observed in many granular materials is explained.

5. High T_c superconducting cuprates may be granular, too.

Not clear yet:

The $\exp(-a/\sqrt{T})$ law is not explained (activation law instead).

What is the origin of the granularity in the superconducting cuprates? Is the granularity related to high values of the critical temperature?