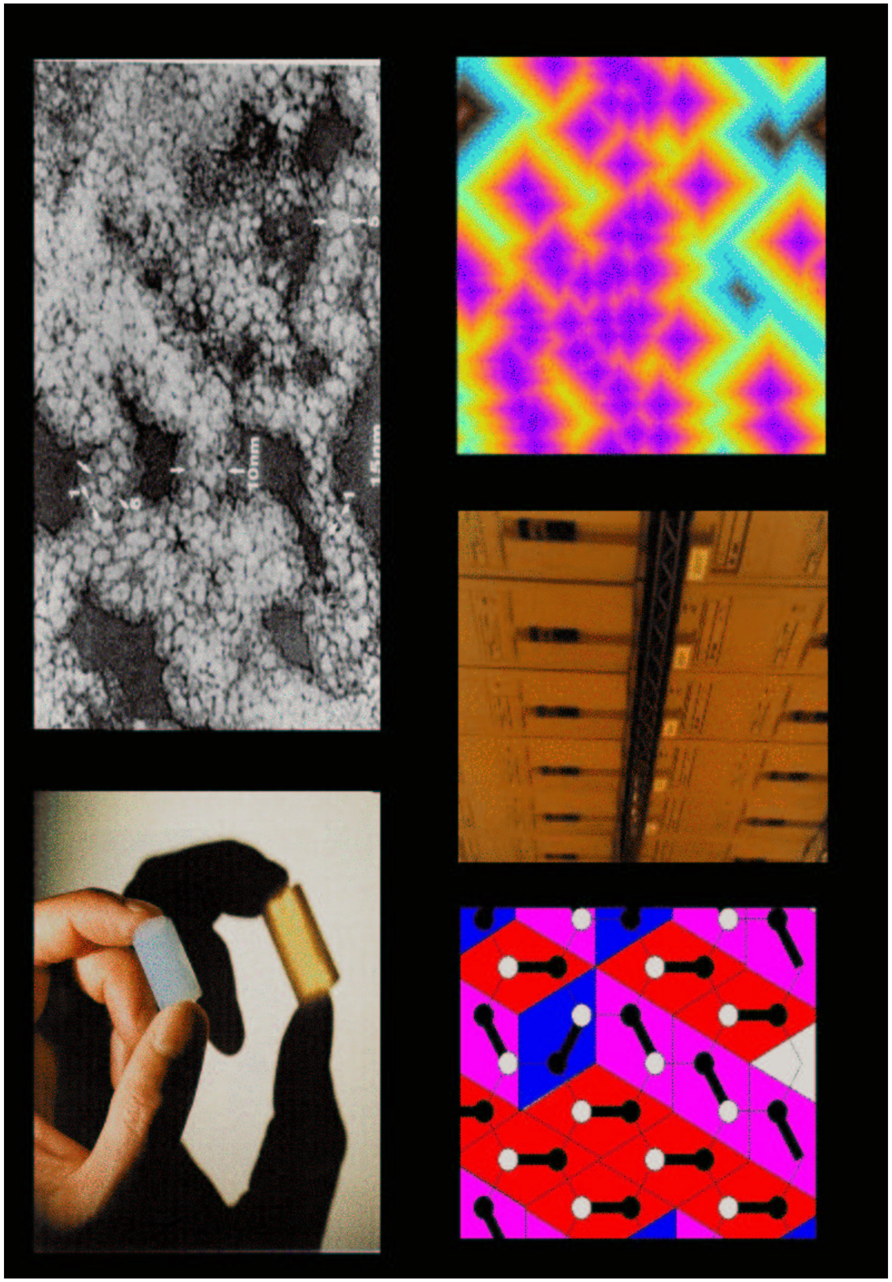
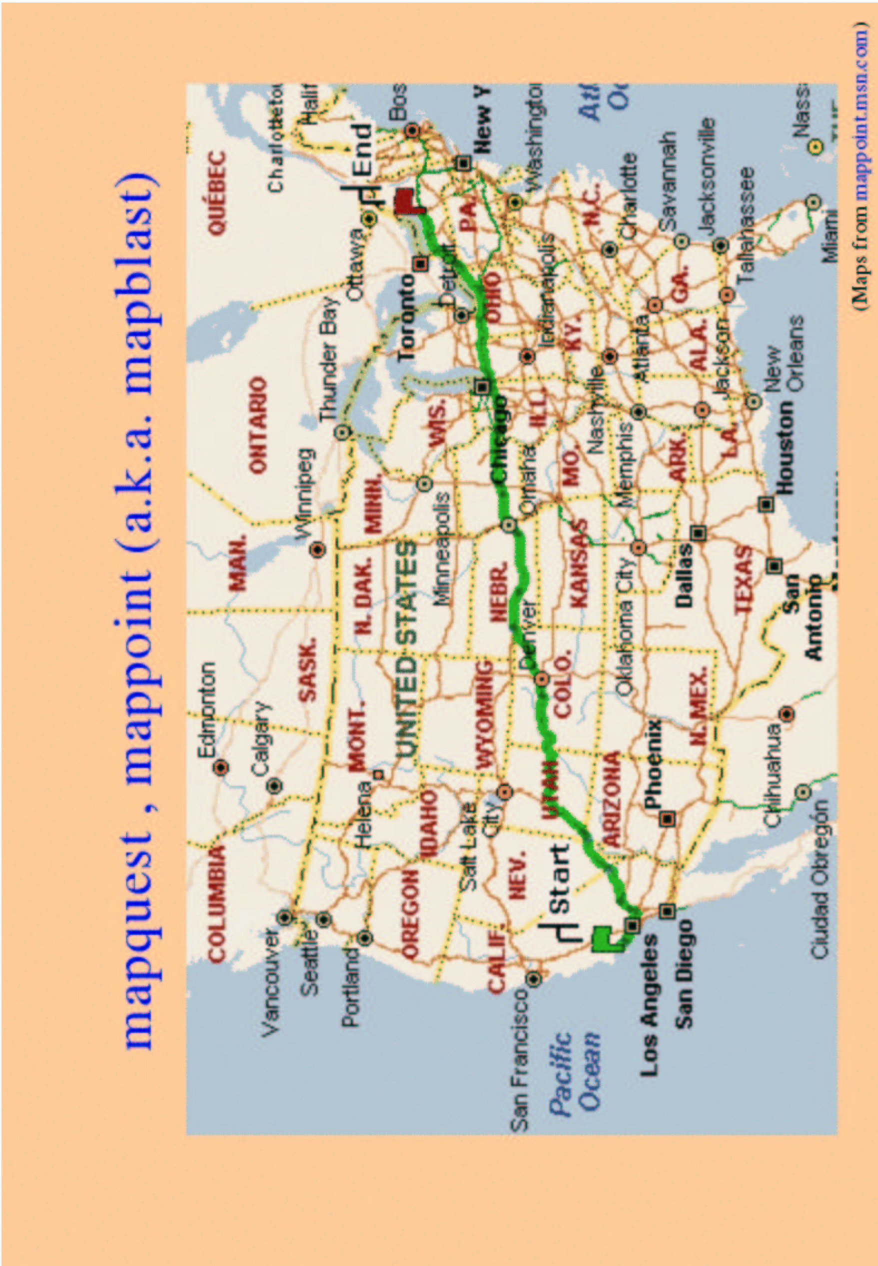


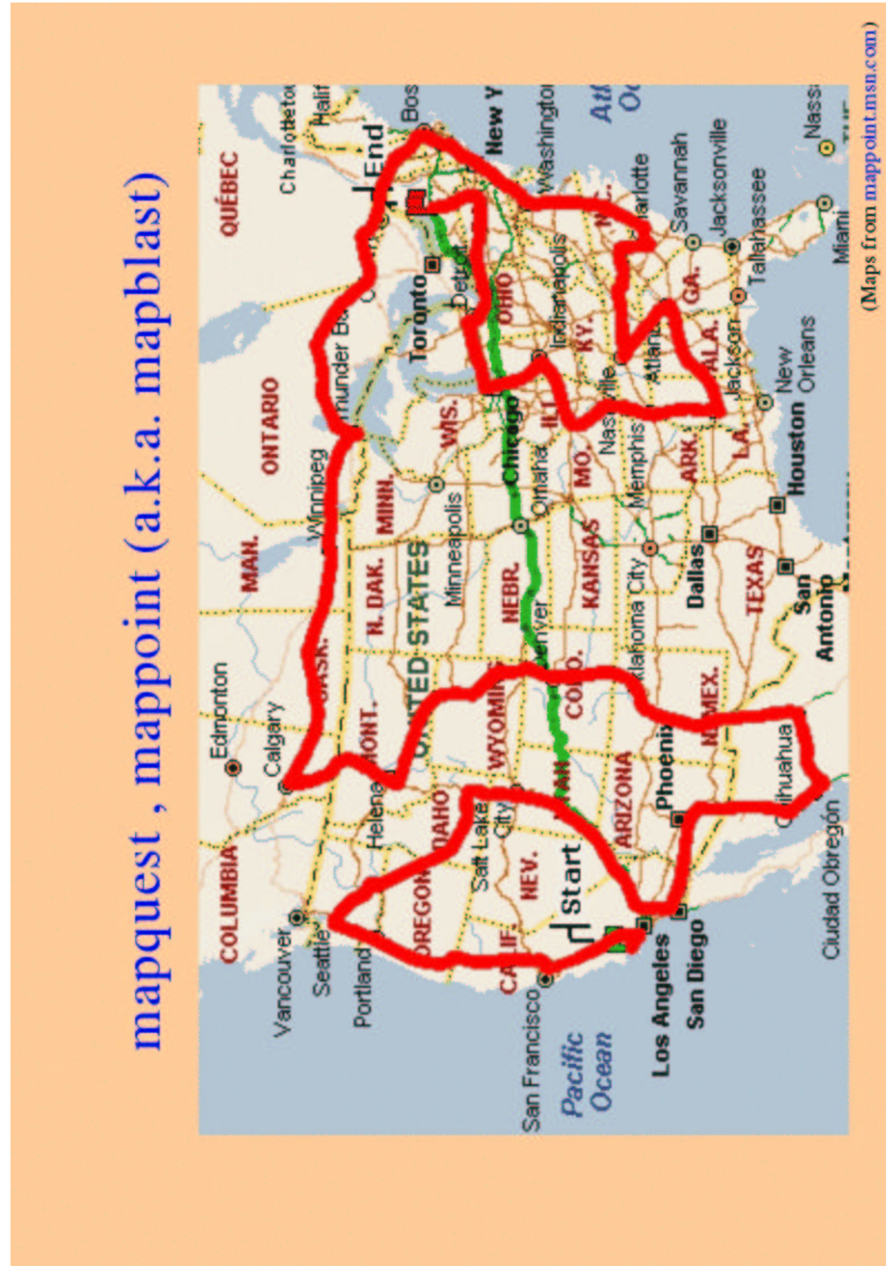
How laziness Can Lead to a Big Ego: Algorithms and Quenched Disorder

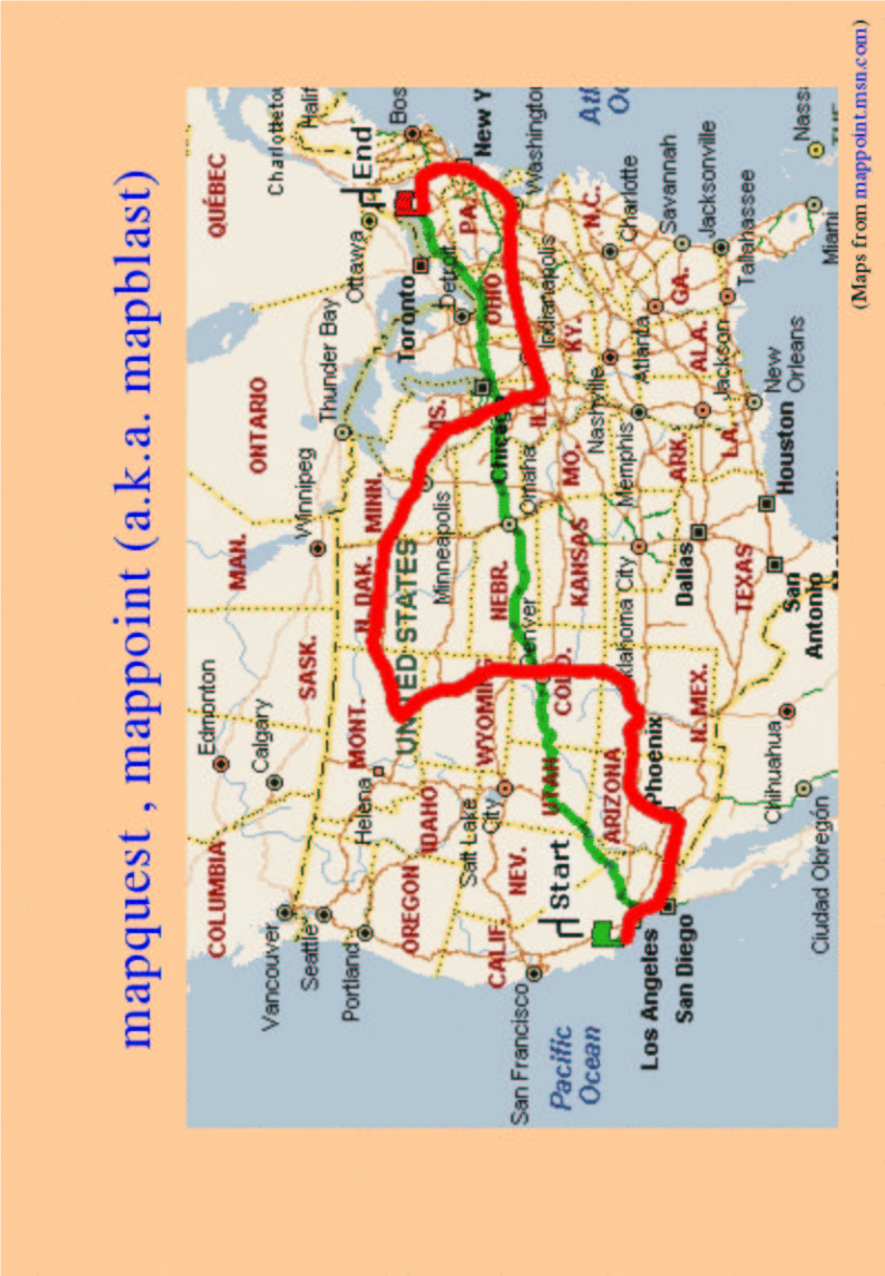
Alan Middleton

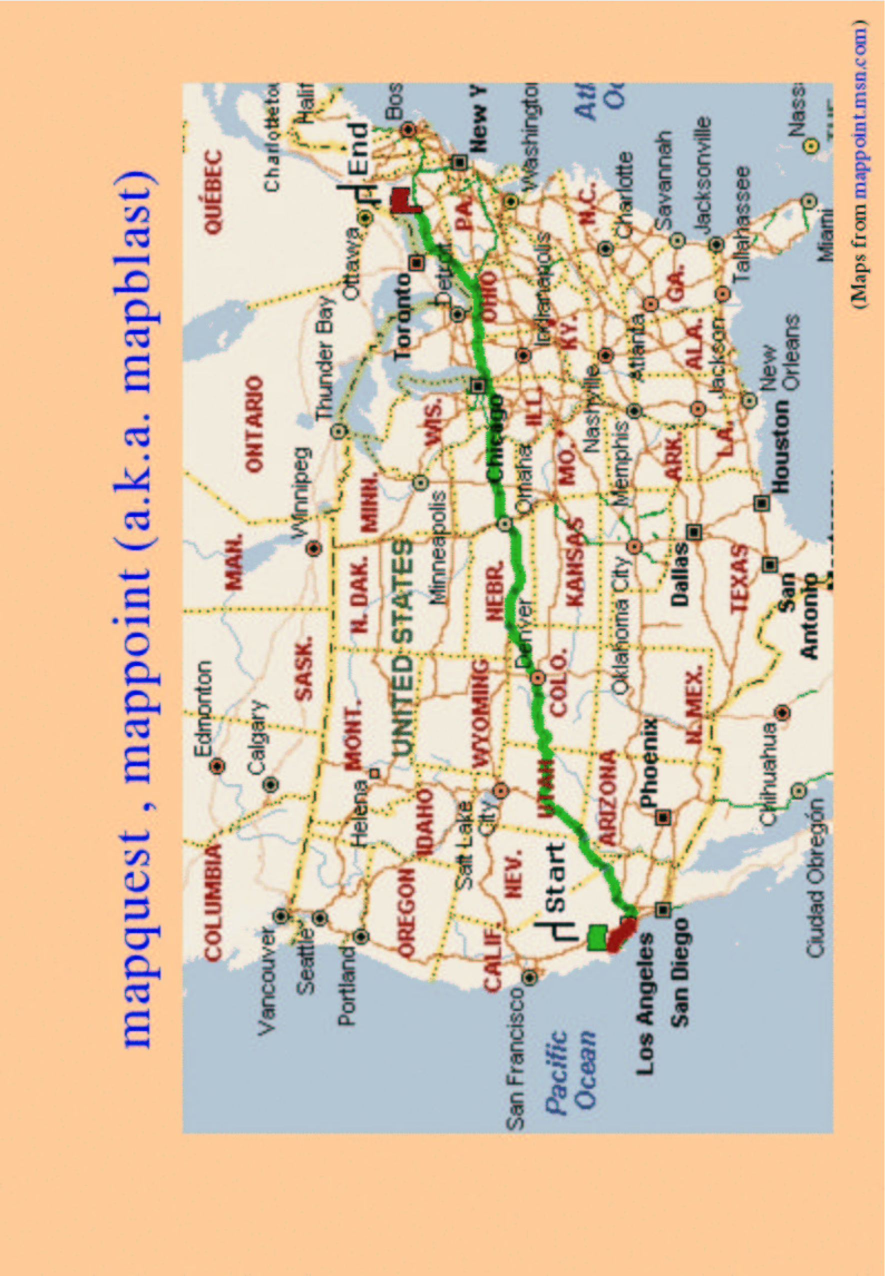
May 1, 2003

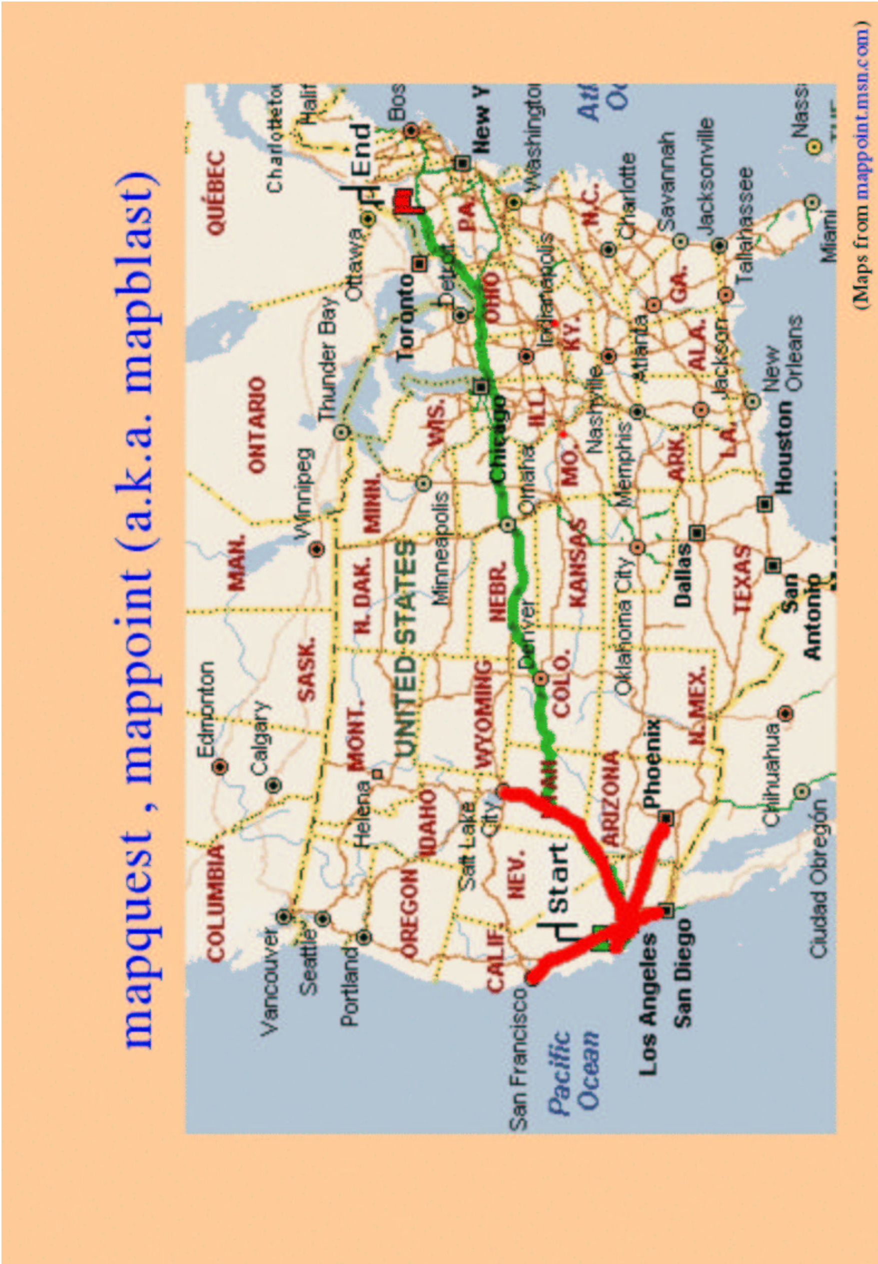
[Much of this work done in collaboration with Chen Zeng, David McNamara, Daniel Fisher, Jennifer Schwarz, Jan Meinke.]

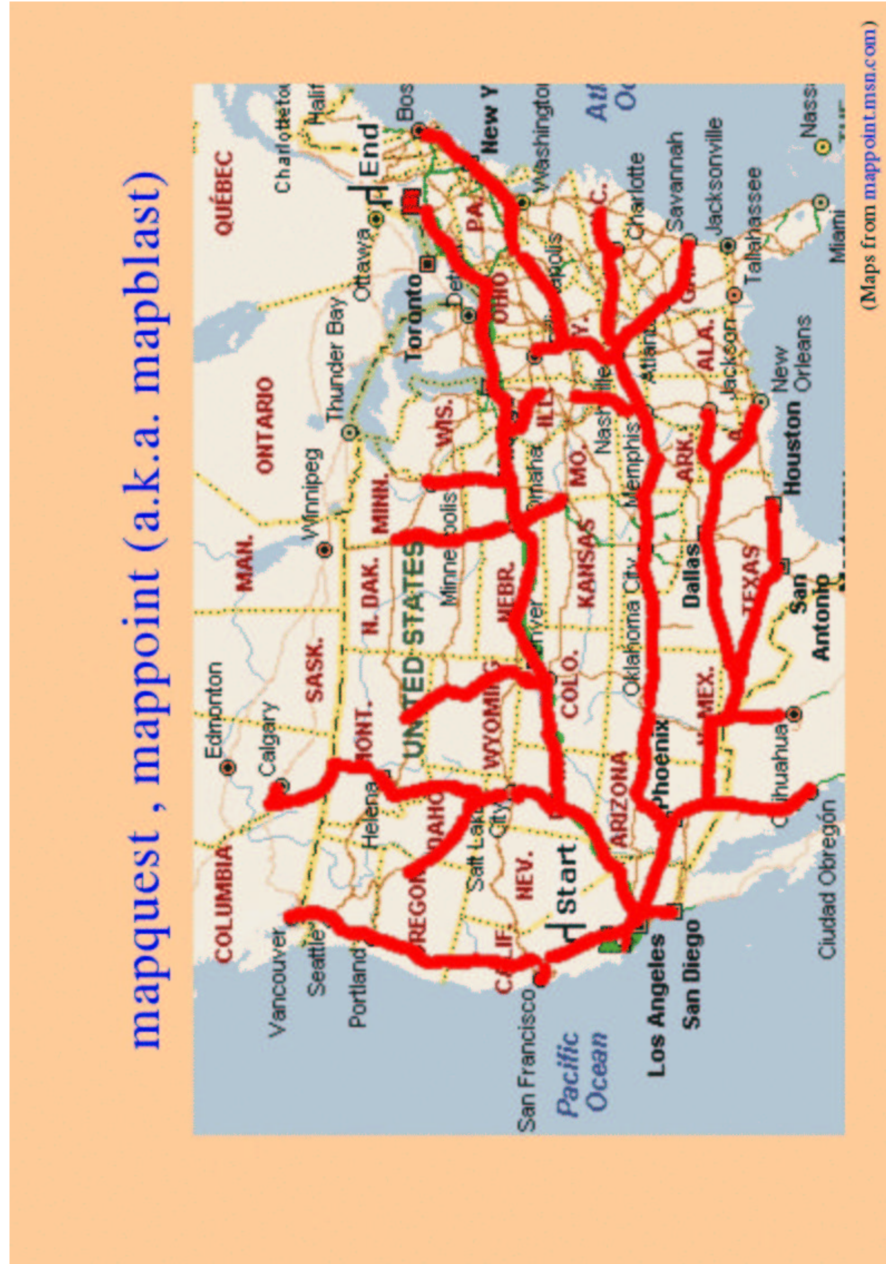

mapquest , mappoint (a.k.a. mapblast)
Start
End
(Maps from mappoint.msn.com)


mapquest , mappoint (a.k.a. mapblast)
Start
End
(Maps from mappoint.msn.com)


mapquest , mappoint (a.k.a. mapblast)
Start
End
(Maps from mappoint.msn.com)


mapquest , mappoint (a.k.a. mapblast)
Start
End
(Maps from mappoint.msn.com)


mapquest , mappoint (a.k.a. mapblast)
Start
End
(Maps from mappoint.msn.com)


mapquest , mappoint (a.k.a. mapblast)
Start
End
(Maps from mappoint.msn.com)

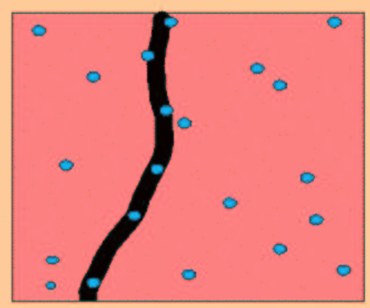
Optimizing Routes
Why should a physicist care?

- Shortest path is the <u>lowest energy state of an extended object in a disordered background.</u> E.g., a vortex line in type-II superconductors.
- (*Dual* problem of breaking a rock: surface of least fracture cost.)

= SC / solid

= defect / pore

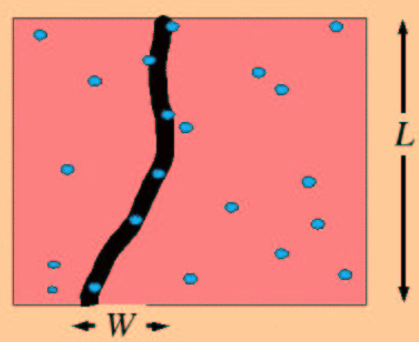
= flux line (fracture)



Statistics of shortest paths

- In 2D, $\zeta = 2/3$, where

$$\langle W \rangle \propto L^{\zeta}$$



- $\zeta = 2/3$ is an exact result.
  - Inspired by simulation.
  - Information about optimal paths (power law) that *wasn't* obvious to computer scientists and mathematicians.

Algorithms for Disordered Materials

- *Find mappings to graph problems.*
- Then, when possible, apply *fast, exact* ground state algorithms.
  - Avoid minima by using nonphysical approximate solutions.
  - Improve solutions by *finding paths*.

What is *tractable*?

Dimensionality, max/min, kinetics/statics, Z (counting)

P

Domain walls, random bond magnet

Partition fn. Z, 2D EA spin glass

Partition fn. Z, 2D elastic (per.)

Percolation

Shortest paths

Any d : many non-intersecting lines in random potential

NP-hard

Ground state, two 2D layers Ising spin glass

Highest energy state or partition fn. Z, random bond magnet

Coulomb glass ground state

Barrier to motion of loop in plane.

In classification, polynomial could be of large degree, but worst case bounds rarely exceed N^3 and practically as fast as $\sim N^{1.2}$.

Can do better, typically? Good heuristics? Able to approximate well? Physical importance?

P: what can we learn?

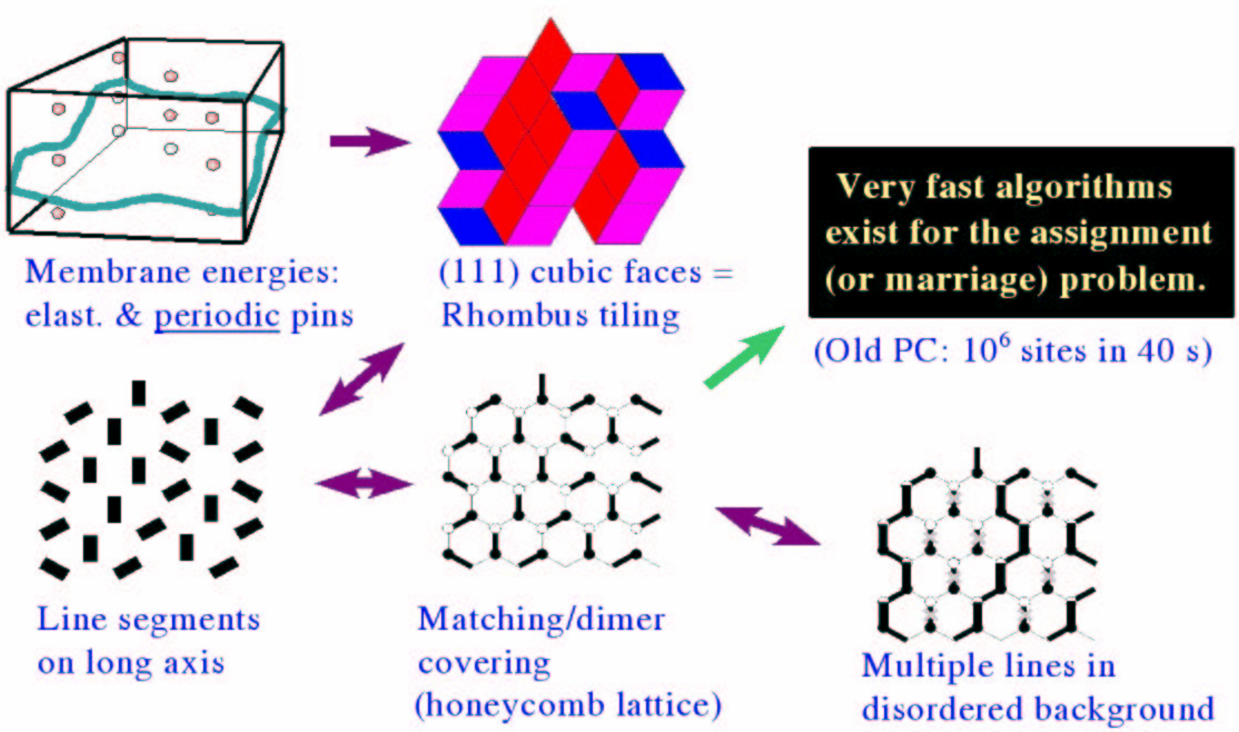
- *Check quantitative predictions,* e.g., Le Doussal, Wiese, Chauve (cond-mat/0304164) $d = 4 - \epsilon$

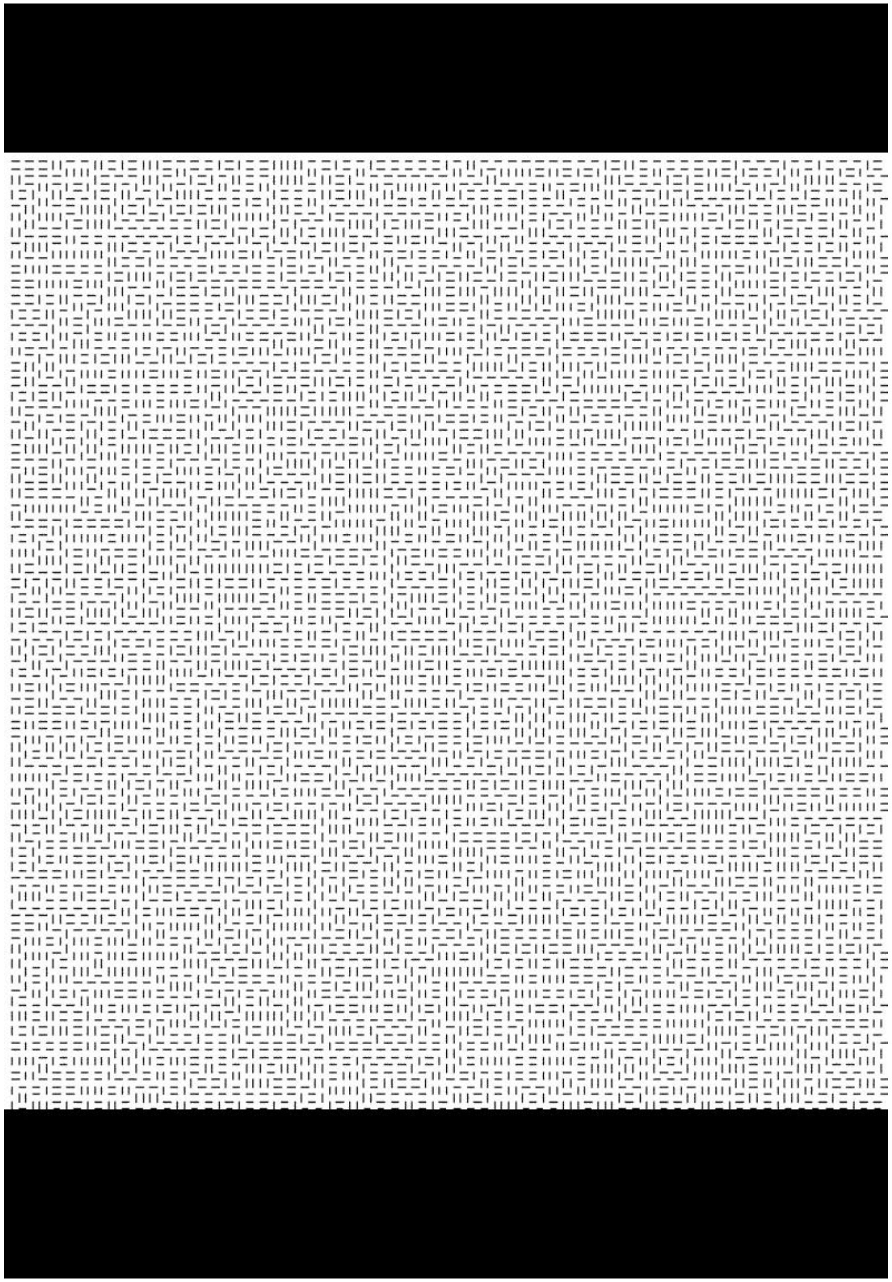
$\zeta = 0.20829804\,\epsilon + 0.006858\,\epsilon^2$ [Non-per. pins]

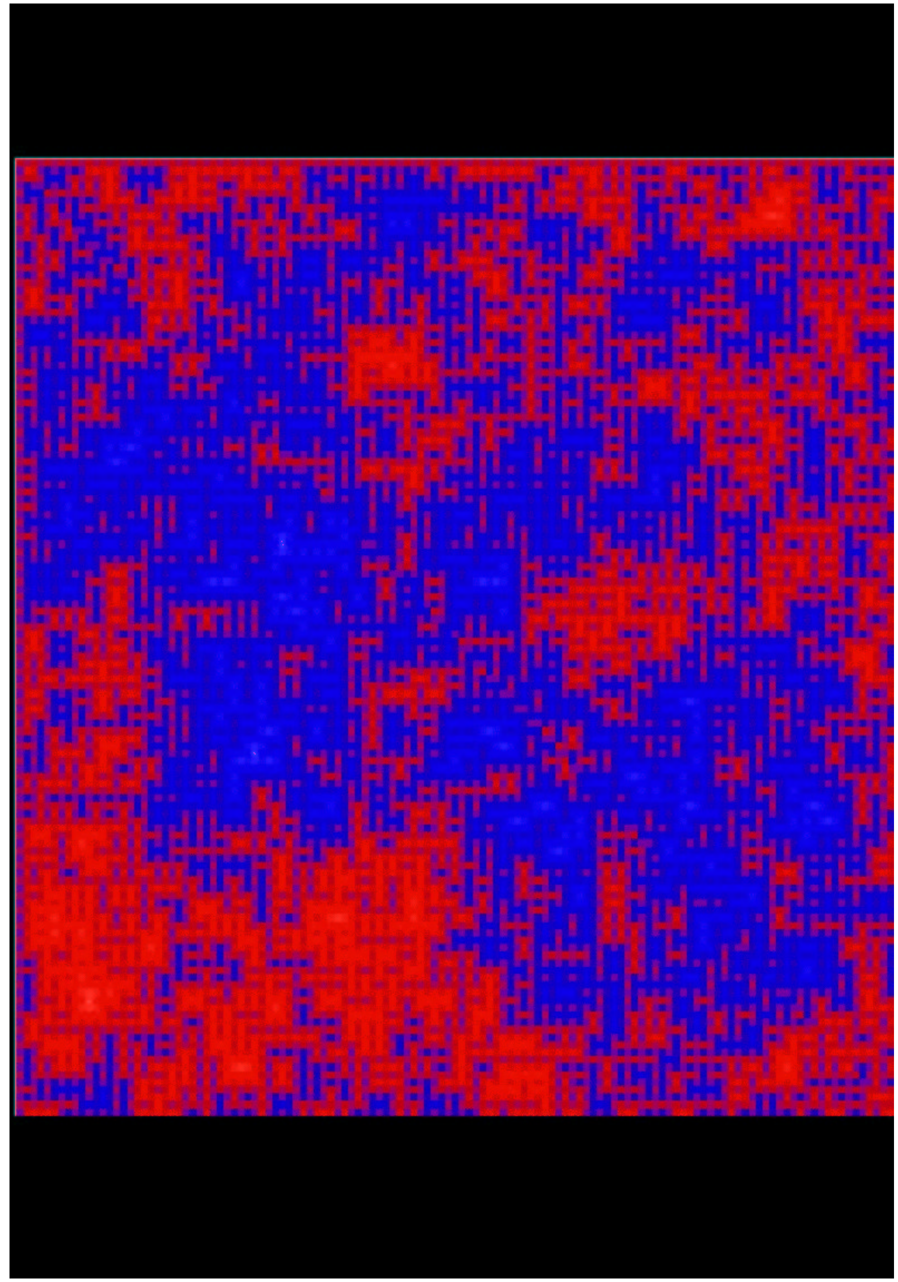
$\left\langle (u(x) - u(0))^2 \right\rangle = \left[\frac{\epsilon}{18} + \frac{\epsilon^2}{108} \right] \ln(|x|)$ [Per. Pins, $2 < d < 4$]

- *More "qualitative" concepts*
  - Number of thermodynamic states
- *Performance* of the algorithm.
  - Length scales and phase transitions.

Map interfaces to matching







Does laziness lead to a big ego?

- How many *states* are there, at T=0, in the thermodynamic limit?
  - A *ground state* in the thermodynamic limit is stable to "flips" of finite clusters.
  - Are there ground states in the thermodynamic limit that are not related by global symmetries?
- One might actually take the limit of large *L*, watching the correlation functions in a finite volume or, equiv., look at changes in interior as change boundary conditions in a large sample. [C. Newman and D. Stein].

Sample expansion

Start with L^d sample.

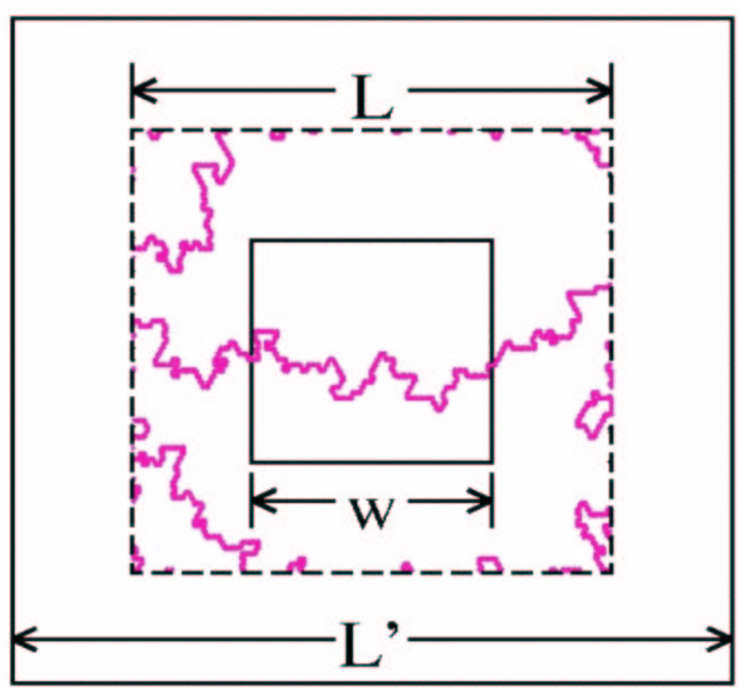
Find ground state config.

Expand by adding spins to volume $(L')^d$.

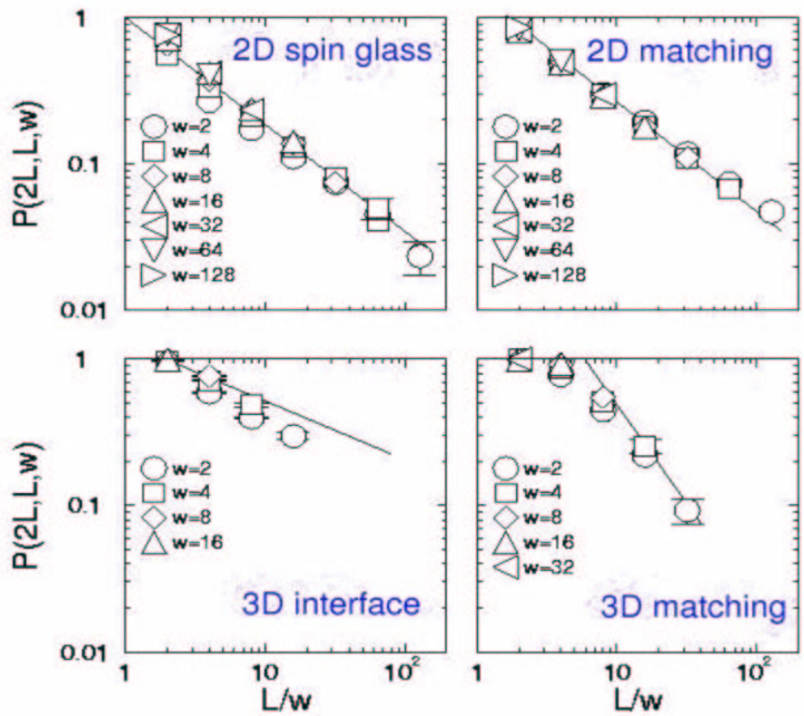
Find ground state config.

Compare bonds (for SG) in common area of size w^d

What is probability $P(L,L',w)$ of <u>any</u> change?



Data from sample expansion



Lines have slope $(d_f - d)$:
The probability of a single domain wall intersecting a cell of size w^d in a sample of size L^d scales with this power.

(d_f determined by separate measurements. Note 2D SG)

(PRL 83, 1672)

3D spin glasses

- Palassini, Liers, Junger, Young (cond-mat/0212551):

  "The ground states are determined *exactly* for systems up to size 12^3 spins ... The data are consistent with a picture where the surface of the excitations is not space-filling, such as the droplet or 'TNT' picture. When allowing for large finite size corrections, the data are also consistent with a picture with space-filling surface, such as replica symmetry breaking."

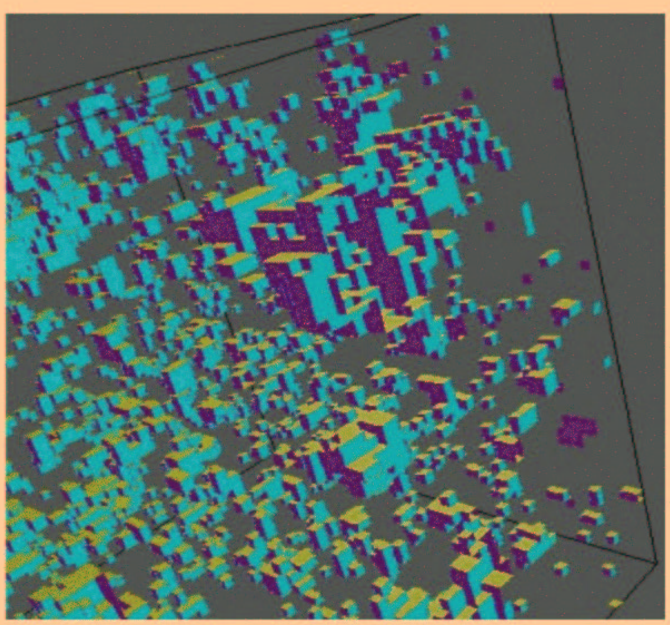
Random Field Ising Magnet (RFIM)

- Ferromagnetically coupled magnetic spins s_i (up/down) subject to random fields h_i.

$$H = -J\sum_{\langle ij\rangle} s_i s_j + \sum_i h_i s_i$$

- As vary $\Delta = \langle h^2\rangle / J$, get a phase transition between FM and PM phases.

3D RFIM (w / D. Fisher)

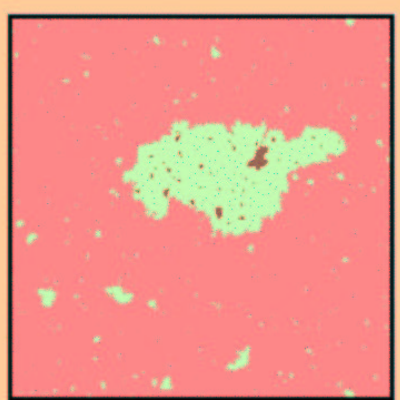


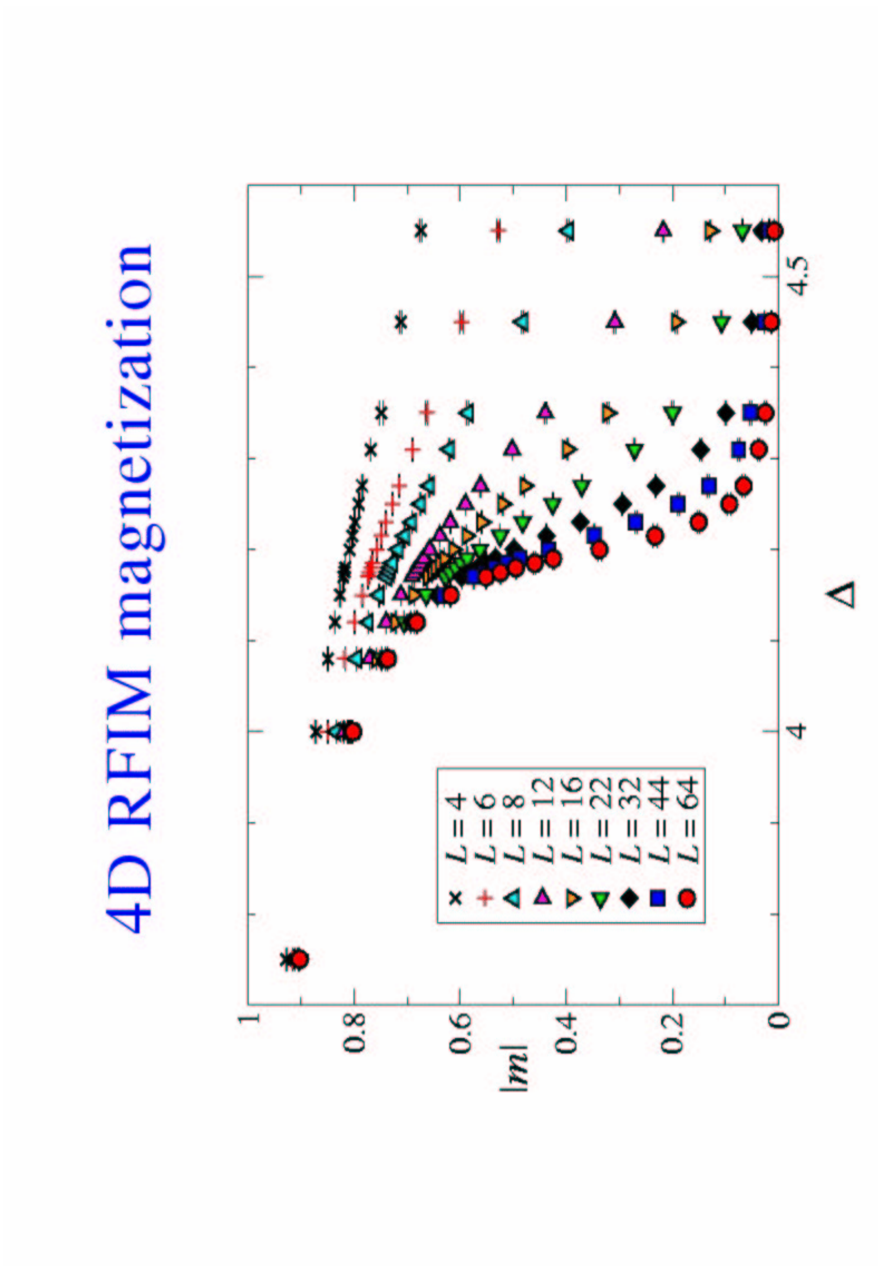
[Part of a ground state for a 64^3 sample, near Δ_c]

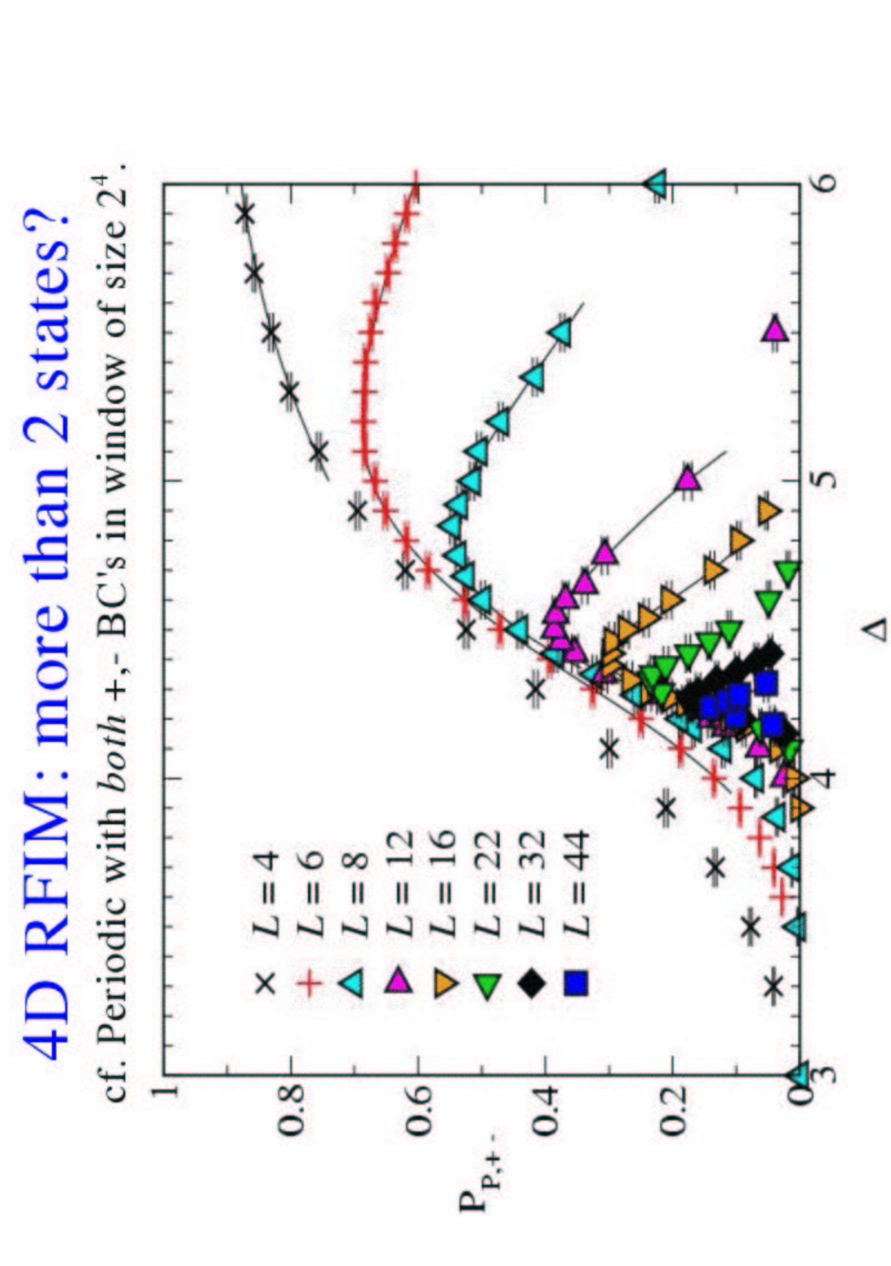
Up to 256^3 ~1 hour.

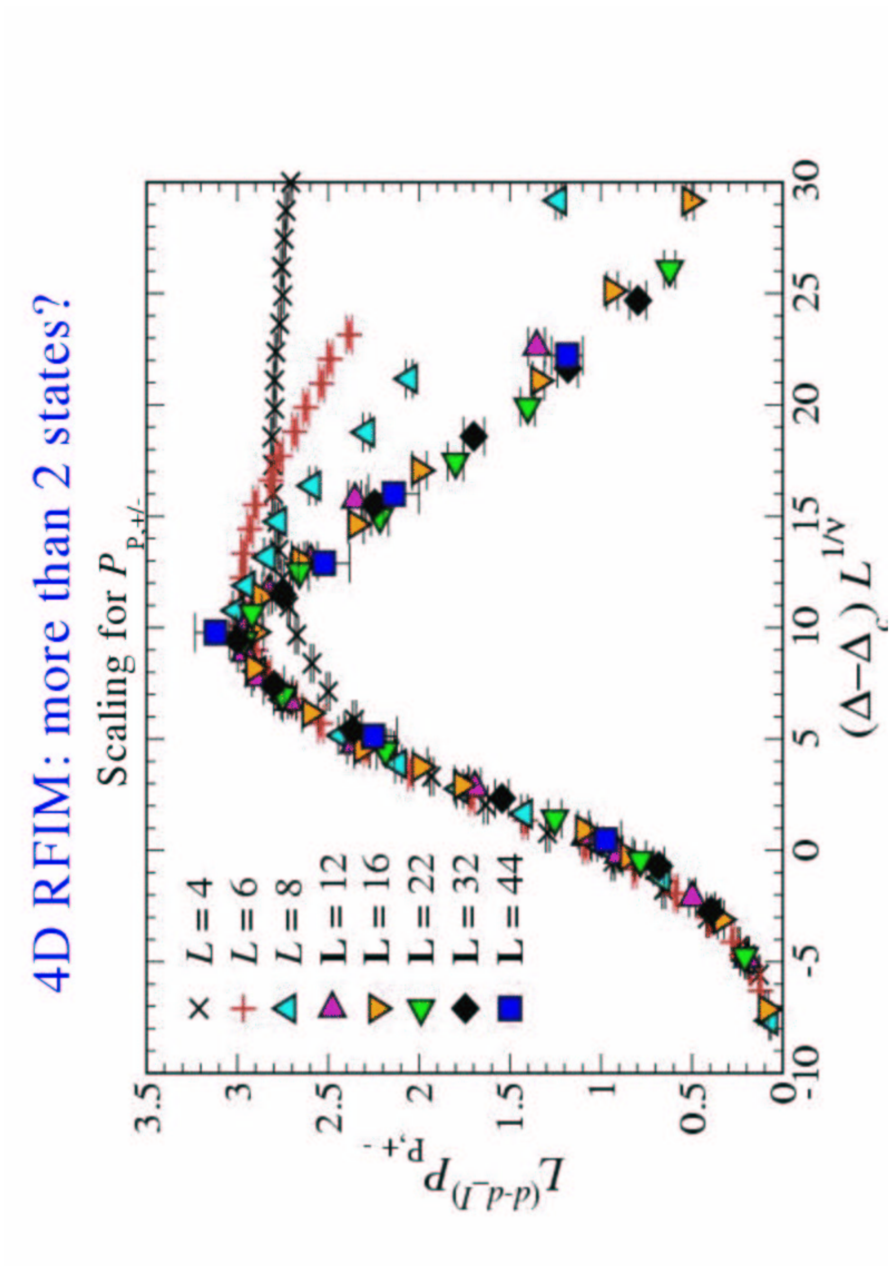
Find β, θ, α, "d_f".
(continuous transition.)

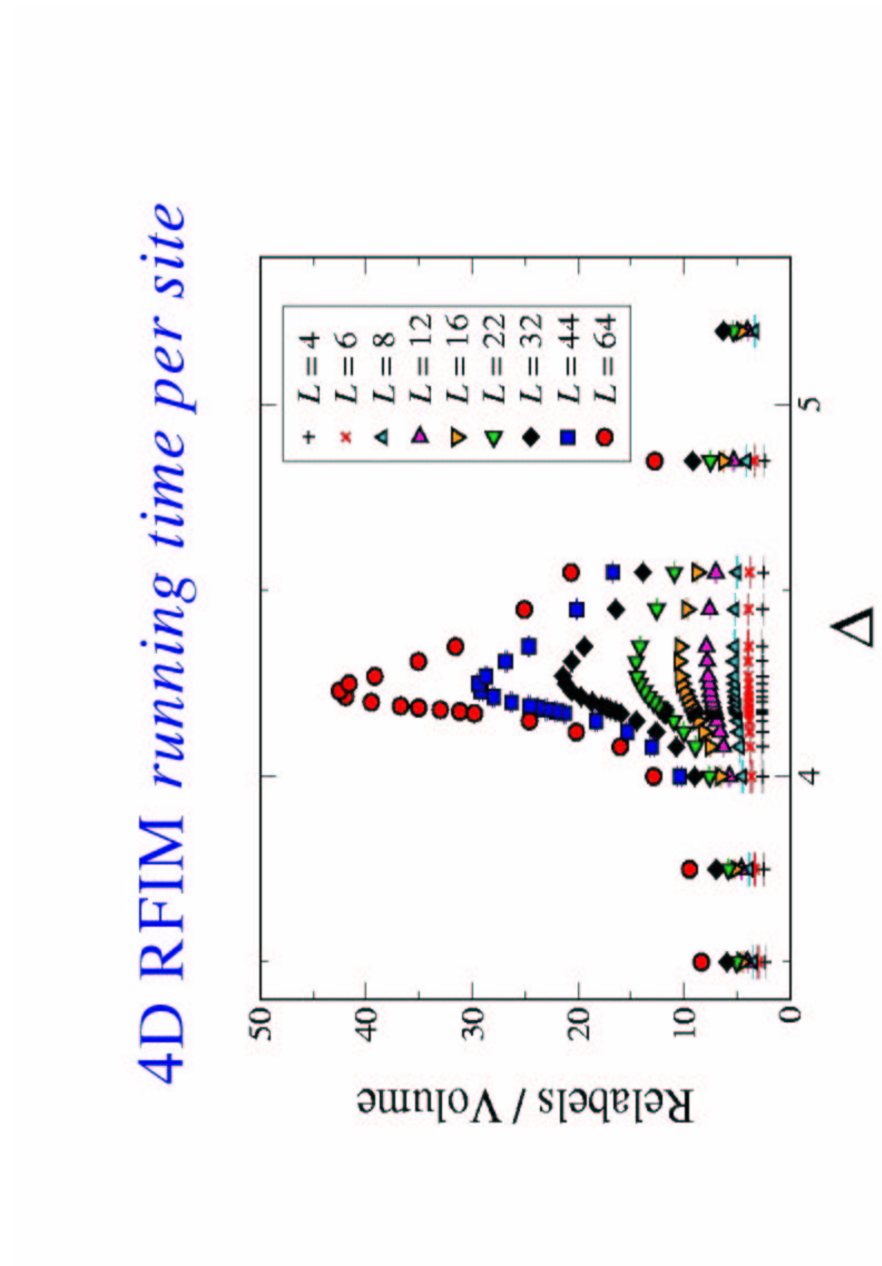
Nested domain walls.




4D RFIM magnetization
|m|
Δ
L = 4
L = 6
L = 8
L = 12
L = 16
L = 22
L = 32
L = 44
L = 64


4D RFIM: more than 2 states?
cf. Periodic with both +,- BC's in window of size 2^4.
P_{P,+-}
Δ
L = 4
L = 6
L = 8
L = 12
L = 16
L = 22
L = 32
L = 44

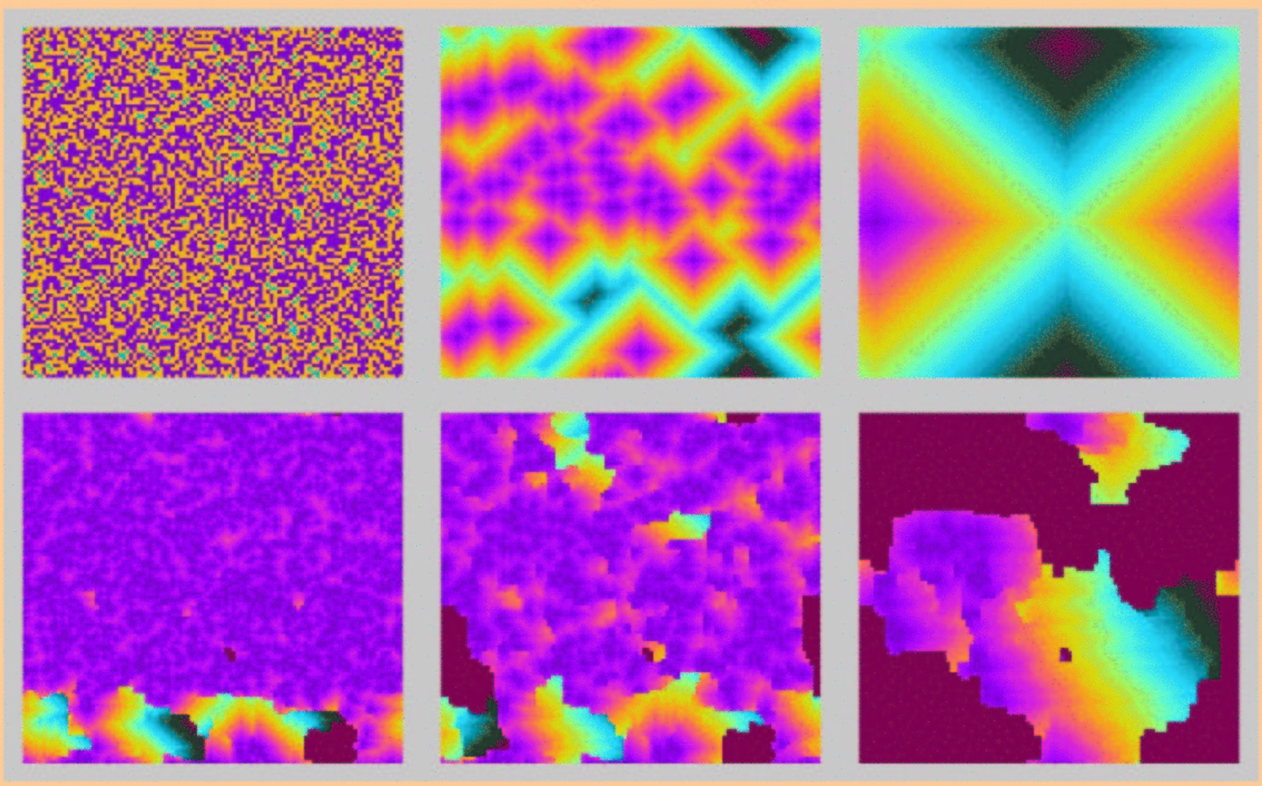

4D RFIM: more than 2 states?
Scaling for $P_{P,+/-}$
L = 4
L = 6
L = 8
L = 12
L = 16
L = 22
L = 32
L = 44
$L^{(d-d_I)}P_{P,+-}$
$(\Delta-\Delta_c)\,L^{1/\nu}$


4D RFIM running time per site
L = 4
L = 6
L = 8
L = 12
L = 16
L = 22
L = 32
L = 44
L = 64
Relabels / Volume
Δ

Running times – RFIM

- Can understand by studying how algorithm grows domains (much faster than physically.)
- Solve RFIM using max-flow approach
  - Based on max-flow / min-cut approach
  - "Push-relabel" (Goldberg & Tarjan)

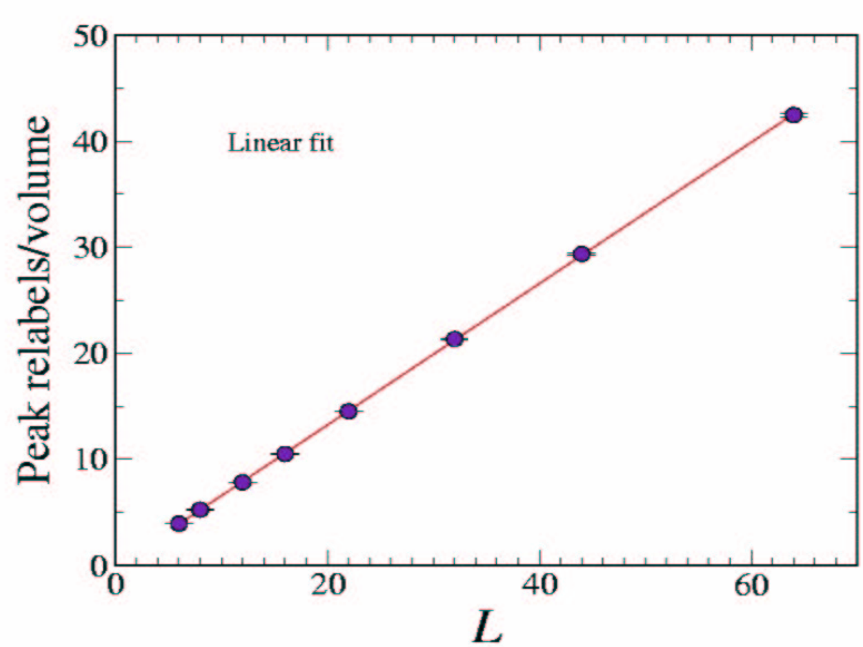
Coarsening in algorithms



Running times

- In the best case, at the worst location (i.e., when $\Delta \approx \Delta_c, \xi \approx L$), *height field* in a correlation volume has constant slope.
- Average number of relabels is then $r \propto L$.

4D RFIM *running time per site*



P, NP & Physics

"The three problems discussed so far in this chapter (REACHABILITY, MAX FLOW, and MATCHING) represent some of the happiest moments in the theory of algorithms."

– C. H. Papadimitriou, *Computational Complexity*

Algorithms & Physics

- What is P or NP-hard can be subtle.
- Find exponents, etc., to "high" precision.
- Study qualitative issues, also subtle.
- Performance of algorithms depends on phase in explicable fashion (?).
- Other distinctions from CS, e.g., (Machta, et al): can growth processes be parallelized?
- Open issues ....