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Can you beat the Second law of thermo-dynamics if you're too dumb too know which way is up? A theory of Live Nematics

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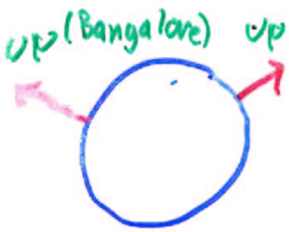


1a

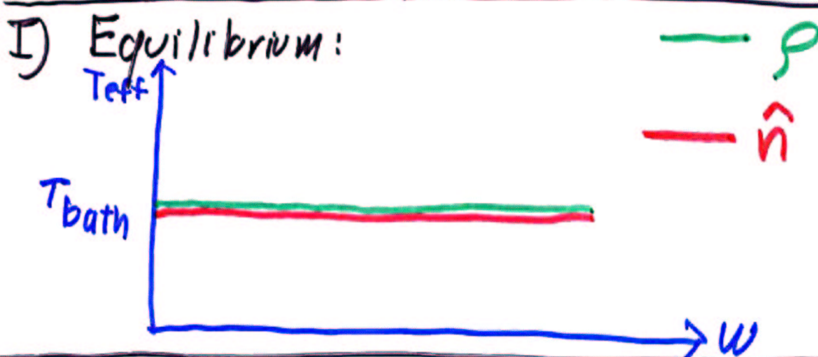
Connection to Glassy dynamics:

"Effective temperature" $\boxed{T_{eff} \equiv \frac{\text{correlation}}{\text{response}}}$

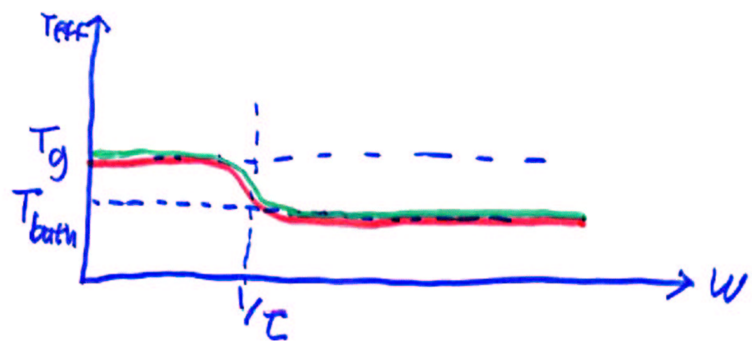
(Thanks to Letitia Cugliandolo + Jorge Kurchan)

3 types of systems:

I) Equilibrium:

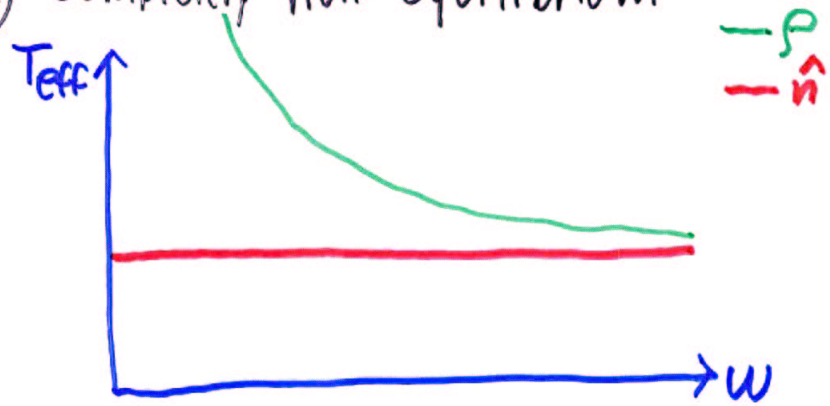


II) "Pseudo-equilibrium"



11b

III) Completely non-equilibrium



I'll describe going directly
I → III (equilibrium → completely non-equilibrium)
by adding one **linear** term
to
completely linear theory
⇓
Understanding limits of I + II?

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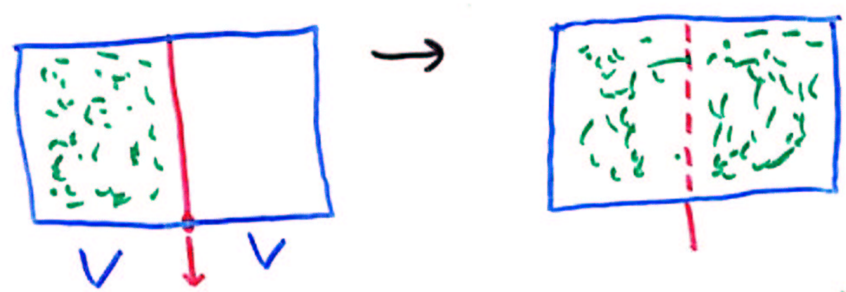
What does Live mean?
⇓
Out of Equilibrium
⇓
Low Entropy steady state

Number fluctuations: (Equilibrium)

N particles (Ideal gas)



$$N_L = N_R \pm O(\sqrt{N}) = \frac{N}{2} \pm O(\sqrt{N}) \quad \ll N$$

$$= \langle N_{L,R} \rangle + \delta N$$

$$\delta N \sim \sqrt{N} \ll N$$

2a

(Just standard stat mech result
(Grand Canonical Ensemble)

$$\sqrt{\langle \delta N^2 \rangle} = \sqrt{k_B T \kappa V} = \sqrt{\frac{k_B T \kappa \langle N \rangle}{\langle \rho \rangle}}$$

$$\left(V = \frac{\langle N \rangle}{\langle \rho \rangle} \right)$$

κ = Isothermal compressibility

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$\boxed{\delta N \propto N^{1/2}}$ holds for all equilibrium systems
(Ideal gases, non-ideal gases, liquids, crystals
dead nematics) except phase transitions ($T = T_c$ only)
and most non-equilibrium systems (e.g., granular materials)

But Not:
Live Nematics

For them, Huge Number fluctuations

$$\boxed{\delta N \propto N^{\frac{1}{2} + \frac{1}{d}} \underset{d=2}{\propto} N^{1}}$$

d = spatial dimension

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Outline

I) What's a dead nematic?
- Phase of (dead) matter (i.e., equilibrium
- Symmetry
- Order (= Orientational)
- Fluctuations: Orientational: Big / Number: Ordinary ($\sqrt{N}$)

II) What's a Live nematic?
- Symmetry and Order: Same as dead
- Orientational fluctuations: Also Big (same as dead)

But: Live ⇒ Big Orientational fluctuations ⇓ Big Number fluctuations

(2a)

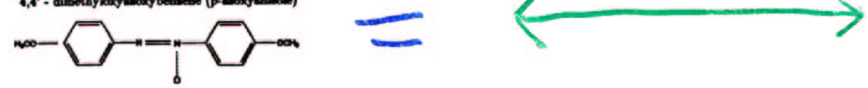
I) What's a dead nematic?

Nematic = Orientationally Ordered + translationally disordered Phase

Constituents: rod-shaped molecules; 2 types:

1) Head-tail symmetric

4,4' - dimethyloxyazoxybenzene (p-azoxyanisole)



or

2) Head-tail asymmetric

4-n-pentylbenzenethio-4'-n-heptyloxybenzoate (7S5)

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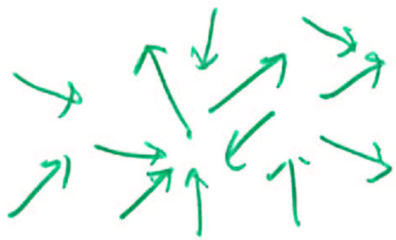
Both types have (among others)
These 2 phases:

Isotropic ⇓ random orientations

Liquid ⇓ random positions



Lower temperature T
Increase pressure P
go to new phase:

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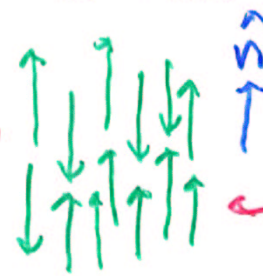
Nematic

Long-ranged orientational order



$\hat{n}$ ← "nematic director"

random positions ⇓ Liquid

But still inversion symmetric
(As many ↑ as ↓)

⇒ No special direction picked out

But special axis picked out
Director $\hat{n}(\vec{r})$ unit vector along that axis

$\hat{n} \to -\hat{n}$ symmetry

But no full rotation symmetry

Defining properties of nematic phase

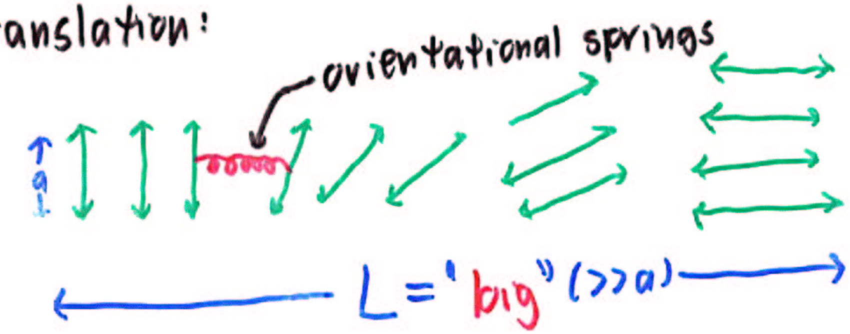
23 (8

Orientational Fluctuations = Fluctuations of $\hat{n}$

Huge!

Why? $\hat{n}$ = "Goldstone mode" of "spontaneously broken continuous rotational symmetry"

Translation:



E small $\left(\propto \frac{1}{L^2}\right)$

$$\frac{E}{\text{Volume}} = \frac{1}{2} K |\vec{\nabla} \hat{n}|^2 \sim \frac{K |\delta \vec{n}|^2}{L^2}$$

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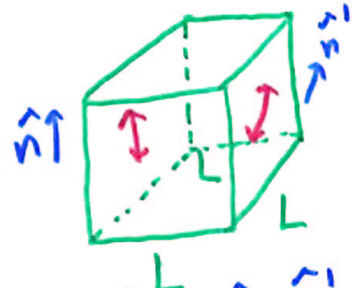


$$\delta \vec{n} \equiv \hat{n} - \hat{n}'$$

$$E = \frac{1}{2} \frac{K |\delta \vec{n}|^2}{L^2} (\text{volume})$$
$$= \frac{1}{2} K |\delta \vec{n}|^2 L^{d-2}$$

Equipartition: $\langle E \rangle_{\text{thermal}} \sim k_B T$

$$\Rightarrow \langle |\delta \vec{n}|^2 \rangle \sim \frac{k_B T}{K L^{d-2}} \propto L^{2-d}$$

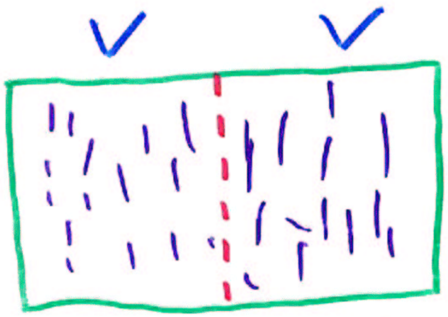
$$\Rightarrow \delta n_{rms} \equiv \sqrt{\langle |\delta \vec{n}|^2 \rangle} \propto L^{1-\frac{d}{2}}$$

Huge!

Compared with what?

Density fluctuations

(25 (10



$N_L = \langle N \rangle + \delta N$, $\delta N \propto \sqrt{N} \propto \sqrt{\langle \rho \rangle V}$

fixed average density

$$\Rightarrow \boxed{\delta \rho_L \propto \frac{\delta N}{V} \propto \frac{1}{\sqrt{V}} \propto L^{-\frac{d}{2}}}$$

vs director fluctuations

$$\boxed{\delta n \propto L^{1-\frac{d}{2}}}$$

δn Bigger by factor of L ($\gg a$)

$\Downarrow$

Huge!

Can big $\hat{n}$ flucs $\Rightarrow$ big ρ flucs? (25 (11

Equilibrium (dead): No!

Why not?

Catch 22 of Equilibrium Stat Mech

Small Energy $E \Rightarrow$ small force $\vec{f}$

Recall: Single Particle conserved dynamics

$$f = -\frac{dU}{dx}$$

Analog for nematics

$$\vec{f}(\vec{r}) = \frac{\delta E}{\delta \vec{n}} \propto \frac{\delta \vec{n}}{L^2} \quad \text{small as } L \to \infty$$

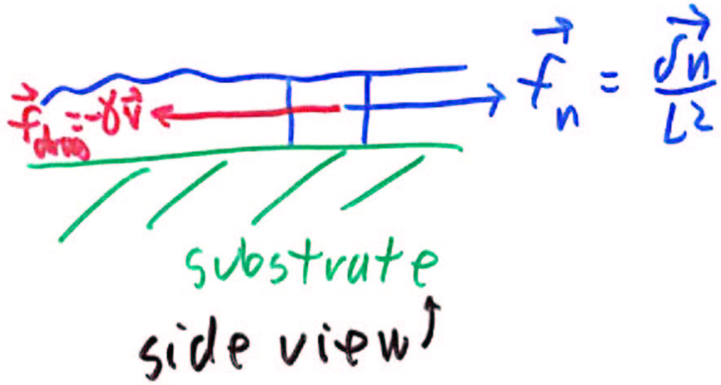
force density

(26)12

Small forces $\vec{f} \Rightarrow$ small velocities $\vec{v}$

Particles on a substrate

$\Rightarrow$ Frictional drag



$\Rightarrow$ Force balance $\Rightarrow$

$$\gamma\vec{v} = \vec{f} \Rightarrow \vec{v} = \frac{1}{\gamma}\vec{f} \equiv \underset{\text{"mobility"}}{\mu}\vec{f}$$

So $\boxed{\vec{v} \propto \frac{\vec{\delta n}}{L^2}}$

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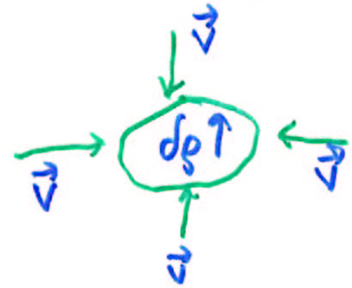
Small velocities $\vec{v} \Rightarrow$ Small density fluctuations $\delta\rho$



Continuity eqn:

$$\partial_t\rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0 \Rightarrow \frac{\delta\rho}{t(L)} \sim \langle\rho\rangle\frac{\delta v}{L}$$

On substrate, particles diffuse $\Rightarrow$ $t(L) = D L^2$

D: Diffusion constant

$$\Rightarrow \delta\rho \sim \langle\rho\rangle\frac{\delta v D L^2}{L} \propto L\delta v \propto \frac{L\delta n}{L^2} \propto \frac{\delta n}{L}$$

Recall: $\delta n \propto L^{1-\frac{d}{2}}$

$$\Rightarrow \delta\rho \propto \frac{\delta n}{L} \propto L^{-\frac{d}{2}}$$

Same as conventional result

(28(14

II) So, what's a live nematic

+

How does it beat this Catch 22?

Live nematic = dead nematic + Self Propulsion

- Same symmetry
- Same Order ($\hat{n}$)
- Same Big $\hat{n}$ fluctuations

But: Propulsion $\Rightarrow$ Big forces $\vec{f}$ from $\hat{n}$ fluctuations

$\Rightarrow$ Big ρ fluctuations

(29(16

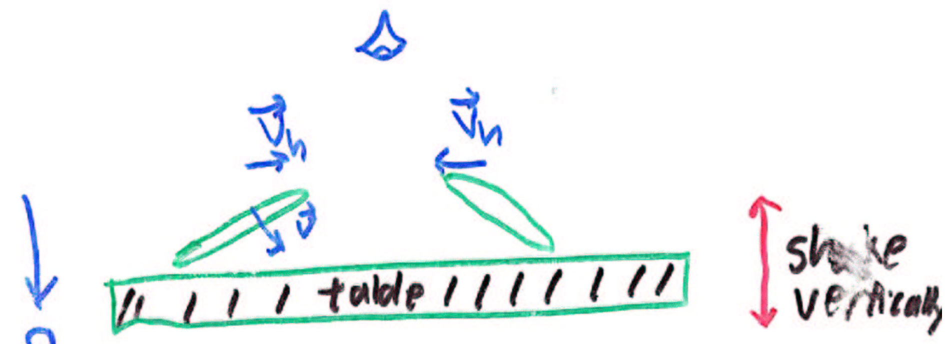
Examples of live nematics (+ Illustration of Propulsion mechanism)

1) Melanocytes (pigment cells in human skin)

"finless spermions"

Flagellae, propellers, jets, rockets, whatever

2) Rod-shaped grains on vibrating table:



side view

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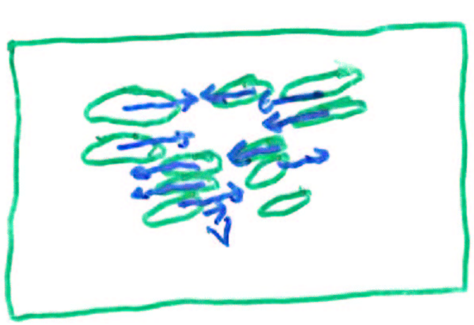


$\langle \vec{v} \rangle = \vec{0}$

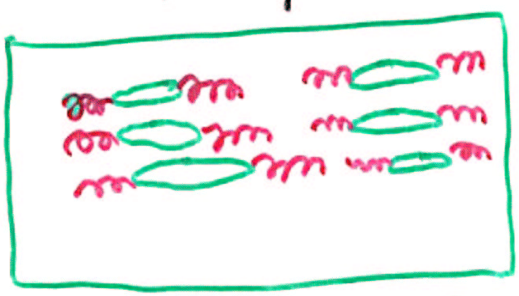
But: anisotropic

$\Downarrow$

nematic order

top view

melanocytes



In both cases, no net motion in uniform state

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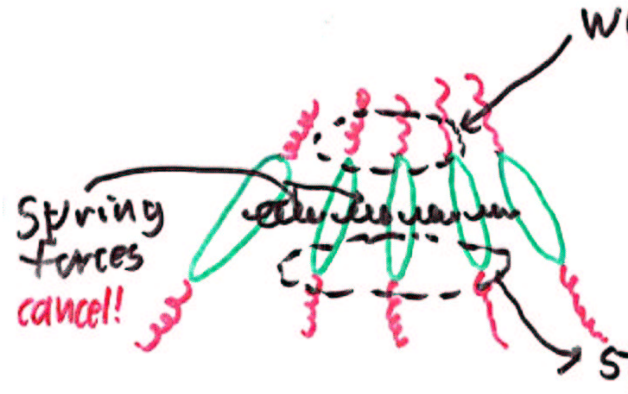
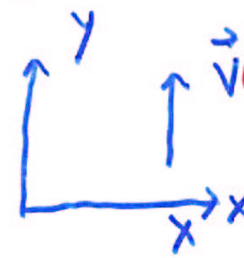
So, how different from dead nematic? How to beat Catch 22?

Motion in non-uniform state:





$\vec{\nabla}\hat{n}$ = constant $\Rightarrow$ No motion in equilibrium (dead) case

$\hat{n}$ along y, $\vec{\nabla}$ along x, $\vec{v}$ along y

$\boxed{\vec{V} = V_0\, \hat{n}(\vec{\nabla}\cdot\hat{n}) \propto \frac{dn}{L}}$ c.f. dead nematic $\boxed{V \propto \frac{dn}{L^2}}$

velocity scale set by motors

(32 (19

$$V \propto \begin{cases} \dfrac{\delta n}{L^2}, & \text{dead nematic} \\[2ex] \dfrac{\delta n}{L}, & \text{live nematic} \end{cases}$$

Connection to number fluctuations:

recall continuity equation

$$\partial_t \rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0 \Rightarrow \quad \delta\rho \sim \langle\rho\rangle \frac{v\,t(L)}{L}$$

Live nematic dynamics still diffusive $\Rightarrow t(L) = DL^2$ (like dead nematic)

$$\Rightarrow \delta\rho \sim \langle\rho\rangle \frac{\delta n}{\cancel{L}} \frac{D\cancel{L^2}}{\cancel{L}} \propto \delta n \quad \text{Live}$$

$$\text{vs} \quad \delta\rho \propto \frac{\delta n}{L} \quad \text{Dead}$$

$$\Rightarrow \delta\rho_{live} \propto \delta n \propto L^{1-\frac{d}{2}} \gg L^{-\frac{d}{2}} \sim \delta\rho_{dead}$$

(33 (20

Back to total number fluctuations:

$$\Rightarrow \delta N \propto V\delta\rho \propto L^{1-\frac{d}{2}} L^d \propto L^{1+\frac{d}{2}}$$

$$\langle N\rangle \propto L^d \Rightarrow L \propto \langle N\rangle^{1/d}$$

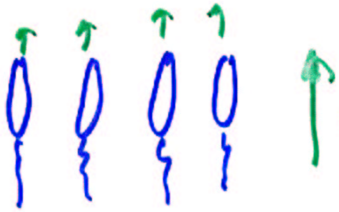
$$\Rightarrow \delta N_{live} \propto \langle N\rangle^{\frac{1}{d}+\frac{1}{2}} \gg N^{1/2} \sim \delta N_{dead}$$

In $d=2$, $\delta N_{live} \propto N$

$\Rightarrow$ Life beats 2nd law

What does this look like?

Picture of ferromagnetic flock:

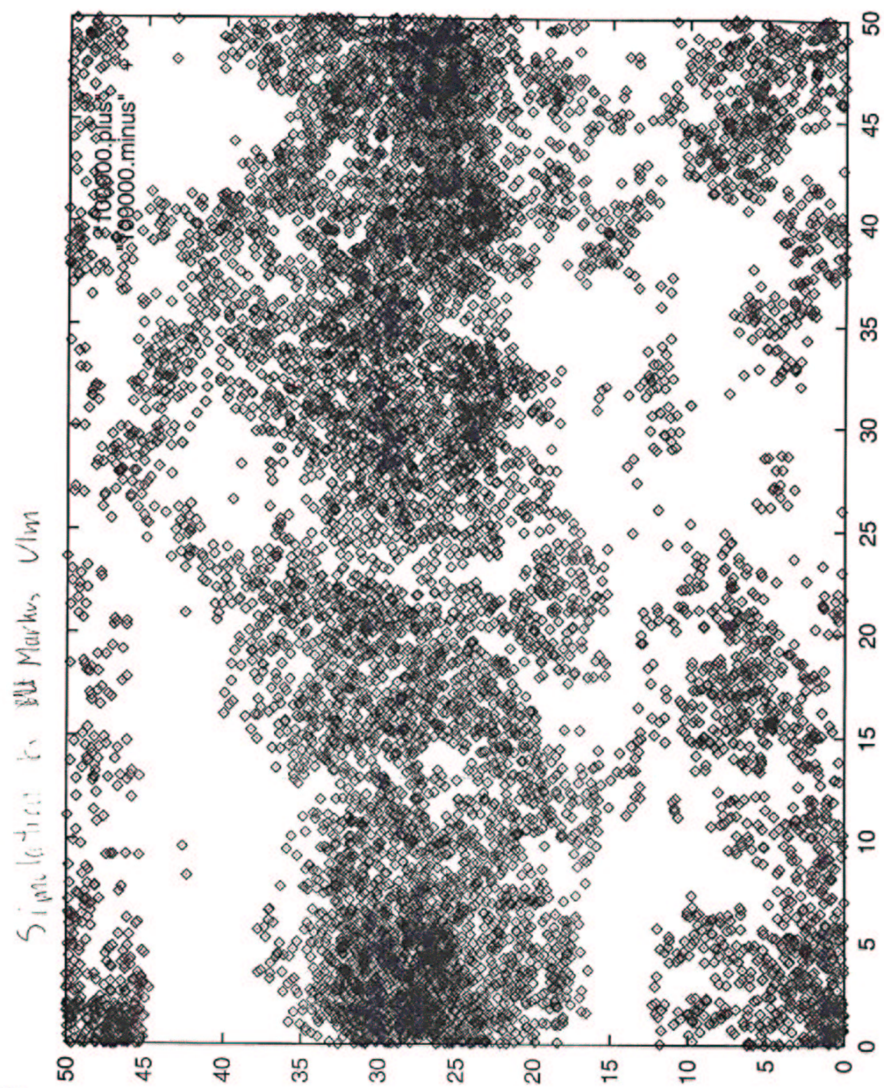


$\langle \vec{v} \rangle \neq 0$

smaller than $\frac{1}{q^2}$

Anisotropic

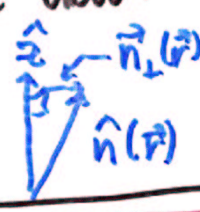
$$\langle |\delta g_q|^2 \rangle \sim \frac{1}{q_x^{6/5} + q_y^2}$$



$\langle \vec{v} \rangle$

Formalism: Hydrodynamic equations for any nematic (dead or alive) on substrate: (21/34)

Linearize about state $\hat{n} = \hat{z}$



linearization

Continuity: $\boxed{\partial_t \rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0}$

Director evolution:

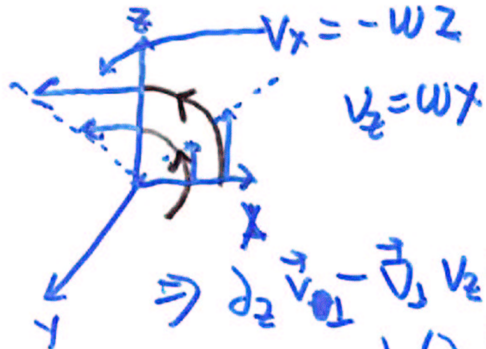
$$\partial_t \vec{n}_\perp = \frac{1}{2}\left(\partial_z \vec{v}_\perp - \vec{\nabla}_\perp v_z\right) \quad \text{rigid body rotation}$$
$$+ \frac{\lambda}{2}\left(\partial_z \vec{v}_\perp + \vec{\nabla}_\perp v_z\right) \quad \text{shear alignment}$$
$$+ \left(D_\perp \nabla_\perp^2 + D_z \partial_z^2\right)\vec{n}_\perp \quad \text{director diffusion}$$
$$+ \vec{W} \quad \text{white noise (Gaussian)}$$

Force balance: $\boxed{V_i = \mu f_i = \mu\, \partial_j \sigma_{ij}}$ ← only difference (dead vs alive)

stress tensor

Where do director evolution terms come from? (22/35)

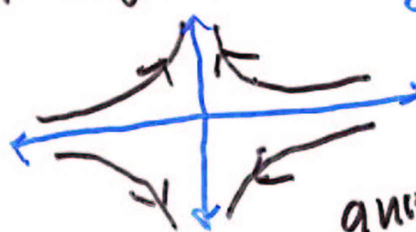
1) Rigid body rotation:



$\Rightarrow \partial_z \vec{v}_\perp - \vec{\nabla}_\perp v_z = -2\omega\hat{x}$

$\Rightarrow \partial_t n_\perp = \frac{1}{2}\left(\partial_z \vec{v}_\perp - \vec{\nabla}_\perp v_z\right) = -\omega\hat{x}$ ✓

2) Shear alignment: $\partial_t \vec{n}_\perp = \frac{\lambda}{2}\left(\partial_z \vec{v}_\perp + \vec{\nabla}_\perp v_z\right)$



anisotropic body will rotate (shape dependent)

$\Rightarrow \lambda$ shape dependent

(23 (36

Difference between dead + alive: Form of σ_{ij}

$$\sigma_{ij}^{dead} = -\delta_{ij} \underset{\text{Pressure}}{P(\rho)} + \underbrace{K_{ijkl}\partial_k n_l}$$

Pressure

only non-uniform state has stresses in equilibrium

$$\sigma_{ij}^{live} = -\delta_{ij} P(\rho) + \left(\frac{v_0}{\mu}\right) n_i n_j$$

[Propulsion)

$$\Rightarrow v_i = \mu \partial_j \sigma_{ij} = v_0 [n_i \partial_j n_j + n_j \partial_j n_i]$$

$$\Rightarrow \boxed{\vec{v} = v_0 [\hat{n}(\vec{\nabla}\cdot\hat{n}) + (\hat{n}\cdot\vec{\nabla})\hat{n}]}$$

✓ As found earlier

(24 (37

Use this in other EOM's, get closed, coupled EOM's for $\delta\rho$, $\vec{n}_\perp$

- Fourier-transform $(t, \vec{r} \to \omega, \vec{k})$
- Solve for

$$\delta\rho(\vec{k},\omega) = L_\rho(\vec{k},\omega)\, W(\vec{k},\omega)$$

$$\vec{n}_\perp(\vec{k},\omega) = \underset{\sim}{L}(\vec{k},\omega)\, \vec{W}(\vec{k},\omega)$$

- Use assumed Gaussian White statistics of $\vec{W}$ to calculate correlations of ρ $\hat{n}$
- Also find eigenfrequencies $\omega(\vec{k})$

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Results:

$\omega_{b2}(\vec{k}) = -i D_{b2}(\hat{k})\, k^2 \Rightarrow t(L) = DL^2$ ✓

$\langle |\delta \vec{n}_\perp(\vec{k},t)|^2 \rangle = \frac{\Delta}{D_n(\hat{k})\, k^2}$ ← noise strength $\Rightarrow \langle |\delta n(\vec{r},t)|^2 \rangle \propto L^{2-d}$ ✓

$\langle |\delta \rho(\vec{k},t)|^2 \rangle = \frac{\Delta}{D_\rho(\hat{k})\, k^2}$

$\Rightarrow \langle |\delta \rho(\vec{r},t)|^2 \rangle \propto L^{2-d}$

$\Downarrow$

$\boxed{\delta N \propto N^{\left(\frac{1}{2}+\frac{1}{d}\right)}}$ ✓

Can also add Non-linearities

check: irrelevant $d>2$

marginal $d=2 \Rightarrow$ logs

$\Rightarrow \boxed{d=2: \ \delta N \propto N/(\ln N)^x}$

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Conclusion:

Live nematics, just by trying to move, allow big $\hat{S}$ fluctuations to cause big N fluctuations

Explicit demonstration of "Life" beating 2nd law of thermo