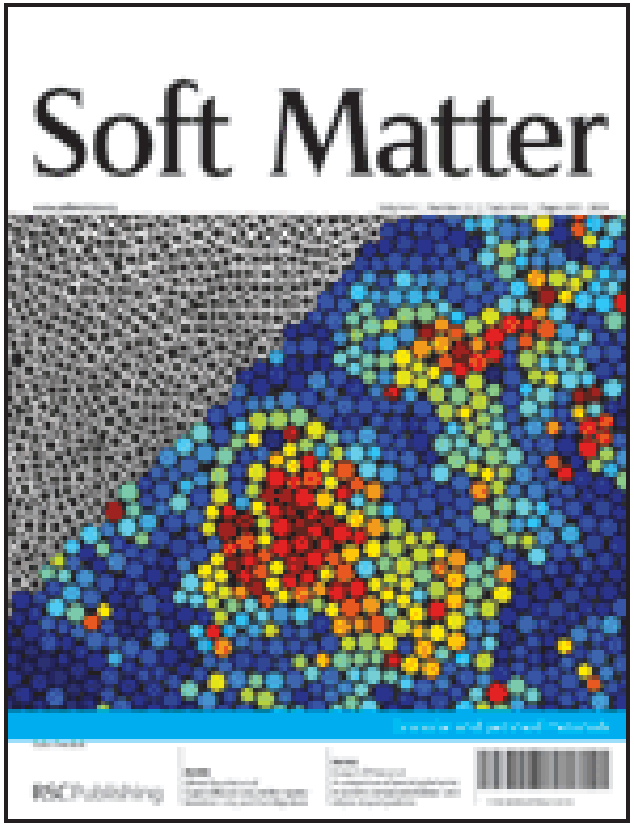


Soft Matter June 2010

Special issue on *granular systems and jammed materials* edited by A.J. Liu and S.R. Nagel

Jamming transition of frictionless grains probed by shear flow

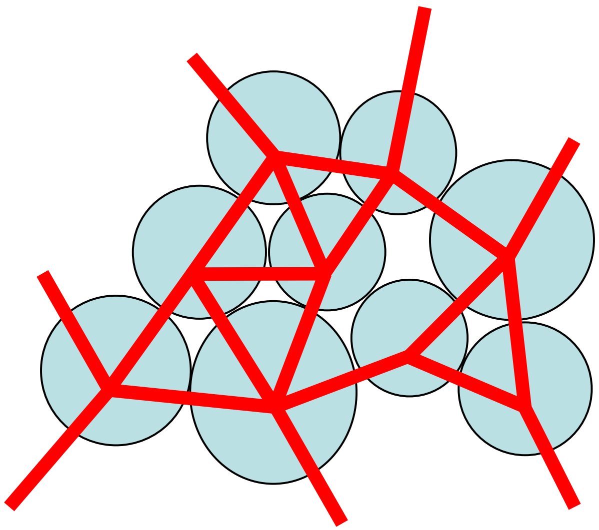
Jean-Louis Barrat and Claus Heussinger
(Univ. Lyon 1)

Ludovic Berthier
(CNRS Montpellier)

Athermal particle systems

Grains (with or without friction, no gravity) ,

Liquid foams



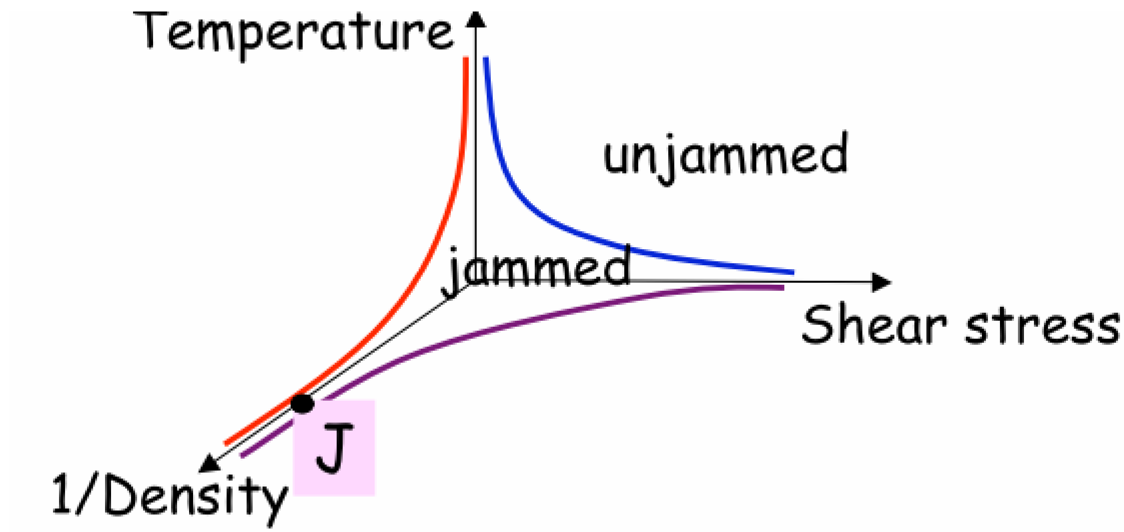
Finite range (contact) interactions
Structure can be described as a contact network

- elastic network models
- rigidity theory (Maxwell)

FLOW => Particles change their contacts

Jamming point (Liu,Nagel, O'Hern,Wyart, Witten)

- Well defined at T = 0 ?
- Critical properties
- Correlation lengths



- **Jamming point marks the appearance of yield stress in typical configurations**
- **Use quasistatic flow probes the system at its yield stress**

The J point corresponds to an isostatic solid

Minimum number of contacts needed for mechanical stability

Match unknowns (# interparticle normal forces) to equations

Frictionless spheres in D dimensions:

Number of unknowns per particle = Z/2

Number of equations per frictionless sphere = D

$\Rightarrow Z_c = 2D$

- -

Maxwell criterion for rigidity: *global* condition.

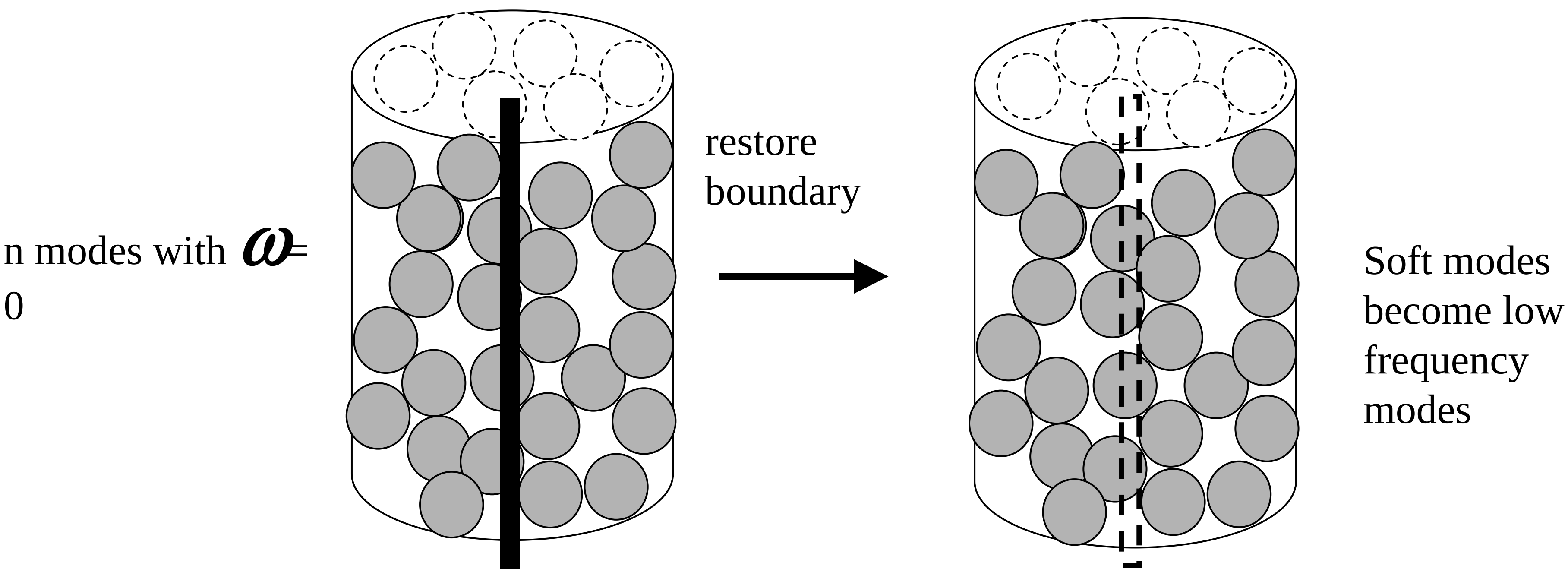
Friction changes z_c

Isostatic solids have an anomalous density of vibrational states at small frequencies

Normal solids: Debye, $g(\omega) \sim \omega^{(d-1)}$

Isostatic packing: excess density of states, $g(\omega) \sim \omega^0$

Construct low-ω modes from soft modes (Matthieu Wyart, Tom Witten, Sid Nagel)



$N(L) \sim L^{D-1}$ floppy modes (from cutting boundaries)

$\omega \sim 1/L$ (variational calculation , Matthieu Wyart)

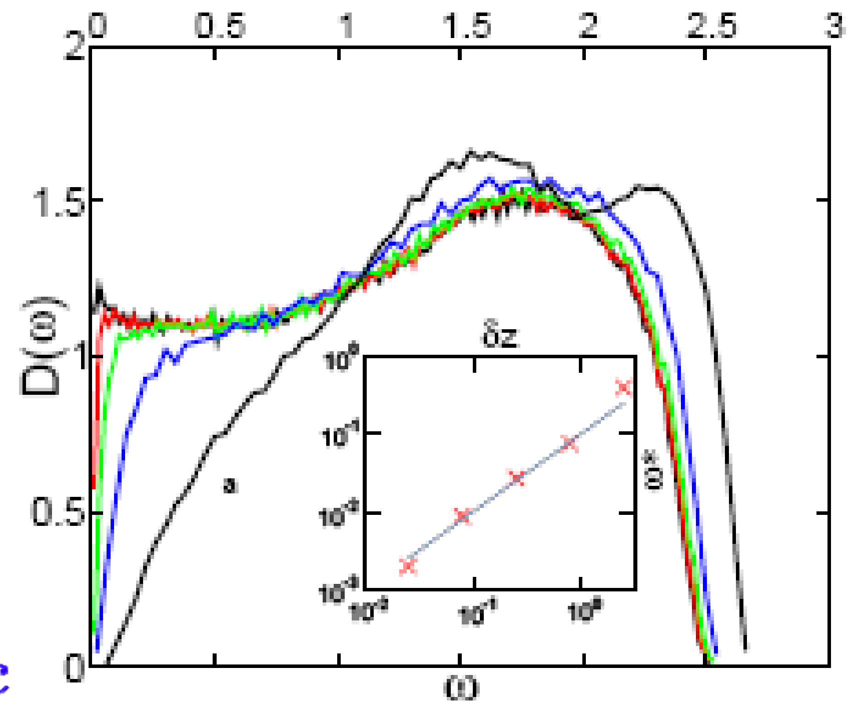
N(L) ~ L^{d-1} floppy modes (from cutting boundaries)

These modes fall within a frequency range $\omega \sim 1/L$ (Wyart)

$g(\omega) \sim N / (\omega L^d) \sim L^0$

(Liu Silbert ,Nagel, O'Hern,Wyart, Witten)

See also recent experiments Mari, Bonn, Kurchan, Soft matter 2010



If $z > z_c$ one has $\delta z = z - z_c$ excess contact per particle

The argument holds for systems smaller than the *isostatic length* $\ell^* \sim 1/\delta z$

Isostatic length

If $z > z_c$ one has $\delta z = z - z_c$ excess contact per particle
The argument holds for systems smaller than the *isostatic length*

$$\ell^* \sim 1/\delta z$$

Above jamming : a system smaller than ℓ^* becomes unstable if contacts are removed at its boundaries

Below jamming : $z < z_c$, a system smaller than $\ell^* = 1/|\delta z|$ becomes solid if its boundaries are blocked.

Questions:

- *Relevance of point J to the glassy state*
- *Relevance of anomalous d.o.s. at point J to the Boson peak and anomalous vibrations in glasses*

- **Rheological behavior in the vicinity of point J?**

Here: quasistatic simulations, probe system at yield stress ; finite strain rate : Olson and Teitel, van Hecke and coworkers

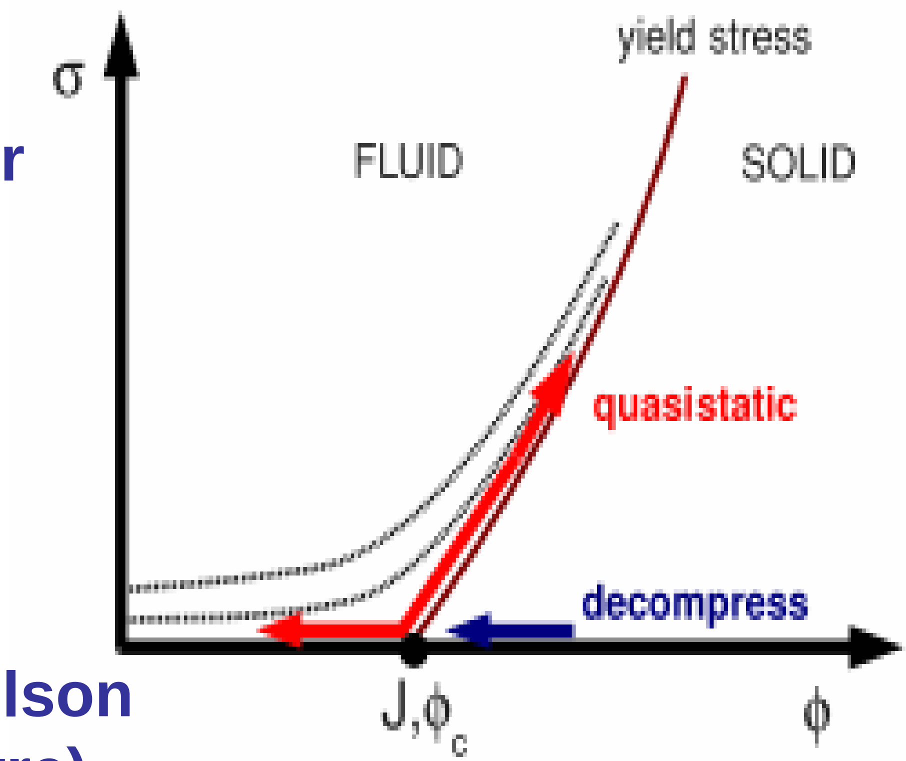
System and methods

•Repulsive, elastic (harmonic) contact interactions. $v(r_{ij}) = \frac{1}{2}k(\sigma - r_{ij})^2\theta(\sigma - r_{ij})$

•Bidisperse, 2d system (O'Hern 2003, Olson Teitel 2007)
•Lees Edwards boundary conditions
•Quasistatic simulations: quench after every increment in the shear strain

step strain $\delta\gamma = 5.10^{-5}$

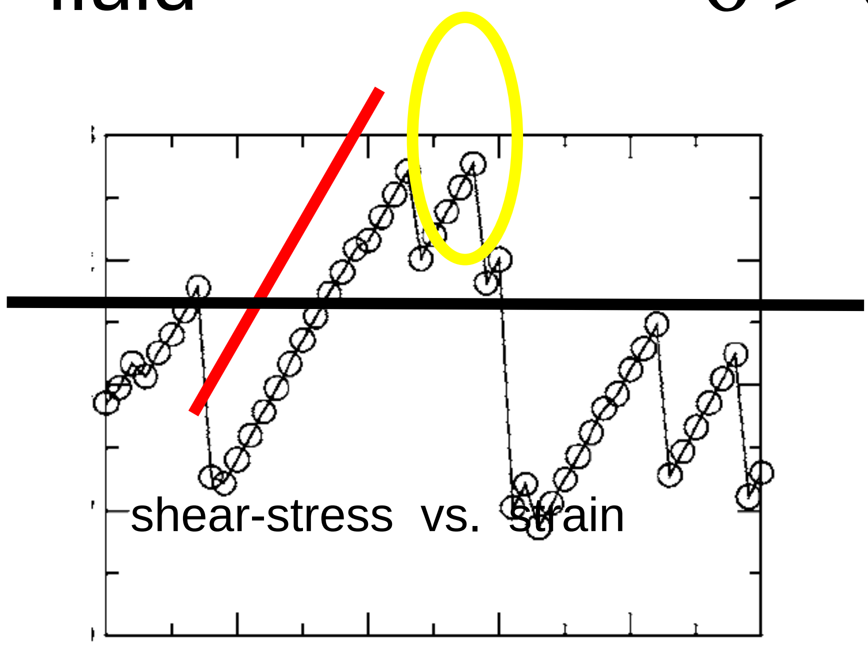
- Temperature T = 0
- Strain-rate -> 0



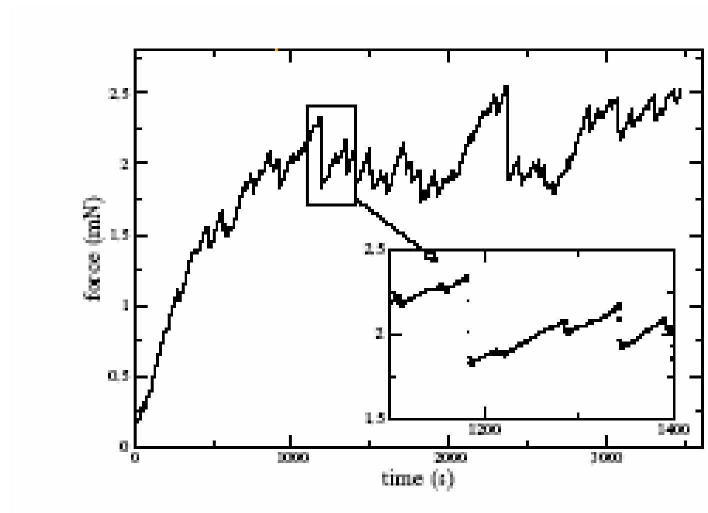
Also: overdamped MD-simulations (Olson Reichhardt, van Hecke, Caroli Lemaître)

Flow in a jammed system at low T

- "yield-stress fluid"
  - solid: shear-stress $\sigma < \sigma_{yield}$,
  - fluid $\sigma > \sigma_{yield}$,



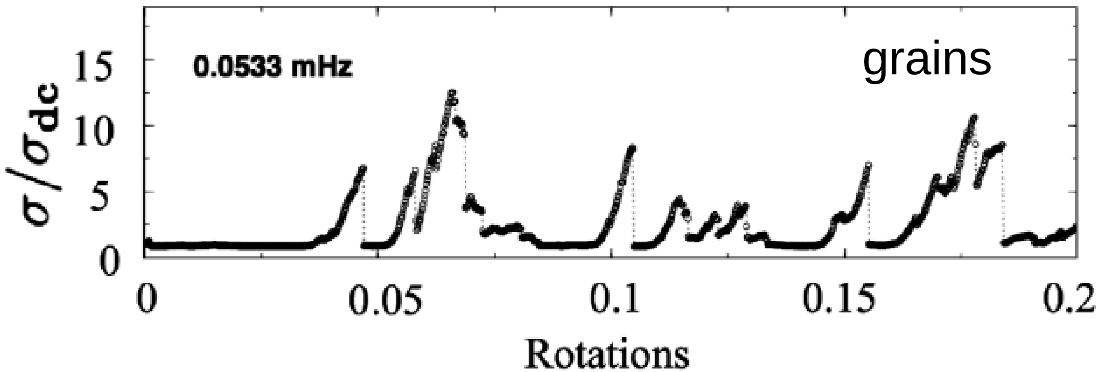
Elastic solid with flow defects



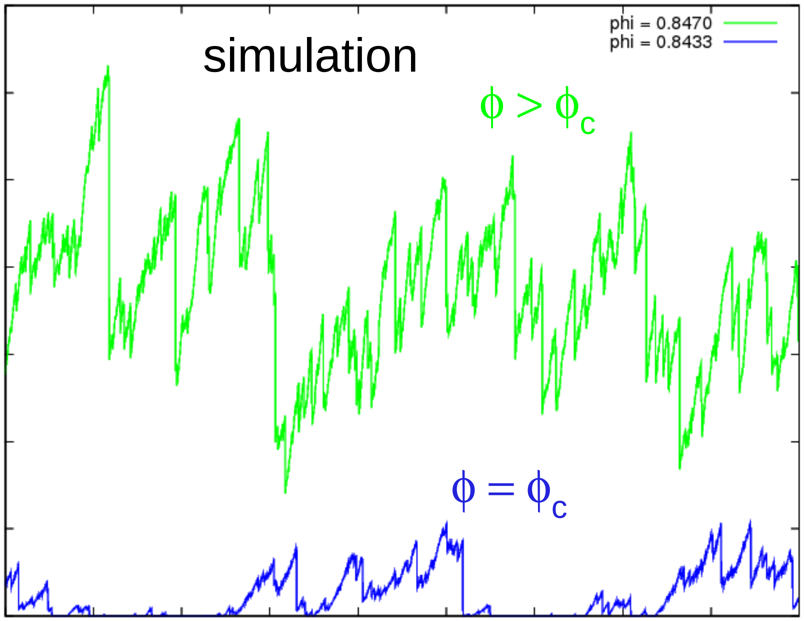
liquid foam (Cantat, Pitois, Phys. Of Fluids 2007)

Jamming point

Change of control parameter (here density) to tune yield stress. At the critical density yield stress vanishes, solid state breaks down



RP Behringer PRL (2008)

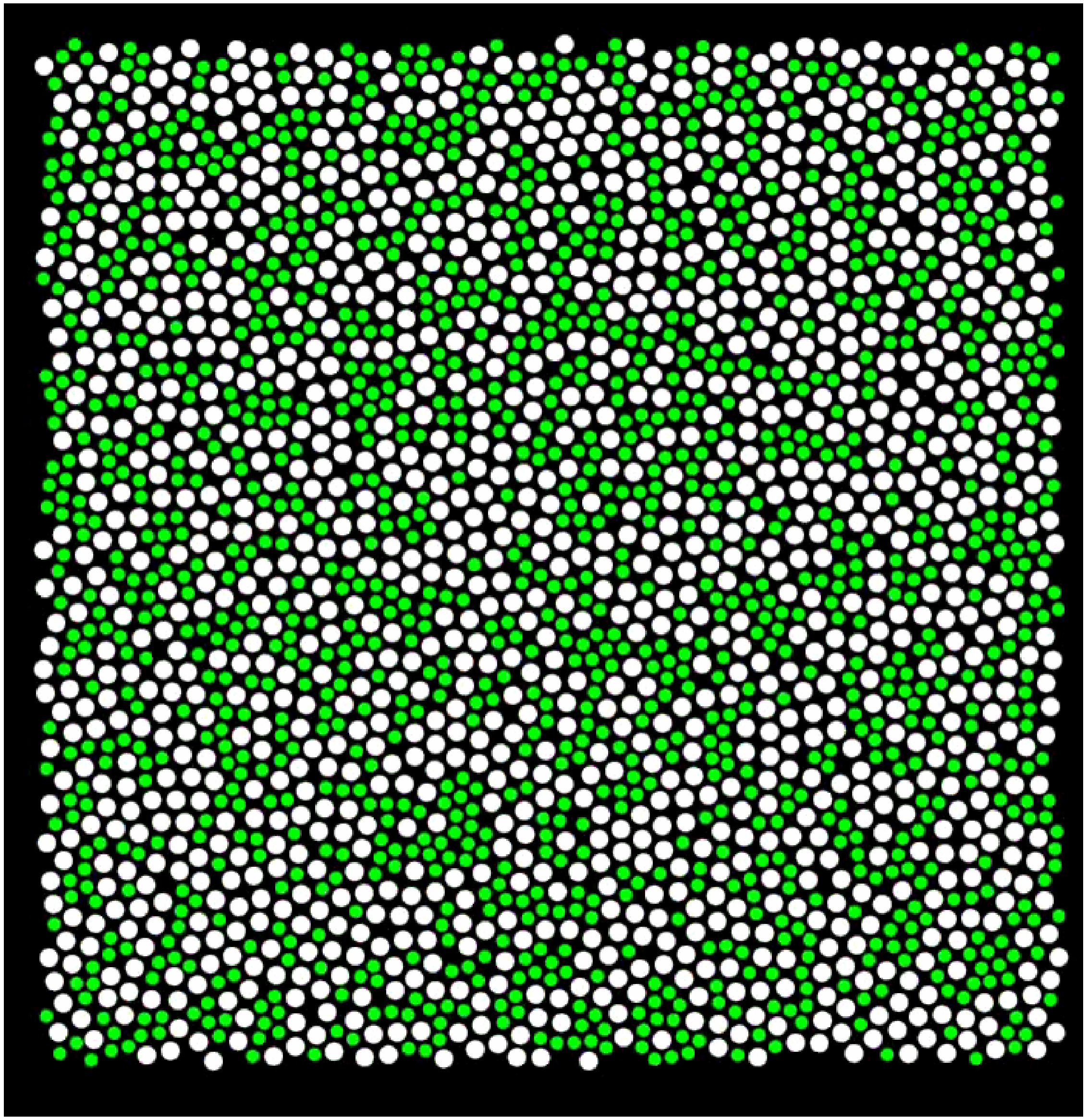


strain

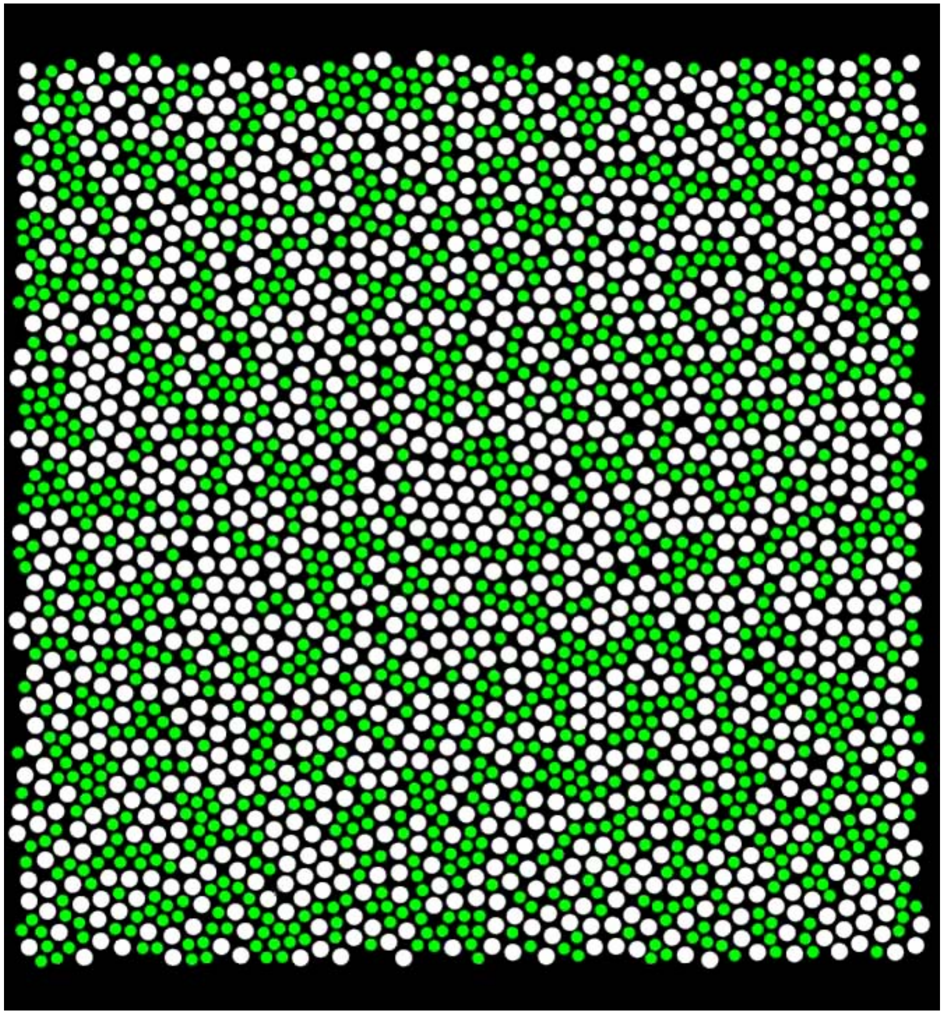
C Heussinger JLB PRL 2009

"solid" clusters in the fluid state

Low density $\phi < \phi_c$

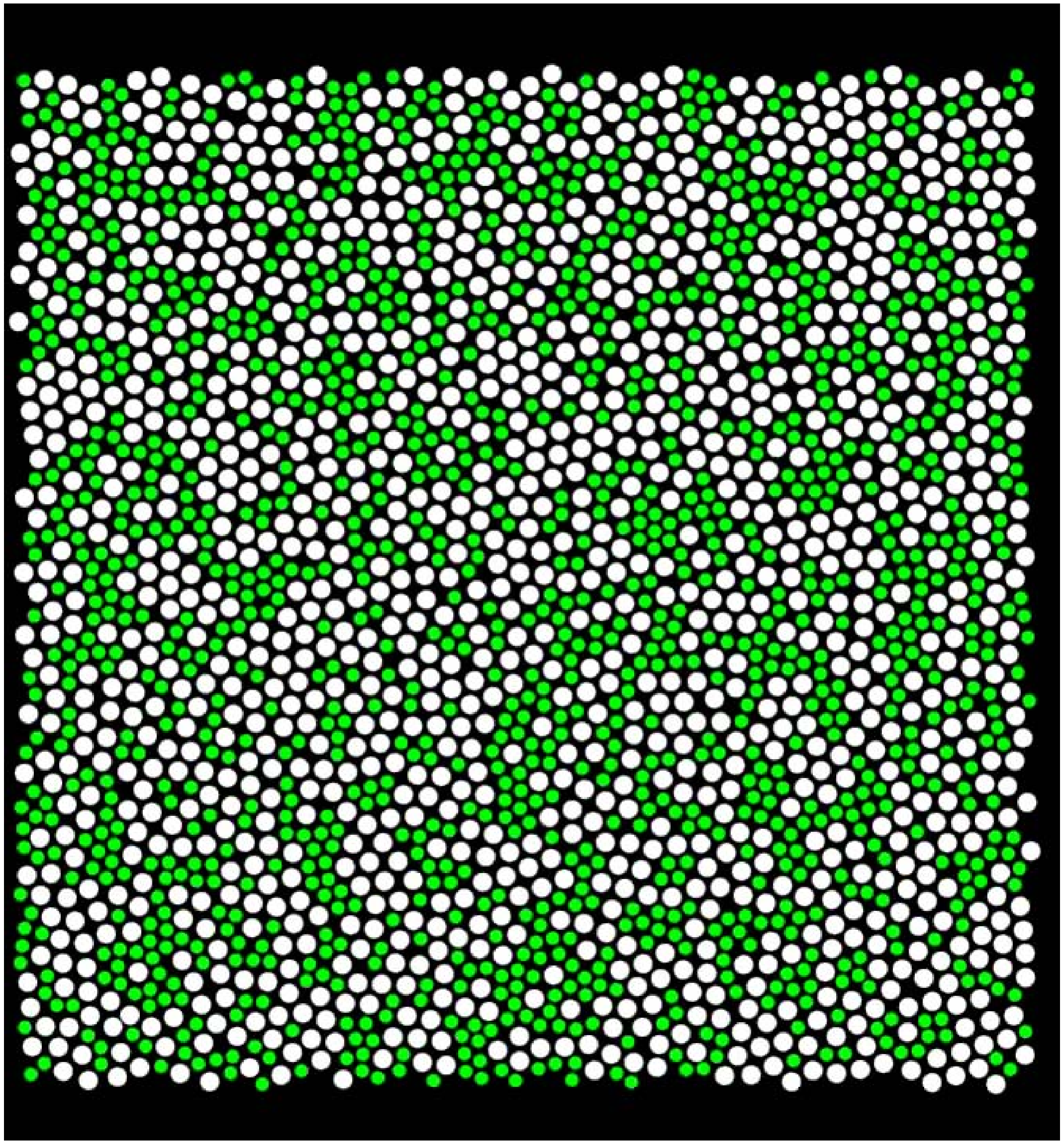


Jamming density $\phi = \phi_c$

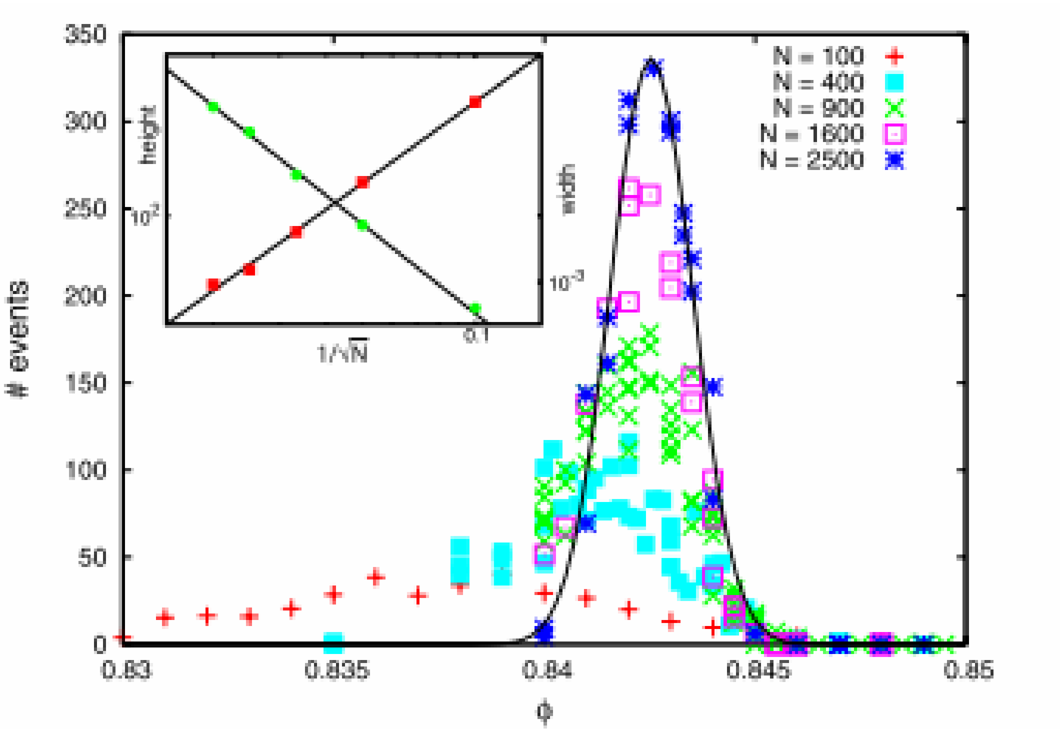


Clusters ---
growing
length-scale

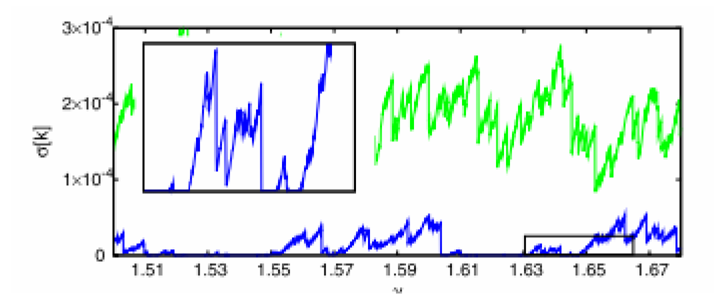
High density $\phi > \phi_c$



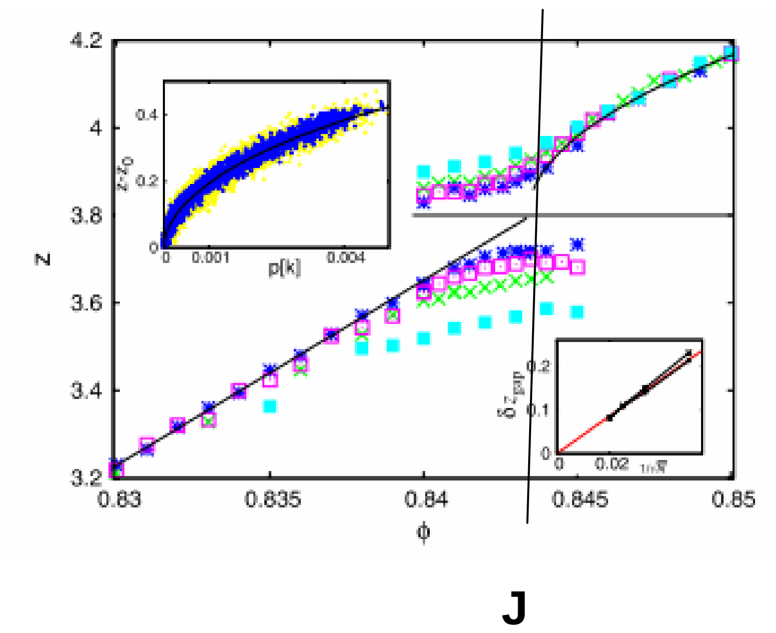
Identification of point J



Number of jamming events
(stress changes from zero to nonzero after strain strep)
as a function of density and system size



Evolution in the number of contacts z



Number of contacts as a function of density for zero stress and nonzero stress states

In contrast to decompression simulation z is continuous

J corresponds to an isostatic condition ; shear flow at point J samples isostatic configurations

Above jamming
(*Wyart et al EPL 2005 Ellenbroek et al PRL 2006*) $\delta z = z - z_c \sim p^{1/2}$

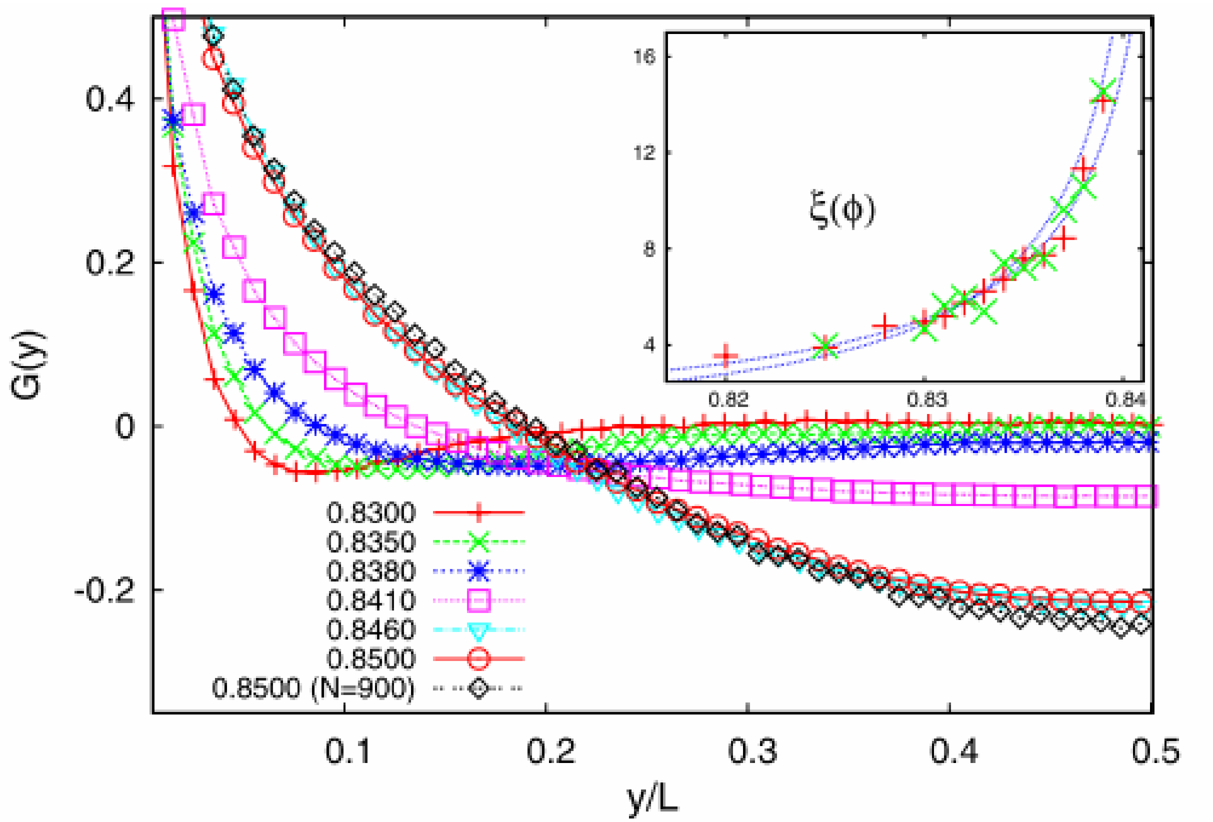
Below jamming: : $\delta z = \phi_c - \phi$

***(numerical result*)**

Isostatic length : $\ell^* \sim |\delta z[^{-1}$

Correlation length during flow

Correlation function of nonaffine displacements (instantaneous « velocities »), similar to Olsson and Teitel PRL 2007



Below jamming: Correlation length increases, independent of system size

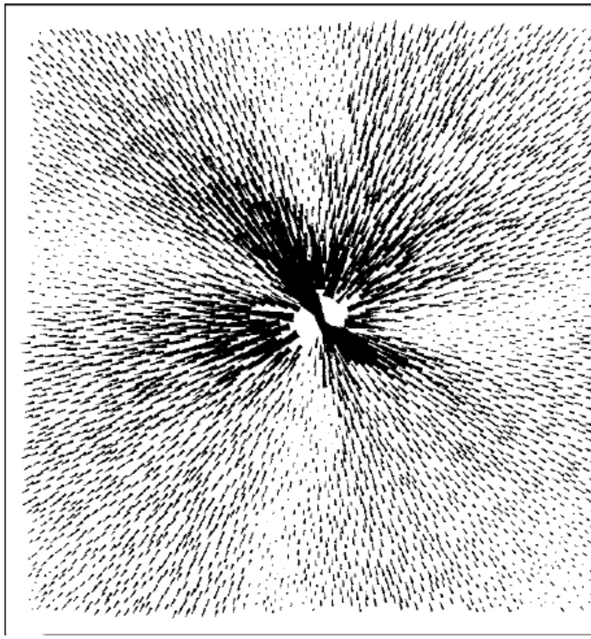
$$\xi(\phi) \sim \ell^*$$

Above jamming: Correlations scale with system size (see C. Maloney, PRL 2007)

General picture

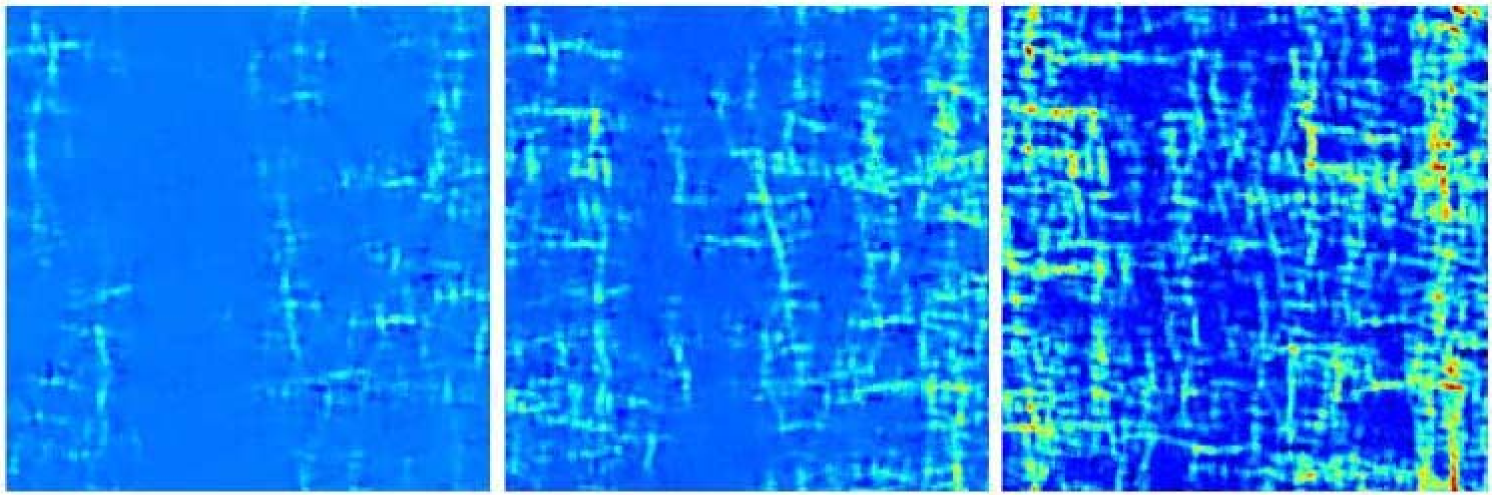
- Below jamming: correlated isostatic clusters, isostatic length is important
- Above jamming: zero temperature elastic solid with plastic flow events, the system size is the important length

Elastic solid at low T (Lennard Jones, Silicon..) : Flow events are shear transformations (Argon)

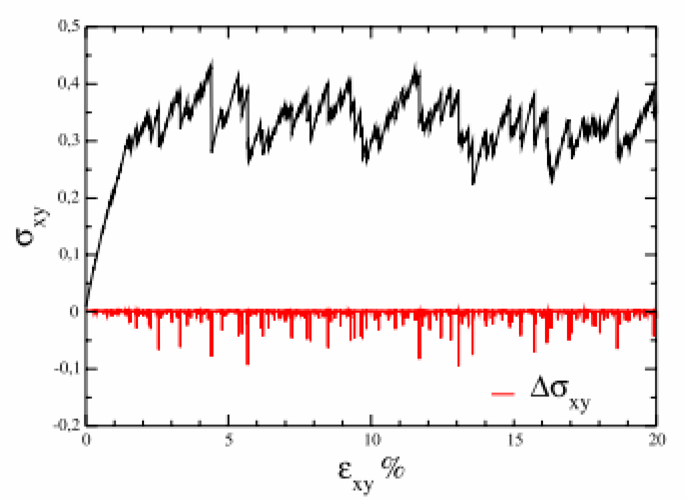


- **Localized "flips"**
- **Nucleation of avalanches**
- **Shear-bands**
- **Models: STZ, SGR, fluidity model (Bocquet PRL 2009)**

Accumulated flow activity (Lemaître Caroli PRL 2009)



In this plastic flow regime stress/energy drops of broadly distributed (from simulations with continuous potentials: , Lemaître et al, Tsamados et al, Procaccia et al)



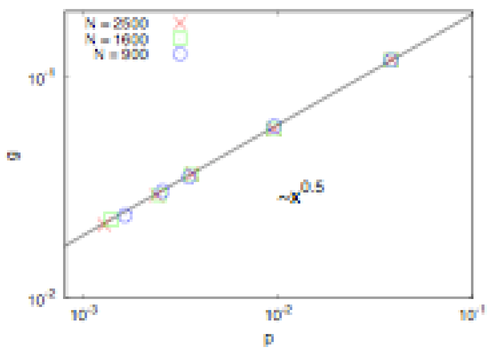
$$\Delta U \sim L^{\alpha}$$

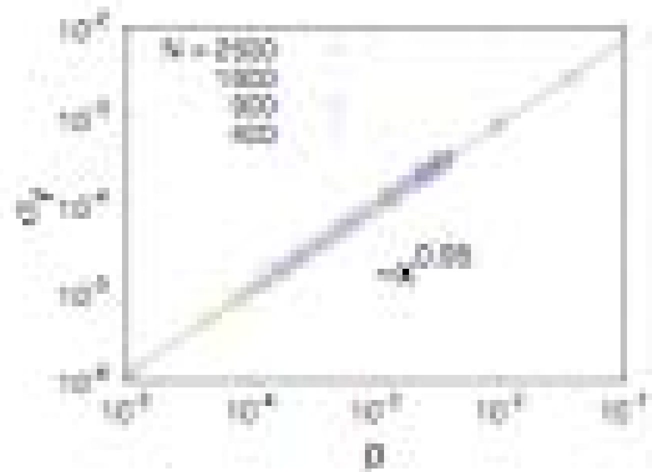
$$\Delta \sigma \sim L^{-\beta}$$

$$\Delta U \sim \Delta\sigma L^{d} \Rightarrow \alpha + \beta = d$$

α may be system dependent ; large events may be cut off by finite shear rate (Caroli, Lemaître) or thermal effects (Lemaitre Caroli, Procaccia)

In this particular system we find (above jamming):





$$G \sim P^{1/2}$$

(Consistent with J point scaling for static packings)

$$\sigma \sim 0.1P \sim (\phi - \phi_c)$$

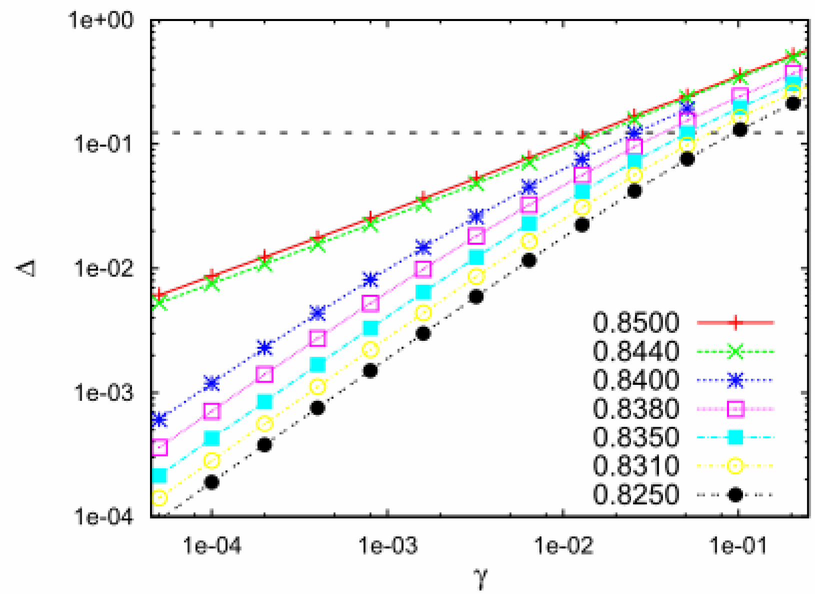
(consistent with simulations of frictionless spheres by Peyneau, Roux, PRE 2008 ; different from Tighe et al who find 3/2 exponent)

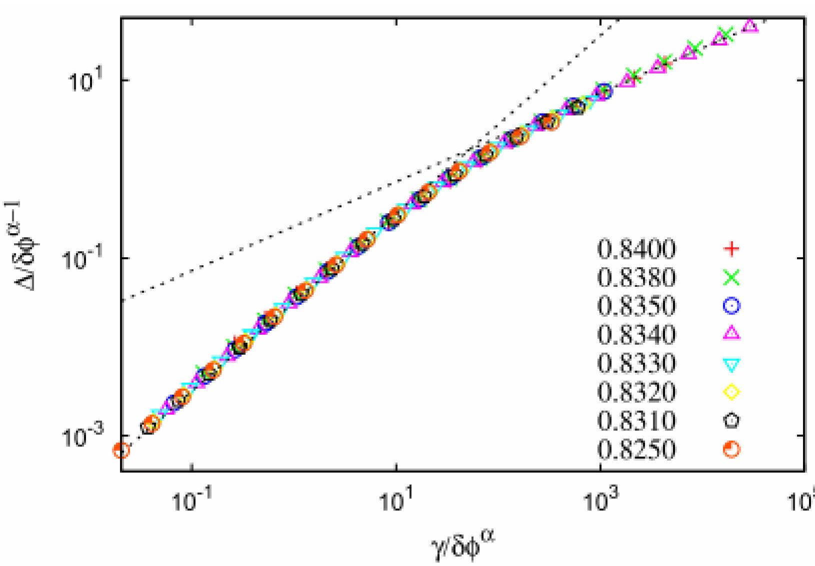
$$< \Delta\sigma > \sim N^{-1/2}$$

In the high P limit, consistent with studies by Maloney, Lemaître, Tanguy et al

Correlations at point J: Single particle displacement

$\Delta(\gamma, \phi)$





RMS displacement $\Delta(\gamma, \phi)$

- diffusive above jamming

- « ballistic » to diffusive crossover below jamming ; note that for fixed strain γ, Δ increases with volume fraction

Rescaling below jamming:

$$\Delta(\gamma, \phi) = (\delta\phi)^{\alpha-1}\mathcal{F}(\gamma/\delta\phi^{\alpha})$$

crossover strain:

$$\gamma_c \sim \delta\phi^{\alpha}, \quad \alpha \simeq 1.5$$

$$\gamma < \gamma_c \Rightarrow \Delta \sim \gamma/\ell^* \text{ with } \ell^* \sim \delta\phi^{-1}$$

Correlations at point J: overlap function

$$Q_i(\gamma, a) = \exp\left[-\frac{u_{ina}(\gamma)^2}{2a^2}\right]$$

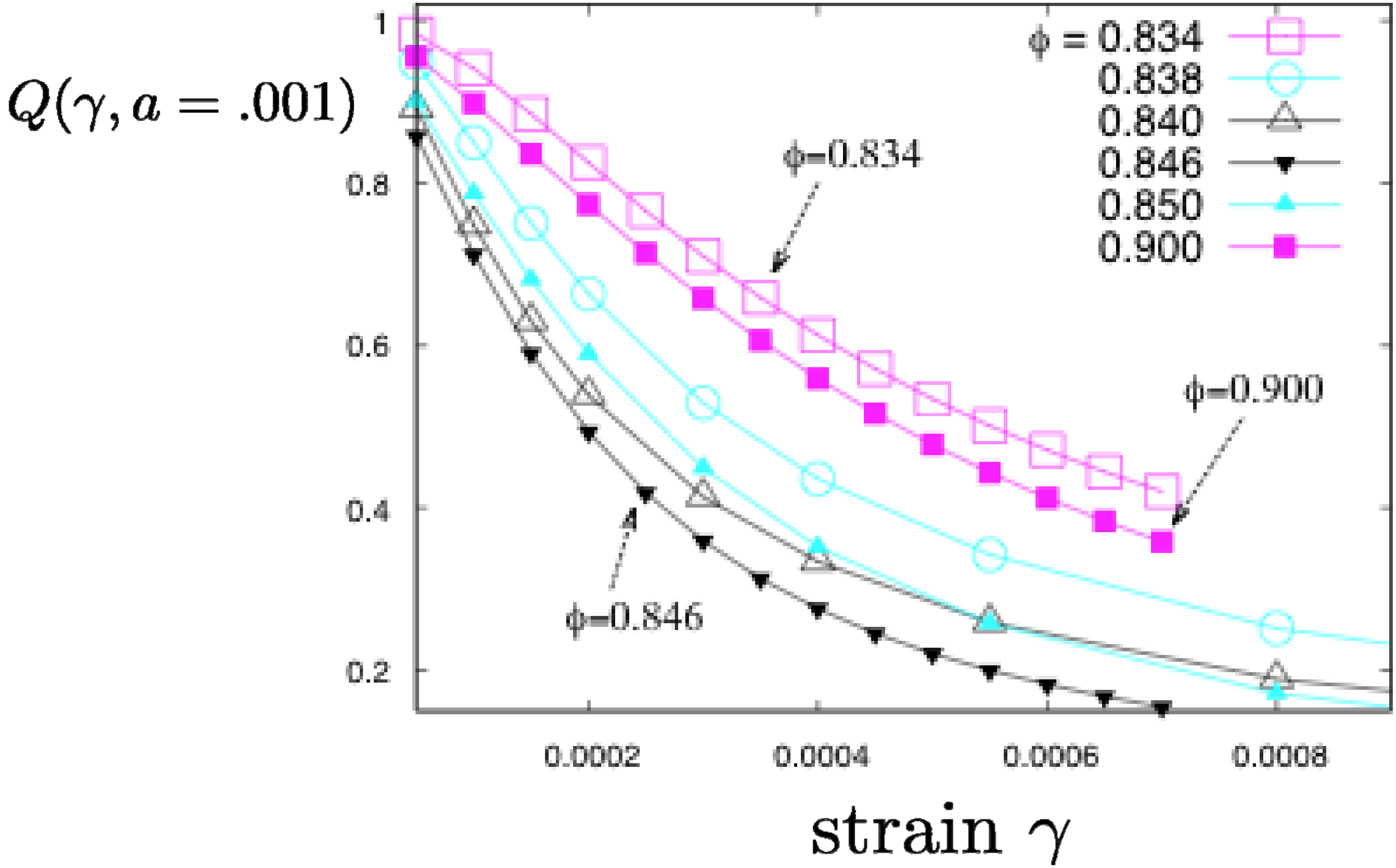
$$Q(\gamma, a) = \frac{1}{N}\sum_{i=1}^{N} Q_i(\gamma, a)$$

Probes displacements on the spatial scale *a* (*1/a* similar to wavevector *k for incoherent scattering*)

Studied in horizontally oscillated granular systems by Lechenaut, Dauchot et al, Europhys Lett 2008

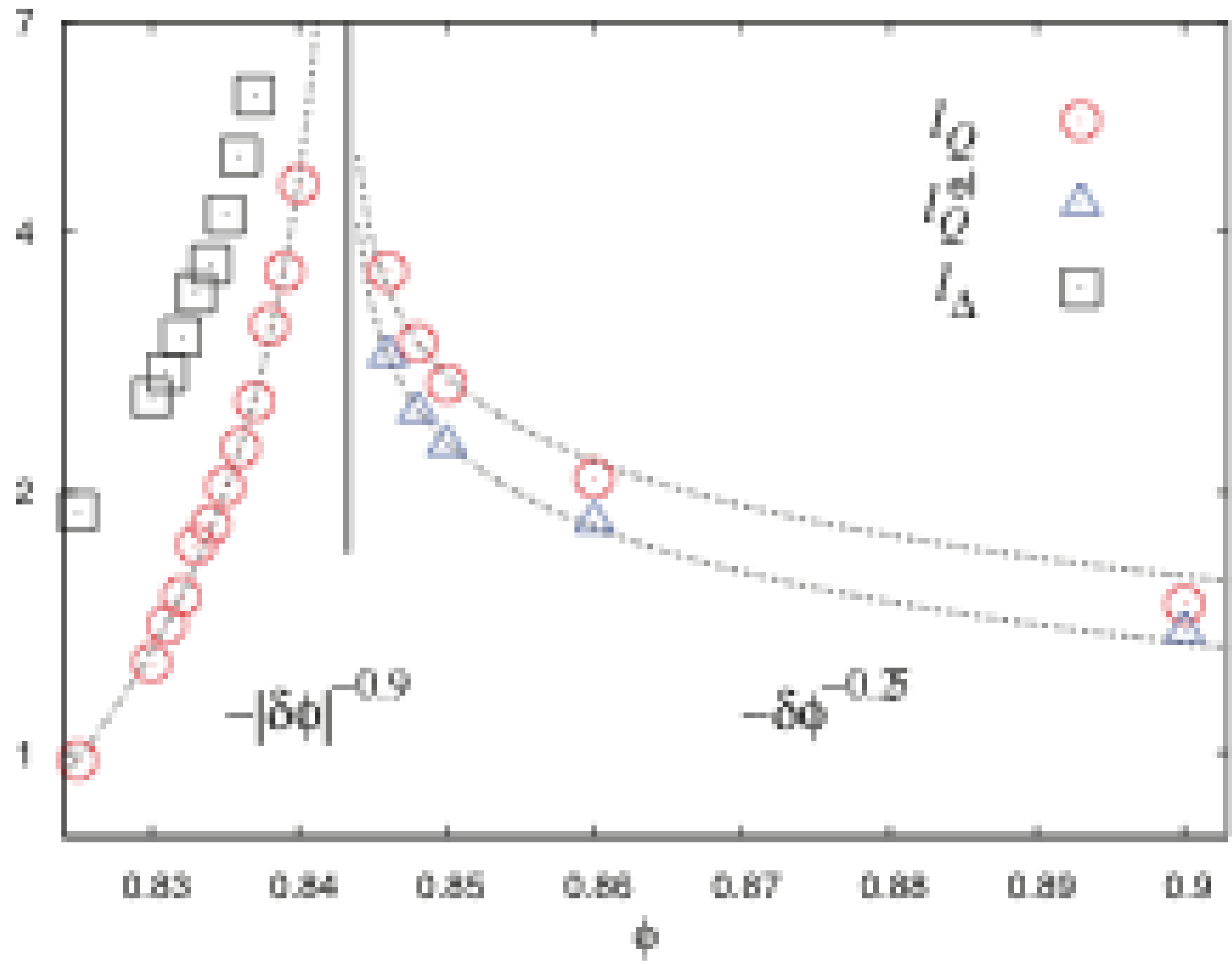
Q decays « faster » as jamming is approached from below (system de-correlates *faster* under compression)

Above jamming system becomes « slower » under compression
=> Identification of point J from the strain dependance of Q



Define $\ell_Q(\phi)$ such that $Q(\gamma, \gamma\ell_Q) = 0.5$
For small strains (ballistic regime) ℓ_Q independent of γ.

And ℓ_Δ defined by $\ell_\Delta = \lim_{\gamma\to 0} \Delta(\gamma)/\gamma$



Cooperativity: correlation length from four points correlations

$$Q(\gamma, a) = \frac{1}{N}\sum_{i=1}^{N} Q_i(\gamma, a)$$

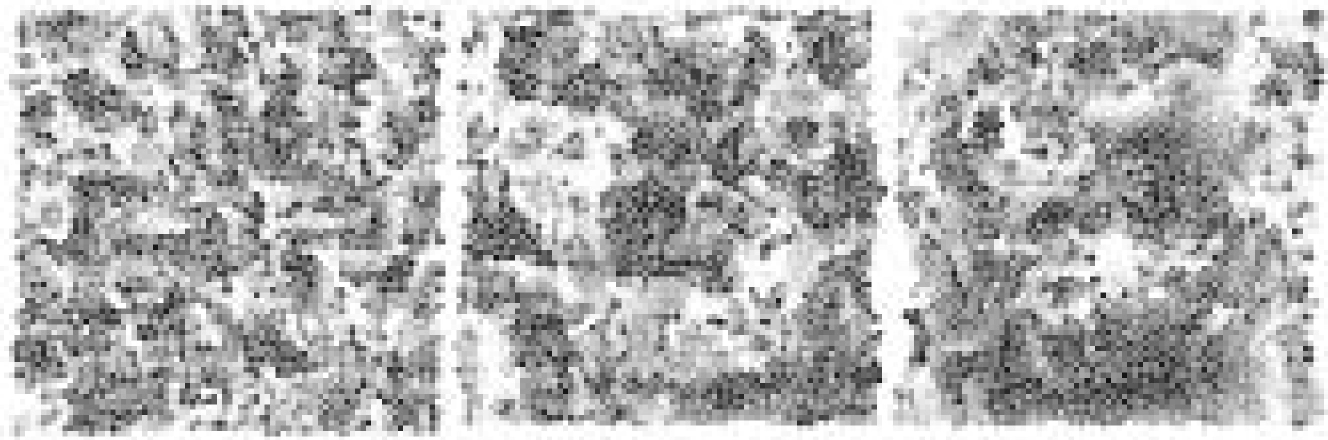
$$\chi_4(\gamma, a) = N\left(\langle Q(\gamma, a)^2\rangle - \langle Q(\gamma, a)\rangle^2\right)$$

Probes cooperativity on the spatial scale *a* (*1/a* similar to wavevector *k*)

Similar studies in vibrated granular systems by Lechenaut, Dauchot et al, Europhys Lett 2008

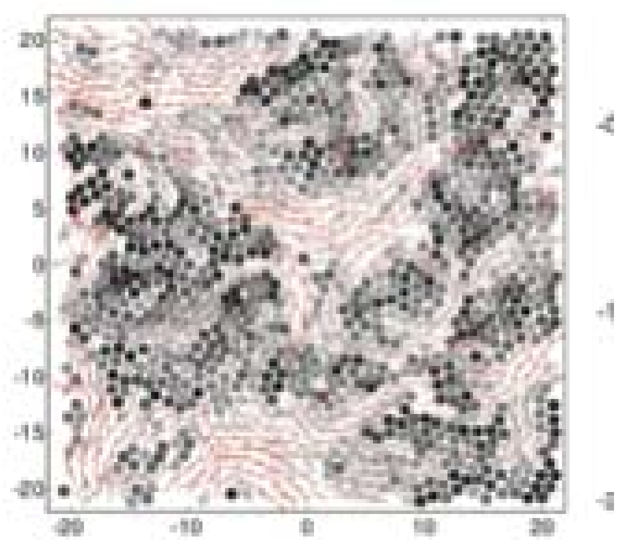
Heterogeneity of Q_4

Simulation
(steady shear)



$\phi < \phi_c$ $\phi \simeq \phi_c$ $\phi > \phi_c$

Experiment (Lechenault, Dauchot, Biroli, Bouchaud EPL 2008) ; horizontally vibrated layer of grains.

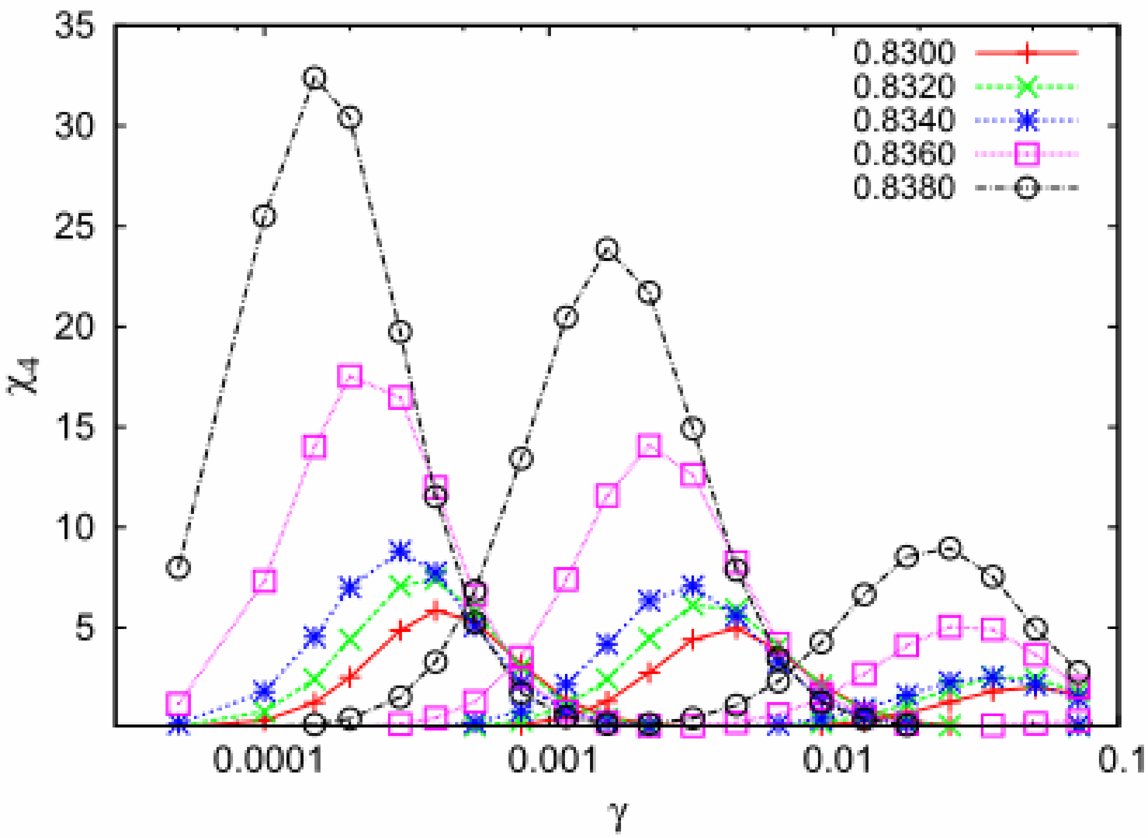


For $\Delta(\gamma) < a$ all particles are "immobile" on scale a, $\chi_4(\gamma, a) \simeq 0$

For $\Delta(\gamma) >> a$ all particles are "mobile" on scale a, $\chi_4(\gamma, a) \simeq 0$

$\Rightarrow$ **maximum of χ_4 for a finite strain $\gamma(a)$**

Behaviour of the 4 points correlations



At fixed a, the peak of χ_4 increases as $\phi \to \phi_c$ from below

Decreasing a

At fixed a, $\chi_4(\gamma, a)$ is maximum for γ^* such that $\Delta(\gamma^*) = a$

No characteristic value for a (different from Lechenaut's experiment)

The maximum can be used to identify N_{coop}, number of particles moving collectively, and length ξ (assuming $N_{coop} \sim \xi^d$)

Argument (Heuer and Doliwa, Phys. Rev E. **61**, 2000)

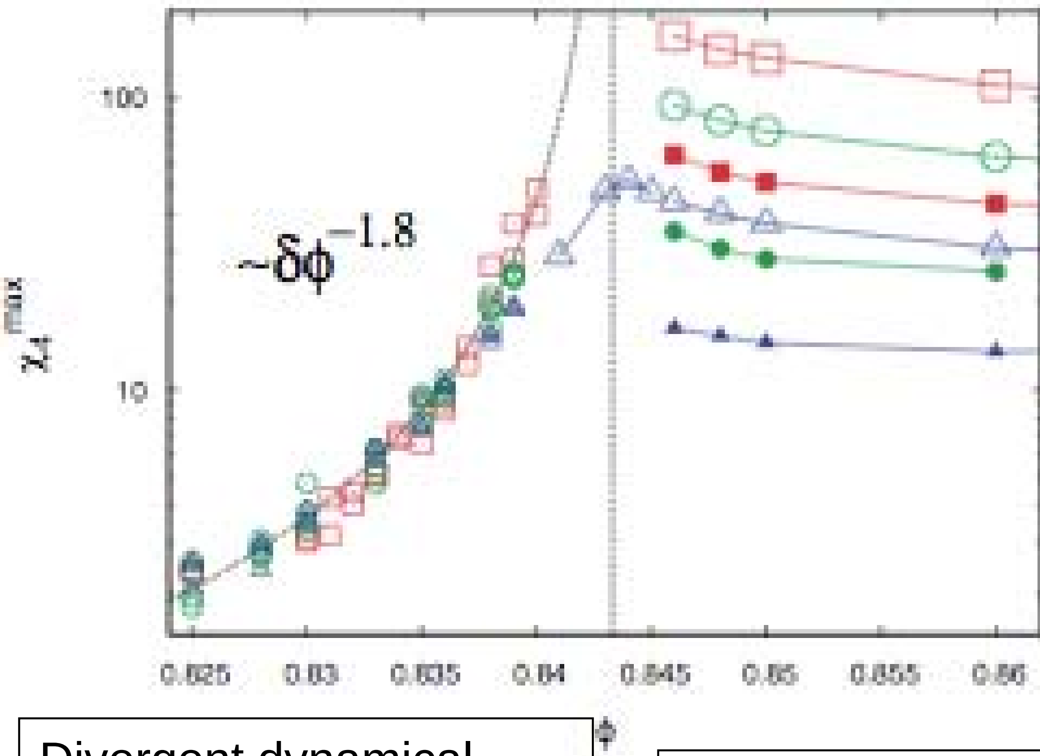
$K = N/N_{coop}$ **blocks of** N_{coop} **particles, each moving collectively**

χ_4/N **is the variance of** $Q = 1/K \sum_{i=1}^{K} s_i$ **with** $s_i = 0$ **if block** i **has moved,** $s_i = 1$ **else.**

When $< s_i >= 1/2$, $var(Q) = 1/K$, $\chi_4 = N_{coop}$

$$\max \chi_4 \sim \xi^2$$

$$\xi \sim \ell^* \sim \delta\phi^{-1}$$

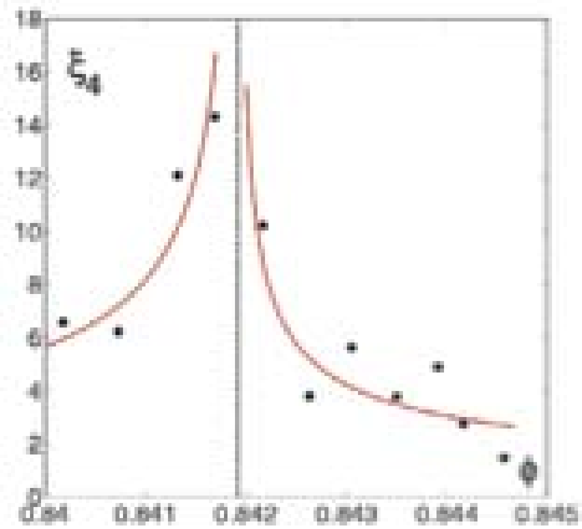


Divergent dynamical correlation length below jamming

System size dependence above jamming

Different from experimentally observed behaviour ?

(note different density intervals)



Conclusion

•Dynamical correlation length diverges at ϕ_c (consistent with Lechenault's experiment) : seen in velocity autocorrelation, mean squared displacement, heterogeneities.

•Below ϕ_c dynamical correlations are associated with isostatic clusters moving rigidly ; « ballistic » (superdiffusive) regime at small strains

$$\xi \sim \ell^* \quad \Delta \sim \gamma \ell^*$$

•Above jamming dynamical correlations dominated by plastic activity, similar to usual amorphous systems at low T ; dynamical correlation length scales with system size, with a finite strain rate cutoff.

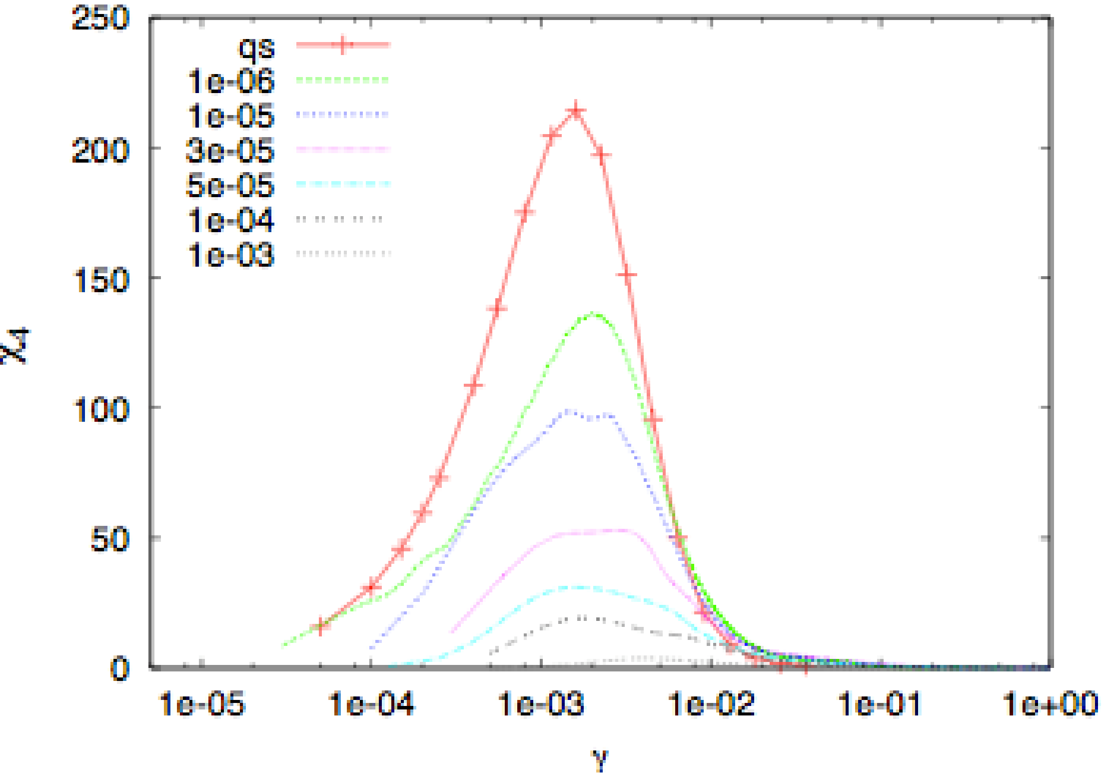
References:

Jamming transition as probed by quasistatic shear flow
C. Heussinger, J-L. Barrat, Phys Rev Lett. 102, 218303 (2009)

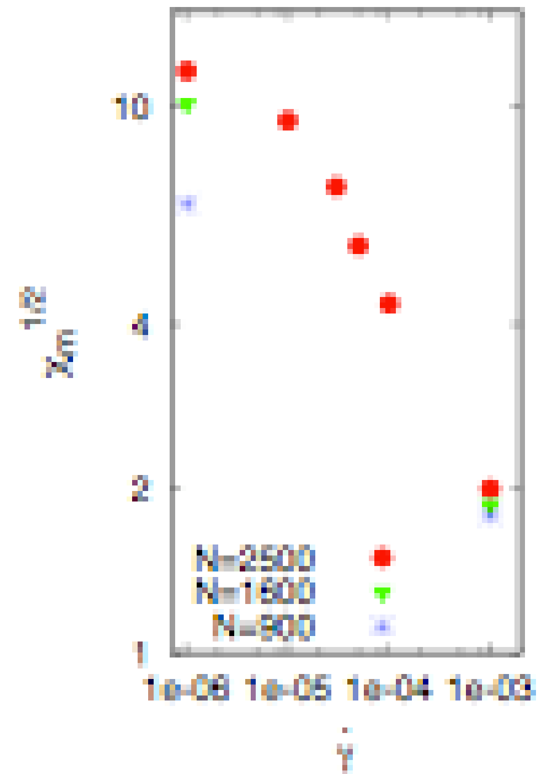
Superdffusive, heterogeneous and ollective particle motion near the fluid solid transition in atherma
disordered materials
C. Heussinger, L. Berthier, J-L. Barrat, EPL 90, 20005 (2010)

Fluctuations and correlations during the shear flow of elastic particles near the jamming transition
C. Heussinger, P. Chaudhuri, J-L. Barrat, Softmatter 6, 3050 (2010)

NB: finite strain rate (with P. Chaudhuri)



χ_4 for different strain rates approaching the quasistatic limit



Peak height for different sizes and strain rates