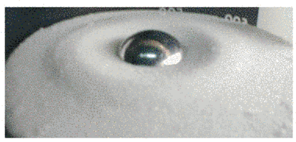
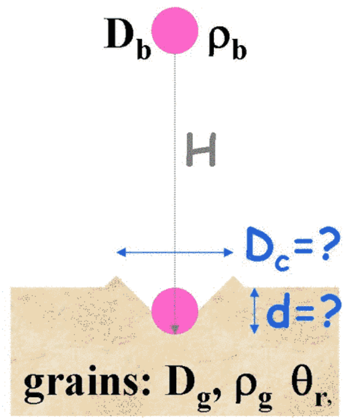
GRANULAR IMPACT CRATERING

Jun Uehara, Katie Newhall, Chris Santore,
Mike Ambroso, Randy Kamien, DJD
UCLA & PENN Physics

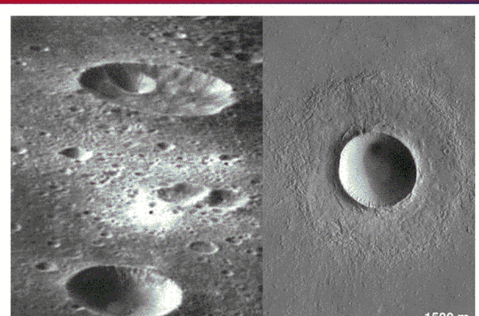
- Crater diameter & depth?
- Dependence on…
  - drop distance?
  - system properties?

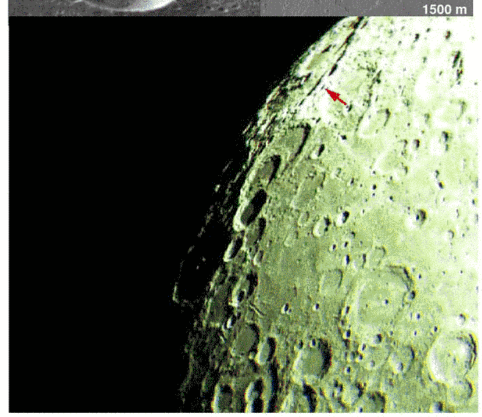




Why impact cratering?

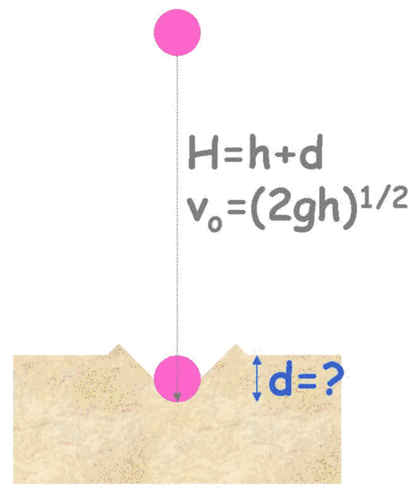
- Natural interest…
  - footprints on the beach
  - golf ball in sandtrap, etc.
- Military…
  - underground nuclear tests
  - high speed ballistics
  - bunker busters
- Geophysics…
  - planetary impact craters
- ***Granular Mechanics***





Outline

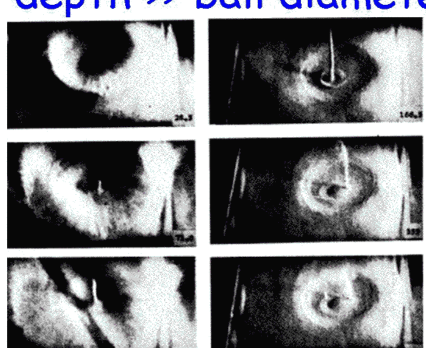
- Penetration Depth, d
  depth-averaged stopping
  force: $\langle F_s \rangle d = mgH$
- Dynamics, z(t)
  instantaneous stopping
  force: $F_s(z,v)$



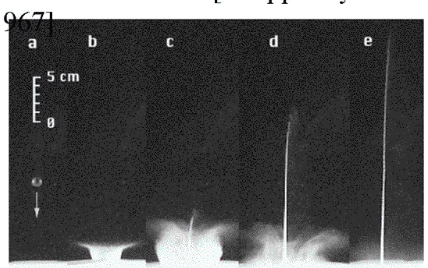
- Challenge for theory:
  to handle rapid flows, slow flows, *and* static regions

Deep impact (NOT OUR REGIME)

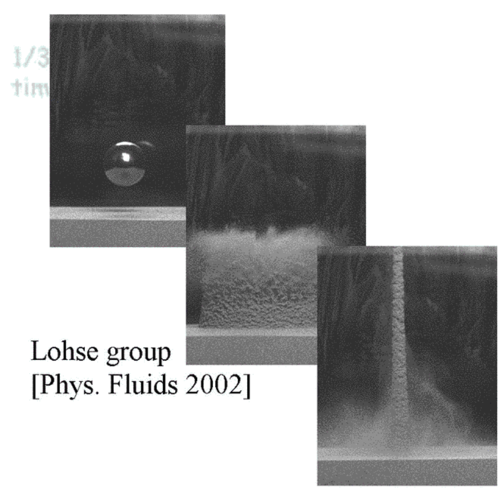
- depth >> ball diameter



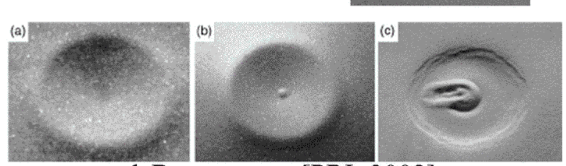
Cook & Mortensen [J. App. Phys. 1967]



Shen & Thoroddsen [Phys. Fluids 2001]



Lohse group [Phys. Fluids 2002]



deBruyn group [PRL 2003]

Shallow impact

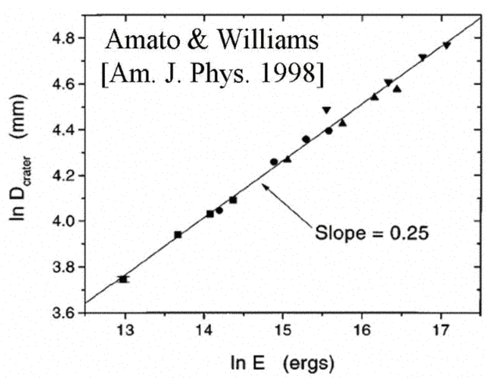
- Our regime:
  - free-fall distance: 0 - 200 cm
  - impact speed: 0 - 600 cm/s
  - penetration depth: 0.1 mm - ball diameter

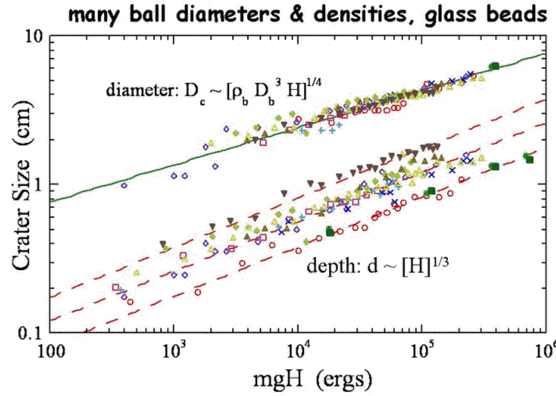
eg golf ball from 30cm

Crater size

Uehara, Ambroso, Ojha, DJD [PRL 2003]

- "gravity-limited" scaling of diameter vs energy





__BUT__: depth does not scale with diameter or with energy!

Penetration depth, repeat

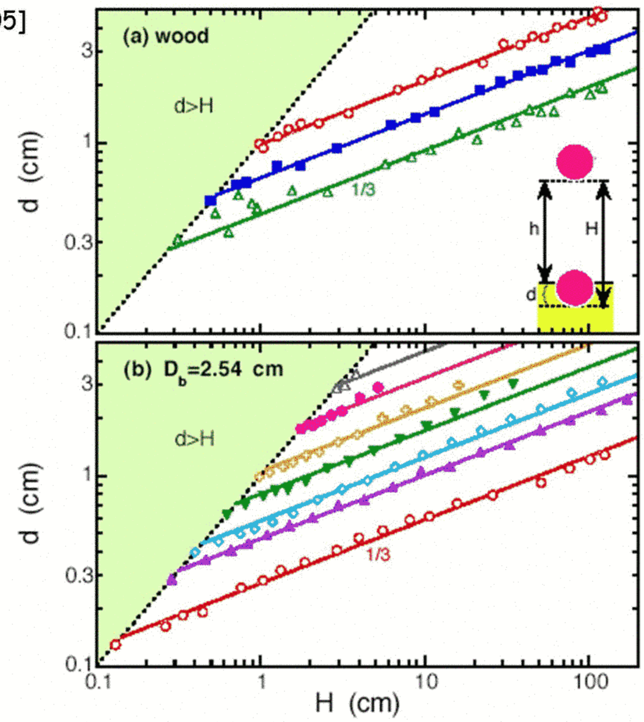
Ambroso, Santore, DJD [PRE 2005]

reaffirm our

$d \sim H^{1/3}$

original conclusion

now examine fits, $d/H^{1/3}$ vs ...



Ball diameter & density

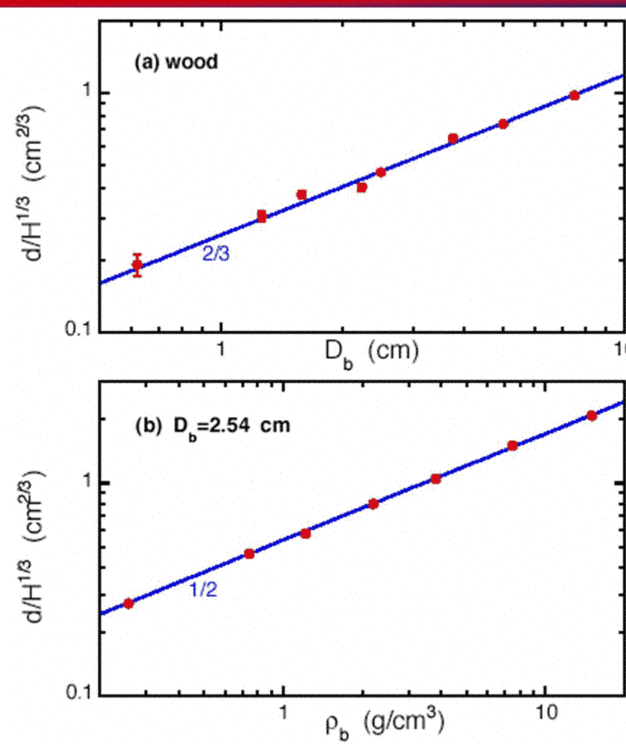
- also power-laws...

(a) $d \sim D_b^{2/3}$

nice: $d \sim D_b^{2/3} H^{1/3}$ is dimensionally correct

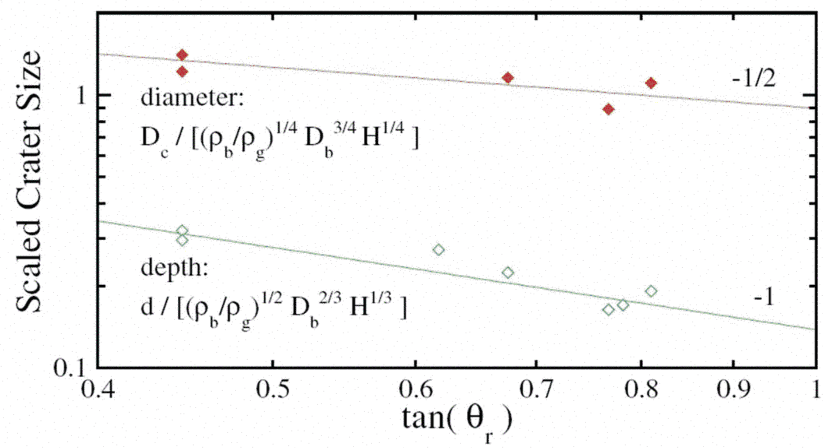
(b) $d \sim \rho_b^{1/2}$

cf liquid jet penetration



Nature of granular medium

- vary ρ_g and μ=tan(repose angle) simultaneously

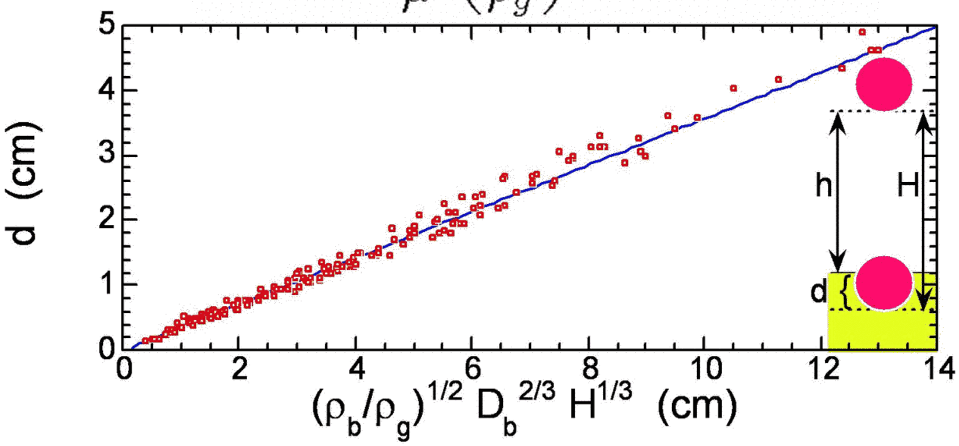


Diameter ~ $(\rho_g\mu^2)^{-1/4}$ and Depth ~ $(\rho_g\mu^2)^{-1/2}$

Penetration depth scaling (I)

- all our data collapse according to

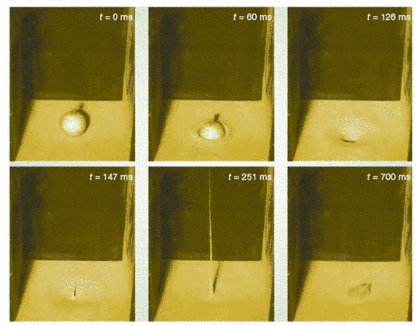
$$d = 0.14\frac{1}{\mu}\left(\frac{\rho_b}{\rho_g}\right)^{1/2} D_b{}^{2/3}H^{1/3}$$



{NB: no dependence on grain size, gravity, air}

Penetration depth scaling (II)

- Our result can be recast as $d/d_o = (H/d_o)^{1/3}$
  - crucial length scale: d_o = penetration depth for $v_o=0$
- Lohse group measured d_o vs ρ_b [Nature 2004]
  - deep penetration into fluffy small-grain sand
  - dynamics are modeled as $\Sigma F = -mg + kz$
  - this predicts $d/d_o = (H/d_o)^{1/2}$



Other models of impact

- Poncelet model [1829]: $\Sigma F = -mg + F_o + cv^2$
  - used for high-speed ballistics
- Granular hydrodynamics: $\Sigma F = -mg + \alpha\rho_g D_g{}^2 v^2$
  - used by Thoroddsen-Shen [2001] for jet height
- Swinney group [2004]: $\Sigma F = -mg + (mg+kd)$
  - gives constant acceleration proportional to v_o
- Bingham model: $\Sigma F = -mg + F_o + bv$
  - used by deBruyn-Walsh [2004] for penetration depth

ALL OF THESE ARE INCONSISTENT WITH $d \sim H^{1/3}$

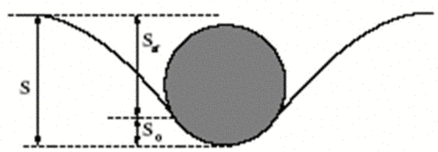
Force law should give $d \sim H^{1/3}$

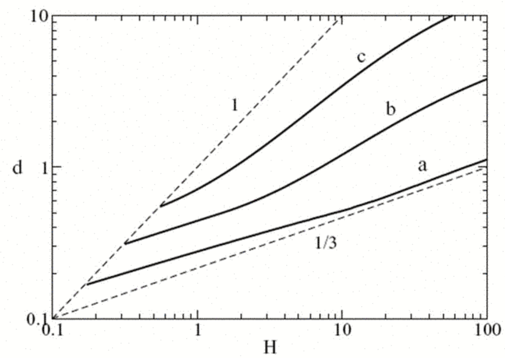
- One possibility: $\Sigma F = -mg + kz^{\alpha} v^{(4-2\alpha)/3}$

  e.g. $F(z,v) \sim v^{4/3}$ or $F(z,v) \sim z^{1/2}v$ or $F(z,v) \sim z^2$

- A better possibility: $\Sigma F = -mg + F(z) + cv^2$
  - suggested by Tsimring & Volfson [preprint 2005]
  - F(z) represents friction, and hence should vary according to hydrostatic pressure and shape of sand around projectile

$$F_s = \begin{cases} \eta \rho_g g s^2 D_b, & s \ll D_b \\ \eta \rho_g g s_0^2 D_b, & s \gg D_b \end{cases}$$

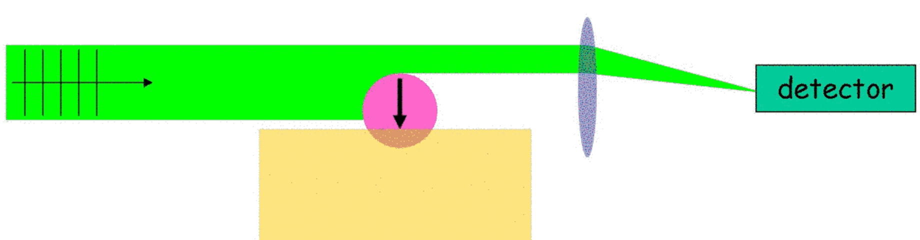




Let's measure impact dynamics

Mike Ambroso, DJD [cond-mat/0503454]

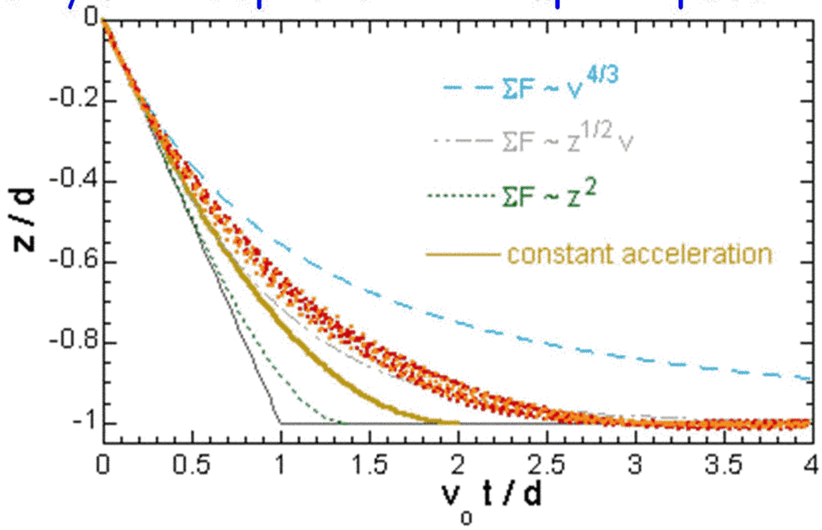
- arrange for ball to block a sheet of light…
  - measure photocurrent to deduce position
  - differentiate to deduce acceleration & ΣF



collect data for 1.5" and 2" balls and x2 range in drop heights…

All data – reasonable collapse

- scale by final depth & initial impact speed



  - acceleration decreases
  - power-law stopping forces are not acceptable

A good model of impact

Randy Kamien, DJD [cond-mat/0503454]

- Modify Poncelet, similar to Tsimring-Volfson

$$\sum F = -mg + F(z) + cv^2$$

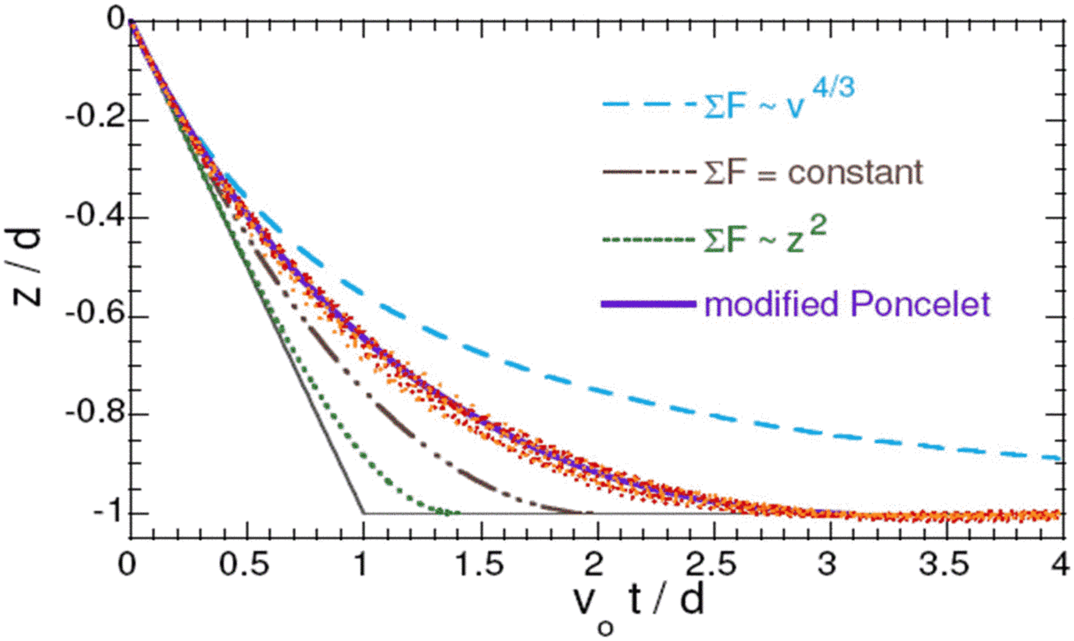
- Use F(z) that gives $d/d_o=(H/d_o)^{1/3}$ exactly:

$$\sum F = mg\left[3\left(\frac{z}{d_o}\right)^2 - 1\right]\exp\left(-2\frac{z}{d_1}\right) + \frac{mv^2}{d_1}$$

$$\left(\frac{v}{v_o}\right)^2 = \left(1 - \frac{z^3 - zd_o^{\,2}}{d^3 - dd_o^{\,2}}\right)\exp\left(-2\frac{z}{d_1}\right)$$

Compare with Z(t) directly

- fit parameter: $d_1 = 2.3d_0$



2 length scales in the model

$$\sum F = mg\left[3\left(\frac{z}{d_o}\right)^2 - 1\right]\exp\left(-2\frac{z}{d_1}\right) + \frac{mv^2}{d_1}$$

- minimum penetration depth, measured:

$$d_0 = (0.14/\mu)^{3/2}(\rho_b/\rho_g)^{3/4} D_b$$

- inertial drag length, kinetic theory prediction:

$$\left\{F_{drag} = \left(\sqrt{1-e^2}\,\rho_g \lambda^2 \dot{\gamma}^2\right) D_b^2, \quad \lambda = D_g, \quad \dot{\gamma} = v/D_g \right\}$$

$$\downarrow$$

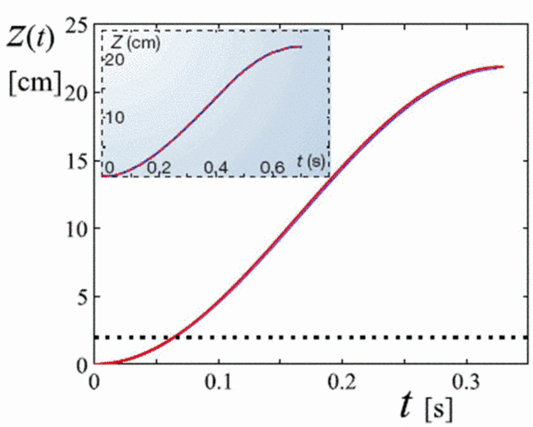
$$d_1 \propto \frac{\rho_b/\rho_g}{\sqrt{1-e^2}} D_b$$

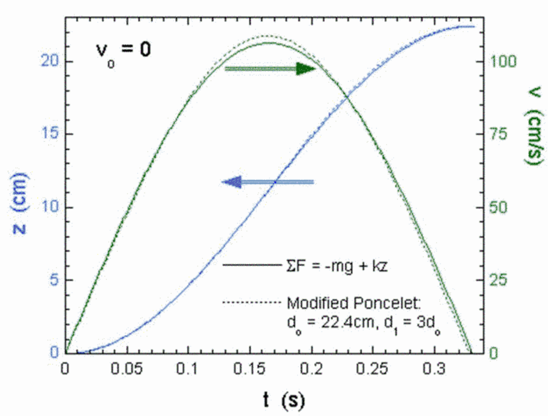
Unfinished business

- Explain scaling of minimum penetration depth…
- Test model for different materials
  - kinetic theory scaling of inertial drag length…
- Test model for deep impacts
  - other, seemingly-contradictory, results…

Lohse group [Nature 2004]

- {R=2cm, m=148g} v_o=0 into fluffy "dry quicksand"
  modeled by Coulomb friction with k=13.3N/m



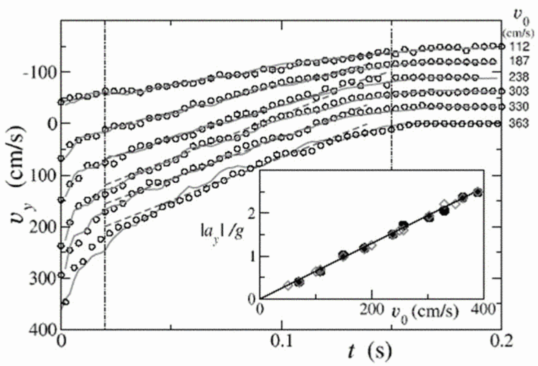


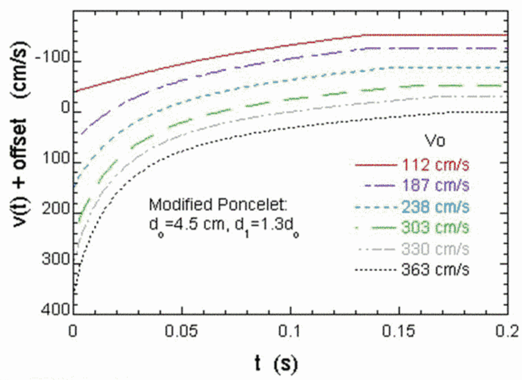
Good description by modified Poncelet

Swinney group [PRL 2004]

- Cylinder dropped side-ways onto pile of rods

  modeled by ΣF = constant, acceleration ~ impact speed

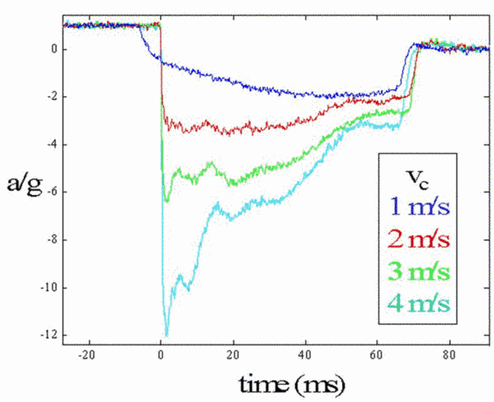


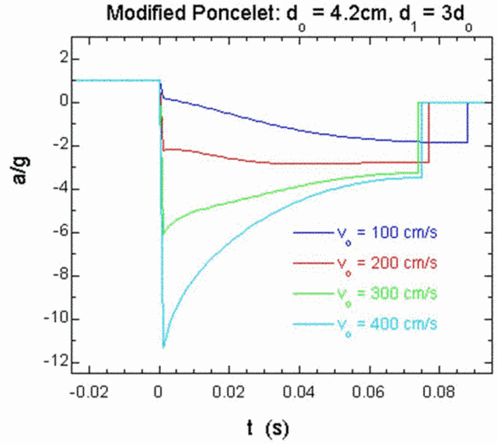


Fair description by modified Poncelet

Sweeney & Umbanhowar ['04]

- {R=2.5cm, steel} dropped into 0.7mm glass beads





Good description by modified Poncelet

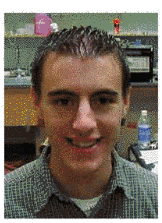
Conclusion

- IMPACT DATA:
  - crater diameter
  - penetration depth and dynamics
  - vs drop height and system properties

- MODIFIED-PONCELET MODEL:
  - explains all of our shallow-impact observations
  - may explain deep-impact data for other systems

Team Impact

- All experiments were done by undergraduates



Mike Ambroso

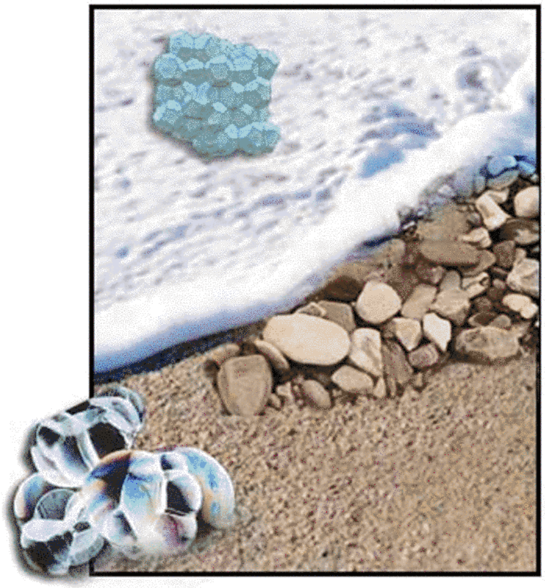
Katherine Newhall

Chris Santore

Jun Uehara

THE END.

- Thank you for your interest.

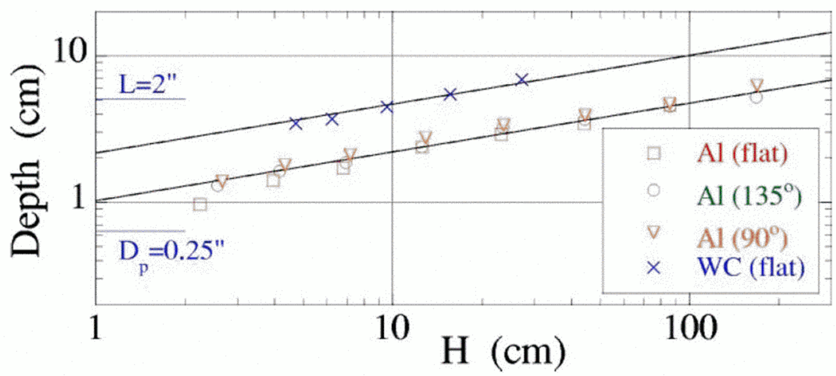


Projectile shape

Katie Newhall, DJD [PRE 2003]

- Drop cylinders of various densities, diameters, lengths, and *tip shapes* into 0.2 mm glass beads

eg:



The d ~ $H^{1/3}$ scaling isn't an accident of projectile shape!

Preparation of medium

- **not crucial**: always get $d \sim [\rho_b/(\rho_g \mu^2)]^{1/2} D_b^{2/3} H^{1/3}$
- only the value of $\mu=\tan(\theta_{repose})$ depends on prep:

swirl and tap until level [Uehara et al. PRL 2003]

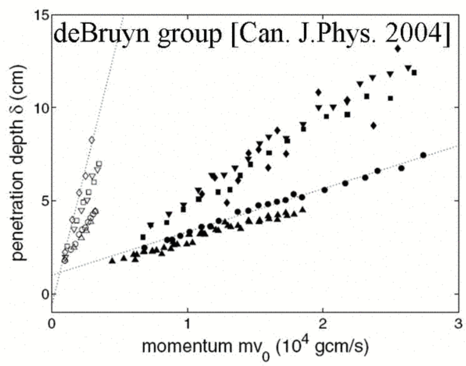
- eg monodisperse glass beads:
  - random-close packing: ϕ= 0.63
  - draining angle of repose: θ = 24°

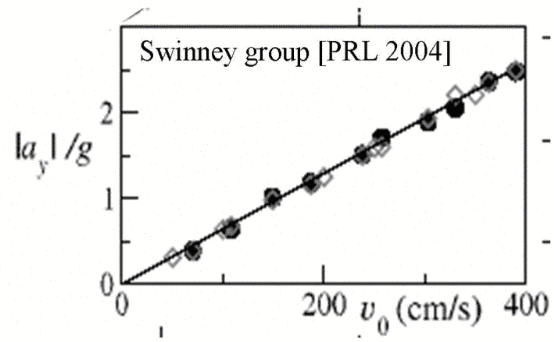
post-gas-fluidize [Ambroso et al. preprints]

- eg monodisperse glass beads:
  - looser packing fraction: ϕ = 0.59 [Ojha, Menon, DJD PRE 2000]
  - draining angle of repose: θ = 21° [inferred]

Depth vs impact speed (I)

- shallow impact (us): $d \sim H^{1/3}$
- deep impact (others): $(d-d_o) \sim v_o = (2gh)^{1/2}$





- deBruyn article states that our data obey $(d-d_o) \sim v_o$
  - could one of us be mistaken? {the forms are similar}
  - could there be different scaling regimes? {eg shallow vs deep}

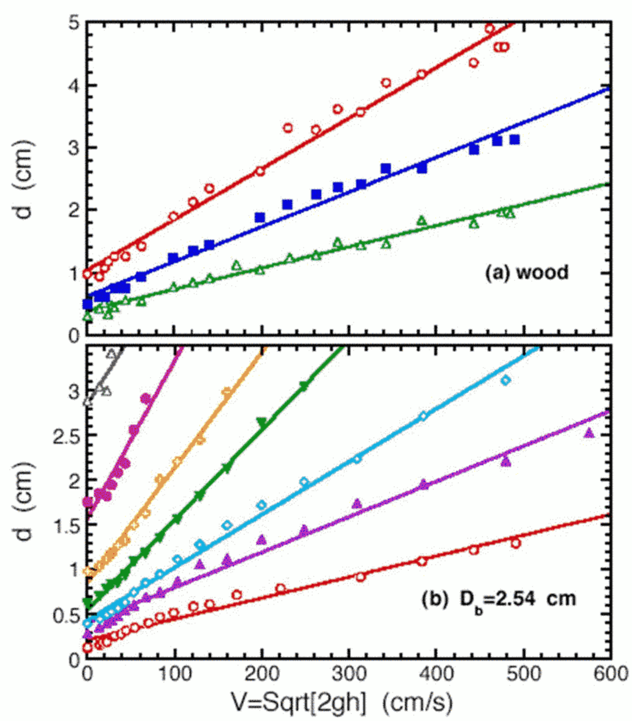
Depth vs impact speed (II)

- all data show subtle deviation from linearity

one might conclude

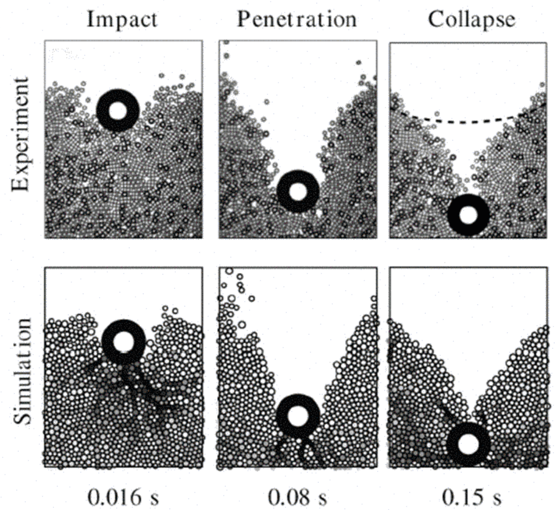
$(d-d_o) \sim V$

over smaller range of impact speeds

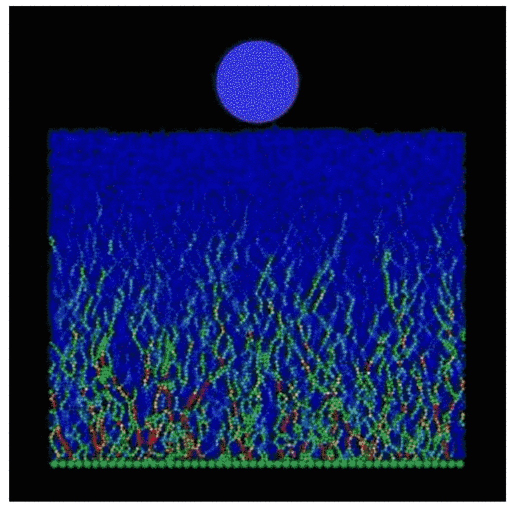


Force chains - simulation

- in two dimensions

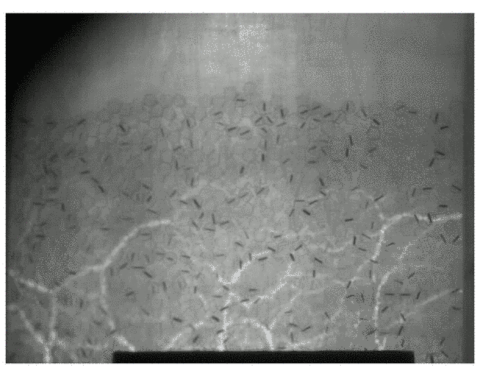


Swinney group [PRL 2004]



Tsimring & Volfson [unpublished]

Force chains - experiment



Behringer group [Chaos 2004]

Planetary craters are circular

- But impacts are not normally normal…

  volcanic activity? NO

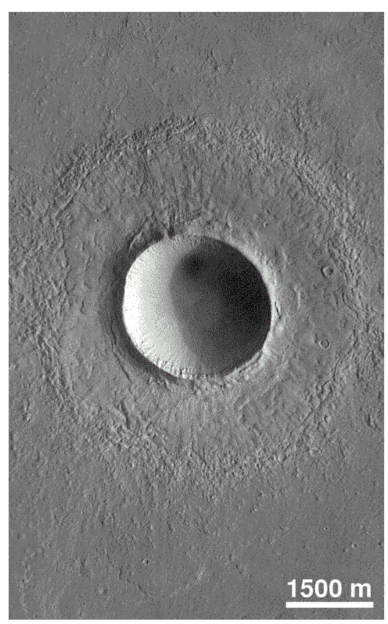
  impact = explosion? YES

Geophysics research focuses on the connection between impact energy and crater diameter

Two scaling regimes

- Assume depth is proportional to diameter

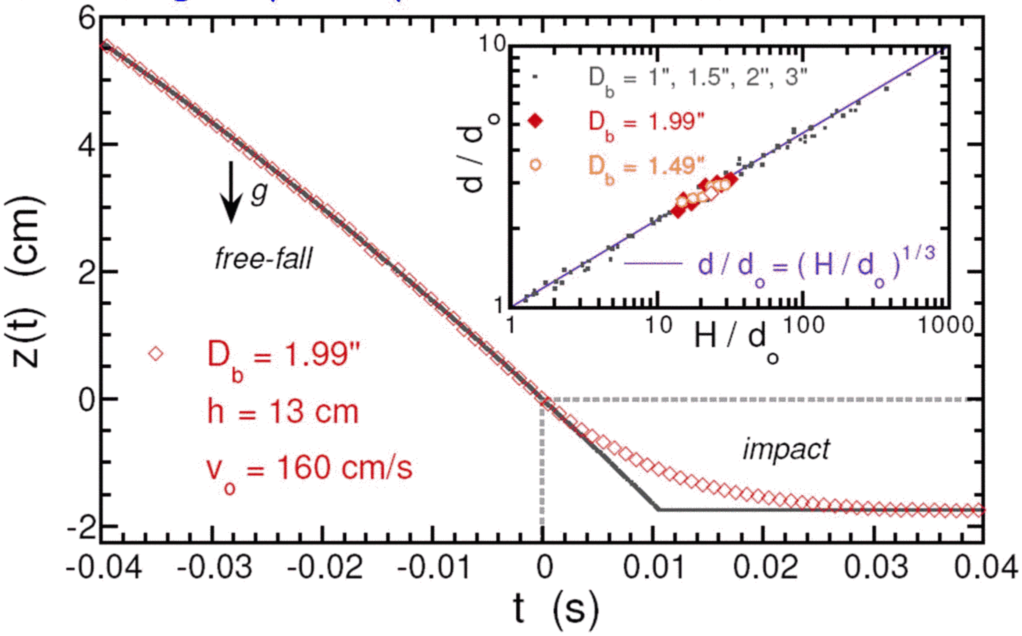
[1] energy is dissipated throughout the whole crater volume…
- diameter ~ $(\text{energy})^{1/3}$
- small "simple" craters, eg:



[2] material must also be lifted against gravity…
- diameter ~ $(\text{energy})^{1/4}$
- large "gravity-limited" craters

Example depth vs time

{showing only every 1/100th datum}

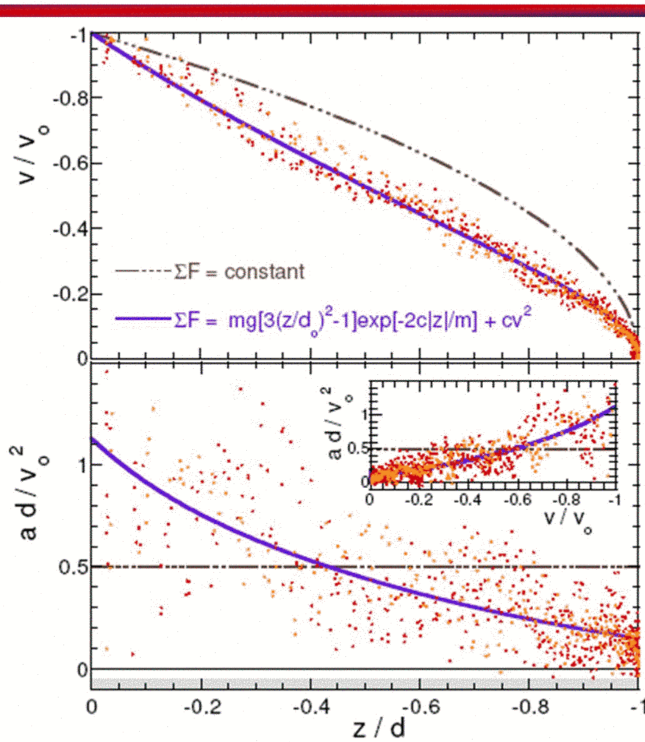


Fit to V vs Z data

- excellent fit!

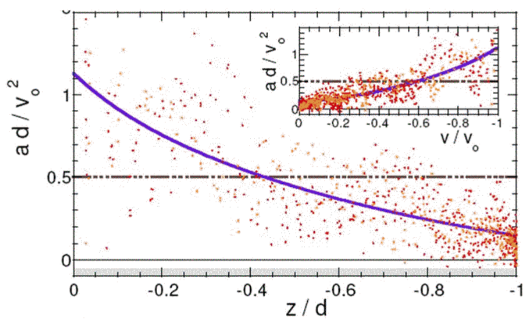
$d/d_o = 2.8$
- measured

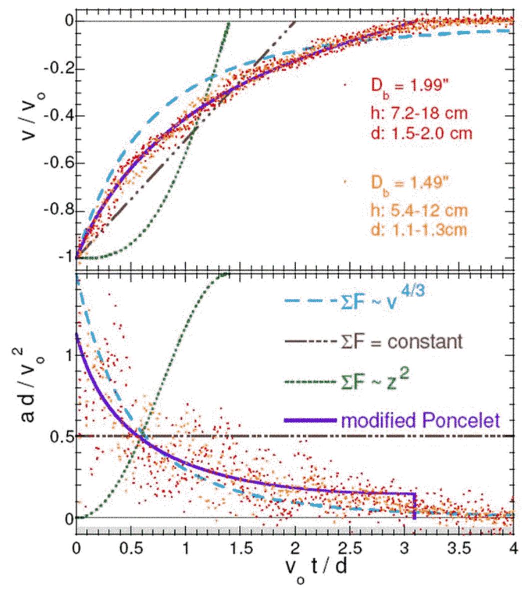
$d/d_1 = 1.2$
- fit parameter



Acceleration Discontinuity

- About 2g





Jun's first experiment…

Uehara, Ambroso, Ojha, DJD [PRL 2003]

- drop various balls into 0.2 mm glass beads
- drop 1" nylon balls into various media

ball	ρ_b (g/cm^3)	D_b (cm)
hollow polypropylene	0.26	2.54
polypropylene	0.82	1.59
wood	0.83	1.59
nylon	1.1	1.59
nylon	1.1	2.54
silicon rubber	1.1	1.52
acrylic	1.2	1.59
live ball	1.2	3.82
dead ball	1.3	3.82
delrin	1.4	1.59
teflon	2.2	1.59
ceramic	3.9	1.00
stainless steel	7.9	2.54
lead	11.3	1.13
tungsten carbide	16.4	1.91

Material	Grain Size (mm)	ρ_g (g/cm^3)	θ_r
sprinkles	2×7	0.76	39°
popcorn	$4 \times 6 \times 7$	0.87	34°
rice	2×7	0.88	32°
salt	0.5	1.30	38°
glass beads	0.2 or 1.0	1.51	24°
beach sand	0.5 ± 0.4	1.59	38°