DENSE GRANULAR FLOWS: QUESTIONS TO KINETIC THEORY

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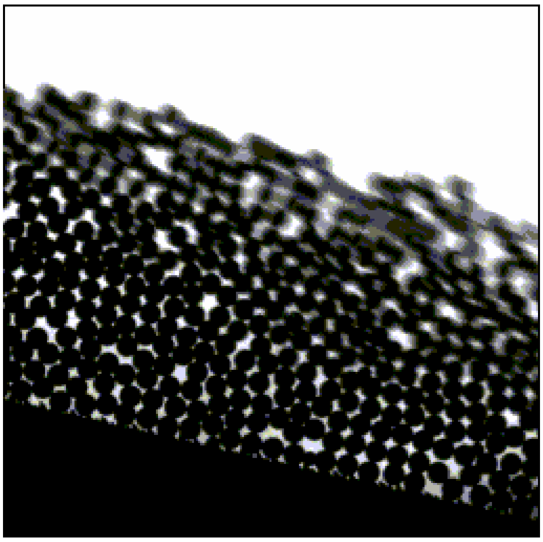
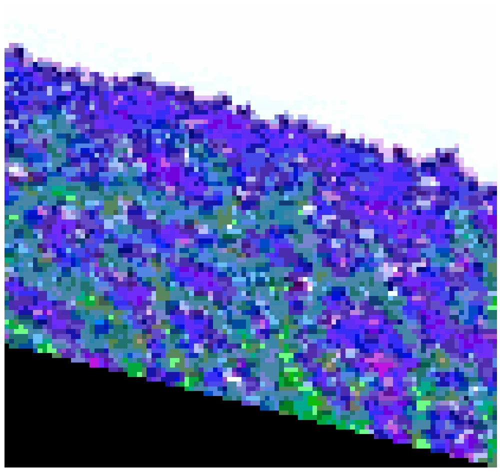


real picture, J.R. 2003



simulations, L. Staron, Ph.D. 2003

KINETIC THEORY

Key theoretical papers

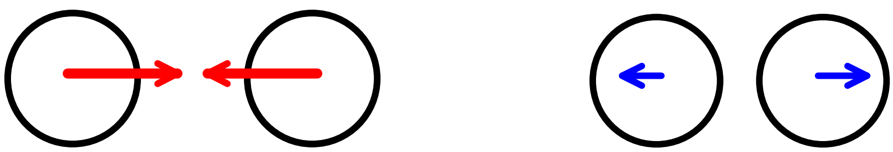
Haff, J. Fluid Mech. 134, 401 (1983)
Jenkins, Richman, Phys. Fluids 28, 3485 (85)
etc....
Extensive reviews: Savage, (1993)
Shen, Babi☝ (1999)
Goldhirsch (2003)

- **Basis hypothesis: binary collisions**

time between collisions >> contact duration

Conservation laws

- **matter as usual**
- **momentum as usual**
- **energy: take into account of inelasticity**



Inelastic sink $\Delta E = 1/4\ M\ (1- e^2)\ n_{12}\ (v_1 - v_2)^2$
(Remark: m**omentum conserved** in the c. o. m. frame)

Constitutive relationship

σ_{xy} corresponds to a viscous process: momentum diffusion associated with free flights and binary collisions.

Granular gas

- *homogeneous shearing*

shear rate $\dot{\gamma} \propto (1-e^2)^{1/2} \lambda^{-1} T^{1/2}$

shear stress $\sigma_{xy} \propto (1-e^2)^{-1/2} \lambda^2 \dot{\gamma}^2$

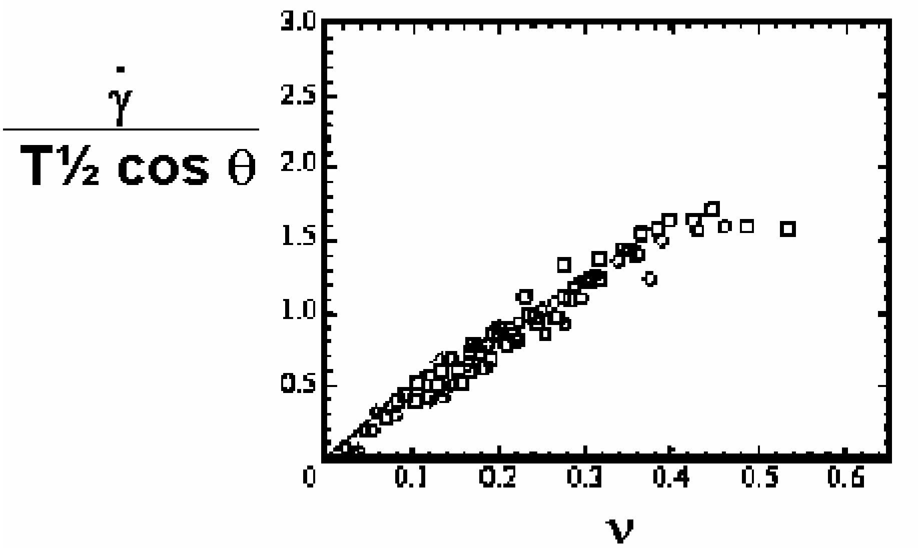
(Bagnold 54)

observed time between collision $\dot{\gamma}^{-1} \approx 0.1$ s
(molecular gas: 10^{-9} sec.)

Bagnold's scaling

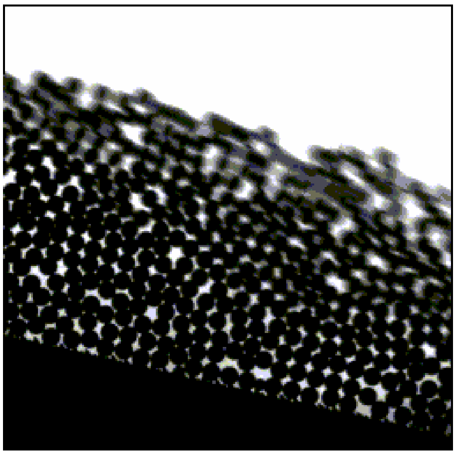
$$\frac{\dot{\gamma}}{T^{1/2}} \propto (1-e^2)^{1/2}\,\lambda^{-1}$$

good agreement with experiments conducted with dilute flows ($\lambda \gg d$)



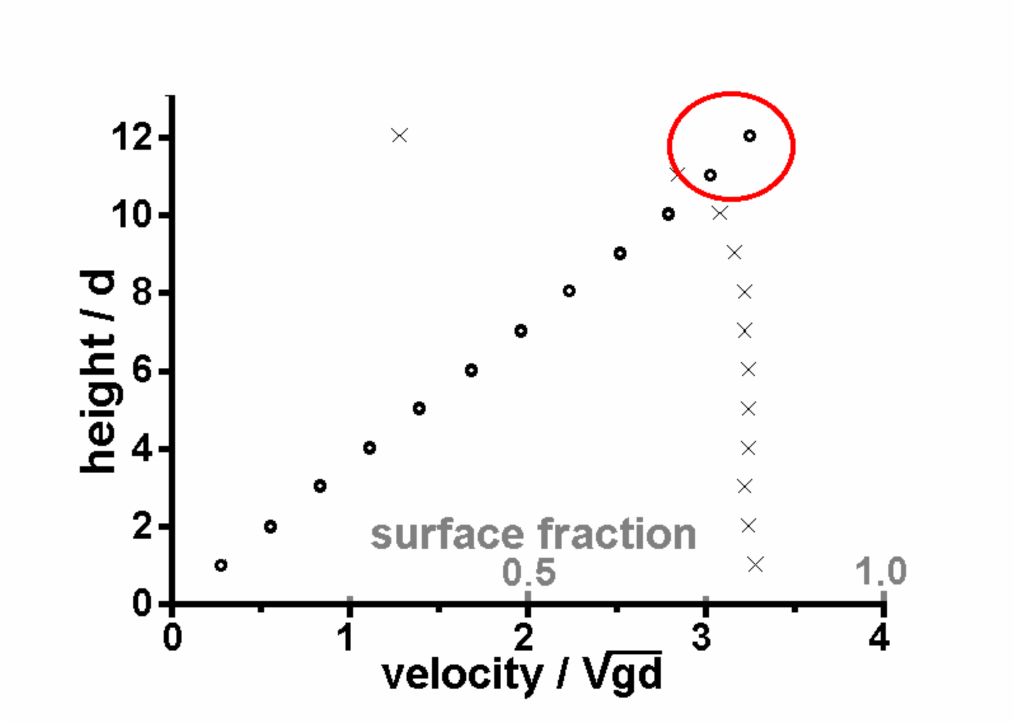
Azanza, Chevoir, Moucheront J. Fluid. Mech.(1999)

Experimental results for gravity-driven concentrated flows



Remark: no rebound ! The effective restitution coefficient is zero (→ dissipation time << $\dot{\gamma}^{-1}$)

Density and velocity field



For observed steady regimes (range of angles θ sloped between 21° and 28°),

- **The density is $\simeq$ constant, $\nu \simeq 0.8$ (random close-packing). All grains are in contact with their nearest neighbors.**

- **velocity fluctuations** (more exactly, sampling of displacements) $\approx$ **constant.**

- **velocity profile is fairly linear in the flowing layer.**

$\dot{\gamma} =$ **const.** (order of magnitude $\sqrt{\frac{g}{d}}$)

- **rheology independent of the elastic restitution coefficient.**

- **nonzero shear rate at the vicinity of the free surface ! !**

in strong opposition with the prediction of kinetic theories

Prediction of kinetic theories

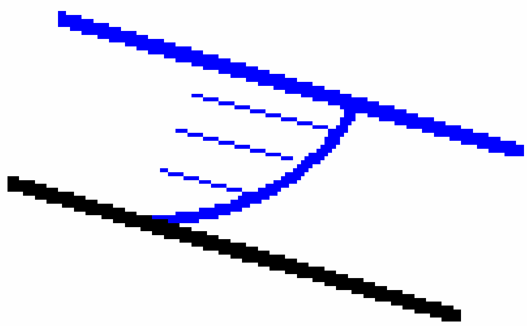
For T = const. and ρ = const.

- shear stress $\sigma_{xz} \propto \dot{\gamma}^2$ (Bagnold's law)
- gravity force = $\rho g \sin\theta$

$$\rightarrow \left(\frac{\partial v_x}{\partial z}\right)^2 \propto \rho g z \sin\theta$$

$$v_x \propto (\rho g h^3 \sin\theta)^{1/2} \left[1 - \left(\frac{z}{h}\right)^{3/2}\right]$$

(Bagnold's profile)



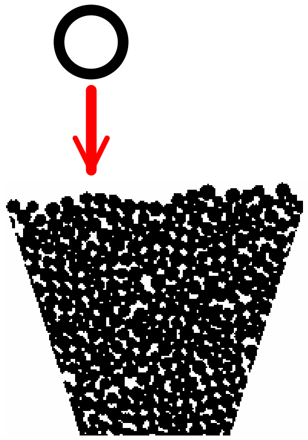
$\dot{\gamma}$ = 0 at the free surface

(σ_{xy} = 0)

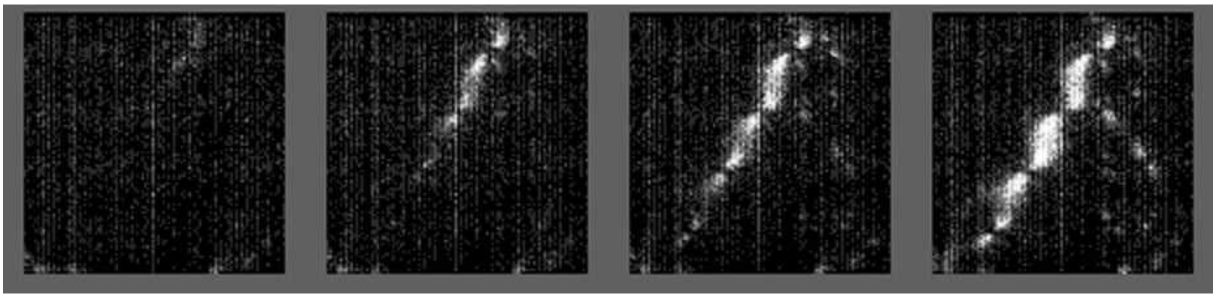
Discussion on the dissipation time.

- First experiment

Let us drop a bead onto a dense piling.



- **no bounce**.
- Visualization of **acoustic waves** (photoelasticity)



sound wave velocity: here 50 m/s (PMC beads)

steel beads 500 m/s < v_S< 1 km/s

The whole substrate is involved in the momentum absorption process

- **Passing time of the sound wave through a grain** (diam. = 1 mm, steel)

τ_S= diam. / v_S= 2 μs (PMC τ_S =20 μs)

to be compared with $\dot{\gamma}^{-1}$ **($\cong$ 0.1 s)** in a gravity driven flow.

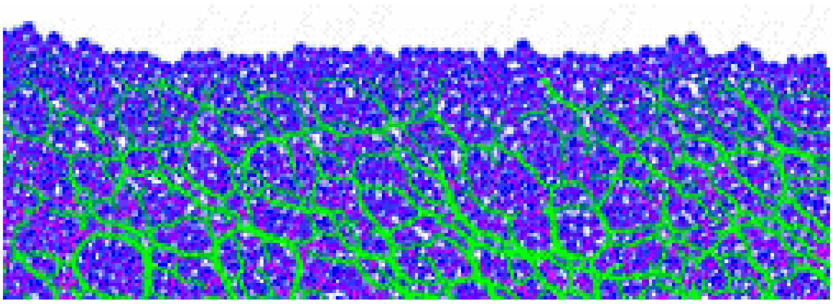
$\rightarrow$ **dissipation time << $\dot{\gamma}^{-1}$**

explains why there is no bounce

($e_{eff.}$ = 0)

- **(Partial) CONCLUSION**

We have to take account of deformation waves through continuous paths of contacts for the long range transport of momentum and for the energy dissipation.



L. Staron 2003

Couples of virtually **colliding particles** cannot be considered as **isolated**.

Dense-packed materials:

- No free flights
- grain motion constrained by steric hindrance
- multibody collisions

• In case of collapsed material, the main channels for dissipation are acoustic wave damping and friction, not viscosity.

• dissipation time << $\dot{\gamma}^{-1}$.
Collisions appear as fully inelastic (no rebound), whatever the elastic restitution.

• the momentum is not conserved in center of mass referential of the couple of virtually colliding particles.

opposes the basis assumptions of the kinetic theory.

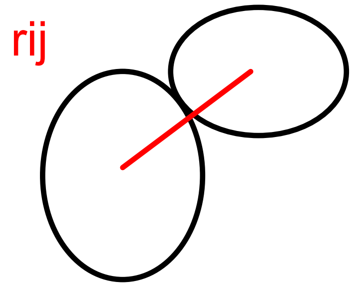
Stress tensor

- **dilute gas**

$$\sigma = (1/2V) \sum_{ij} \mathbf{p}_i \otimes \mathbf{v}_i$$

- **dense-packed materials**

contact force contribution



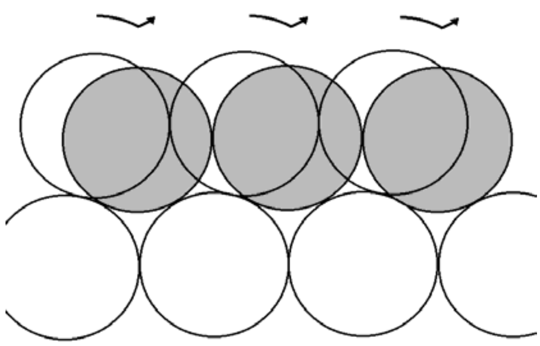
$$\sigma = (1/2V) \sum_{ij} \mathbf{f}_{ij} \otimes \mathbf{r}_{ij}$$

f_{ij} contact force

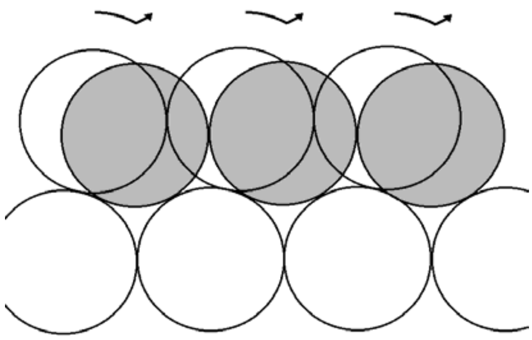
$$\frac{\sigma_{xy}}{\sigma_{yy}} = \tan \theta_c \qquad \text{(rate - independent)}$$

dynamic contribution:

Take into account impulsive transfer of momentum



Skip the hypothesis of **binary collisions**. Simplifying assumption: the **momentum** gained between two successive collisions (delay time $\dot{\gamma}^{-1}$) is totally **transferred** to **acoustic** waves



momentum discontinuity $\propto$ $m\, d\, \dot{\gamma}$

frequency of collision $\dot{\gamma}$

damping term as

$$\left(\frac{Dp}{Dt}\right)/\mathbf{coll} \propto -d\, \dot{\gamma}^2$$

to be compared with the Bagnoldian form (stipulating binary collisions and local momentum conservation)

$$\sigma_{xy} = f(\nu)\,(1-e^2)^{-1/2}\, d^2\, \dot{\gamma}^2 \quad \text{(Bagnold's stress)}$$

$$\rightarrow \quad \frac{Dv}{Dt}\,/\mathbf{coll} \propto -d^2\, \frac{\partial}{\partial z}\, \dot{\gamma}^2$$

Frictional effects

Experimental outcomes

• increasing intergrain friction **decreases** the flow rate, but the velocity profile remains **linear.**

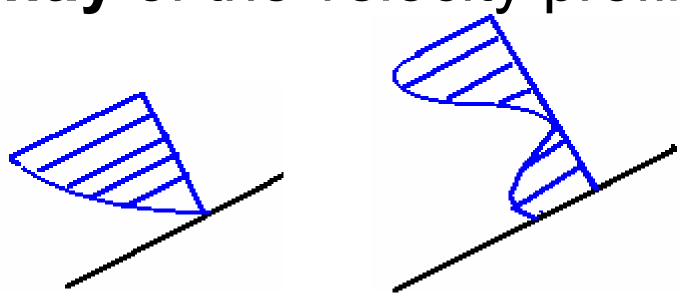
(decrease of the flow rate ← larger part taken by friction losses vs inelastic losses)

• friction losses are **proportional** to the **flow rate**,

$$W_{frict} = \int k\rho gz \cos\theta \; \frac{\partial v_x(z)}{\partial z} \, dz \quad (\rho \cong Cte)$$

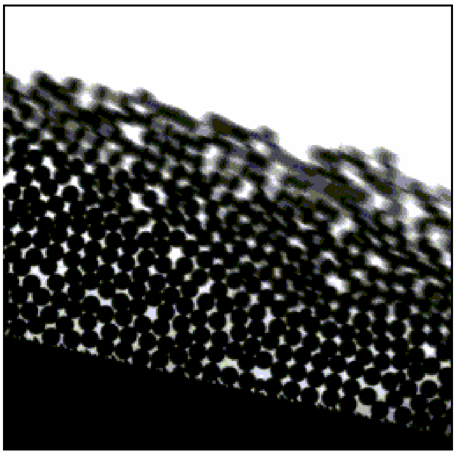
$$= - (k\rho g\cos\theta) \int v_z \, dz$$

independently of the velocity profile



so that friction **cannot impose** the velocity profile shape.

Dense gravity-driven flow down an incline plane ($\theta \to \theta_C$).



Momentum equation

$$\frac{Dv}{Dt} = g \sin\theta - g \tan\theta_C \cos\theta - d\,\dot{\gamma}^2$$

gravity friction impulsive transfer

steady regime solution

$$\dot{\gamma} = \left[\frac{\sin(\theta-\theta_C)}{\cos\theta_C}\right]^{1/2} \sqrt{\frac{g}{d}}$$

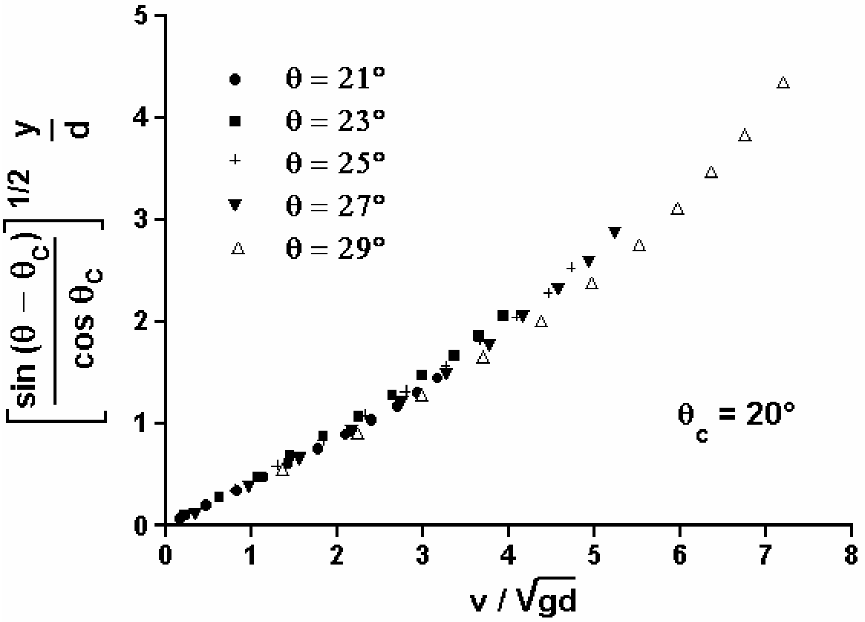
- **linear** velocity profile (rotational momentum neglected).

- **explains why the shear rate** is **nonzero** at the vicinity of the free surface.

Proposed scaling parameters

$$\frac{v}{\sqrt{gd}} = g\left[\left(\frac{\sin(\theta-\theta_c)}{\cos\theta_c}\right)^{1/2}\frac{y}{d}\right]$$

Comparison with experimental data



- **good collapse of experimental data.** No fitting parameters.

- **deviation**, but **unsteadiness** for θ **= 29°.**

- **the closer the slope to** θ_c**, the more linear the velocity profile** (rotational momentum neglected in the above simple modeling).

ratio $\dfrac{\mathbf{W_{frict}}}{\mathbf{E_{inel}}} = \dfrac{\sin\theta_C \cos\theta}{\sin(\theta-\theta_C)}$

divergence as $(\theta - \theta_C)^{-1}$ for $\theta \to \theta_C$.

spontaneous flows in Nature $\theta \cong \theta_C$

for $\theta_C = 20°$
$\theta = 21°$

$$\frac{W_{frict}}{E_{inel}} \approx 18$$

• main part of dissipation of frictional origin.

Conclusion.

- kinetic theory does not hold in the **dense** limit (**multicontact** collisions).

- **rheology of rapid flows:** monitored by **long range momentum transmission** through paths of contacts and **fast dissipation processes** through the bulk and by **dry friction** (not by **viscosity**).

- velocity profile **approximately linear** $\dot{\gamma} \propto \sqrt{\frac{g}{d}}$

prefactor $[\sin(\theta-\theta_C)/\cos\theta_C]^{1/2}$, depends on friction.

- main part of **dissipation** of **frictional** origin.

- **transition** between the **dilute collisional** regime and the **dense-packed regime** governed by the shortest time relevant to transport momentum; i.e. by the **ratio**

$$\frac{\textbf{shearing time}}{\textbf{transit time of sound wave through a grain}}$$

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