The Dynamics of Avalanches and Gravity Currents

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Vallée de la Sionne

Acknowledgments

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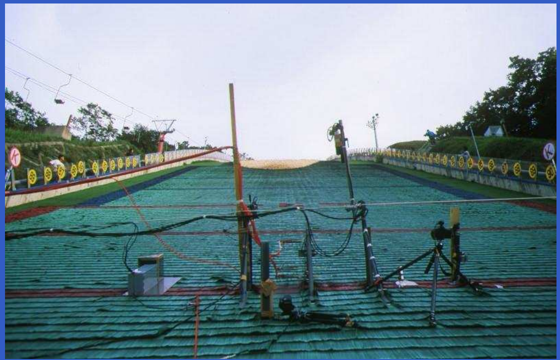
Plan of Talk



- Ping-Pong Balls
- Internal Dynamics
- Lab Experiments
- Field Measurements
- Future Work

Current Avalanche Research

- Huge variety:

speeds	$25\text{–}250\,\mathrm{km/h}$
densities	$5\text{–}500\,\mathrm{kg/m^3}$
masses	$10^2\text{–}10^9\,\mathrm{kg}$

- Three dimensional terrain and structure
- Snow properties are complicated and ill-defined
- Unpredictable, destructive, unreproducible
- Current theories are phenomenological
- Genesis of powder snow avalanches not understood

Questions

- How do powder snow avalanches start ?
- What determines lateral spreading ?
- When is shallow water approximation valid ?
- What are the essential dynamics ?
- What measurements are necessary for validation?

Ping-pong ball avalanches

terminal velocity $v_T = 7.5\,\mathrm{m/s}$

characteristic length $v_T^2/g^* = 6\,\mathrm{m}$

ball-ball restitution 0.8

ball-ground restitution $= 0$

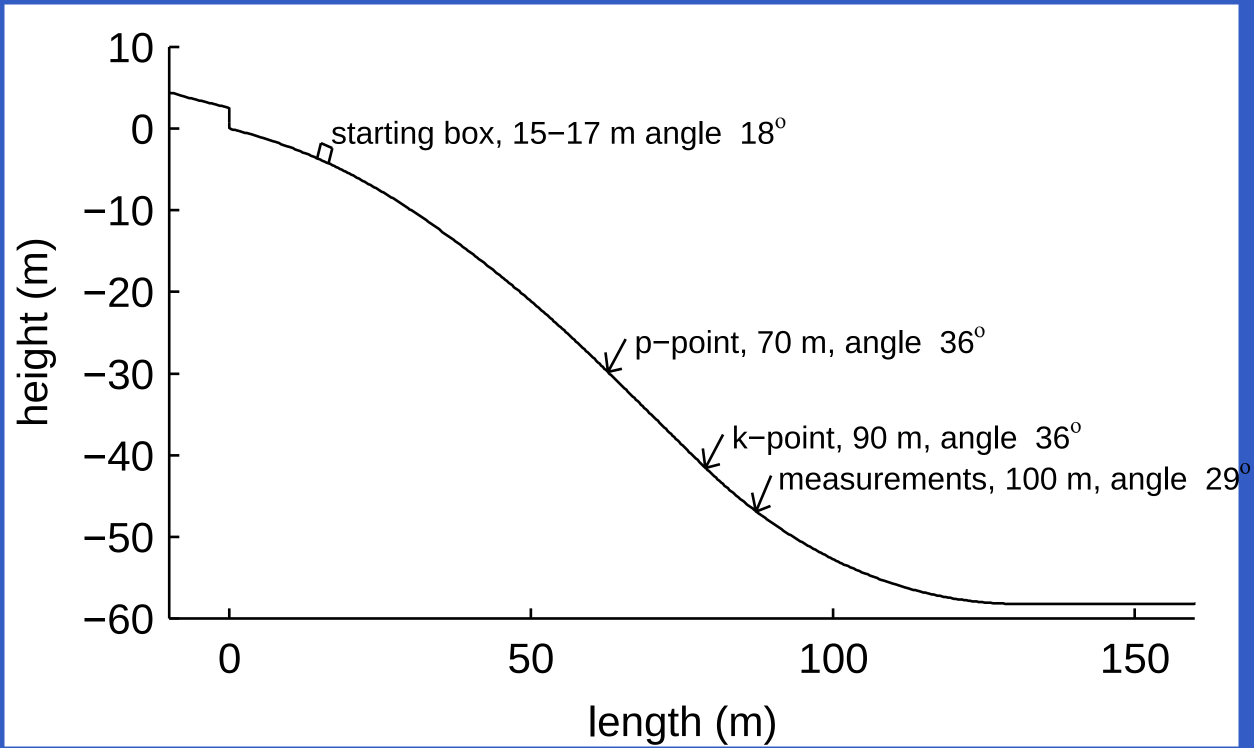
collision time $O(10^{-3}\,\mathrm{s})$

mass $m = 25\,\mathrm{g}$

radius $r = 19\,\mathrm{mm}$

up to 550,000 balls

friction $\mu = 0.3$

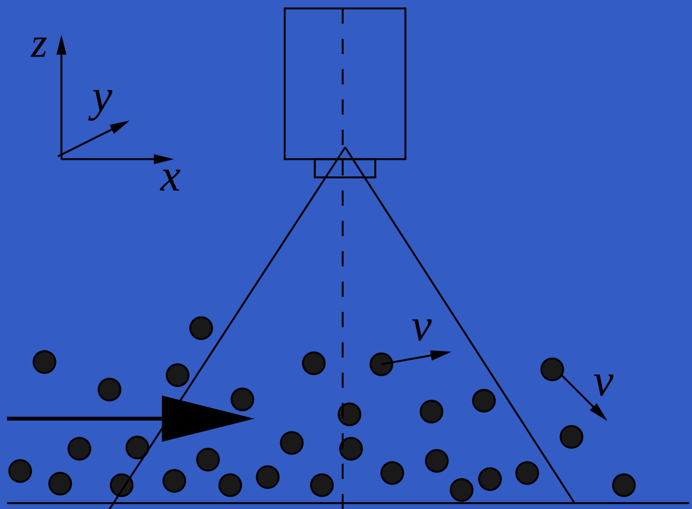


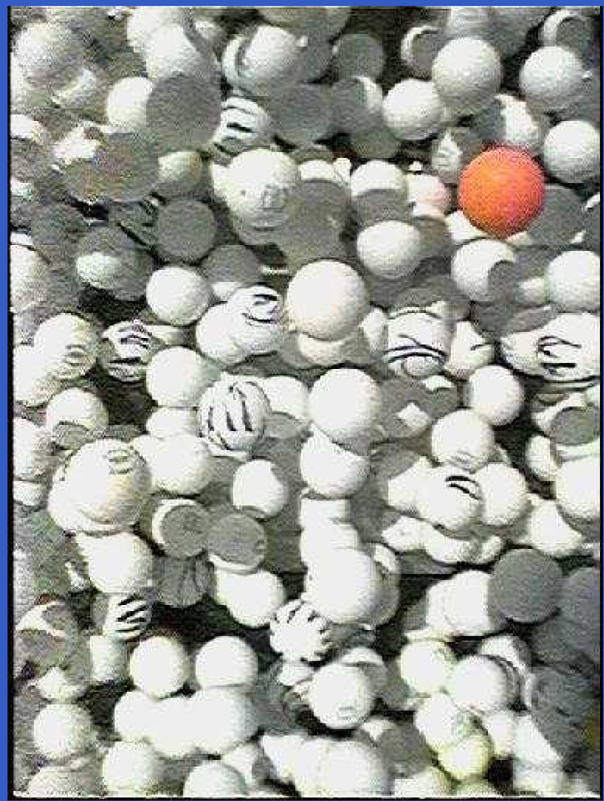
10,000 balls

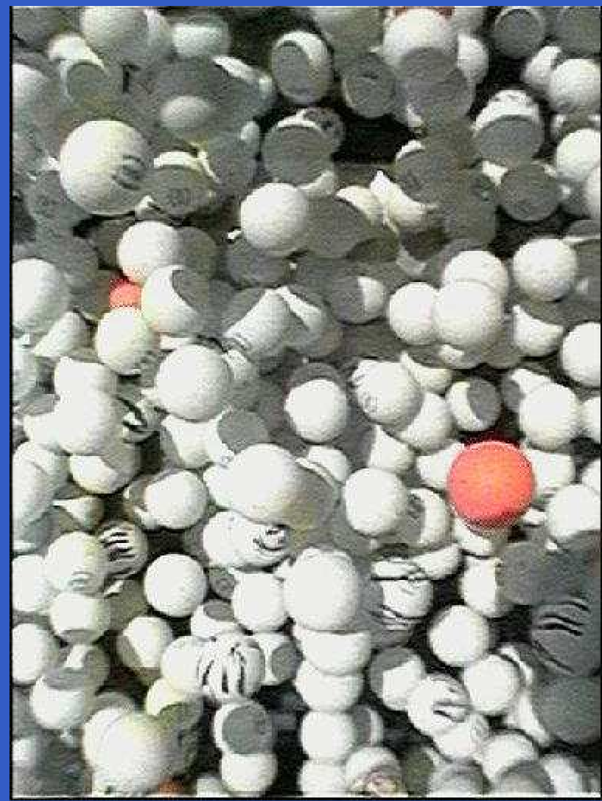
350,000 balls

Individual Ball Measurements

Ball positions are calculated in 3d with a video camera

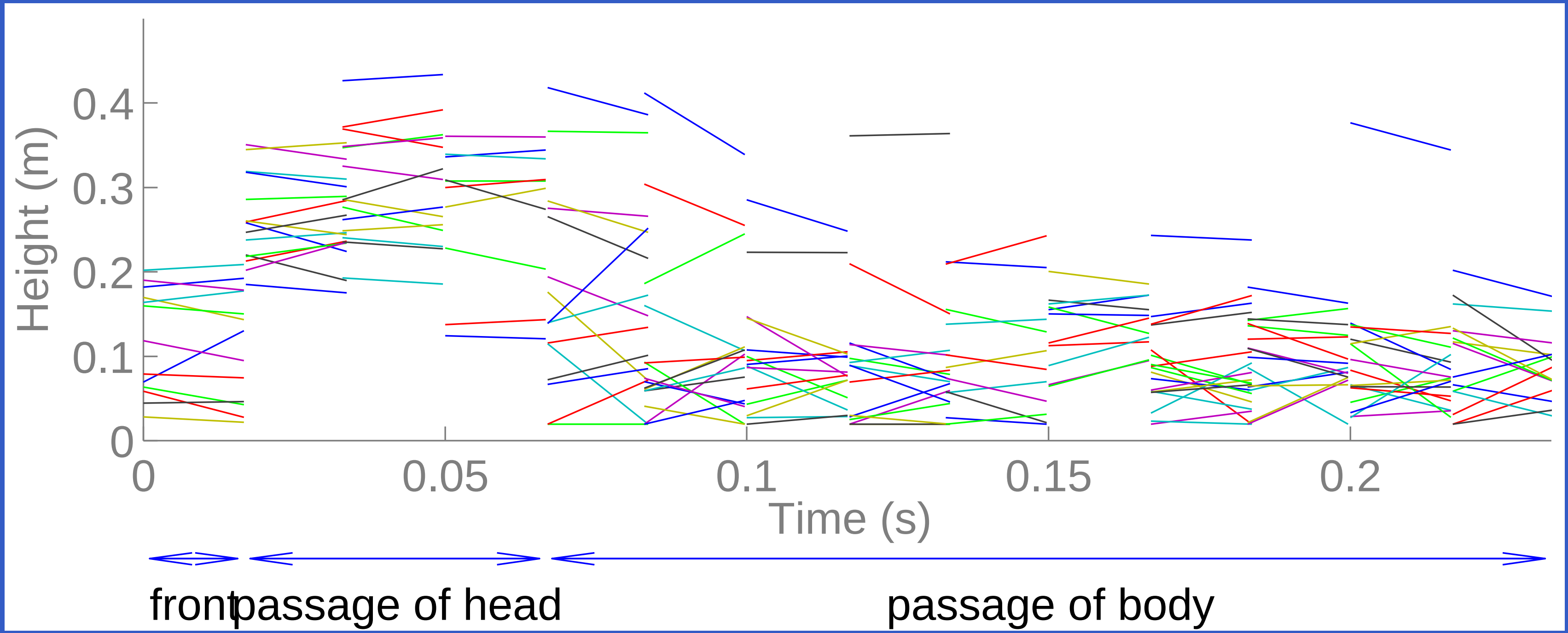






Velocity Distribution

Ball heights in a 200,000 ball avalanche



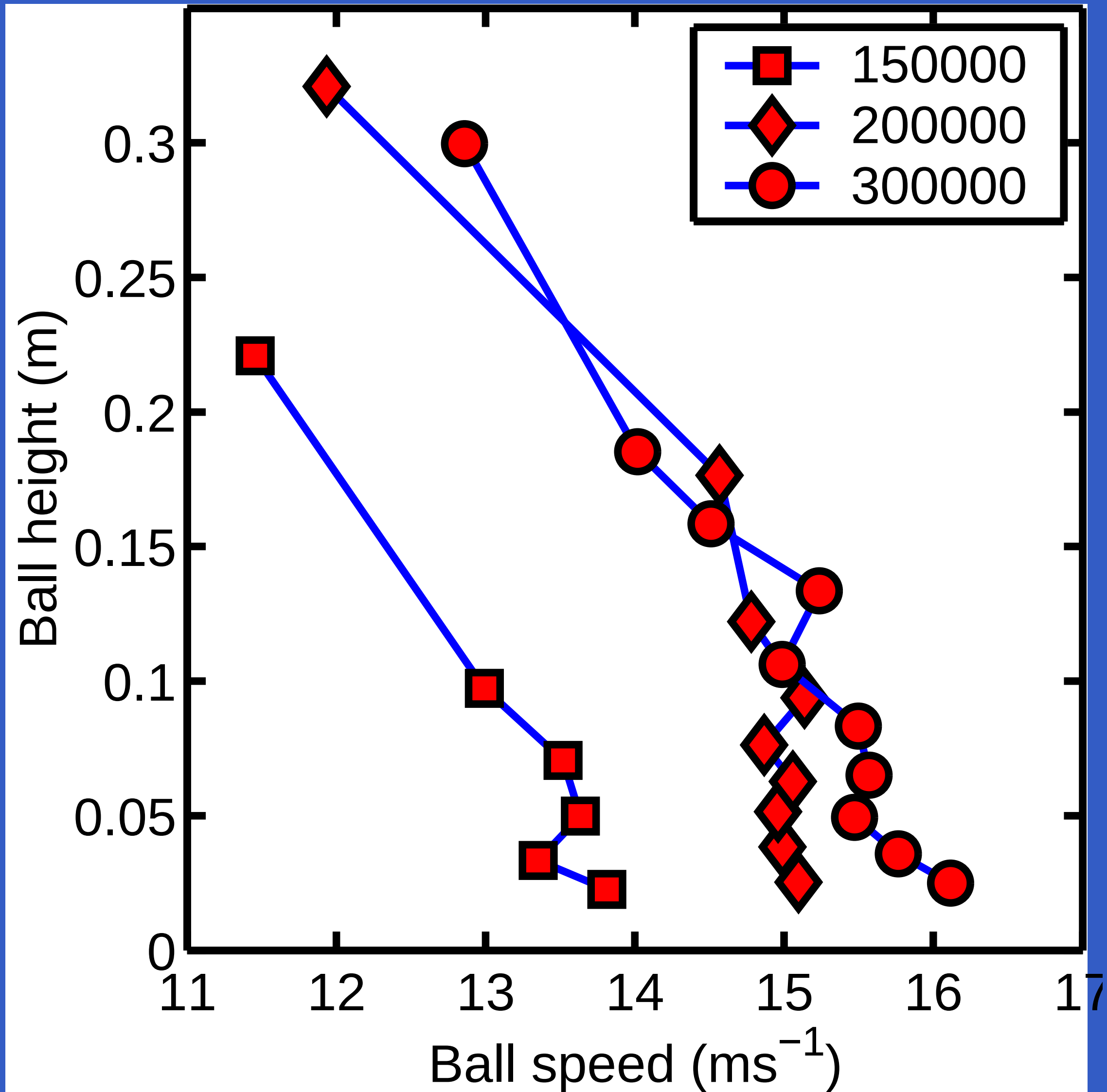
Mean flow velocity is $15\,\mathrm{ms}^{-1}$

Head is $1\,\mathrm{m} \times 0.4\,\mathrm{m}$

Body is $10\,\mathrm{m} \times 0.2\,\mathrm{m}$

Tail is $50\,\mathrm{m} \times$ 1 ball

Vertical profile of average ball x-velocity



Speed highest near ground

Air drag dominates

Kinetic Theory of Granular Matter

Extension of kinetic theory of gases to include inelasticity

$f(\mathbf{c}, \mathbf{x}, t)\, d\mathbf{c}\, d\mathbf{x}$	particle distribution function
	velocity $\mathbf{c}$, position $\mathbf{x}$, time t
$\phi(\mathbf{x}, t) = \pi d^3/6 \int f\, d\mathbf{c}$	volume density
$\mathbf{u}(\mathbf{x}, t) = \langle \mathbf{c} \rangle = \int \mathbf{c} f\, d\mathbf{c}$	mean velocity field
$\mathbf{C}(\mathbf{x}, t) = \mathbf{c} - \mathbf{u}$	fluctuation velocity
$K(\mathbf{x}, t) = \langle \mathbf{C}\mathbf{C} \rangle$	second moment of fluctuation velocity
$T(\mathbf{x}, t) = \mathrm{Tr}[K]$	granular temperature

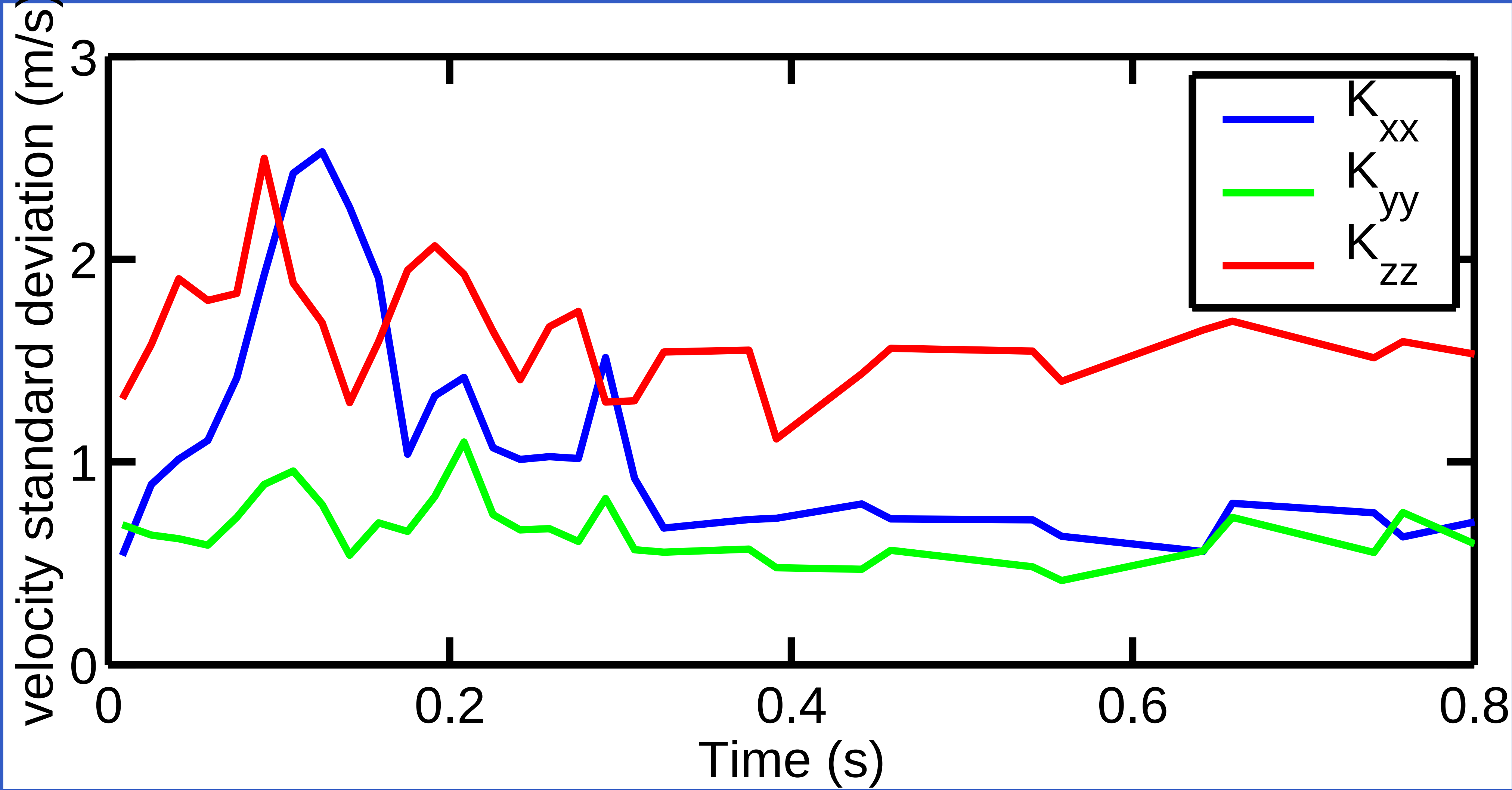
Granular stress tensor

Stress given by

$$\sigma = \phi \rho_b K + m\boldsymbol{\Theta}[\mathbf{C}],$$

ϕ	volume density
ρ_b	particle density
K	second moment of fluctuation velocity
m	particle mass
$\boldsymbol{\Theta}[\mathbf{C}]$	collisional transport

Fluctuation velocities for 300,000 ball experiment

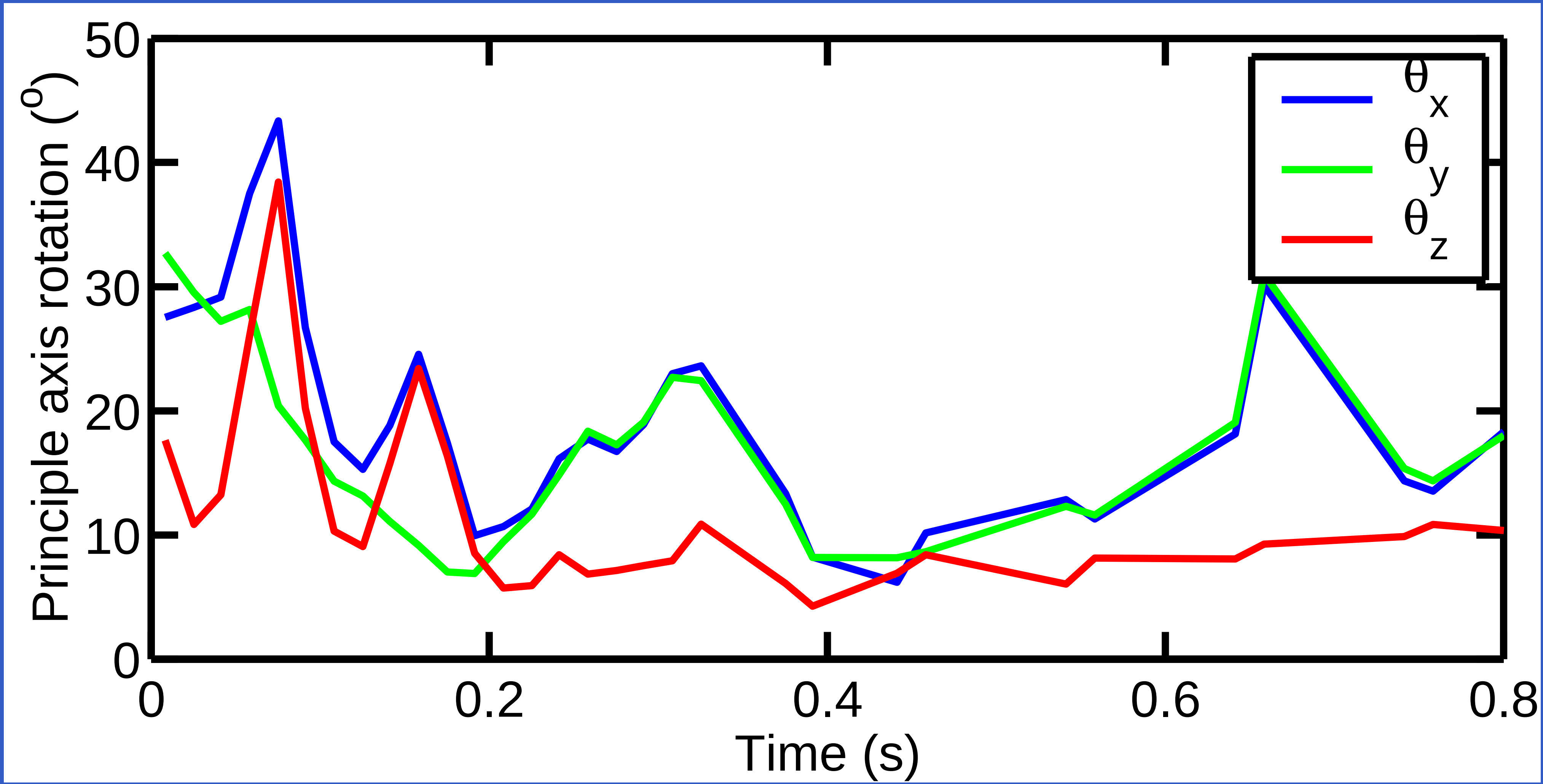


$\sqrt{K_{xx}}$ down-slope $\sqrt{K_{yy}}$ cross-slope

$\sqrt{K_{zz}}$ slope-normal

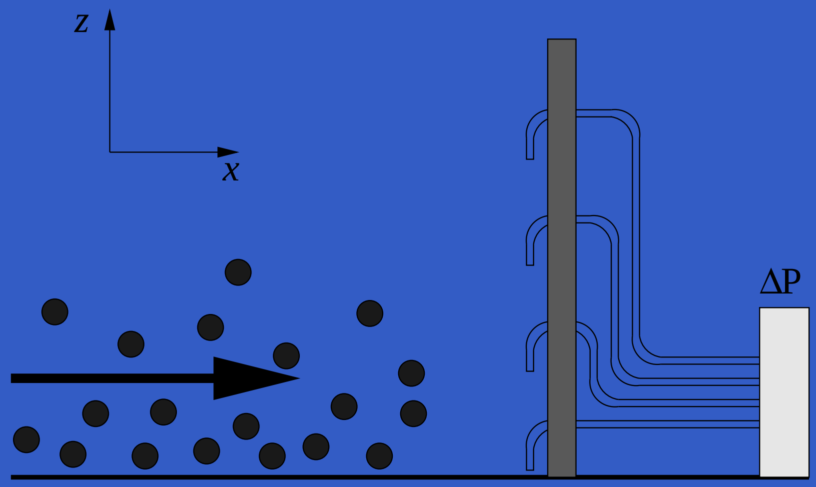
Anisotropy of Granular Stress

Rotation angles of the principle axes of the granular stress tensor for a 300,000 ball experiment

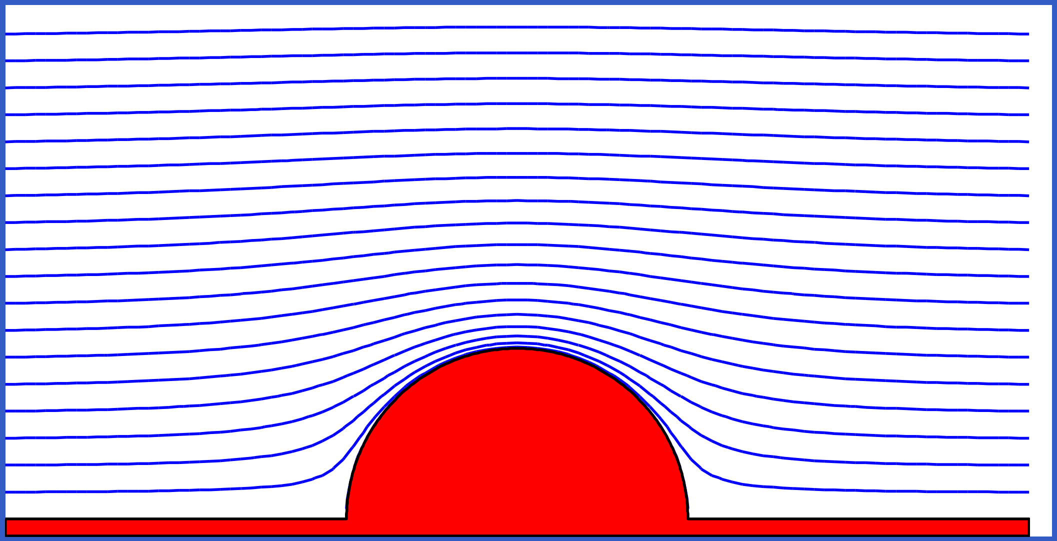


Air Pressure Measurements

Air pressure sensors



flow around a sphere



dipole flow $$\mathbf{v}(\mathbf{x}) = -\mathbf{v} + \frac{R^3}{2x^3}\left(\frac{3\mathbf{x}(\mathbf{x}\cdot\mathbf{v})}{x^2} - \mathbf{v}\right)$$

pressure $$\Delta p(\mathbf{x}) = \frac{\rho_a v^2}{2}\frac{R^3}{x^3}\left[2 - \frac{R^3}{x^3} - \frac{3}{4}\sin^2\theta(4 - \frac{R^3}{x^3})\right]$$

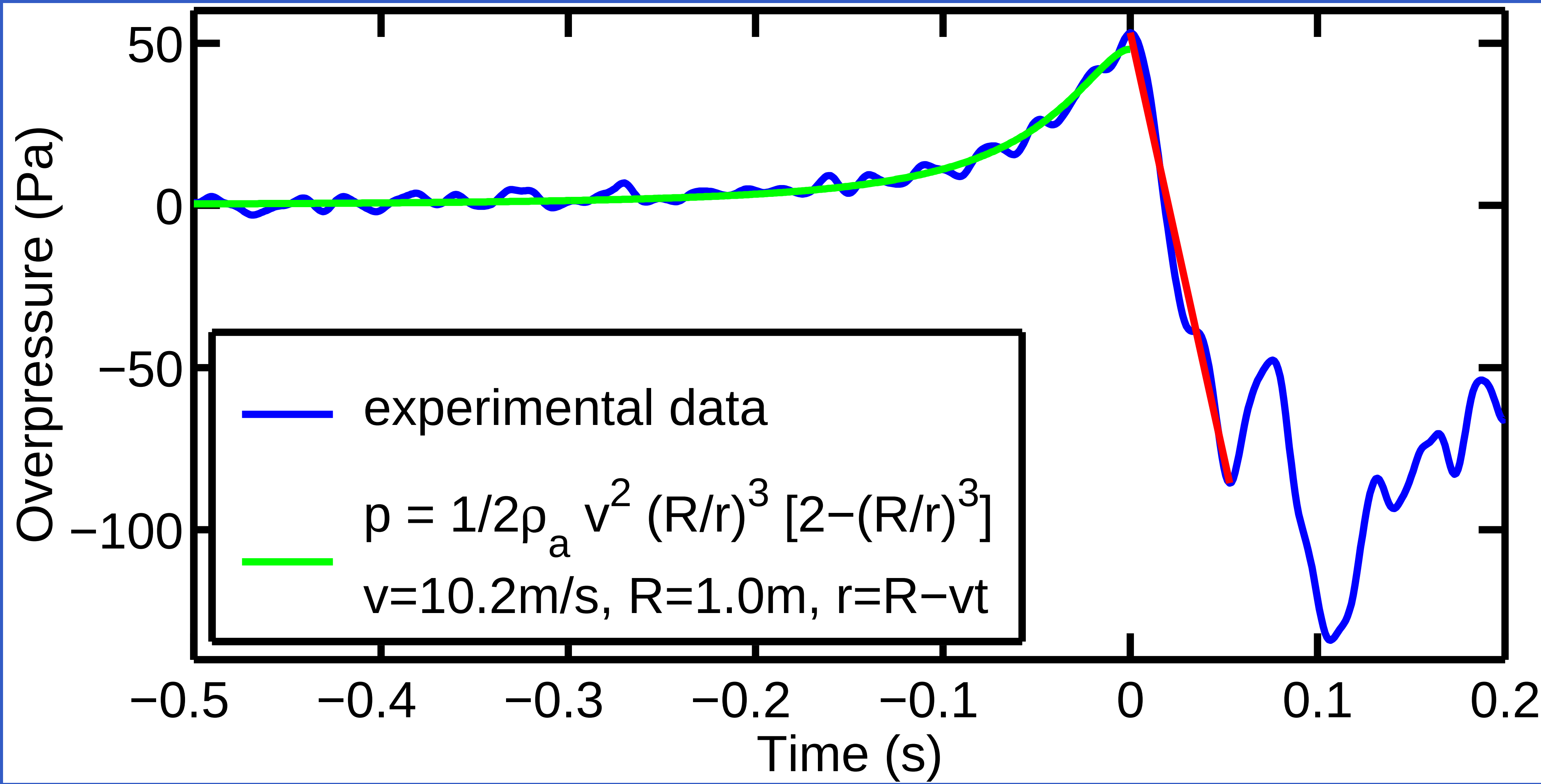
$$x\cos\theta = \mathbf{x}\cdot\mathbf{v}$$

$$\mathbf{x}(t) = \mathbf{v}(R/v - t)$$

Fit to the data

Find the line of best fit to calculate v and R.

Static air pressure change as the front of a 300,000 ball avalanche is advected past the sensor



Air Pressure Results

height	v_1	v_2	$\tilde{v}_1/\tilde{v}_2$	R_1	R_2	$\tilde{R}_1/\tilde{R}_2$	$2^{\gamma'} R_1/R_2$
0.01	7.55	8.16	1.04	0.76	0.88	1.09	1.02
0.15	8.21	9.66	0.95	0.68	0.80	1.06	1.00
0.30	8.88	10.13	0.98	0.87	1.02	1.08	1.01
0.45	6.81	8.96	0.85	0.98	1.08	1.14	1.07

$L_i = dN_i^{1/3}$ length scale

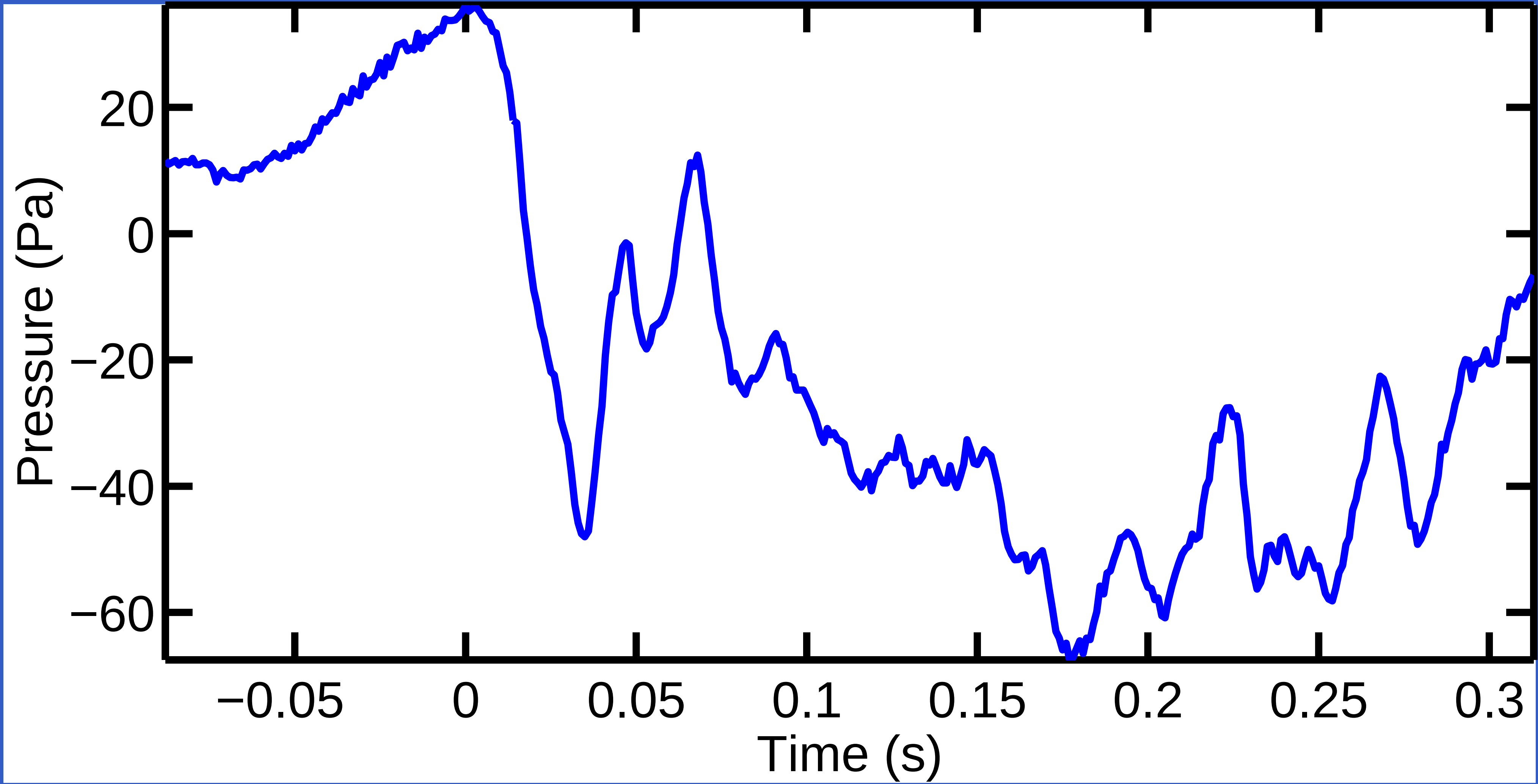
$\tilde{v}_i = v_i/\sqrt{gL_i}$ non-dimensionalised velocity

$\tilde{R}_i = R_i/L_i$ non-dimensionalised radius

$i = 1$ 150,000 balls

$i = 2$ 300,000 balls

Air Pressure Through the Front



Front reaches sensor at $t = 0\,\mathrm{s}$

Decrease due to drag or a vortex?

Steady Equations for the Air Flow

$\nabla \cdot (1-\phi)\mathbf{v} = 0$ mass

$\rho_a(1-\phi)(\mathbf{v}\cdot\nabla)\mathbf{v} + (1-\phi)\nabla p = -\mathbf{f}$ momentum

$\mathbf{f} \approx \phi(1-\phi)\rho_a \mathbf{v}|\mathbf{v}|/\alpha \qquad \alpha = 0.08\,\mathrm{m}$ ball-air drag

$$\frac{\Delta p}{\Delta x} = \frac{\Delta p}{v_b \Delta t} = -\frac{\rho_a \phi (v_b - v_a)^2}{\alpha}$$

$\Delta p \approx -84\,\mathrm{Pa} \quad \Delta t \approx 0.035\,\mathrm{s}$

$v_b \approx 15\,\mathrm{ms}^{-1} \quad v_a \approx 8\,\mathrm{ms}^{-1}$

Therefore $\phi \approx 0.2$

Steady Equations for the balls

$$\nabla(\cdot\phi\rho_b K) + \phi\nabla p = \phi\rho_b \mathbf{g} + \mathbf{f}$$

$$\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} + \frac{\partial\sigma_{xz}}{\partial z} + \phi\frac{\partial p}{\partial x} = f_x - \phi\rho_b g\sin\theta.$$

Symmetry about $y = 0 \Rightarrow \partial/\partial y = 0$
Assume $\partial\sigma_{xz}/\partial z = 0$

$$\frac{\partial\sigma_{xx}}{\partial x} + \phi\frac{\partial p}{\partial x} = f_x - \phi\rho_b g\sin\theta.$$

Integrated to provide a variant of Bernoulli’s law

$$K_{xx}(x) = \frac{p(0) - p(x)}{\rho_b} + x\frac{\rho_a v_0^2}{\rho_b \alpha} - xg\sin\theta$$

Integrated stress balance

$$K_{xx}(x) = \frac{p(0) - p(x)}{\rho_b} + x\frac{\rho_a v_0^2}{\rho_b \alpha} - xg\sin\theta$$

At $t = 0.035\,\mathrm{s}$

$$K_{xx} = 1.9\,\mathrm{m}^2\mathrm{s}^{-2}$$

$$[p(0) - p(x)]/\rho_b = 0.96\,\mathrm{m}^2\mathrm{s}^{-2}$$

$$x\frac{\rho_a v_0^2}{\rho_b \alpha} = 4.5\,\mathrm{m}^2\mathrm{s}^{-2}$$

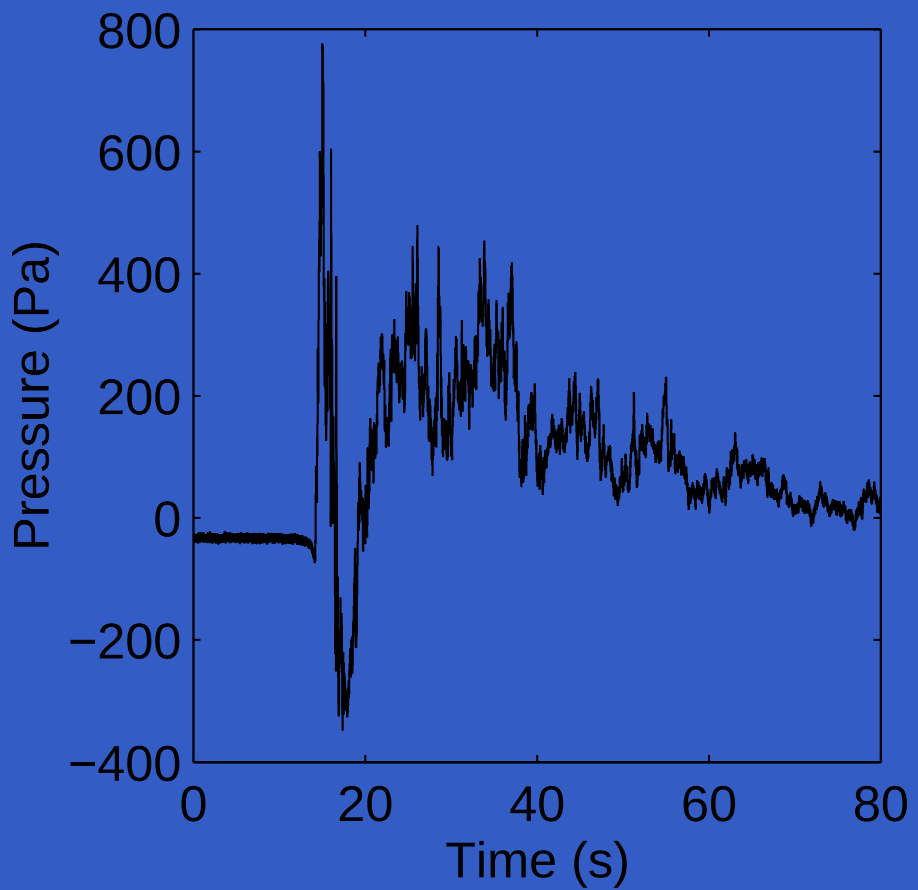
$$xg\sin\theta = 2.5\,\mathrm{m}^2\mathrm{s}^{-2}$$

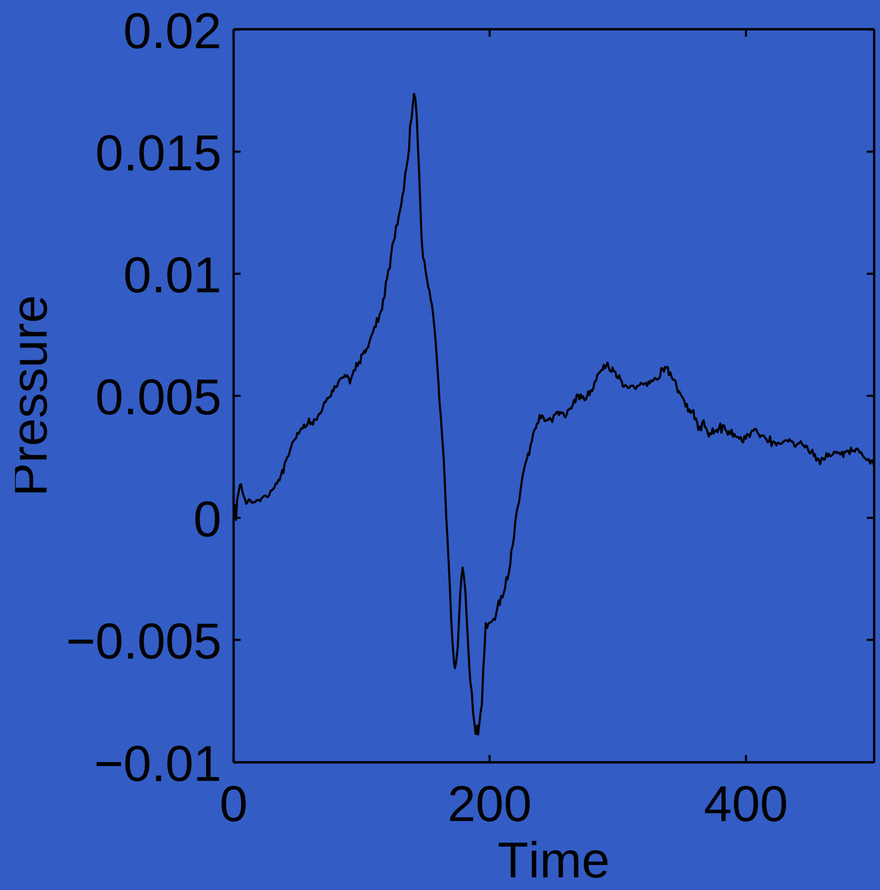
All are same order of magnitude

In the head: interior air drag, granular stress and gravity

In the body: air mixing on the top surface and gravity

Other Air Pressure Measurements

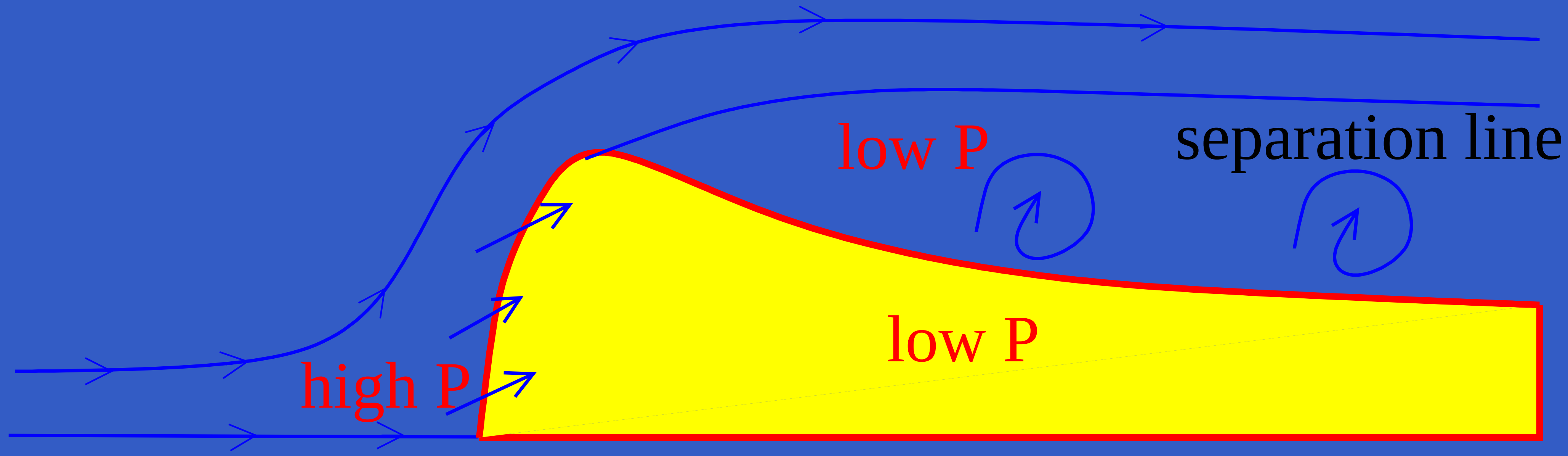




Air pressure data from a powder snow avalanche in Kurobe Canyon (Kouichi Nishimura 1996)

Air pressure data from a direct numerical simulation of a gravity current (Jocelyne Etienne 2003)

Air Flow Around the Avalanche

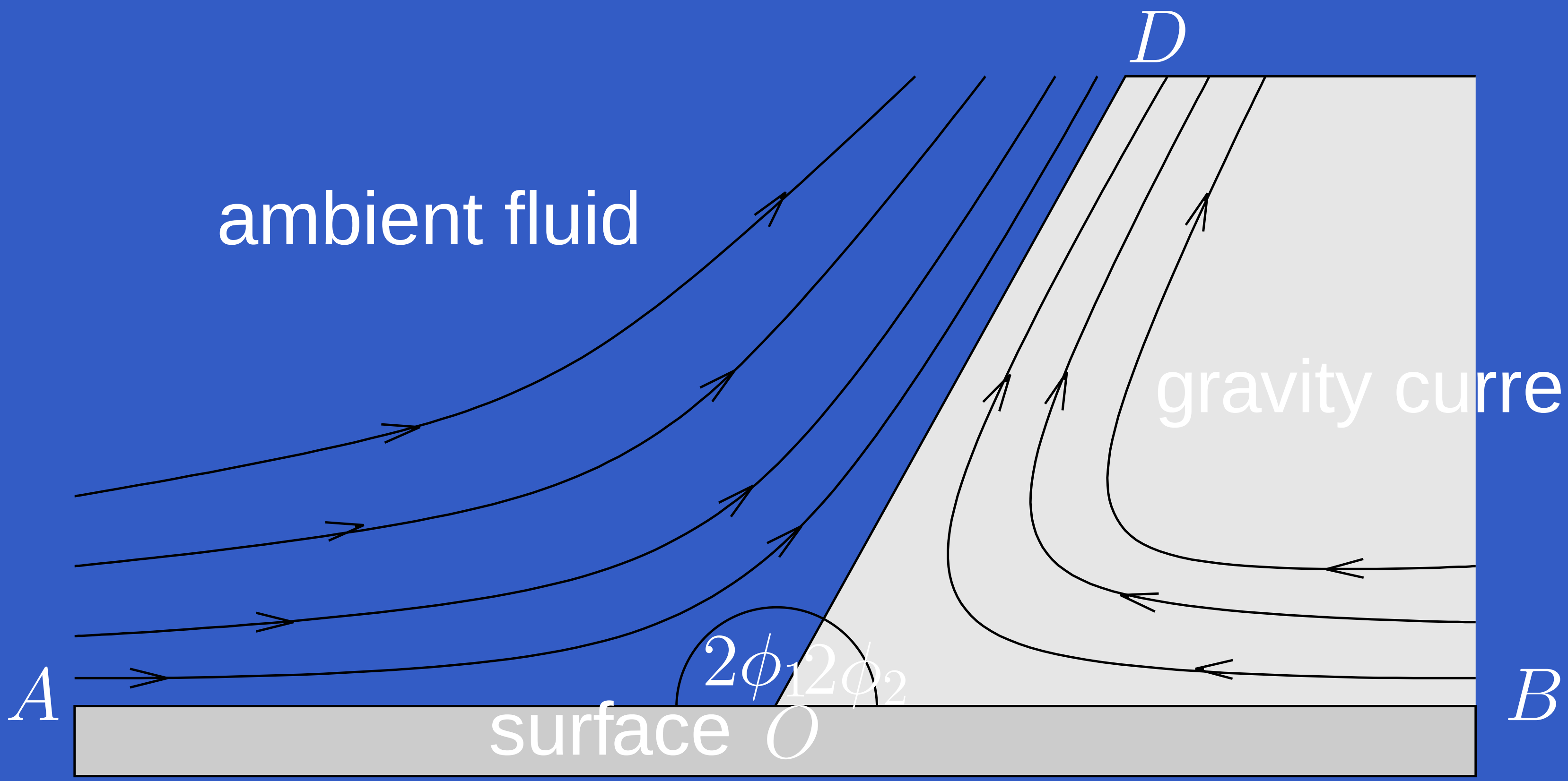


But no slip when flow is much larger than particles
If interior is static then porous flow gives

$$\nabla^2 p = 0 \quad \Rightarrow p = \int^2 p \nabla G \cdot \mathbf{dA}$$

p is average of external pressure

Flow in a wedge



Schematic in the vicinity of the front.

- pressure continuous
- velocity discontinuous
- high Reynolds number

Flow in a wedge

Lowest order term in stream function

$$\psi(r,\theta) = r^{\alpha} f(\theta)$$

Steady Euler equation is

$$0 = \mathbf{u}\cdot\nabla\omega = \alpha h \dddot{f} + (2-\alpha)\dot{f}\ddot{f} + 2\alpha^2 f\dot{f}$$

Integrates to

$$h_\theta^2 = a - h^2 + bh^{2-2\alpha}, \quad h = f^{1/\alpha}$$

$$\int_h^1 \frac{\mathrm{d}\,h}{\sqrt{1-h^2+b(h^{2-2\alpha}-1)}} = |\theta|$$

Special Solutions

α	ϕ	ψ	note
α	$\pi/2$	$(r\cos\theta)^\alpha$	shear
α	$\pi/(2\alpha)$	$r^\alpha \cos(\alpha\theta)$	irrotational
α	$\phi \ll 1$	$1 - \frac{\theta}{\phi} = I_{h^{2(\alpha-1)}}\left(\frac{\alpha}{2(\alpha-1)}, \frac{1}{2}\right) + O(\phi^2)$	small angle
$\ll -1$	ϕ	$r^\alpha \frac{\sin(\phi-\theta)}{\sin\phi} + O\left(\frac{1}{\sqrt{-\alpha}}\right)$	singular
$1/2$	ϕ	$\sqrt{r}\sqrt{1 - \frac{\sin^2\frac{\theta}{2}}{\sin^2\frac{\phi}{2}}}$	singular
1	ϕ	$\approx r\frac{(1+b)\cos\left(\theta\sqrt{1-b}\right)-2b}{1-b}{}^{\dagger}$	
$3/2$	ϕ	elliptic	linear pressure
2	ϕ	$r^2\left(1 - \frac{\sin^2\theta}{\sin^2\phi}\right)$	
$\gg 1$	ϕ	$r^\alpha \cos\theta + O\left(\frac{1}{\alpha}\right)$	$\lvert\theta\rvert < \phi - O\left(\frac{1}{\alpha}\right)$

Boundaries

Can calculate exactly on boundaries

$$\mathbf{u}(\pm\phi) = \mp c\alpha r^{\alpha-1}\hat{\mathbf{r}}$$

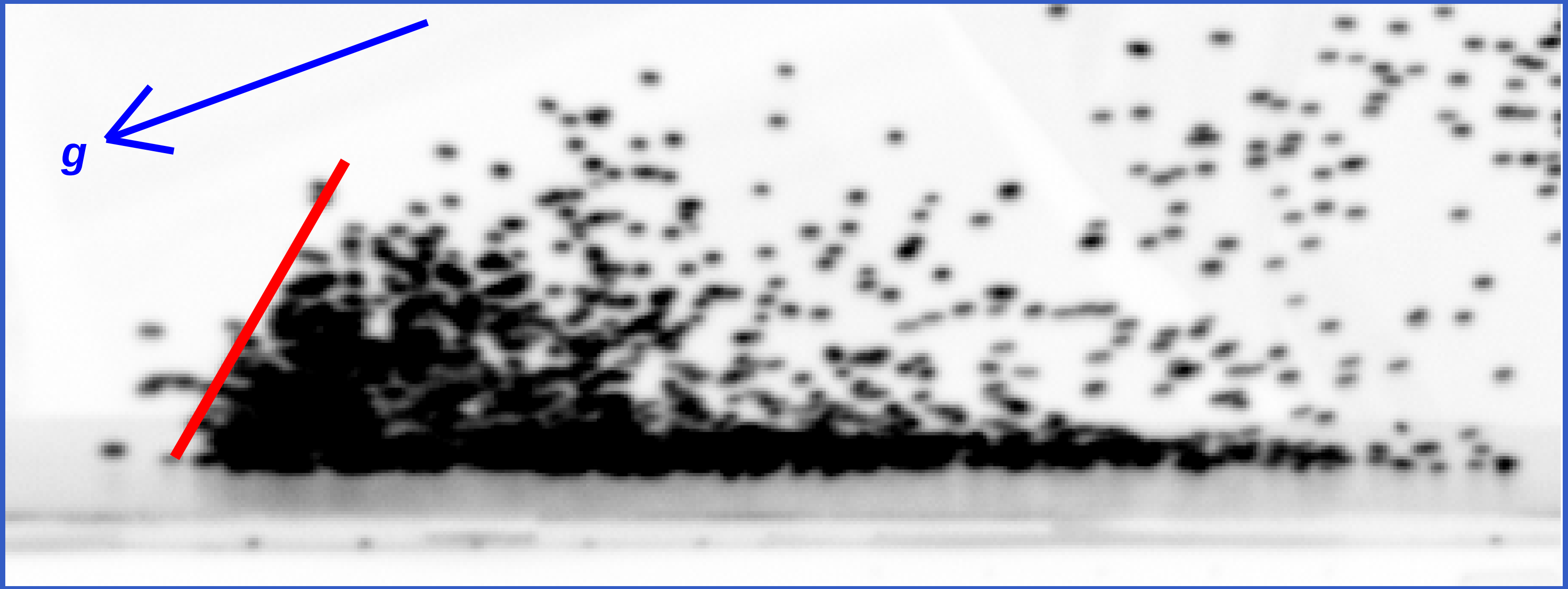
Pressure is only function of r

$$p = \rho r\hat{\mathbf{r}} \cdot \mathbf{g} - \frac{1}{2}\rho c^2\alpha^2 r^{2\alpha-2}$$

External flow must be irrotational therefore

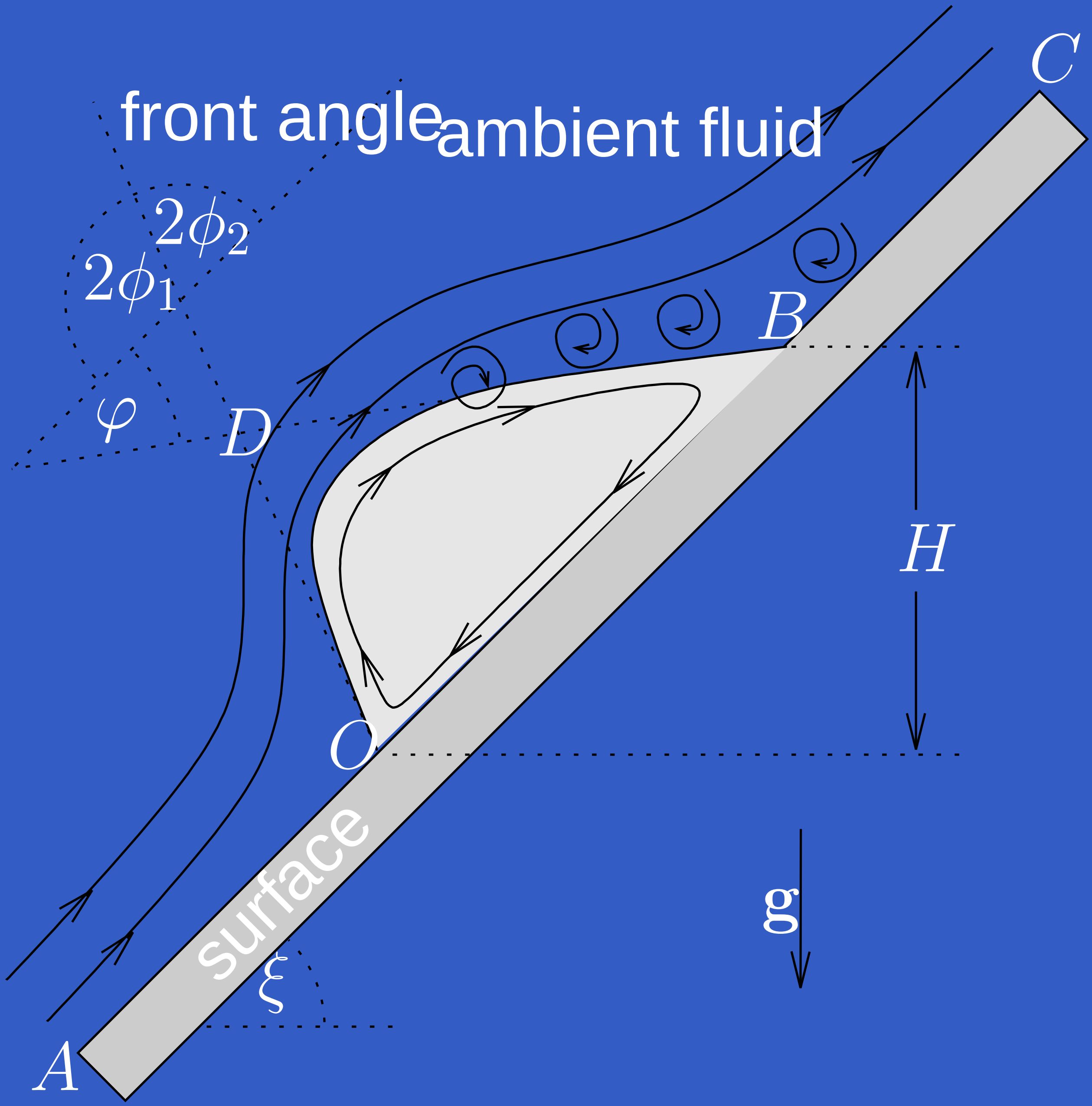
Matching across front forces $\alpha = 3/2$ and 60°

Polystyrene balls

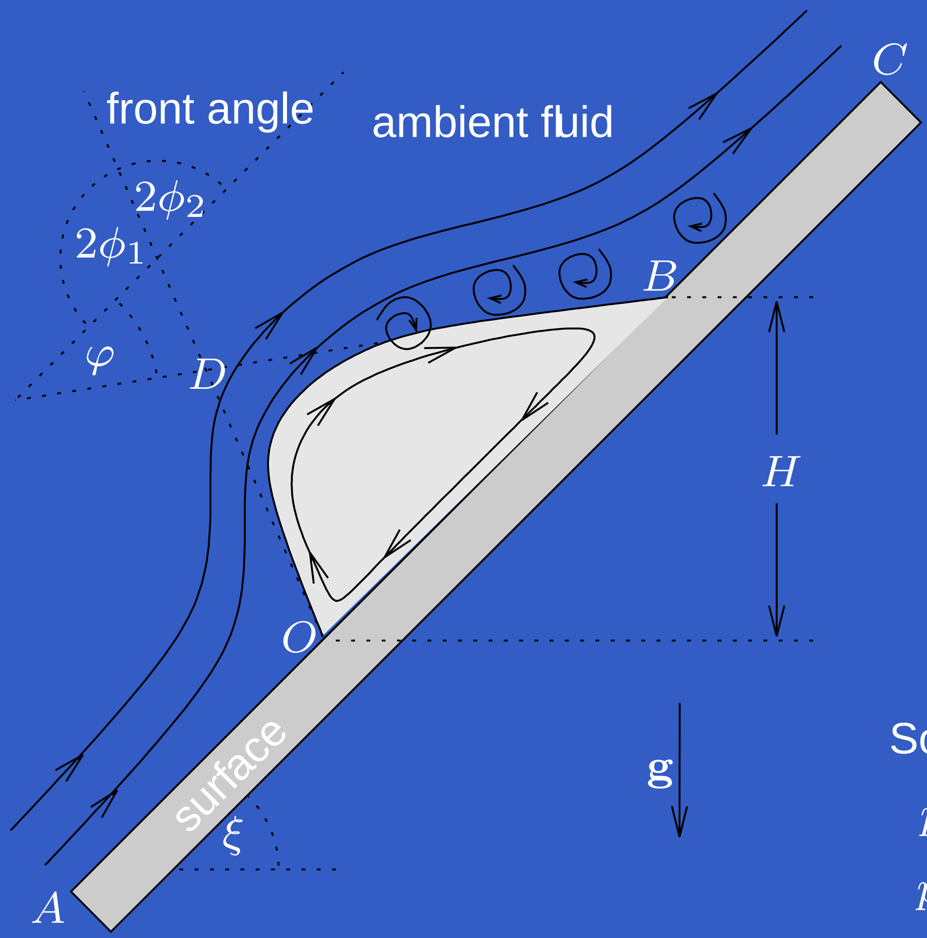


angle is constant

Schematic of a gravity current on an incline



Flow reflection

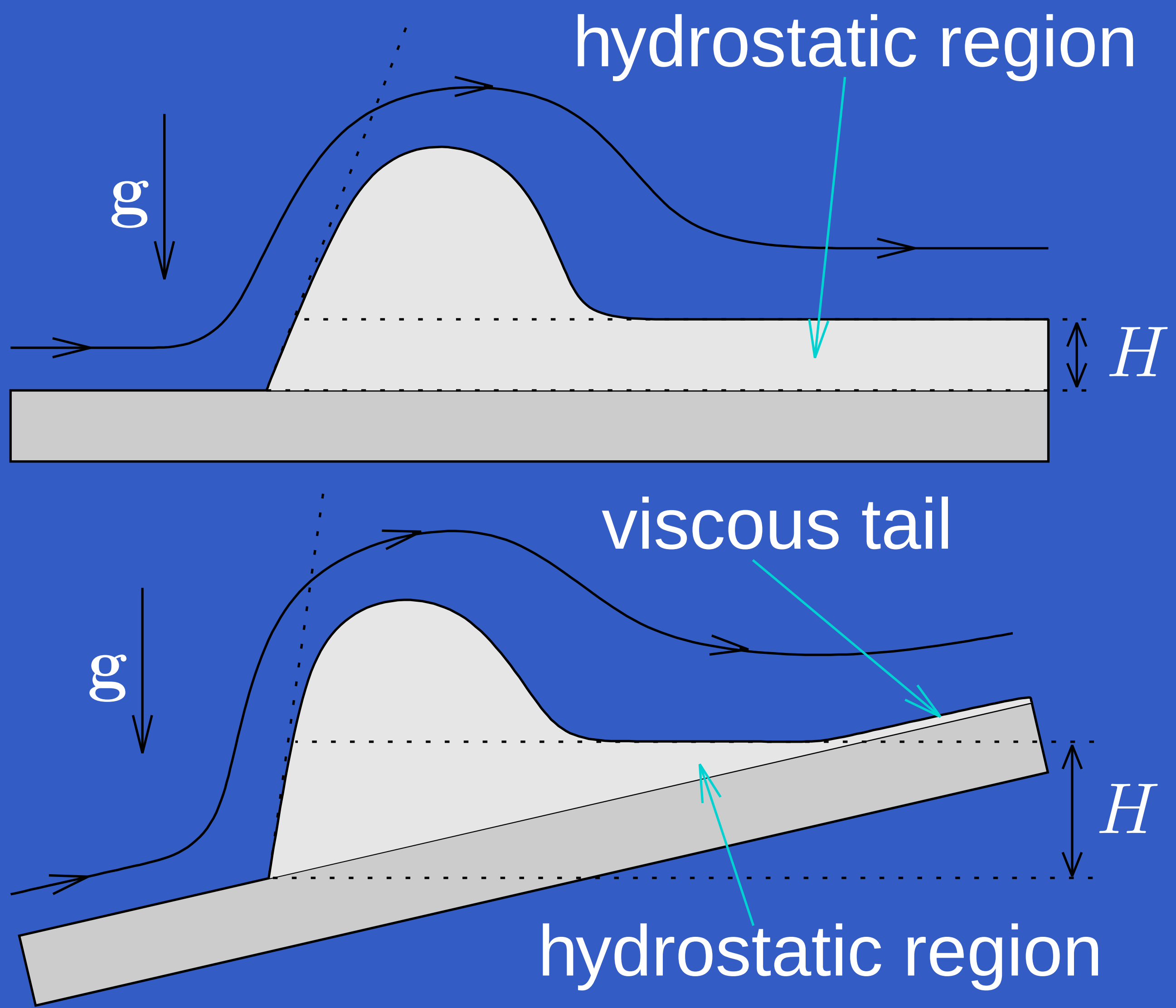


Solutions are symmetric in a wedge

$$
\begin{aligned}
p_{OA}(r) &= p_1(r,\phi_1) &&= -\tfrac{1}{2}\rho_1 u_1^2 - g\rho_1 r\sin(\xi+\pi) \\
p_{OD}(r) &= p_1(r,-\phi_1) &&= -\tfrac{1}{2}\rho_1 u_1^2 - g\rho_1 r\sin(\xi+\Phi) \\
&= p_2(r,\phi_2) &&= -\tfrac{1}{2}\rho_2 u_2^2 - g\rho_2 r\sin(\xi+\Phi) \\
p_{OB}(r) &= p_2(r,-\phi_2) &&= -\tfrac{1}{2}\rho_2 u_2^2 - g\rho_2 r\sin(\xi).
\end{aligned}
$$

So that $p_{OB} = p_{OA} - 2g\rho_1 r\sin\xi + g(\rho_2-\rho_1)r[\sin(\xi+\Phi) - \sin\xi]$.

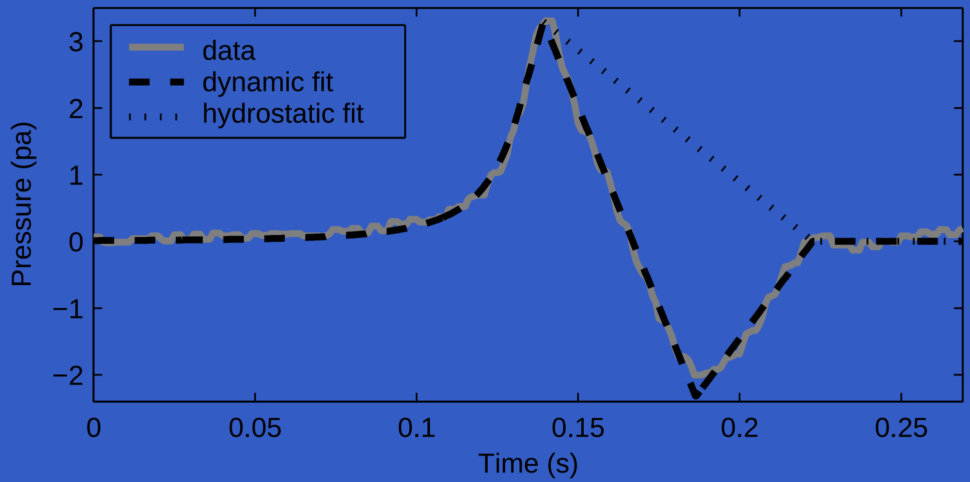
Global flow



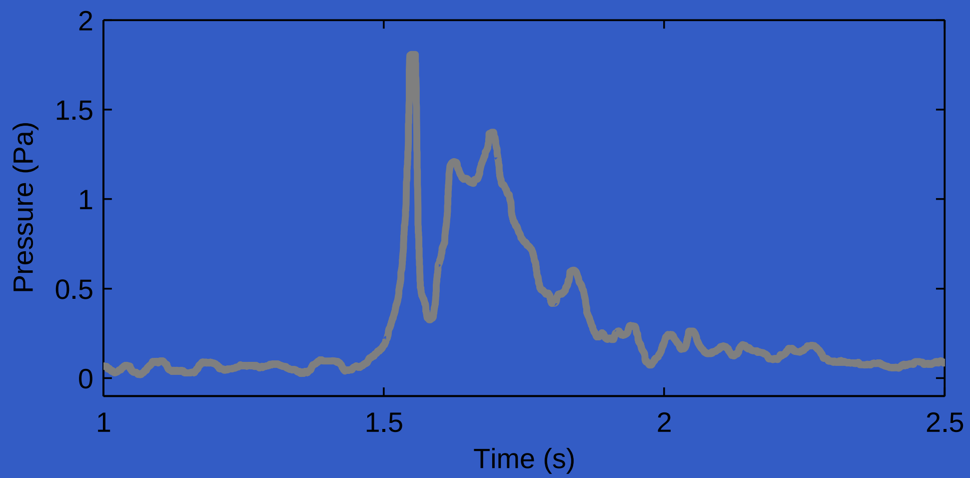
Global flow

$$p = \begin{cases} \frac{1}{2}\rho_1 U^2 \frac{R^3}{(r+R)^3}\left(2 - \frac{R^3}{(r+R)^3}\right) & OA \quad r/R > \lambda - 1 \\ \frac{1}{2}\rho_1 U^2 \left(1 - \kappa \frac{r}{R}\right) & OA \quad r/R < \lambda - 1 \\ \frac{1}{2}\rho_1 U^2 \left(1 - \kappa \frac{r}{R}\right) + 2g(\rho_2 - \rho_1) r \sin(\phi_2)\cos(\xi + \phi_2) & OB \quad r/R < O(\lambda - 1) \\ \text{unknown} & OB \quad r/R = O(\lambda - 1) \\ \approx 0\text{, but turbulent fluctuations} & BC. \quad \kappa \approx 1.01,\ \lambda \approx 1.39 \end{cases}$$

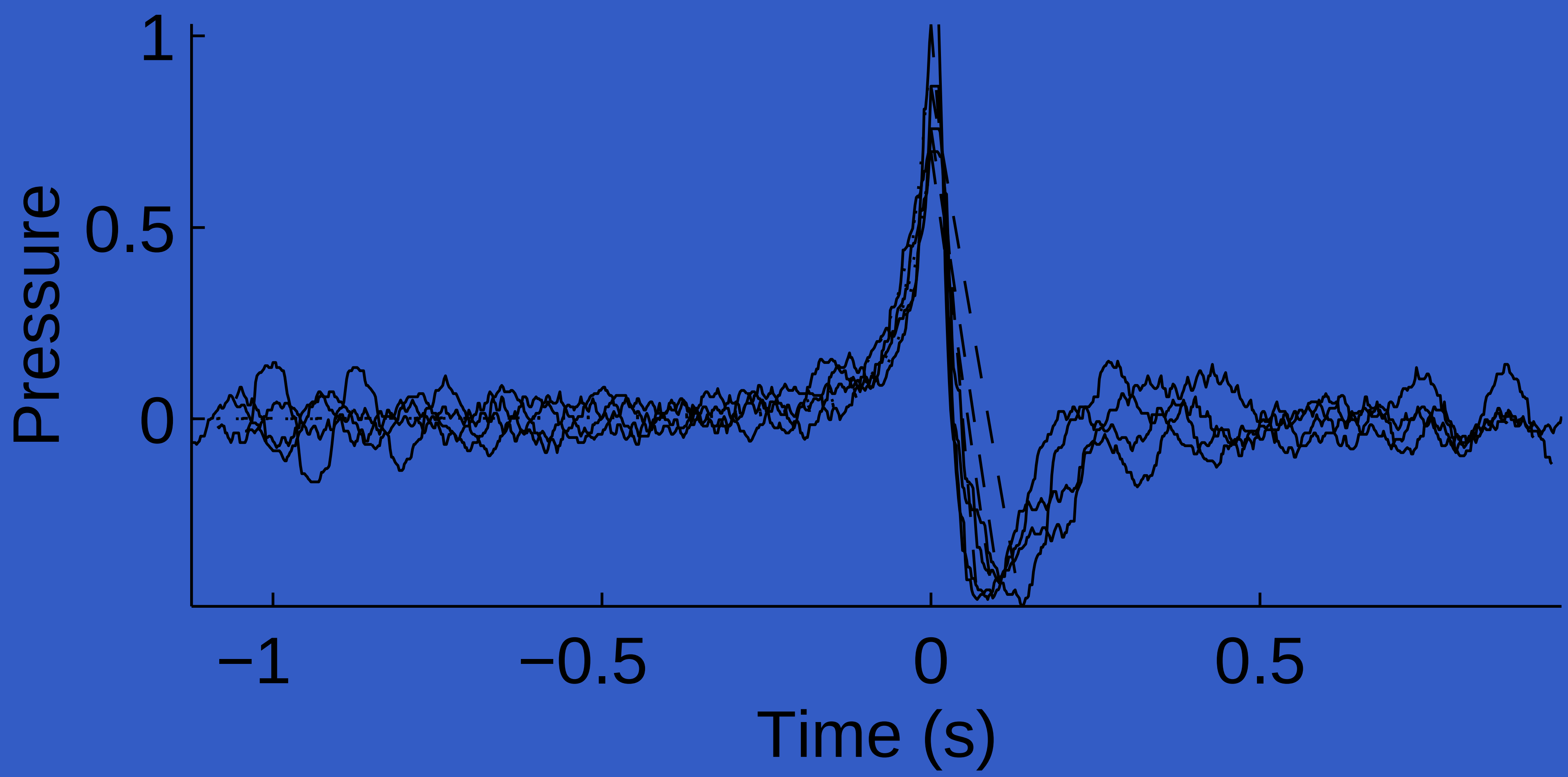
Air pressure in small powder cloud



Air pressure in small dense fbw



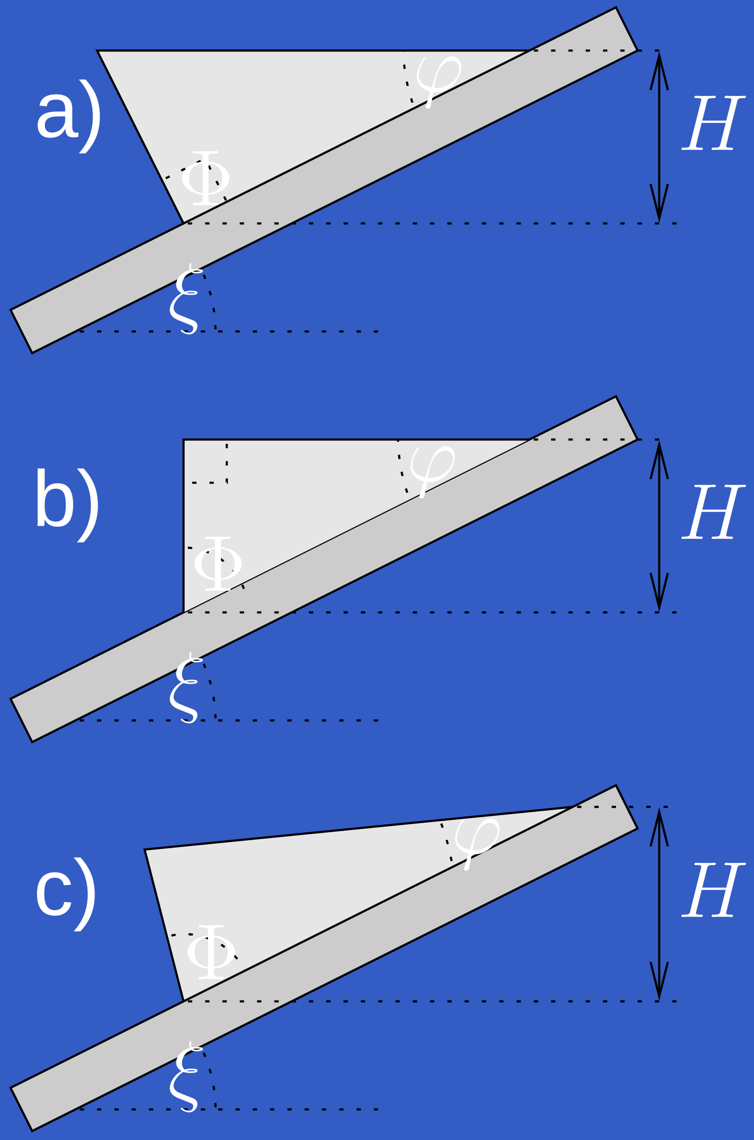
Pressure In Snow Experiments



Conclusions

- extends result of von Kármán (1940) and Benjamin (1968)
- any angle
- internal motion
- front angle is 60°
- Froude number $\sqrt{2}$
- explains large negative pressures
- unreasonable effectiveness of shallow water equations

Dependence of flow speed on flow volume



volume V, width W

$$V = \frac{1}{2} W H^2 \frac{\sin\Phi \sin\varphi}{\sin^2\xi \sin(\Phi+\varphi)}.$$

$$U = \left[2g\left(\frac{\rho_2}{\rho_1} - 1\right)\right]^{\frac{1}{2}} \left[\frac{2V}{W} \frac{\sin^2\xi \sin(\Phi+\varphi)}{\sin\Phi \sin\varphi}\right]^{1/4}$$

Extension to three-dimensional flows

External potential flow

$$\psi = r^{\frac{3}{2}} P_{\frac{3}{2}}(\cos\theta),$$

Solution possible *only* on vertical slopes

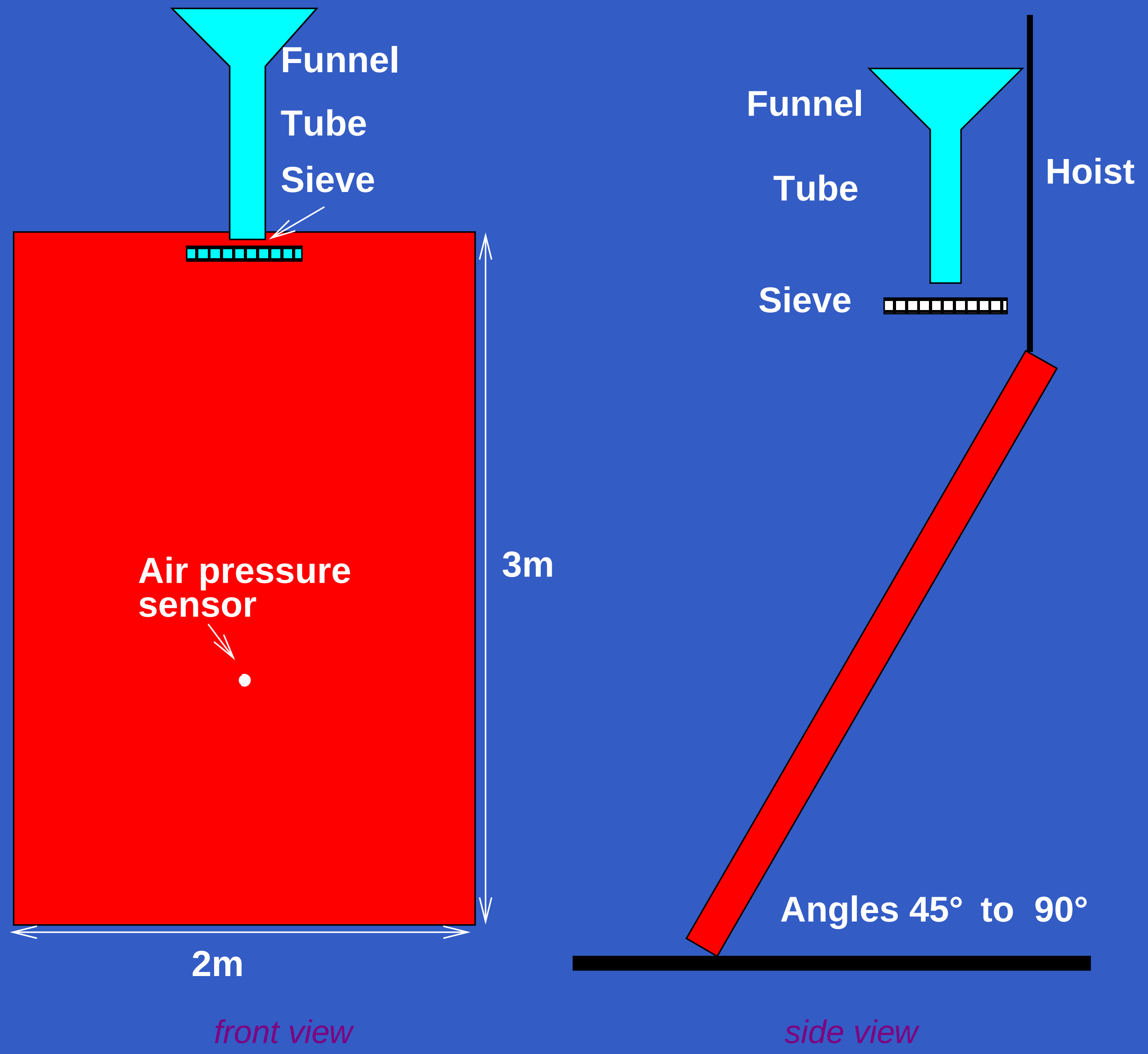
Front angle of 65°.

Aims: Investigate flow structure

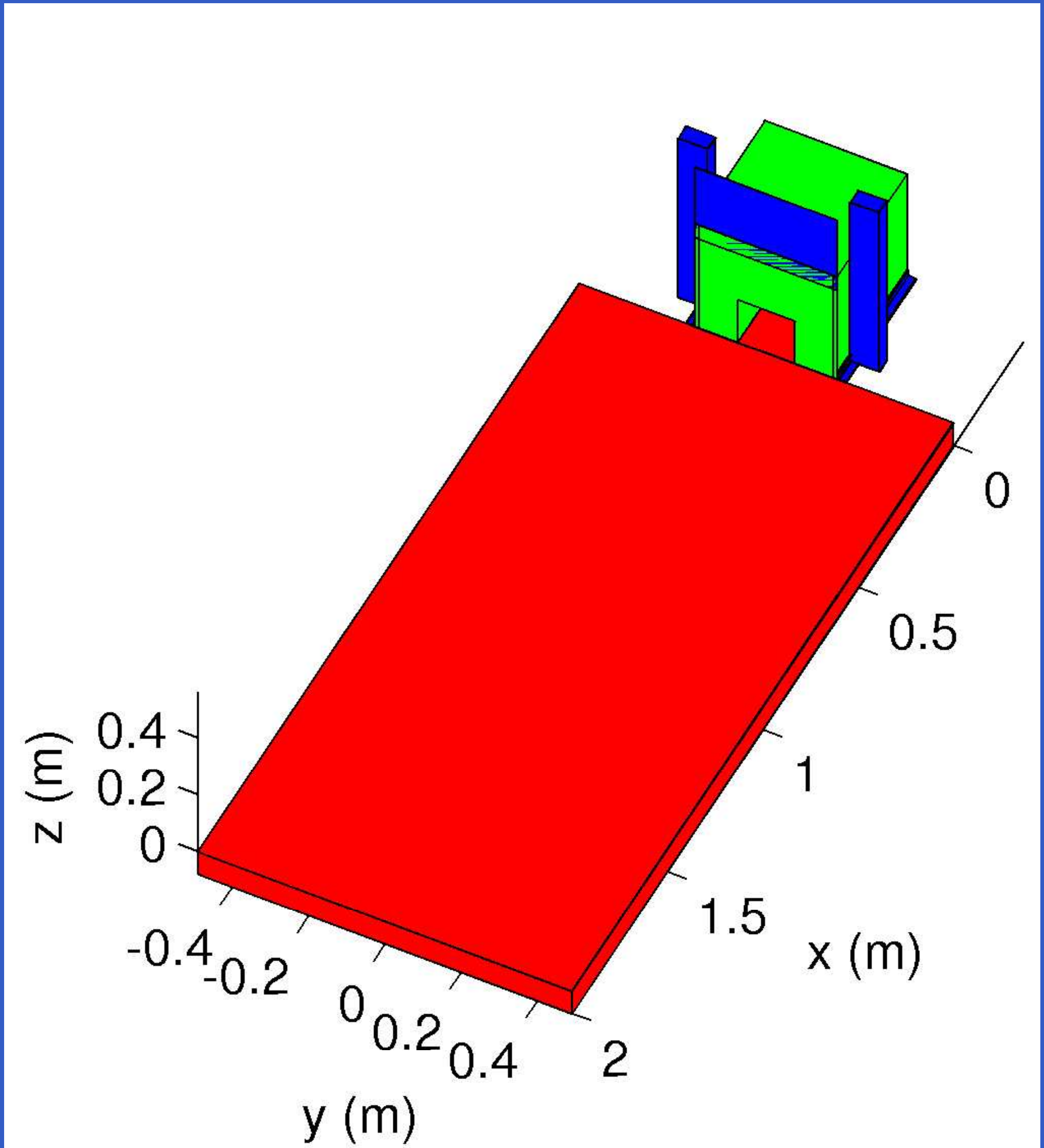
Use steep slopes to give a low Richardson number for large density difference.

- Transition to suspension
- Spreading
- Limited entrainment
- Steady flows

Experimental Setup Davos



Experimental Setup Cambridge



Material Properties

	snow	polystyrene
temperature	-16^o C	25^o C
bulk density	$200\,\mathrm{kg/m^3}$	$19\,\mathrm{kg/m^3}$
density	$300\,\mathrm{kg/m^3}$	$32\,\mathrm{kg/m^3}$
diameter	0.25–0.75 mm	2–4 mm
velocity	0.1–0.5 m/s	0.3–0.7 m/s
$\mathrm{Re_p}$	2–30	40–150
Length	1–28 mm	9–35 mm
Half length	1–4 mm	1–5 mm
Half time	0.01–0.03 s	0.02–0.03 s

Flow Properties

volume	0.1–$8.0\,\mathrm{l}$
duration	0.5–$2\,\mathrm{s}$
velocity	2–$6\,\mathrm{m/s}$
width	0.1–$1\,\mathrm{m}$
height	0.01–$0.5\,\mathrm{m}$
Fr	1–10
Ri	0–100
Re	10^4–10^6

Videos of Results

100 ml side
8000 ml side
100 ml front
8000 ml front

angle	31.5	45.0	58.5	75.5	91.0
100 ml	f s	f s	f s	f s	f s
8000 ml	f s	f s	f s	f s	f s

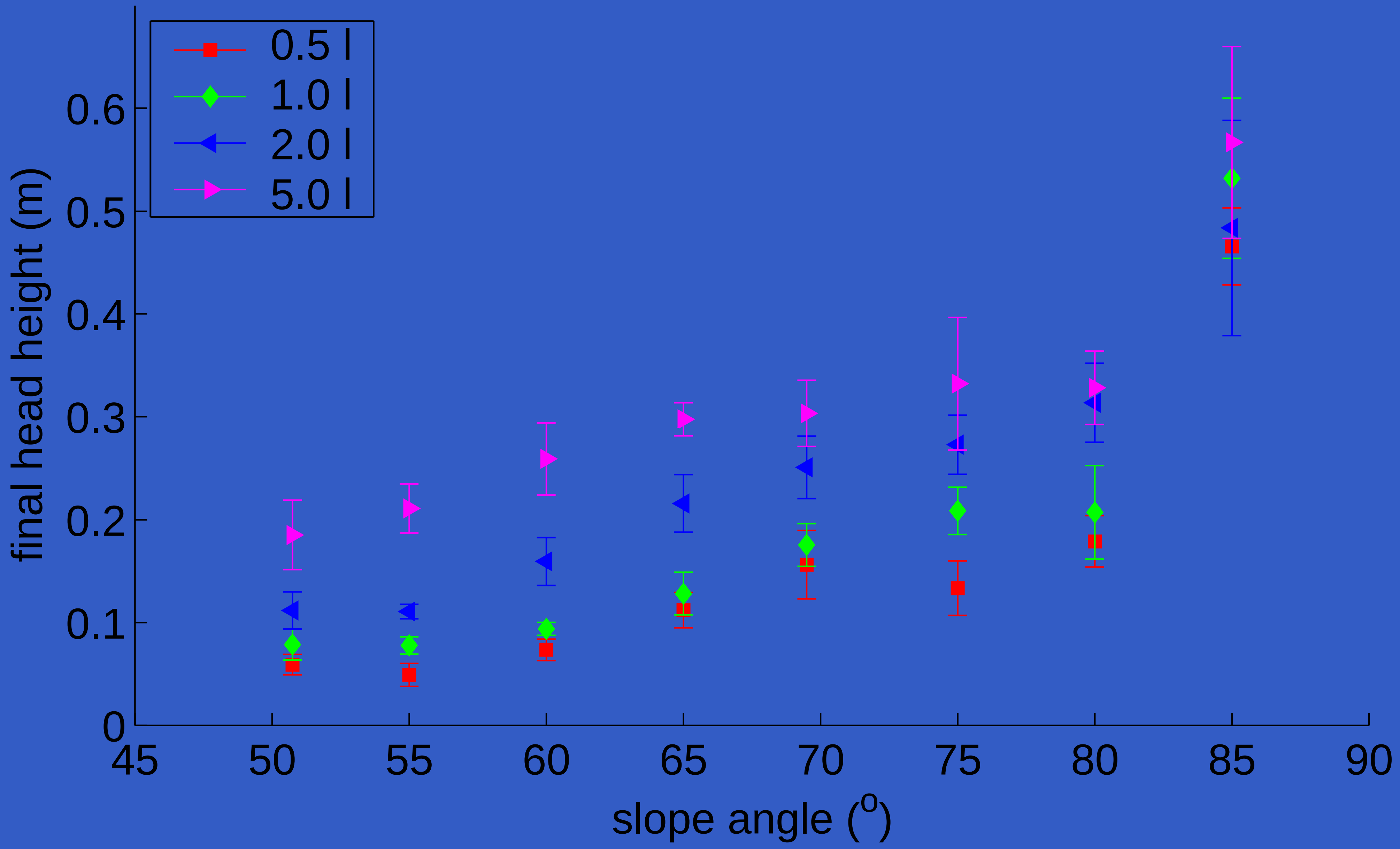
Side View 8 Litre Avalanches

31.5^o slope

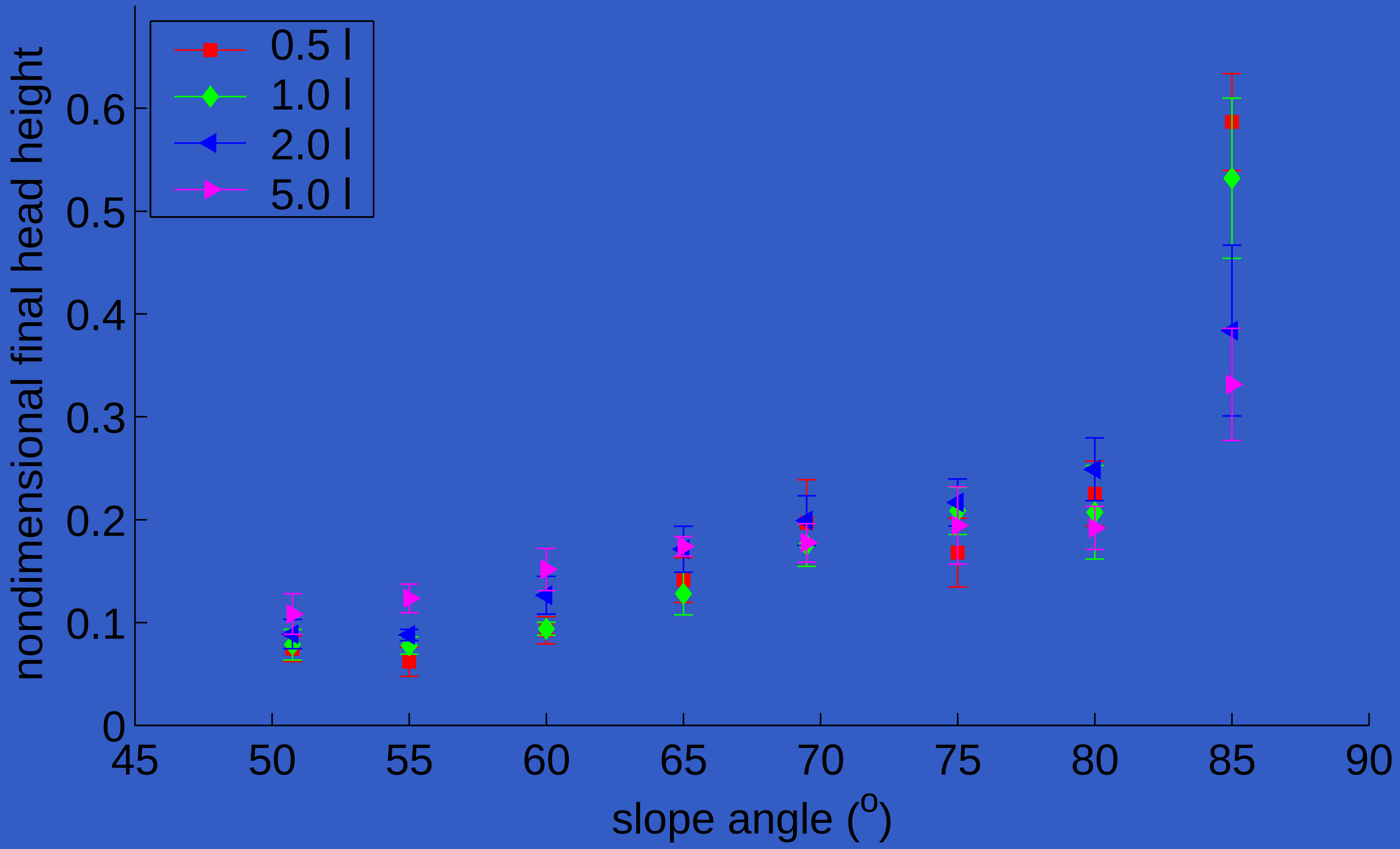
58.5^o slope

91.0^o slope

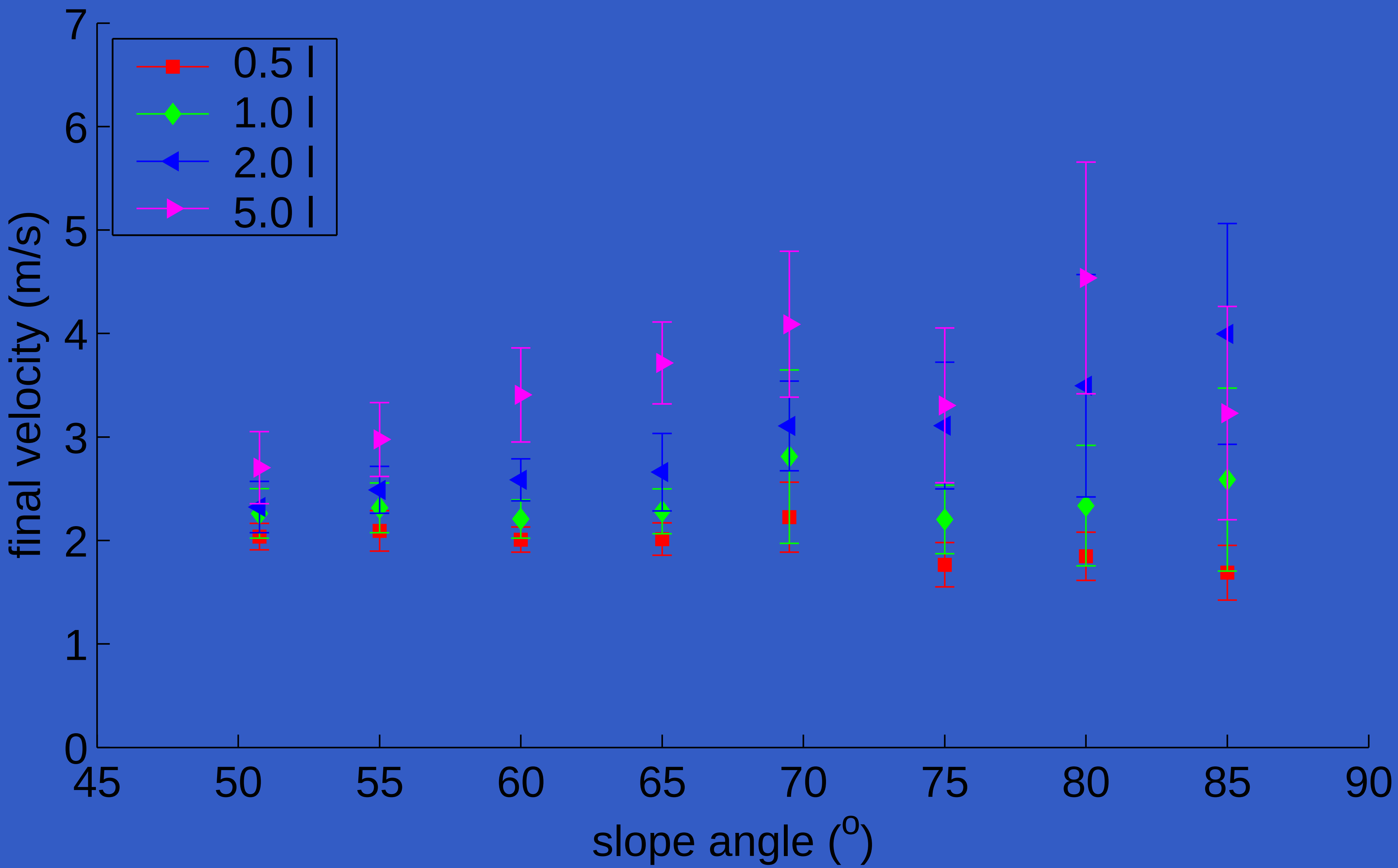
Polystyrene Final Height



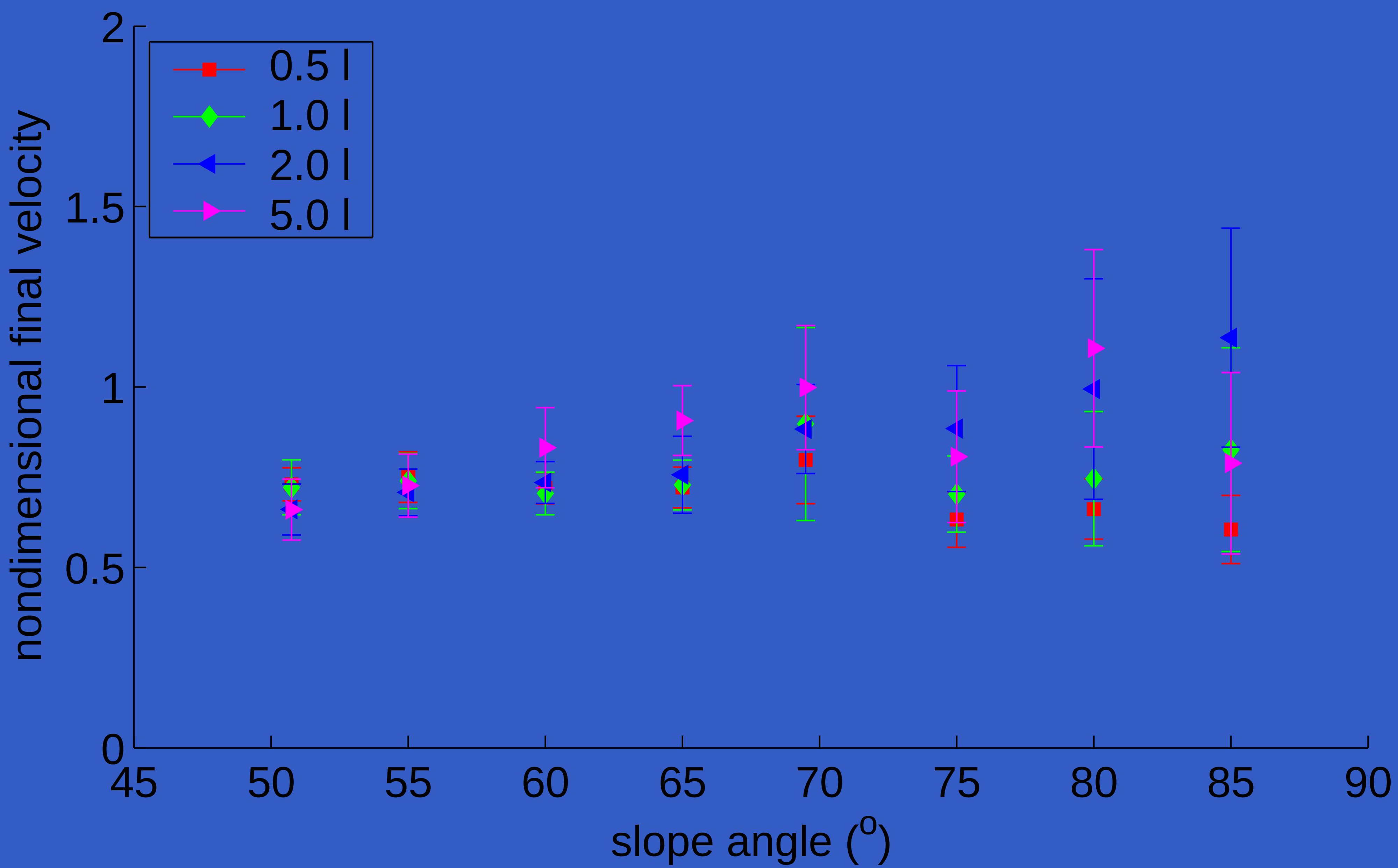
Polystyrene Final Nondimensional Height



Polystyrene Final Velocity



Polystyrene Final Nondimensional Velocity



Continuous 2d Currents   Britter & Linden (1980)

$$v = (1.5 \pm 0.2)\,(g'Q)^{1/3}$$

v	front velocity	$[LT^{-1}]$
Q	volume flux per unit area	$[L^2T^{-1}]$
g'	density adjusted gravity	$[LT^{-2}]$

Key results

- velocity is constant
- velocity independent of slope angle
- mean flow 60% larger than front velocity

Problems

- Extend to 3d
- Fixed buoyancy

Point Release 2d Beghin, Hopfinger & Britter (1981)

$$v = \alpha(\theta)\left(\frac{g'Q}{x_f}\right)^{1/2} \qquad \alpha(\theta) = \begin{array}{ll} 2.6 \pm 0.2 & \theta = 15^o \\ 1.5 \pm 0.2 & \theta = 90^o \end{array}$$

v	front velocity	$[LT^{-1}]$
Q	flow volume	$[L^2]$
g'	density adjusted gravity	$[LT^{-2}]$
x_f	front position	$[L]$

Assumptions

- Boussinesq
- Ignores sedimentation
- α is function of θ not Ri
- No drag

Non-Boussinesq With Drag

$$\frac{d}{dt}[v(1+h^2)] = 1 - \beta v^2 h$$

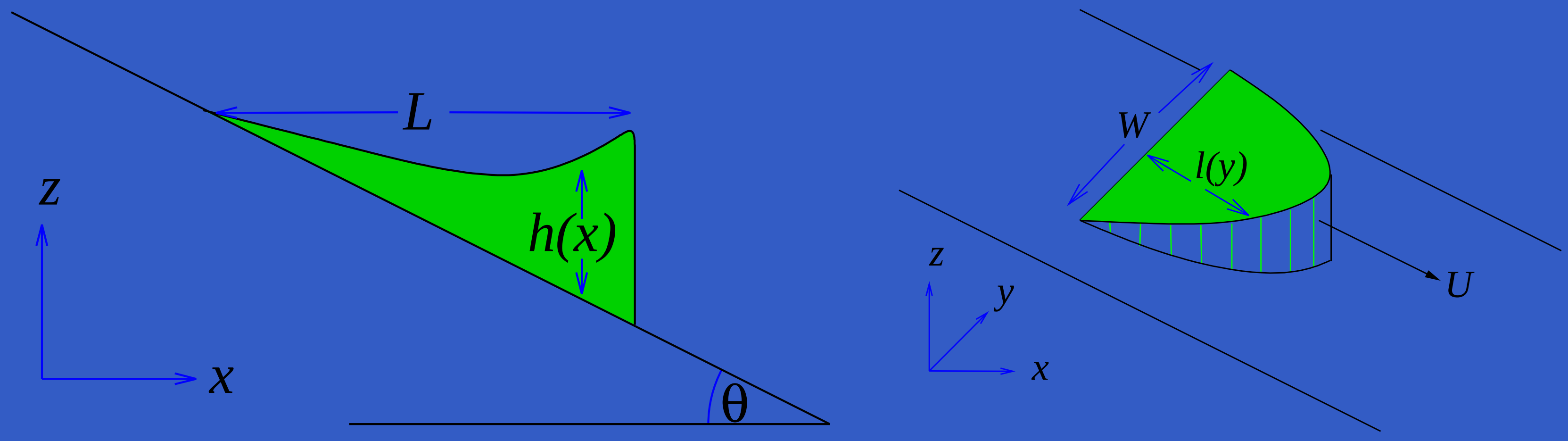
$$\frac{d}{dt}h = v\alpha - \delta$$

v	front velocity
h	height
β	drag
α	entrainment
δ	sedimentation

(all Nondimensional)

Neglects lateral spreading

Webber *et al.* (1993)



$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = 0 \quad h = 0 \quad \text{rear}$$

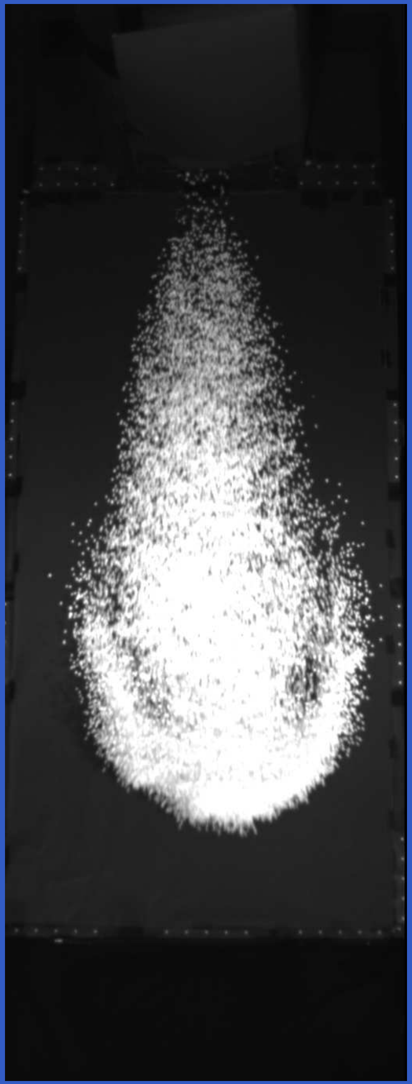
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + g''(\nabla h - \tan\theta \hat{\mathbf{z}}) = 0 \quad u = K\sqrt{g'h}\sqrt{1 + \left(\frac{dl}{dy}\right)^2} \quad \text{front}$$

Steady solution possible

$W/L = \pi$, $U \propto (\tan\theta)^{1/3} V^{1/6}$, V is volume

Front View 8 Litre Avalanches

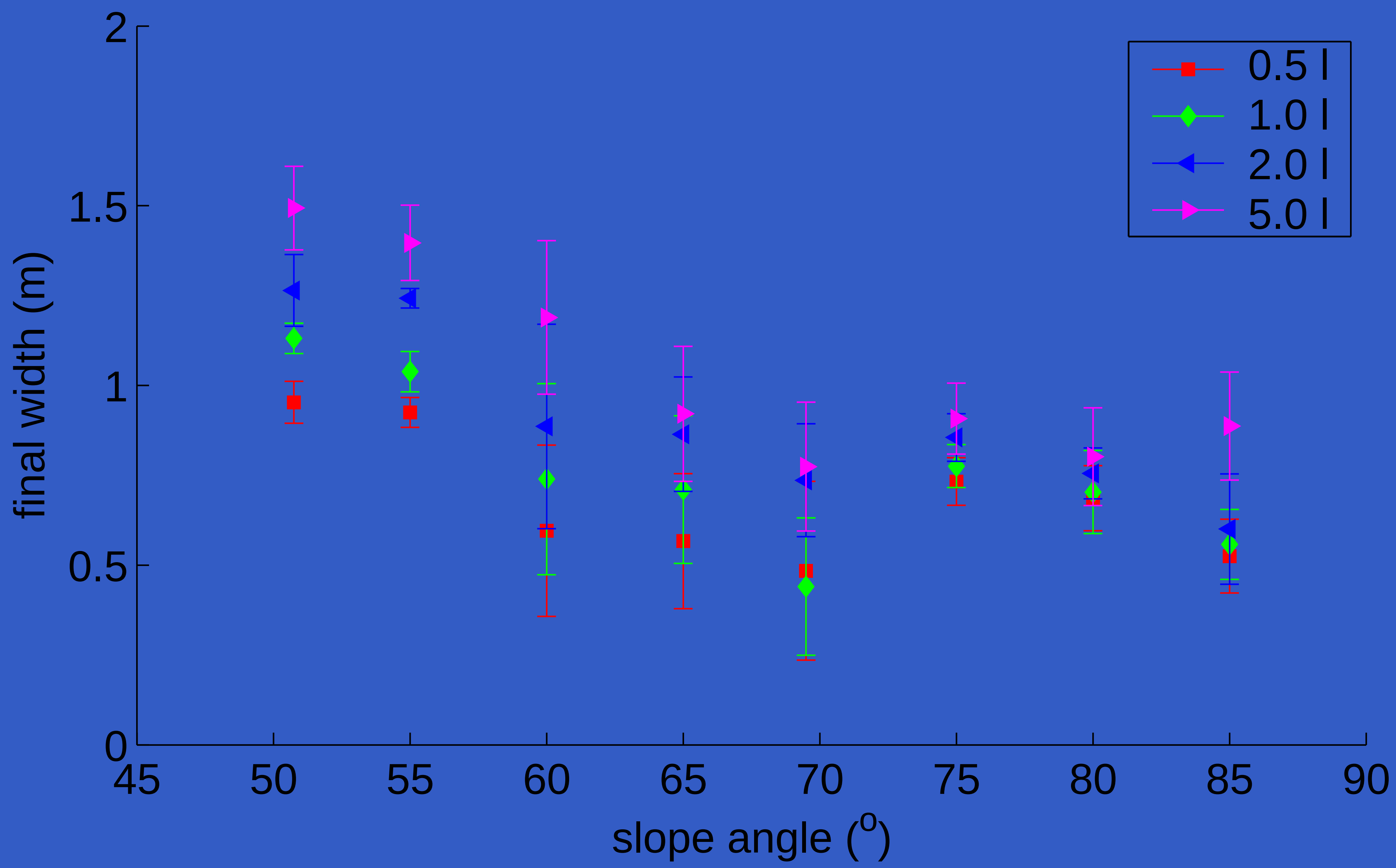
31.5^{o} *slope*



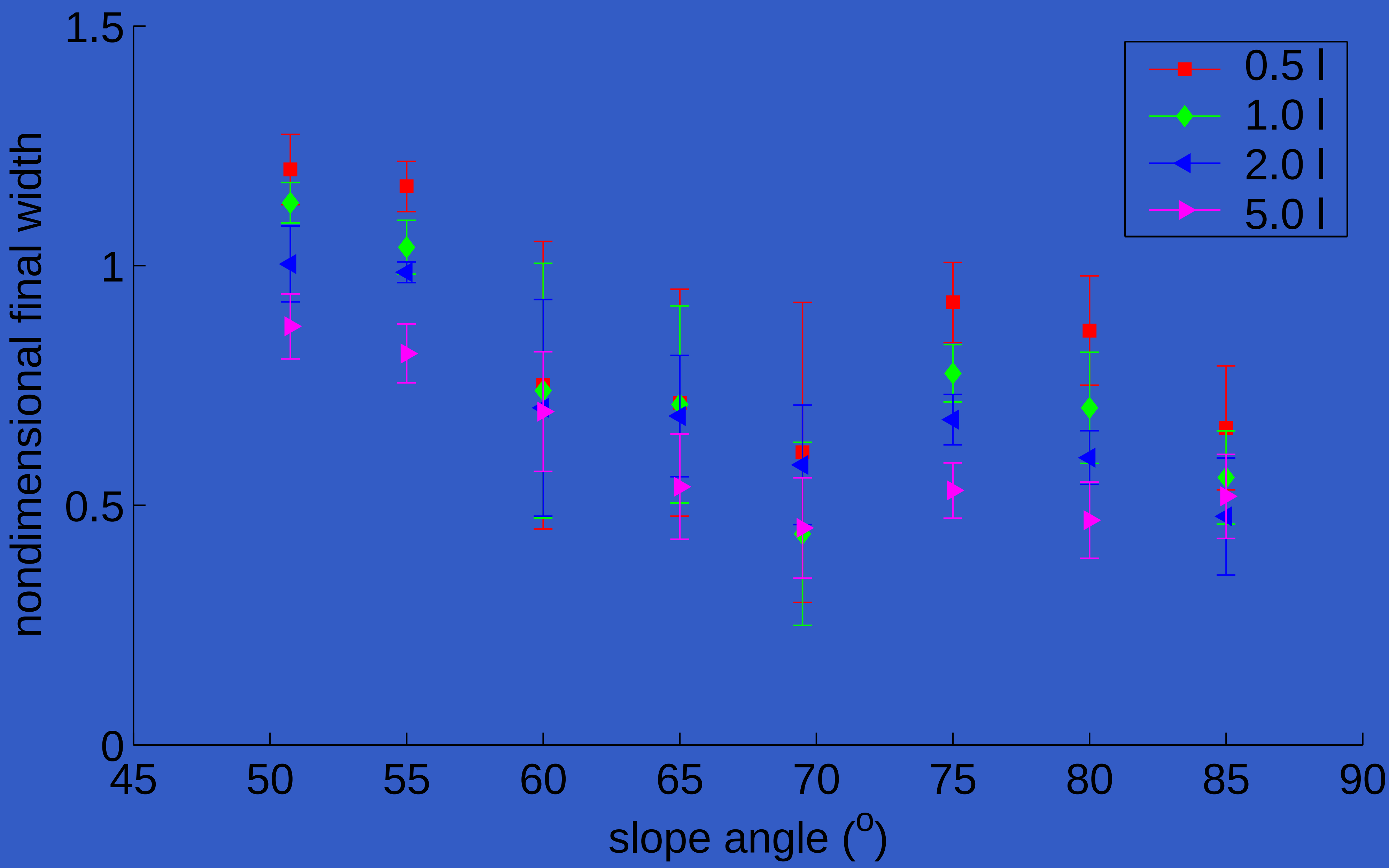
58.5^{o} *slope*

91.0^{o} *slope*

Polystyrene Final Width



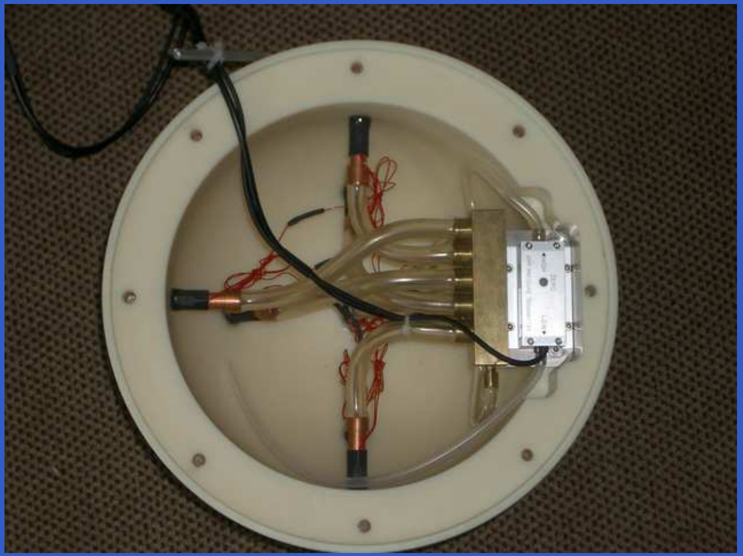
Polystyrene Final Nondimensional Width



Lateral Spreading

- Hydrostatic pressure
- Dynamic pressure
- Inertia
- Ambient pressure field
- Ambient entrainment
- Surface drag

Sensor design



Mast Mounting

Flow Around the Sensor

The pressure is effected by the sensor

$$p(\mathbf{x}_i) = p_0 + \frac{1}{8}\rho[9(\mathbf{u}\cdot\hat{\mathbf{x}}_i)^2 - 5u^2].$$

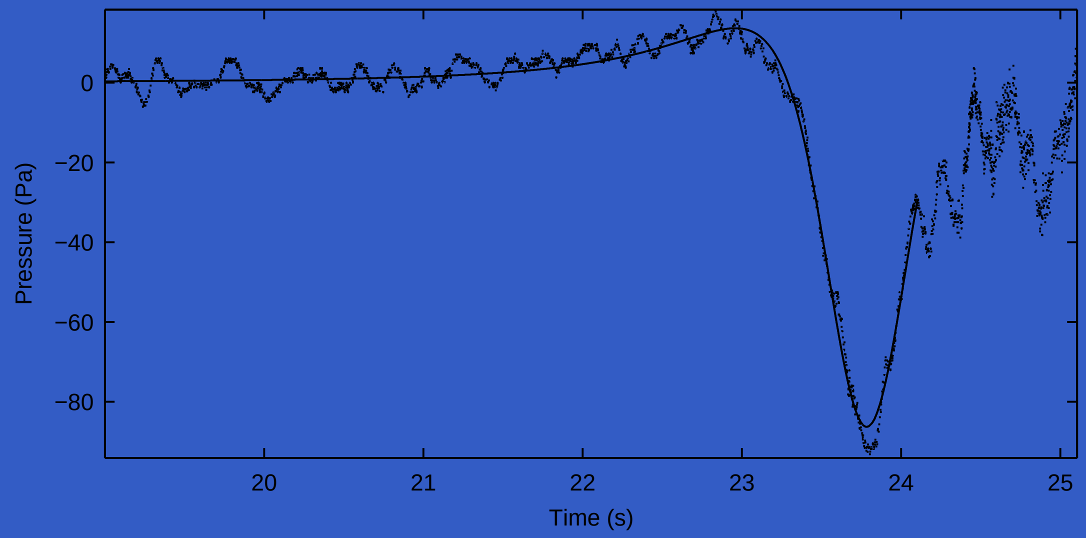
Average over 8 holes is

$$p = p_0 - \frac{1}{64}\rho[13u^2 + 9(\mathbf{u}\cdot\hat{\mathbf{z}})^2].$$

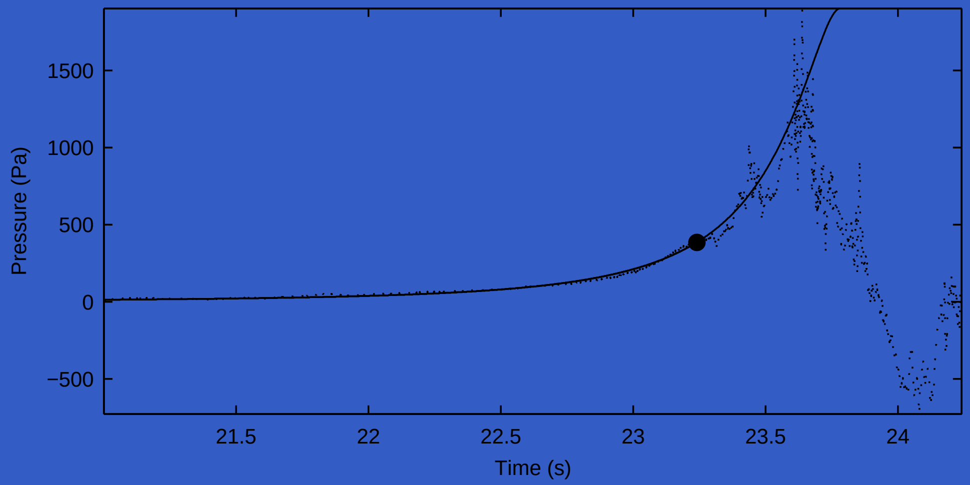
Measured pressure is always lower

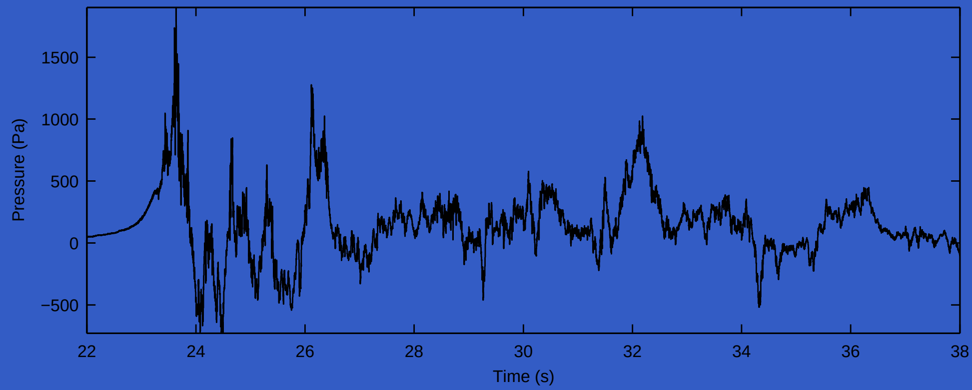
For irrotational flow $-1/2\rho u^2 = p_0$ so increase observed pressures by between $1 + \frac{13}{32}$ and $1 + \frac{22}{32}$

Avalanche no. 628

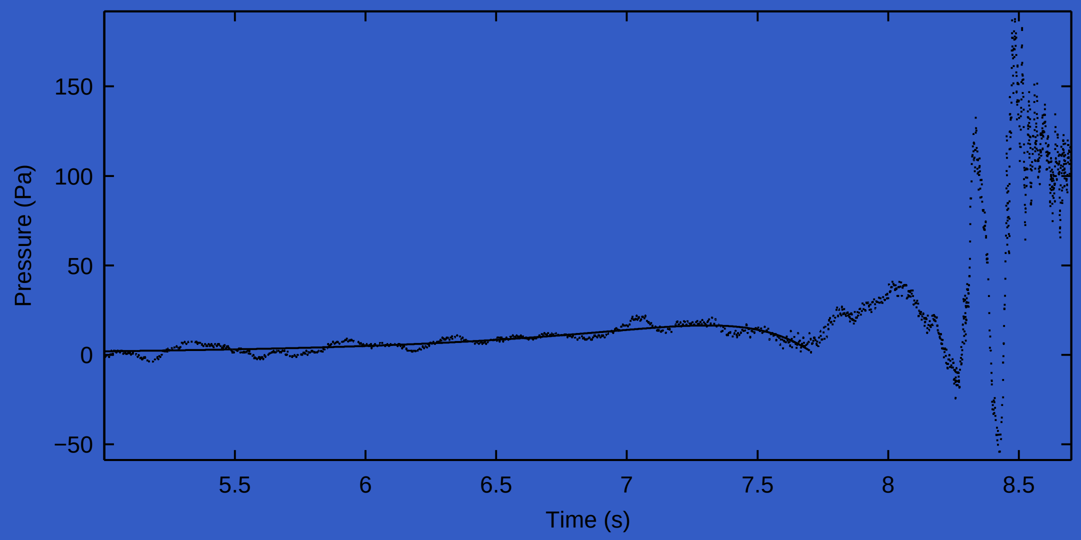


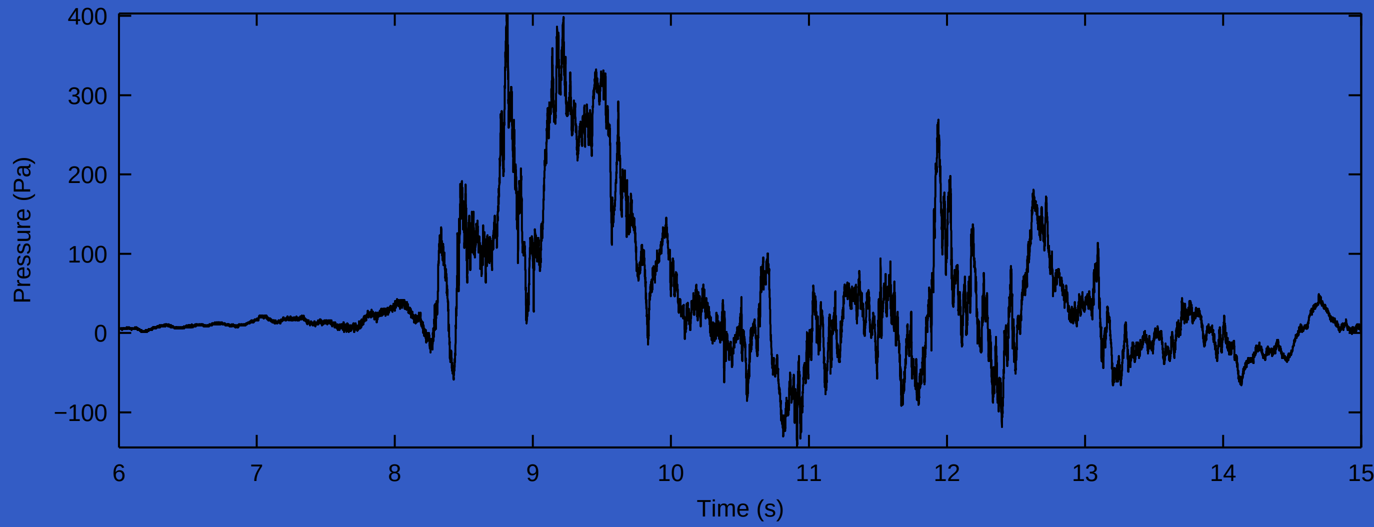
Avalanche no. 629



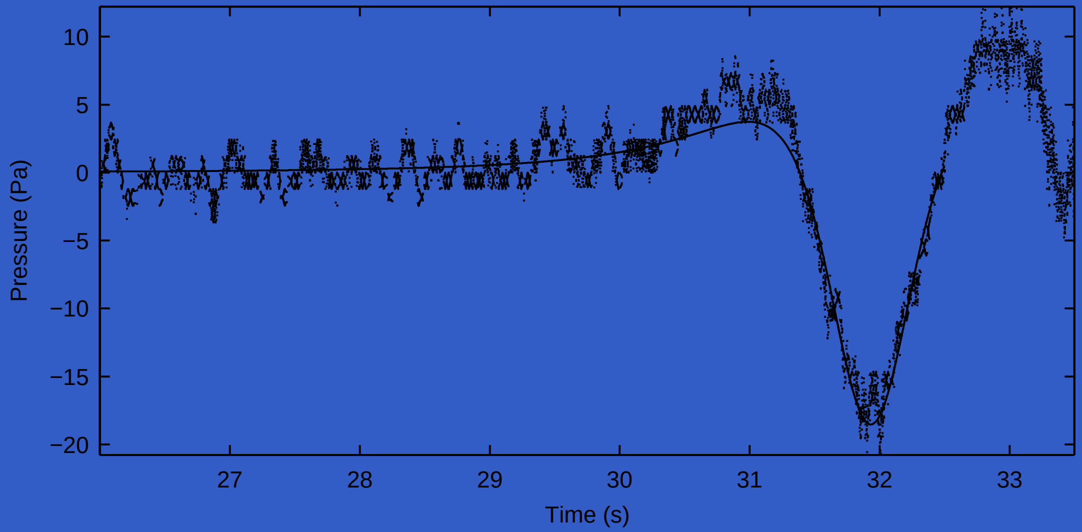


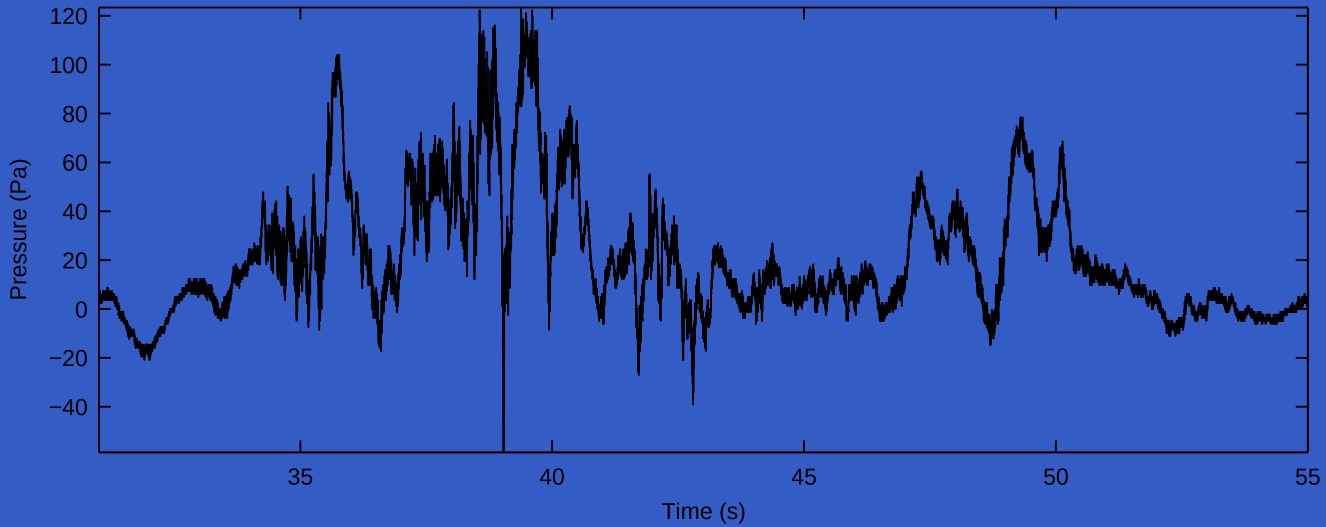
No. 6236


150
100
50
0
−50
Pressure (Pa)
5.5
6
6.5
7
7.5
8
8.5
Time (s)


400
300
200
100
0
−100
Pressure (Pa)
6
7
8
9
10
11
12
13
14
15
Time (s)

Avalanche no. 6237





Results

Avalanche	628	629	6236	6237
Max p velocity (m/s)	16.2	59.1	27.2	15.1
Min p velocity (m/s)	13.1	36.6	16.3	10.4
Dipole velocity u(m/s)	15.8 ± 1.0	59.1 ± 0.0	26.4 ± 5.7	69.7 ± 15.5
Dipole radius R(m)	8.2 ± 0.3	28.9 ± 0.0	12.9 ± 0.9	10.3 ± 0.8
Dipole offset L(m)	10.0 ± 0.8	0.0 ± 0.0	21.1 ± 4.7	53.8 ± 12.0
Centre time t_c(m)	23.78 ± 0.00	24.27 ± 0.00	8.32 ± 0.02	31.93 ± 0.00
Residual (Pa)	2.8	9.2	2.6	1.4

Mean fitted parameters with standard deviations calculated by refitting after adding on Gaussian noise with standard deviation of the residual and perturbing the initial choice

Future Work

- Experiments to separate entrainment from lateral spreading
- Study of turbulent structure in flows
- Stability of wedge flows near the front
- Development of theory for 3d external flow

Thanks !