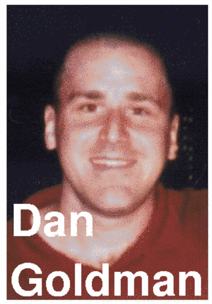
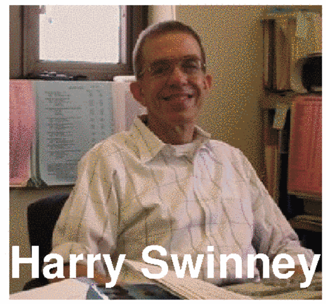
Measuring the configurational temperature of granular media

Matthias Schröter





Center for Nonlinear Dynamics
University of Texas, Austin

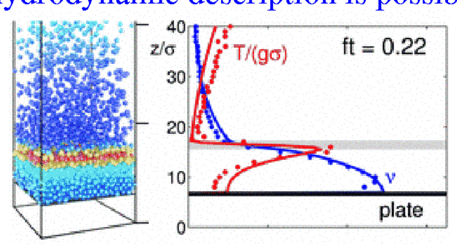
Kinetic granular temperature

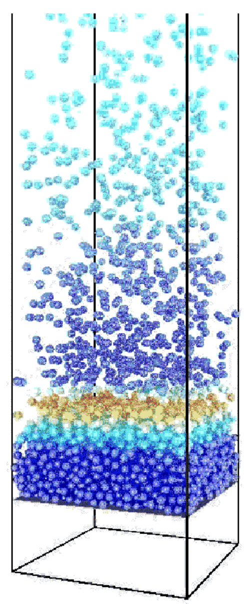
is proportional to the average kinetic energy of the random motion of the particles:

$$T_{kin} \sim \langle v^2 \rangle - \langle v \rangle^2$$

collisions are dissipative ->
T_{kin} has complex space and time dependence.

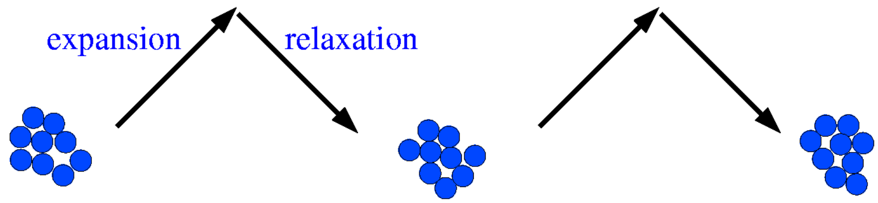
A hydrodynamic description is possible.





Bougie, J. *et al.* Phys. Rev. E **66**, 051301 (2002)

Exploring the phase space of stable configurations ...



.... requires periodic driving:

Driving has to assure *ergodicity*: all mechanical stable configurations are sampled with equal probability.

necessary condition for ergodicity: history independence

Edwards & Oakeshott, Physica A **157**, 1080 (1989)

Configurational granular temperature

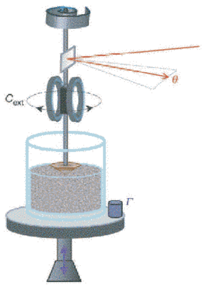
	Granular statistical mechanics	Classical statistical mechanics
	Number of jammed configurations to fill N particles in volume V	Number of micro states
Entropy	$S = k_E \ln(\Omega)$	$S = k_B \ln(\Omega)$
Extensive variable	Volume V	Energy E
Intensive variable	Compactivity X $X = \frac{\partial V}{\partial S}\Big\|_N$	Temperature T $T = \frac{\partial E}{\partial S}\Big\|_{V,N}$

Edwards & Oakeshott, Physica A **157**, 1080 (1989)

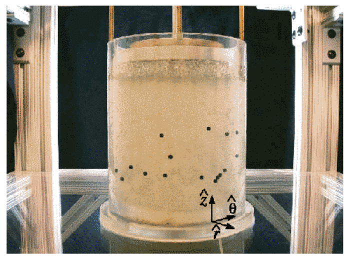
Other granular temperatures

Fluctuation-dissipation temperature:

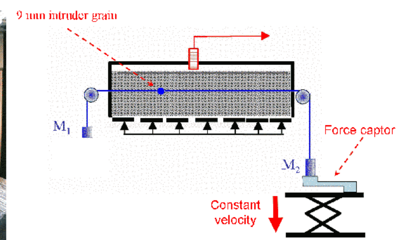
D'Anna *et al.* Nature **424** 909 (2003)



Song, Wang & Makse, PNAS **102** 2299 (2005)



Caballero, Lindner & Clement, ESPCI

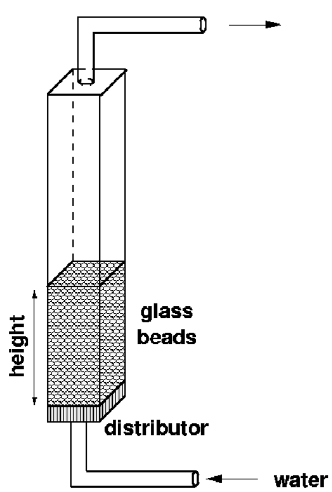


Generalized granular temperature:

includes kinetic and elastic energies.

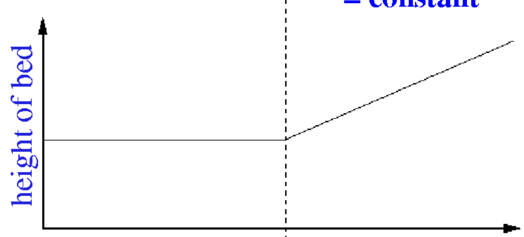
Kondic & Behringer, EPL **67** 205 (2004)

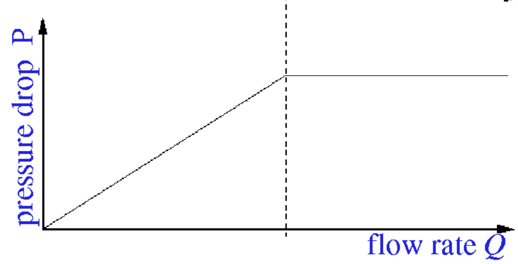
Driving grains with a fluidized bed



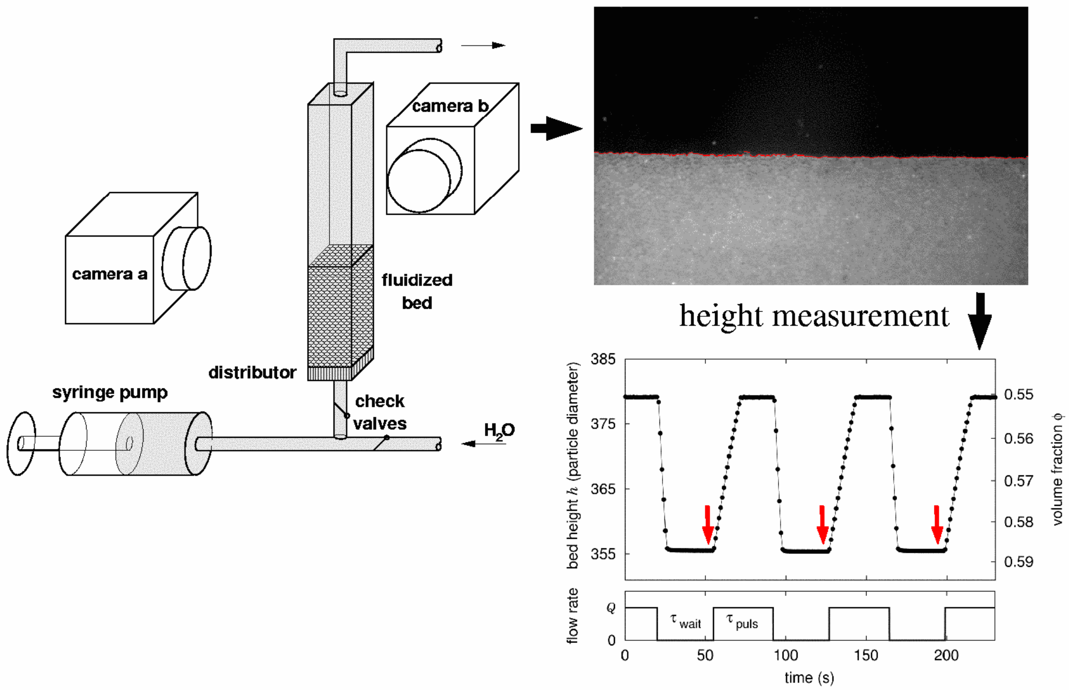
Darcy's law: $Q \sim k(\phi)\, \Delta P$

below fluidization	above fluidization
k = constant	ΔP **= buoyant weight = constant**

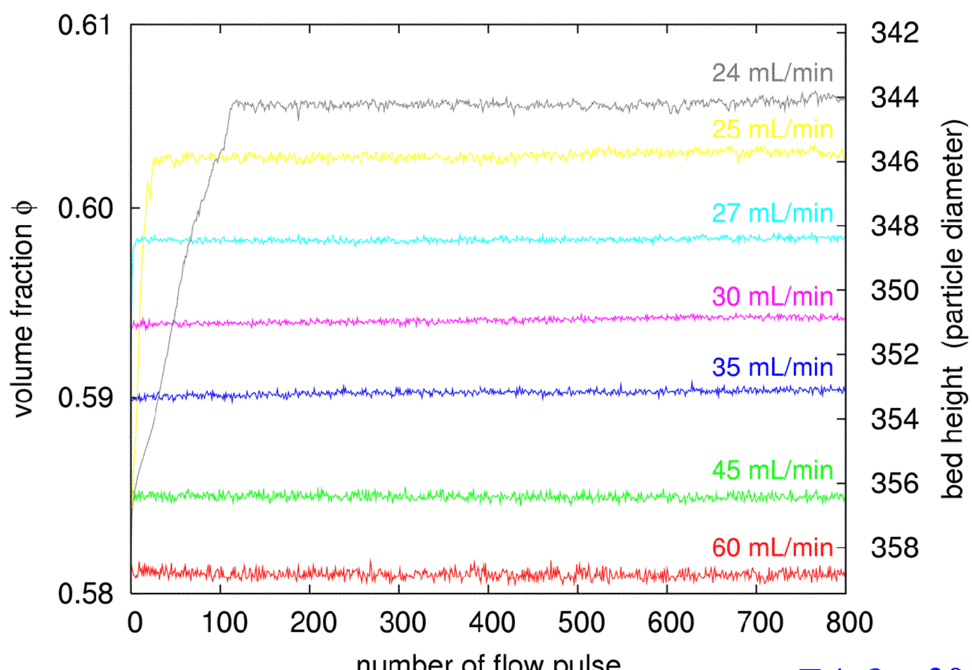




Pulsing flow through a fluidized bed

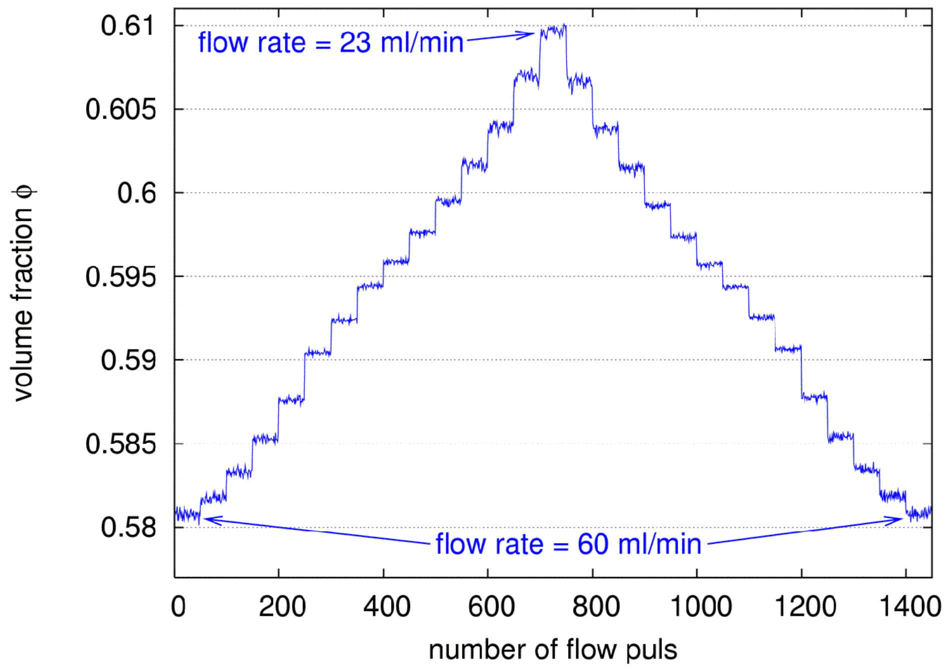


Steady state

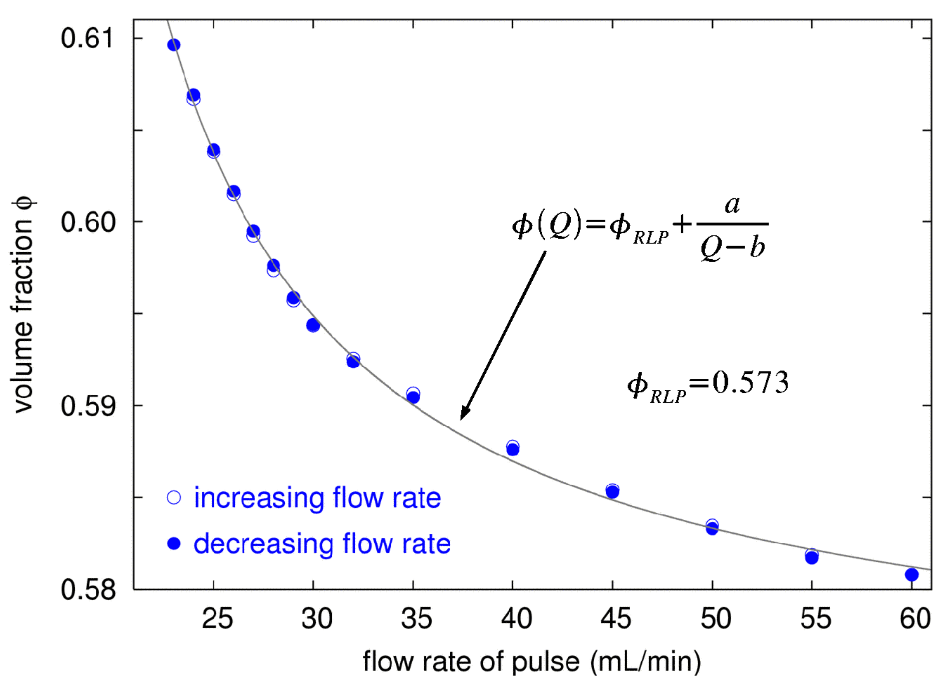


$T * Q = 30$ ml

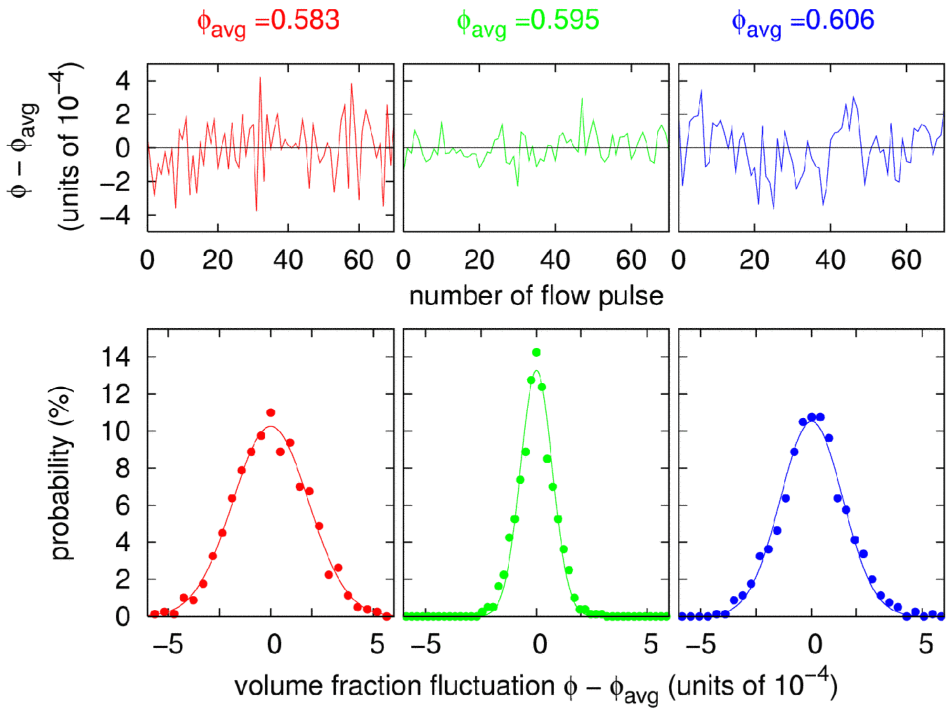
Ramping the flow rate down and up



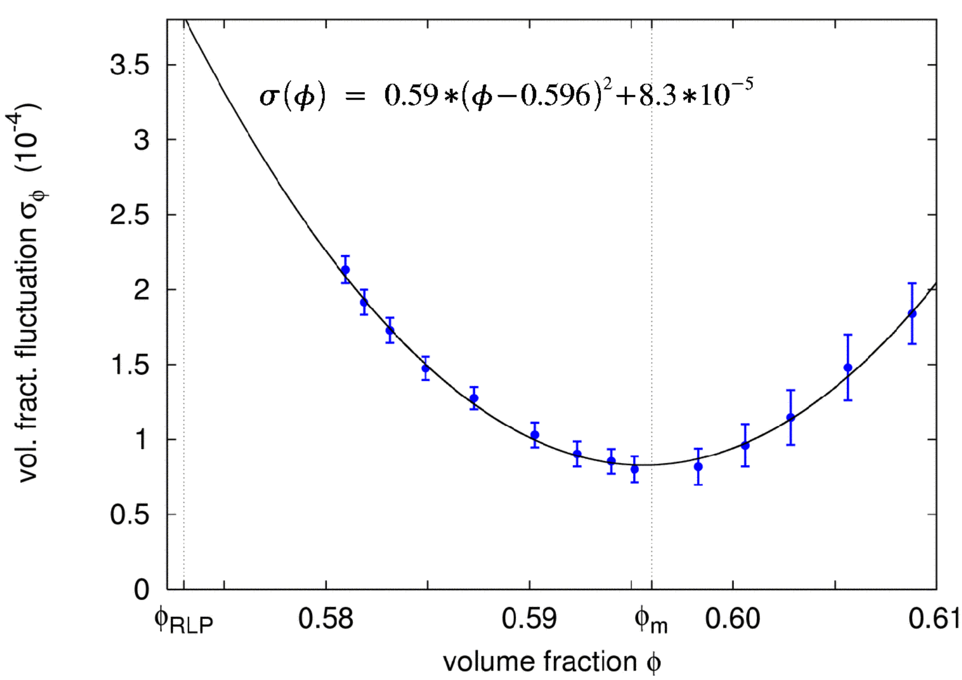
Fluidized beds are history independent



Gaussian fluctuations



Volume fraction fluctuations



Statistical independent regions

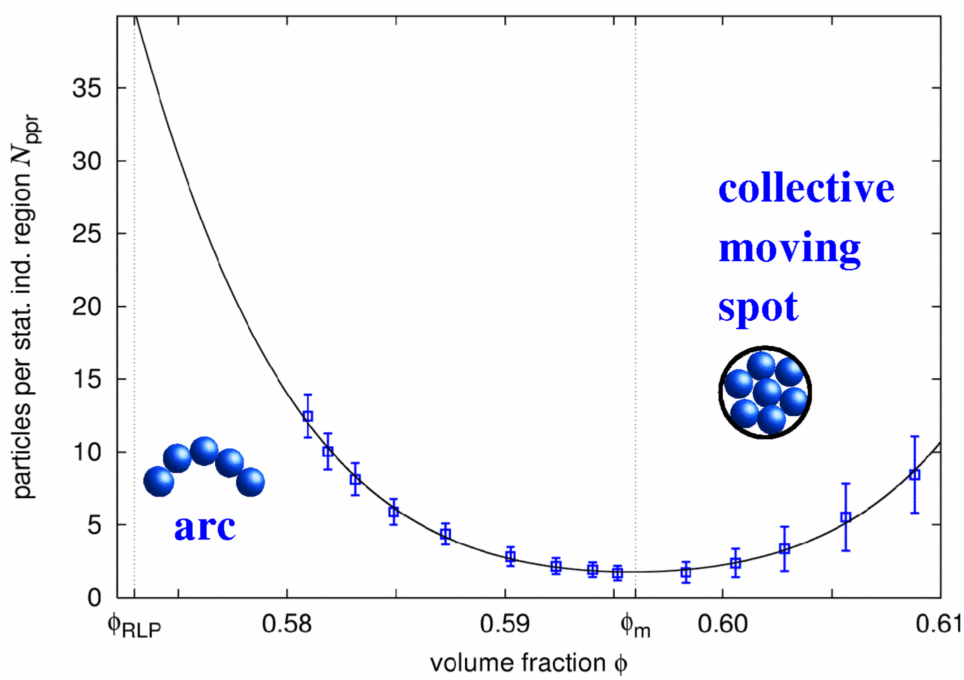
Statistical mechanics: relative fluctuations of an extensive variable are proportional to one over square root of the *number of statistical independent regions.*

$$\frac{\sigma_\phi}{\phi} \approx \frac{0.2}{\sqrt{N_{sir}}}$$

Nowak, E. R. *et al.* Phys. Rev E **57**, 1971 (1998)

-> How many particles form a statistical independent region at a given packing fraction?

Size of a statistical independent region



Measuring compactivity

Classical thermodynamics:

$$C_V = \frac{\partial E_{avg}}{\partial T} = \frac{<(E - E_{avg})^2>}{k_B T^2}$$

Granular volume capacity:

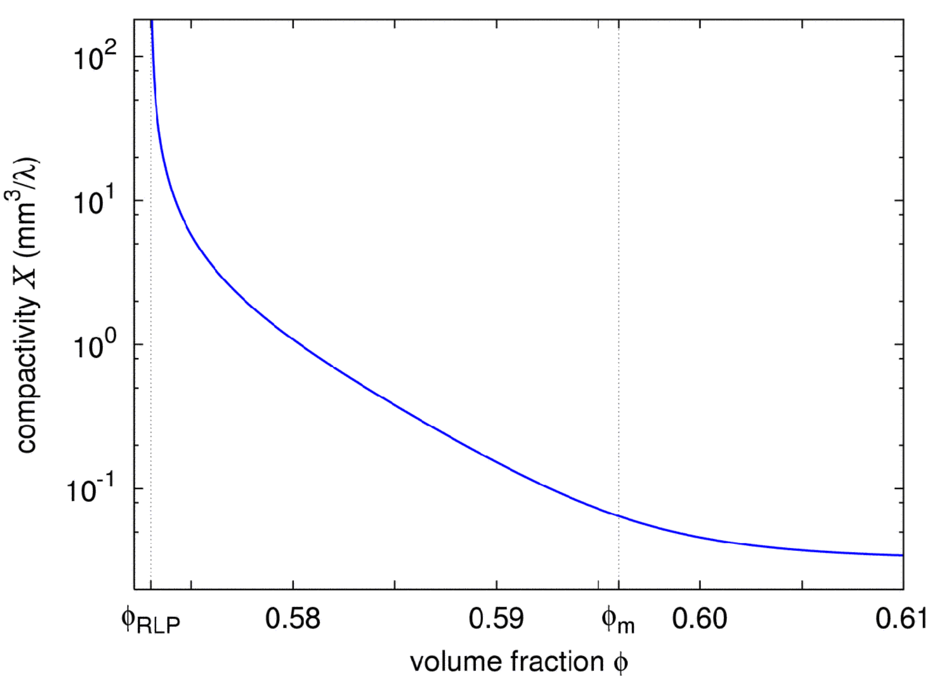
$$C = \frac{\partial V_{avg}}{\partial X} = \frac{<(V - V_{avg})^2>}{k_E X^2}$$

X of RLP is defined as ∞.

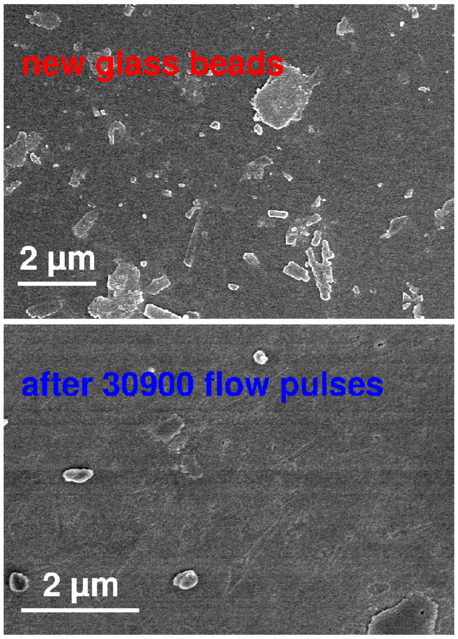
$$\frac{1}{X(\phi)} = \frac{k_E \rho}{m} \int_{\phi_{RLP}}^{\phi} \left(\frac{\phi'}{\sigma_{\phi'}} \right)^2 d\phi'$$

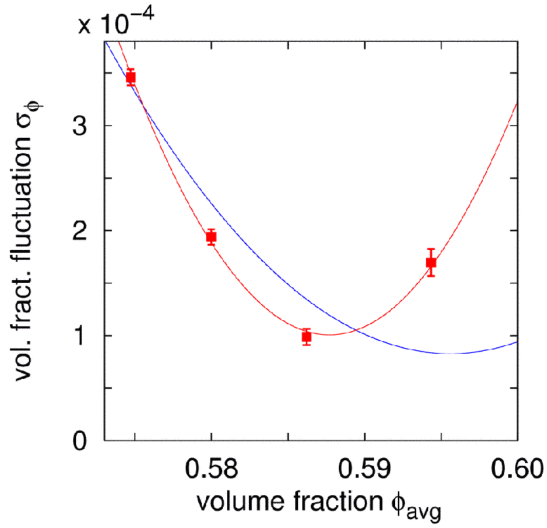
Nowak, E. R. *et al.* Phys. Rev E **57**, 1971 (1998)

Compactivity

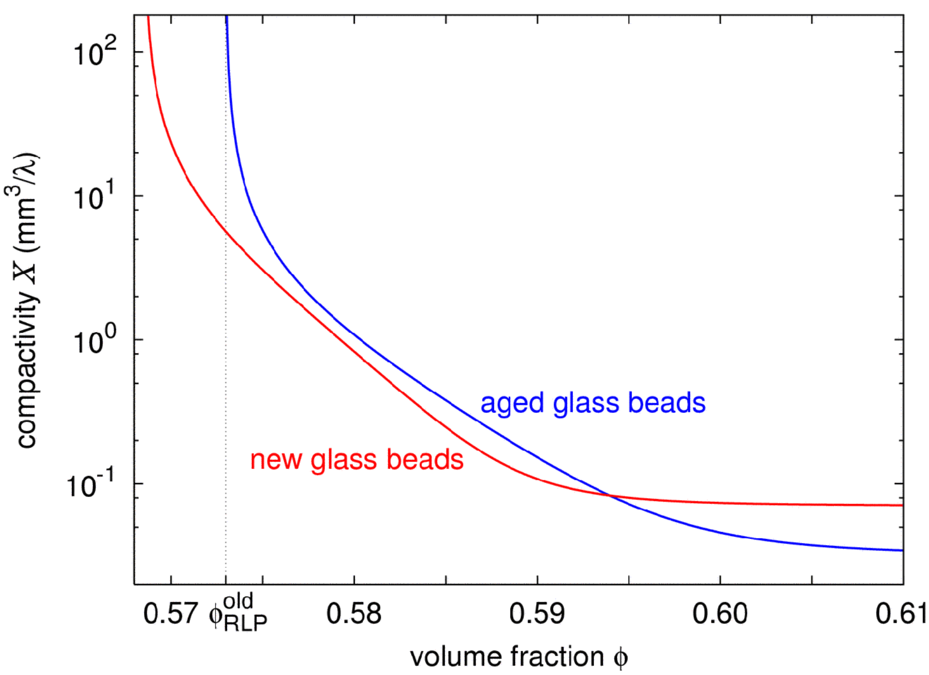


Influence of surface friction





Compactivity



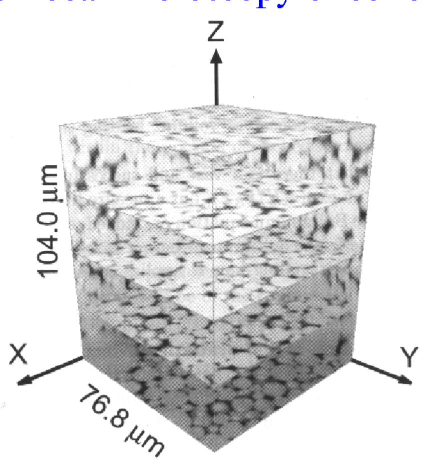
We measured compactivity ....

So what ?

- Did we measure compactivity the right way?
- Is the underlying Edwards hypothesis correct?
- Does compactivity have any predictive value?
- What is the relation between compactivity and the other granular temperatures?

Compactivity from volume distribution ?

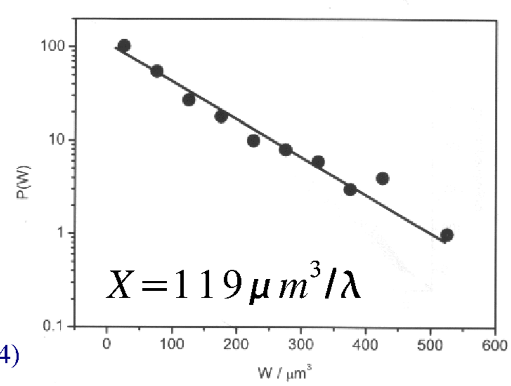
confocal microscopy of colloid



Edwards, Brujić & Makse in *Unifying concepts in Granular Media and Glasses,* Elsevier (2004)

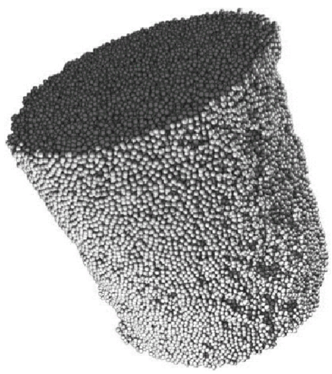
partitioning into first coordination shells of each particle -> volume distribution

$$P(V) \sim \exp\left(\frac{-V}{\lambda X}\right)$$



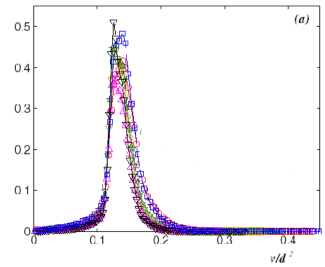
Results from X-ray tomography

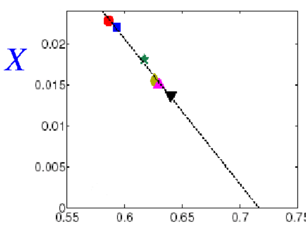
Tomaso Aste, to appear in J. Phys.: Condens. Matter



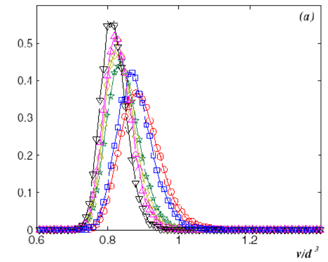
six samples with between 0.586 and 0.640

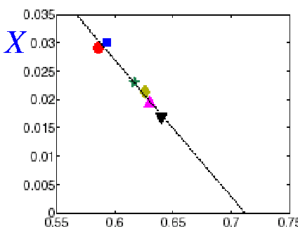
Distribution of Delaunay volumes:





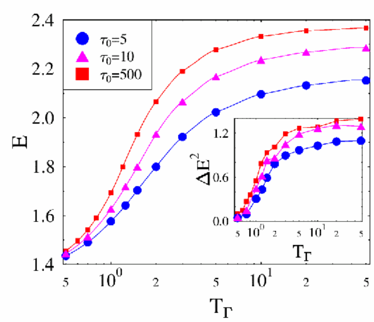
Distribution of Voronoï volumes:

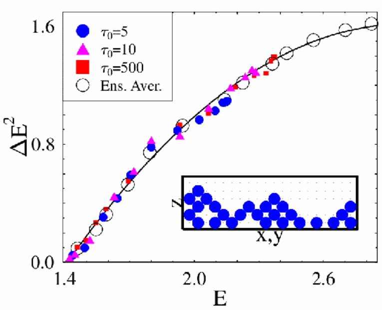




Is there a thermodynamic description?

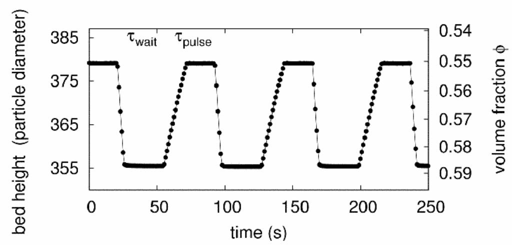
Are the stationary states characterized by few parameters?





Monte Carlo "tap dynamics" simulations

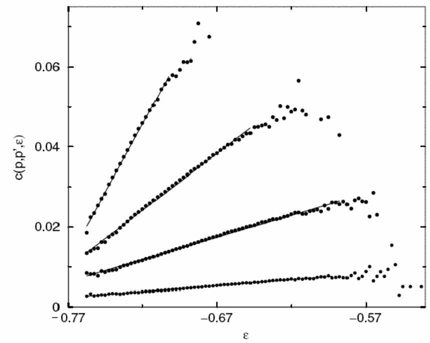
Nicodemi *et al.* in *Unifying concepts in Granular Media and Glasses*, Elsevier (2004)



Pulse duration is an inept parameter for this test. However a more radical test: do fluctuations from different ways of driving coincide?

A weak test of the Edwards measure

Dean & Lefèvre,
PRL **90** 198301 (2003)

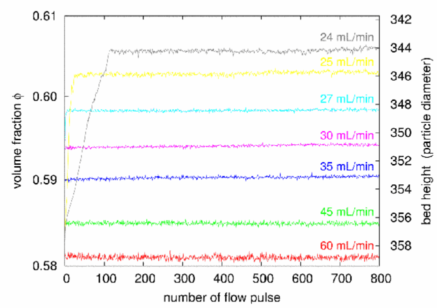


$$c(p, p', E) = \ln\left(\frac{N_D(E, p)}{N_D(E, p')}\right)$$

$$= -[\beta(p) - \beta(p')]E + f(N, p, p')$$

we replace: $E \rightarrow V$

$p, p' \rightarrow Q, Q' \rightarrow V_{avg}, V_{avg}'$

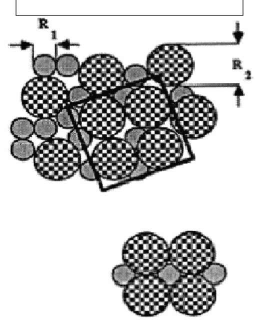


N = 4 000 000,
fluctuations are too small

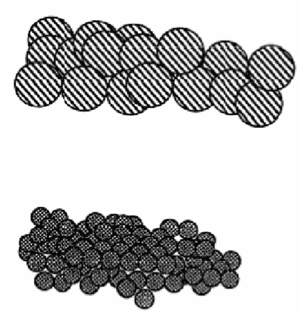
Does compactivity control segregation ?

A. Mehta & S. F. Edwards, *Physica A* **157,** 1091 (1989)

mixed state:



segregated state:



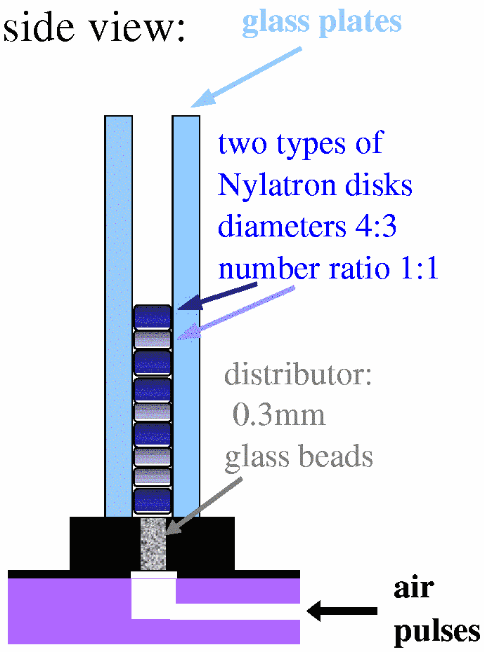
mapped on the Ising model:

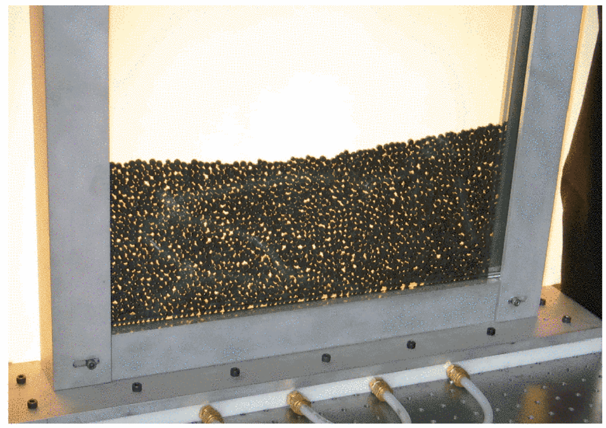
$X > X_C$ $X < X_C$

More recent models, e.g. Tarzia *et al.* cond-mat/0505724

Tapping a quasi-two dimensional bed

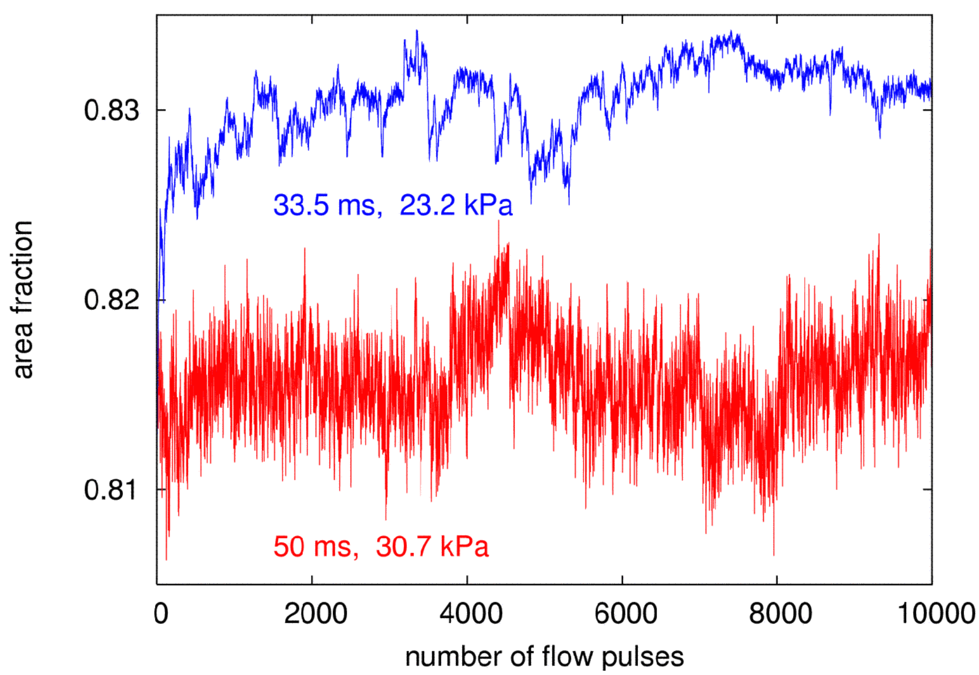
side view:



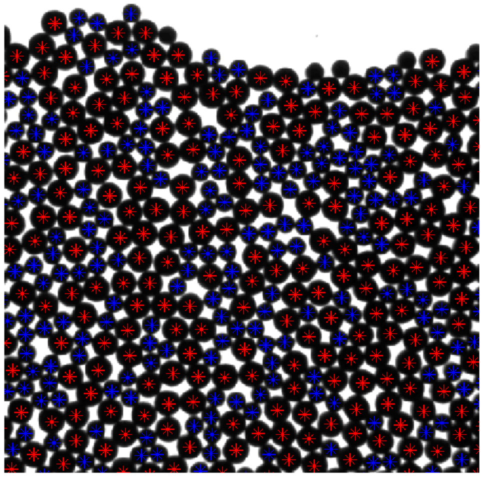


Experiment: Sabrina Nagel

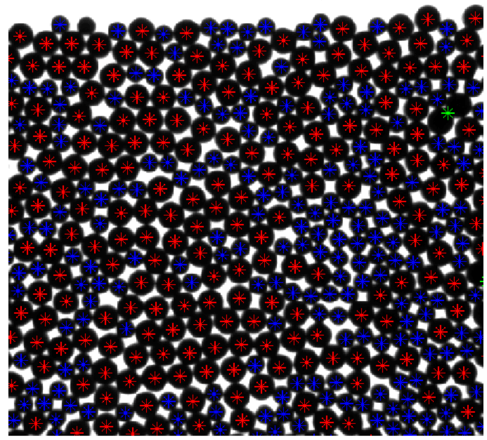
Area fluctuations



Characterizing the segregation



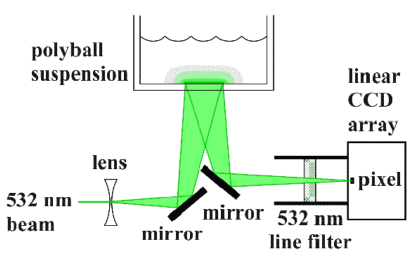
initial state

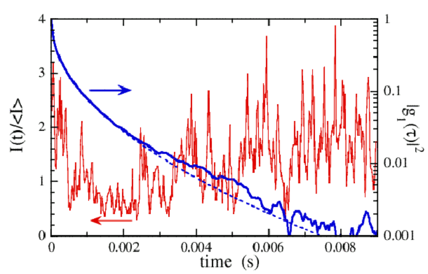


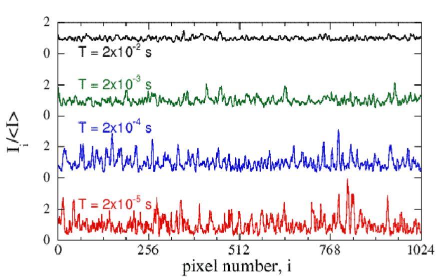
after 10000 flow pulses

Speckle-visibility spectroscopy

Dixon & Durian, PRL **90**, 184302 (2003)
Bandyopadhyay *et al.* cond-mat/0506081

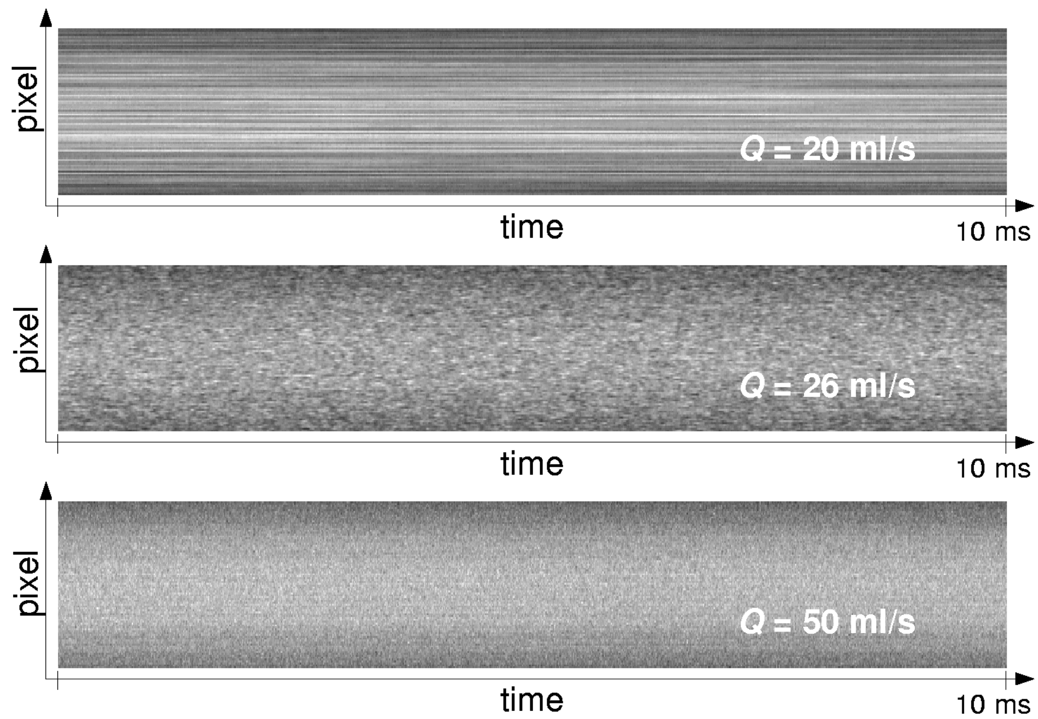




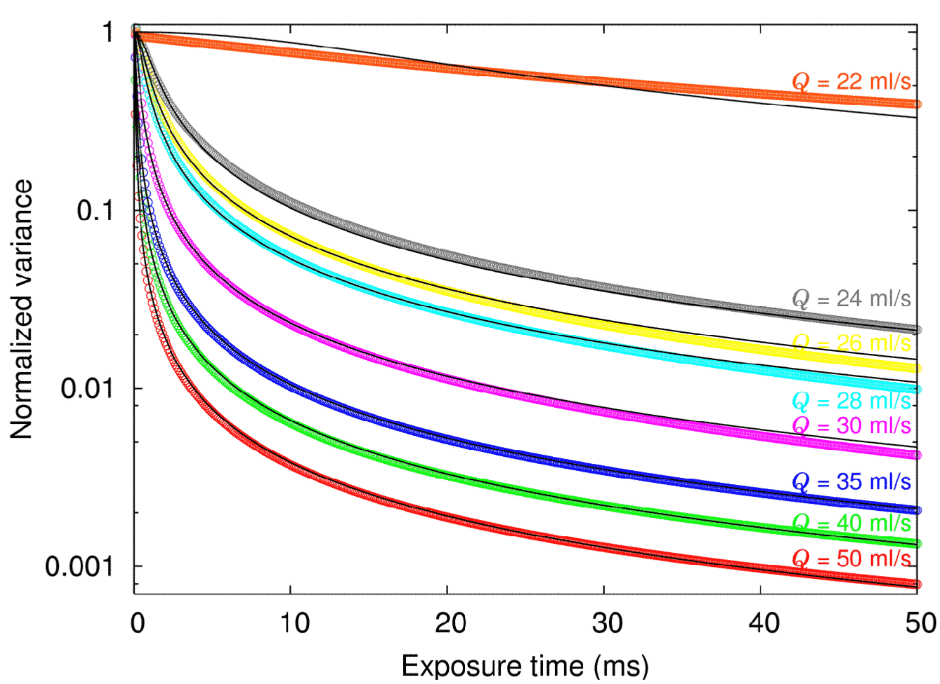


Variance becomes smaller
with longer exposure time.

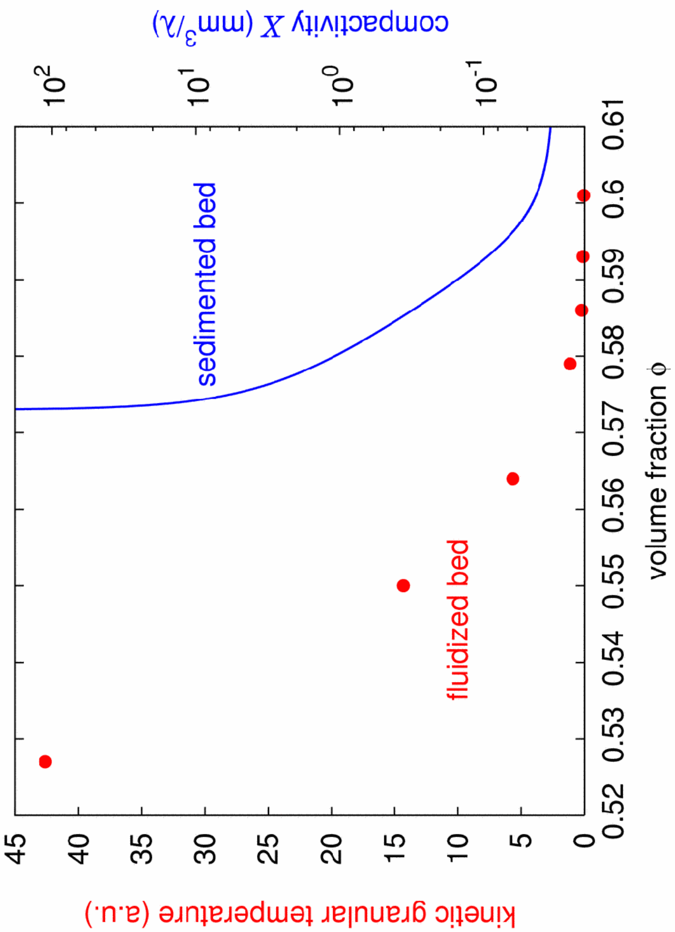
Temporal evolution of speckle pattern



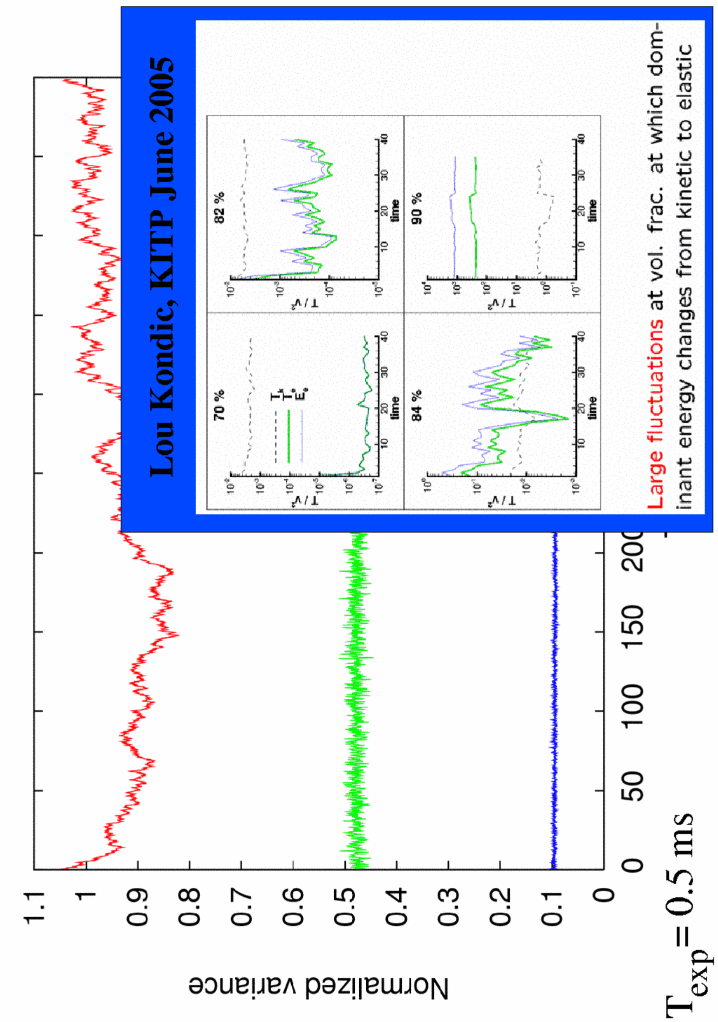
Speckle variance vs exposure time



Kinetic temperature of fluidized state



Time resolved measurements



Conclusions

- Fluidized beds are great tools to study granular media.
- We have now two competing "thermometers" to measure compactivity.
- A possible way to check the Edwards hypothesis is the comparison of the volume fluctuations for different forms of driving (flow pulses, shear, horizontal/vertical shaking).