Ensembles for Stress Transmission in Isostatic Assemblies

David Wu
Colorado School of Mines

KITP Granular Physics Program
June 2, 2005

II. Statistics

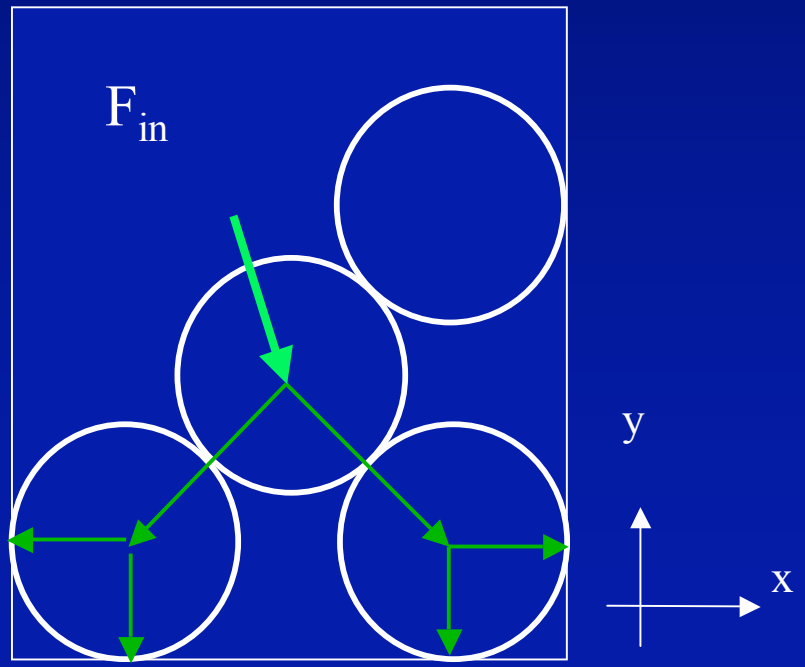
Let me count the ways….

Geometry to Mechanics

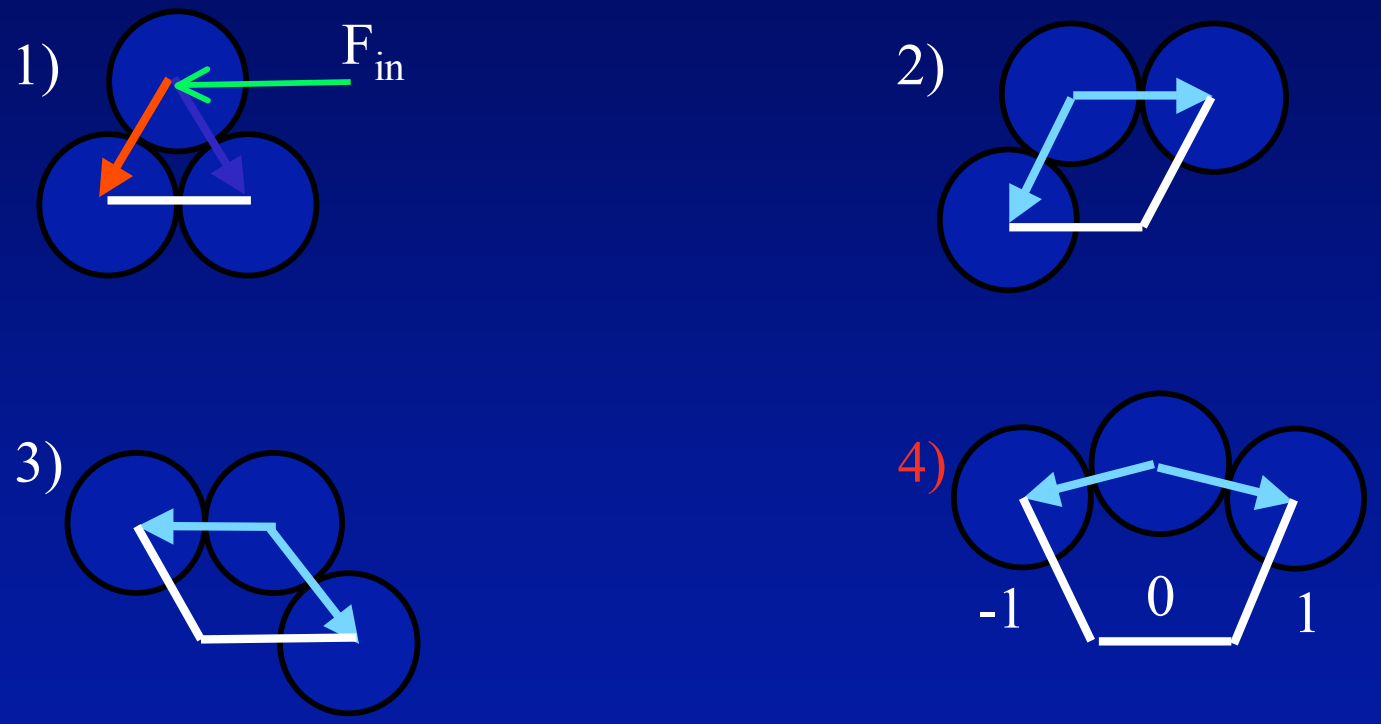
- We can thus exactly relate packing geometry to mechanical properties
- The “scattering centers” are characterized by the 2 special outgoing legs
- Need the spatial field of scattering centers
  - Quenched distribution
  - Spatial correlations
- Seek a simplified model system

Specific Model: Random Sequential Deposition of Smooth Hard Disks

- Each particle deposited is a little larger than the previous one-> well defined contact geometry
- Approximately a lattice



Analytic Theory: Response Function

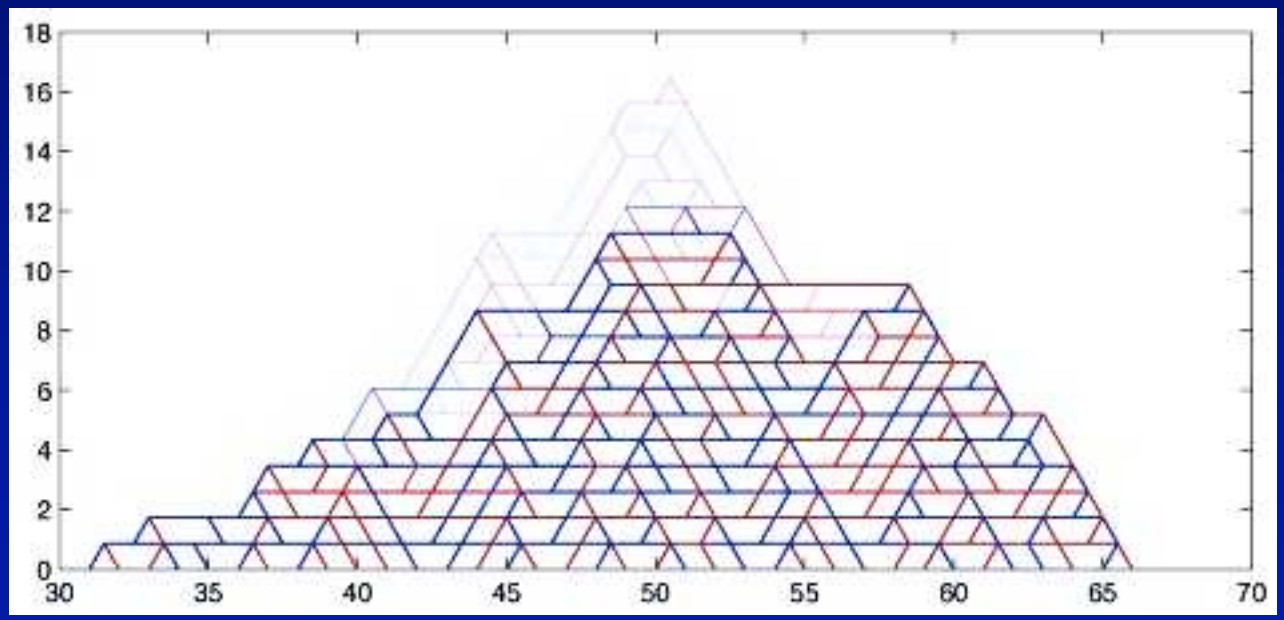


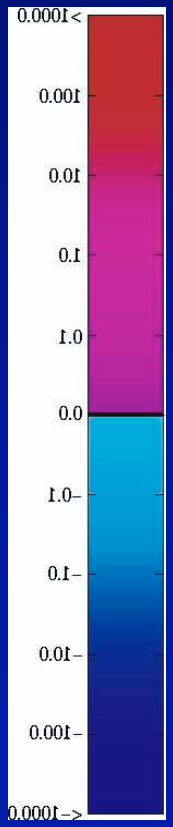
The four support geometries (types of scattering center)

The steady-state pack surface growth statistics can also be calculated.

Direct Simulation of Pack

- Single Instance Response Function



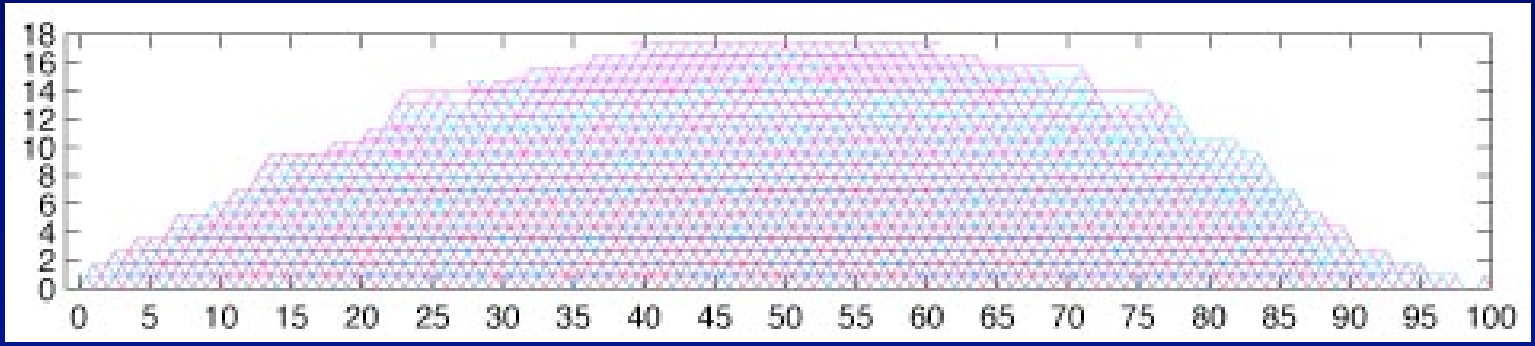


Compression

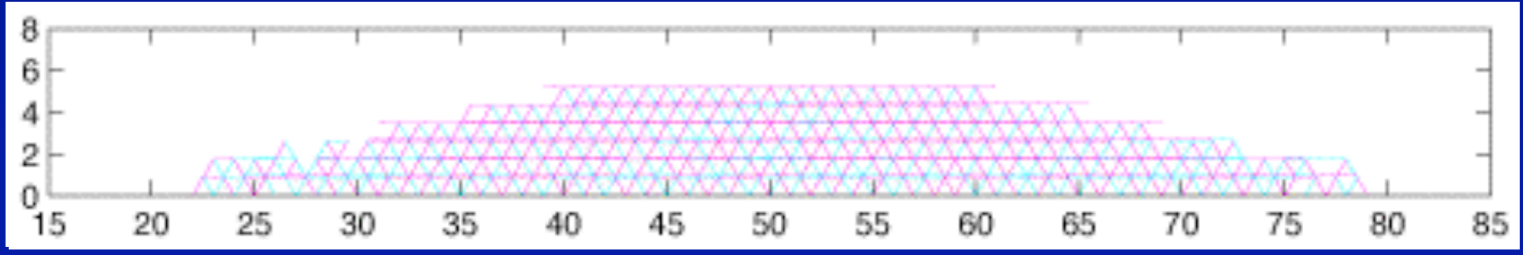
Decompression

100 x 20 pack

Ensemble Average: Large Fluctuations



100,000 instances

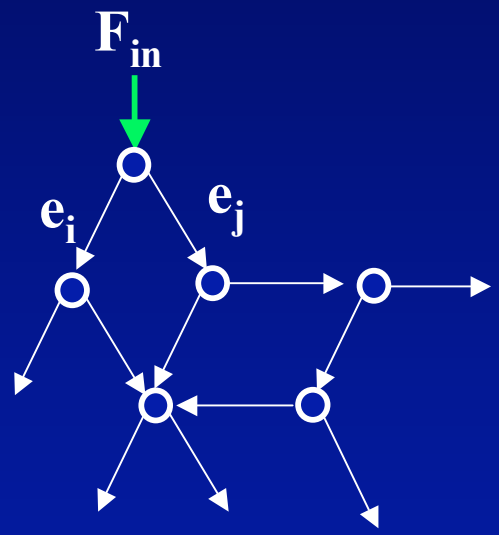


1,000,000 instances

Force Scattering: Boltzmann Equation on Quenched Fields

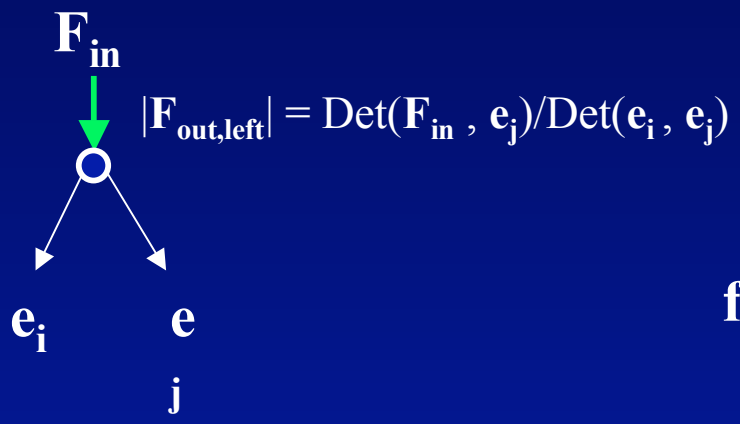
A force applied in the system is thus resolved sequentially by "scattering centers" fixed in space:

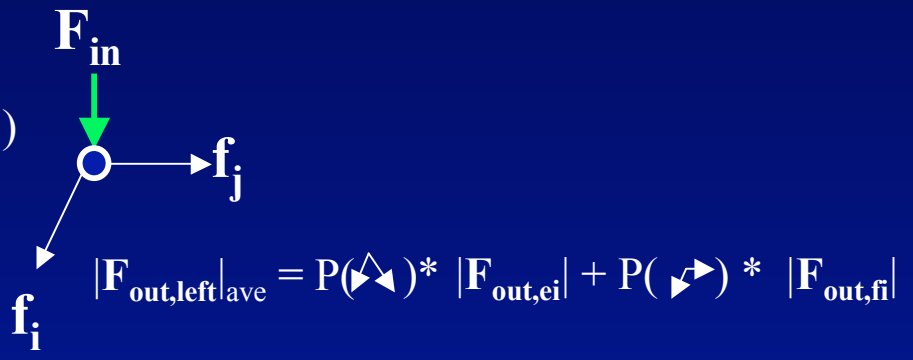
Boltzmann Equation on Quenched Fields

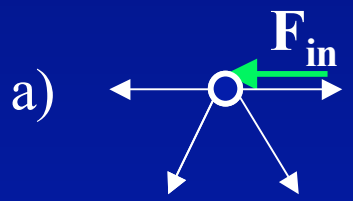


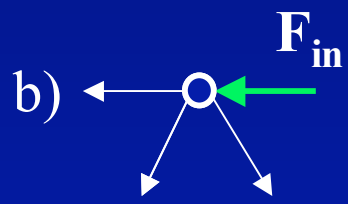
Analogous to wave scattering in inhomogeneous media, except "sequentially" doesn't correspond to any time:
Summing over sequential events gives force response function

Force Scattering: Analytic Approximations







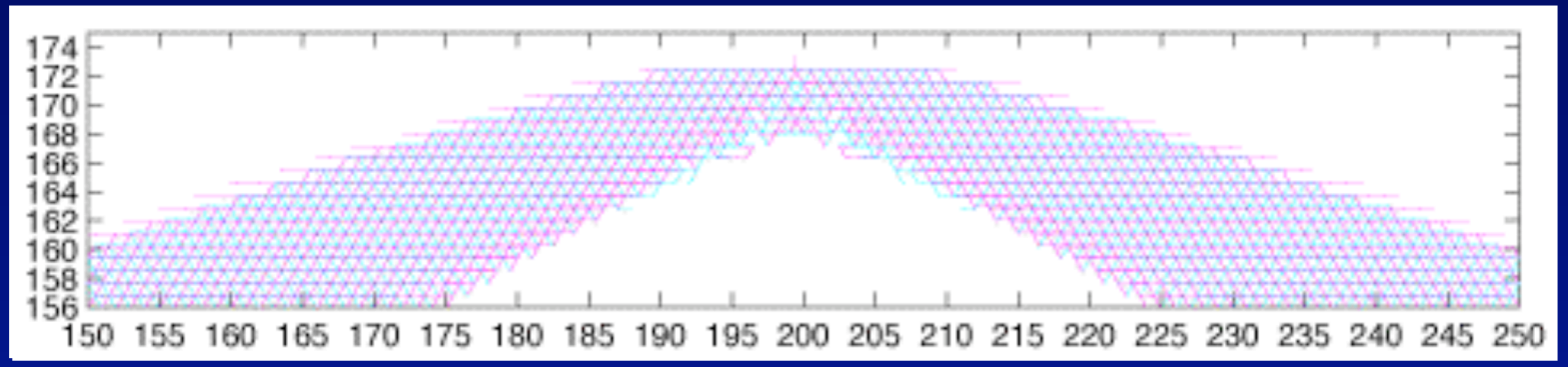


Uncorrelated Scattering

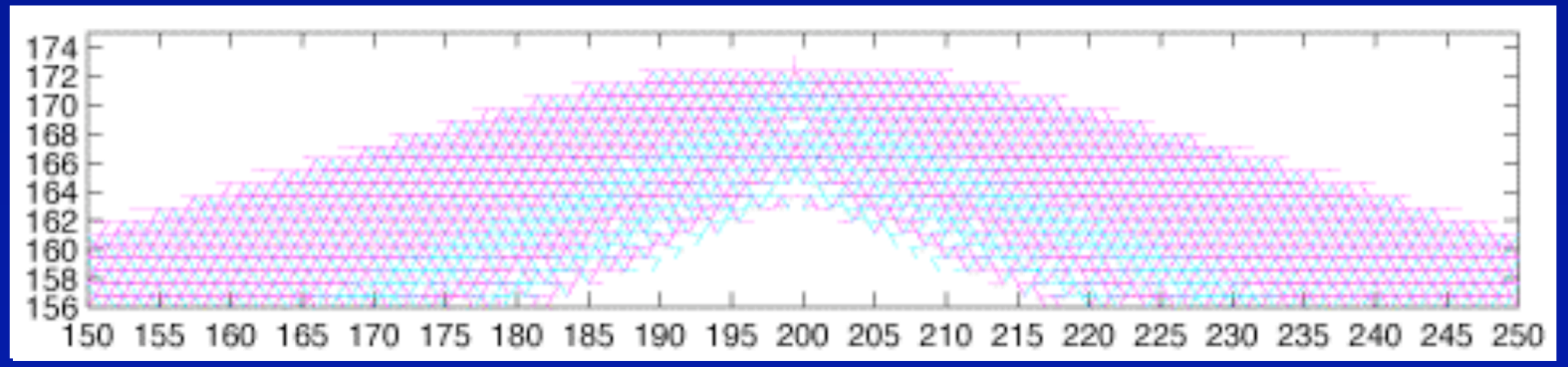
Correlated Scattering

1) We assume only incoming direction is needed.
2) Correlations treated at nearest neighbor, sequential scattering.

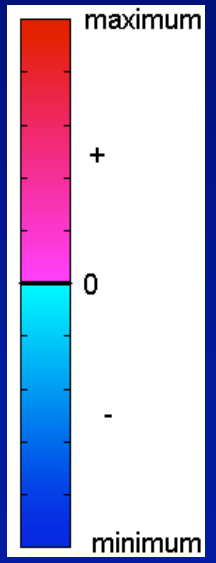
Average Response Function (Analytic Approximations)



Uncorrelated Scattering

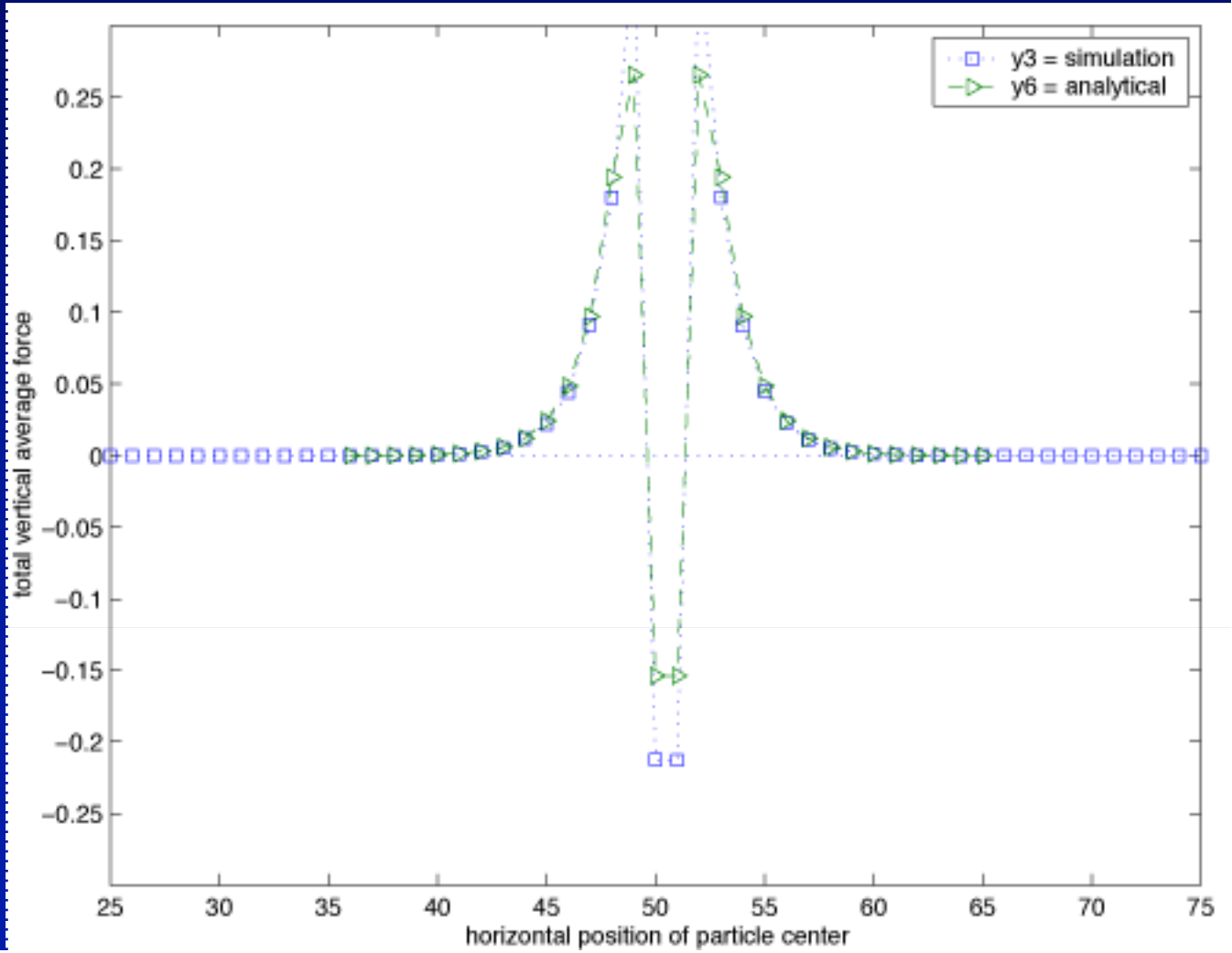


Correlated Scattering



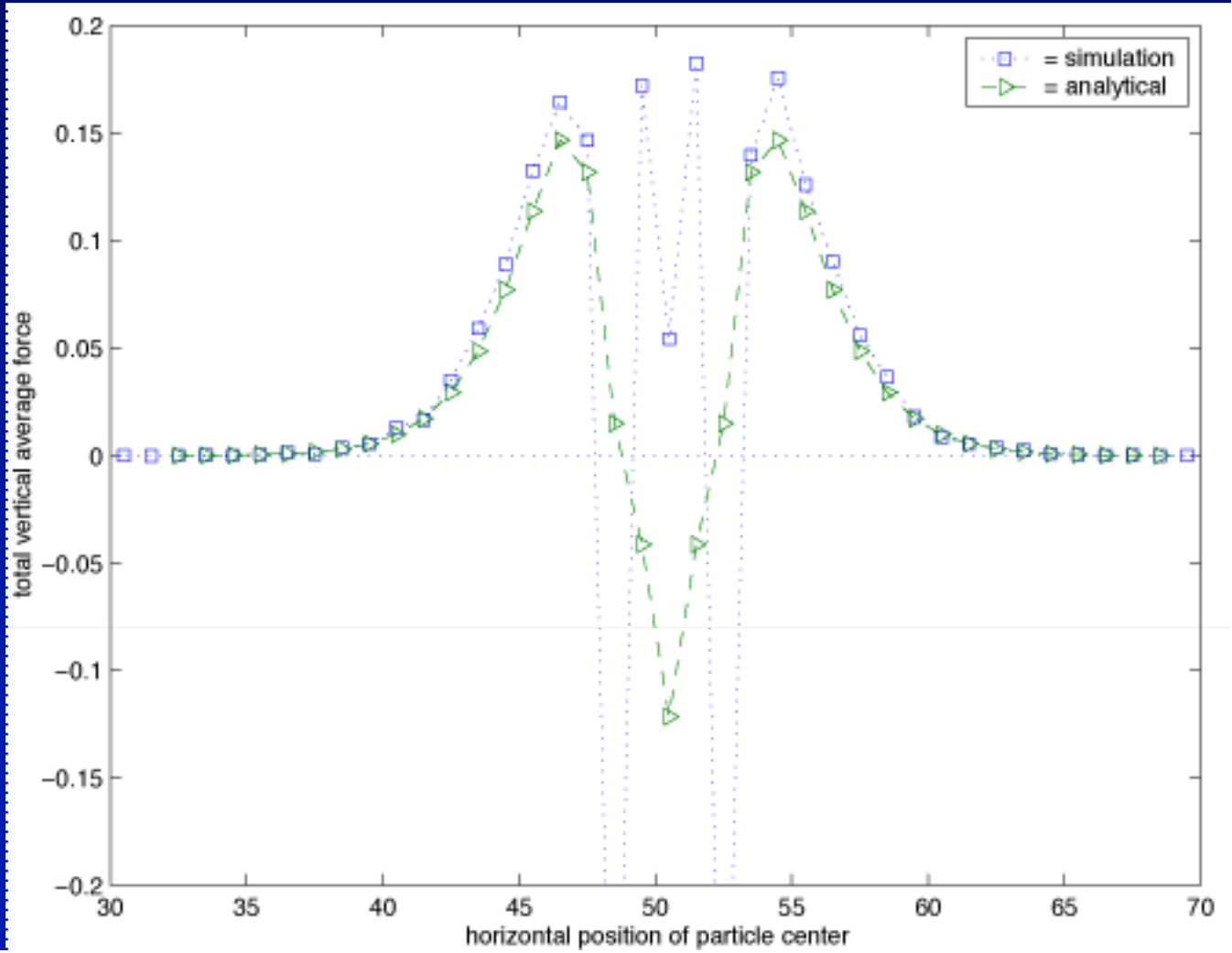
Comparison with Simulation

Normal Force Magnitude at a Depth of 1 particle



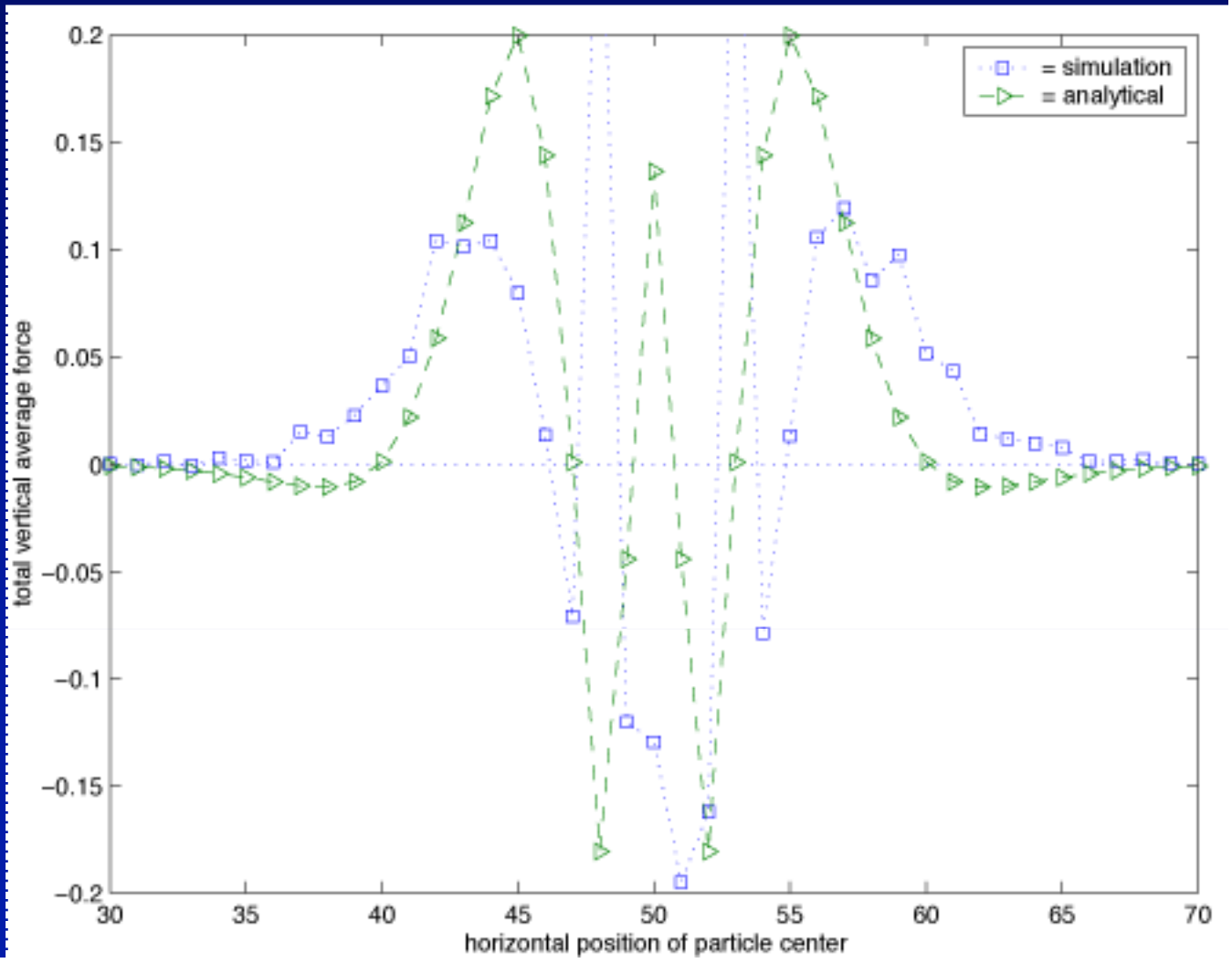
Comparison with Simulation

Normal Force Magnitude at a Depth of 2 particle



Comparison with Simulation

Normal Force Magnitude at a Depth of 3 particle



Summary for Lattice Model

- A simple model was chosen where the spatial distribution of scattering types is known
- The steady-state probabilities of scattering types are known, and used to iteratively propagate a force
  - Assuming no spatial correlation
  - Assuming nearest neighbor correlation for consecutive scattering
- The response function was compared to numerical simulation of the same model

Conclusions for Lattice Model

- Fluctuations are exponentially large
  - consistent with random multiplicative process Moukarzel J. Phys '02
  - but stably formed packs likely reorganize to avoid large stresses
- Off-axis bimodal response function was found
- Spatial correlations can significantly alter the response function
- Nearest neighbor correlation was inadequate for matching experiment beyond a couple of layers

Acknowledgements

- Tawiwan Kangsadan & Jeremy Lechman
- Particle Science & Technology Group, CSM

- Research Corporation
  - Research Innovation Award RI0161, 1997