Interacting massless Dirac fermions in 2D

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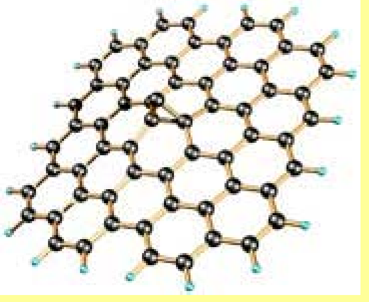
Examples of 2D Dirac fermions in condensed matter

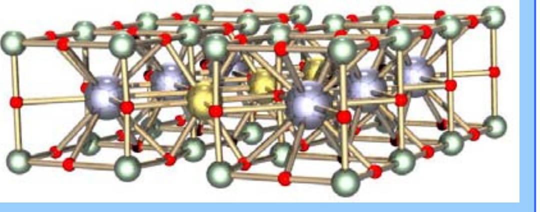
Graphene



Dirac nodes due to:
C_{3v} symmetry of the wavevector (w/out SO)

- must **fine-tune** E_F to sit at Dirac point
- breaking lattice symmetries (e.g. reflection about the bond) generally lifts the degeneracy
- definite charge => cyclotron orbits + QHE

***d*-wave superconductors**



Dirac nodes due to:
the topology of the gap

- need <u>not</u> **fine-tune** E_F to sit at Dirac point
- generally, breaking lattice symmetries DOES NOT lift the degeneracy (e.g. YBCO is orthorhombic=> d+s)
- indefinite charge => no cyclotron orbits

Interacting massless Dirac fermions in graphene

Interactions do not break the lattice symmetries

$$\mathcal{H} = \mathcal{H}_0 + \hat{V}$$

$$\mathcal{H}_0 = \sum_{j=1}^{N} \int d^2\mathbf{r} \left[\psi_j^\dagger(\mathbf{r}) v_F \mathbf{p} \cdot \sigma \psi_j(\mathbf{r}) \right]$$

"relativistic"

$$\hat{V} = \frac{1}{2} \int d^2\mathbf{r} d^2\mathbf{r}' \left[\delta\hat{n}(\mathbf{r}) \frac{e^2}{4\pi\epsilon} \frac{1}{|\mathbf{r} - \mathbf{r}'|} \delta\hat{n}(\mathbf{r}') \right]$$

non-relativistic

- *Short range interactions perturbatively irrelevant*
- *Disorder + interactions interesting, but will not be considered*

$$r \to br \not\Rightarrow \mathcal{H} \to \frac{1}{b}\mathcal{H}$$

Interacting massless Dirac fermions in graphene: weak coupling approach

Interaction strength is given by $\alpha = \dfrac{e^2}{\epsilon \hbar v_F}$ "fine structure constant"

In weak coupling:
(equivalently H-F)

$$\frac{dv_F}{d \ln k} = \frac{e^2}{\epsilon \hbar} \Rightarrow \boxed{v_F(k) = v_F^{(0)} + \frac{e^2}{8\pi\hbar} \ln \frac{\Lambda}{k}}$$

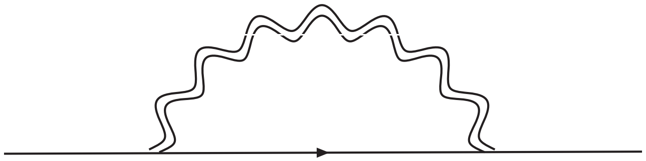
Gonzalez *et. al.* Nucl. Phys. B 424, 595 (1994)

Since $\alpha \sim 1$, its hard to justify the weak coupling approach.

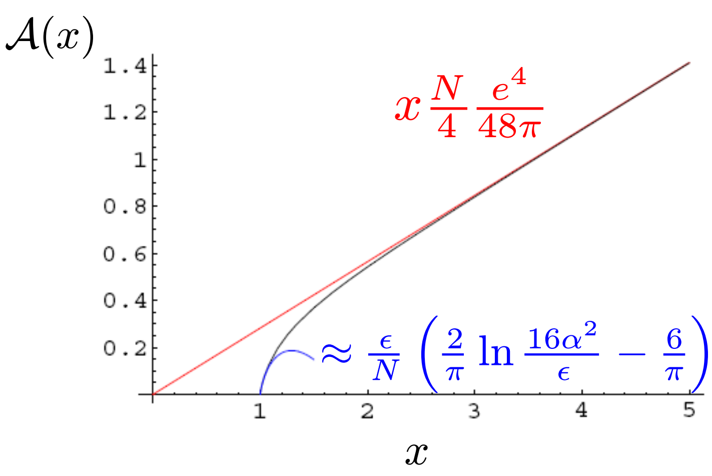
Note the divergent group velocity: the non-relativistic approximation invalid when $v_F \sim c$. Must include retardation and current-current coupling. Ultimate NFL fixed point @ $v_F = c$.

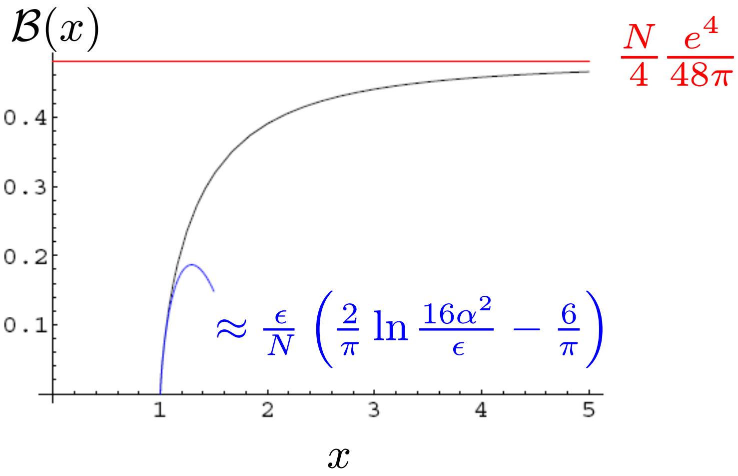
Interacting massless Dirac fermions in graphene: RPA

RPA (includes screening effects)



$$\Im m\Sigma_{RPA}(\omega,k,T=0) = -\left[k\mathcal{A}\left(\frac{\omega}{k}\right) + \mathbf{k}\cdot\sigma\mathcal{B}\left(\frac{\omega}{k}\right)\right]$$





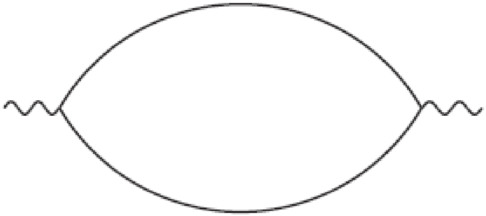
Interacting massless Dirac fermions in graphene: thermodynamics

To the leading order in large N expansion, which includes the screening effects, the free energy density is

$$f = -Nk_B T^3 \frac{3\zeta(3)}{4\pi v_F^2} + \delta f$$

$$\delta f = \int_0^\Lambda \frac{dqq}{2\pi} \int_0^\infty \frac{d\Omega}{2\pi} \coth\frac{\Omega}{2T} \left\{ \tan^{-1}\left[\frac{\Im m \Pi_0^{ret}(q,\Omega,T)}{\frac{q}{e^2} + \Re e \Pi_0^{ret}(q,\Omega,T)}\right] - \tan^{-1}\left[\frac{\Im m \Pi_0^{ret}(q,\Omega,0)}{\frac{q}{e^2} + \Re e \Pi_0^{ret}(q,\Omega,0)}\right] \right\}$$

$$\Pi_0^{ret}(q,\Omega,T) =$$



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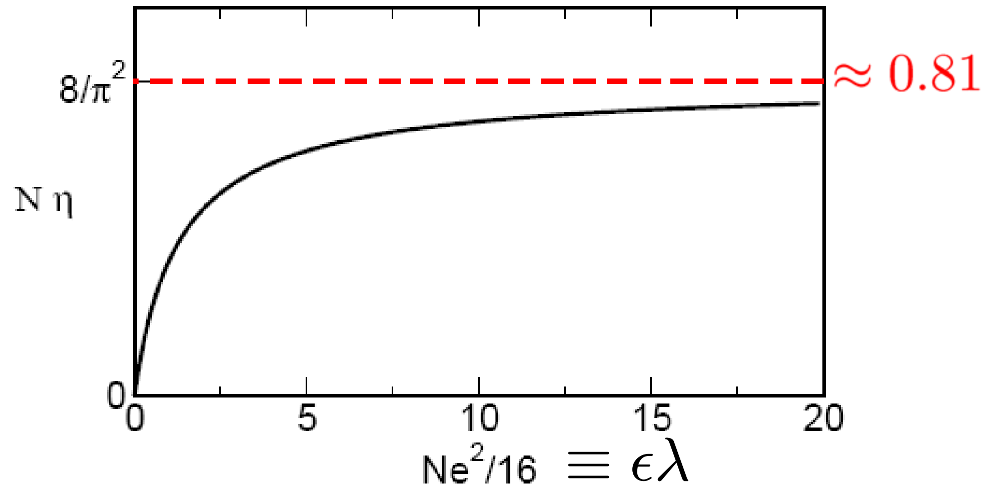
Interacting massless Dirac fermions in graphene: thermodynamics

The free energy correction:

$$\frac{\delta f}{f_0} = -2\eta \ln\left(\frac{T_{UV}}{T}\right)$$

$$\eta = \frac{1}{N}\left(\frac{16}{\pi^2}\lambda^2 g(\lambda) - \frac{2}{\pi\lambda}\right)$$



$$g(x) = \frac{1}{2x^2} + \frac{\tan^{-1}\left[\frac{\sqrt{1-x^2}}{x}\right]}{2x^3\sqrt{1-x^2}}; \quad 0 < x < 1$$

$$g(x) = \frac{1}{2x^2} + \frac{1}{4x^3\sqrt{x^2-1}} \ln\left[\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}\right]; \quad x > 1$$

For $N = 4$, $v_F \approx 10^6 m/s$, $\epsilon \approx 1$ we get $\eta \approx 0.06$ $(T_{UV} = \frac{\hbar v_F \Lambda}{k_B} \approx 1eV)$

The correction is $\mathcal{O}(1)$ at $T \approx 2K$

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The free energy suppression

The expression $\frac{\delta f}{f_0} = -2\eta \ln \frac{T_{UV}}{T}$ can be understood as the first non-trivial term in the Taylor expansion of

$$f(T, \Lambda, \eta) = -2Nk_B T \int_0^\infty \frac{dk}{2\pi} k \ln \left[1 + \exp\left(-\frac{\epsilon_\eta(k)}{k_B T} \right) \right].$$

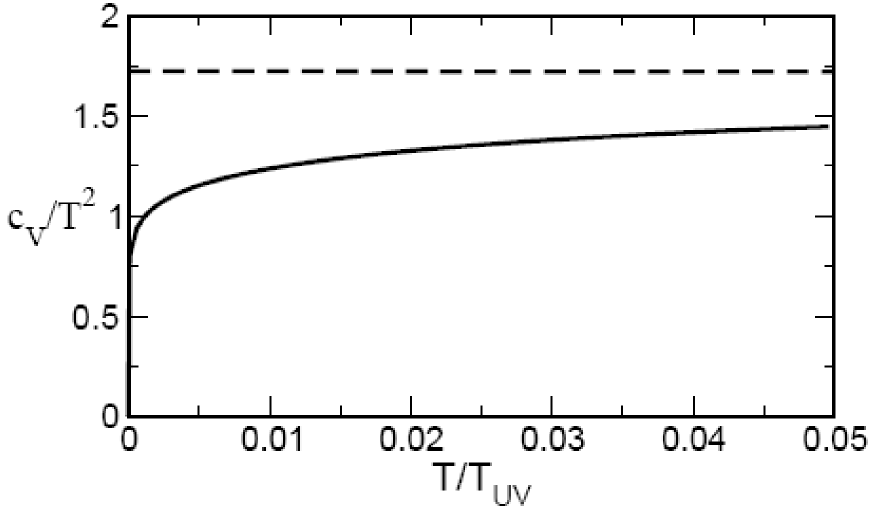
with

$$\epsilon_\eta(k) = \hbar v_F k \left(1 + \eta \ln \left[\frac{\Lambda}{k} + 1 \right] \right)$$

In the limit of $e^2 \to 0$, the above expression coincides with the free energy calculated within the Hartree-Fock approximation. Thus the suppression of the specific heat $c_V = -T\partial^2 f/\partial T^2$ relative to the non-interacting case *persists* when the polarization effects are included.

The Coulomb interaction effectivelly suppresses the density of states.

The specific heat suppression



One of the non-trivial predictions of this theory is the dependence of the suppression of c_V on the dielectric constant ε.

In the strict large *N* limit, the dependence on e^2 drops out. We can compare the resulting suppression to the gauge theory **without** the time component of the gauge field (Kim, Lee, and Wen PRL 1997) where the specific heat is *enhanced*.
This enhancement is *exactly* compensated by the suppression found here.
This follows from the Lorenz invariance of 2+1D QED where the two effects must cancel to each order in large N.

Plasmons at the Dirac point

In 2 dimensions the plasma oscillations obey the dispersion relation

$$\omega_{pl} \sim \sqrt{\frac{q}{\xi}},$$

where ξ is the screening length.
At the Dirac point, the screening length diverges as $\xi \sim \frac{v_F}{T}$ which gives

$$\omega_{pl} \sim \sqrt{Tq}.$$

The full RPA expression for the plasma mode and its decay rate is

$$\omega_{pl}(q,T) = \sqrt{Tv_F q}\left[\frac{\frac{v_F q}{T} + \alpha_F \frac{N \ln 2}{2\pi}}{\sqrt{\frac{v_F q}{T} + \alpha_F \frac{N \ln 2}{\pi}}}\right]$$

$$\frac{1}{\tau_{pl}} = \frac{\pi}{4 \ln 2} \frac{\omega_{pl}^2(q,T)}{T} \mathrm{th} \frac{\omega_{pl}(q,T)}{4T}$$

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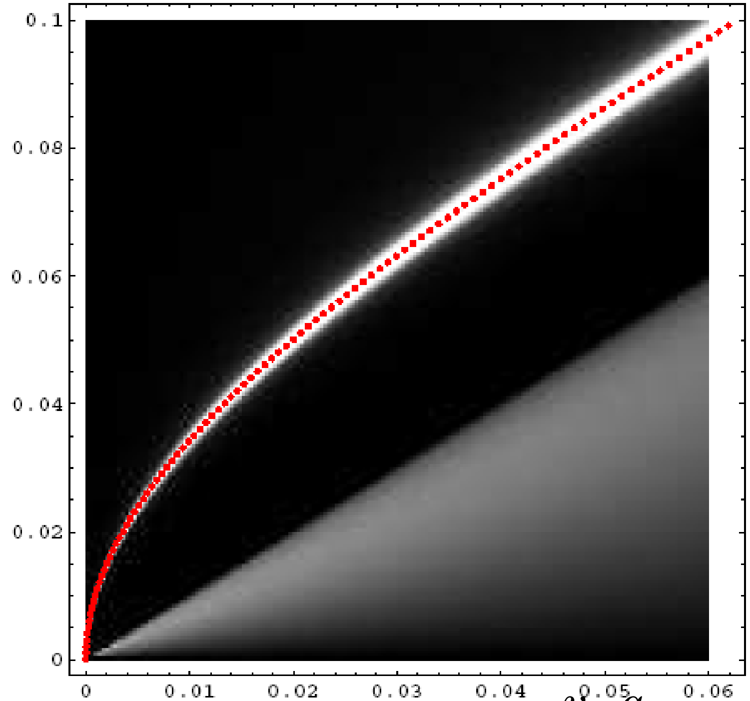
Plasmons at the Dirac point

Density plot of $\Im m D^{ret}_{RPA}(\Omega, q, T) \sim S(\Omega, q, T)$



$\frac{\Omega}{T}$

$\frac{v_F q}{T}$

At room temperature and for $q^{-1} \approx 400\text{nm}$, $\omega_p \approx 6\text{THz}$.

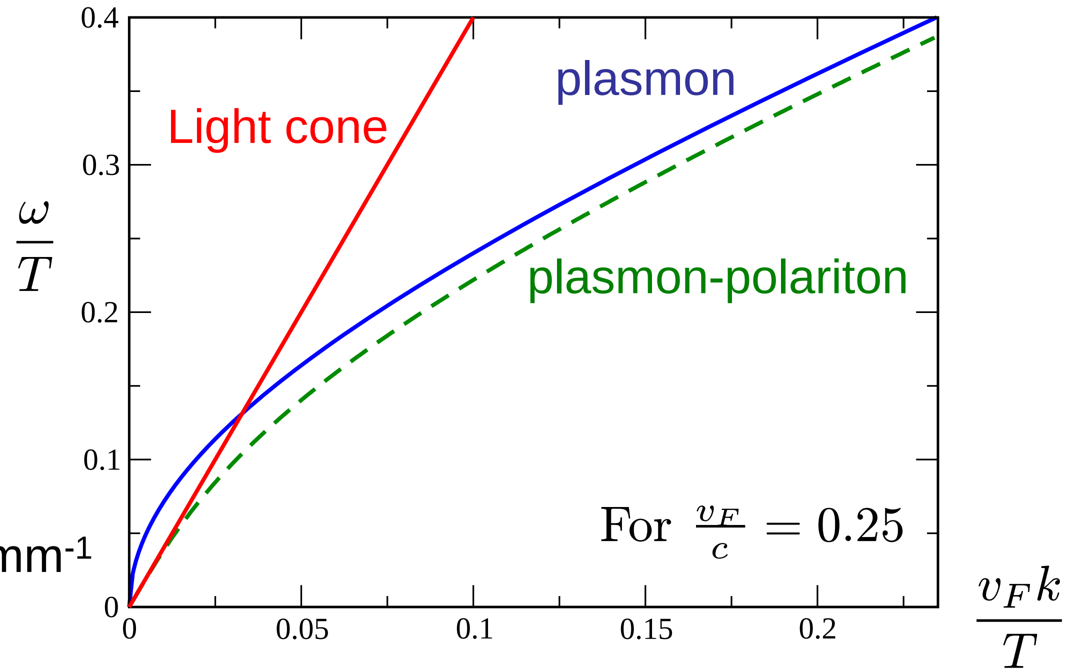
Finite T plasmon and its coupling to light at Dirac point

Since the Hamiltonian is scale-free we can measure all energy scales in units of $k_B T$ and all length in units of $\ell_T = \frac{\hbar v_F}{k_B T}$. The coupling of the thermal quasi-particles to the three-dimensional electromagnetic radiation leads to a thermo-plasma polariton mode. In dimensionless variables $s = \frac{\omega_p}{T}$ and $t = k\ell_T$, the thermoplasma polariton frequency $\omega_p(k,T)$ satisfies

$$\alpha_F^2 \left(\frac{N \ln 2}{2\pi}\right)^2 \frac{1}{t^2} \left(1 - \frac{v_F^2}{c^2}\frac{s^2}{t^2}\right) \left(1 - \frac{|s|}{\sqrt{s^2 - t^2}}\right)^2 = 1$$

Plasmon-photon mixing

The mixing (at room T) @ k~1mm^{-1}



Experimental observation of the plasmon-polariton in 2DEG

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Observation of Retardation Effects in the Spectrum of Two-Dimensional Plasmons

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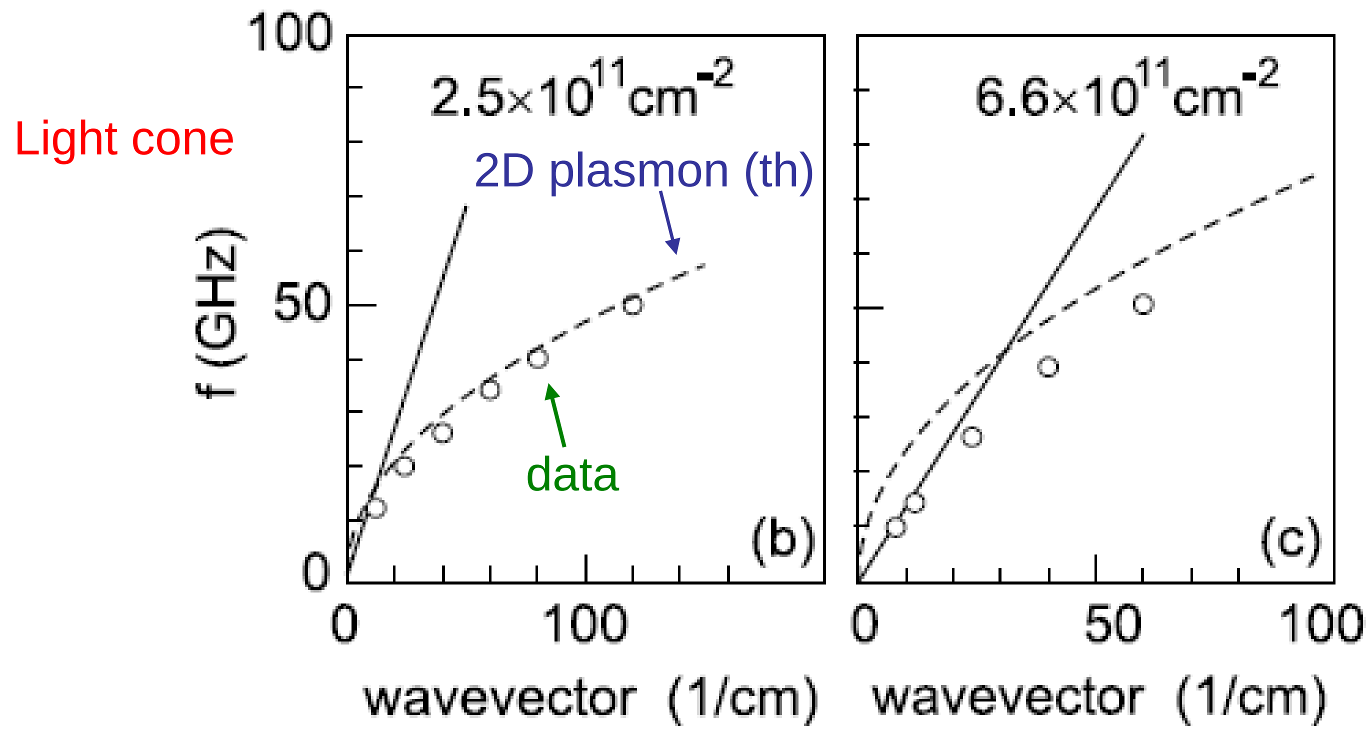
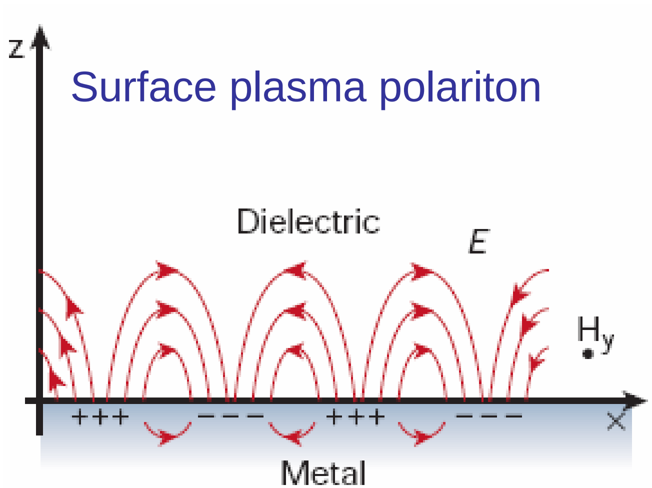
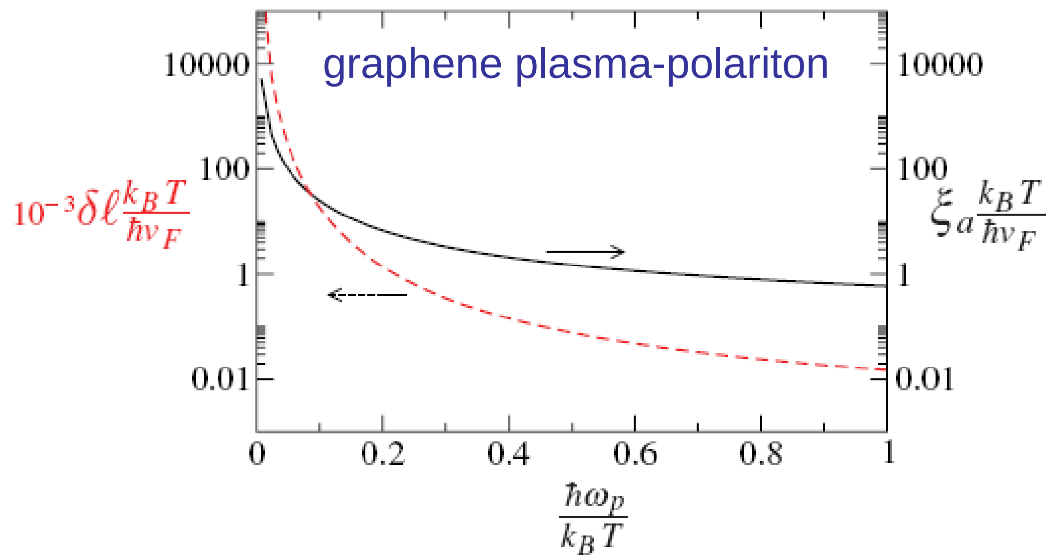


Frequency dependence of the atttenuation length





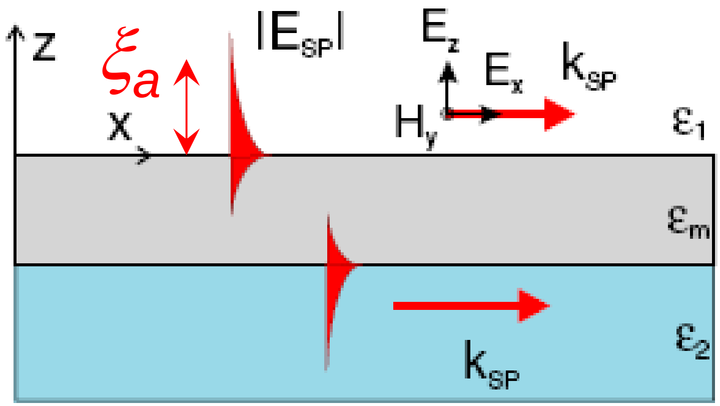


FIG. 2 (color online). The thermoplasma polariton (in-plane) propagation length $\delta\ell$ (dashed red curve) and the (out-of-plane) attenuation length ξ_a (solid black curve) normalized to thermal length [$\ell_T = \hbar v_F/(k_B T)$] vs the mode frequency ω_p normalized by $k_B T/\hbar$. At room temperature and for $\omega_p = 20$ THz, $\delta\ell \approx 2\ \mu$m.

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Near-field characterization of guided polariton propagation and cutoff in surface plasmon waveguides

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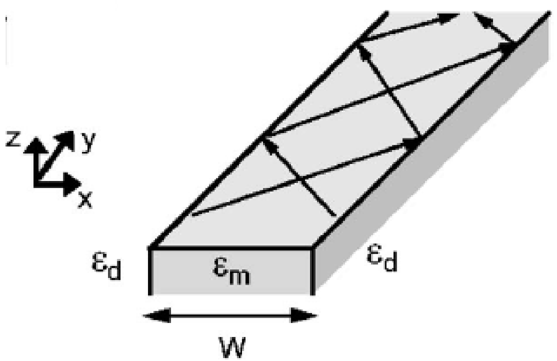

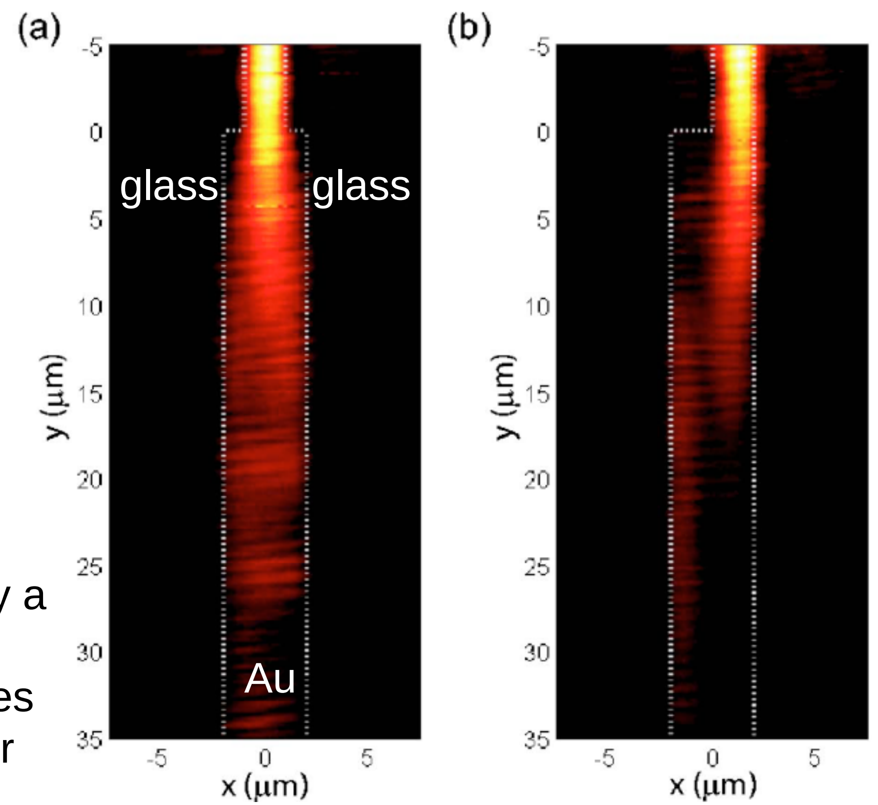

Experimental demonstration of multimode interference between the two guided modes supported by a 4μ wide Au stripe as excited by a 2μ wide input stripe. The dashed white lines indicate the outline of the Au structures. Frames (a) and (b) show near-field images acquired for symmetric and asymmetric alignment of the input stripe, respectively.

Guiding graphene thermo-plasmons with temperature?

The group velocity $v_g = \frac{\partial \omega_p(q,T)}{\partial q} \sim \sqrt{\frac{T}{q}}$