Interaction-driven states in bilayer graphene

Maxim Kharitonov

Center for Materials Theory, Rutgers University

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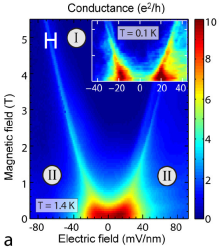
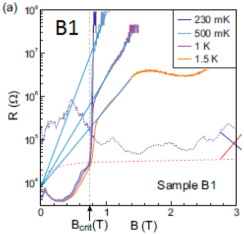
Transport experiments on bilayer graphene at zero doping:

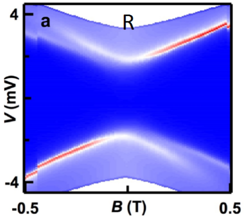
Evidence for interaction-induced states! Several scenarios reported!

R. T. Weitz *et al.*, Science **330** (2010) (Harvard=H)
F. Freitag *et al.*, arXiv:1104.3816 (2011) (Basel=B)
A. S. Mayorov, *at al.* Science **333** (2011) (Manchester=M)
J. Velasco Jr.*et al.*, arXiv:1108.1609 (2011) (Riverside=R)

	insulating	"metallic"
QH ($B \gtrsim 1$T):	H, B1,R ($G \sim \exp(-\Delta/T)$, $\Delta \sim 10 B_\perp$[T]K)	B2 ($0 < G < 0.5 e^2/h$)







Earlier in BLG: B. E. Feldman, *et al*, Nature Phys. **5** (2009); Y. Zhao *et al.*, Phys. Rev. Lett. **104** (2010).

Earlier, in MLG: J. G. Checkelsky, *et al*, Phys. Rev. Lett. **100**, 206801 (2008) (Princeton); Xu Du *et al.*, Nature 462 (2009) (Rutgers) K. I. Bolotin *et al.*, Nature 462 (2009) (Columbia)

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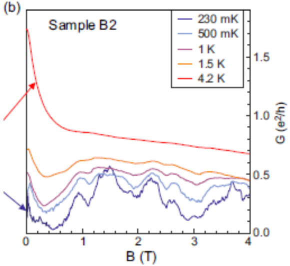

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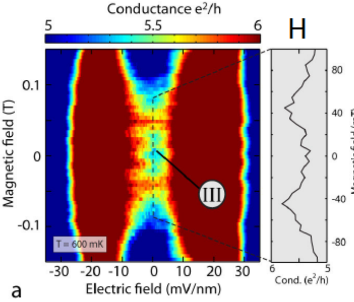

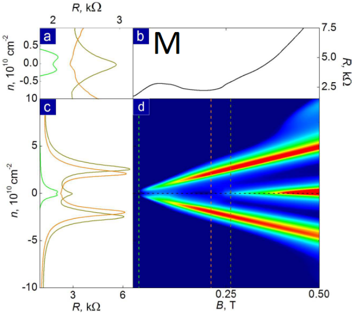

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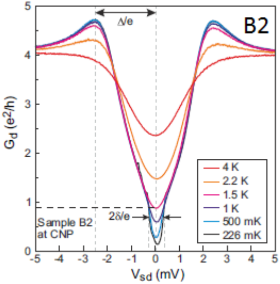
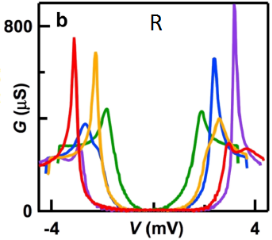
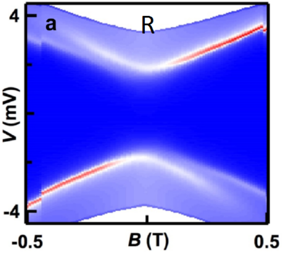




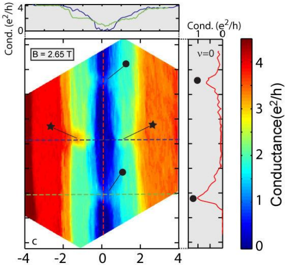


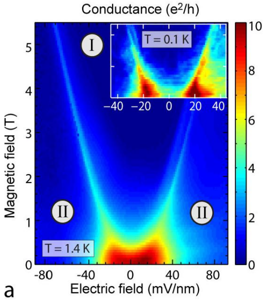
Dual-gated BLG devices:
Perpendicular electric field E = "Zeeman effect" for "layer isospin"

$\nu = 0$ state in QH regime:

phase transitions to another insulating phase observed as E applied!

R. T. Weitz *et al.*, Science **330**, 812 (2010) (Harvard)





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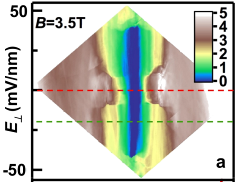


Challenge (theoretical+experimental):

understand the nature of and unambiguously identify the interaction-induced states realized in experiments

Theory:

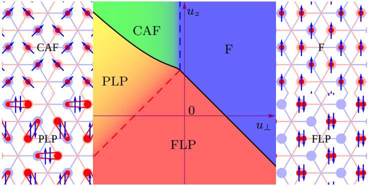
H. Min *et al.*, Phys. Rev. B **77**, 041407(R) (2008).
O. Vafek and K. Yang, Phys. Rev. B **81**, 041401(R) (2010).
F. Zhang *et al.,* Phys. Rev. B **81**, 041402(R) (2010).
R. Nandkishore and L. Levitov, Phys. Rev. Lett. **104**, 156803 (2010).
Y. Lemonik *et al.,* Phys. Rev. B **82**, 201408(R) (2010).
O. Vafek, Phys. Rev. B **82**, 205106 (2010).
R. Nandkishore and L. Levitov, Phys. Rev. B **82**, 115124 (2010).
F. Zhang *et al.* Phys. Rev. Lett. **106**, 156801 (2011).
J. Jung, F. Zhang, and A. H. MacDonald, Phys. Rev. B **83**, 115408 (2011).
Y. Barlas *et al.,* PRL **101** (2008)
D. A. Abanin, S. A. Parameswaran, and S. L. Sondhi, PRL **103**, (2009)

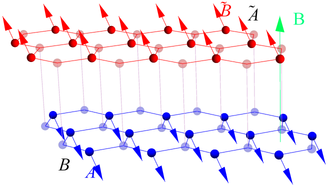
Challenge (theoretical+experimental):

understand the nature of and unambiguously identify the interaction-induced states realized in experiments

This work:

- Can we (realistically) list all possible phases of the $\nu = 0$ state – generic phase diagram? YES (arXiv:1105.5386)
- Do we have sufficient data to identify the phase realized experimentally in the insulating $\nu = 0$ state?
  Claim 1: Thanks to phase transitions in dual-gated devices, YES – canted antiferromagnetic (arXiv:1105.5386)
- Claim 2: If insulating state persists down to $B = 0$ [J. Velasco Jr.*et al.*, arXiv:1108.1609 (2011)]], then it is (canted) antiferromagnetic down to $B = 0$ (arXiv:1109.1553)





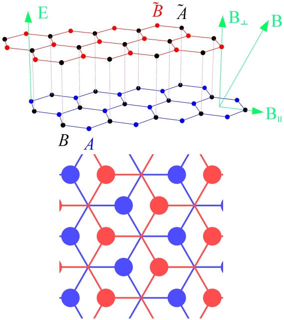
Low-energy Hamiltonian: two-band model

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{e-e},\circ} + \hat{H}_{\text{e-e},\diamond} + \hat{H}_{\text{e-ph}} + \hat{H}_Z + \hat{H}_V$$
$$\hat{\psi}_\sigma(\mathbf{r}) = (\hat{\psi}_{KA\sigma}(\mathbf{r}), \hat{\psi}_{K\tilde{B}\sigma}(\mathbf{r}), \hat{\psi}_{K'\tilde{B}\sigma}(\mathbf{r}), -\hat{\psi}_{K'A\sigma}(\mathbf{r}))^{\mathrm{t}}_{KK'\otimes\bar{A}\bar{B}}.$$

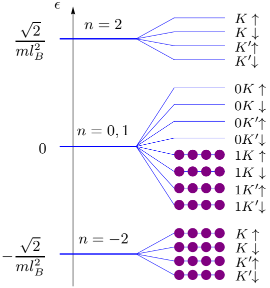
$$\hat{H}_0 = \int d^2\mathbf{r}\, \hat{\psi}^\dagger(\mathbf{r}) \frac{1}{2m} (\mathcal{T}_{z+}\tilde{\hat{p}}_+^2 + \mathcal{T}_{z-}\tilde{\hat{p}}_-^2) \hat{\psi}(\mathbf{r}),\ \tilde{\hat{p}}_\alpha = \hat{p}_\alpha - \frac{e}{c}A_\alpha$$
$$\hat{H}_Z = -\int d^2\mathbf{r}\, \hat{\psi}^\dagger(\mathbf{r})\epsilon_Z S_z \hat{\psi}(\mathbf{r}),\ \hat{H}_V = -\int d^2\mathbf{r}\, \hat{\psi}^\dagger(\mathbf{r})\epsilon_V \mathcal{T}_{zz}\hat{\psi}(\mathbf{r}),$$

$$\hat{H}_{\text{e-e},\circ} = \frac{1}{2}\int d^2\mathbf{r} d^2\mathbf{r}'\, [\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})] \frac{e^2}{|\mathbf{r}-\mathbf{r}'|} [\hat{\psi}^\dagger(\mathbf{r}')\hat{\psi}(\mathbf{r}')]$$
$$\hat{H}_{\text{e-e},\diamond} = \frac{1}{2}\int d^2\mathbf{r} \sum_{\alpha,\beta}{}' g_{\alpha\beta}[\hat{\psi}^\dagger(\mathbf{r})\mathcal{T}_{\alpha\beta}\hat{\psi}(\mathbf{r})]^2,\ \text{8 ind. constants},\ g_{\alpha\beta} \sim e^2 a$$
$$\hat{H}_{\text{e-ph}} = \int d^2\mathbf{r} \sum_{\alpha,\beta}{}'' F_{\alpha\beta}\hat{\psi}^\dagger(\mathbf{r})\mathcal{T}_{\alpha\beta}\hat{u}_{\alpha\beta}(\mathbf{r})\hat{\psi}(\mathbf{r}),\ \mathcal{T}_{\alpha\beta} = \tau_\alpha^{KK'} \otimes \tau_\beta^{\bar{A}\bar{B}} \otimes \hat{1}^s$$

$\nu = 0$ quantum Hall state: half-filled zero energy Landau level (LL)



$$\epsilon_n = \frac{1}{ml_B^2}\sqrt{|n|(|n|-1)}\,\mathrm{sgn}\, n, \;\; n \neq -1$$



$\epsilon = 0$ LL

- in each valley, wfs reside on only one sublattice=layer
  $\Rightarrow$ KK' valley = $\tilde{B}A$ sublattice = layer "isospin"
- $|0\rangle$ and $|1\rangle$ oscillator states belong to $\epsilon = 0$ LL $\Rightarrow$ 01-"pseudospin"

quenched kinetic energy $\Rightarrow$ system very susceptible to interactions

$\nu = 0$ state: quantum Hall ferromagnet

Starting point: For $KK' \otimes s$-symmetric interactions

ground* state(s) can be found *exactly* (Hund's rule idea):

$$\Psi = \prod_{p,n=0,1} \left(\sum_{\lambda\sigma} \chi^*_{a,\lambda\sigma} c^\dagger_{n\lambda\sigma,p} \right) \left(\sum_{\lambda'\sigma'} \chi^*_{b,\lambda'\sigma'} c^\dagger_{n\lambda'\sigma',p} \right) |0\rangle,$$

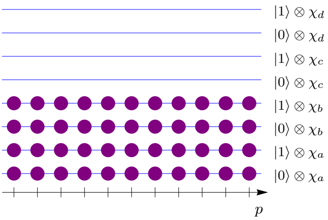
$\{\chi_a, \chi_b, \chi_c, \chi_d\}$ - arbitrary(!) basis in $KK' \otimes s$, $\lambda, \lambda' = K, K'$ $\sigma, \sigma' = \uparrow, \downarrow$

Ground* state Ψ is degenerate according to the choice of $\chi_{a,b}$

Order parameter for isospin-spin degrees of freedom

$$P = \chi_a \chi_a^\dagger + \chi_b \chi_b^\dagger; \; P^\dagger = P, \; P^2 = P, \; \mathrm{tr} P = 2.$$

transforms according to U(4)/[U(2) × U(2)]



anisotropy in 01 space: Y. Barlas *et al.,* PRL **101** (2008); D. A. Abanin, S. A. Parameswaran, and S. L. Sondhi, PRL **103**, (2009)

$KK' \otimes s$ SU(4)-symmetry-breaking effects

Single-particle mechanisms

- Zeeman effect $\Rightarrow$ spin polarization (breaks spin symmetry)

$$\mathcal{E}_Z(P) = \langle\Psi|\hat{H}_Z|\Psi\rangle/N = -\epsilon_Z \,\mathrm{tr}[S_z P],$$

$S_z = \hat{1}^{KK'} \otimes \tau_z^s$, $\epsilon_Z = \mu_B B = 0.65B[\mathrm{T}]\mathrm{K}$, $B = \sqrt{B_\perp^2 + B_\parallel^2}$

- Perpendicular electric field $\Rightarrow$ layer charge polarization (breaks isospin symmetry)

$$\mathcal{E}_V(P) = \langle\Psi|\hat{H}_V|\Psi\rangle/N = -\epsilon_V \,\mathrm{tr}[\mathcal{T}_z P],$$

$\mathcal{T}_\alpha = \tau_\alpha^{KK'} \otimes \hat{1}^s$, $\epsilon_V \approx E a_z/2$, $a_z \approx 0.35\mathrm{nm}$

$KK' \otimes s$ SU(4)-symmetry-breaking effects

Many-body mechanisms

valley/sublattice=layer asymmetry of e-e and e-ph interactions
$\Rightarrow$ isospin anisotropy:

$$\mathcal{E}_\diamond(P) = \langle \Psi | \hat{H}_{\text{e-e}} + \hat{H}_{\text{e-ph}} | \Psi \rangle / N = \frac{1}{2} \sum_{\alpha=x,y,z} u_\alpha \{ \text{tr}^2[\mathcal{T}_\alpha P] - \text{tr}[\mathcal{T}_\alpha P \mathcal{T}_\alpha P] \},$$

$\mathcal{E}_\diamond(P)$ fully characterized by two signed energies $u_\perp = u_x = u_y$ and u_z

$$u_\alpha = u_\alpha^{(\text{e-e})} + u_\alpha^{(\text{e-ph})}, \; u_\alpha^{(\text{e-e})} = \frac{g_{\alpha z}(l_B) + g_{\alpha 0}(l_B)}{2\pi l_B^2}, \; u_\alpha^{(\text{e-ph})} = -\frac{f_{\alpha z}(l_B)}{2\pi l_B^2}, \; \alpha = \perp, z$$

- bare $u_{\perp,z} \sim e^2 a / l_B^2 \sim 1 - 10 B_\perp[\text{T}]\text{K}, \; 1/l_B^2 = eB_\perp/c$

Energy of SU(4)-symmetry-breaking effects

$$\mathcal{E}(P) = \mathcal{E}_\diamond(P) + \mathcal{E}_V(P) + \mathcal{E}_Z(P), \; P = \chi_a\chi_a^\dagger + \chi_b\chi_b^\dagger$$

- isospin anisotorpy: $\mathcal{E}_\diamond(P) = \frac{1}{2}\sum_{\alpha=x,y,z} u_\alpha \{\mathrm{tr}^2[\mathcal{T}_\alpha P] - \mathrm{tr}[\mathcal{T}_\alpha P \mathcal{T}_\alpha P]\}$,
- Zeeman effect: $\mathcal{E}_Z(P) = -\epsilon_Z \, \mathrm{tr}[S_z P]$
- electric field: $\mathcal{E}_V(P) = -\epsilon_V \, \mathrm{tr}[\mathcal{T}_z P]$

Four parameters: ϵ_Z, ϵ_V, $u_\perp$, u_z

- ϵ_V, ϵ_Z – known, $\epsilon_{V,Z}/u_{\perp,z}$ can be varied in experiment
- $u_{\perp,z}$ – not reliably known; $u_\perp/u_z$ can hardly be varied in experiment (not at all, if renormalizations are negligible)

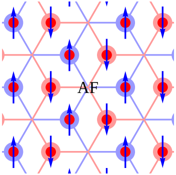
Minimization of $\mathcal{E}(P) \Rightarrow$ “classical” (Hartree-Fock mean-field) phase diagram

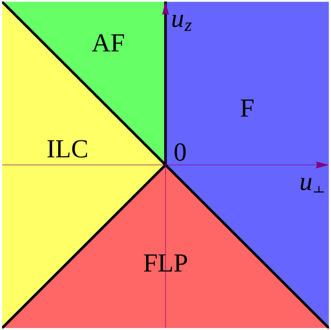
Minimization of anisotropy energy $\mathcal{E}_\diamond(P)$

antiferromagnetic (AF),

$\chi_a = |K\rangle \otimes |\mathbf{s}\rangle$,

$\chi_b = |K'\rangle \otimes |-\mathbf{s}\rangle$

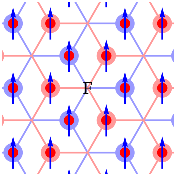




ferromagnetic (F)

$\chi_a = |K\rangle \otimes |\mathbf{s}\rangle$,

$\chi_b = |K'\rangle \otimes |\mathbf{s}\rangle$

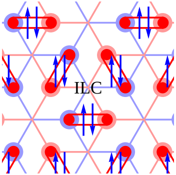


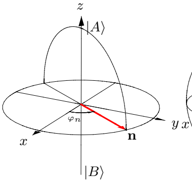
interlayer-coherent (ILC)

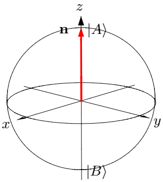
$\chi_a = |\mathbf{n}_\perp\rangle \otimes |\uparrow\rangle$,

$\chi_b = |\mathbf{n}_\perp\rangle \otimes |\downarrow\rangle$,

$\mathbf{n}_\perp = (\cos\varphi_n, \sin\varphi_n, 0)$





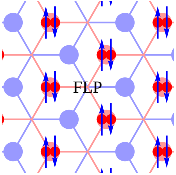


fully layer-polarized (FLP)

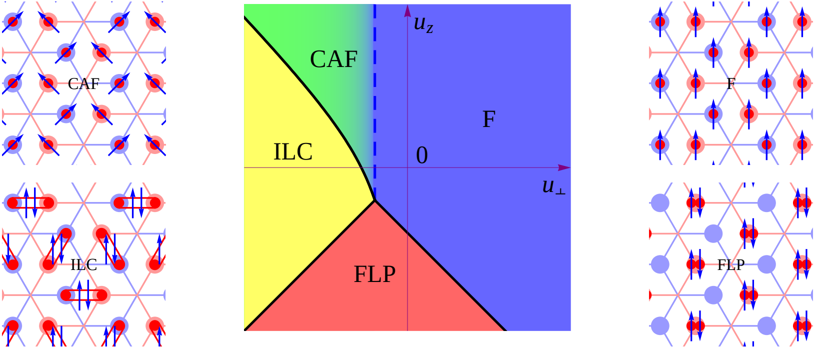
$\chi_a = |\pm\mathbf{n}_z\rangle \otimes |\uparrow\rangle$,

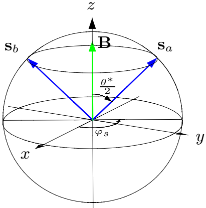
$\chi_b = |\pm\mathbf{n}_z\rangle \otimes |\downarrow\rangle$,

$\mathbf{n}_z = (0, 0, 1)$



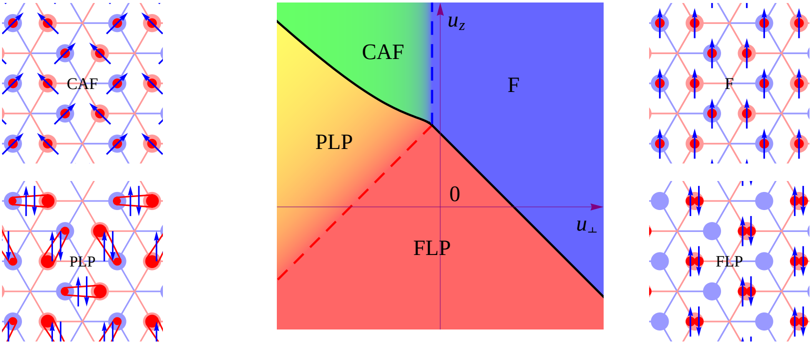
Minimization of $\mathcal{E}_\diamond(P) + \mathcal{E}_Z(P)$: Phase diagram for $\nu = 0$ QHFM in BLG at zero el. field (and in MLG!)

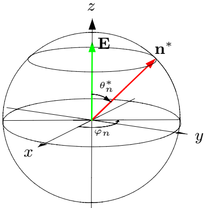




- F phase: $\mathbf{s} \to \mathbf{s}_z = (0,0,1)$,
- AF $\to$ canted anti-ferromagnetic (CAF)
  $\chi_a = |K\rangle \otimes |\mathbf{s}_a^*\rangle$, $\chi_b = |K'\rangle \otimes |\mathbf{s}_b^*\rangle$
  $\mathbf{s}_{a,b}^* = (\pm \sin\theta_s^* \cos\varphi_s, \pm \sin\theta_s^* \sin\varphi_s, \cos\theta_s^*)$,
  $s_z^* = \cos\theta_s^* = \epsilon_Z/(2|u_\perp|)$

Minimization of $\mathcal{E}_\diamond(P) + \mathcal{E}_Z(P) + \mathcal{E}_V(P)$: Phase diagram for the $\nu = 0$ QHFM in BLG at finite electric field

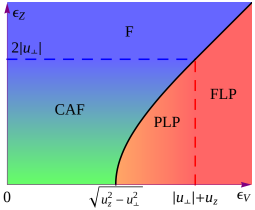




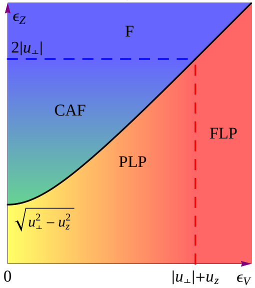
- FLP: $\pm\mathbf{n}_z \to \mathbf{n}_z$,
- ILC $\to$ partially layer-polarized phase (PLP),
  $\chi_a = |\mathbf{n}^*\rangle \otimes |\uparrow\rangle$, $\chi_b = |\mathbf{n}^*\rangle \otimes |\downarrow\rangle$,
  $\mathbf{n}^* = (\sin\theta_n^* \cos\varphi_n, \sin\theta_n^* \sin\varphi_n, \cos\theta_n^*)$
  $n_z^* = \cos\theta_n^* = \epsilon_V/(u_z + |u_\perp|)$.

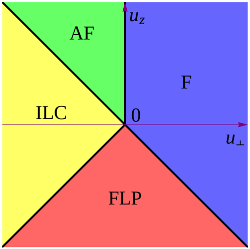
Tilting m. field and applying el. field, phase diagram(s) in (ϵ_V, ϵ_Z)

AF



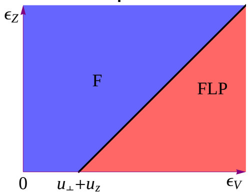
ILC



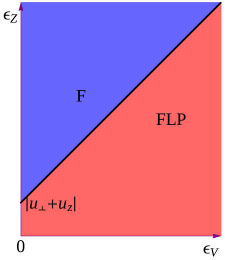


$\epsilon_Z(B)/u_{\perp,z}(B_\perp)$ is changed by tilting the magnetic field

F



FLP

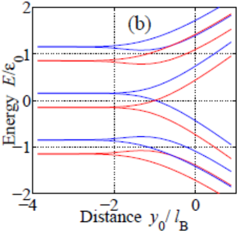


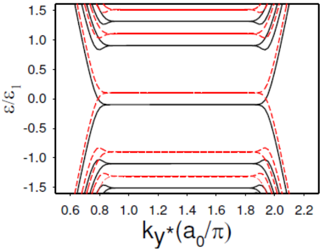
Edge charge excitations of the $\nu = 0$ QHFM (bulk charge excitations: gapped for any order of any QHFM, skyrmions)

Studies in MLG for specific edges (armchair and zigzag):

- F: gapless counterpropagating edge excitations $\Rightarrow$ metallic edge conductance
- other phases: gapped edge excitations $\Rightarrow$ whole sample insulating

D. A. Abanin, *et al*, PRL **96** (2006); H.A. Fertig and L. Brey, PRL **97** (2006); V. P. Gusynin *et al.*, PRB **77** (2008); J. Jung and A.H. MacDonald, PRB **80** (2009)





Edge charge excitations of the $\nu = 0$ QHFM (bulk charge excitations: gapped for any order of any QHFM, skyrmions)

Studies in MLG for specific edges (armchair and zigzag):

- F: gapless counterpropagating edge excitations $\Rightarrow$ metallic edge conductance
- other phases: gapped edge excitations $\Rightarrow$ whole sample insulating

D. A. Abanin, *et al*, PRL **96** (2006); H.A. Fertig and L. Brey, PRL **97** (2006); V. P. Gusynin *et al.*, PRB **77** (2008); J. Jung and A.H. MacDonald, PRB **80** (2009)

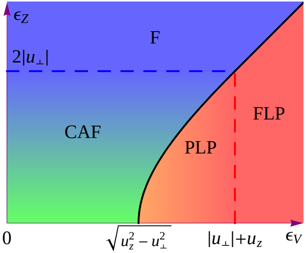
Topological arguments, BLG and MLG

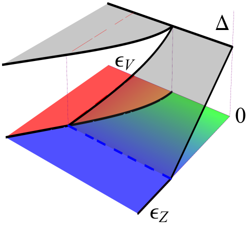
- F phase of $\nu = 0$ QHFM,

$$\langle : \psi\psi^\dagger : \rangle = \frac{1}{2\pi l_B^2} \hat{1}^{KK'} \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}^{\bar{A}\bar{B}} \otimes \tau_z^s = \frac{1}{2\pi l_B^2} \hat{1}^{KK'} \otimes \frac{1}{2} (\hat{1}^{\bar{A}\bar{B}} - \tau_z^{\bar{A}\bar{B}}) \otimes \tau_z^s$$

  - mixture of ferromagnetic and quantum spin Hall (!) order parameters
- Other phases: "topologically trivial" (many-body) insulators

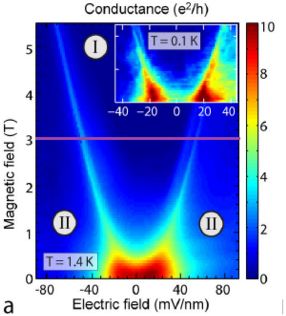
Edge transport



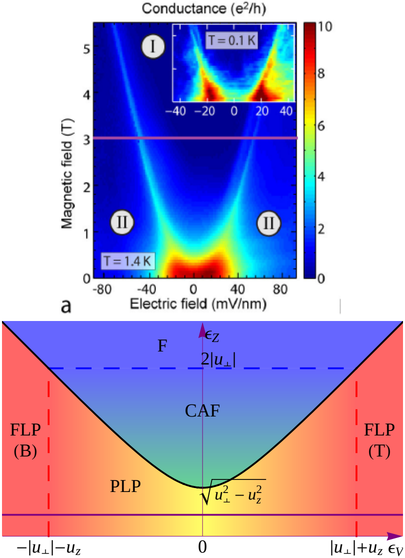


- F: gapless collective charge excitations: $G = 4e^2/h$
- continuous interpolation:
  PLP ($0 < n_z^* < 1$): ILC ($n_z^* = 0$), FLP ($n_z^* = 1$); electric field (ϵ_V)
  CAF ($0 < s_z^* < 1$): AF ($s_z^* = 0$), F ($s_z^* = 1$); tilting magnetic field (ϵ_Z),
- no phase transitions at the CAF-F and PLP-FLP boundaries; no conductance spike.
- PLP to FLP crossover: system remains insulating,
- CAF to F crossover: edge transport gap $\Delta_{\mathrm{CAF}}(s_z^*)$ monotonically decreases with $s_z^* = \epsilon_Z/(2|u_\perp|)$; from finite $\Delta_{\mathrm{CAF}}(s_z^* = 0) = \Delta_{\mathrm{AF}}$ at $\epsilon_Z = 0$ to zero $\Delta_{\mathrm{CAF}}(s_z^* = 1) = \Delta_{\mathrm{F}} = 0$ at CAF-F boundary $\epsilon_Z = 2|u_\perp|$.

Insulating $\nu = 0$ state in BLG at zero electric field is ...

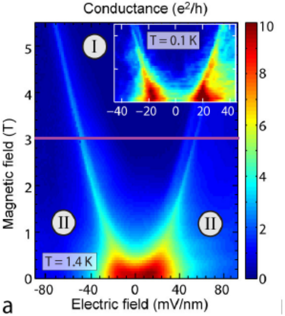


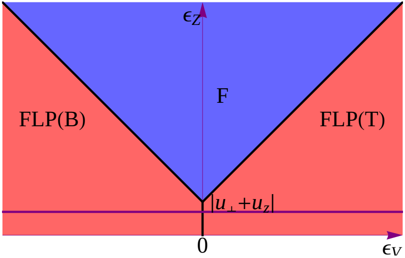
Insulating $\nu = 0$ state in BLG at zero electric field is ...



- not ILC: no phase transitions at all

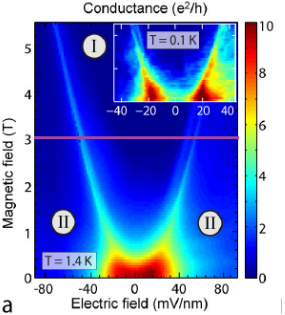
Insulating $\nu = 0$ state in BLG at zero electric field is ...

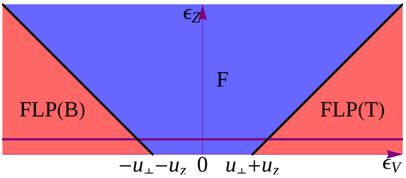




- not ILC: no phase transitions at all
- not FLP: only one phase transition, at $\epsilon_V = 0$ or hysteresis

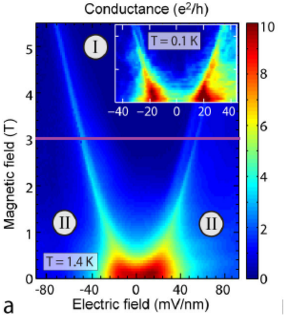
Insulating $\nu = 0$ state in BLG at zero electric field is ...

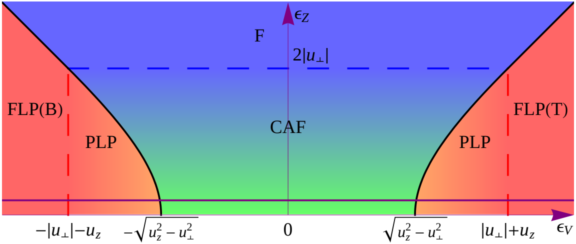




- not ILC: no phase transitions at all
- not FLP: only one phase transition, at $\epsilon_V = 0$ or hysteresis
- not F: metallic edge conductance, $G \sim 4e^2/h$

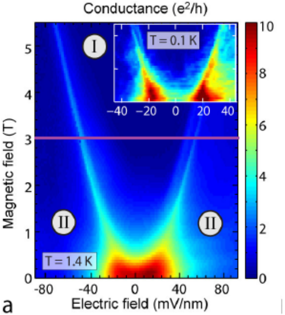
Insulating $\nu = 0$ state in BLG at zero electric field is ...

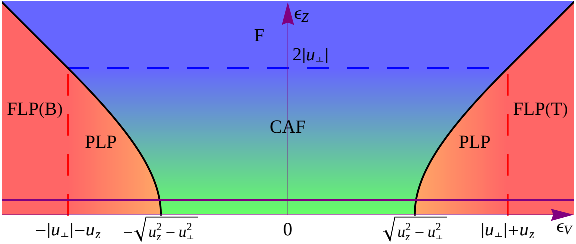




- not ILC: no phase transitions at all
- not FLP: only one phase transition, at $\epsilon_V = 0$ or hysteresis
- not F: metallic edge, $G \sim 4e^2/h$
- $\Rightarrow$ CAF!

Insulating $\nu = 0$ state in BLG at zero electric field is ...





- AF favored by anisotropy $\Rightarrow u_z > -u_\perp > 0$: consistent with micro considerations - ok
- $\epsilon_V^* \approx \sqrt{u_z^2 - u_\perp^2} \approx 20 B_\perp$[T]K - ok
- linear $B_\perp$-dependence at $B_\perp \gtrsim 2$T - ok

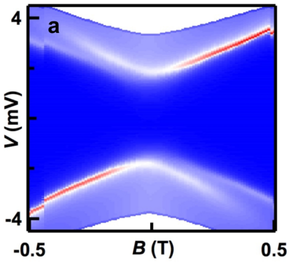
Insulating state at all B (J. Velasco Jr.*et al.* (2011)): AF all the way?

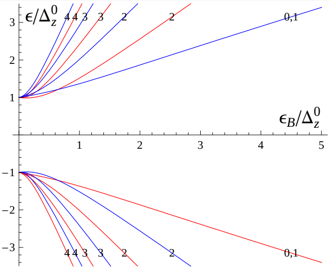
Continuous crossover between $B = 0$ AF state and AF phase of $\nu = 0$ QHFM

Mean-field theory at arbitrary B: AF state can persist at all B

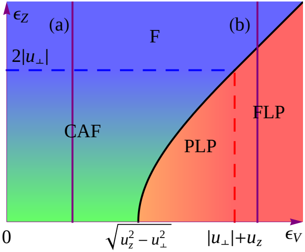
$$E_{n\lambda\sigma} = s_\lambda s_\sigma(\Delta_0 - \Delta_z),\ n = 0, 1;\ E_{n\pm\lambda\sigma} = \Delta_0 s_\lambda s_\sigma \pm \sqrt{\epsilon_n^2 + \Delta_z^2},\ n \geq 2.$$

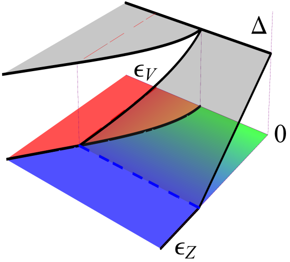
$$\Delta_z = \frac{g^{zzz}}{4\pi l_B^2}\left(\sum_{n=2}^{n_0} \frac{\Delta_z}{\sqrt{\epsilon_n^2 + \Delta_z^2}} + 1\right),\ \Delta_0 = -\frac{g^{z0z}}{4\pi l_B^2}.$$





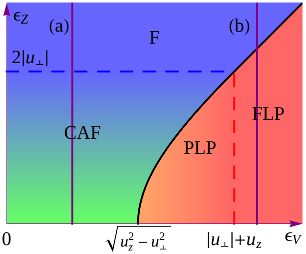
Transport "smoking gun": Tilted-field experiment in the QH regime

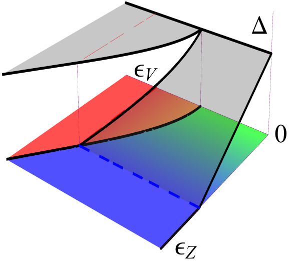




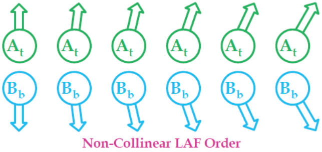
- (a): CAF-F insulator-metal crossover
- (b): FLP-F insulator-metal transition
- required $B/B_\perp = 2|u_\perp(B_\perp)|/(\mu_B B_\perp)$ not really known for smaller $|u_\perp| \lesssim u_z/2$;
  for example, $B/B_\perp \approx 30$ at $|u_\perp| = u_z/2$
- available $B/B_\perp \approx 50$ at $B_\perp \approx 1$T and $B \approx 45$T

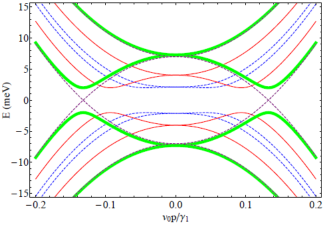
Transport “smoking gun”: Tilted-field experiment in the QH regime





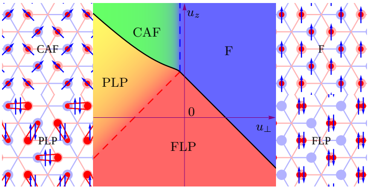
In contrast with $B_\perp = 0$, finite $B_{||}$, F. Zhang and A. H. MacDonald, arXiv:1107.4727 (2011): AF $\rightarrow$ CAF, but transport gap remains constant and pure F is never reached.

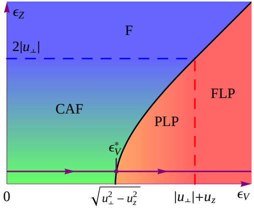


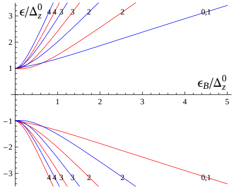


Summary

- Generic phase diagram for the $\nu = 0$ QHFM in BLG and MLG obtained
- Claim 1: insulating $\nu = 0$ state in BLG – canted antiferromagnetic phase of the $\nu = 0$ QHFM
- Claim 2: insulating zero-density state at all B (J. Velasco Jr.*et al.*, arXiv:1108.1609 (2011), Riverside) – (canted) antiferromagnetic
- Transport smoking gun: tilted-field experiment in QH regime ($B_\perp \gtrsim 1$T)

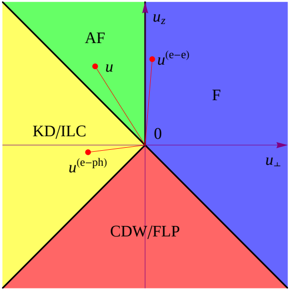






THANK YOU!

Isospin anisotropy



$$u_\alpha = u_\alpha^{(\text{e-e})} + u_\alpha^{(\text{e-ph})}, \ \alpha = \perp, z$$

$$u_\alpha^{(\text{e-e})} = \frac{1}{2\pi l_B^2}[g_{\alpha z}(l_B) + g_{\alpha 0}(l_B)], \ u_\alpha^{(\text{e-ph})} = -\frac{f_{\alpha z}(l_B)}{2\pi l_B^2}$$

$u_\alpha \gtrless 0 \Rightarrow$ repulsive/attractive interactions in α

- e-ph (effectively attractive): $u_\alpha^{(\text{e-ph})} < 0, \ -u_z^{(\text{e-ph})} \ll -u_\perp^{(\text{e-ph})}$;
- e-e (bare interactions eff. repulsive): $u_\alpha^{(\text{e-e},0)} > 0$, but can change sign under RG; $u_z^{(\text{e-e},0)} \gg u_\perp^{(\text{e-e},0)}$;
  for BLG most definitely: $u_z^{(\text{e-e})} > 0$ – “capacitance” anisotropy (interactions stronger within than between the layers)
- Hence $u_\perp < 0$, $u_z > 0$, i.e., AF or KD/ILC phases seem most physically natural