

# Electromechanical properties of suspended graphene membranes

Eros Mariani

Centre for Graphene Science  
University of Exeter (UK)

in collaboration with Felix von Oppen (FU Berlin) and Alex Pearce (Exeter)

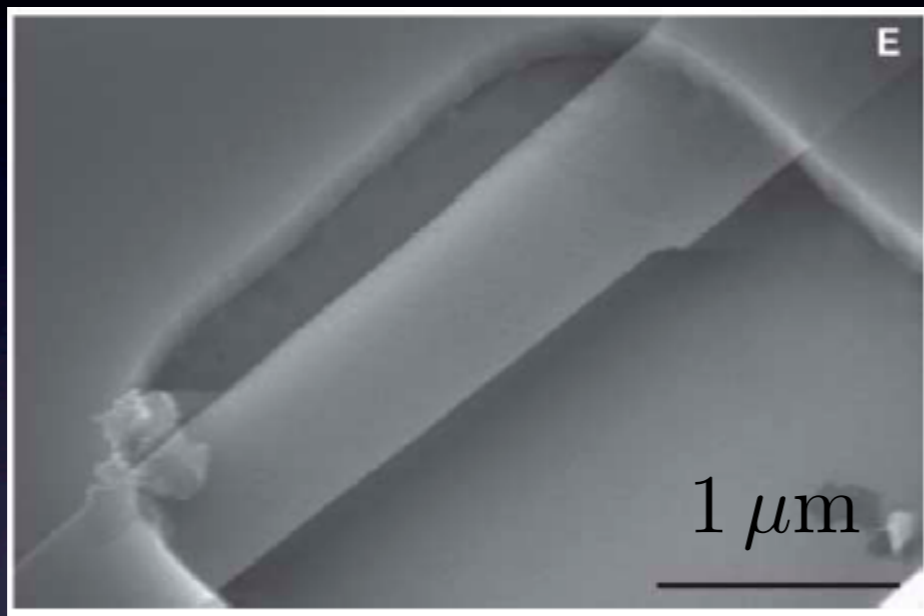
# Suspended graphene membranes

Graphene in between QED, hard and soft condensed matter

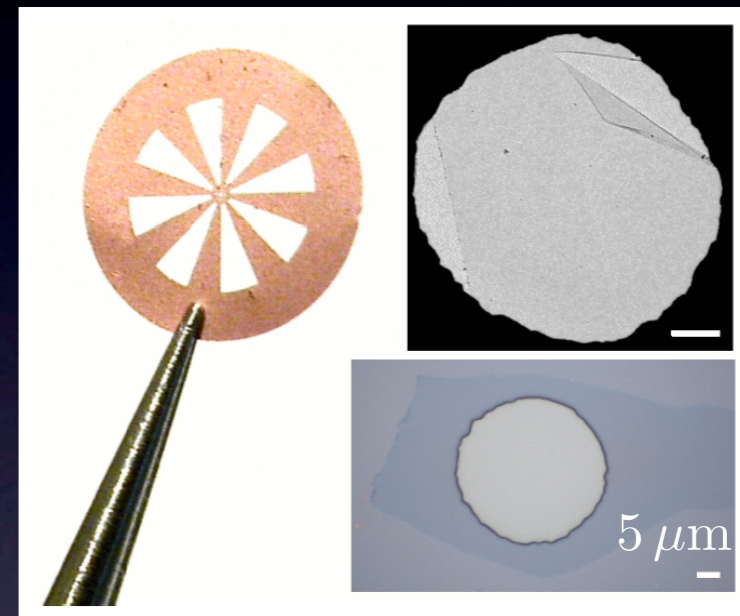


# Suspended graphene membranes

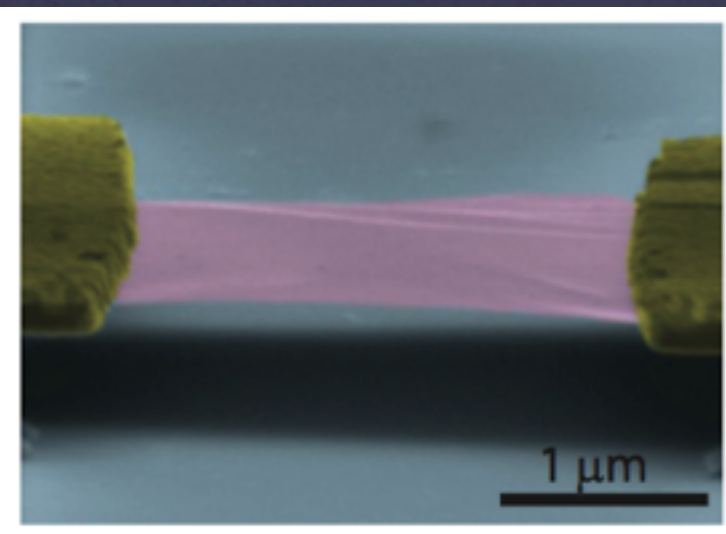
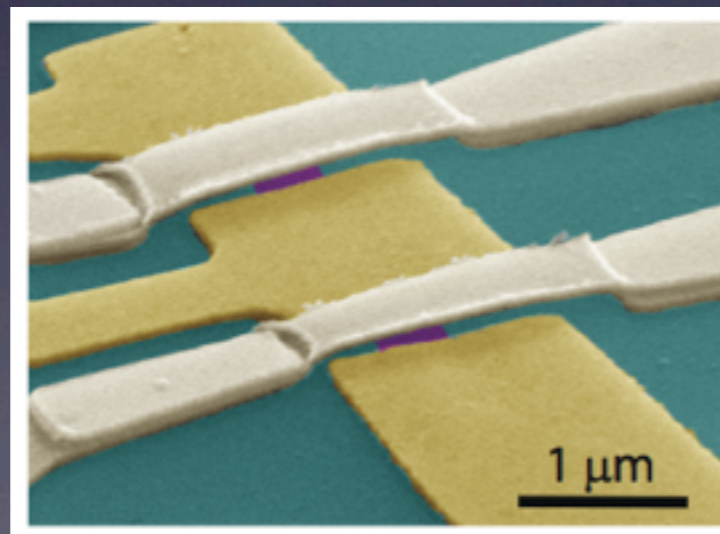
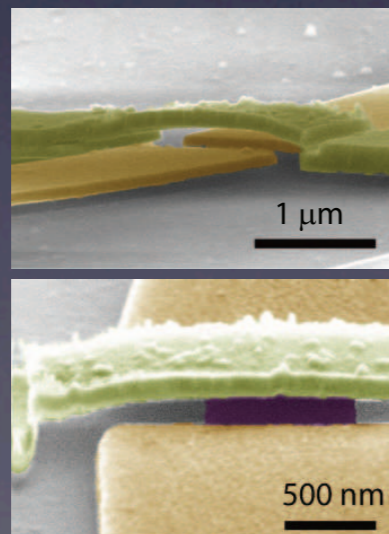
Graphene in between QED, hard and soft condensed matter



Scott-Bunch 07



Geim group 08



S. Russo, Exeter 2011

# Questions

Flexural phonons VS in-plane ones?

How do phonons affect transport? Resistivity vs. T?

Deformations in mono- and bilayer membranes?

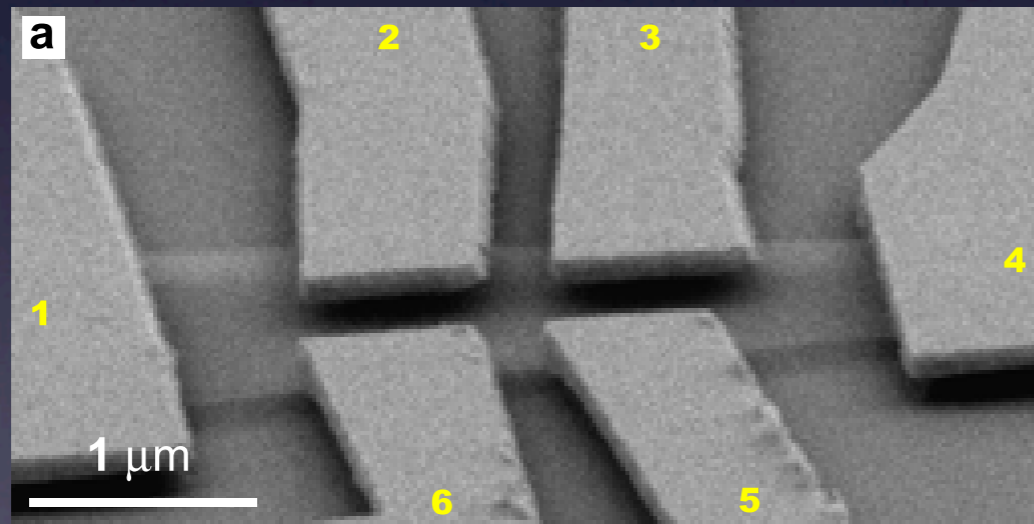


# Questions

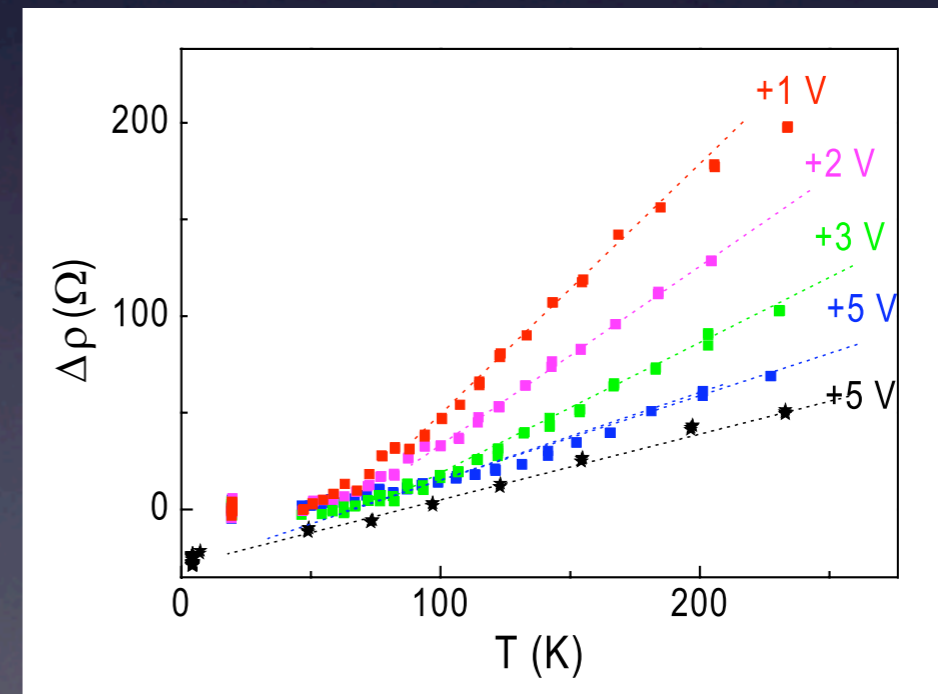
Flexural phonons VS in-plane ones?

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Bolotin et al, PRL 08

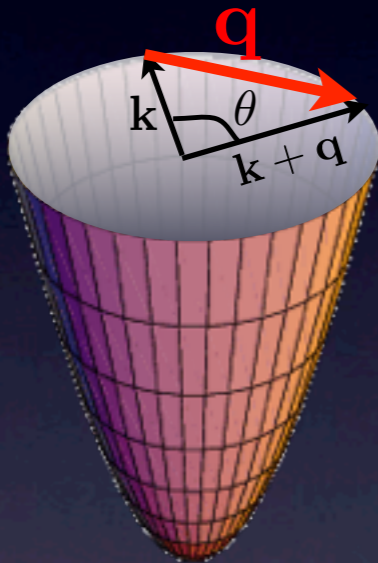


See also Morozov 08 and Chen 08 in non-suspended samples

# TEXTBOOK

## In-plane Phonon resistivity (in 2D)

$$\rho = \frac{m}{ne^2\tau_{\text{tr}}} \quad \frac{1}{\tau_{\text{tr}}} = \frac{2\pi}{\hbar} \int d\mathbf{q} |M_{\text{FI}}|^2 (1 - \cos \theta) \delta(\epsilon_{\mathbf{k}+\mathbf{q}} - \epsilon_{\mathbf{k}} \pm \hbar\omega_{\mathbf{q}})$$



$$d\mathbf{q} \sim q^2$$

$$|M_{\text{FI}}|^2 \sim q N_q^{(\text{Bose})} \sim q \frac{T}{\omega_q}$$

$$\delta(\dots) \sim \frac{1}{q}$$

$$(1 - \cos \theta) \sim q^2$$

$$T_{\text{BG}} = \omega_{2k_{\text{F}}}$$

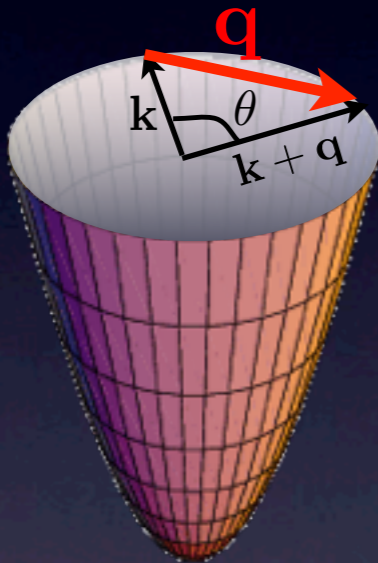


# TEXTBOOK

## In-plane Phonon resistivity (in 2D)

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$$\delta(\dots) \sim \frac{1}{q}$$

$$(1 - \cos \theta) \sim q^2$$

$$T_{\text{BG}} = \omega_{2k_{\text{F}}}$$

$$T \ll T_{\text{BG}}$$

$$q \in [0, \frac{T}{\hbar v_{\text{ph}}}]$$

$$\rho \propto T^4$$

$$T \gg T_{\text{BG}}$$

$$q \in [0, 2k_{\text{F}}]$$

$$\rho \propto T$$

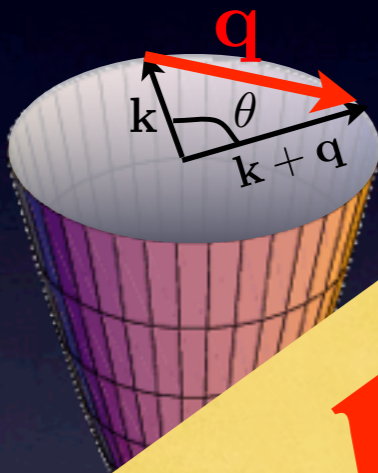
Bloch law

TEXTBOOK

# In-plane Phonon resistivity (in 2D)

$$\rho = \frac{m}{ne^2\tau_{\text{tr}}}$$

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ARE WE FINISHED?

$$T_{\text{BG}} = \omega_{\text{ph}}$$

$$\rho \propto T^4$$

$$T \gg T_{\text{BG}}$$

$$q \in [0, 2k_{\text{F}}]$$

$$\rho \propto T$$

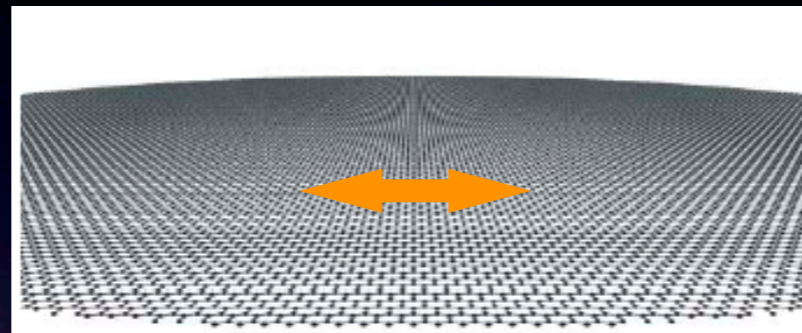
Bloch law



# Phonons in graphene

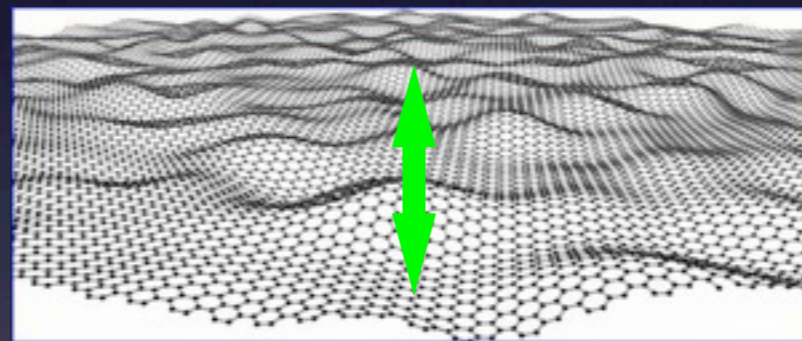
Woods '00  
Katsnelson '07  
Castro-Neto '07

In-plane modes



Hard to excite  
Good coupling to electrons

Flexural modes



Soft to excite  
Weak coupling to electrons

Q?

How do they couple to electrons?

What is their dispersion?

# Electron-phonon coupling

(basic mechanism)

$$H = v \begin{bmatrix} & p_x - ip_y \\ p_x + ip_y & \end{bmatrix} \begin{matrix} A \\ B \end{matrix}$$



# Electron-phonon coupling

(basic mechanism)

Membrane  
Distortions



variation of areas  
(and density)  
 $\rho \rightarrow \rho + \delta\rho$



Diagonal coupling  
 $H_{e-ph}^{(Def)} = \delta t_{on-site} c_j^\dagger c_j$

$$H = v \begin{bmatrix} \delta t_{on-site} & p_x - ip_y \\ p_x + ip_y & \delta t_{on-site} \end{bmatrix} \begin{matrix} A \\ B \end{matrix}$$

Deformation  
potential  
(screened)

# Electron-phonon coupling

(basic mechanism)

Membrane Distortions

variation of areas  
(and density)

$$\rho \rightarrow \rho + \delta\rho$$

Diagonal coupling

$$H_{e-ph}^{(\text{Def})} = \delta t_{\text{on-site}} c_j^\dagger c_j$$

Modified bond length,  
correction to hopping

$$t \rightarrow t + \delta t$$

Off-diagonal coupling

$$H_{e-ph}^{(\text{Gauge})} = \delta t_{\text{hop}} c_A^\dagger c_B$$

$$H = v \begin{bmatrix} \delta t_{\text{on-site}} & p_x - ip_y + \delta t_{\text{hop}} \\ p_x + ip_y + \delta t_{\text{hop}} & \delta t_{\text{on-site}} \end{bmatrix} \begin{matrix} A \\ B \end{matrix}$$

Bond-length variation (like a gauge field)

Deformation potential (screened)

$$H = \Pi_z \otimes \sigma \cdot \left( \mathbf{p} - \frac{e}{c} \mathbf{A} \right) + V(\mathbf{r})$$



# Electron Phonon Coupling

Woods & Mahan 00  
Suzuura & Ando '02  
Vozmediano review 2010

$$H_{e-ph} = \begin{pmatrix} g_1(u_{xx} + u_{yy}) & g_2(2u_{xy} - i(u_{xx} - u_{yy})) \\ g_2(2u_{xy} + i(u_{xx} - u_{yy})) & g_1(u_{xx} + u_{yy}) \end{pmatrix}$$

$$g_1 \simeq 30 \text{ eV}$$
$$g_2 \simeq 1.5 \text{ eV}$$

fictitious gauge field

$$\text{Tr}[u_{ij}] = \frac{\delta S}{S}$$

relative area  
variation

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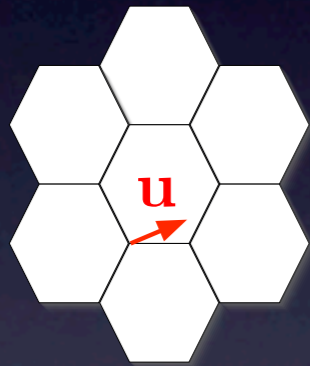
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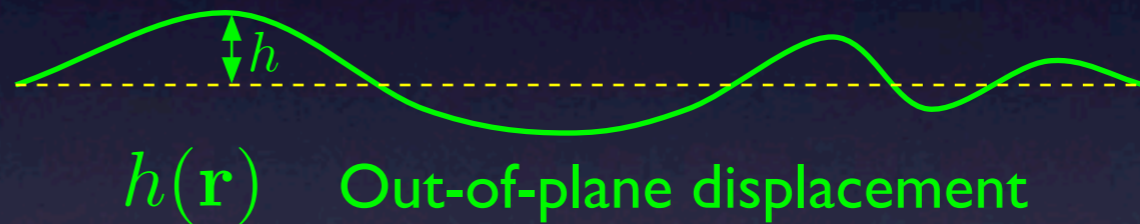
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$\mathbf{u}(\mathbf{r})$   
In-plane  
displacement



$h(\mathbf{r})$  Out-of-plane displacement

$$u_{ij} = \frac{1}{2} [\partial_i u_j + \partial_j u_i + (\partial_i h)(\partial_j h)]$$



# Electron Phonon Coupling

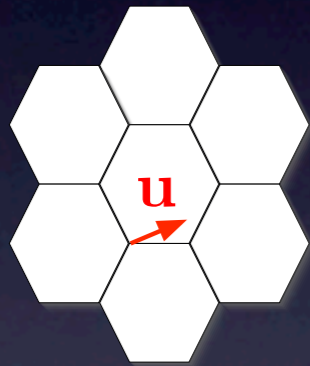
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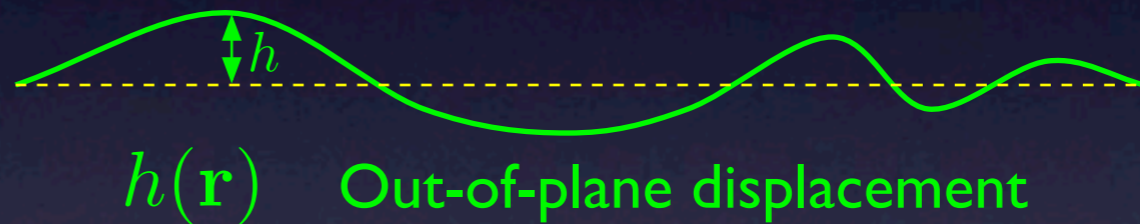
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fictitious gauge field

$\text{Tr}[u_{ij}] = \frac{\delta S}{S}$  relative area variation



$\mathbf{u}(\mathbf{r})$   
In-plane displacement

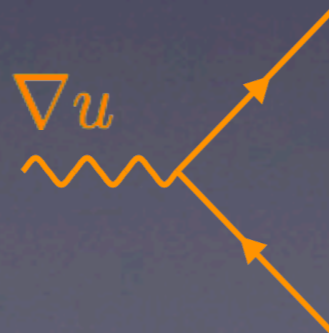


$h(\mathbf{r})$  Out-of-plane displacement

$$u_{ij} = \frac{1}{2} [\partial_i u_j + \partial_j u_i + (\partial_i h)(\partial_j h)]$$

Protected by symmetry & Ensures that a rotation implies no strain

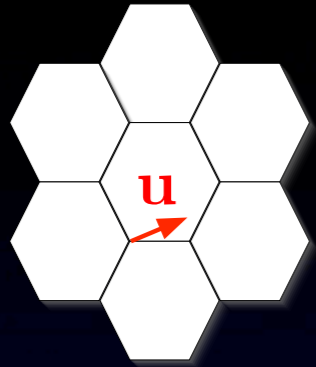
1st order in u



2nd order in h



# In-plane vs Flexural phonons

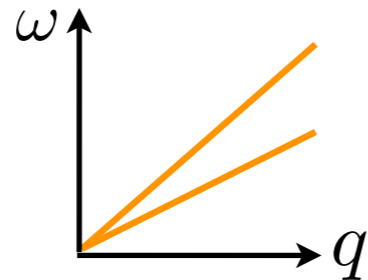


$\mathbf{u}(\mathbf{r})$   
In-plane  
displacement

## In-plane phonons

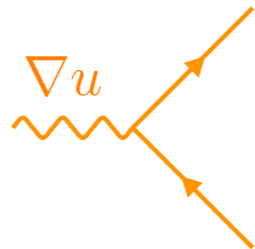
Linear dispersion

$$\omega^{(l,t)} = v^{(l,t)} q$$



Linear coupling

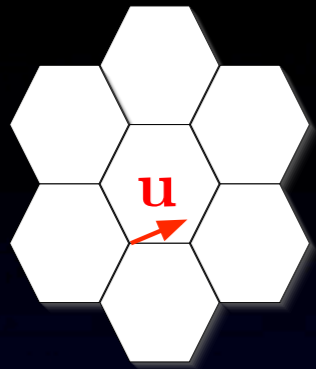
$$H_{e-ph} \propto u_{ij} \sim \nabla u$$



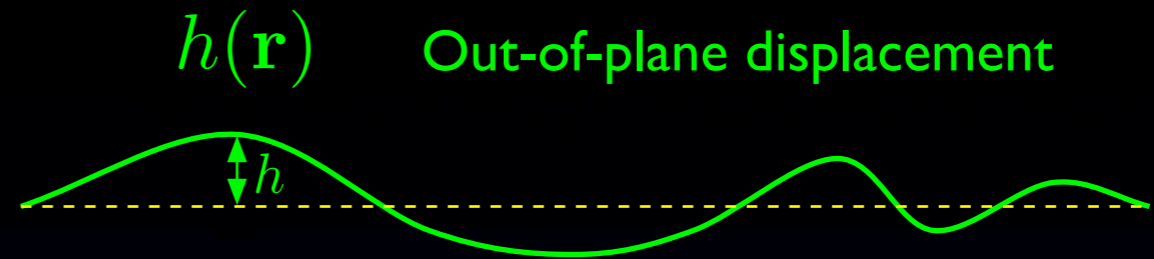
Small DOS, Good coupling



# In-plane vs Flexural phonons



$\mathbf{u}(\mathbf{r})$   
In-plane  
displacement

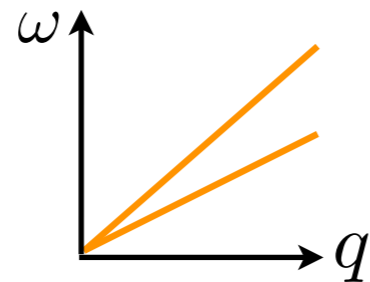


$$\mathcal{L} = \frac{\rho}{2} \dot{h}^2 - \frac{\kappa}{2} (\nabla^2 h)^2$$

## In-plane phonons

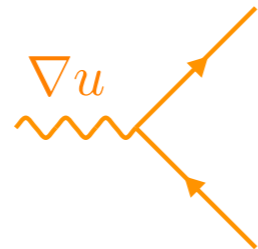
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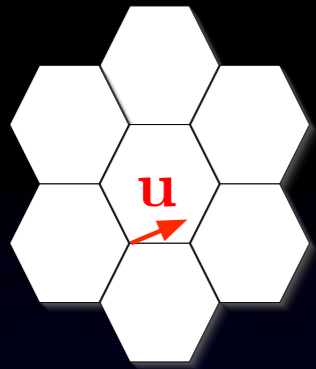
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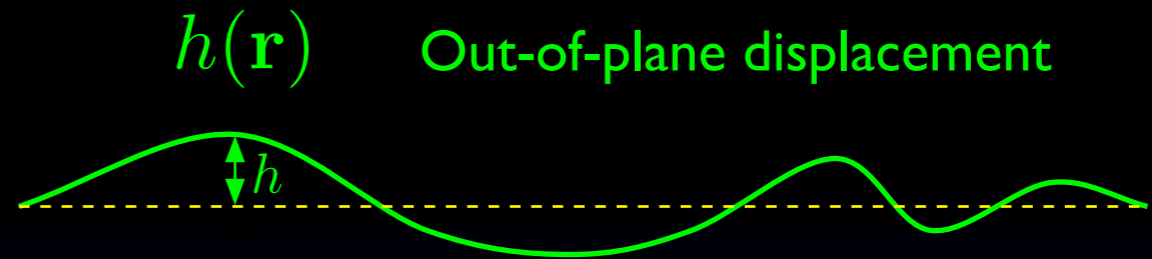


Small DOS, Good coupling

# In-plane vs Flexural phonons



$\mathbf{u}(\mathbf{r})$   
In-plane  
displacement

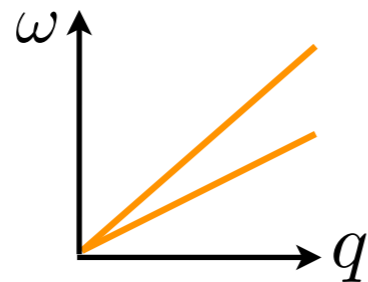


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## In-plane phonons

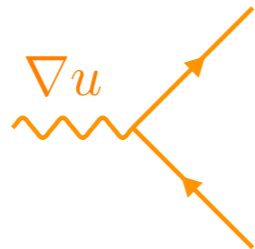
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$$\omega^{(l,t)} = v^{(l,t)} q$$



Linear coupling

$$H_{e-ph} \propto u_{ij} \sim \nabla u$$

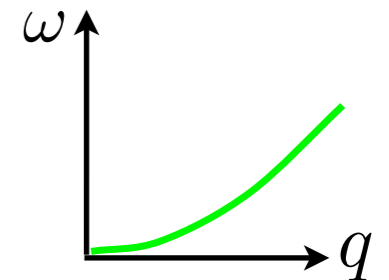


Small DOS, Good coupling

## Flexural phonons

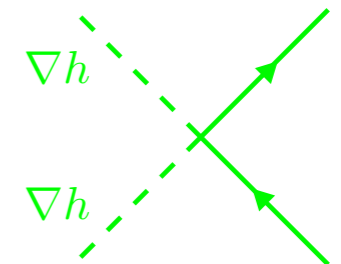
Quadratic dispersion

$$\omega^{(h)} = \alpha q^2$$



Quadratic coupling

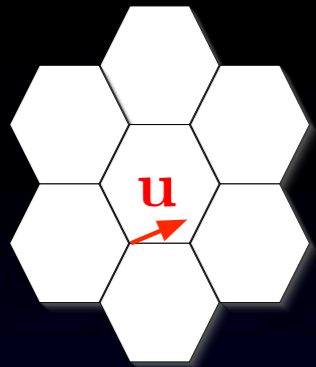
$$H_{e-ph} \propto u_{ij} \sim (\nabla h)^2$$



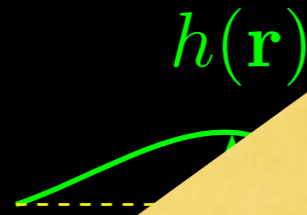
Big DOS, Poor coupling



# In-plane vs Flexural phonons



$\mathbf{u}(\mathbf{r})$   
In-plane displacement



In-plane phonons

Linear dispersion

$$\omega(l,t) \propto q$$

**WHO WINS?**

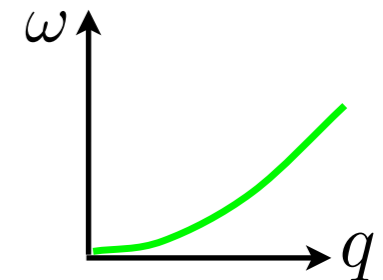
$$H_{e-ph} \propto u_{ij}$$

Small DOS, Strong coupling

Flexural phonons

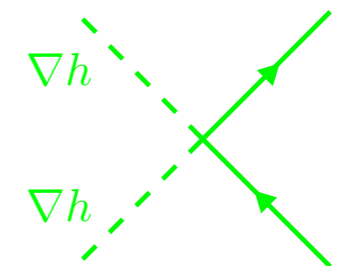
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Quadratic coupling

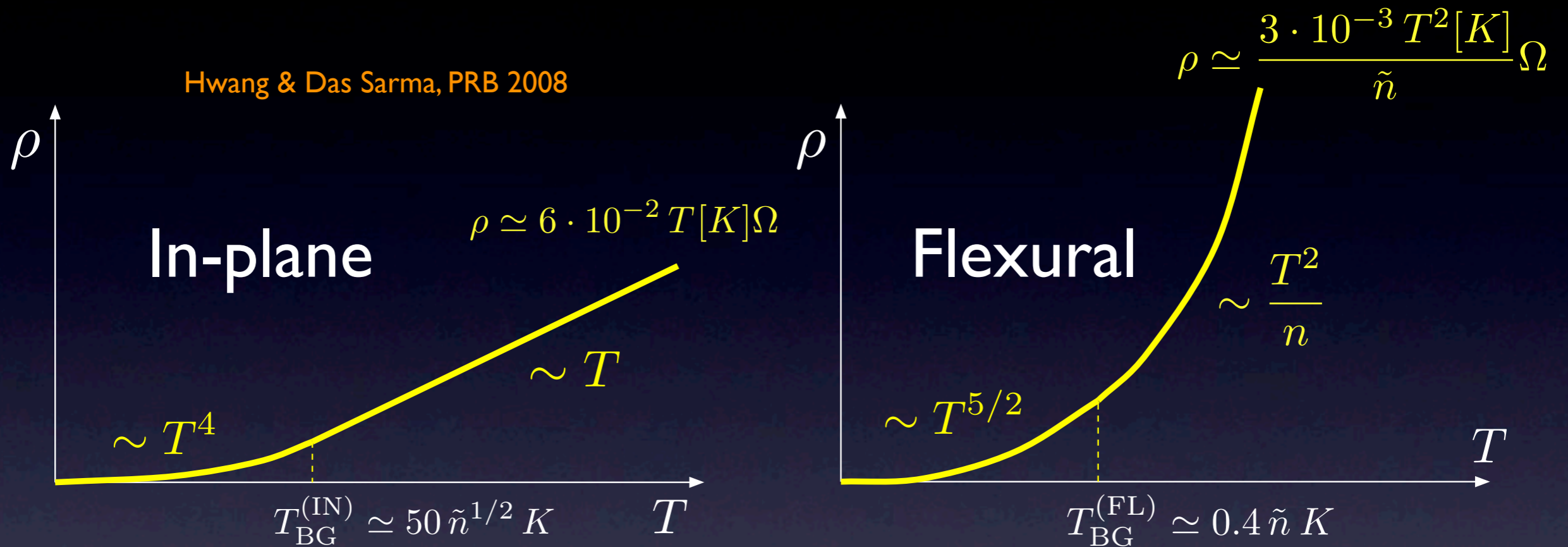
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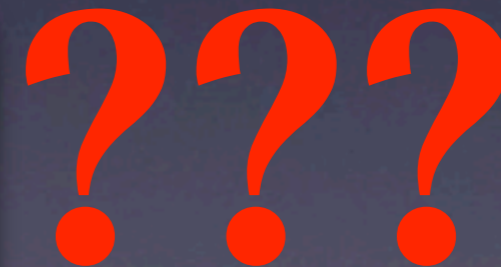
Big DOS, Poor coupling

# Temperature-dependent Resistivity

Hwang & Das Sarma, PRB 2008



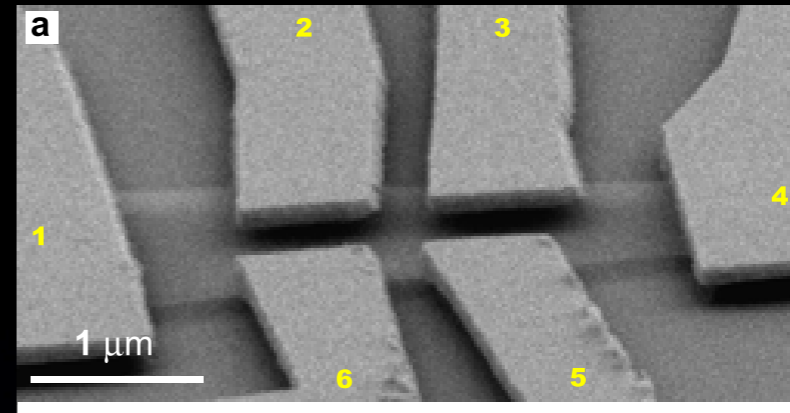
Flexural modes should dominate in-plane ones at present dopings



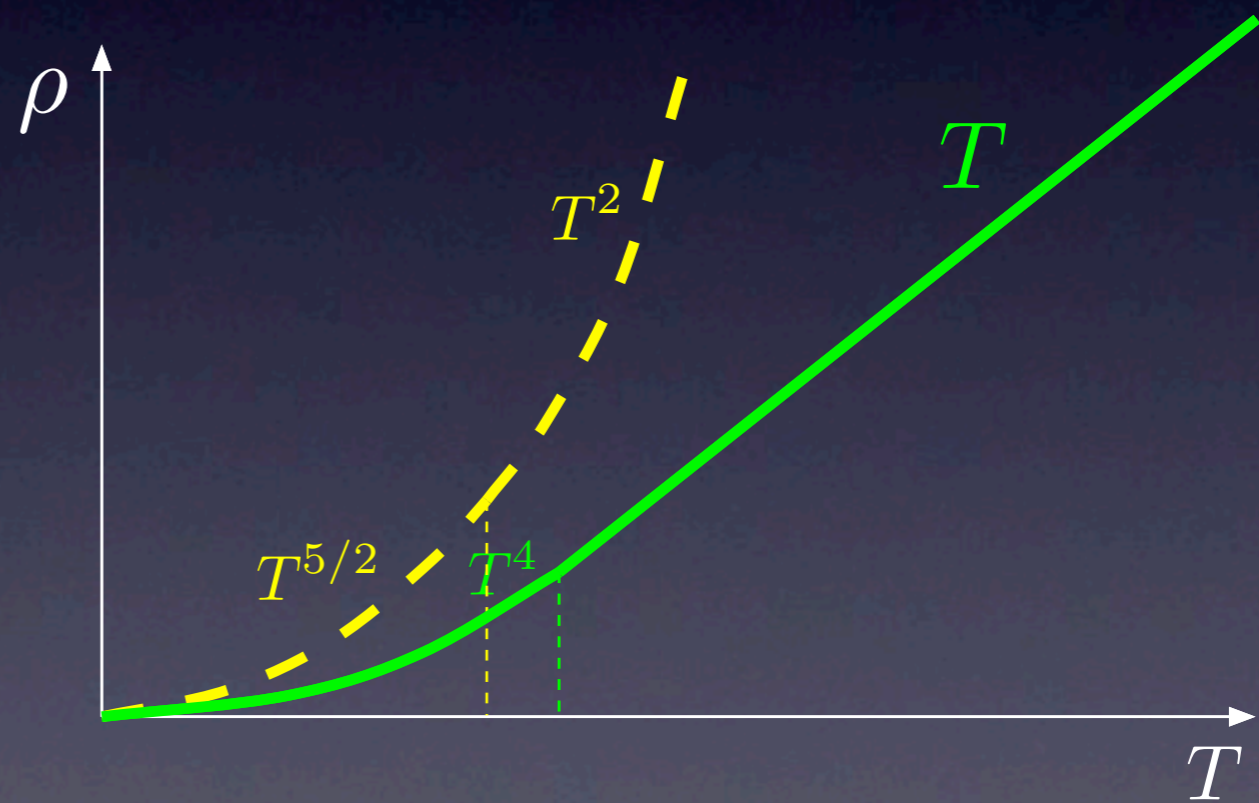
E. Mariani and F. von Oppen, Phys. Rev. Lett. **100**, 076801 (2008)  
 Phys. Rev. Lett. **100**, 249901 (2008)  
 Phys. Rev. B **82**, 195403 (2010)



# Role of tension?



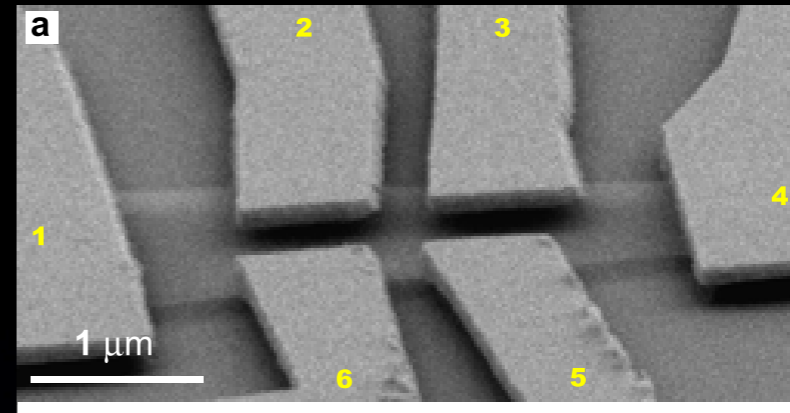
$$F_{\text{bend}} \rightarrow \frac{1}{2} \int d\mathbf{r} \kappa (\nabla^2 h)^2$$



E. Mariani and F. von Oppen, Phys. Rev. B **82**, 195403 (2010)



# Role of tension?

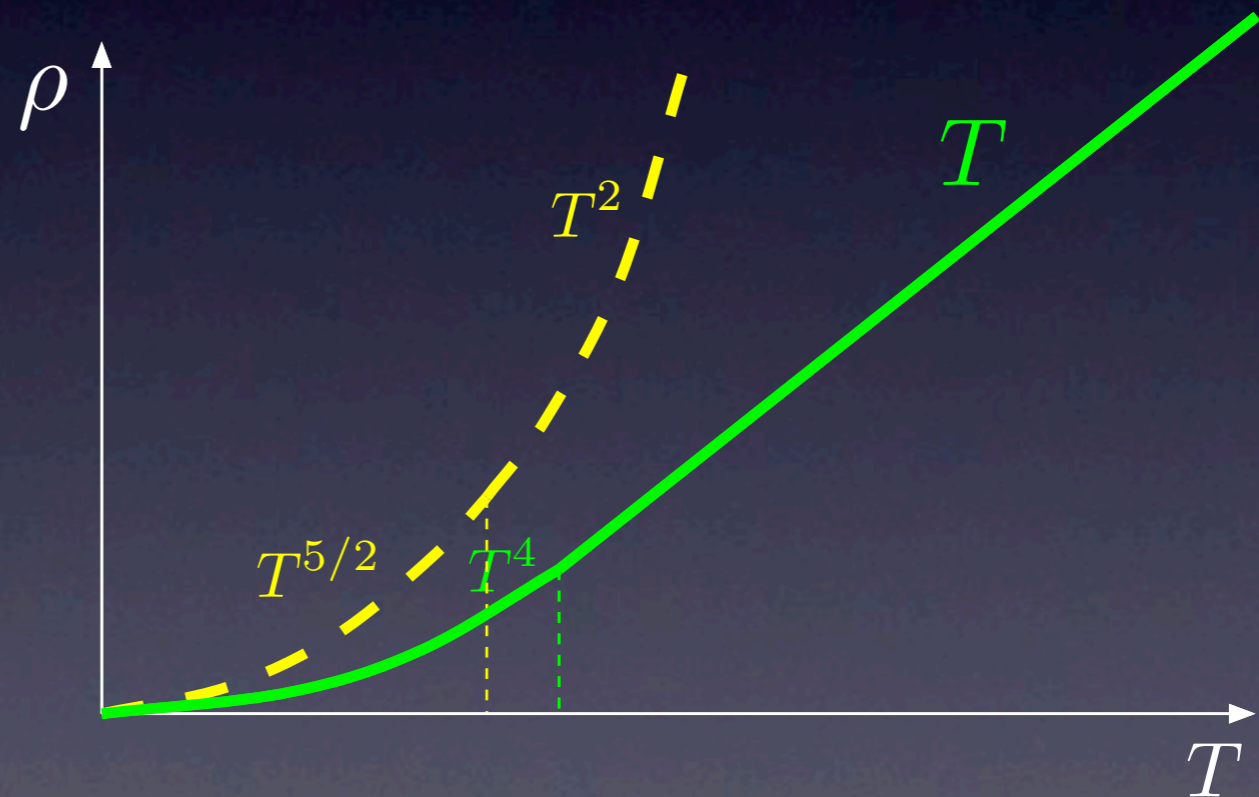


$$F_{\text{bend}} \rightarrow \frac{1}{2} \int d\mathbf{r} \left[ \kappa (\nabla^2 h)^2 + \gamma (\nabla h)^2 \right]$$

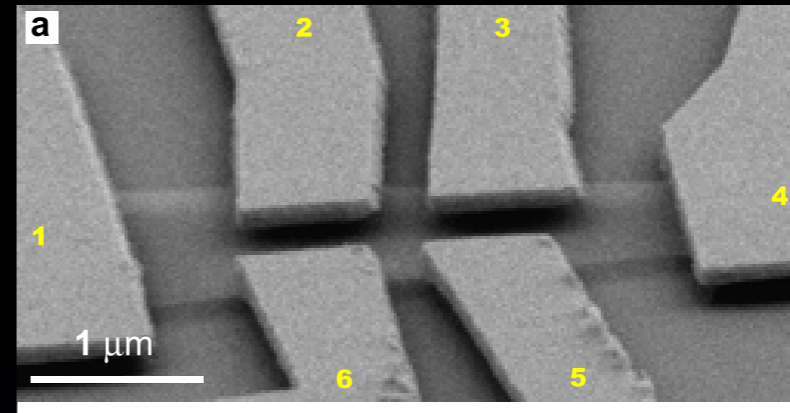
Flexural dispersion  
stiffening

$$\omega_q \sim q^2 \rightarrow q$$

Reduced  
DOS



# Role of tension?

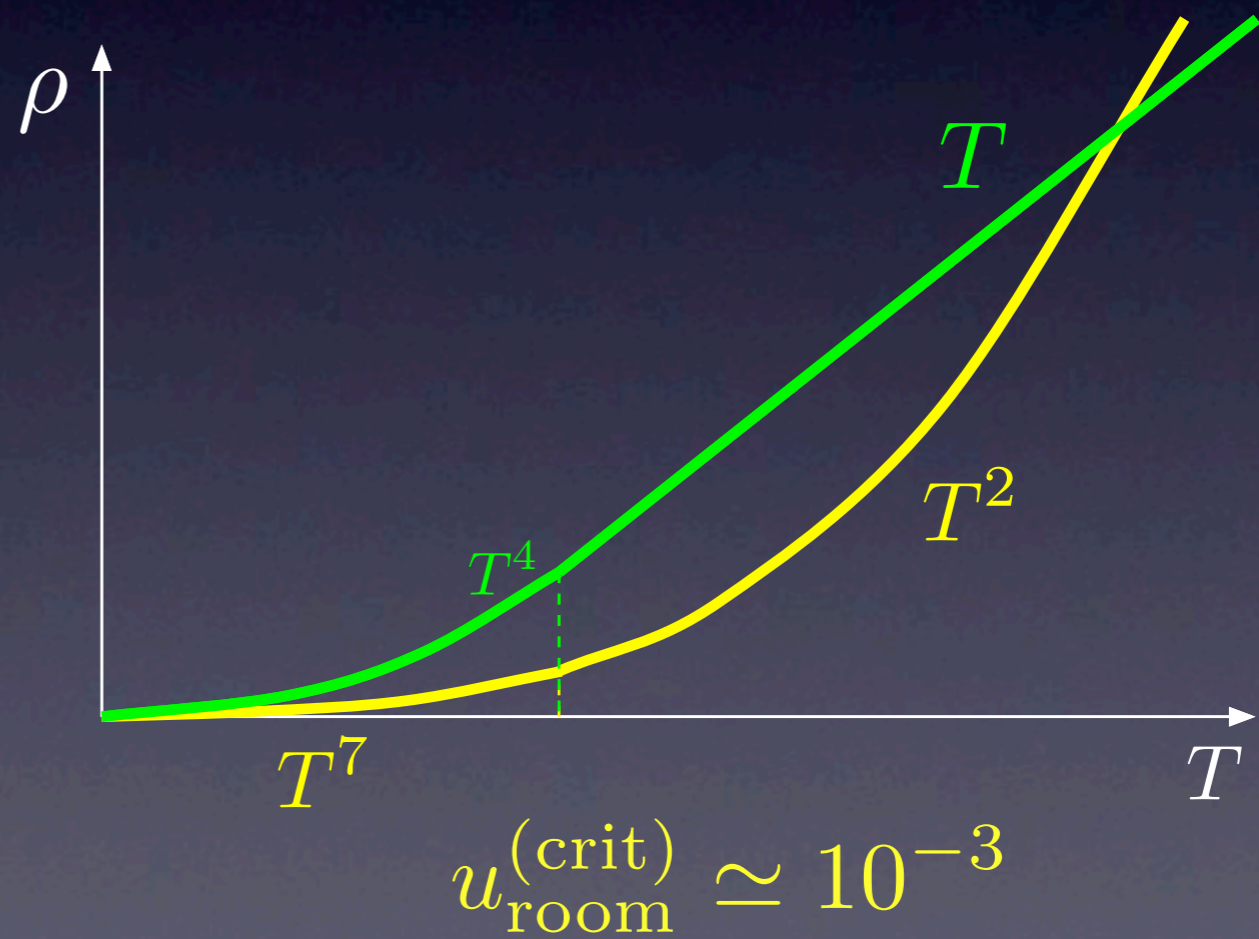


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Flexural dispersion  
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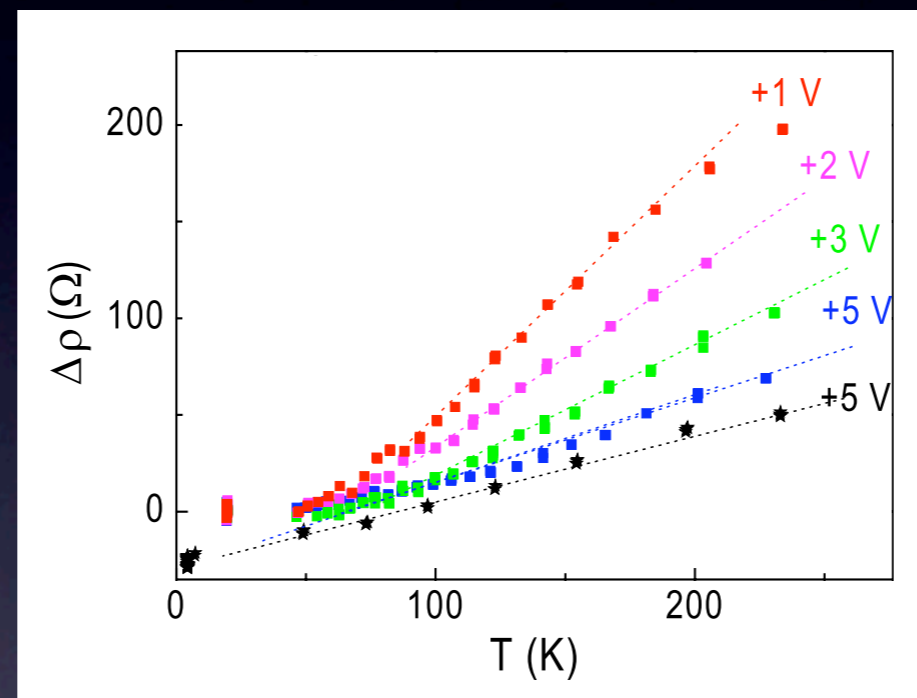
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Reduced  
DOS



E. Mariani and F. von Oppen, Phys. Rev. B **82**, 195403 (2010)

# What experiments tell us



Bolotin et al, PRL 08

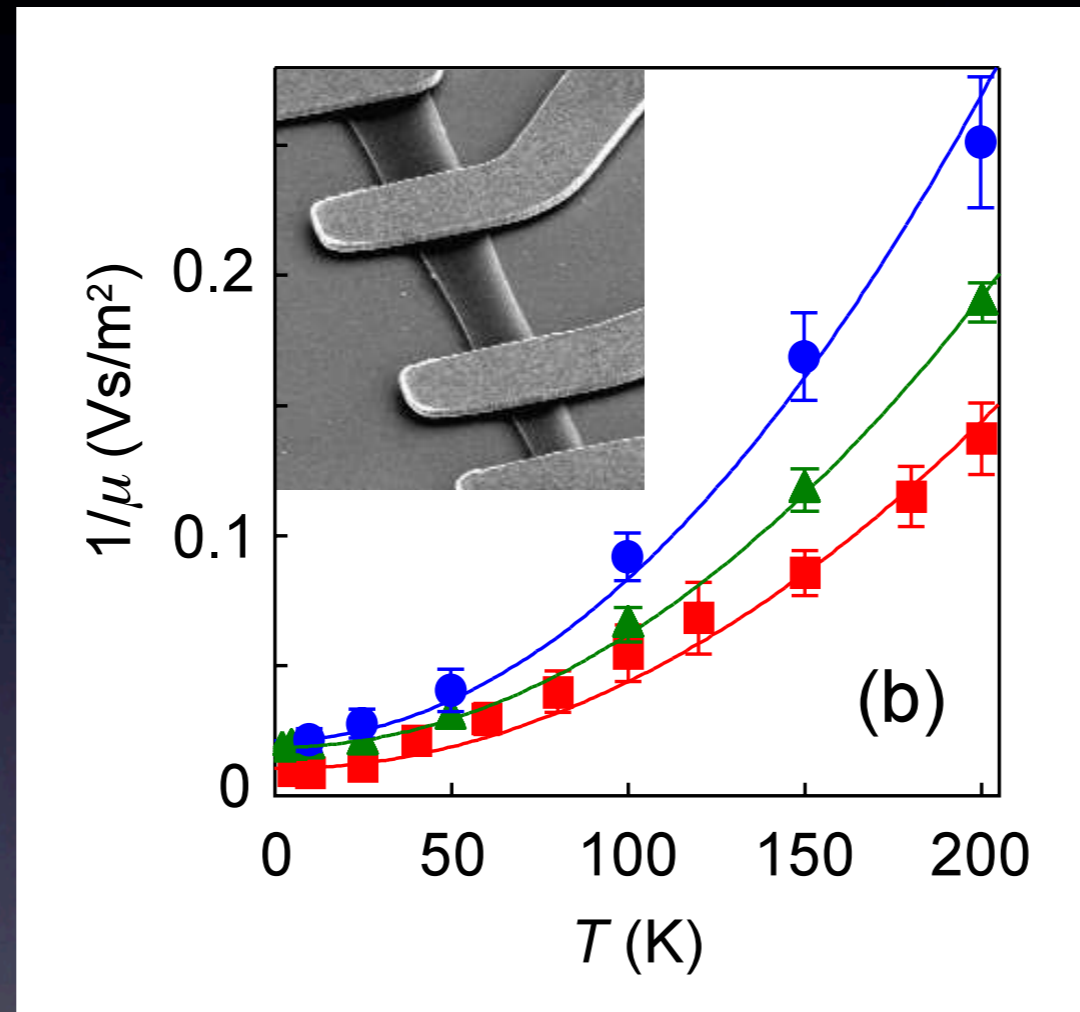
The linear  $T$ -dependence implies the presence of tension  
Crossover to  $T^2$  would allow to know the strength of tension

E. Mariani and F. von Oppen, Phys. Rev. B **82**, 195403 (2010)



One more thing...

# One more thing...



E. Castro et al., Phys. Rev. Lett. **105**, 266601 (2010)

Quadratic temperature dependence observed!

...and what about  
deformations in  
Bilayer Graphene Membranes?

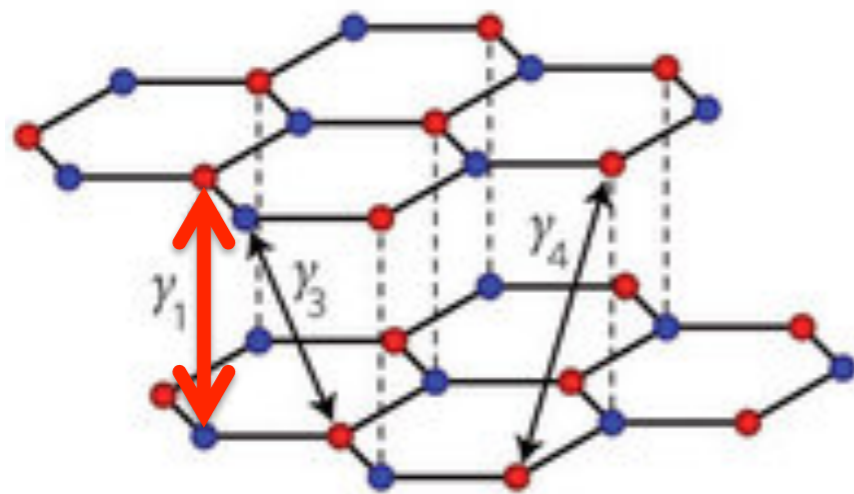
EM, A. Pearce and F. von Oppen, arXiv:1110.2769



# Electronic Properties of Graphene

(McCann PRL 2006)

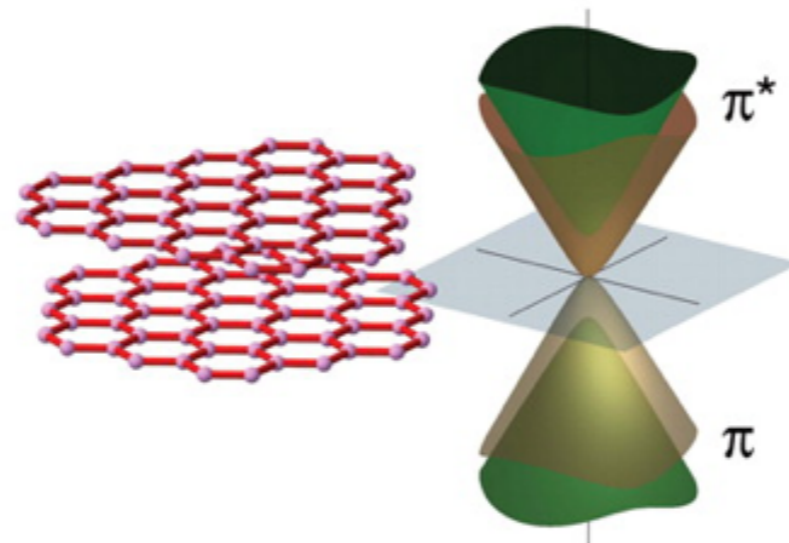
## Bilayer Graphene (dominant hopping)



$$\mathcal{H} = \frac{1}{2m} \begin{bmatrix} 0 & (p^\dagger)^2 \\ p^2 & 0 \end{bmatrix} + \frac{\Delta}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\epsilon_{\vec{p}} = \pm \frac{1}{2m} (p_x^2 + p_y^2)$$

$\Delta=0$

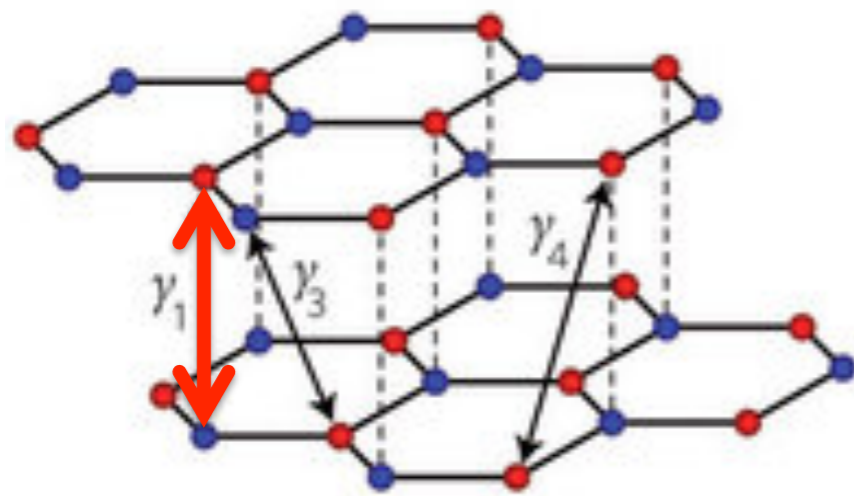


**Massive Dirac Fermions**  
Tuneable bandgap  
Berry Phase of parabolic bands  $\phi=2\pi$

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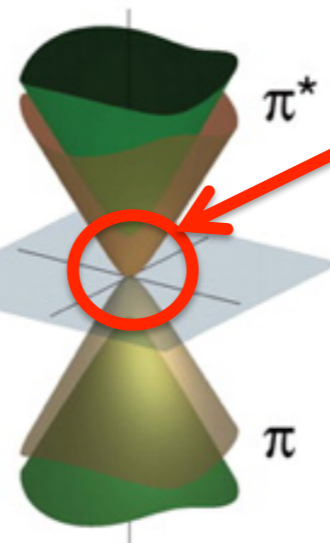
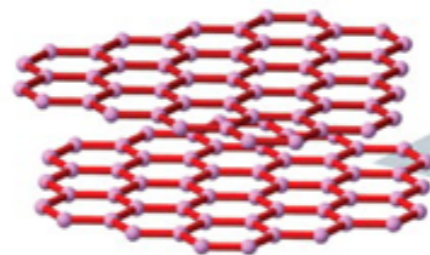
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**ZOOM**

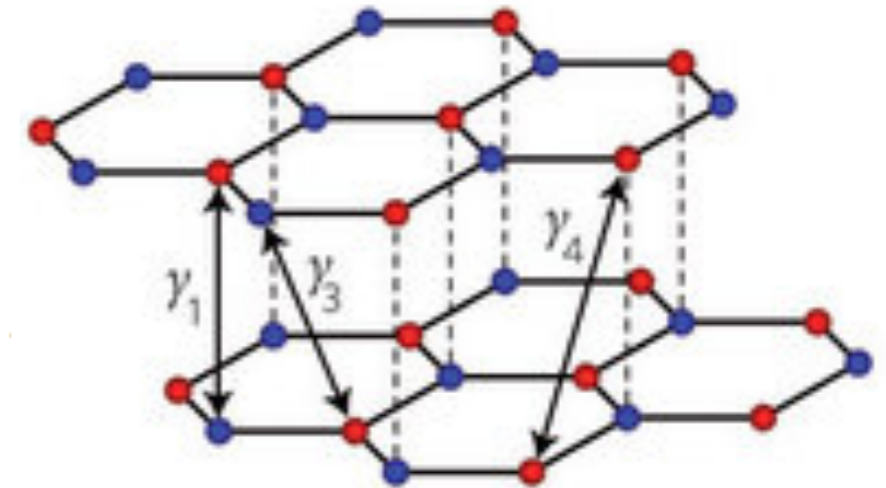
**Massive Dirac Fermions**  
Tuneable bandgap  
Berry Phase of parabolic bands  $\phi=2\pi$

# Low Energy bandstructure of Bilayer Graphene

(McCann PRL 2006)

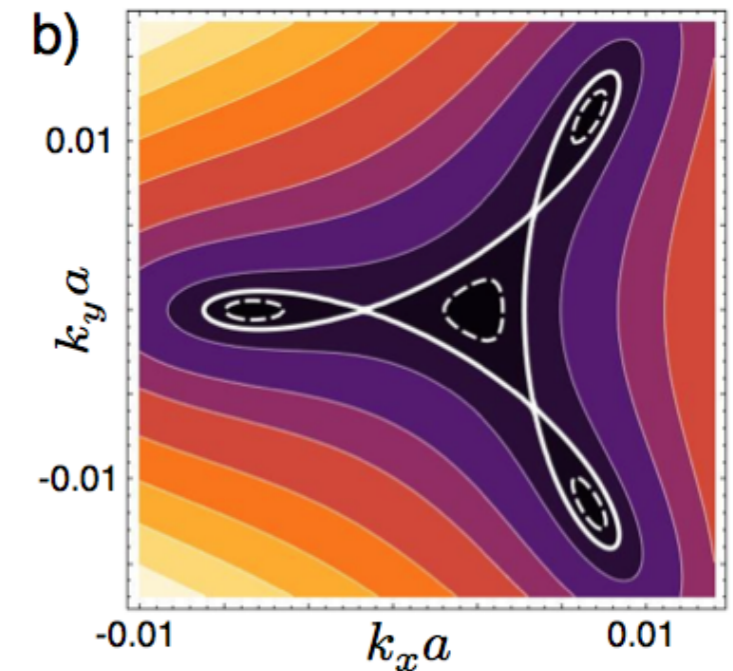
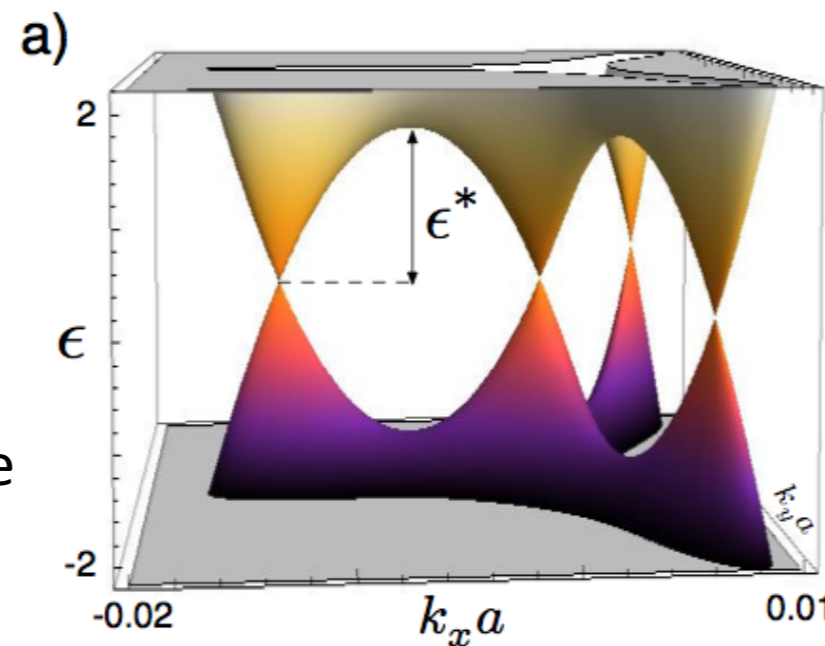
Bilayer Graphene (subdominant hoppings)  
Trigonal warping

$$\mathcal{H} = \frac{1}{2m} \begin{bmatrix} 0 & (p^\dagger)^2 \\ p^2 & 0 \end{bmatrix} + v_3 \begin{bmatrix} 0 & p \\ p^\dagger & 0 \end{bmatrix}$$



Four massless Dirac cones  
at low energy ( $\epsilon^* \approx 1.6\text{meV}$ )

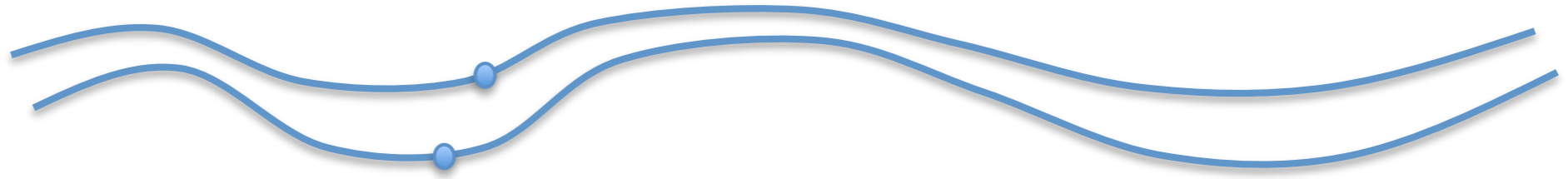
Lifshitz Transition  
Topological Transition in the  
Fermi surface



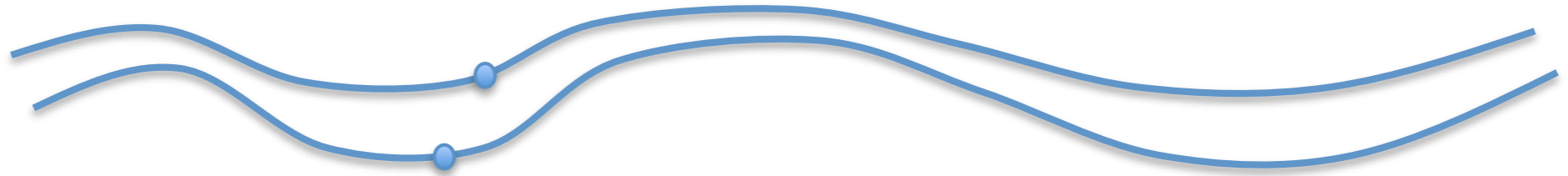
Effects in compressibility and transport measurements



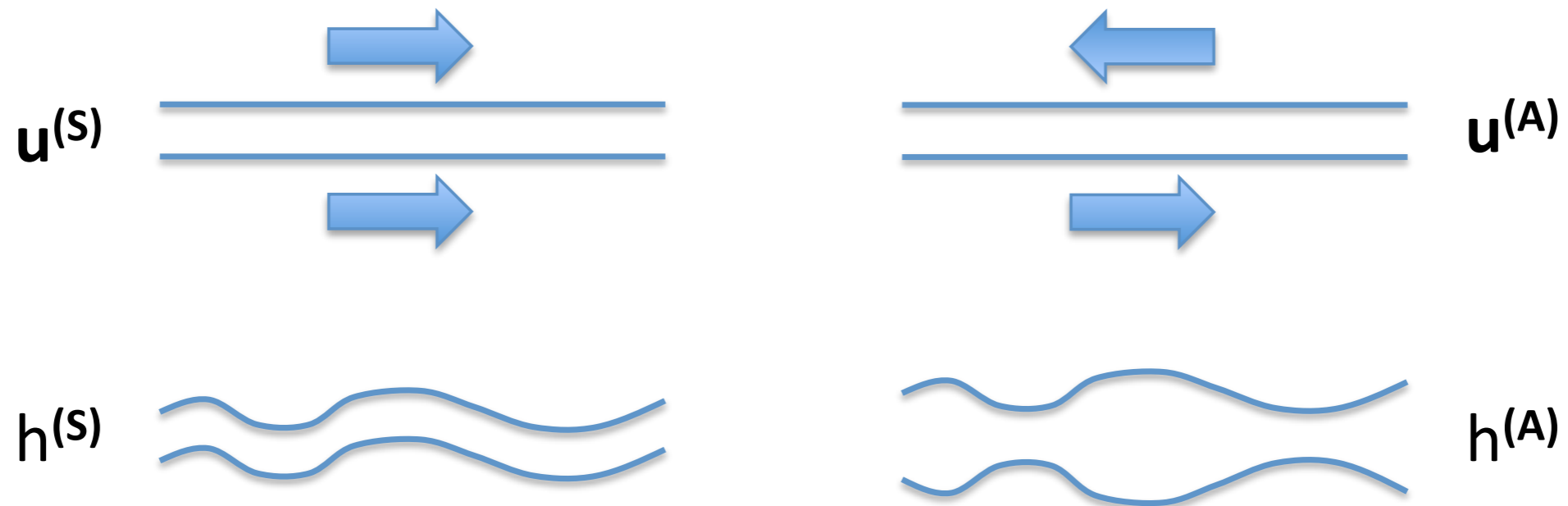
# Deformations in bilayer membranes



# Deformations in bilayer membranes



Generic elastic deformations decomposed into  
symmetric and antisymmetric  
In-plane and Flexural deformations



# Deformations in bilayer membranes

(symmetric and antisymmetric in-plane and flexural channels)



$$H_{\text{eff}}^{(+)} = \begin{pmatrix} D^{(S)} - D^{(A)} + \frac{\Delta}{2} & v_3 P_3^{(+)} \\ v_3 P_3^{(+)\dagger} & D^{(S)} + D^{(A)} - \frac{\Delta}{2} \end{pmatrix} + \frac{1}{\gamma} \begin{pmatrix} v_1 v_4 \left( P_4^{(+)\dagger} P_1^{(+)} + P_1^{(+)\dagger} P_4^{(+)} \right) & v_4^2 \left( P_4^{(+)\dagger} \right)^2 + v_1 v_2 P_1^{(+)\dagger} P_2^{(+)\dagger} \\ v_4^2 \left( P_4^{(+)} \right)^2 + v_1 v_2 P_2^{(+)} P_1^{(+)} & v_2 v_4 \left( P_2^{(+)} P_4^{(+)\dagger} + P_4^{(+)} P_2^{(+)\dagger} \right) \end{pmatrix}$$

Where  $P_j = p + F_j / v_j$

$$D_{l=1,2} = g \text{Tr}[u_{ij}^{(l)}]$$

$$F_{l=1,2}^{(\tau)} = \frac{3}{4} a \frac{\partial t_l}{\partial a} \left[ u_{xx}^{(l)} - u_{yy}^{(l)} - i\tau \left( u_{xy}^{(l)} + u_{yx}^{(l)} \right) \right]$$

$$F_3^{(\tau)} = \frac{3}{2\tilde{c}} \frac{\partial \gamma_3}{\partial \tilde{c}} \mathcal{F}[\mathbf{u}^{(S)}, \mathbf{u}^{(A)}, h^{(S)}, h^{(A)}]$$

$$F_4^{(\tau)} = \frac{3}{2\tilde{c}} \frac{\partial \gamma_4}{\partial \tilde{c}} \mathcal{F}[\mathbf{u}^{(S)}, -\mathbf{u}^{(A)}, -h^{(S)}, h^{(A)}]$$

$$F_\gamma = -2 \frac{\partial \gamma}{\partial c} \left[ h^{(A)} + \frac{\mathbf{u}^{(A)2}}{c} \right].$$

$$\mathcal{F}[\mathbf{u}^{(S)}, \mathbf{u}^{(A)}, h^{(S)}, h^{(A)}] = ac \left( \partial_y h^{(S)} - i\tau \partial_x h^{(S)} \right)$$

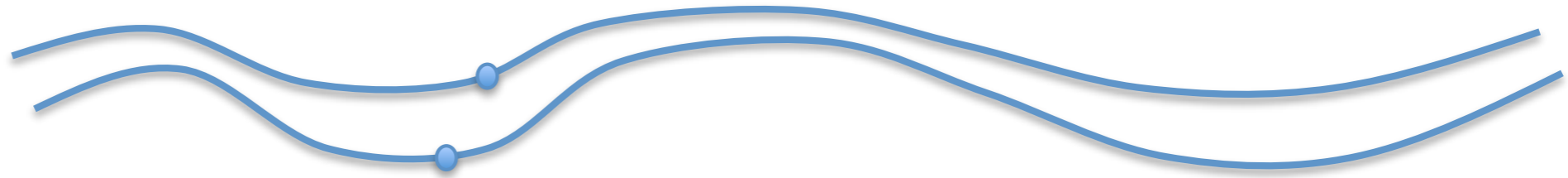
$$+ \frac{a^2}{2} \left( u_{xx}^{(S)} - u_{yy}^{(S)} - i\tau \left( u_{xy}^{(S)} + u_{yx}^{(S)} \right) \right)$$

$$+ 2a \left( u_y^{(A)} - i\tau u_x^{(A)} \right).$$



# Deformations in bilayer membranes

(symmetric and antisymmetric in-plane and flexural channels)



$$H_{\text{eff}}^{(+)} = \begin{pmatrix} D^{(S)} - D^{(A)} - \frac{\Delta}{2} & v_3 P_3^{(+)} \\ v_3 P_3^{(+)\dagger} & D^{(S)} + D^{(A)} - \frac{\Delta}{2} \end{pmatrix} + \frac{1}{\gamma} \begin{pmatrix} v_1 v_4 (P_4^{(+)\dagger} P_1^{(+)} + P_1^{(+)\dagger} P_4^{(+)}) & v_4^2 (P_4^{(+)\dagger})^2 + v_1 v_2 P_1^{(+)\dagger} P_2^{(+)\dagger} \\ v_4^2 (P_4^{(+)} )^2 + v_1 v_2 P_2^{(+)} P_1^{(+)} & v_2 v_4 (P_2^{(+)} P_4^{(+)\dagger} + P_4^{(+)} P_2^{(+)\dagger}) \end{pmatrix}$$

Where  $P_j = p + F_j / v_j$

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$$F_\gamma = -2 \frac{\partial \gamma}{\partial c} \left[ h^{(A)} + \frac{\mathbf{u}^{(A)2}}{c} \right]$$

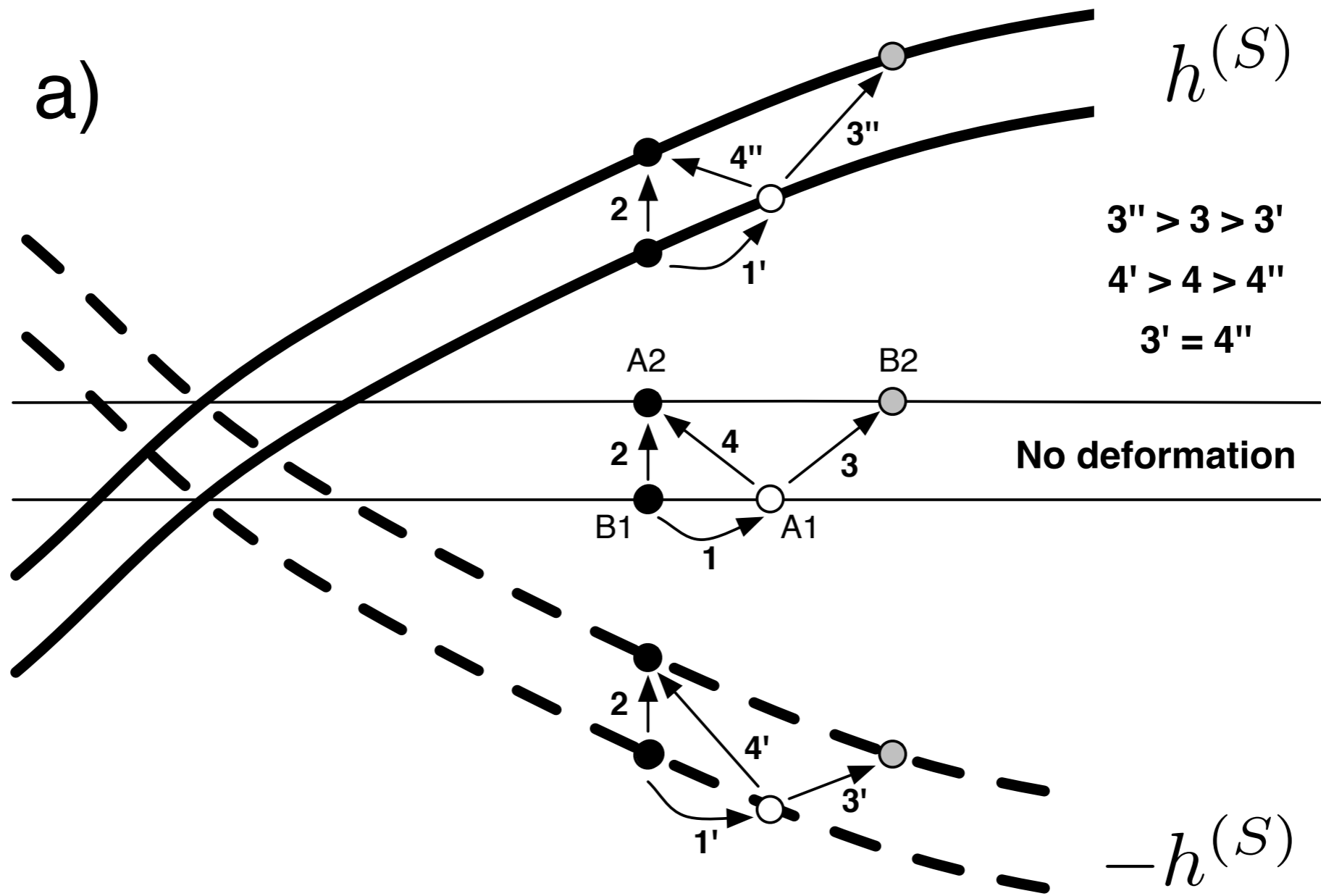
$$\mathcal{F}[\mathbf{u}^{(S)}, \mathbf{u}^{(A)}, h^{(S)}, h^{(A)}] = ac \left( \partial_y h^{(S)} - i\tau \partial_x h^{(S)} \right)$$

$$+ \frac{a^2}{2} \left( u_{xx}^{(S)} - u_{yy}^{(S)} - i\tau (u_{xy}^{(S)} + u_{yx}^{(S)}) \right)$$

$$+ 2a \left( u_y^{(A)} - i\tau u_x^{(A)} \right)$$

Anti-symmetric deformation Potential opens band gap

Unexpected linear coupling for flexural modes

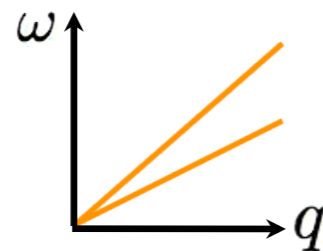


# Temperature-dependent resistivity

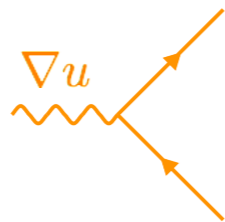
Intralayer coupling

In-plane phonons (Viljas 2010, Min 2011)

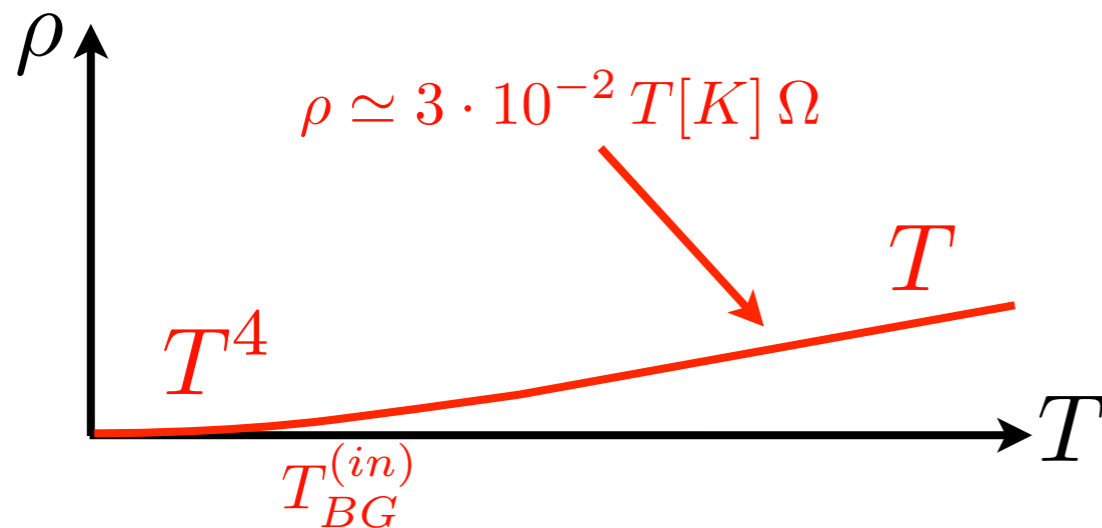
Linear dispersion  
 $\omega^{(l,t)} = v^{(l,t)} q$



Linear coupling  
 $H_{e-ph} \propto u_{ij} \sim \nabla u$



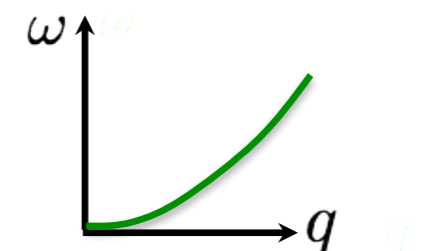
Small DOS, Good coupling



Flexural phonons (H. Ochoa 2011)

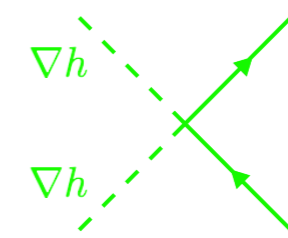
Quadratic dispersion

$$\omega^{(h)} = \alpha q^2$$

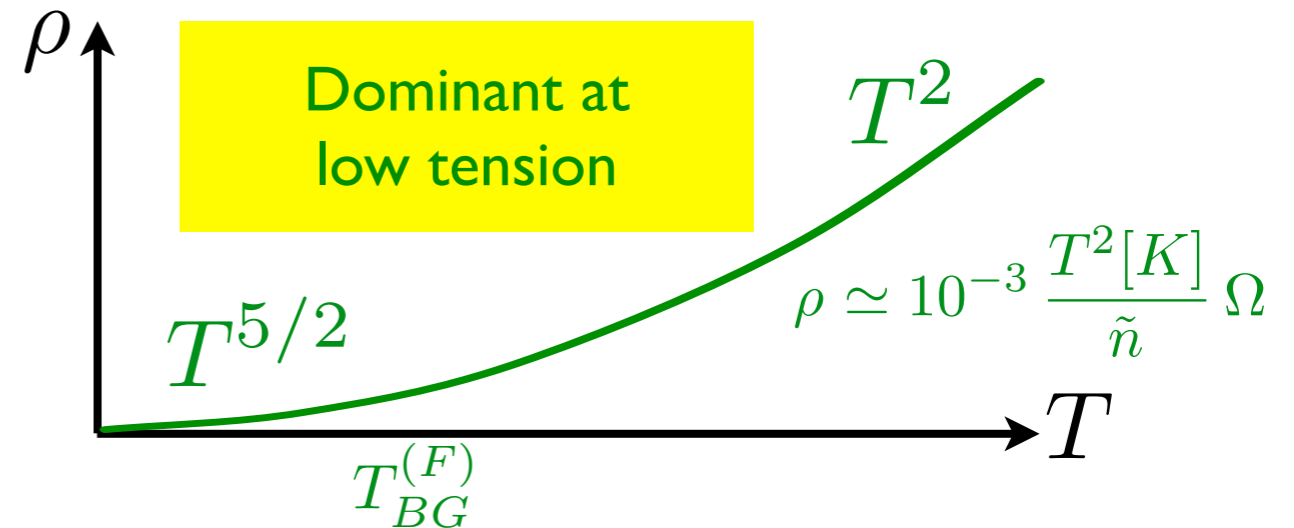


Quadratic coupling

$$H_{e-ph} \propto u_{ij} \sim (\nabla h)^2$$



Big DOS, Poor coupling

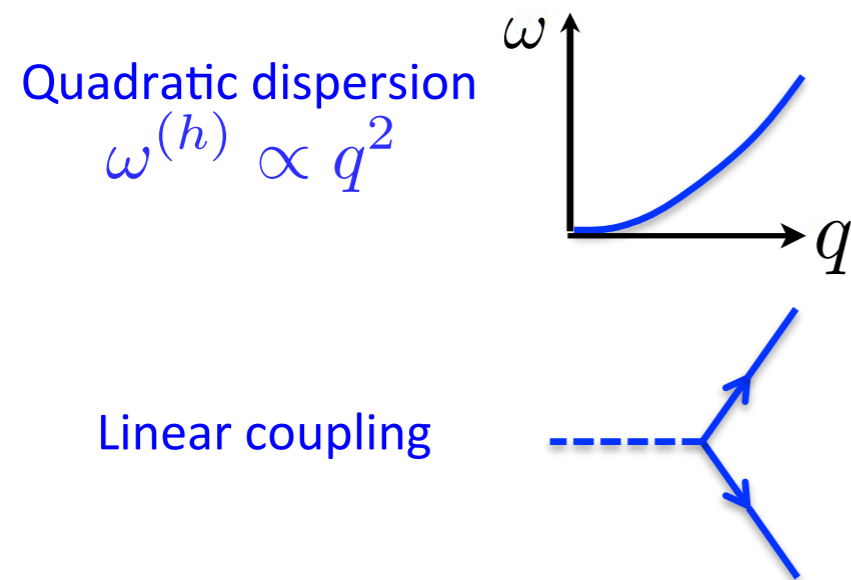




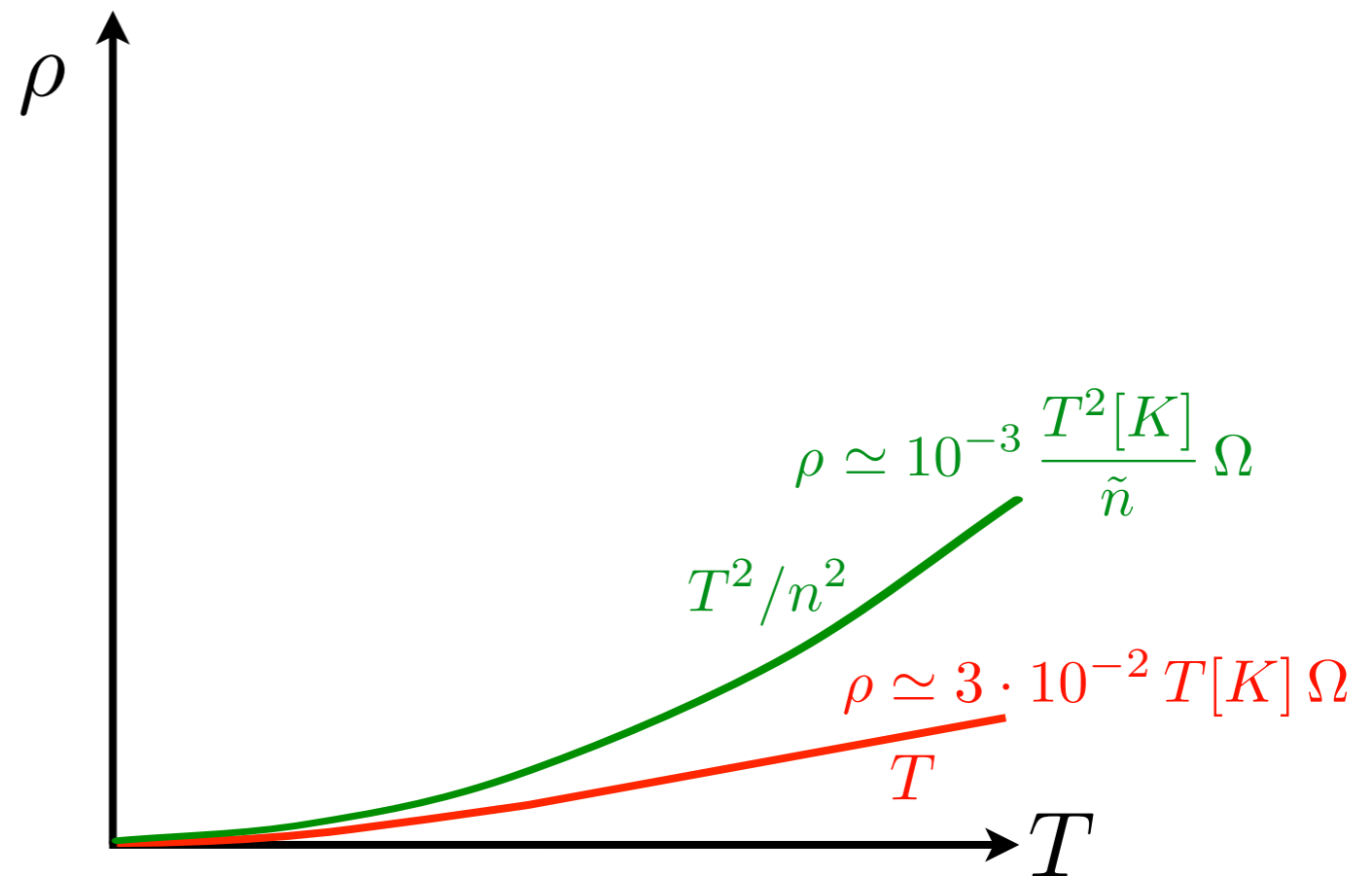
# Temperature-dependent resistivity (new interlayer gauge field contribution)

Interlayer  
coupling

Symmetric Flexural phonons  
(new linear coupling)



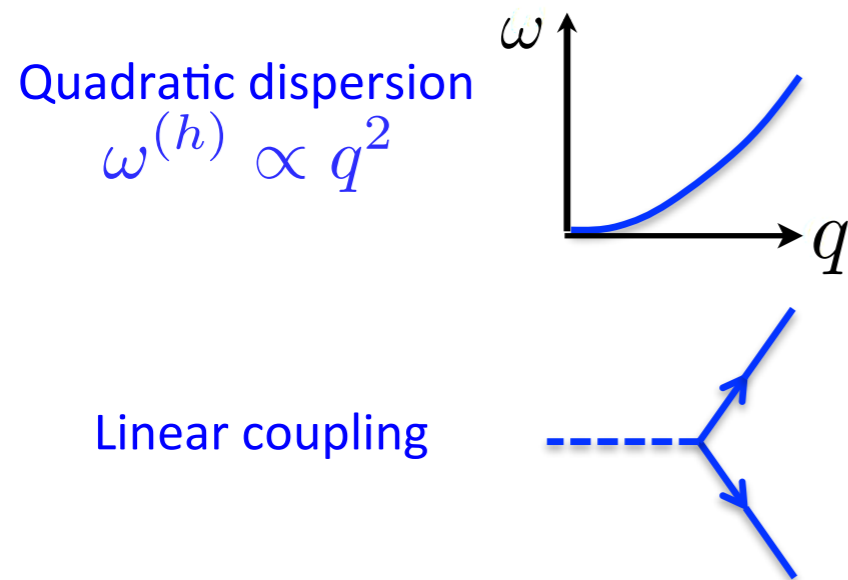
Big DOS, Strong coupling



# Temperature-dependent resistivity (new interlayer gauge field contribution)

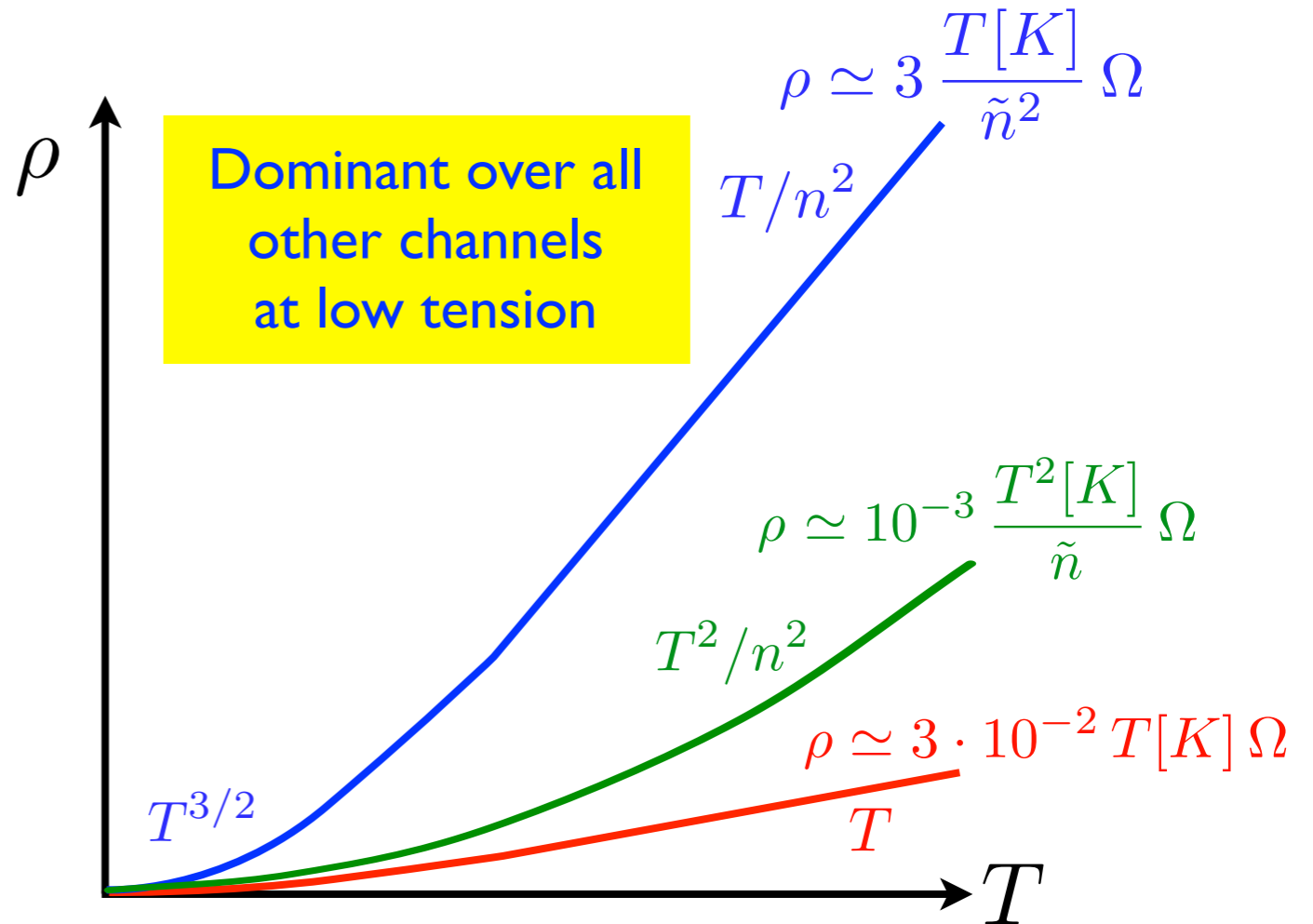
Interlayer coupling

Symmetric Flexural phonons  
(new linear coupling)



Big DOS, Strong coupling

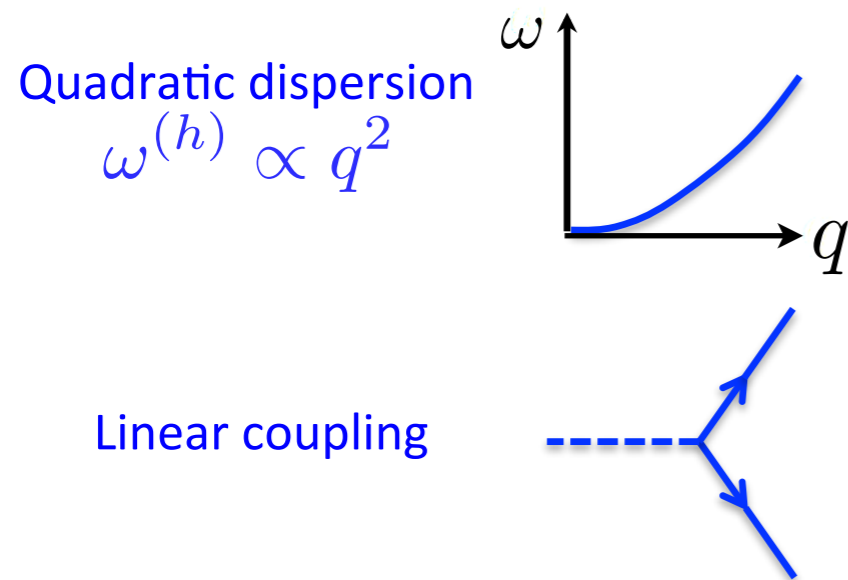
$$\rho_{q_* \ll 2k_F} \simeq 3 \frac{T[K]}{\tilde{n}^2} \Omega$$



# Temperature-dependent resistivity (new interlayer gauge field contribution)

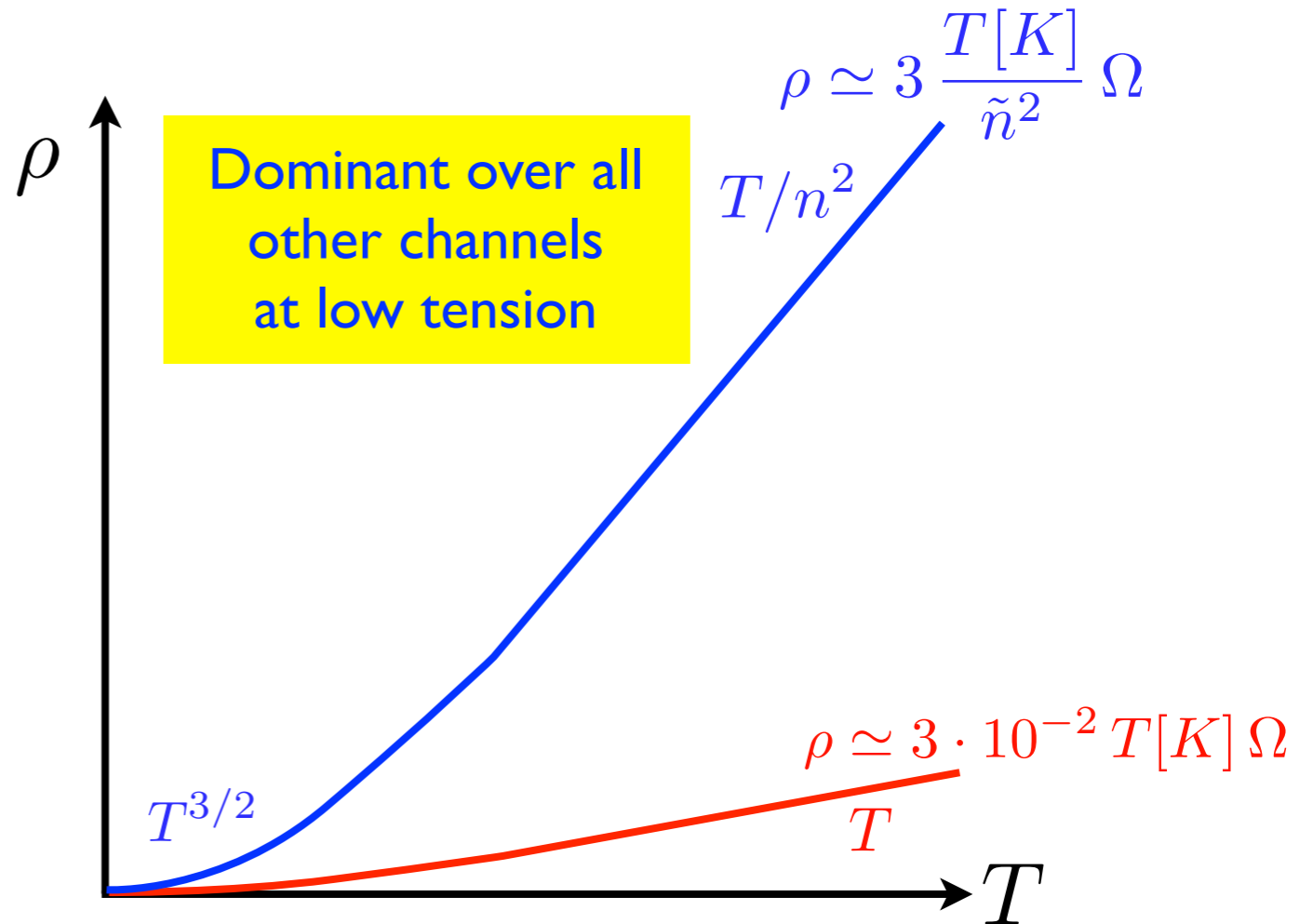
Interlayer coupling

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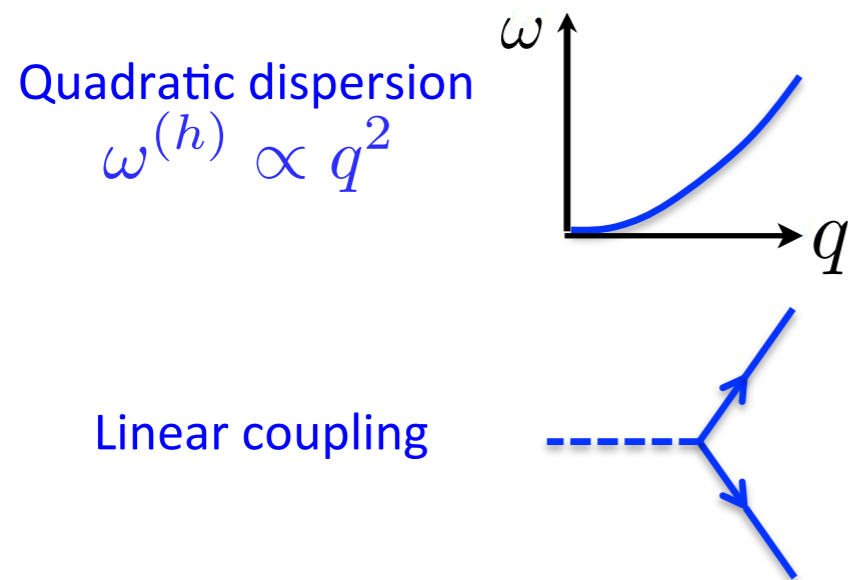




# Temperature-dependent resistivity (new interlayer gauge field contribution)

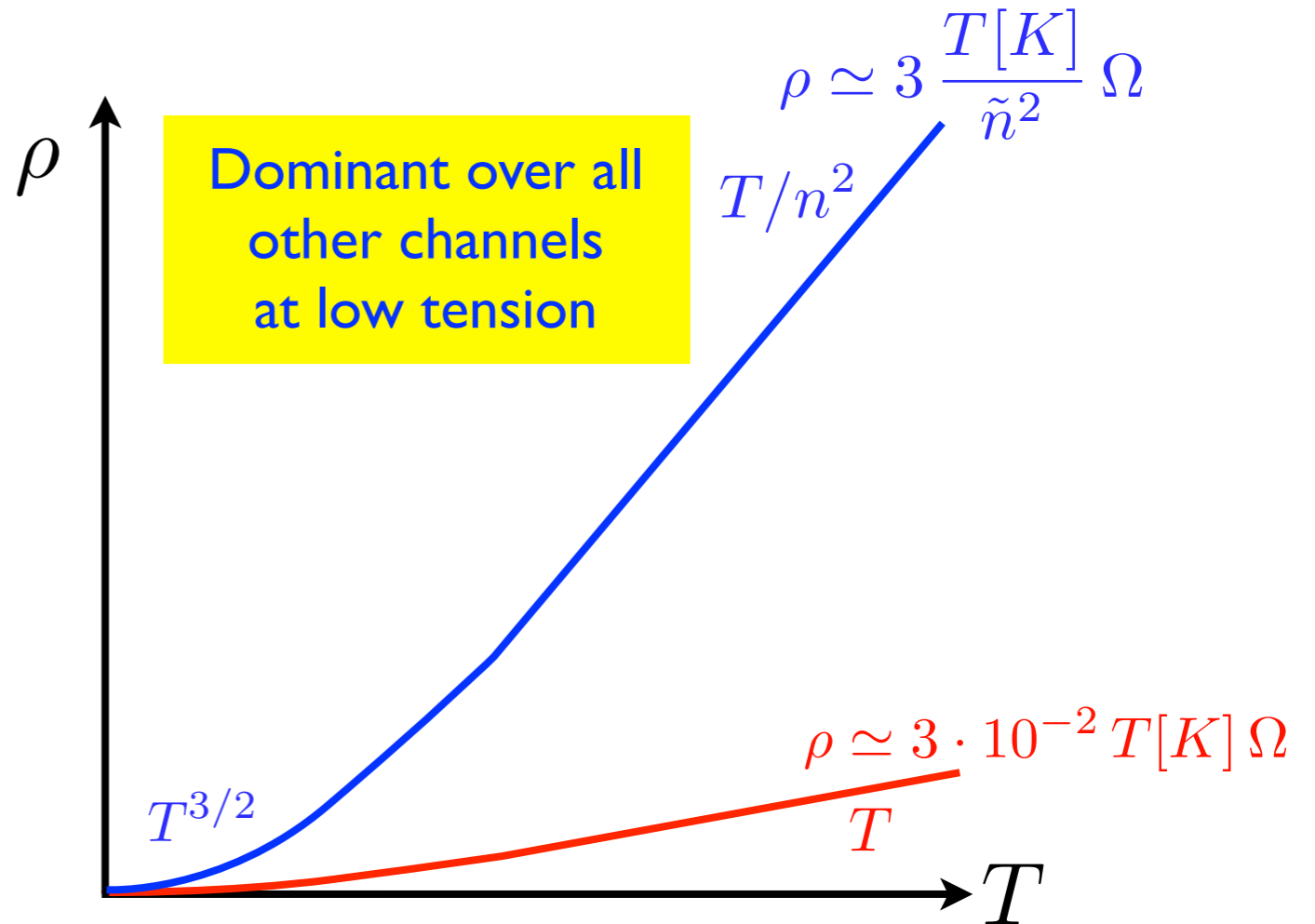
Interlayer coupling

Symmetric Flexural phonons  
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Big DOS, Strong coupling

$$\rho_{q_* \ll 2k_F} \simeq 3 \frac{T[K]}{\tilde{n}^2} \Omega$$



**Tension**  
Flexural dispersion stiffening  
 $\omega_q \sim q^2 \rightarrow q$   
Reduced DOS

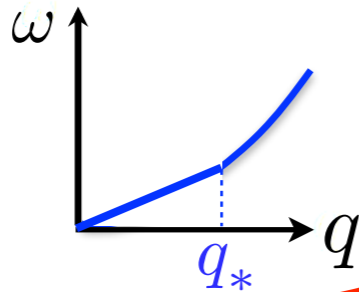
# Temperature-dependent resistivity (new interlayer gauge field contribution)

Interlayer coupling

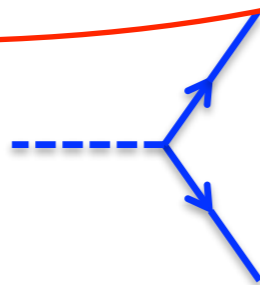
Symmetric Flexural phonons  
(new linear coupling)

$$\omega^{(h)} \propto q \quad \text{if } q \ll q_*$$

$$\omega^{(h)} \propto q^2 \quad \text{if } q \gg q_*$$

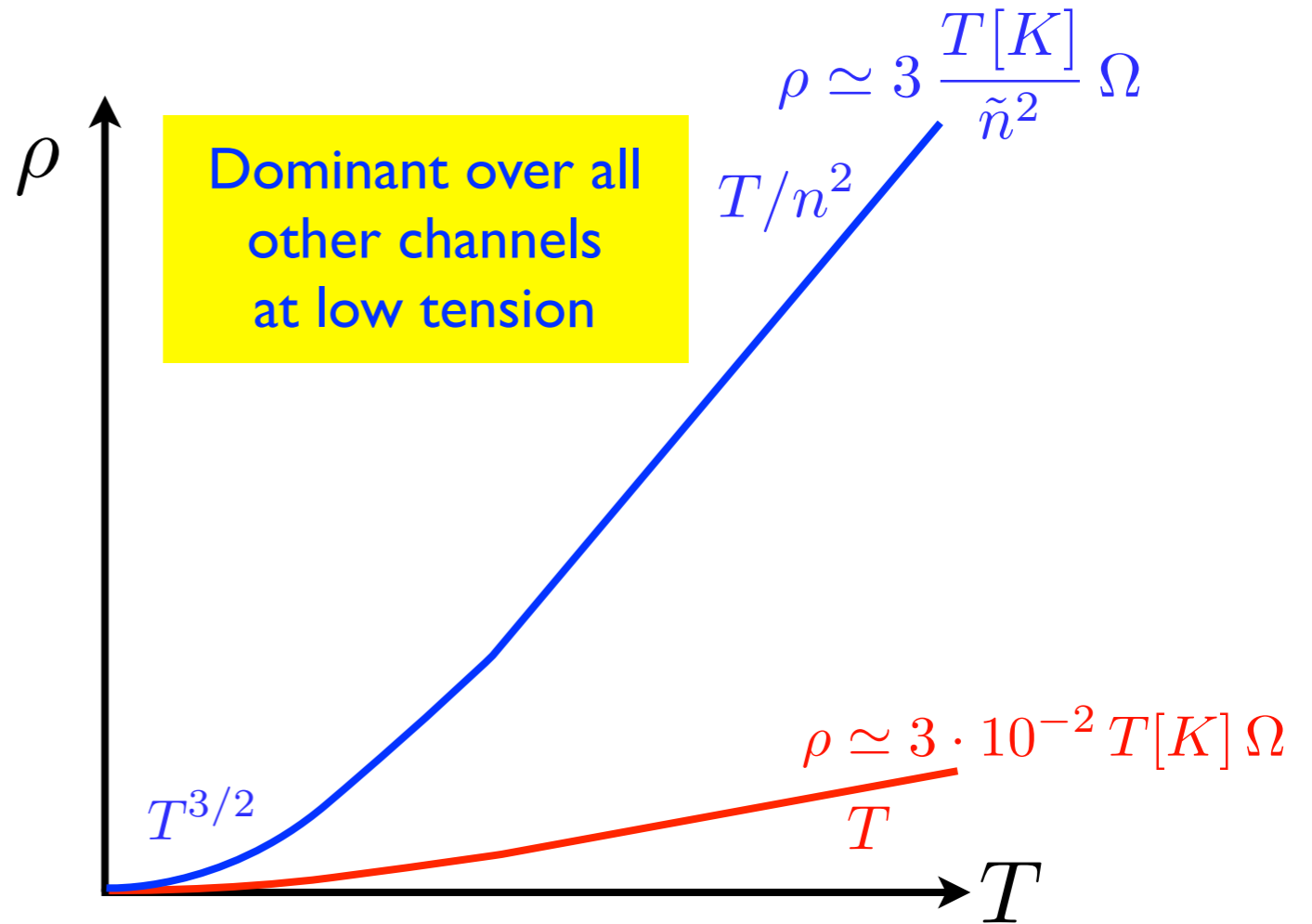


Linear coupling



Big DOS, Strong coupling

$$\rho_{q_* \ll 2k_F} \simeq 3 \frac{T[K]}{\tilde{n}^2} \Omega$$



**Tension**  
Flexural dispersion stiffening  
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Reduced DOS

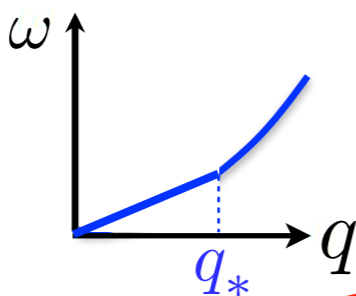
# Temperature-dependent resistivity (new interlayer gauge field contribution)

Interlayer coupling

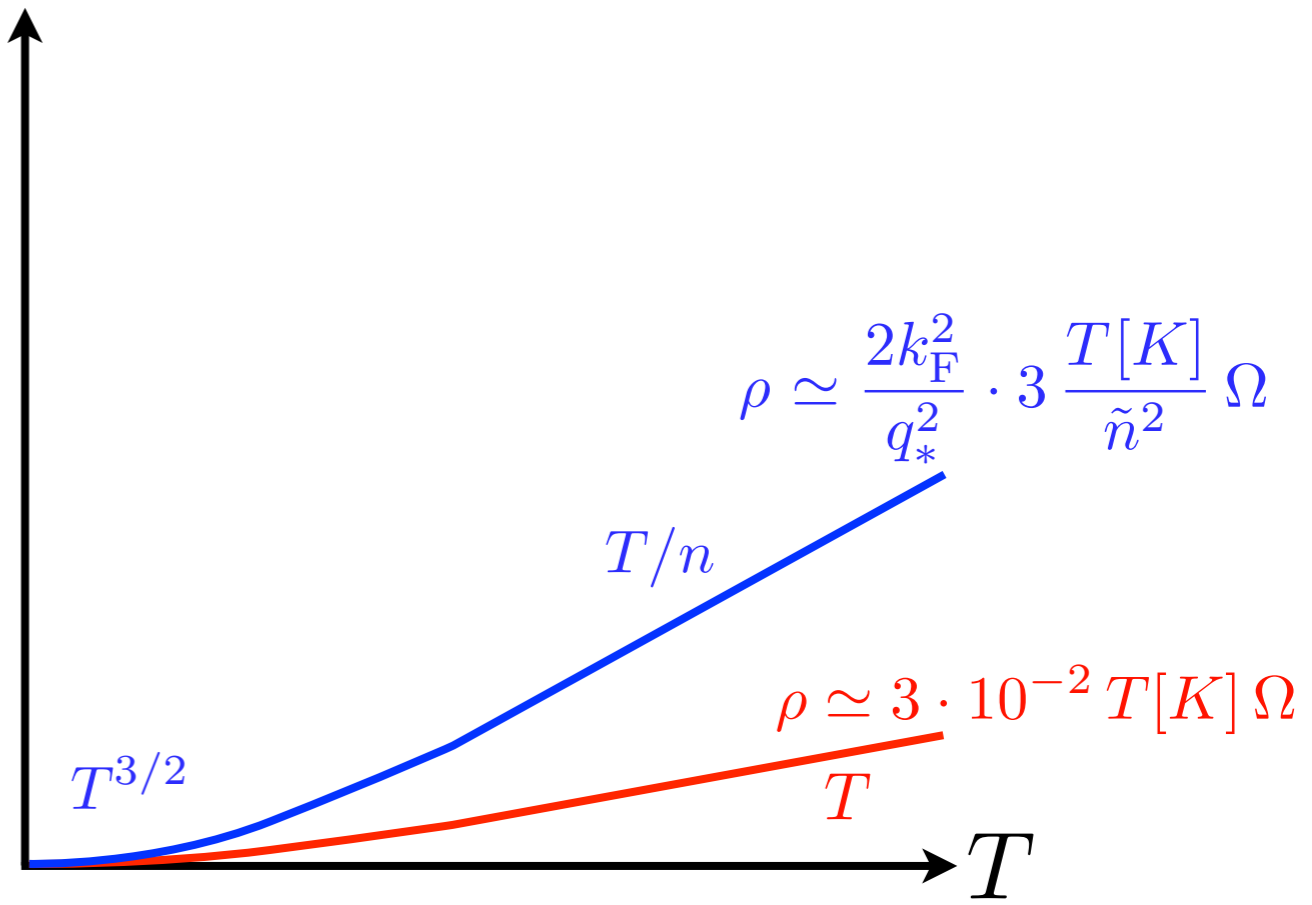
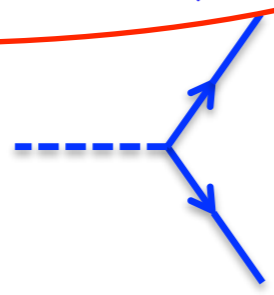
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$$\rho_{q_* \gg 2k_F} \simeq \frac{2k_F^2}{q_*^2} \cdot 3 \frac{T[K]}{\tilde{n}^2} \Omega$$

**Tension**  
Flexural dispersion stiffening  
 $\omega_q \sim q^2 \rightarrow q$   
Reduced DOS



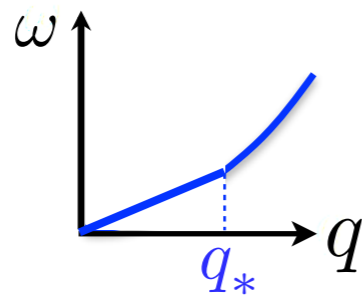
# Temperature-dependent resistivity (new interlayer gauge field contribution)

Interlayer coupling

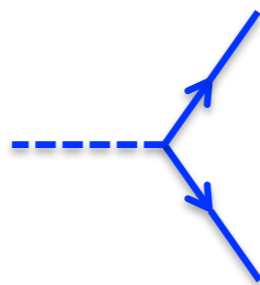
Symmetric Flexural phonons  
(new linear coupling)

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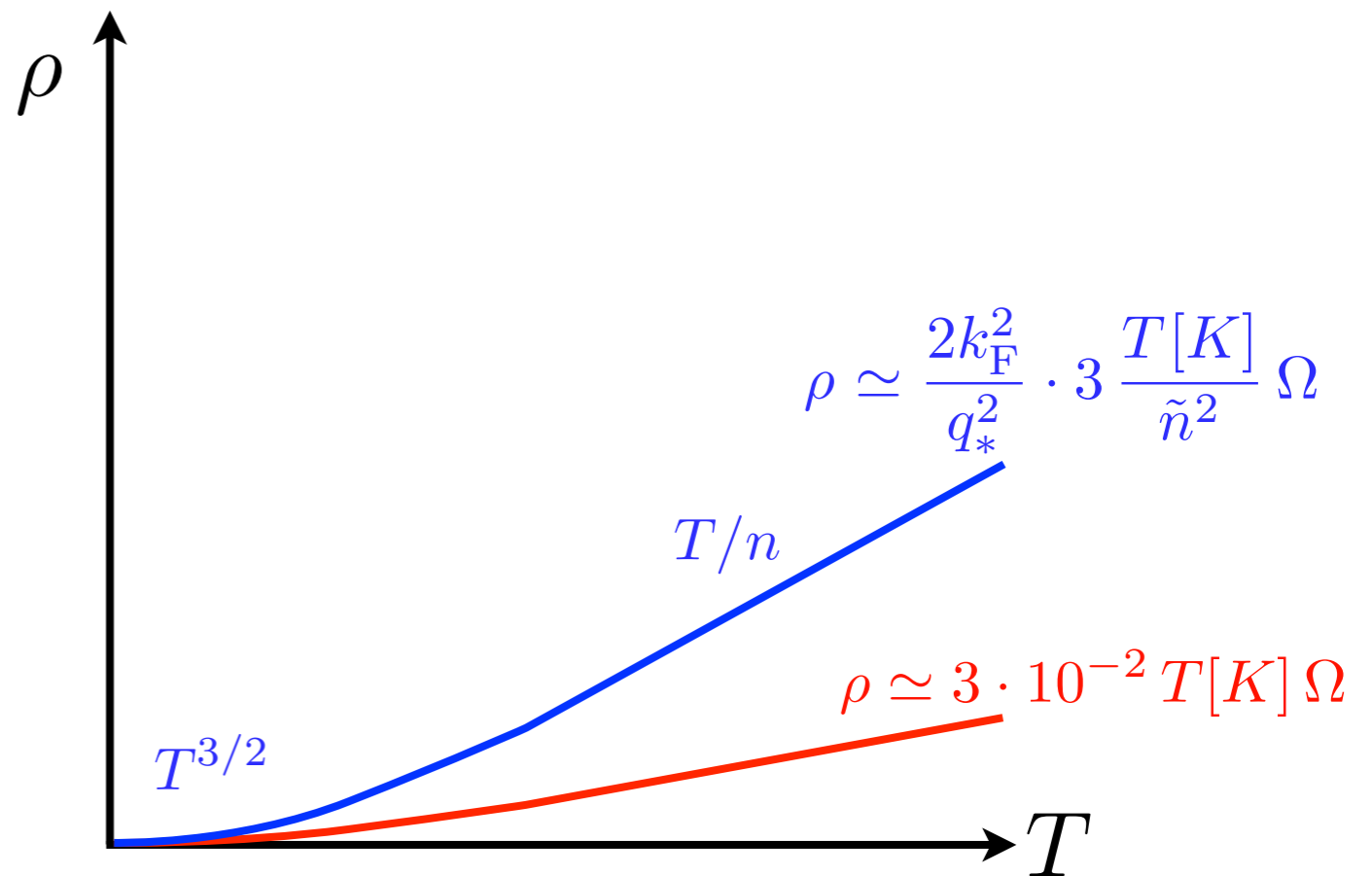
Linear coupling



Big DOS, Strong coupling

$$\rho_{q_* \ll 2k_F} \simeq 3 \frac{T[K]}{\tilde{n}^2} \Omega$$

$$\rho_{q_* \gg 2k_F} \simeq \frac{2k_F^2}{q_*^2} \cdot 3 \frac{T[K]}{\tilde{n}^2} \Omega$$



Further increase tension  
beyond a critical strain  
 $u \sim 3 \cdot 10^{-3} / \tilde{n}$   
**Flexural modes  
suppressed!**

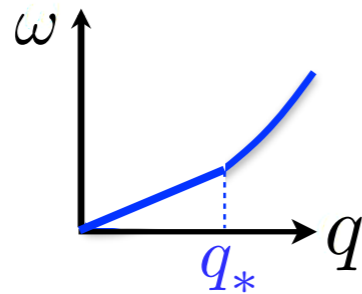
# Temperature-dependent resistivity (new interlayer gauge field contribution)

Interlayer  
coupling

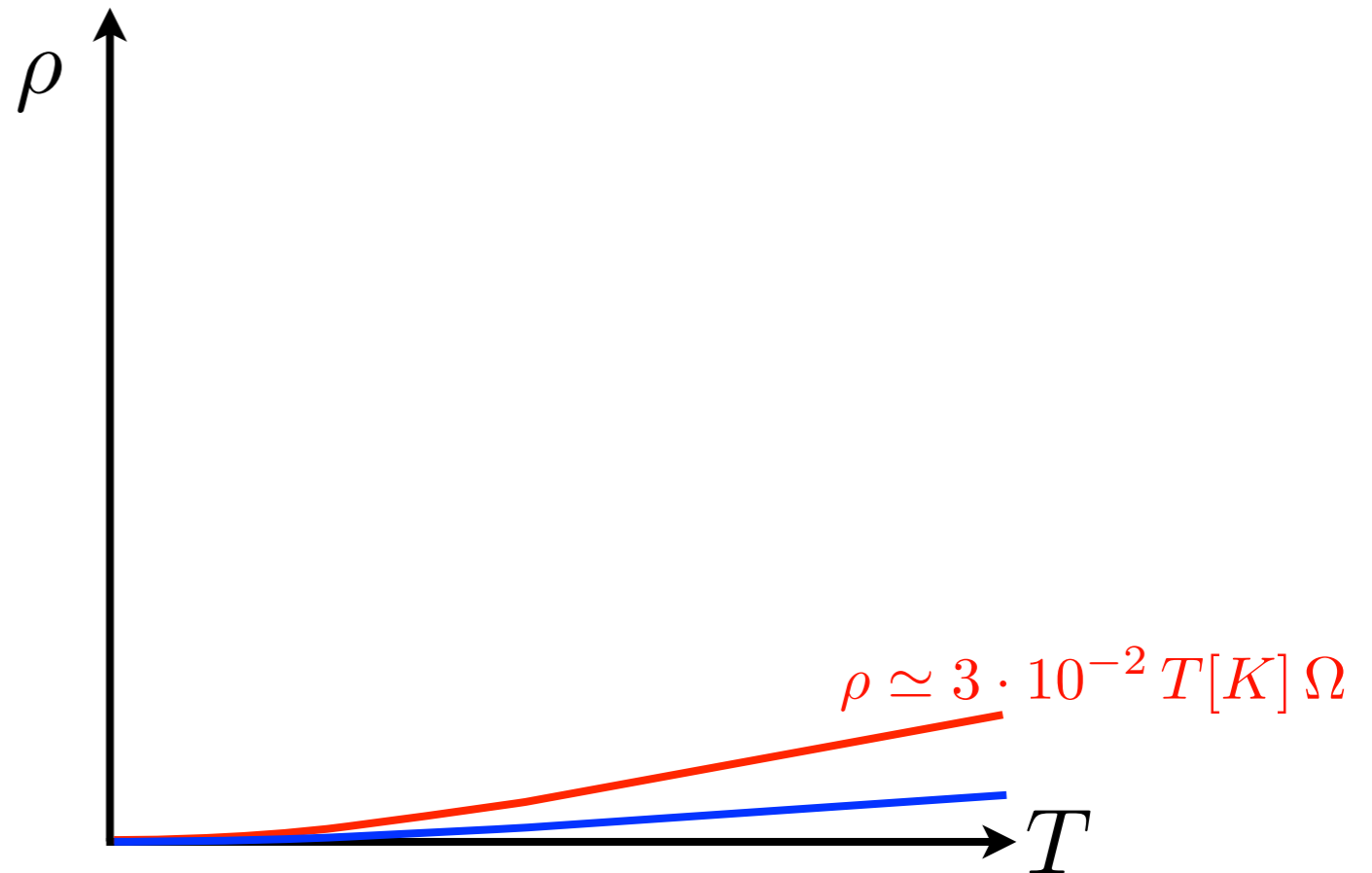
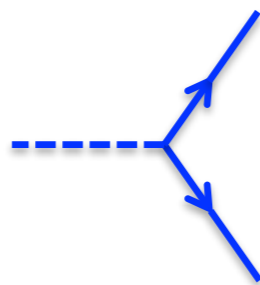
Symmetric Flexural phonons  
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Linear coupling



Big DOS, Strong coupling

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$$\rho_{q_* \gg 2k_F} \simeq \frac{2k_F^2}{q_*^2} \cdot 3 \frac{T[K]}{\tilde{n}^2} \Omega$$

Further increase tension  
beyond a critical strain

$$u \sim 3 \cdot 10^{-3} / \tilde{n}$$

Flexural modes  
suppressed!

# Summary

## Electron-phonon resistivity in suspended bilayers

$$\rho \propto T \quad \text{for} \quad T \gg T_{\text{BG}}^{(\text{in})}$$

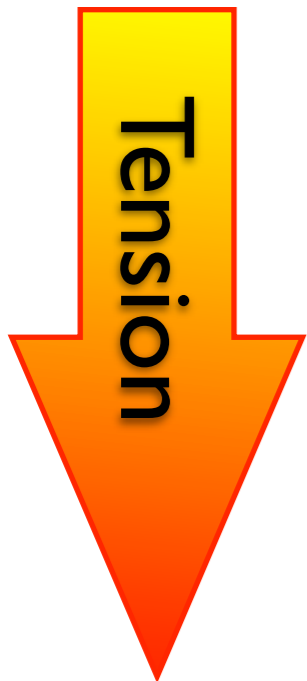
**Flex**  $\rho \propto \frac{T}{n^2}$  if  $q_* \ll 2k_{\text{F}}$

..... strain  $\sim 5 \cdot 10^{-5} \tilde{n}$

**Flex**  $\rho \propto \frac{T}{n}$  if  $q_* \gg 2k_{\text{F}}$

..... strain  $\sim \frac{3 \cdot 10^{-3}}{\tilde{n}}$

**In**  $\rho \propto T$



# Conclusions

## Monolayers

In-plane VS flexural phonons

Flexural modes dominate  
the resistivity at low tension

$$\rho \propto \frac{T^2}{n}$$

## Bilayers

Fictitious gauge fields for  
generic deformations

New linear coupling  
for flexural modes

Resistivity linear in T:  
density dependence reveals tension

$$\rho \propto \frac{T}{n^\alpha}$$



E.Mariani@exeter.ac.uk

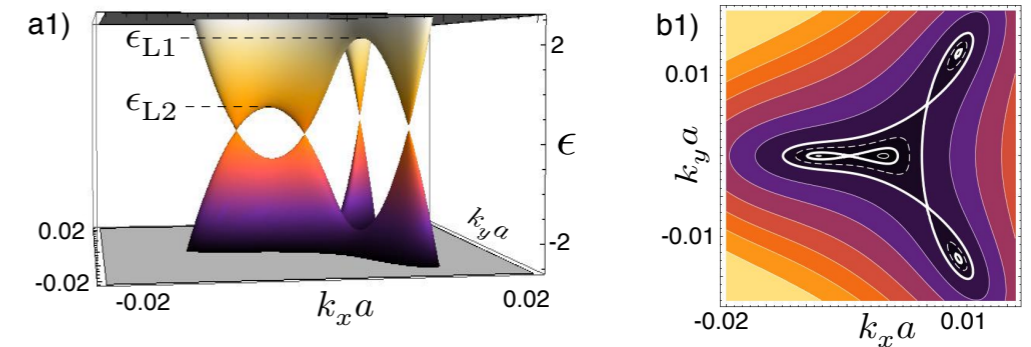
Thank you!

# Uniform gauge fields and the Lifshitz transition

Uniaxial strain, pure shear and sliding layers

J.W. Son, PRB 2011

M. Mucha-Kruczynski PRB 2011

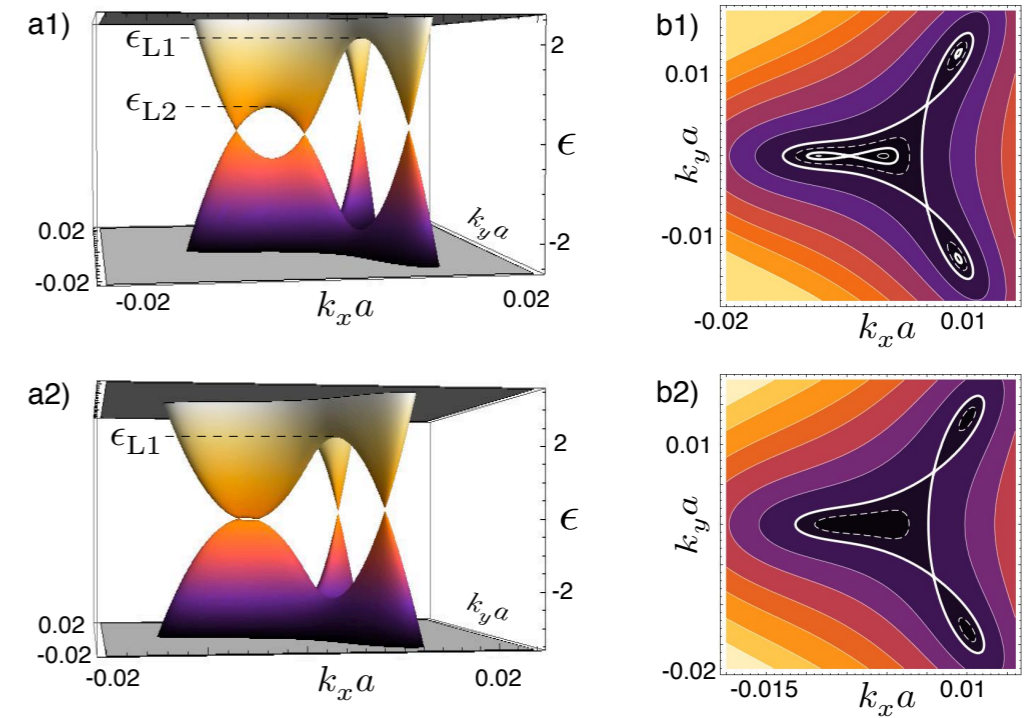
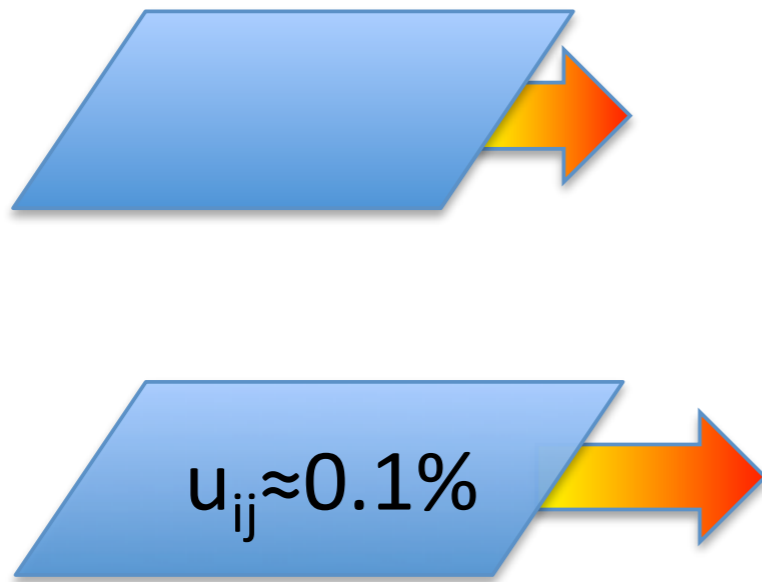


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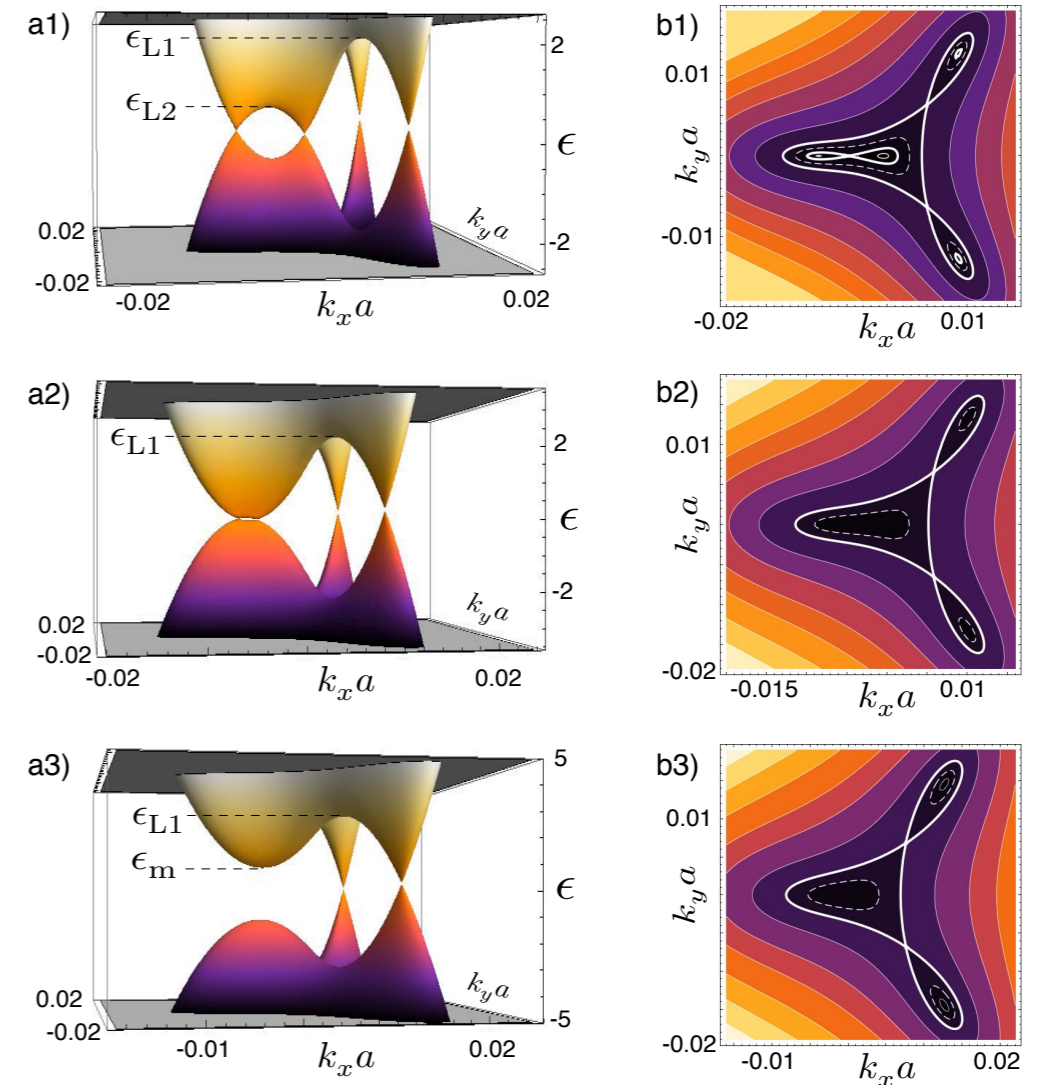
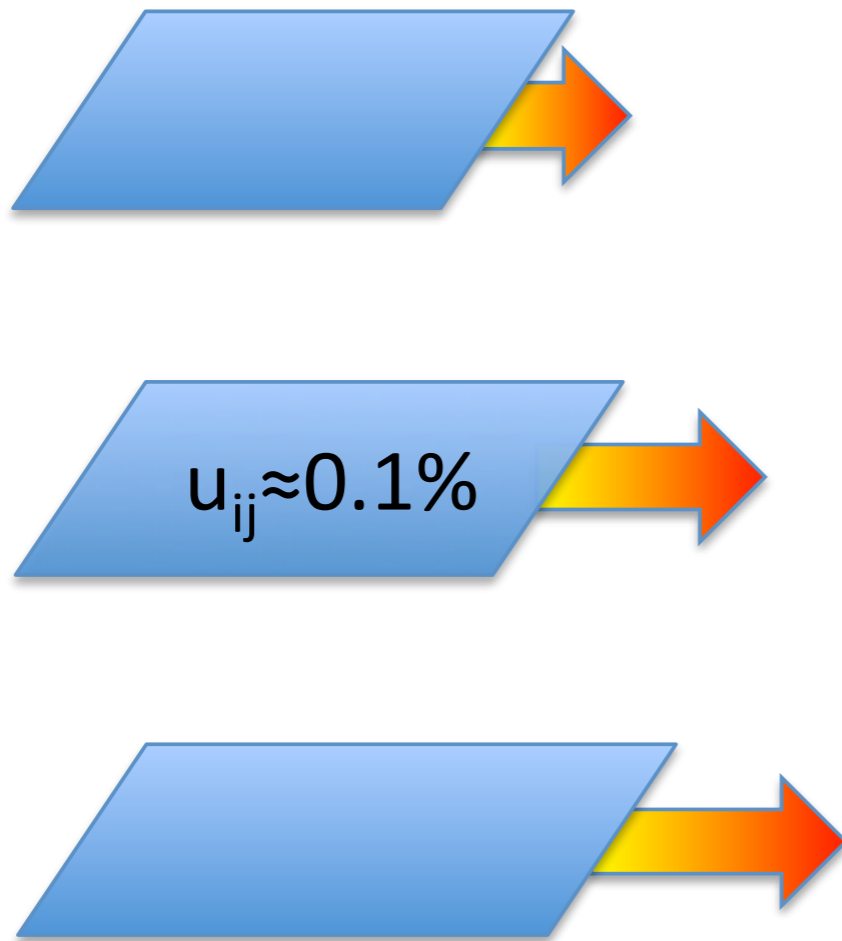


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Uniaxial strain, pure shear and sliding layers

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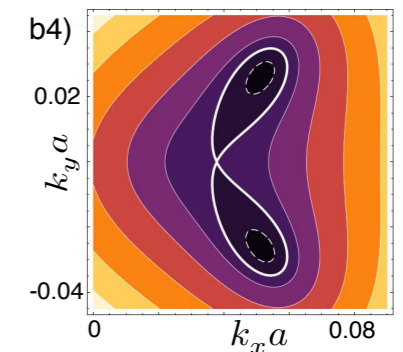
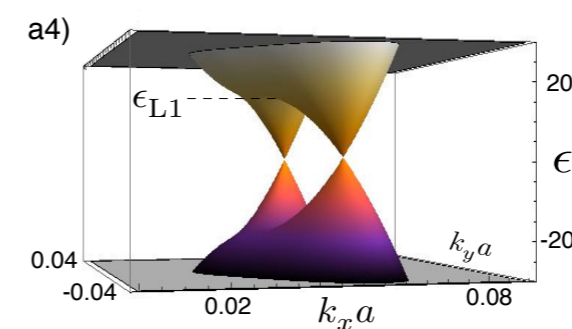
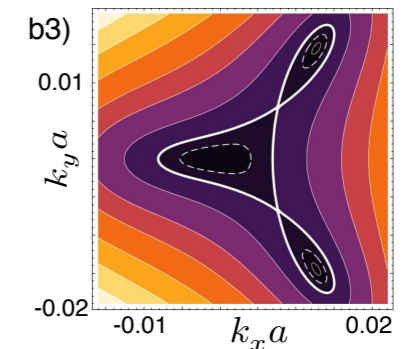
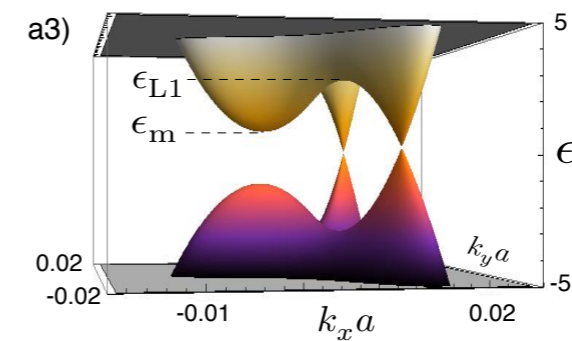
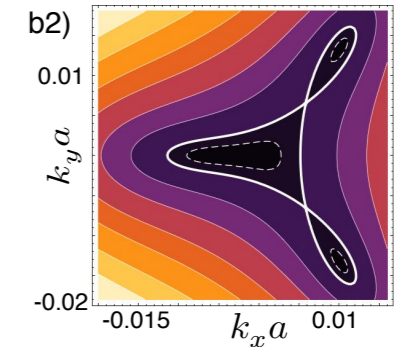
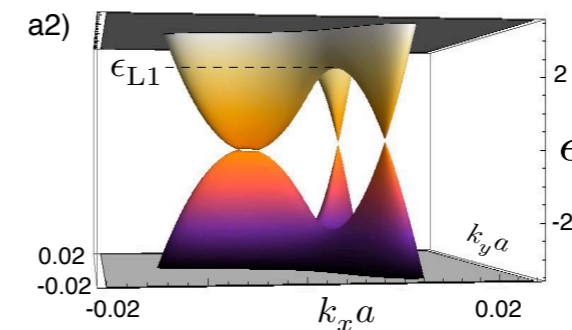
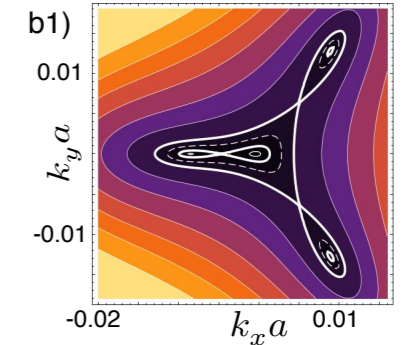
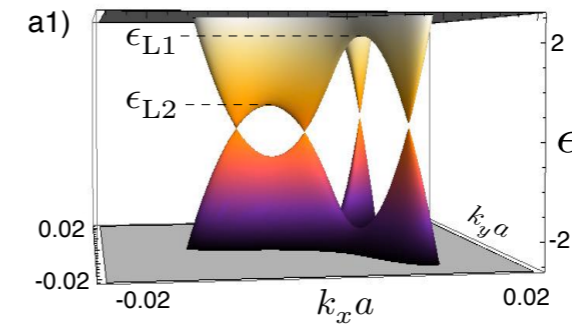
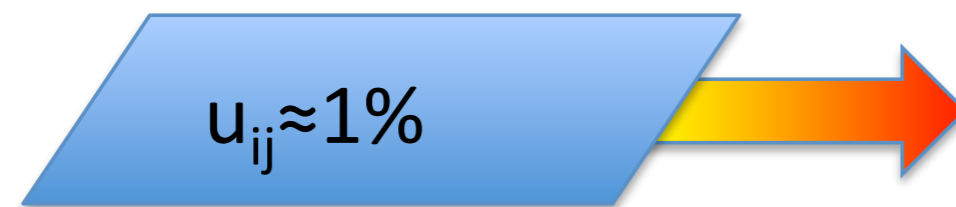


# Uniform gauge fields and the Lifshitz transition

Uniaxial strain, pure shear and sliding layers

J.W. Son, PRB 2011

M. Mucha-Kruczynski PRB 2011



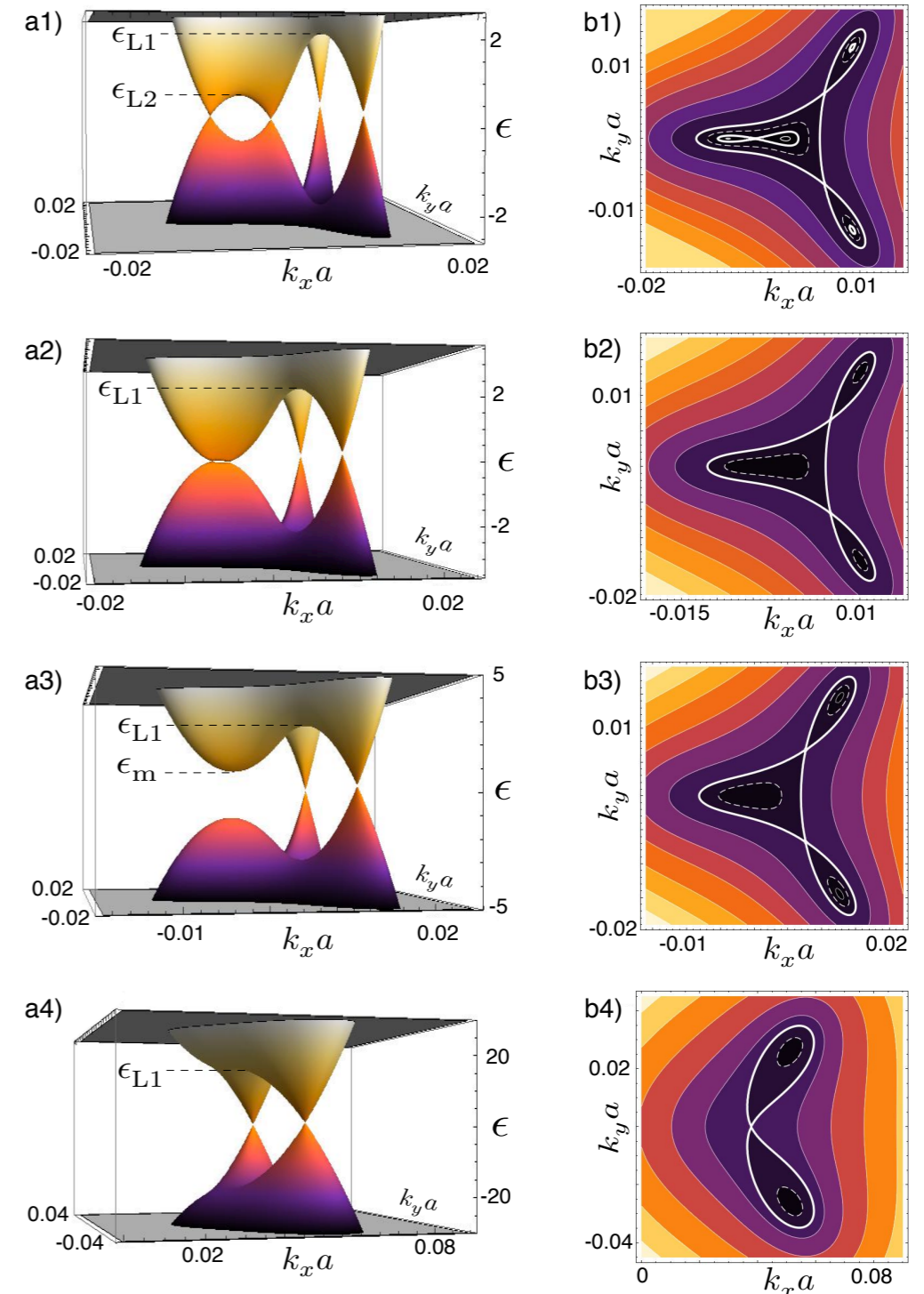
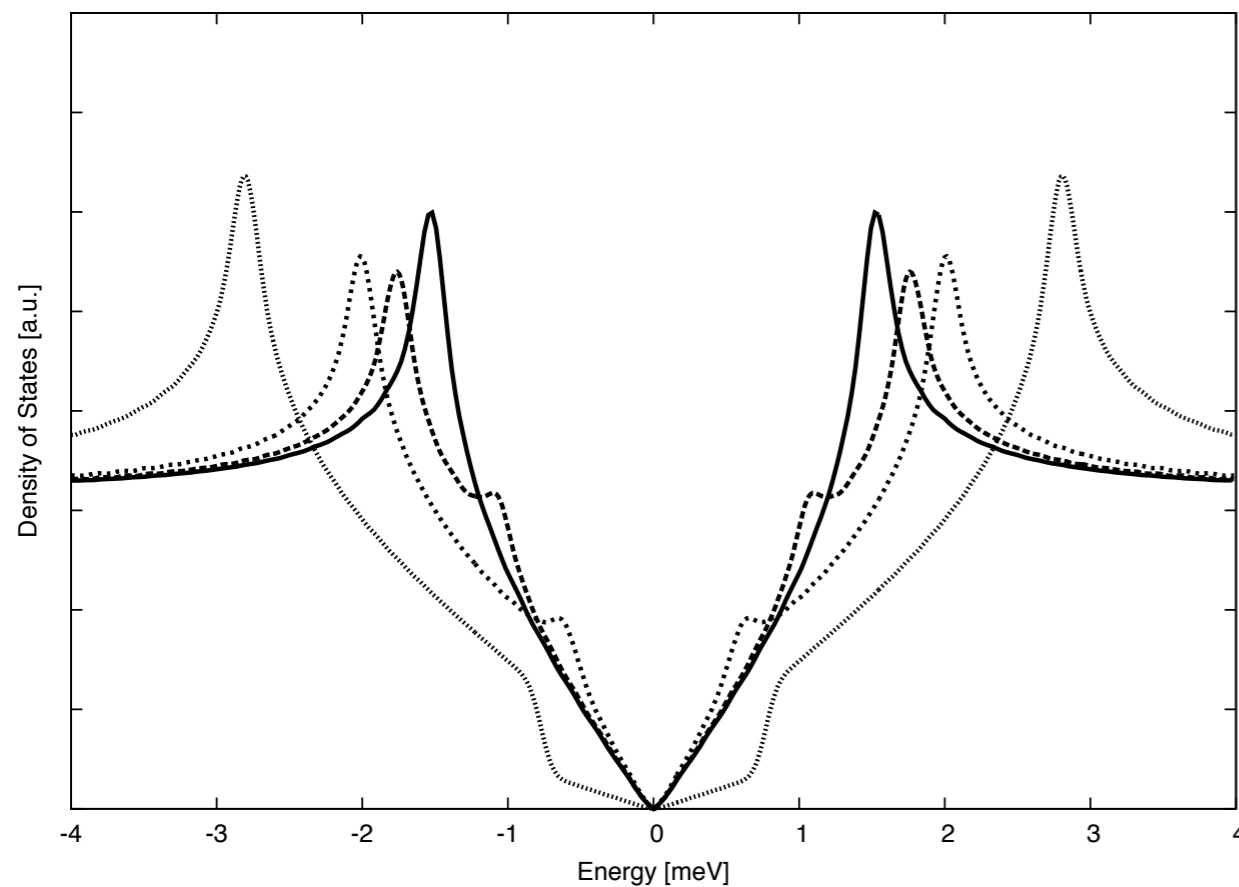
# Uniform gauge fields and the Lifshitz transition

Uniaxial strain, pure shear and sliding layers

J.W. Son, PRB 2011

M. Mucha-Kruczynski PRB 2011

- Dramatic changes in the bandstructure
- Dirac points drift with strain – annihilation of two Dirac points!
- Tuneable Lifshitz transition



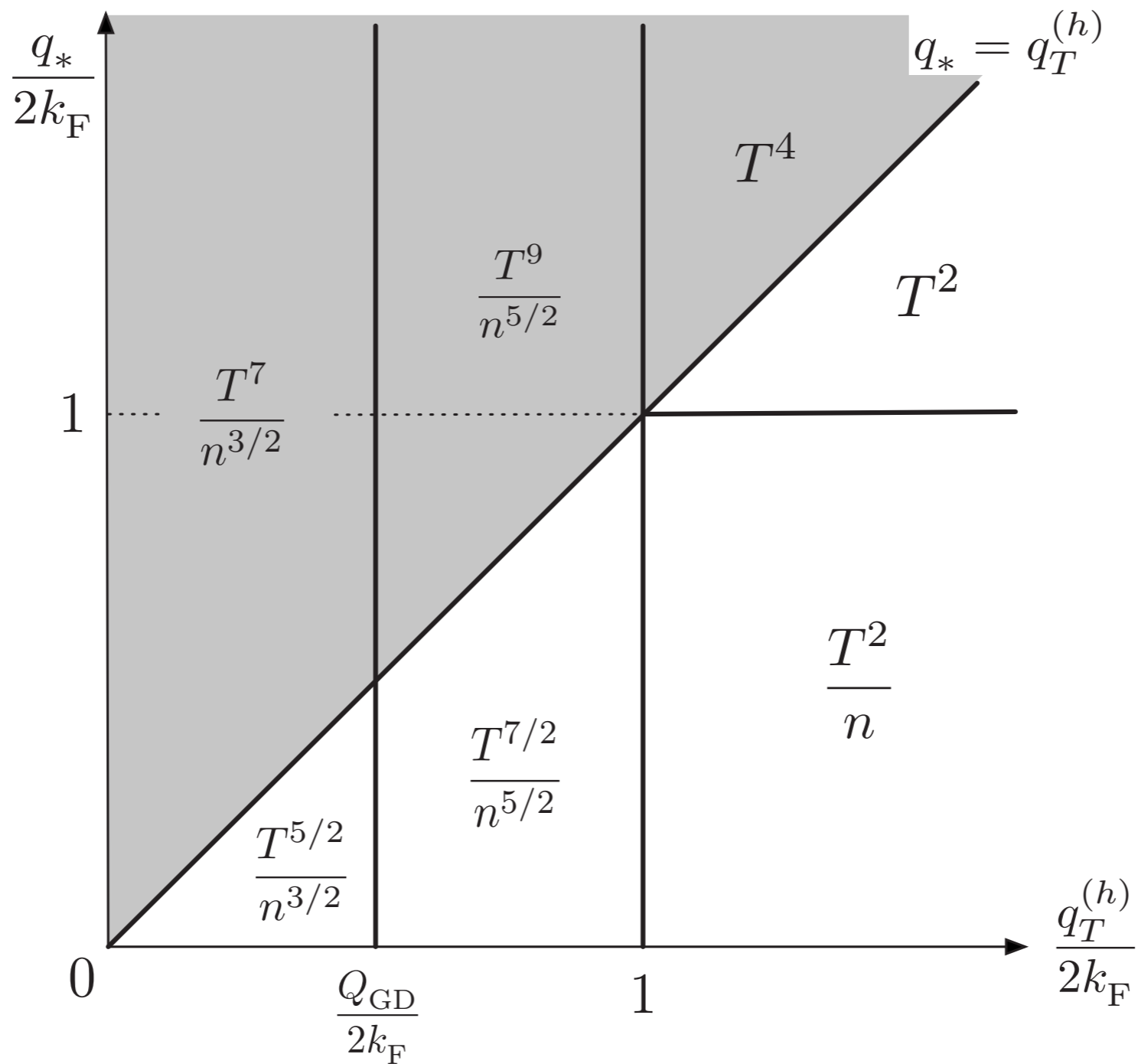


FIG. 2. The dependence of the resistivity due to scattering off flexural modes on temperature  $T$  and electron density  $n$ . The gray area identifies the region  $q_* > q_T^{(h)}$  where the relevant flexural phonon dispersion is dominated by tension and  $\omega_{\mathbf{q}}^{(h)} \simeq \alpha q$ .

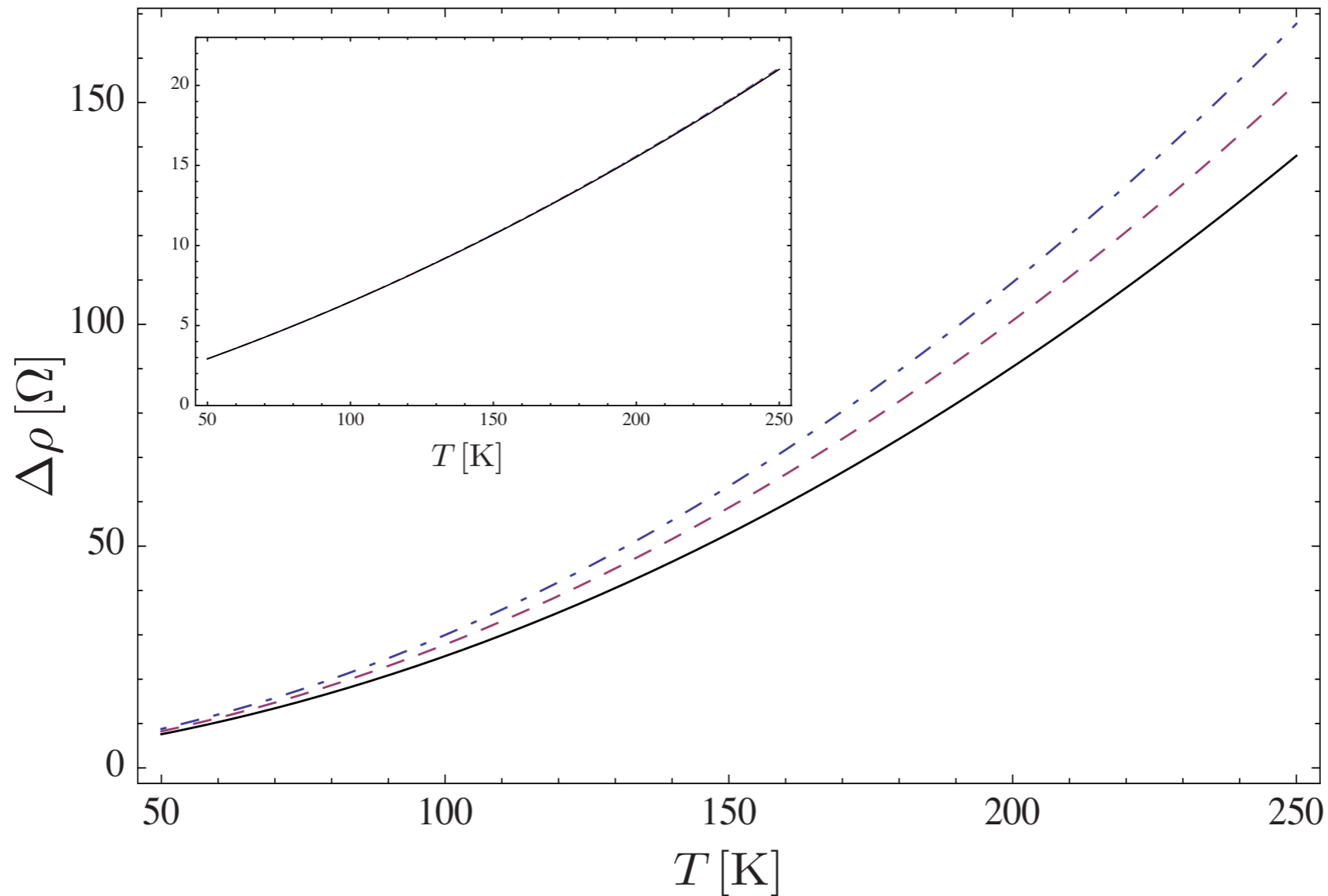


FIG. 3. (Color online) The combined contributions to the resistivity due to in-plane and flexural-phonons  $\Delta\rho$  as a function of the temperature  $T$  for three different electron densities  $\tilde{n} = 0.05, 0.15, 0.3$  (dashed-dotted, dashed, and continuous line, respectively). Here we assume a tension  $\tilde{\gamma} = 1$  and a deformation potential coupling  $\tilde{g}_1 = 10$ . Inset: same plot as in the main figure but for stronger tension  $\tilde{\gamma} = 20$ . Notice the almost perfect linear- $T$  scaling, independent of density.