GENERAL RELATIVISTIC MAGNETOHYDRODYNAMICS

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OUTLINE

1. Astrophysical Motivation

2. Introduction to GRMHD

3. Numerical Methods

4. Accreting Black Hole

5. Conclusions

Astrophysical Motivation

Astrophysical plasmas are commonly magnetized, with

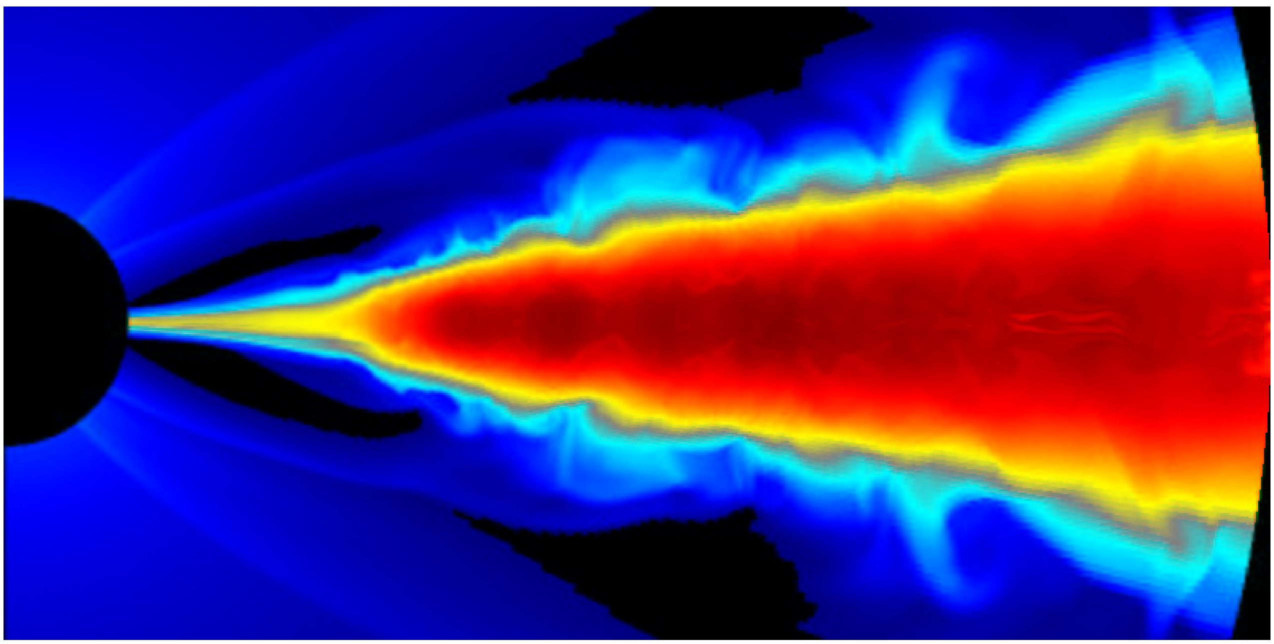
- $\rho v^2/2 \sim B^2/8\pi$
- MHD approximation

Broad class of luminous astrophysical sources likely accreting black holes

- galactic microquasars
- active galactic nuclei
- gamma-ray bursts

Dynamics of accretion flows controlled by magnetic fields

- Balbus-Hawley instability
- Disk winds
- Blandford-Znajek effect



Thin disk simulation

Note event horizon, plunging region, centrifugally supported disk

GRMHD Equations

Particle number conservation:

$$\partial_t(\sqrt{-g}\,\rho u^t) = -\partial_i(\sqrt{-g}\,\rho u^i) \qquad \partial_t\rho = -\nabla\cdot(\rho\mathbf{v})$$

Ideal MHD:

$$u_\mu F^{\mu\nu} = 0 \qquad \mathbf{E} + \mathbf{v}\times\mathbf{B}/c = 0$$

Momentum and energy conservation:

$$\partial_t\left(\sqrt{-g}\,T^t{}_\nu\right) = -\partial_i\left(\sqrt{-g}\,T^i{}_\nu\right) + \sqrt{-g}T^\kappa{}_\lambda\Gamma^\lambda{}_{\nu\kappa}$$

$$\partial_t(\rho\mathbf{v}) = -\nabla\cdot\mathbf{T} - \rho\nabla\phi$$

$$T_{\mu\nu} = (\rho + u + p + \frac{1}{4\pi}b^2)u_\mu u_\nu + (p + \frac{1}{8\pi}b^2)g_{\mu\nu} - \frac{1}{4\pi}b_\mu b_\nu$$

$$b^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\kappa\lambda}u_\nu F_{\lambda\kappa} \qquad B^i \equiv F^{*it}$$

$$T_{ij} = \rho v_i v_j + (p + \frac{1}{8\pi}B^2)\delta_{ij} - \frac{1}{4\pi}B_i B_j$$

Induction equation:

$$\partial_t(\sqrt{-g}B^i) = -\partial_j(\sqrt{-g}(b^j u^i - b^i u^j)) \qquad \partial_t\mathbf{B} = \nabla\times(\mathbf{v}\times\mathbf{B})$$

$$\partial_t\mathbf{B} = -\nabla(\mathbf{vB} - \mathbf{Bv})$$

No monopoles constraint:

$$\partial_i(\sqrt{-g}B^i) = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

Free Oscillations

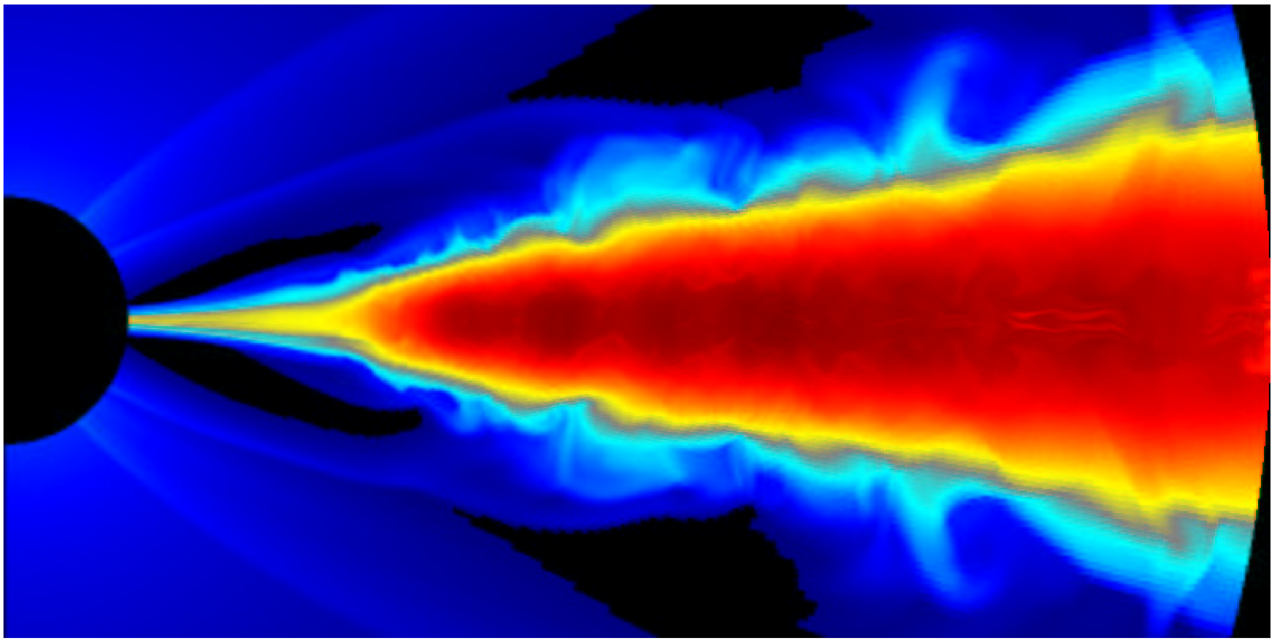
Nonrelativistic MHD supports 8 modes:

ω^2	entropy, monopole
$\times[\omega^2 - (\mathbf{k}\cdot\mathbf{V}_\mathsf{A})^2]$	Alfven
$\times[\omega^4 - \omega^2 k^2(\mathbf{V}_\mathsf{A}^2 + c_s^2) + k^2 c_s^2 (\mathbf{k}\cdot\mathbf{V}_\mathsf{A})^2]$	slow, fast
$= 0$	$\mathbf{V}_\mathsf{A} \equiv \frac{\mathbf{B}}{\sqrt{4\pi\rho}}$

Relativistic MHD also supports 8 modes:

ω^2	entropy, monopole
$\times[\omega^2 - (\mathbf{k}\cdot\mathbf{V}_\mathsf{A})^2]$	Alfven
$\times[\omega^4 - \omega^2(k^2(\mathbf{V}_\mathsf{A}^2 + c_s^2 - \mathbf{V}_\mathsf{A}^2 c_s^2/c^2) + c_s^2(\mathbf{k}\cdot\mathbf{V}_\mathsf{A})^2/c^2)$	
$+k^2 c_s^2 (\mathbf{k}\cdot\mathbf{V}_\mathsf{A})^2]$	slow, fast
$= 0$	$\mathbf{V}_\mathsf{A} \equiv \frac{\mathbf{b}}{\sqrt{4\pi(\rho+u+p+b^2/(4\pi))}}$

Inflow Solution



Quasi-analytic model for flow in plunging region.

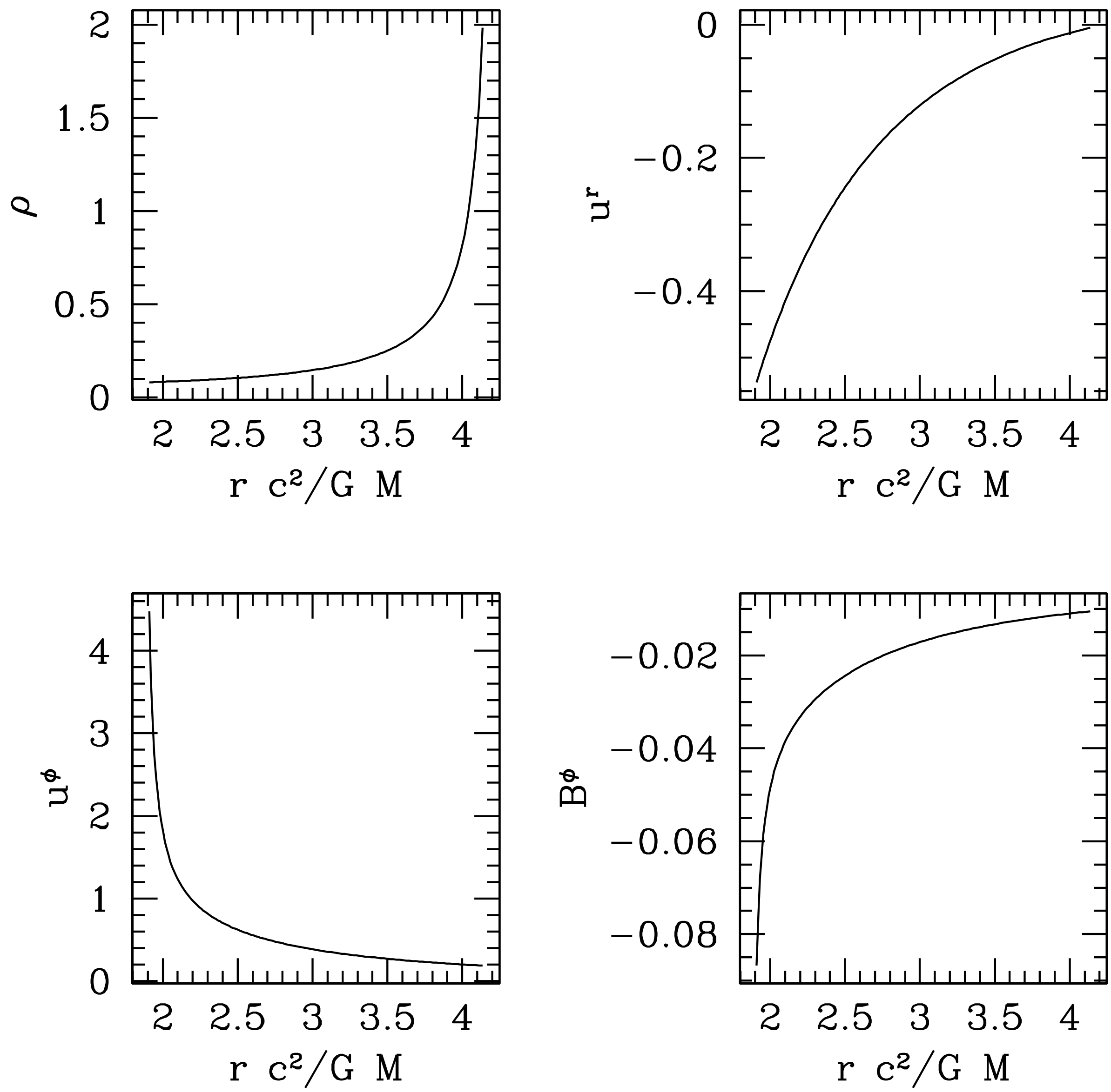
Assume flow is:
stationary
close to equatorial plane
along lines of constant θ
cold (zero pressure)

Fully integrable

Find fast critical point, solve.

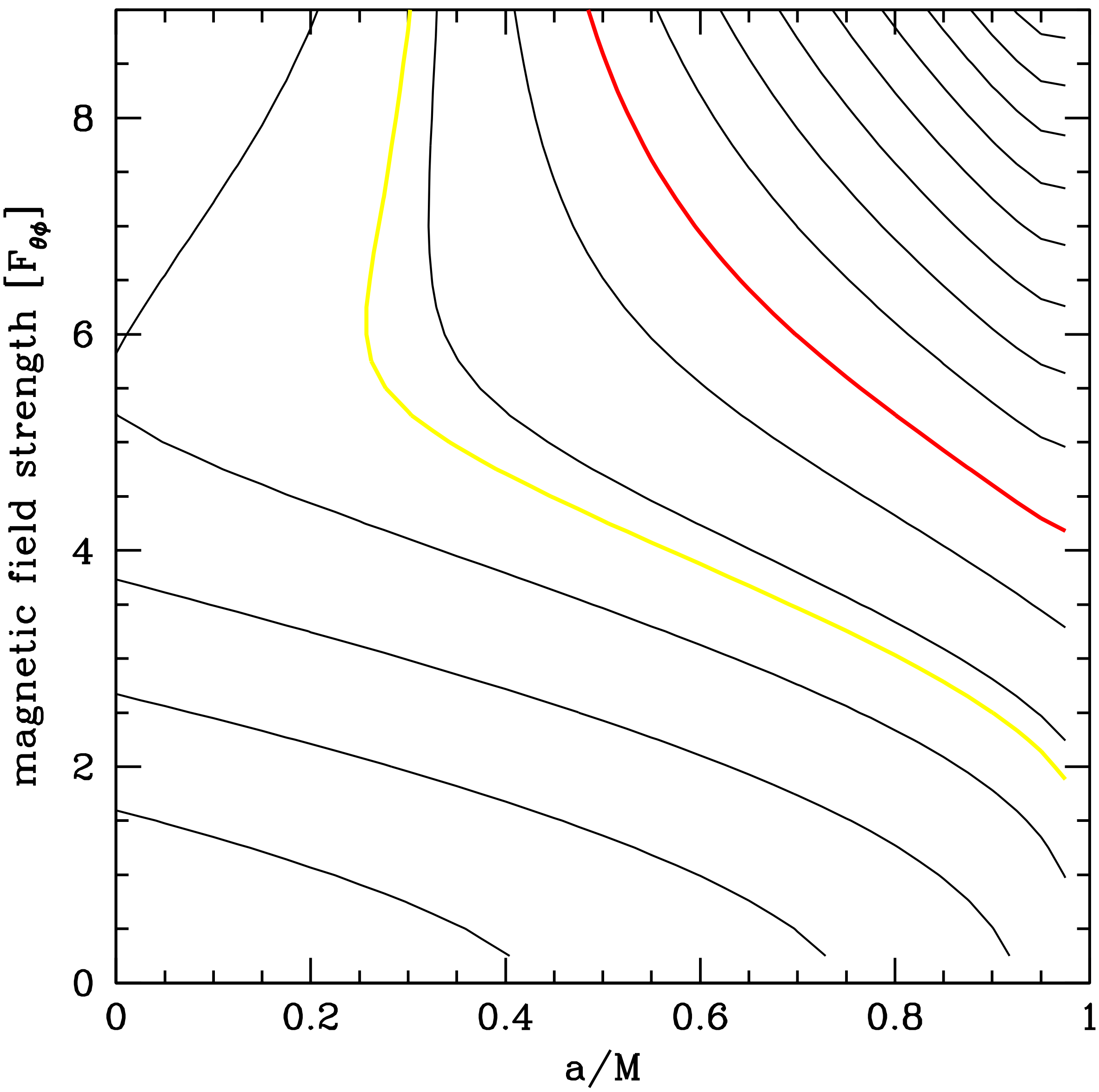
Analogous to Weber-Davis model for solar wind.

Gammie (1999), Takahashi et al. (1990), Li (2003)



Magnetized, equatorial inflow for $a/M = 0.5$

(Boyer-Lindquist coordinates)



specific angular momentum of inflow solutions

Algorithm: HARM

Physics

geometry described by line element

ideal fluid dynamics

magnetohydrodynamics

No cooling

Algorithm

conservative, shock-capturing (HLL solver)

zone-centered

constrained transport, $\nabla \cdot \mathbf{B} = 0$ to machine precision

second order on smooth flows

54K zone cycles/second on 2.4GHz PIV

Gammie, McKinney, Tóth (2003)

also DeVilliers & Hawley (2003), Koide et al. (1999)

Code Verification

Nonrelativistic Tests

Ryu & Jones shock tubes

Orszag-Tang vortex

Special Relativistic Tests

Linear modes, up to $b^2/\rho = 10^6$

Komissarov's shocks

Transport

General Relativistic Tests

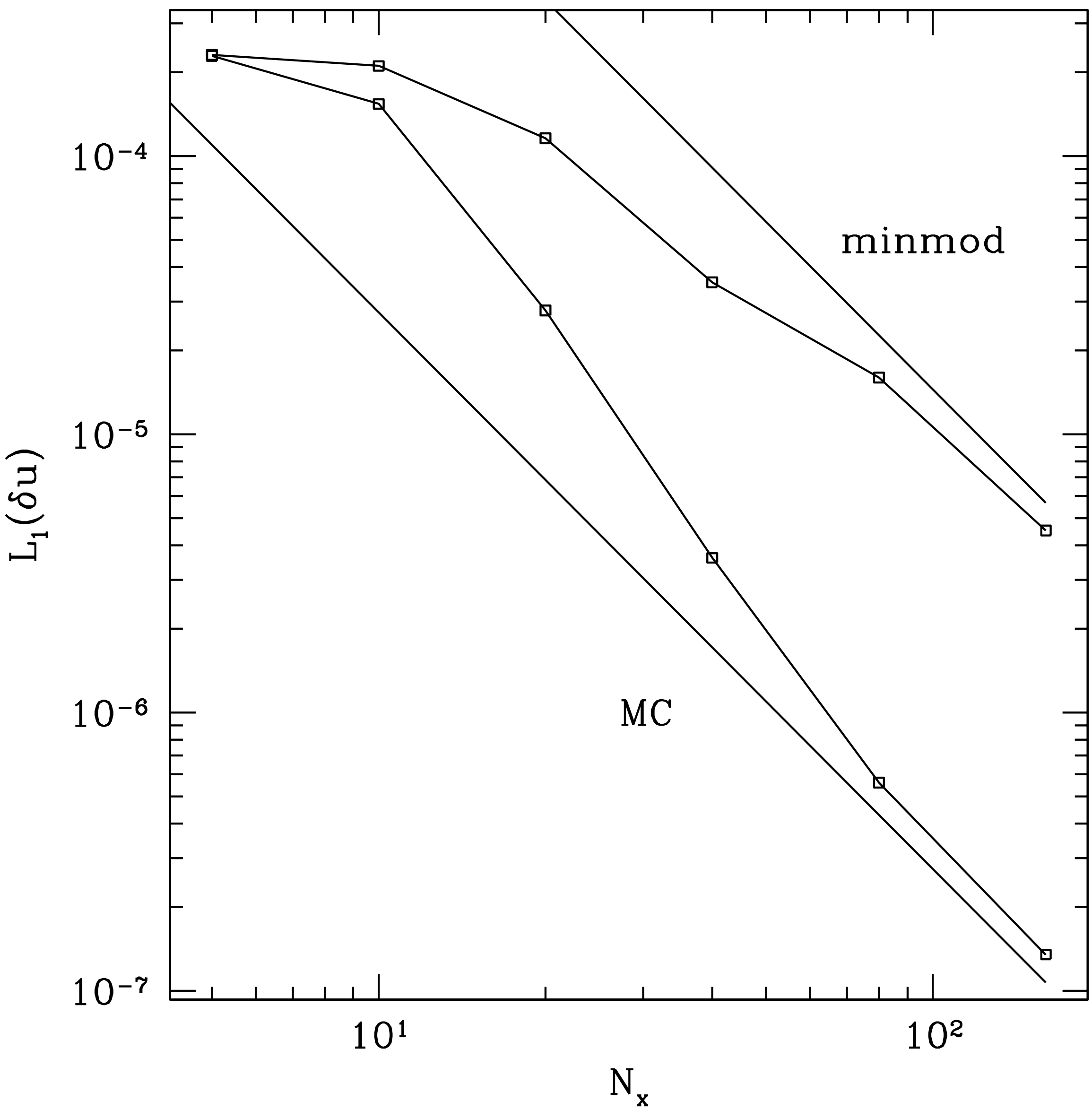
Bondi flow ($a/M = 0$)

Magnetized Bondi flow ($a/M = 0$)

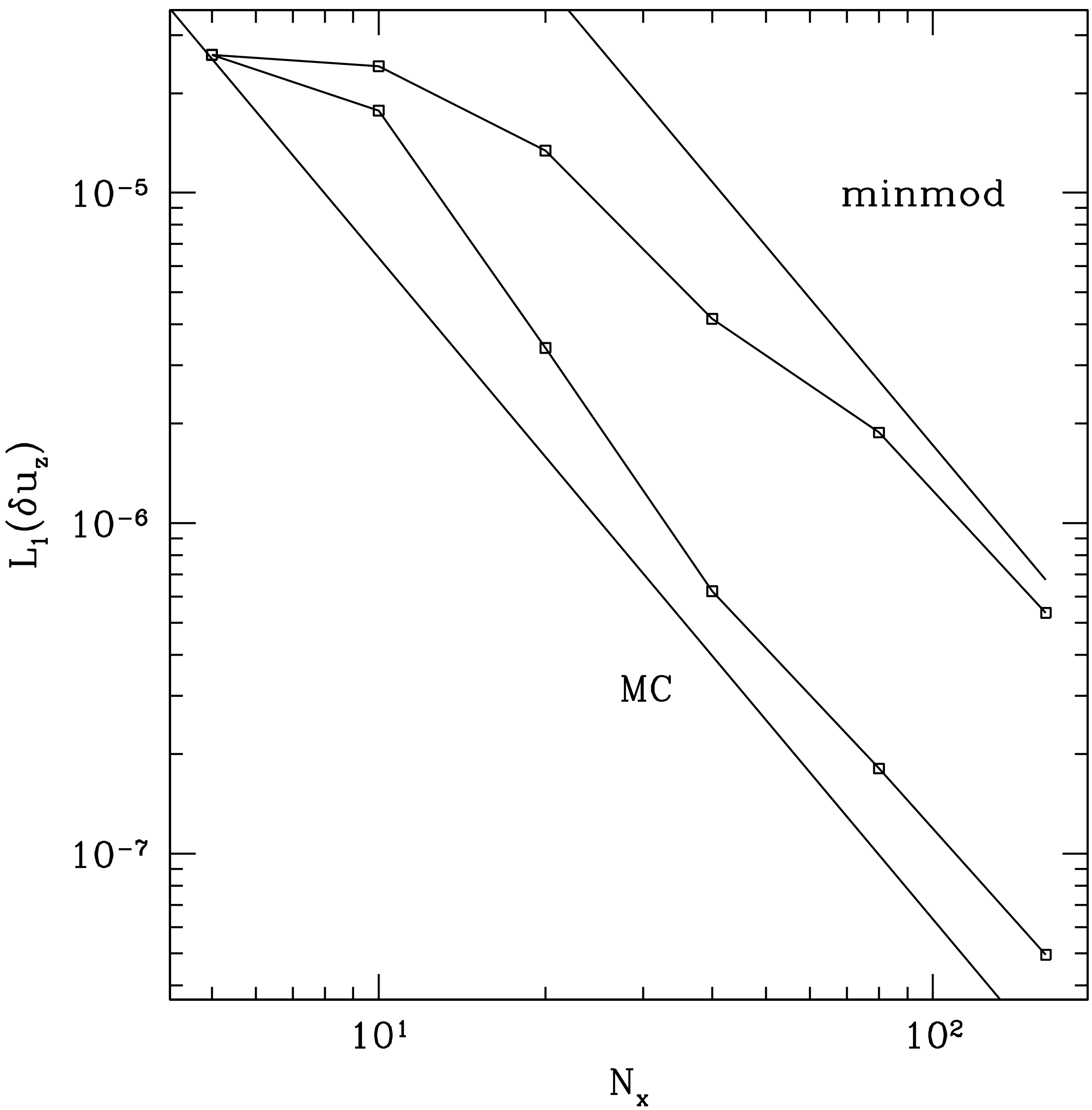
up to $b^2/\rho = 10^3$

Magnetized equatorial inflow ($a/M = 0.5$)

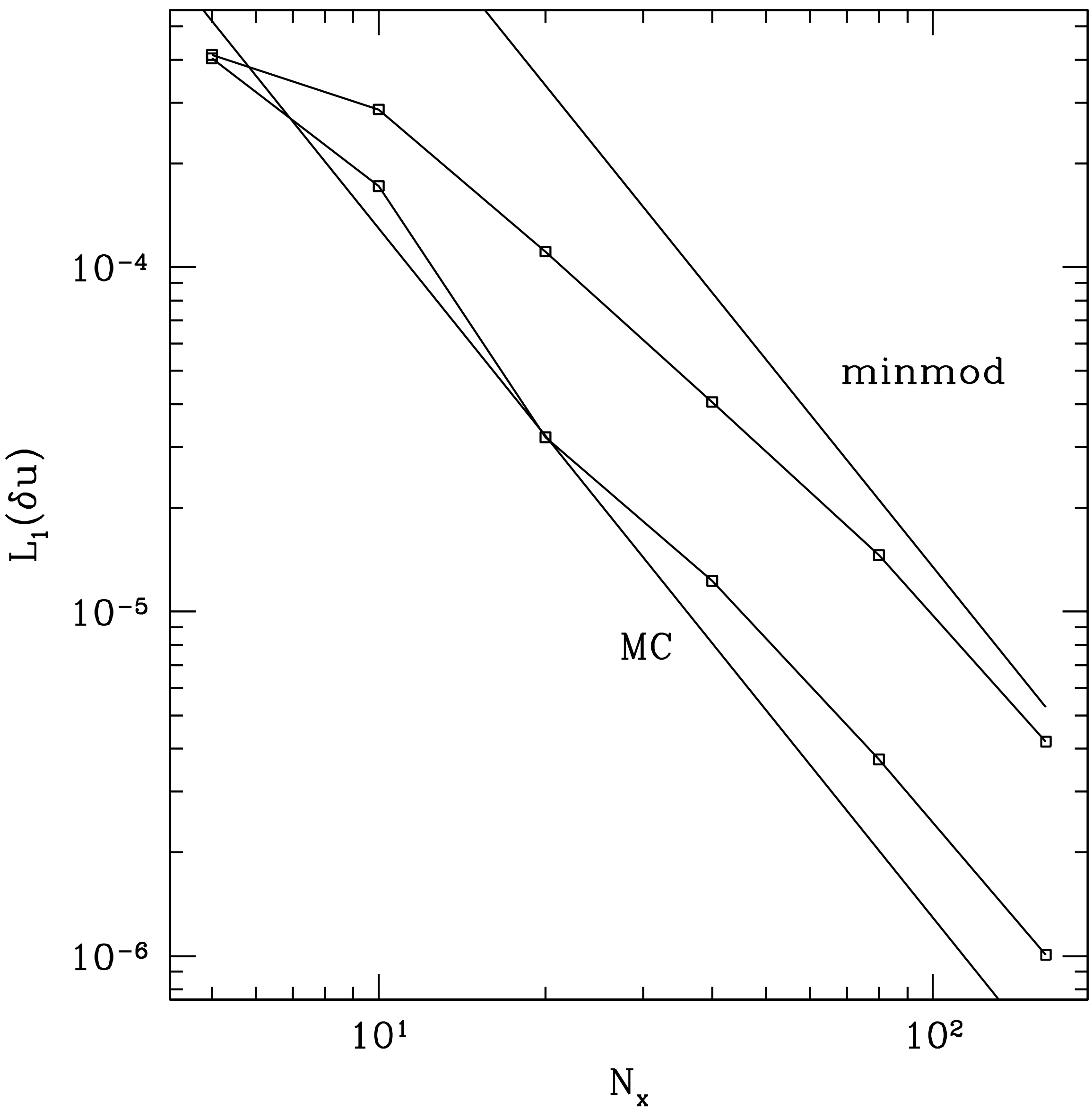
Fishbone-Moncrief torus ($a/M = 0.9$)



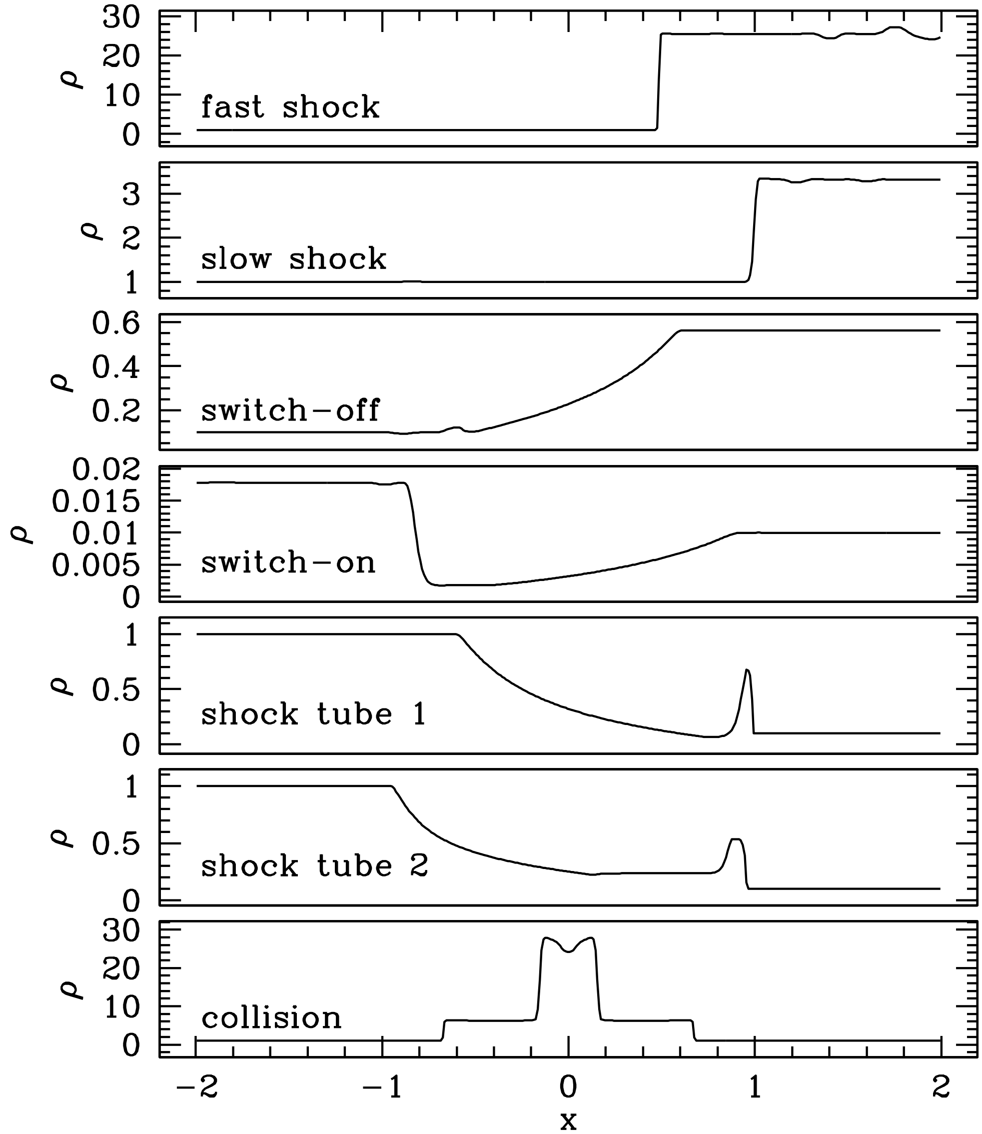
Slow wave convergence



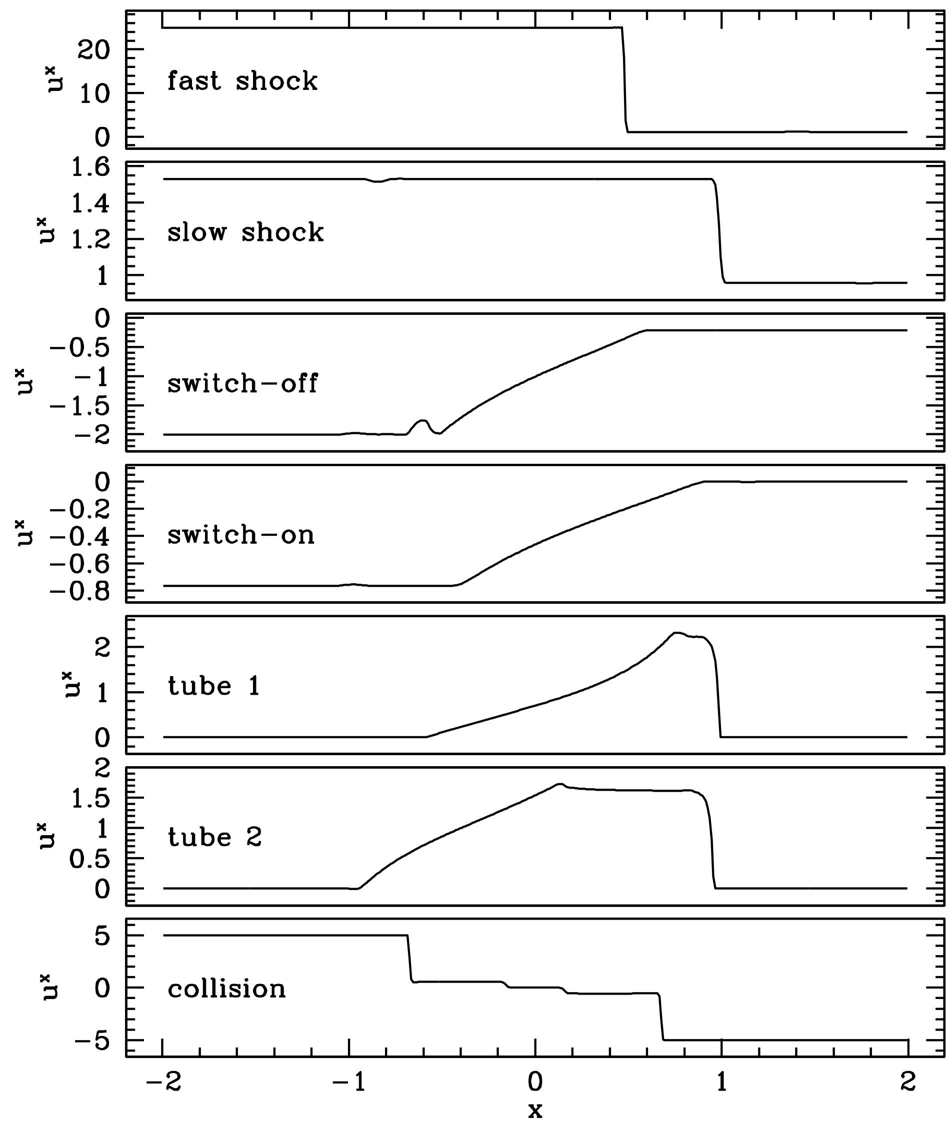
Alfvén wave convergence



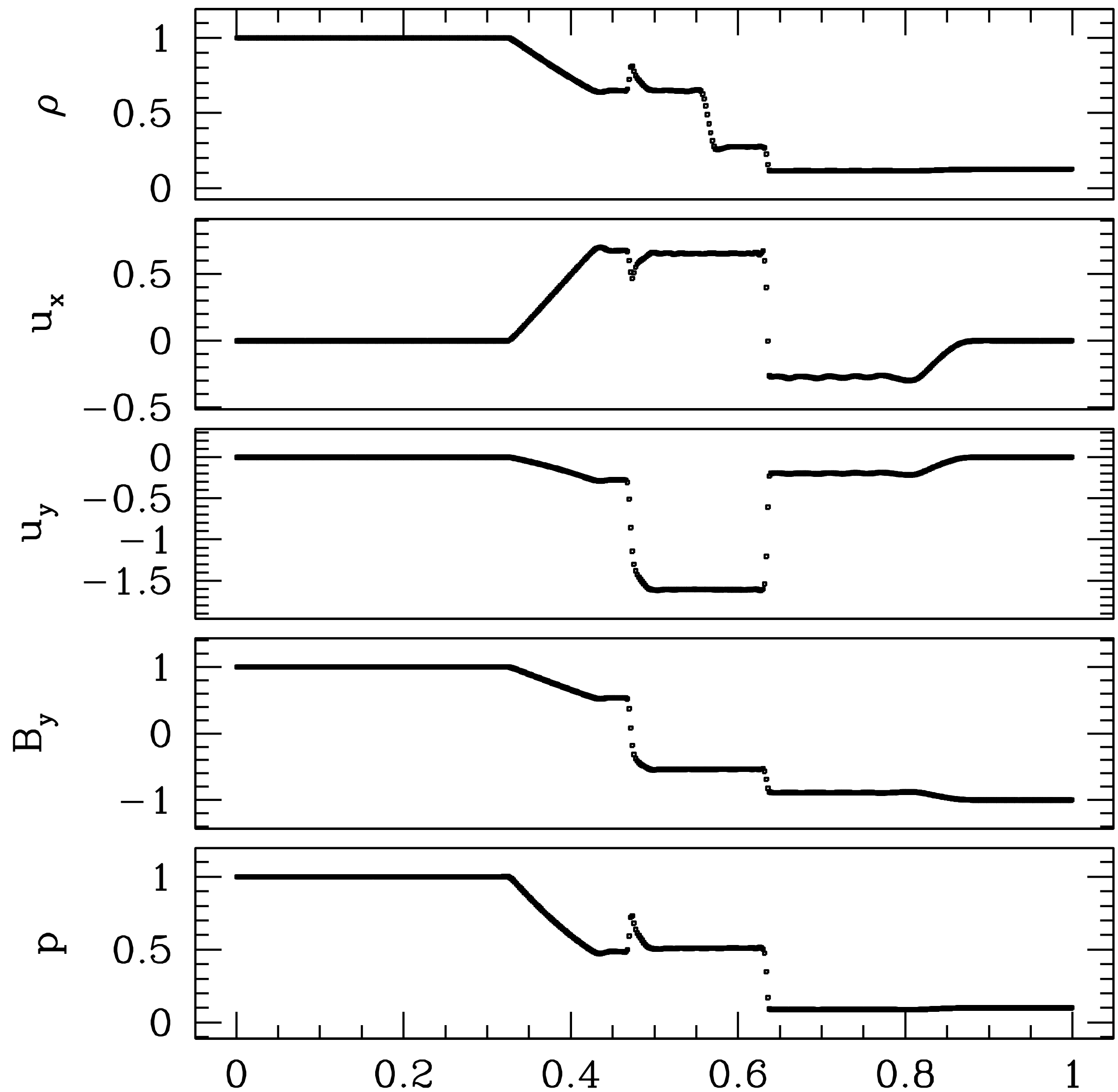
Fast wave convergence



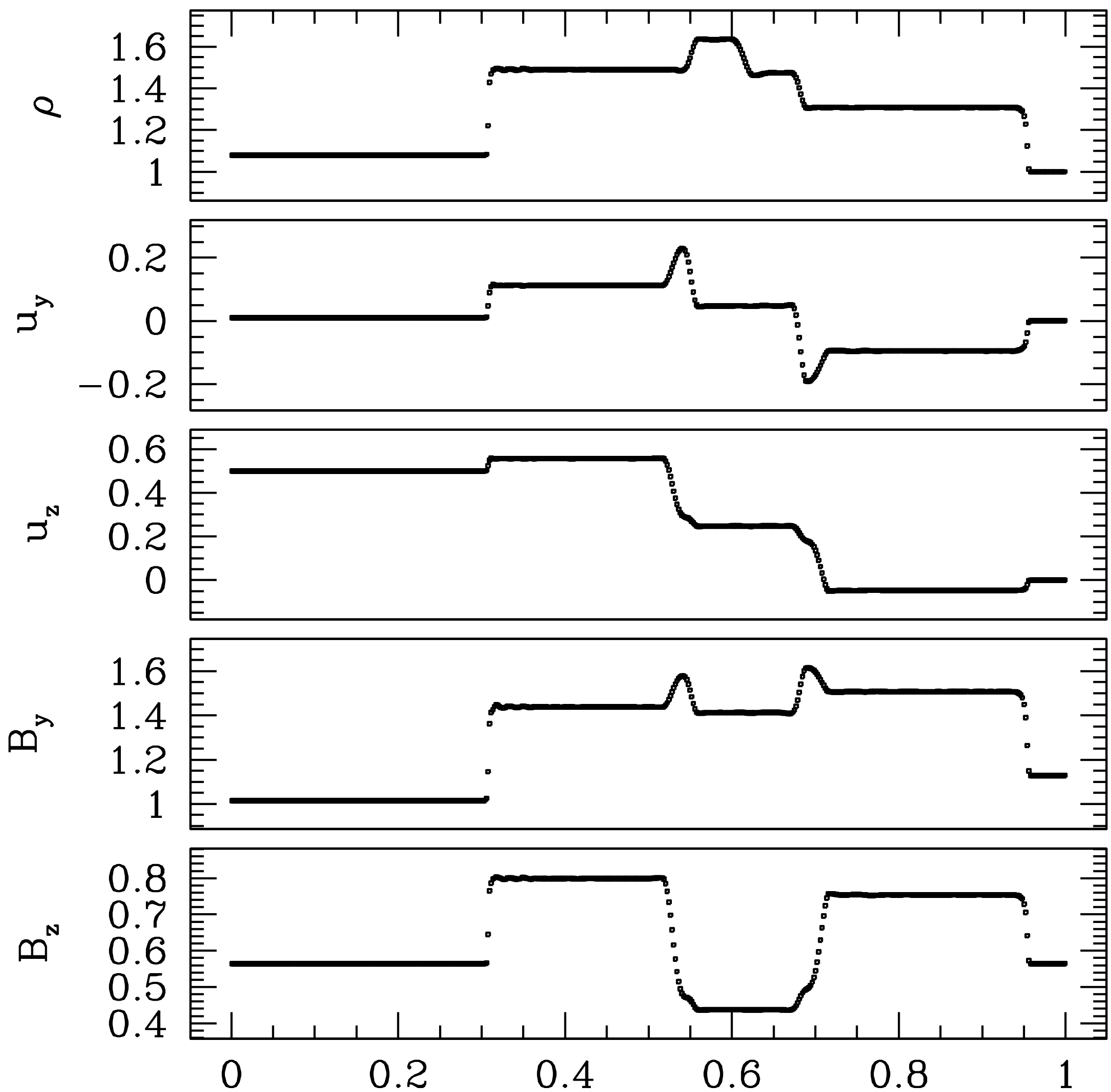
Density for Komissarov's nonlinear waves



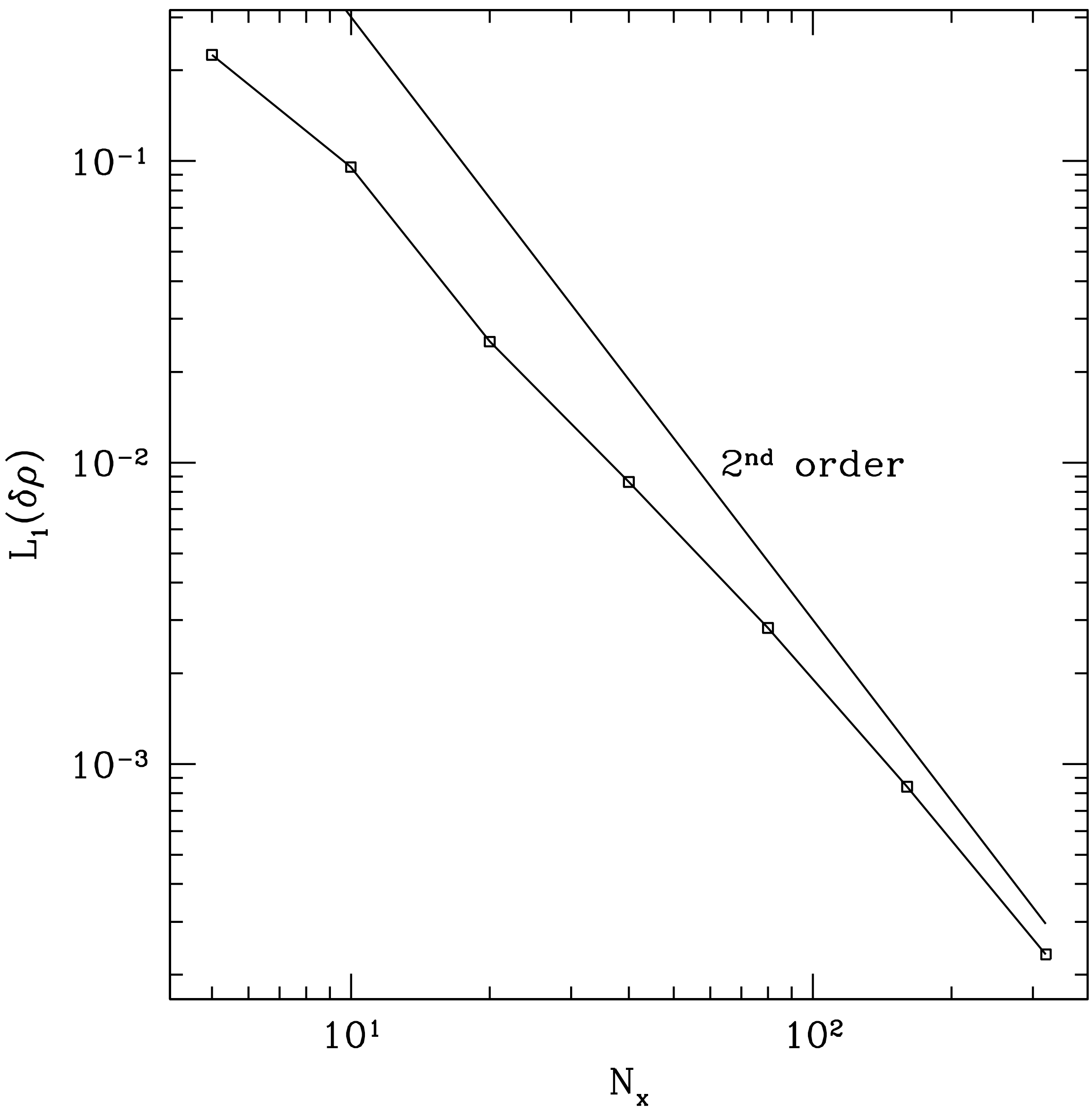
Internal energy for Komissarov's nonlinear waves



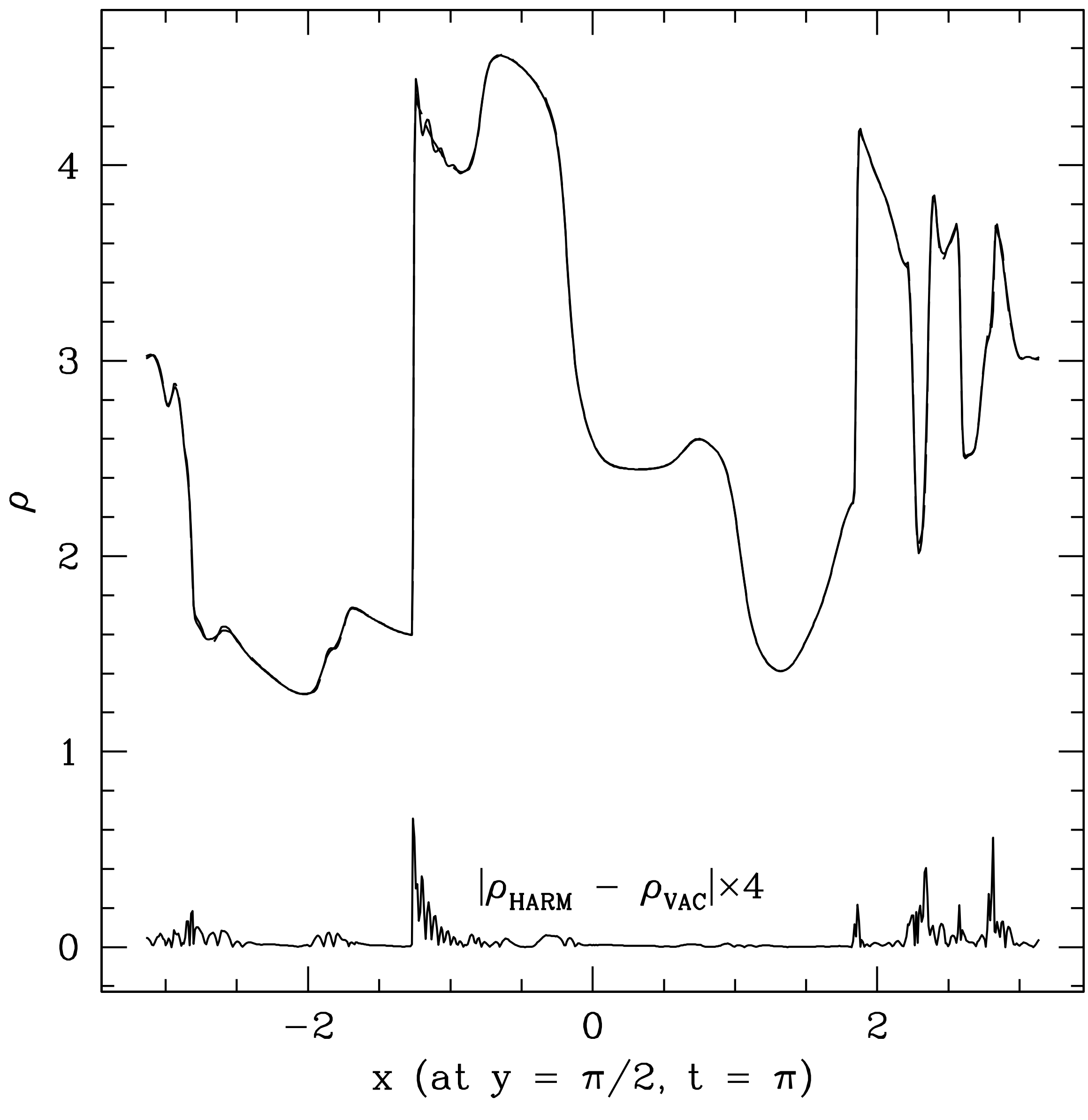
Ryu & Jones test 5A (Brio & Wu) with $c = 100$



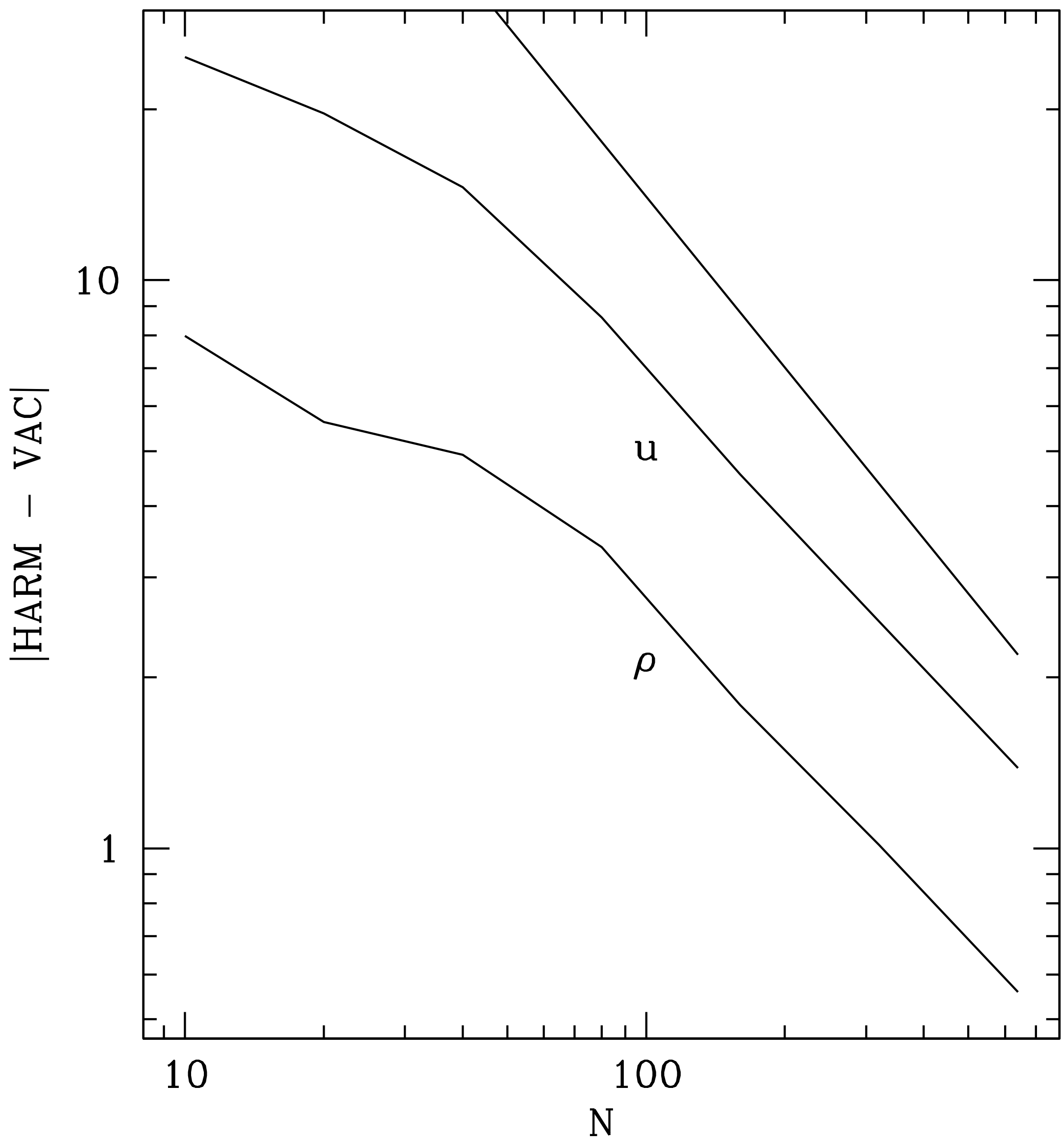
Ryu & Jones test 2A with $c = 100$



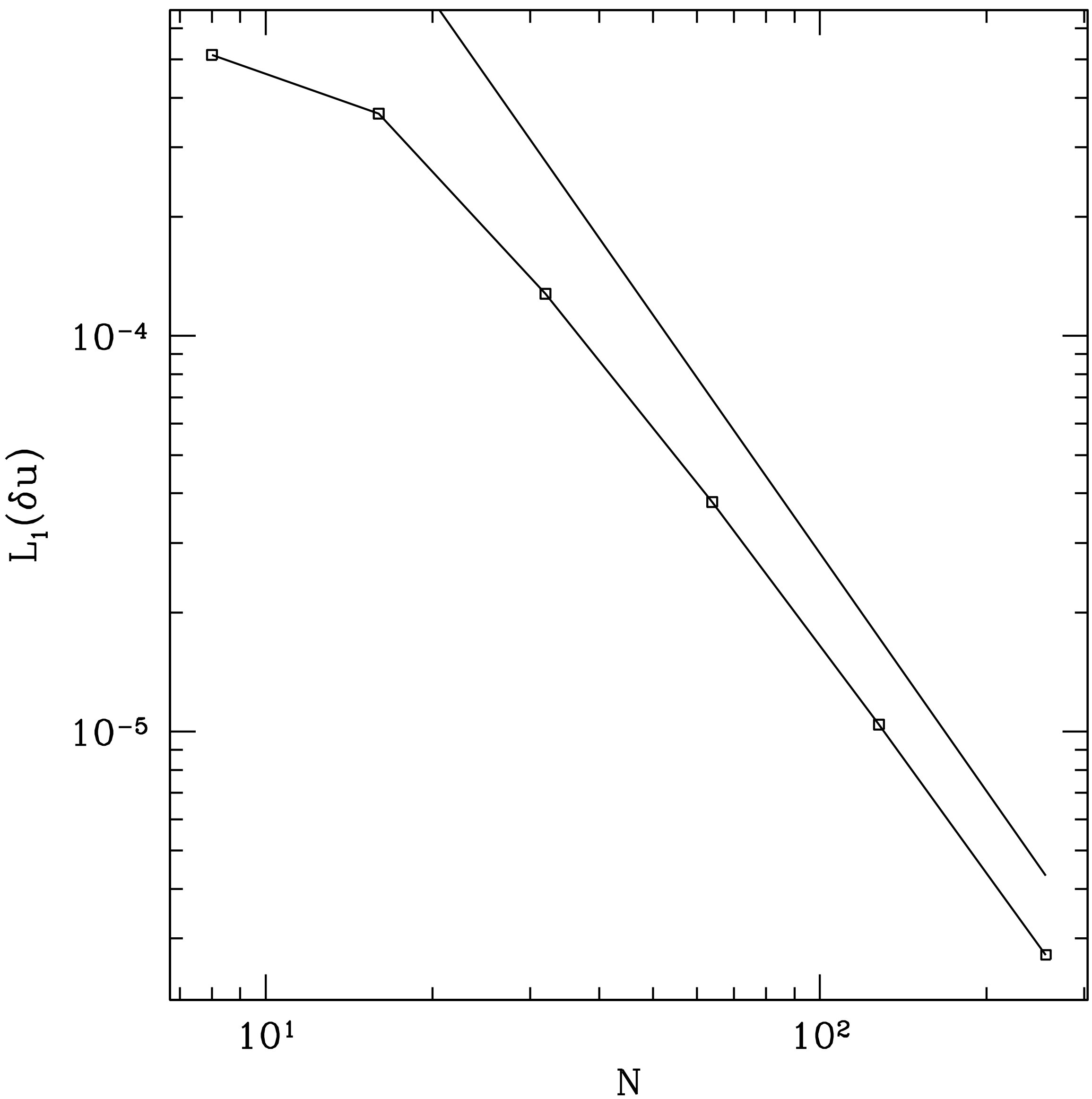
Transport test convergence



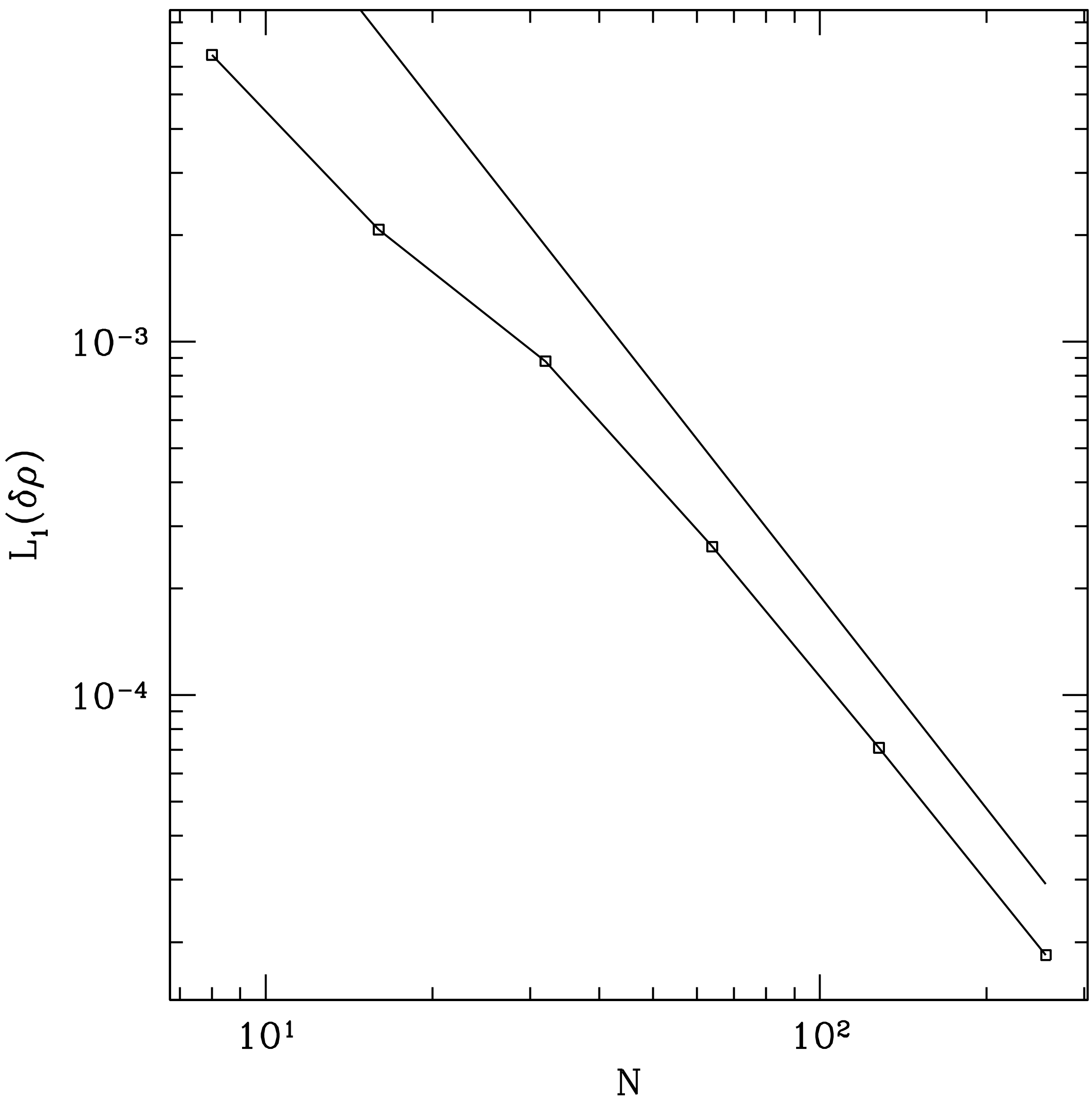
Orszag-Tang vortex, HARM vs. VAC, with $c = 100$



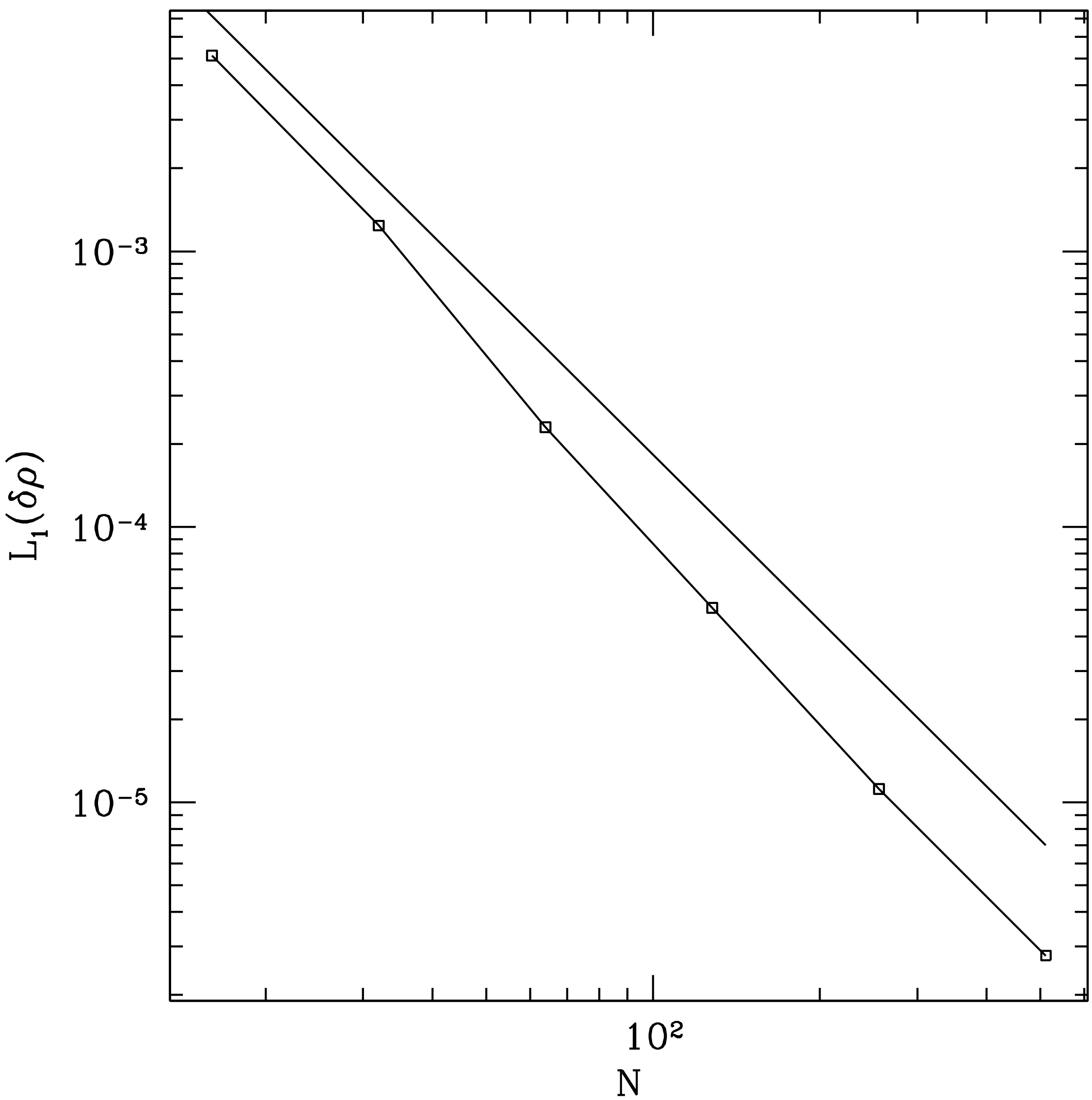
Orszag-Tang vortex, HARM vs. VAC, with $c = 100$



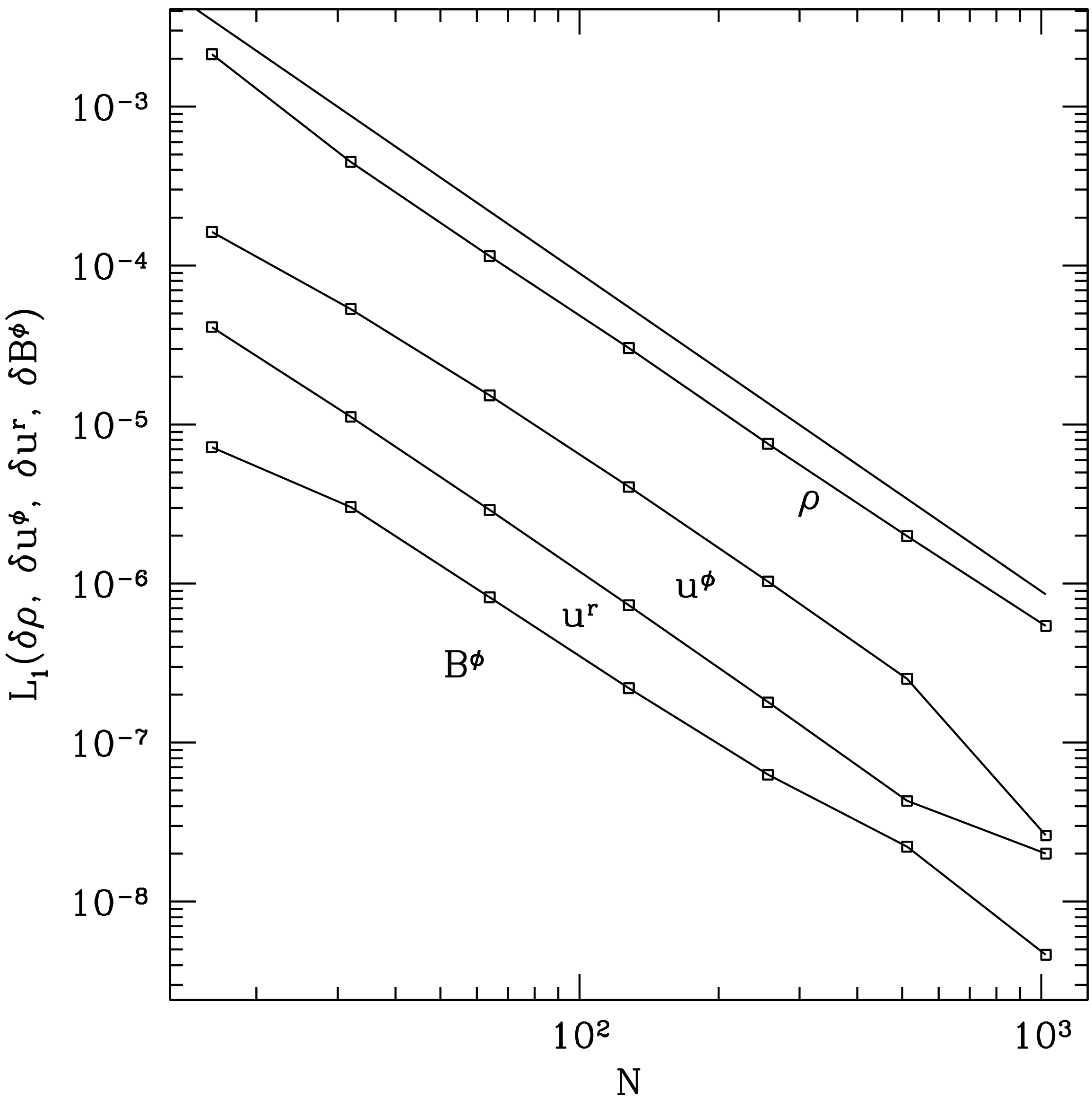
Convergence results for Bondi flow



Convergence results for magnetized Bondi flow



Convergence results for a Fishbone-Moncrief donut in the Kerr metric with $a/M = 0.9$



Convergence results for magnetized equatorial inflow in the Kerr metric with $a/M = 0.5$

Black Hole Accretion

Model

Kerr metric

Kerr-Schild coordinates (reg. on horizon)

$a/M = 0.7$

Fishbone-Moncrief torus

small ($\beta_{min} = 100$) poloidal field

Numerics

r, θ transformed to refine disk

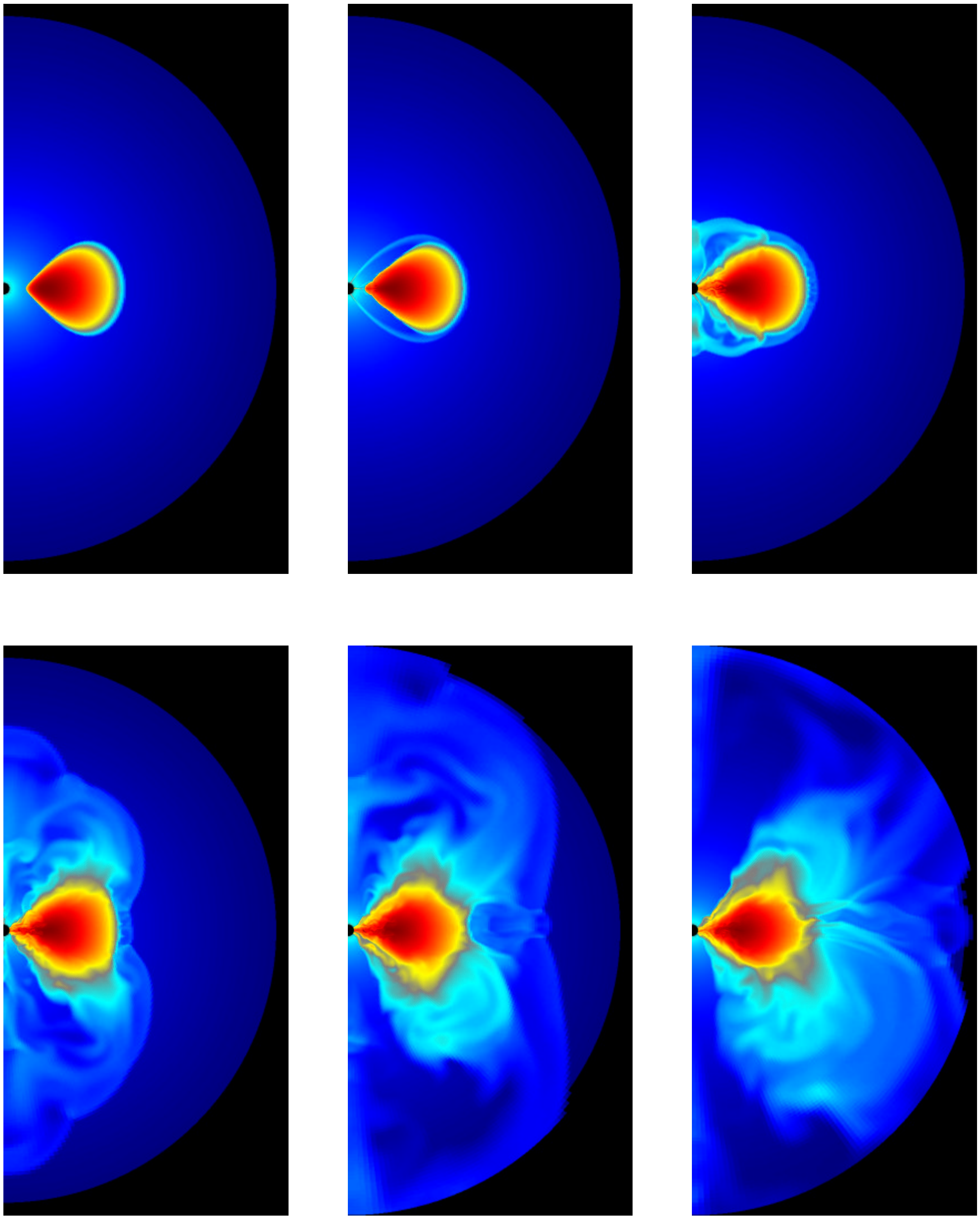
resolution 256^2

$r_{in} = 1.68M$, inside horizon

$r_{out} = 80M$

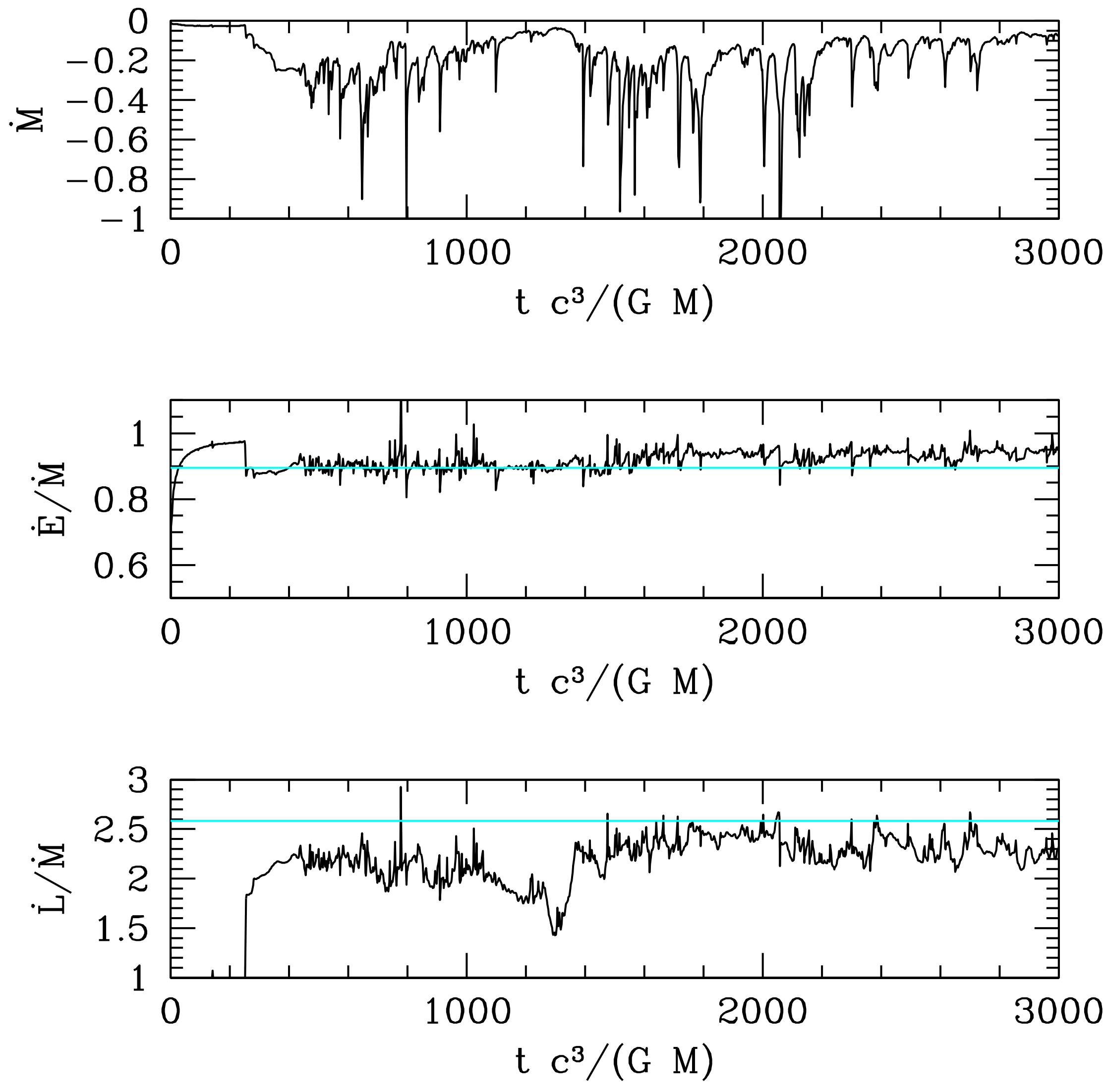
floor in low density regions

evolved for $3000M$

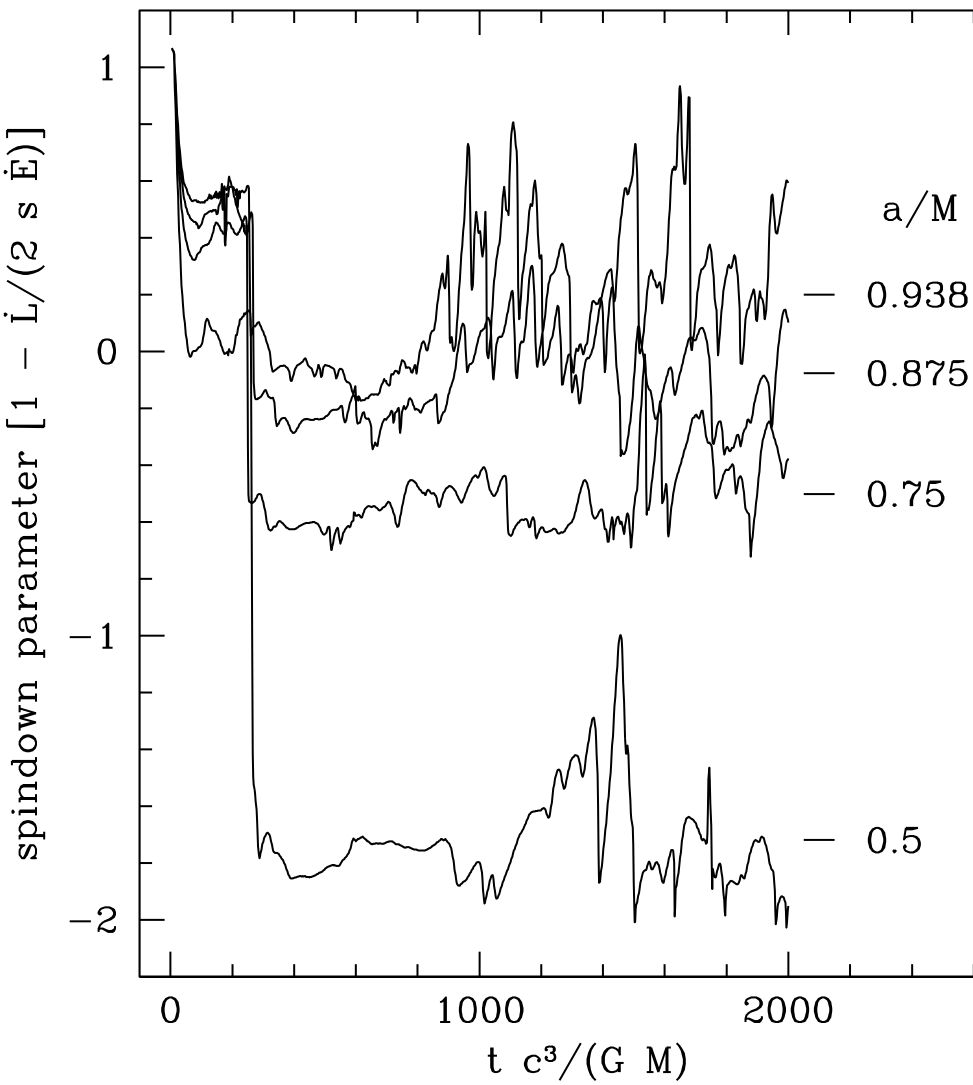


color shows log(density)

Results: Torque on Inner Disk



Spin Equilibrium at $a/M \sim 0.9$



Summary

Code

Fully relativistic MHD code

verified on wide range of problems

numerical difficulties for $b^2/\rho \gg 1$

Spin Equilibrium

$d(a/M)/dt = 0$ at $a/M \sim 0.9$

for thick (high accretion rate) flows

thin flows differ

Future

several GRMHD solvers now exist

measure Blandford-Znajek effect

spacetime evolution with elecromagnetic sources