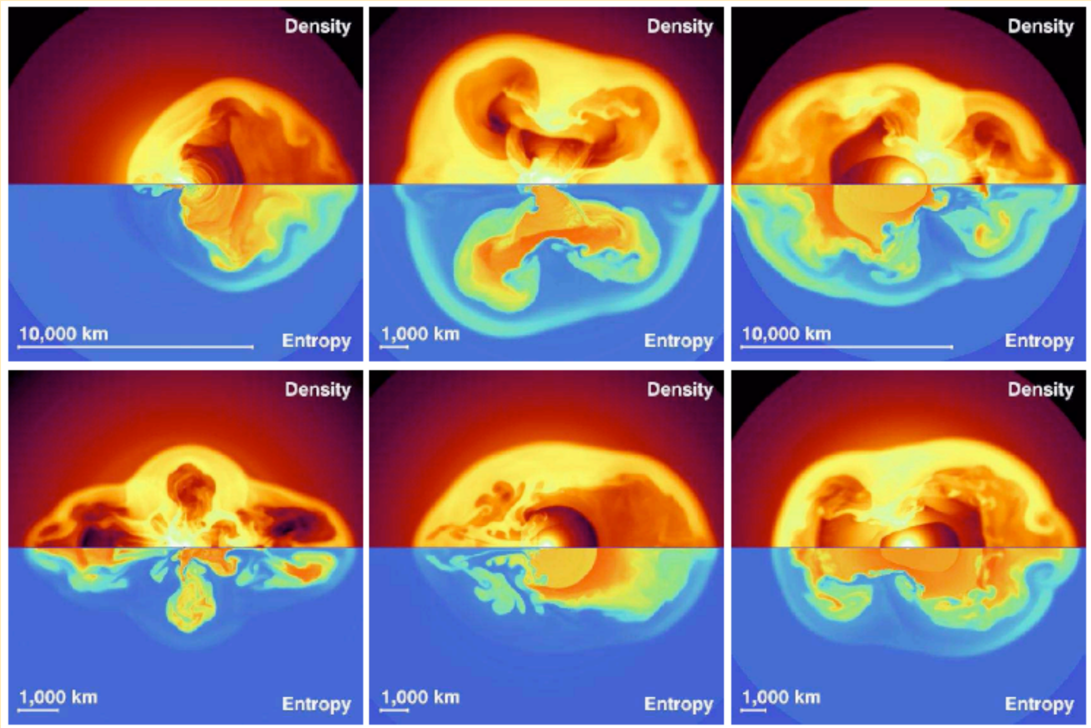
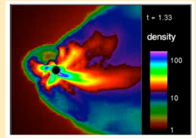
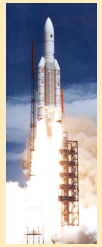
The basics of the Advective-Acoustic Cycle



Janka et al. (2004)





Chanaud & Powell (1965)

Thierry Foglizzo, Pascal Galletti (CEA Saclay)
Thomas Janka, Leonhard Scheck (MPA Garching)

Outline

1. Instability of the stalled accretion shock during core collapse

2. The advective-acoustic cycle: a new instability ?

3. Understanding simple toy models
   - why is there an advective-acoustic coupling ?
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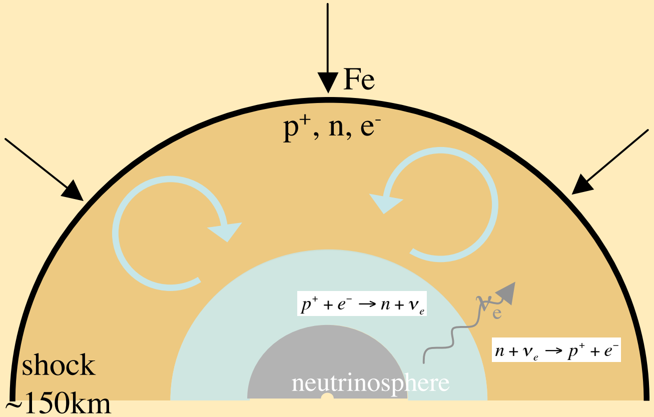
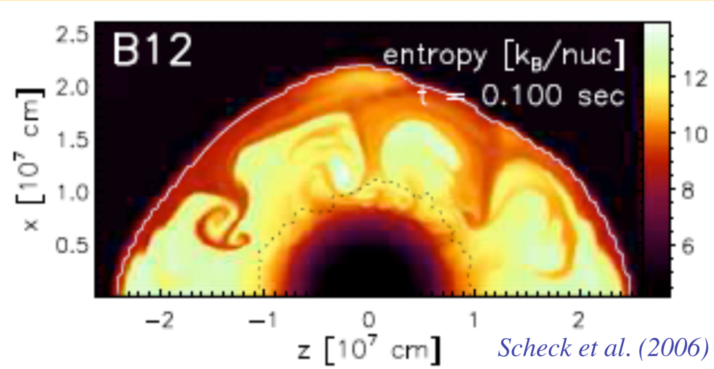
4. Conclusion: back to the core-collapse problem

Instabilities during the phase of stalled accretion shock





Scheck et al. (2006)

- Convection in the gain region, low l
(Herant, Benz & Colgate 1992)

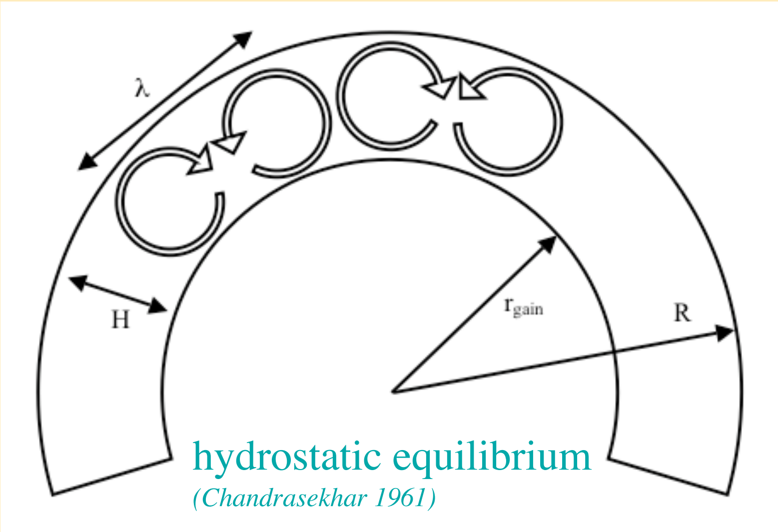
- l=1,2 SASI in an adiabatic flow:
vortical-acoustic cycle *(Blondin, Mezzacappa & DeMarino 2003)*
or purely acoustic mechanism *(Blondin & Mezzacappa 2006)* ?

-Neutron star kick resulting from the l=1 instability of convection and/or vortical-acoustic cycle
-*(Scheck et al. 2004, Janka et al. 2004, Scheck et al. 2006)*

- New explosion mechanism driven by acoustic waves, initiated by the advective-acoustic cycle *(Burrows et al. 2005)*

convection ?
SASI 2003 ?
SASI 2006 ?
advective-acoustic cycle ?

Contribution of the convective instability to a mode l=1 ?

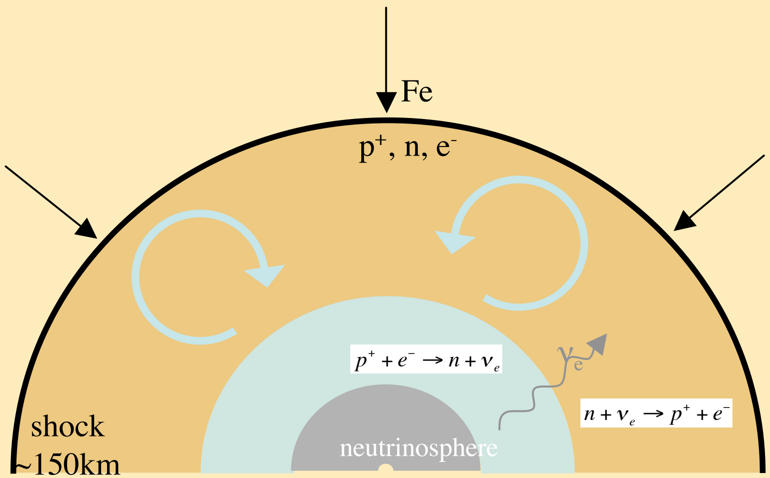


hydrostatic equilibrium
(Chandrasekhar 1961)

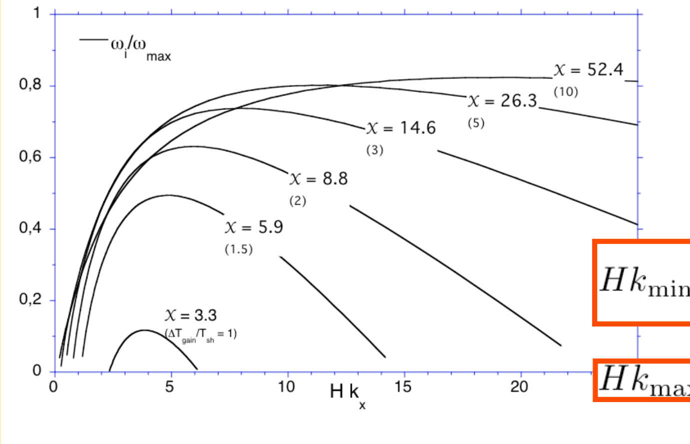
$$\frac{H\omega_{\rm buoy}}{v} \sim \left(\frac{\mathcal{G}M}{r_{\rm sh}v_2^2}\right)^{\frac{1}{2}}\left(\frac{H}{r_{\rm sh}}\right)^{\frac{1}{2}}$$

$$\sim 3.1\left(\frac{v_1}{7v_2}\right)\left(\frac{H}{0.4r_{\rm sh}}\right)^{\frac{1}{2}}$$

!



Foglizzo, Scheck & Janka (2005)



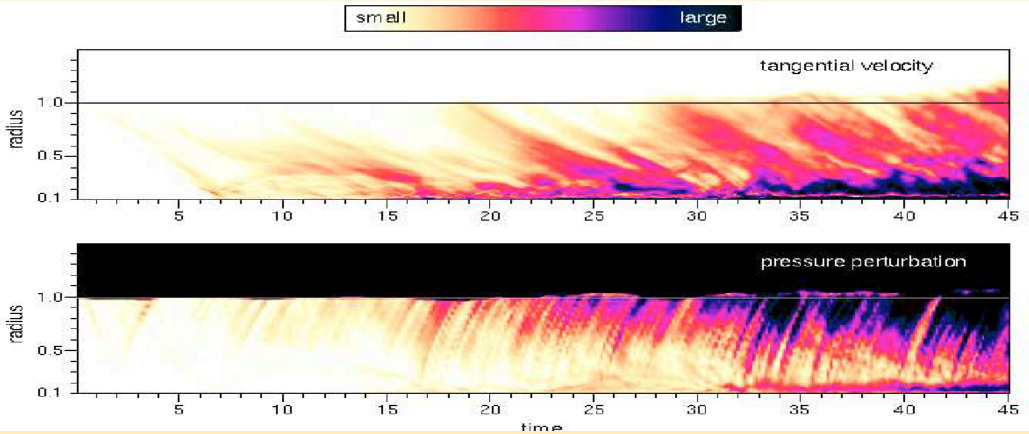
$$\boxed{Hk_{\rm min} \propto \frac{1}{\chi}}$$

$$\boxed{Hk_{\rm max} \propto \chi}$$

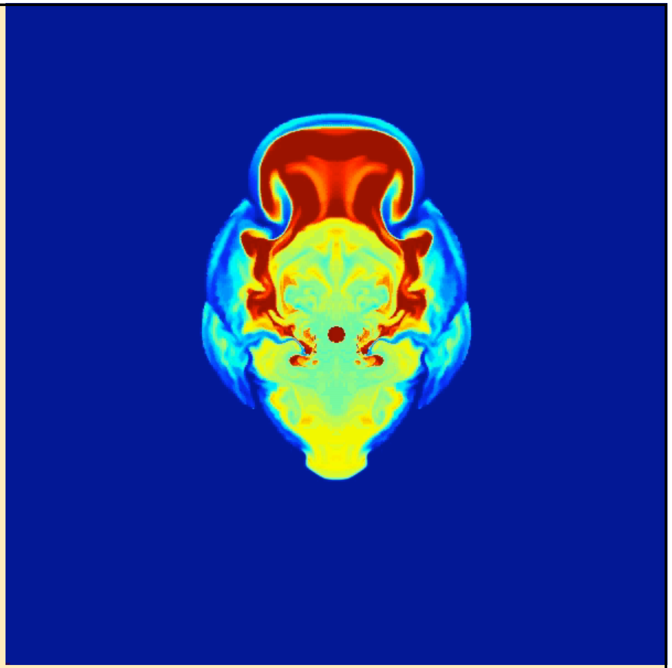
$$\chi \equiv \int_{z_{\rm sh}}^{z_{\rm gain}} \omega_{\rm buoy}\frac{{\rm d}z}{v}$$

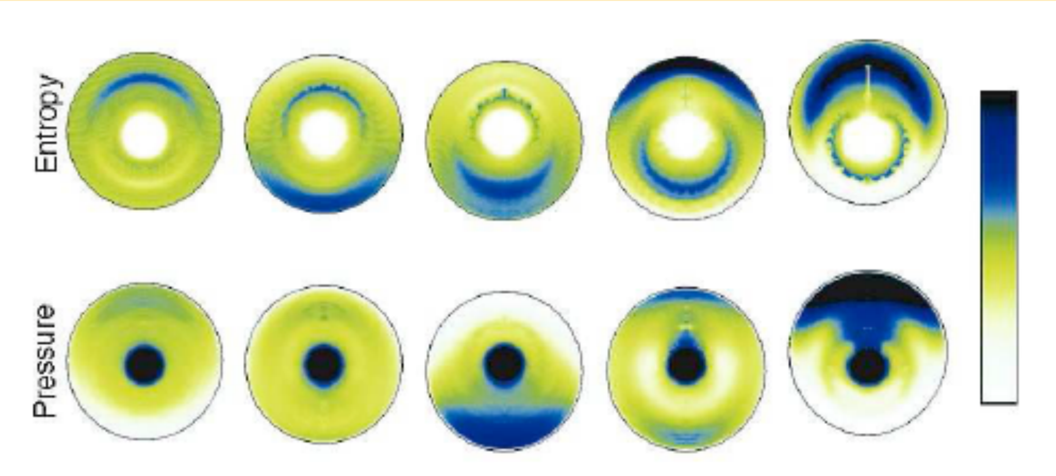
$$\boxed{\chi \sim 3}$$

Asymmetry without convection:
adiabatic simulation of a stalled accretion shock
on a neutron star



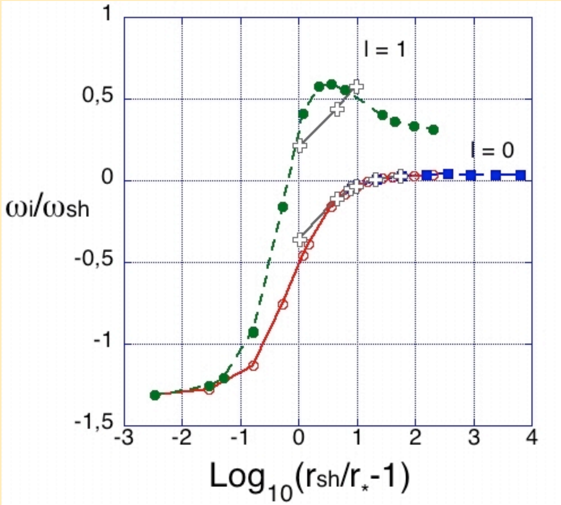
Evidence for a vortical-acoustic cycle *(Blondin, Mezzacappa & DeMarino 2003)*





A purely acoustic cycle ? *(Blondin & Mezzacappa 2005)*

Linear stability analysis



l=0: *Houck & Chevalier (1992)*
l=0,1: *Galletti (PhD Thesis 2005)*
Blondin & Mezzacappa (2005)

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Aero-acoustic instabilities

- advected perturbations
- acoustic feedback

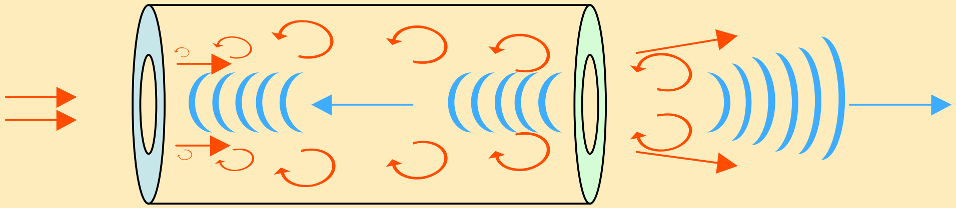
vibrations in Ariane 5:
segmented solid propergol

Mettenleiter, Haile & Candel (2000)
J. of Sound and Vibration 230, 761

impinging shear layers

Rockwell, D. 1983, AIAA J., 21, 645

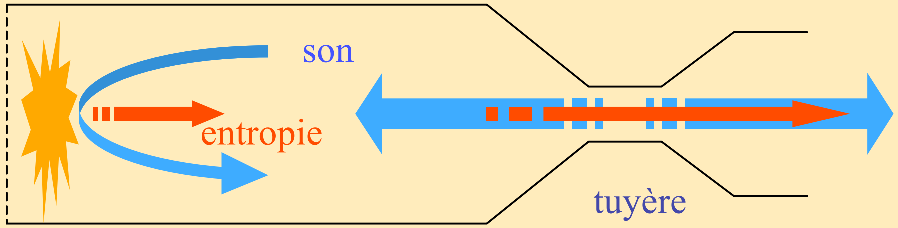
- vortical-acoustic cycle



whistling kettle

Chanaud & Powell (1965)
J. Acoust.Soc. Am. 37, 902

- entropic-acoustic cycle



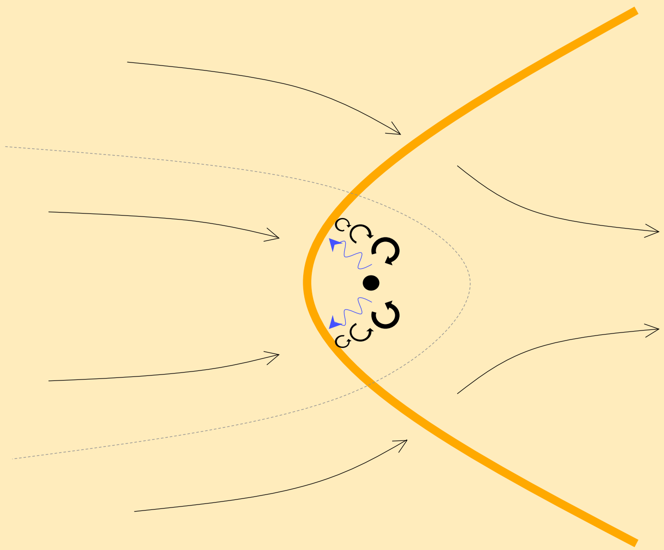
combustion

rumble instability of ramjet combustors

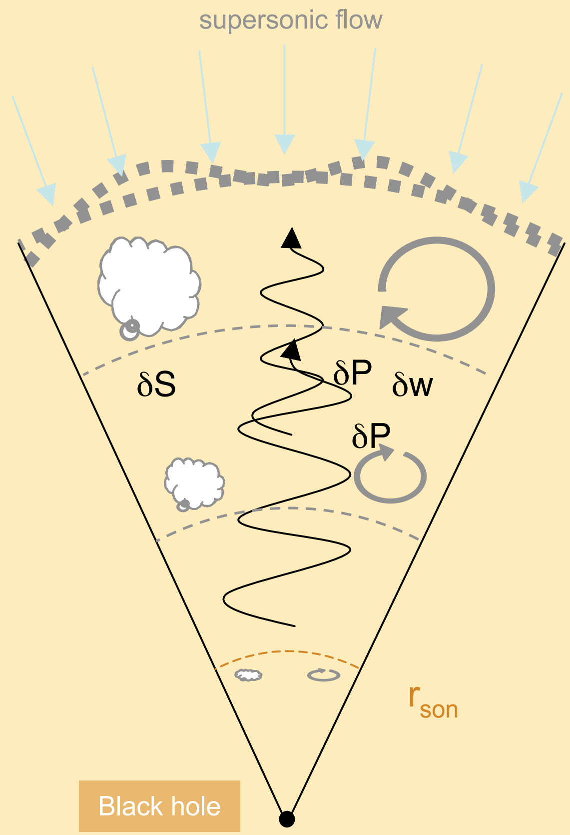
Abouseif, Keklak & Toong (1984)
Combustion Science and Technology, 36, 83

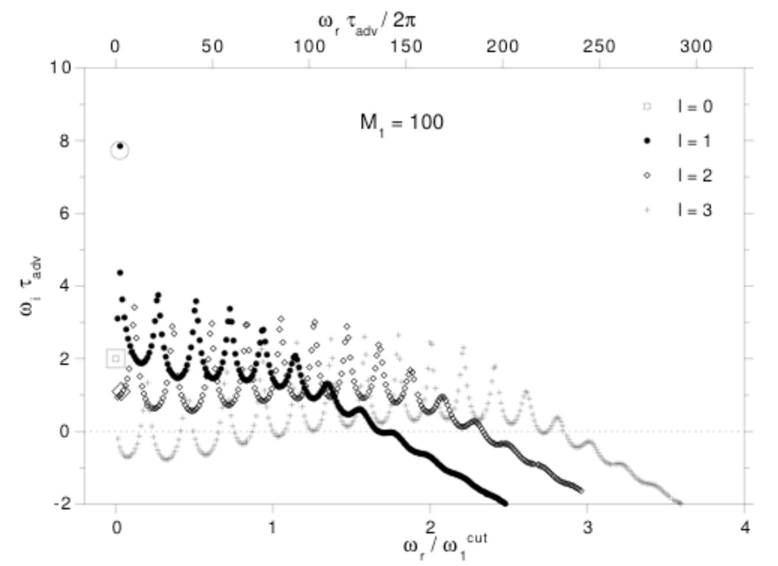
Analytical study of the advective-acoustic coupling in a radial, accelerated flow

Foglizzo (2001, 2002), Foglizzo, Galletti & Ruffert (2005)



Bondi-Hoyle-Lyttleton accretion; unstable

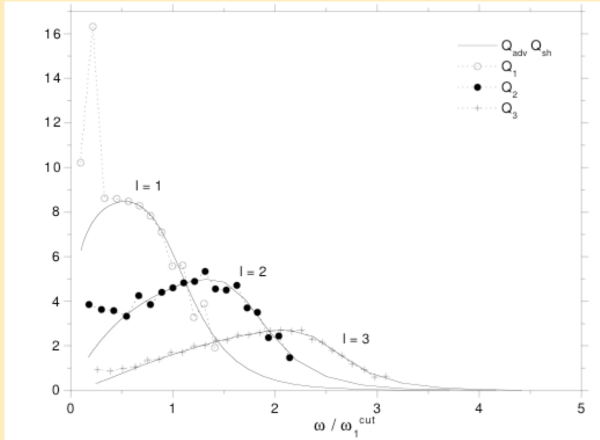




isothermal flow *(Foglizzo 2002)*

Beyond the eigenspectrum

- identification of the 2 cycles (Q, τ_Q) and (R, τ_R)



$$\begin{array}{ccccc} & \xrightarrow{Q_{sh}} & \boxed{\delta S} & \xrightarrow{Q_\nabla} & \boxed{\delta p^-} \\ \delta p^- & & & & \\ & \xrightarrow{R_{sh}} & \boxed{\delta p^+} & \xrightarrow{R_\nabla} & \boxed{\delta p^-} \end{array}$$

advective-acoustic cycle

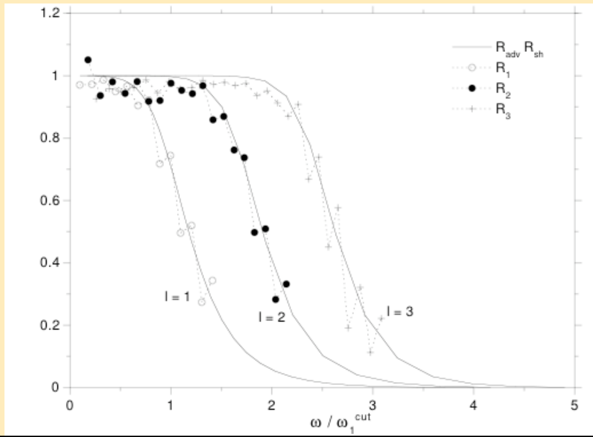
efficiency $\mathcal{Q} \equiv \mathcal{Q}_{sh}\mathcal{Q}_\nabla$

duration $\tau_\mathcal{Q}$

purely acoustic cycle

efficiency $\mathcal{R} \equiv \mathcal{R}_{sh}\mathcal{R}_\nabla$

duration $\tau_\mathcal{R}$



$$\mathcal{Q}e^{i\omega\tau_\mathcal{Q}} + \mathcal{R}e^{i\omega\tau_\mathcal{R}} = 1$$

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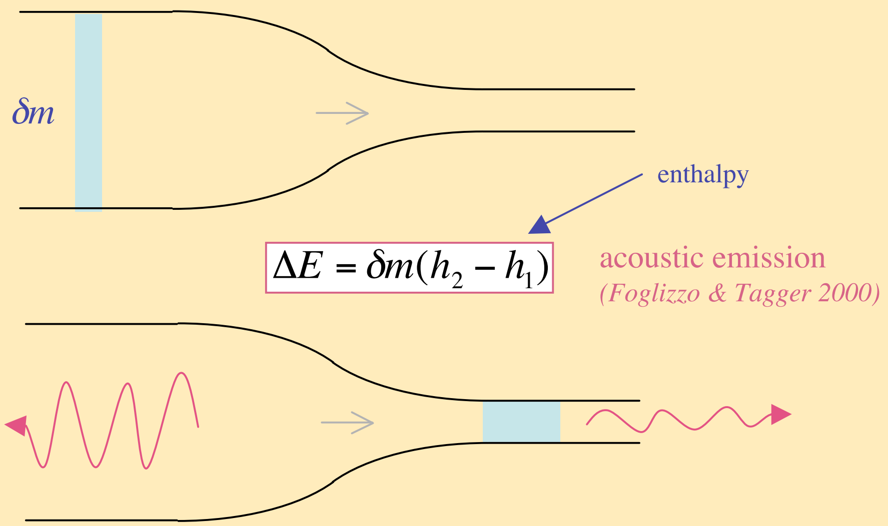
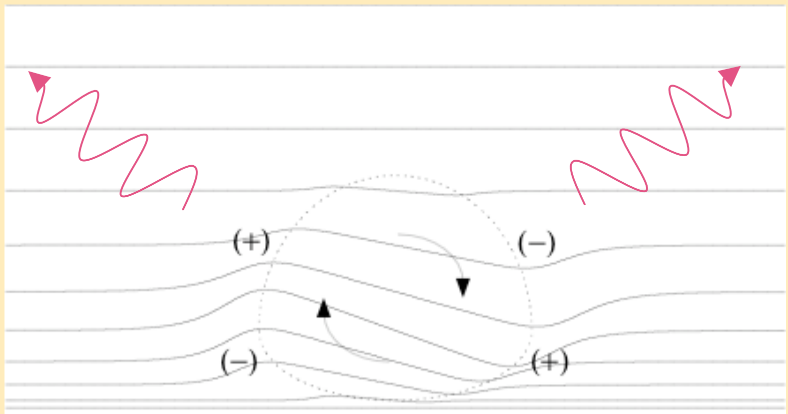
Advective-acoustic coupling: hand waving



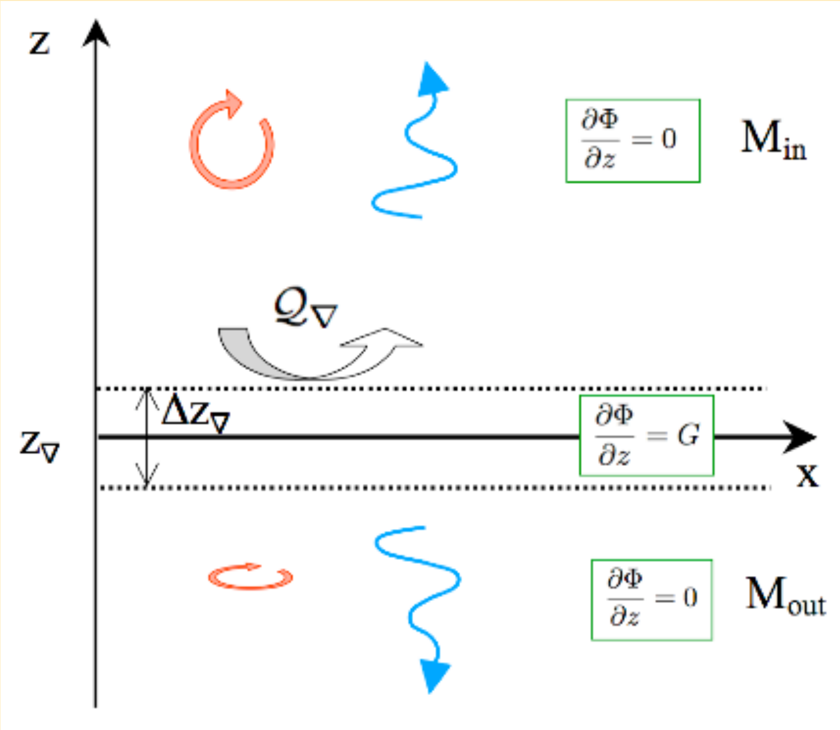
$$\boxed{\Delta E = \delta m(h_2 - h_1)}$$

advection of entropy

advection of vorticity



Advective-acoustic coupling: illustrative simulations



2 isothermal simulations *(thanks to F. Masset)*

-> accelerated flow (M_{in}=0.1, M_{out}=0.7)

$$\omega < k_x c(1-\mathcal{M}_{\rm out}^2)^{\frac{1}{2}} < k_x c(1-\mathcal{M}_{\rm in}^2)^{\frac{1}{2}}$$

in: evanescent
out: evanescent

-> decelerated flow (M_{in}=0.7, M_{out}=0.1)

$$k_x c(1-\mathcal{M}_{\rm in}^2)^{\frac{1}{2}} < \omega < k_x c(1-\mathcal{M}_{\rm out}^2)^{\frac{1}{2}}$$

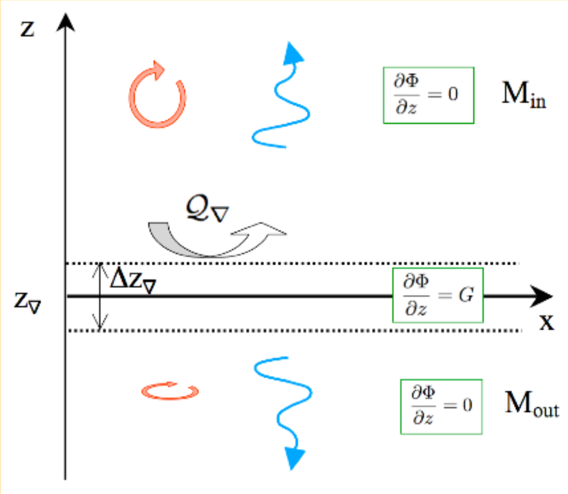
in : propagate
out : evanescent

propagating wave: $\omega > k_x c(1-\mathcal{M}^2)^{\frac{1}{2}}$

evanescent wave: $\omega < k_x c(1-\mathcal{M}^2)^{\frac{1}{2}}$

$$\omega^2 = (k_x^2 + k_z^2)c^2$$

$$k_z^2 = \frac{\omega^2}{c^2} - k_x^2$$



Compact approximation

$$\Delta z_\nabla \to 0$$

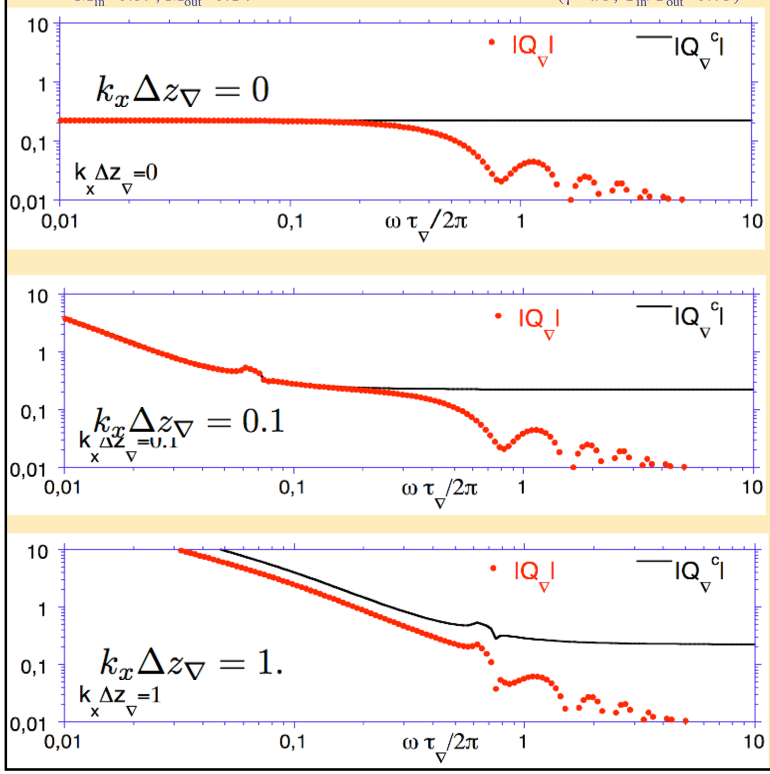
scale free

$$\tau_\nabla \equiv \int_{z_\nabla - \frac{\Delta z_\nabla}{2}}^{z_\nabla + \frac{\Delta z_\nabla}{2}} \frac{\mathrm{d}z}{|v|}$$

validity of the compact approximation

$$\begin{cases} \tau_\nabla \ll \dfrac{2\pi}{\omega} \\ k_x \Delta z_\nabla \ll 1 \end{cases}$$

$M_{in}=0.39$, $M_{out}=0.14$ $(\gamma=4/3,\ T_{in}/T_{out}=0.75)$



$$\mathcal{Q}_\nabla^c = \frac{\mathcal{M}_{\rm out} + \mu_{\rm out}}{1 + \mu_{\rm out}\mathcal{M}_{\rm out}} \frac{1}{\mu_{\rm out}\frac{c_{\rm in}^2}{c_{\rm out}^2} + \mu_{\rm in}\frac{\mathcal{M}_{\rm out}}{\mathcal{M}_{\rm in}}} \left[1 - \frac{c_{\rm in}^2}{c_{\rm out}^2} + \frac{k_x^2 c_{\rm in}^2}{\omega^2}(\mathcal{M}_{\rm in}^2 - \mathcal{M}_{\rm out}^2)\right]$$

$$\mu^2 \equiv 1 - \frac{k_x^2 c^2}{\omega^2}(1 - \mathcal{M}^2)$$

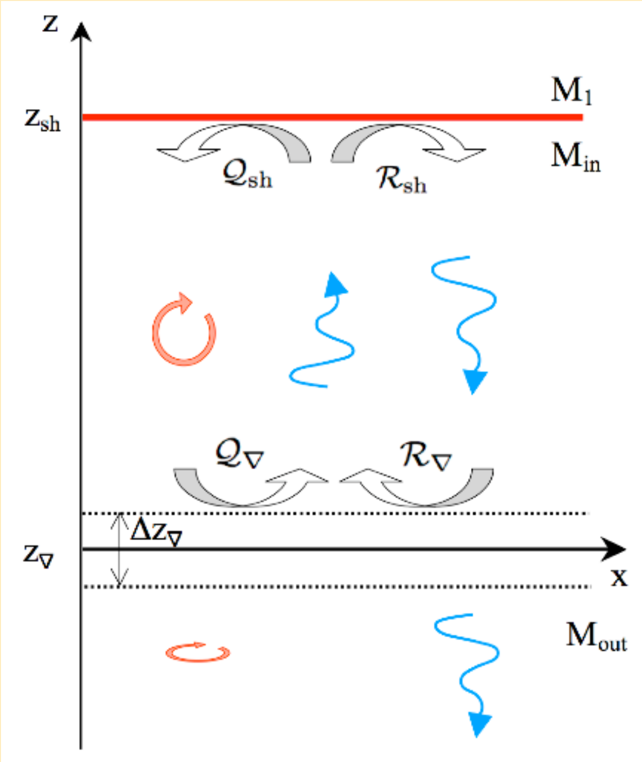
longitudinal transverse

cut-off frequency $\omega_\nabla \equiv \dfrac{1}{\tau_\nabla}$

-> analytical upper bound of the coupling efficiency

The simplest example of an advective-acoustic instability

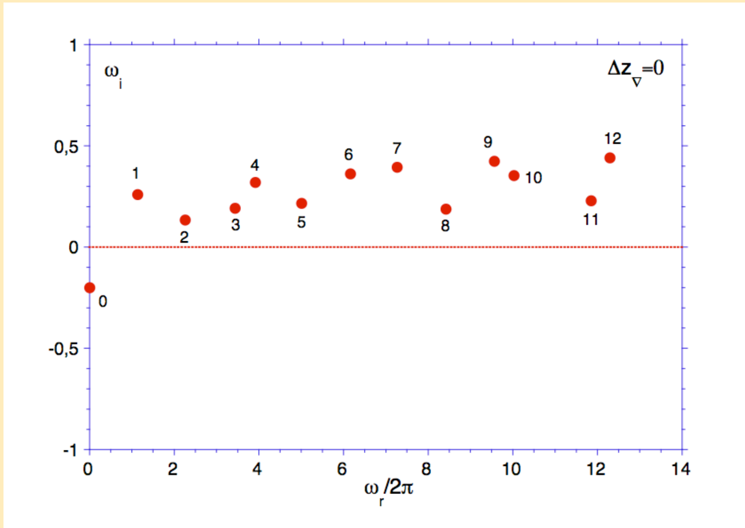
-parallel adiabatic flow, localized coupling (γ, M_1, $\Delta\Phi$, Δz_∇)
-2-D perturbations $\omega, k_\perp$



$$\tau_{\mathcal{Q}} \equiv \frac{1+\mu_{\rm in}\mathcal{M}_{\rm in}}{1-\mathcal{M}_{\rm in}^2}\frac{z_{\rm sh}-z_\nabla}{|v_{\rm in}|}$$

$$\tau_{\mathcal{R}} \equiv \frac{2\mu_{\rm in}}{1-\mathcal{M}_{\rm in}^2}\frac{z_{\rm sh}-z_\nabla}{c_{\rm in}}$$

M_1=5, γ=4/3, T_{in}/T_{out}=0.75 (M_{in}=0.39, M_{out}=0.14)



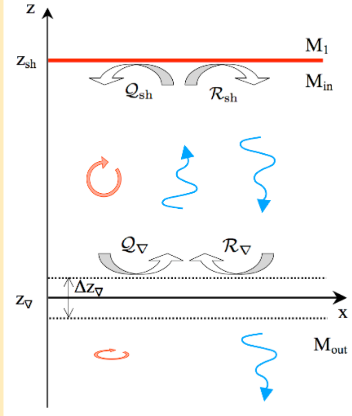
$$\mathcal{Q}e^{i\omega\tau_{\mathcal{Q}}} + \mathcal{R}e^{i\omega\tau_{\mathcal{R}}} = 1$$

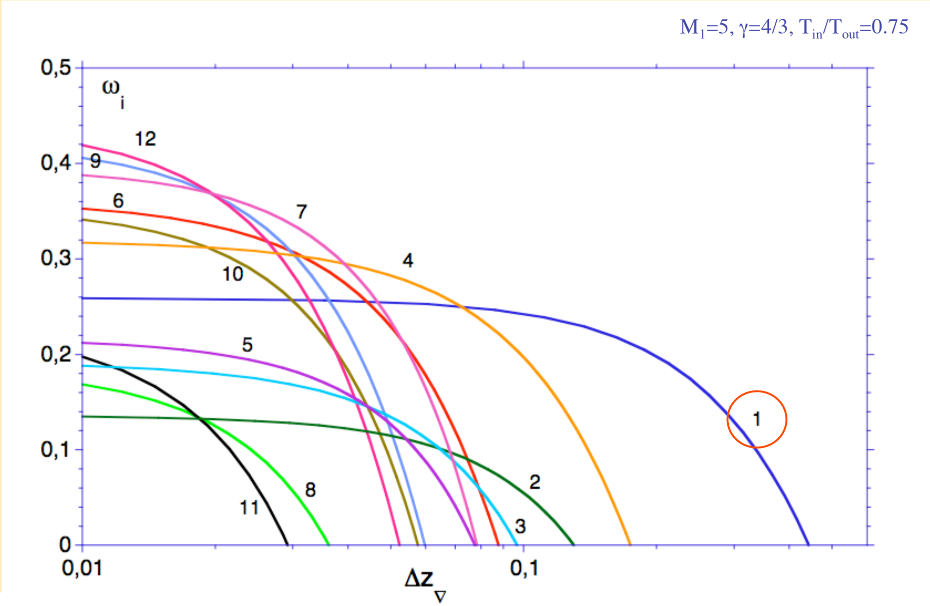
$$\mu^2 \equiv 1 - \frac{k_x^2 c^2}{\omega^2}(1-\mathcal{M}^2)$$

$$\mathcal{Q}_c \equiv \frac{\frac{4\mu_{\rm in}}{\mathcal{M}_{\rm in}}(1-\mathcal{M}_{\rm in}^2)\left(1-\frac{1}{\mathcal{M}_1^2}\right)}{\mu_{\rm out}\frac{c_{\rm in}^2}{c_{\rm out}^2}+\mu_{\rm in}\frac{\mathcal{M}_{\rm out}}{\mathcal{M}_{\rm in}}}\,\frac{\mathcal{M}_{\rm out}+\mu_{\rm out}}{1+\mu_{\rm out}\mathcal{M}_{\rm out}}\left[\frac{1-\frac{c_{\rm in}^2}{c_{\rm out}^2}+\frac{k_x^2 c_{\rm in}^2}{\omega^2}(\mathcal{M}_{\rm in}^2-\mathcal{M}_{\rm out}^2)}{(\gamma+1)(1-\mu_{\rm in}\mathcal{M}_{\rm in})(\mu_{\rm in}^2+2\mu_{\rm in}\mathcal{M}_{\rm in}+\mathcal{M}_1^{-2})}\right]$$

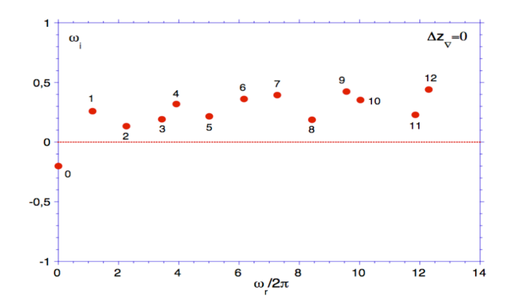
$$R = \frac{\mu_{out}M_{in}c_{in}^2-\mu_{in}M_{out}c_{out}^2}{\mu_{out}M_{in}c_{in}^2+\mu_{in}M_{out}c_{out}^2}\left(\frac{1+\mu_{in}M_{in}}{1-\mu_{in}M_{in}}\right)\frac{\mu_{in}^2-2\mu_{in}M_{in}+\frac{1}{M_1^2}}{\mu_{in}^2+2\mu_{in}M_{in}+\frac{1}{M_1^2}}$$

Effect of the size Δz_∇ of the coupling region

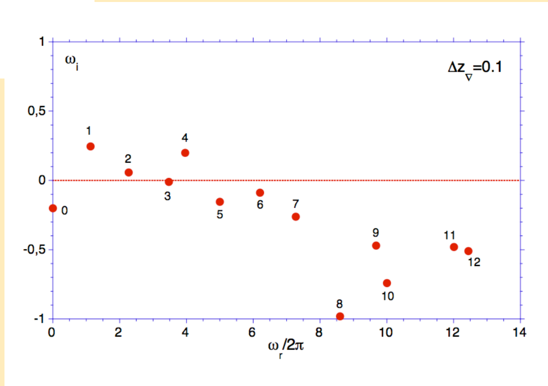




compact approximation confirmed



a possible benchmark test



a low frequency instability

$$k_x c(1-\mathcal{M}^2)^{\frac{1}{2}} \leq \omega \ll \omega_\nabla$$

a low n_x instability

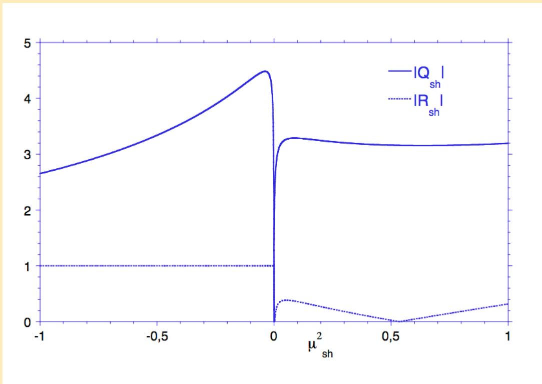
why not n_x=0 ?
why so much irregularity ?

Comparison of n_x=0 and n_x>0 modes

- no vorticity in 1-D: the instability of the mode n_x=0 relies only on temperature gradients
- transverse modes benefits from the vortical-acoustic coupling

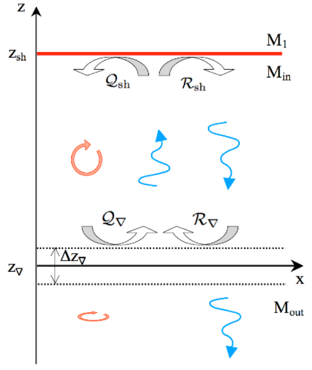
$$\mathcal{Q}_c \equiv \frac{\frac{4\mu_{\rm in}}{\mathcal{M}_{\rm in}}(1-\mathcal{M}_{\rm in}^2)\left(1-\frac{1}{\mathcal{M}_1^2}\right)}{\mu_{\rm out}\frac{c_{\rm in}^2}{c_{\rm out}^2}+\mu_{\rm in}\frac{\mathcal{M}_{\rm out}}{\mathcal{M}_{\rm in}}}\,\frac{\mathcal{M}_{\rm out}+\mu_{\rm out}}{1+\mu_{\rm out}\mathcal{M}_{\rm out}}\left[\frac{1-\frac{c_{\rm in}^2}{c_{\rm out}^2}+\frac{k_x^2 c_{\rm in}^2}{\omega^2}(\mathcal{M}_{\rm in}^2-\mathcal{M}_{\rm out}^2)}{(\gamma+1)(1-\mu_{\rm in}\mathcal{M}_{\rm in})(\mu_{\rm in}^2+2\mu_{\rm in}\mathcal{M}_{\rm in}+\mathcal{M}_1^{-2})}\right]$$

M_1=5, γ=4/3



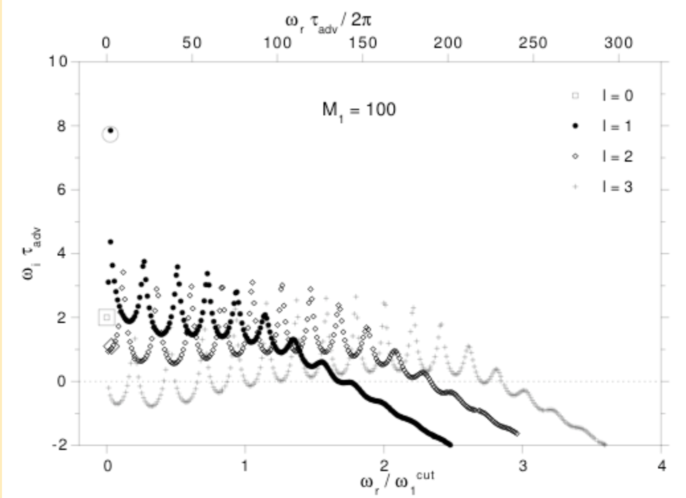
At the shock, the coupling efficiency is also maximum for transverse perutrbations:

$$\mu_{\rm sh}^2 \propto \pm\frac{1}{\mathcal{M}_1^2}$$



Contribution of the acoustic cycle

(Foglizzo 2002)



$$\mathcal{Q}e^{i\omega\tau_{\mathcal{Q}}} + \mathcal{R}e^{i\omega\tau_{\mathcal{R}}} = 1$$

global dispersion relation

The acoustic cycle alone is stable: $|\mathcal{R}| \leq 1$

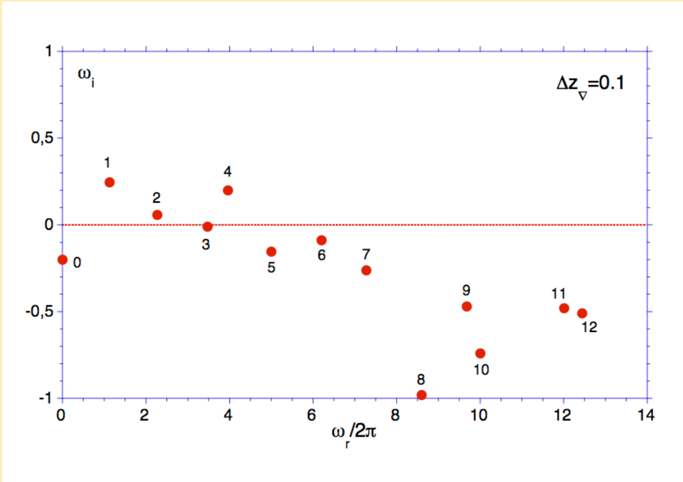
Its contribution can be either constructive or destructive

$|\mathcal{Q}| + |\mathcal{R}| < 1$ stable

$|\mathcal{Q}| - |\mathcal{R}| > 1$ unstable

$$\alpha \equiv \frac{|\mathcal{R}|}{|\mathcal{Q}|^{\frac{\tau_{\mathcal{R}}}{\tau_{\mathcal{Q}}}}}$$

$$\frac{1}{\tau_{\mathcal{Q}}} \log\left\{(1-\alpha)|\mathcal{Q}|\right\} \leq \omega_i \leq \frac{1}{\tau_{\mathcal{Q}}} \log\left\{\frac{|\mathcal{Q}|}{1-\alpha}\right\}$$



->the stability threshold is very sensitive to geometrical factors

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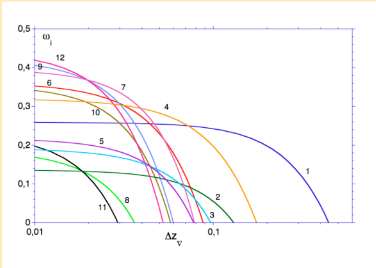
Conclusion

Simple toy model of the advective-acoustic instability in a decelerated flow

- gradient cut-off -> low frequency

$$k_x c(1-\mathcal{M}^2)^{\frac{1}{2}} \le \omega \ll \omega_\nabla$$

- acoustic evanescence -> low n_x
- vorticity -> transverse rather than longitudinal
- acoustic cycle -> sensitive to geometrical parameters



Benchmark test for numerical simulations

- advection of vorticity, numerical viscosity ?

Relevance to the core-collapse problem ?

- gravity, geometry, photodissociation, heating, cooling
- detailed comparison with numerical simulations: ongoing effort (Blondin, Scheck et al. 2006)