Using GW to infer formation channels for binary (BH) mergers: Prospects and procedures

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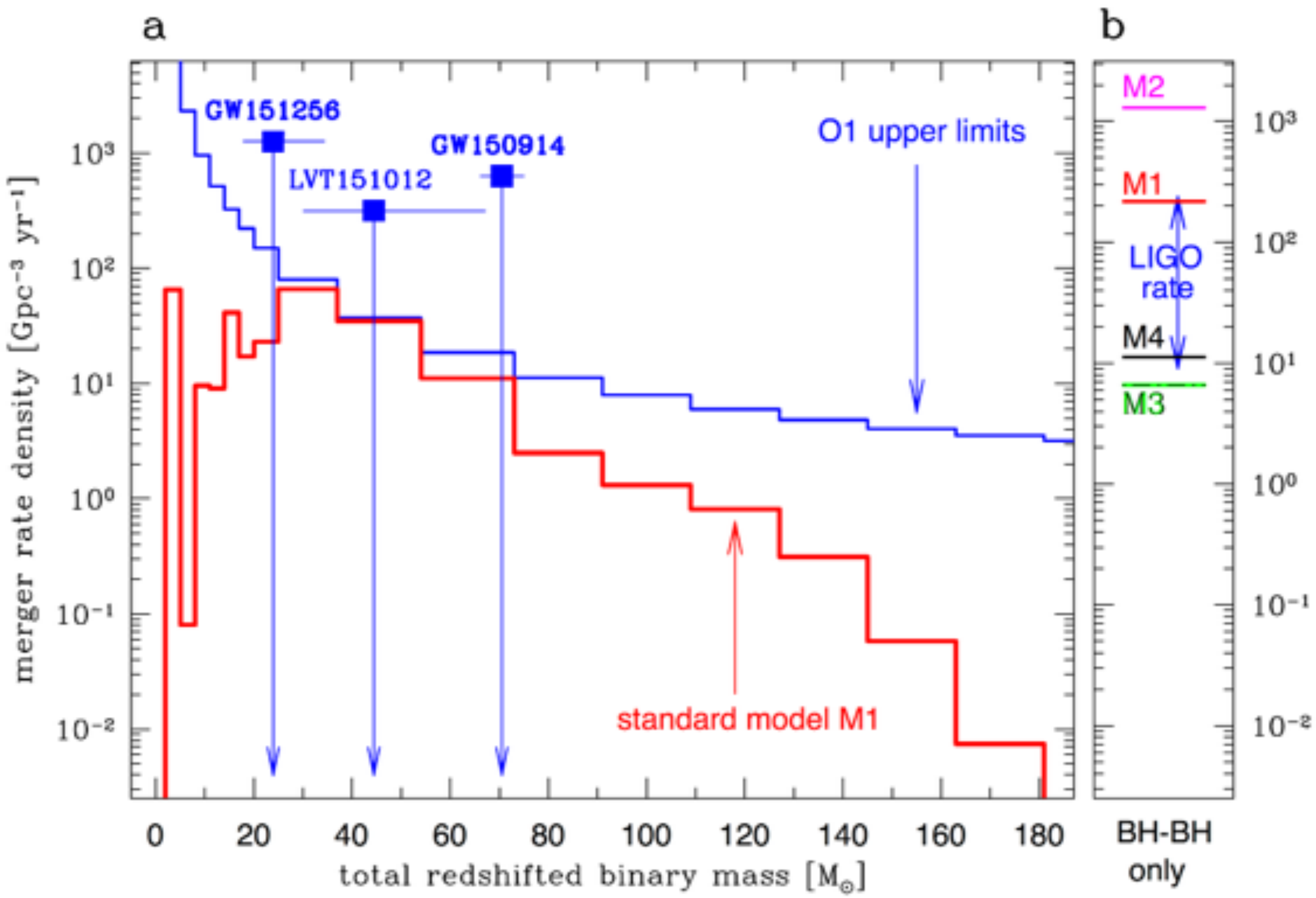
R·I·T

KITP: Astrophysics from LIGO's first black holes
Aug 3, 2016

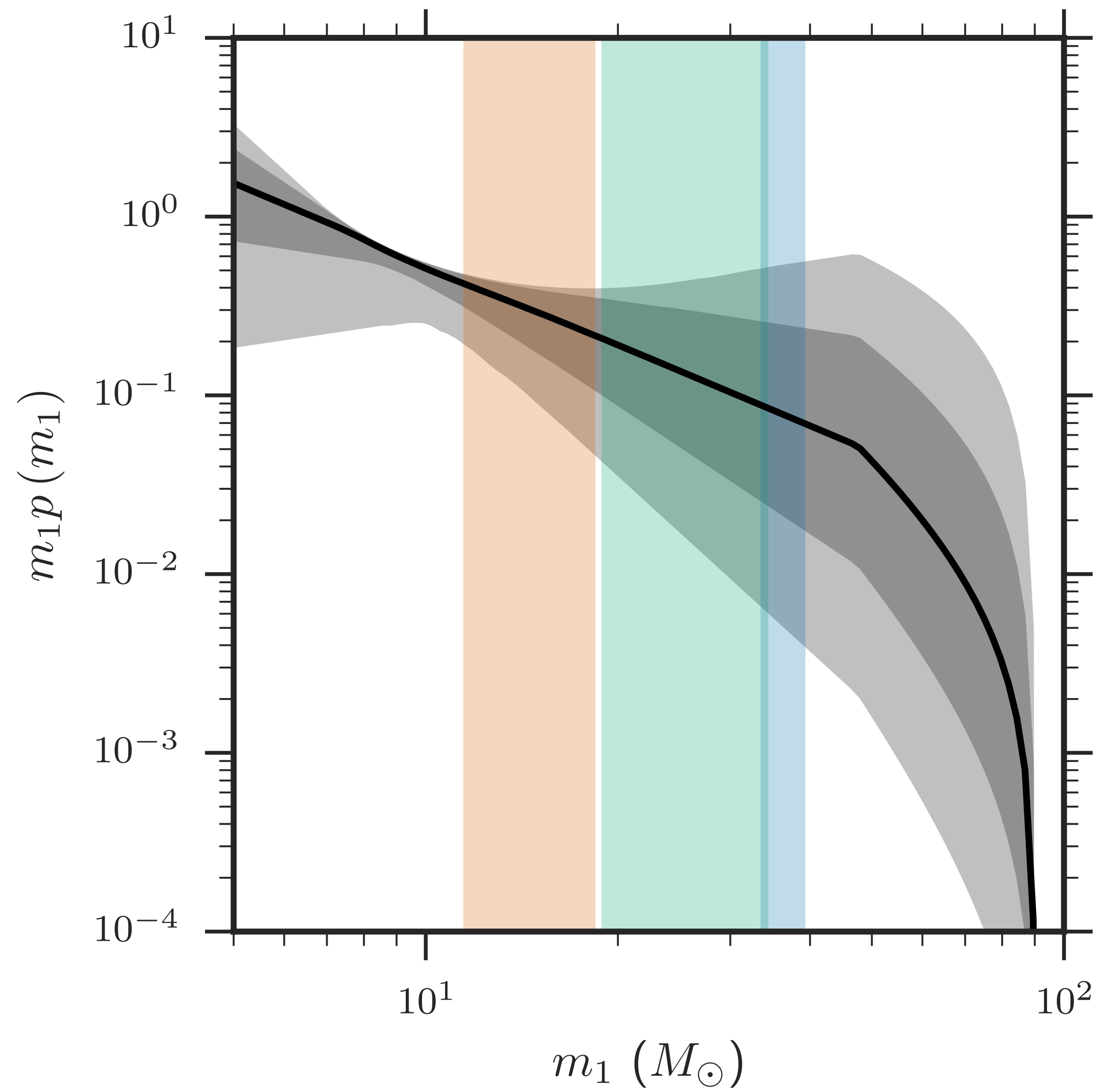
Confronting theory with observations

(Intrinsic distribution)



Belczynski et al *Nature* 2016

(Intrinsic distribution)



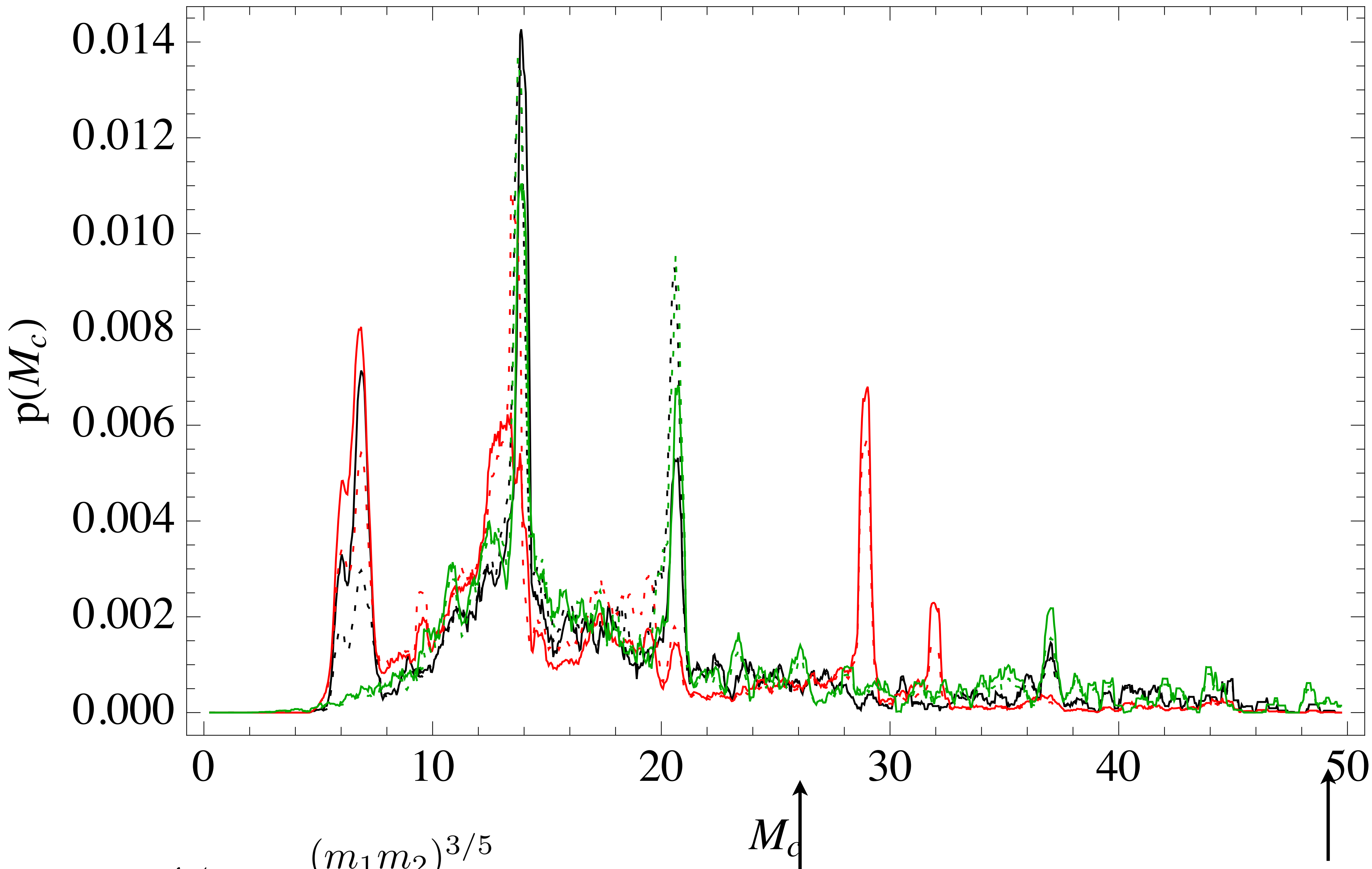
Abbott et al O1 BBH (1606.04856)

A function has infinitely many degrees of freedom

Distributions vary significantly…

(**Detected** distribution)

Dominik et al (2015: 1405.7016)



$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

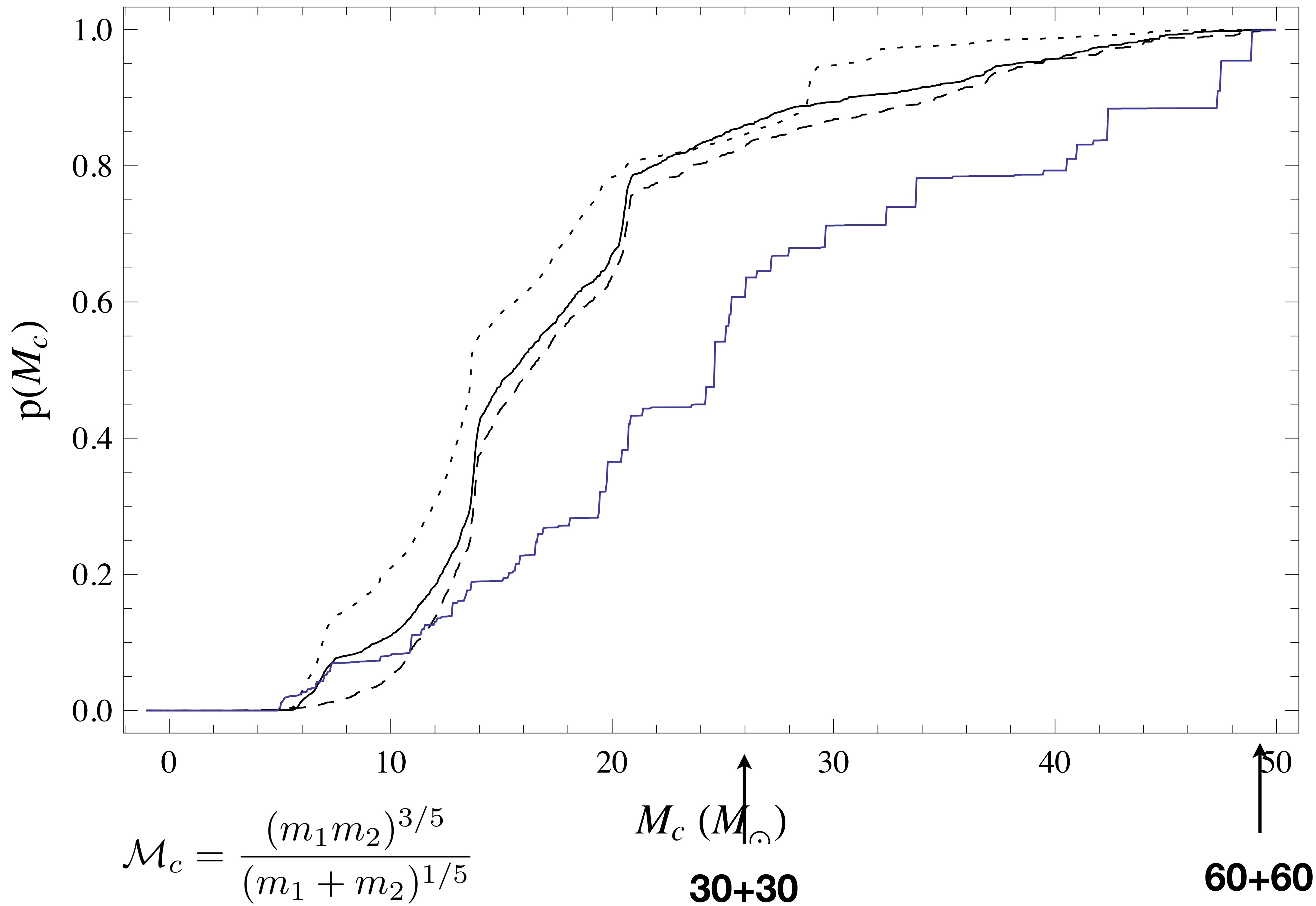
30+30

60+60

Distributions vary significantly…

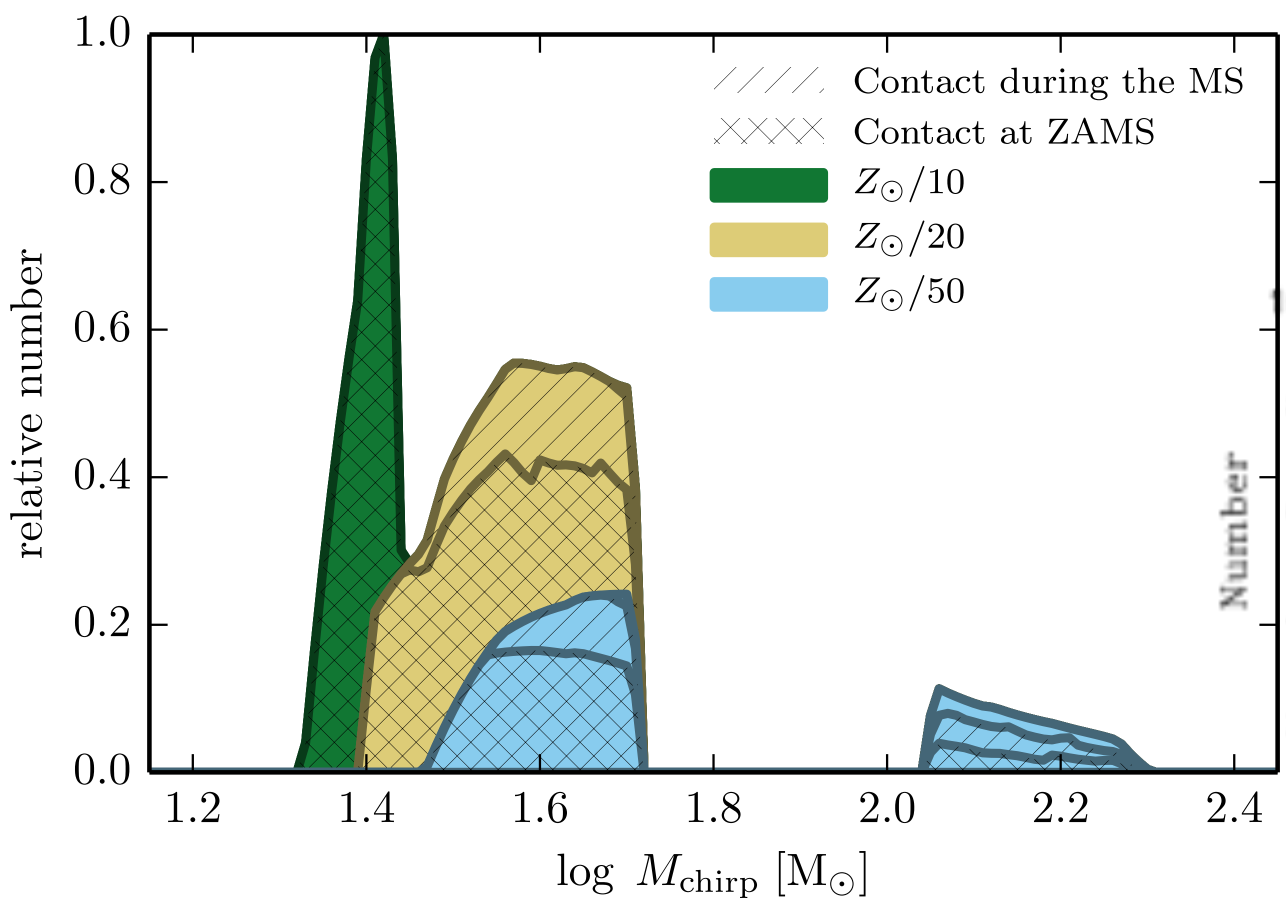
(**Detected** distribution)

Dominik et al (2015: 1405.7016)



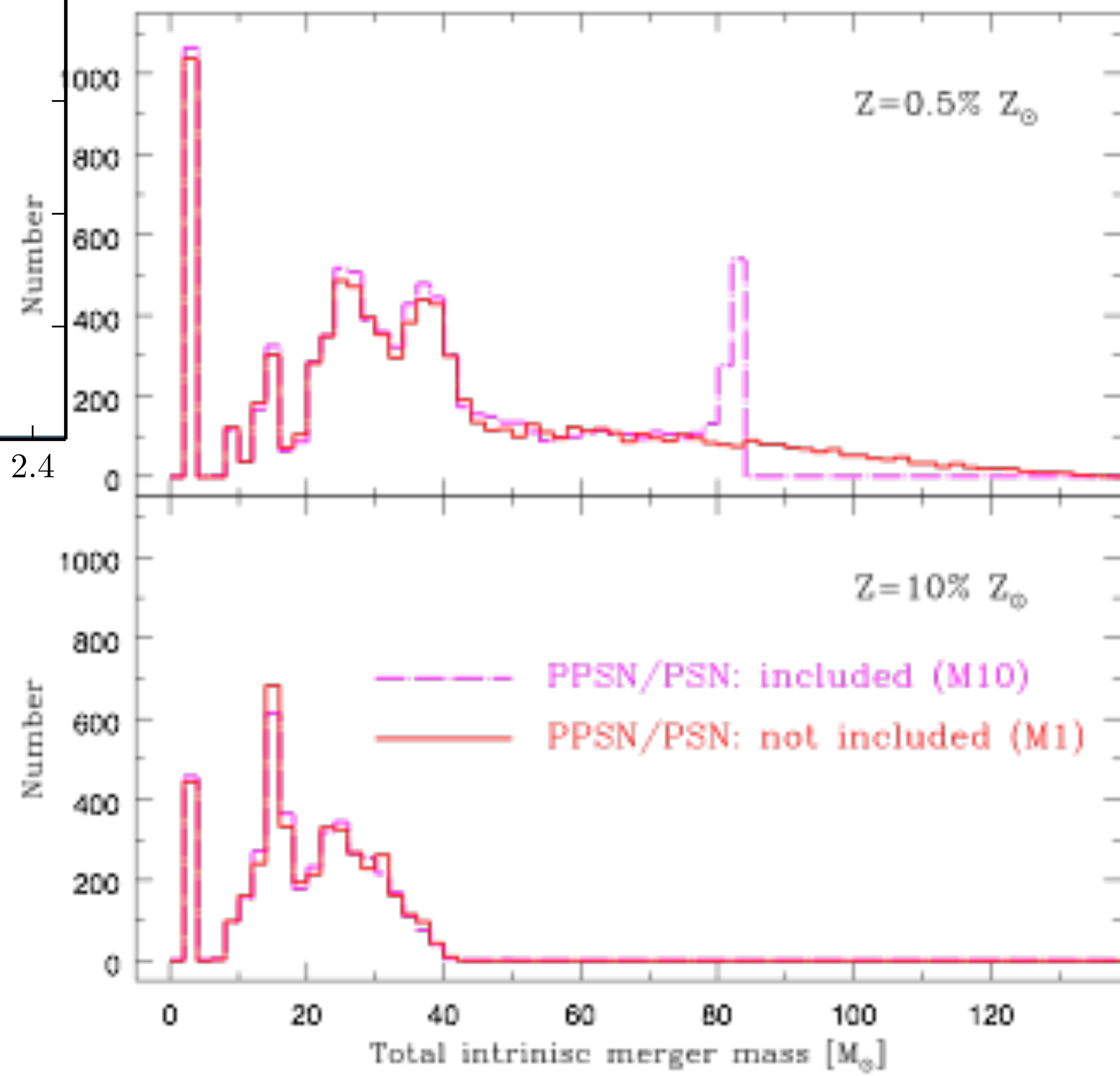
$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

…and for physical reasons, like pair instability



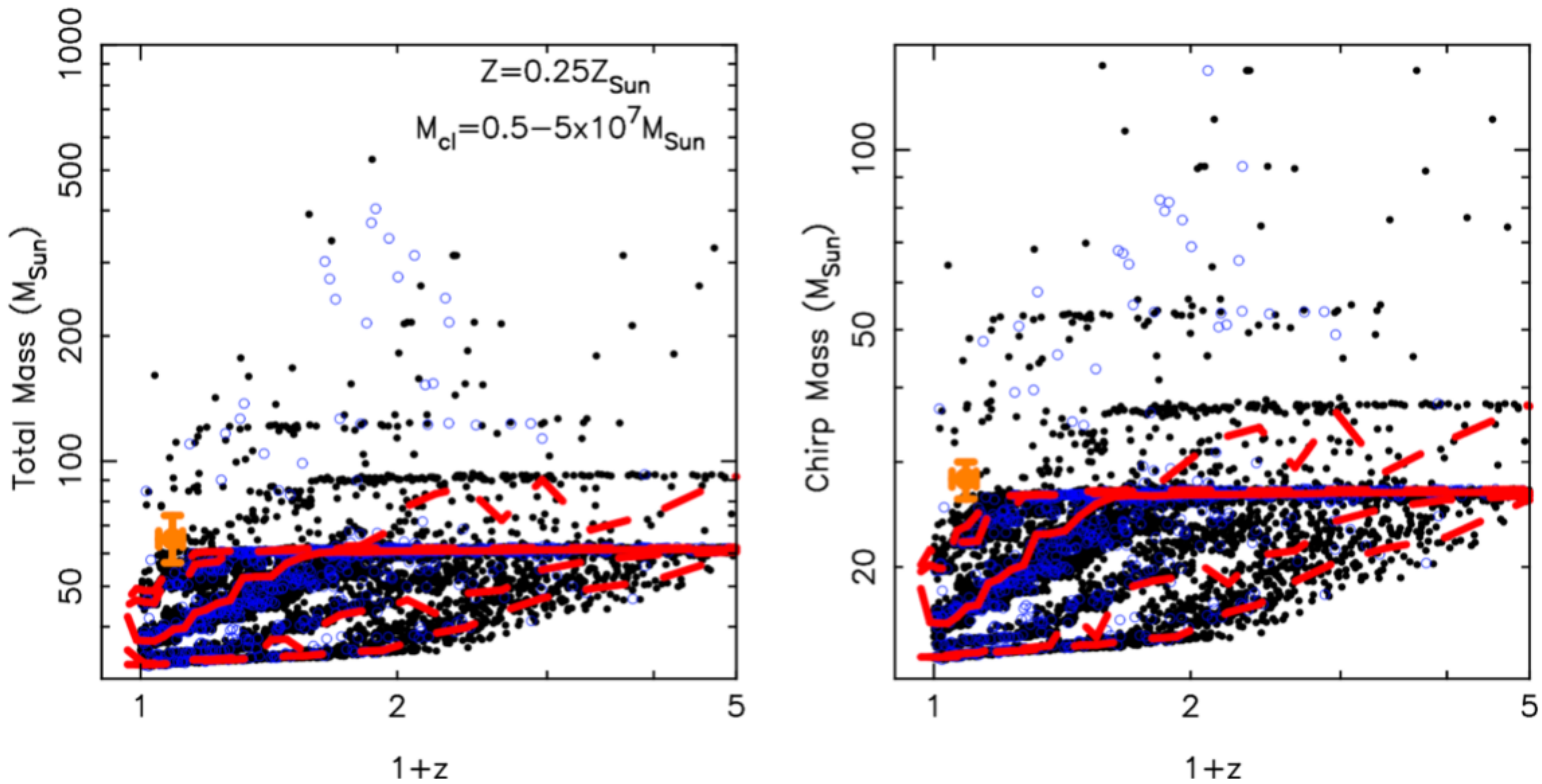
Marchant et al A&A 2016 (1601.03718)

(Intrinsic distribution)



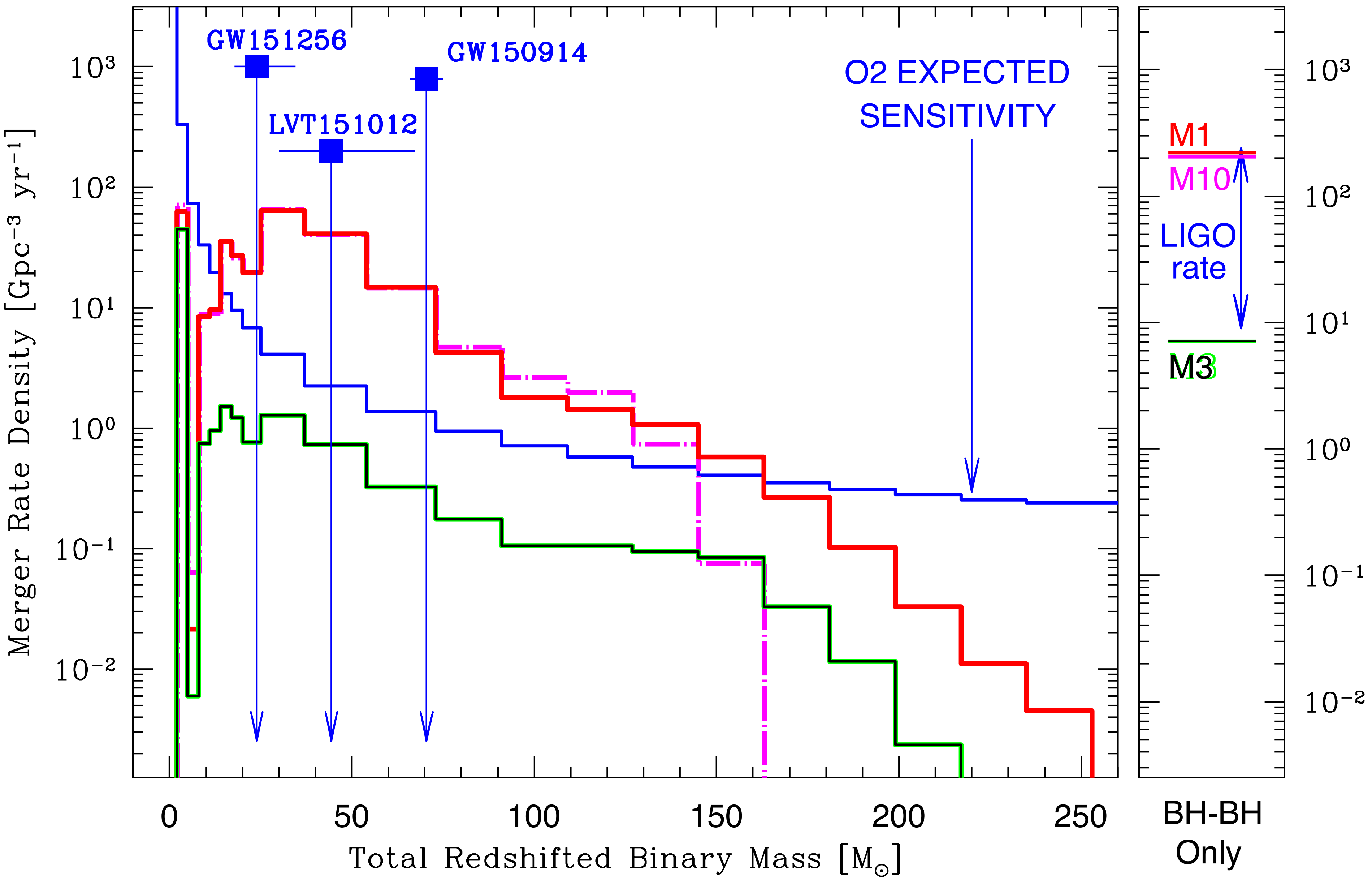
Belczynski et al 1607.03116

…or multiple mergers and single star evolution



Antonini and Rasio 2016
[see Carl Rodriguez talk]

…that may be observationally accessible soon

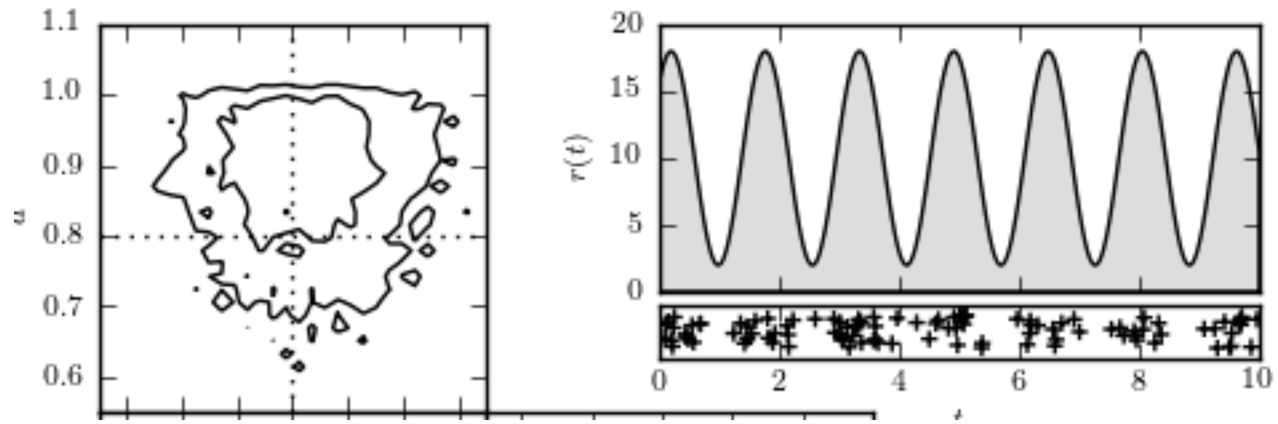


Belczynski et al 1607.03116

Familiar statistical challenge

- Inference via Poisson likelihood + bayes

$$L(\Lambda) = e^{-\mu}\frac{\mu^n}{n!}\prod_k \int d\lambda_k p(d_k|\lambda_k)p(\lambda_k|\Lambda)$$



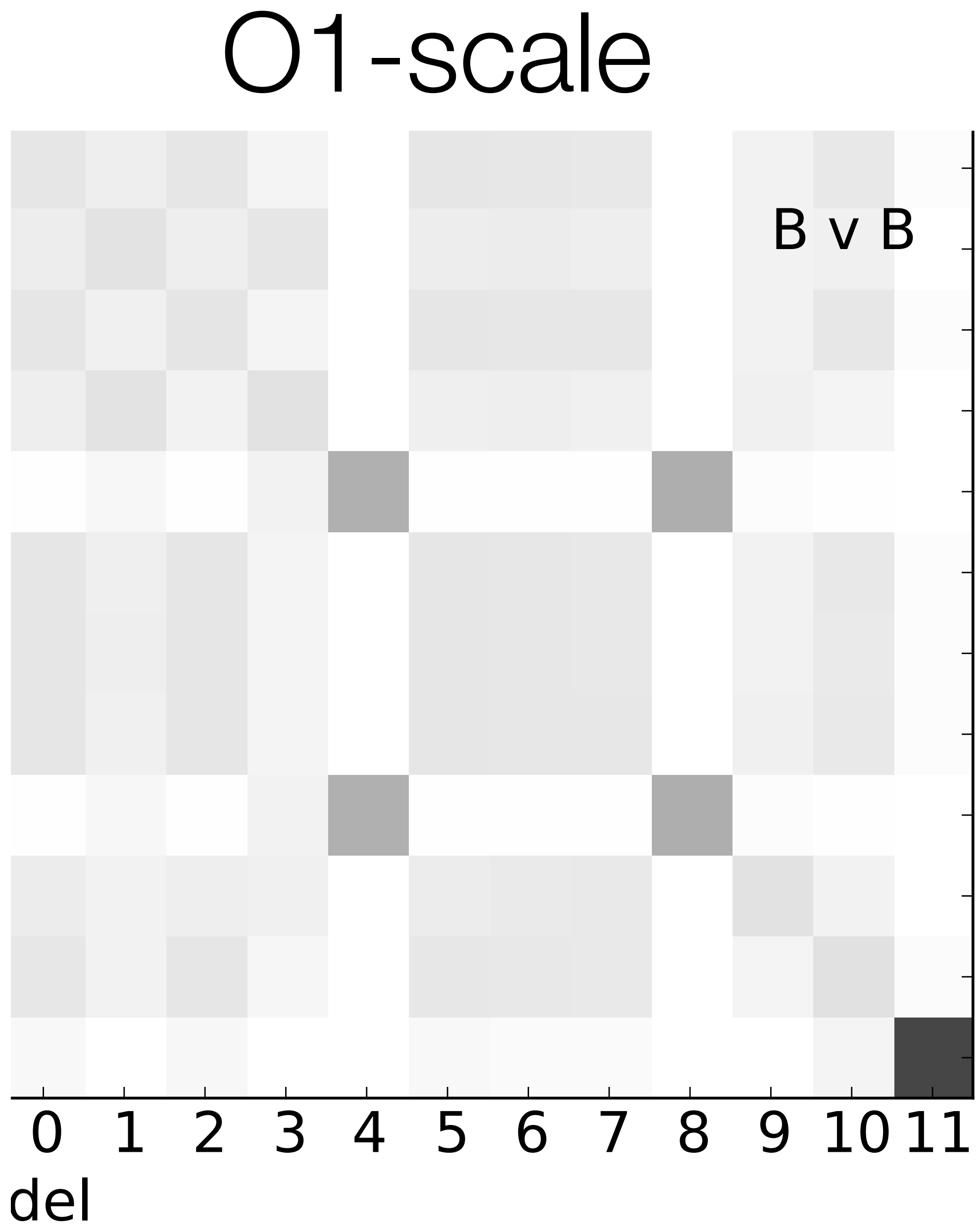
  - Same likelihood for nonparametric, parametric, and physical models
  - μ expected n (selection bias)
  - $p(d_k|\lambda_k)$ measurements and error
  - $p(\lambda_k|\Lambda)$ binary parameter distribution, given model parameters

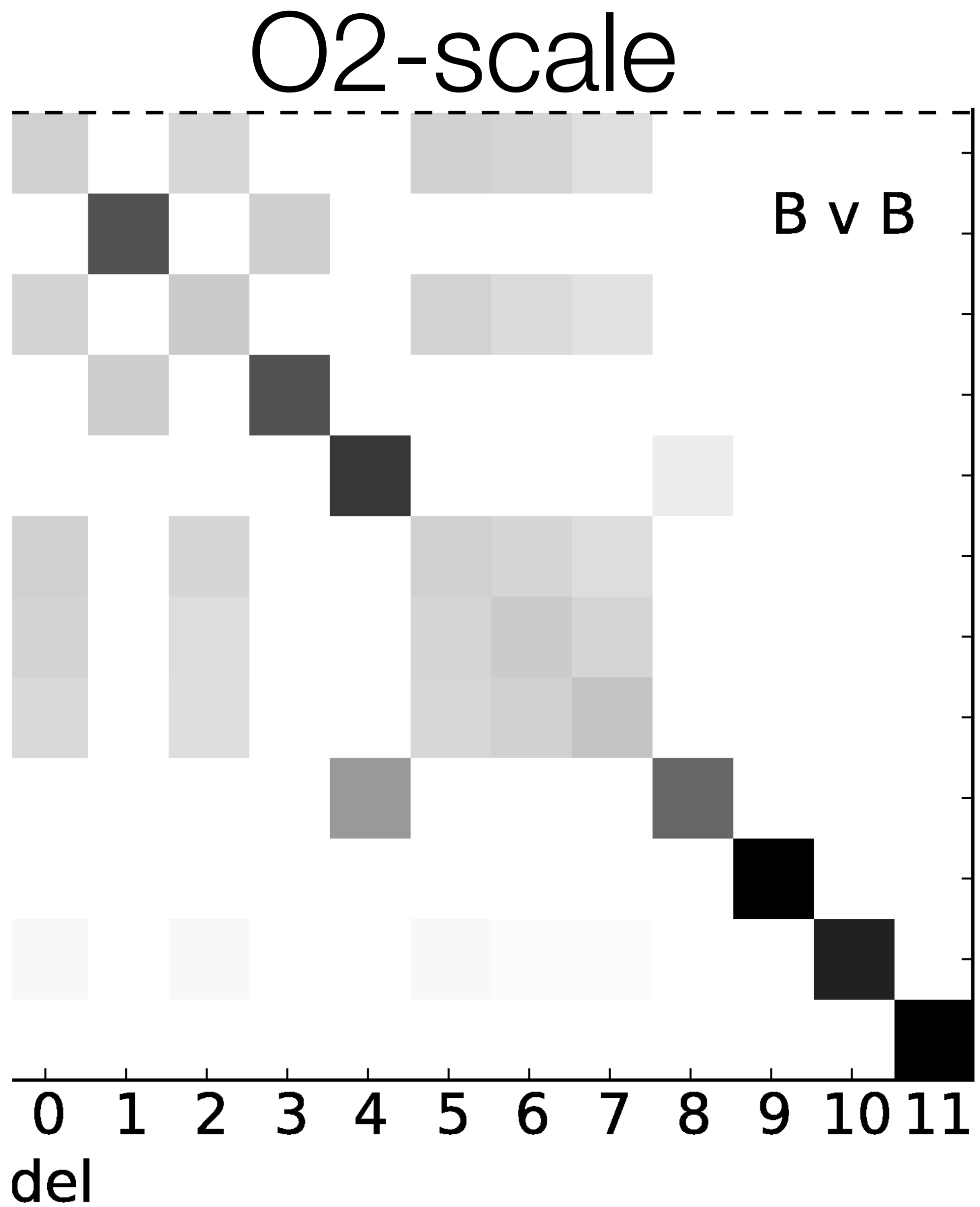
- Informal approaches: weighted histograms (=gaussian mixture models)

Ivezic et al, *Statistics, data mining, and machine learning in astronomy*
Gregory and Loredo (discrete photon light curves)

ROS PRD 2013
Hogg and Bovy
W. Farr, LIGO LIGO-T1600562; Mandel, Farr, Gair LIGO-P1600187
ROS LIGO T1600208

Distinguishing a discrete model set straightforward





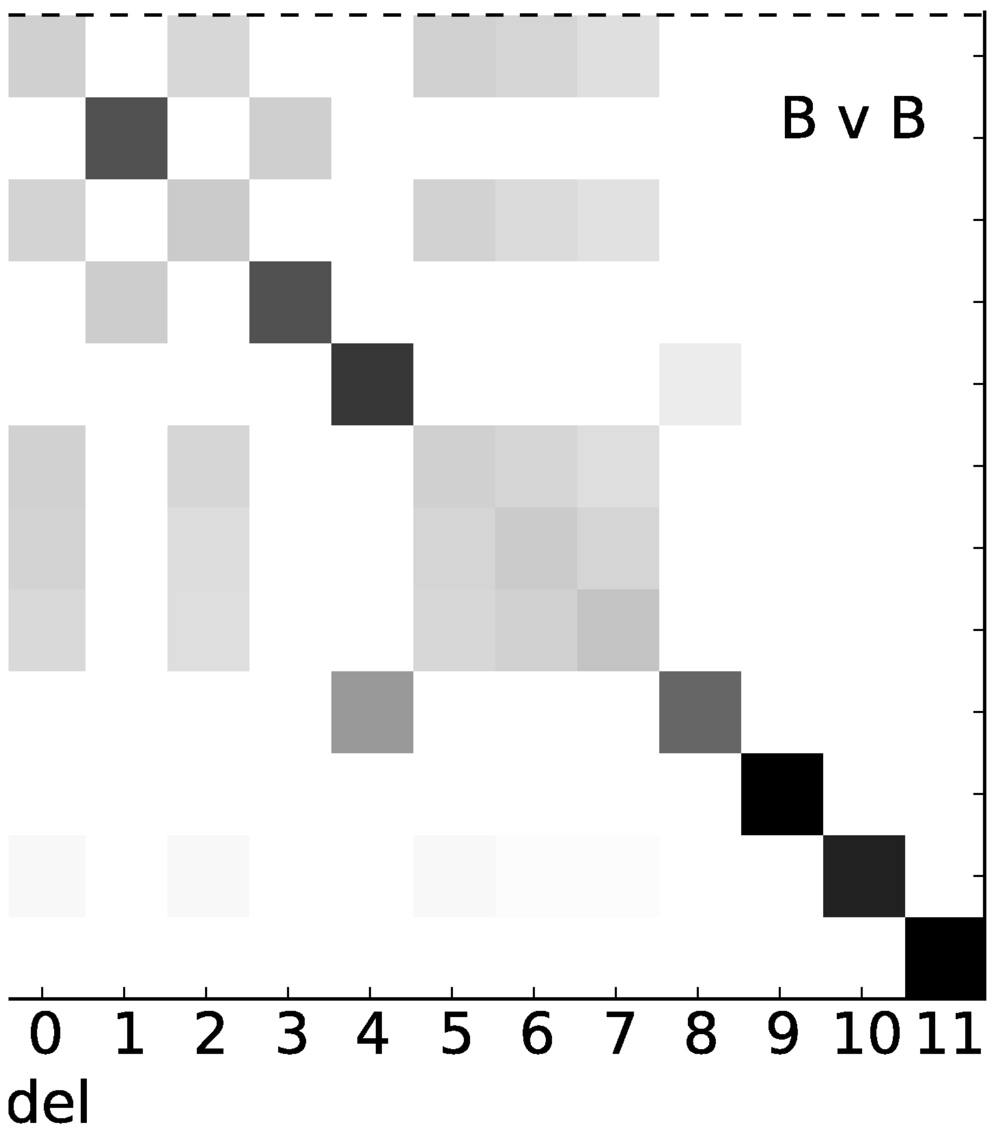
Stevenson, Ohme, Fairhurst (1504.07802), based on Dominik et al 2012
See also Miyamoto et al, GWPAW 2016; Dhani, Mukerjee et al 2016 (LVC meeting)

but this is driven by large rate differences. Rate is highly degenerate with other factors…

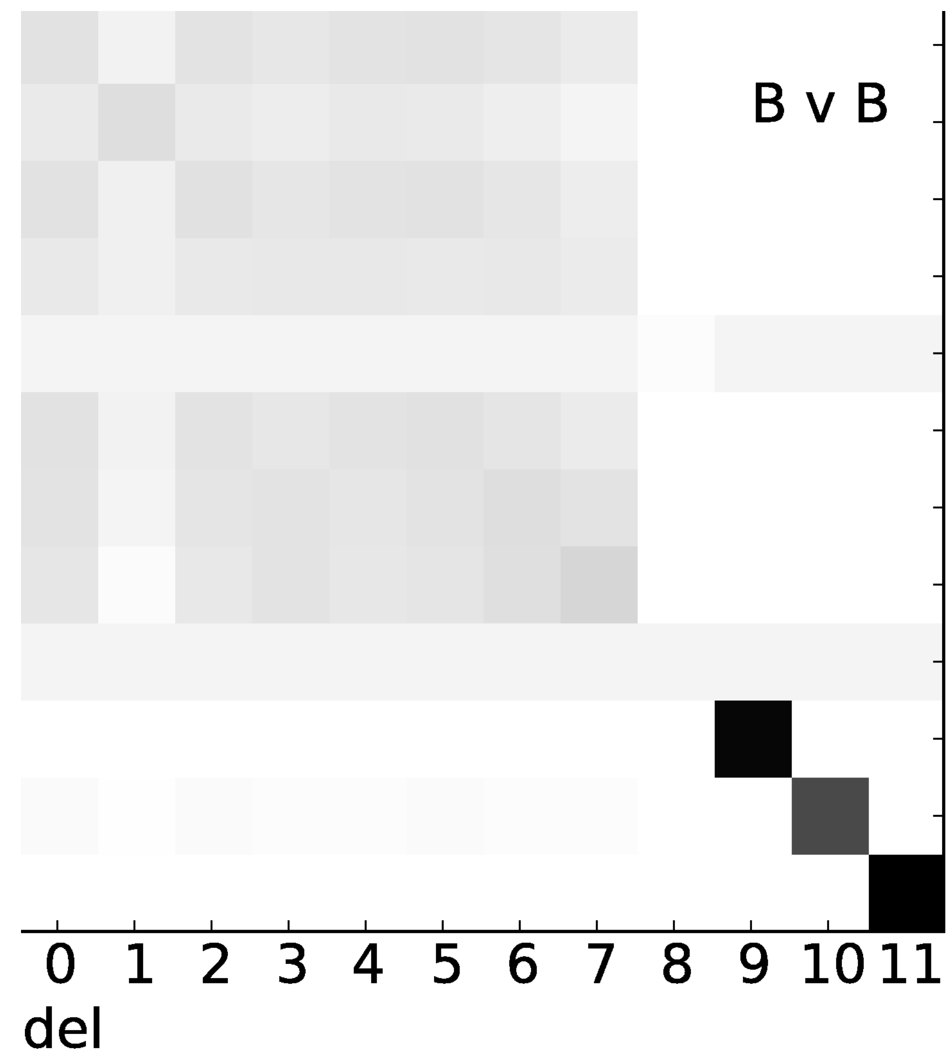
Distinguishing a discrete model set straightforward

- Mass distributions alone are more similar, given measurement error

O2-scale, as before



O2-scale, no rate info



Theory and modeling challenge

- Robust theoretical control (or parameterization) over everything?
  - Massive evolution and J transport (with rotation, winds, extra mass loss)?
  - SN
  - Binary physics:tidal coupling, common envelope, supereddington accretion, …
  - Initial conditions:
    - IMF (at low Z ?)
    - SFR over all time
    - Z distribution

- How much progress can we make in 5 years?

Theory and modeling challenge: by force?

- **Computational limits example: isolated evolution** ROS et al 2010,2008
  - ~ 1000 binaries/hour/core
  - 20 M binaries for an accurate result -> 20k CPU-hours (kSU)
  - With 50 MSU, limited to 2500 simulations (!)…
    - But: easy to optimize: ~ 4000 distinct simulations by ROS et al 2008 (0706.4139), with <<1 MSU

- **Can we find a model matching the data**?
  - “Understood” model with d parameters:
    - use hierarchical search + likelihood interpolation
    - $d(d+1)/2$ new simulations per refinement (factor 4 in n)

$$\text{cost} = \frac{d(d+1)}{2}\log_4 n \quad \sim 200$$ (d=10,n=1000)

    - Conservative — assumes all parameters always significant
  - “Complicated”: brute-force grid in d parameters: **impossible** unless ~ universal

$$\text{cost} = n^{d/2}$$

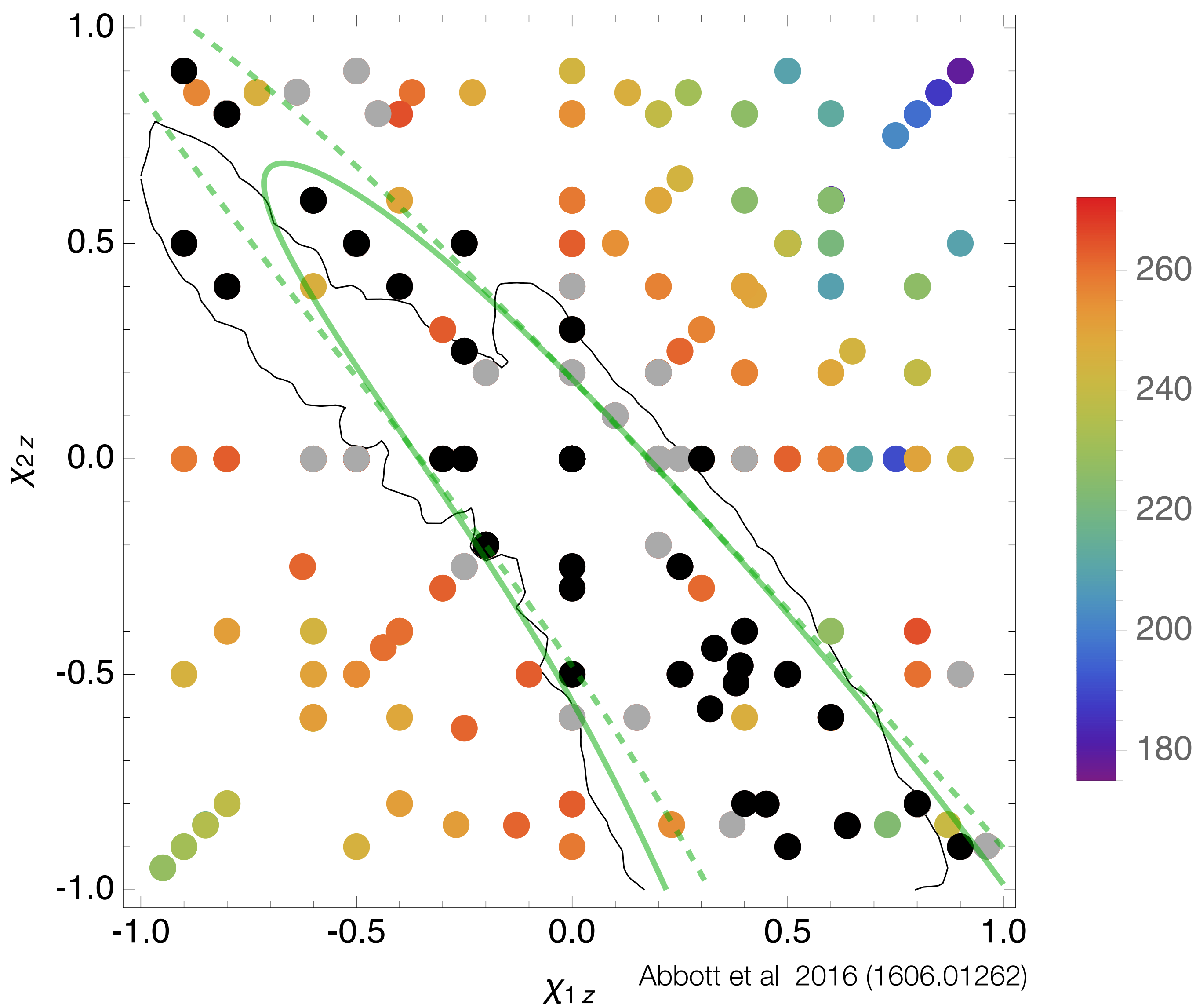
Mass ratio/spin: degenerate or not? [B. Farr talk]

- "Nonprecessing" binaries:
  - Strong degeneracy between q, spin at low(er) mass
  - Limits ability to probe mass-ratio dependent questions:
    - "mass gap" between BH, NS [Farr talk]
    - "Deconvolution" may be possible…requires high #s.

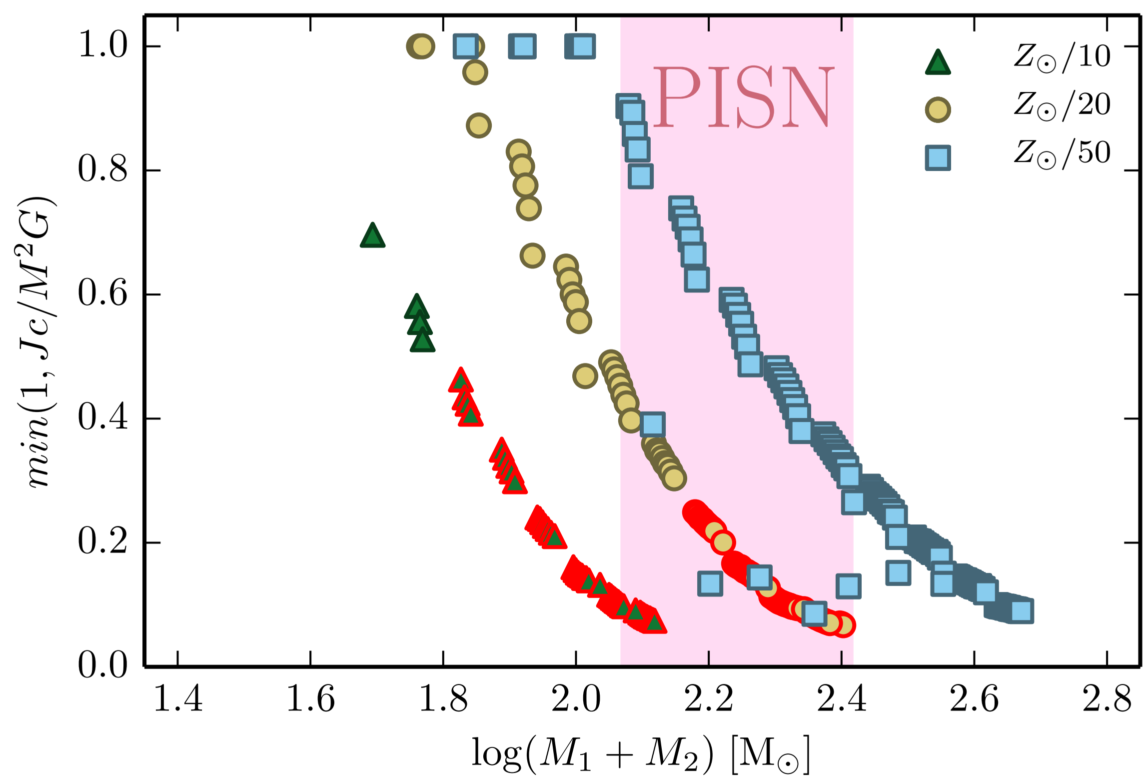
- Identifiably precessing binaries (e.g., BH-NS)
  - Precession **not** always identifiable…but…
  - Spin measurements enable very informative spin distribution
  - Mass ratio accuracy lets you probe mass gaps, NS mass function, …

Beyond the mass distribution: Power of spin

- High mass binaries may be strictly and positively aligned (fallback)
- Low spins required for GW150914…possible? [Kushnir et al]
  - Tells us something about how massive stars evolve? About tides?
  - Or favors dynamics?



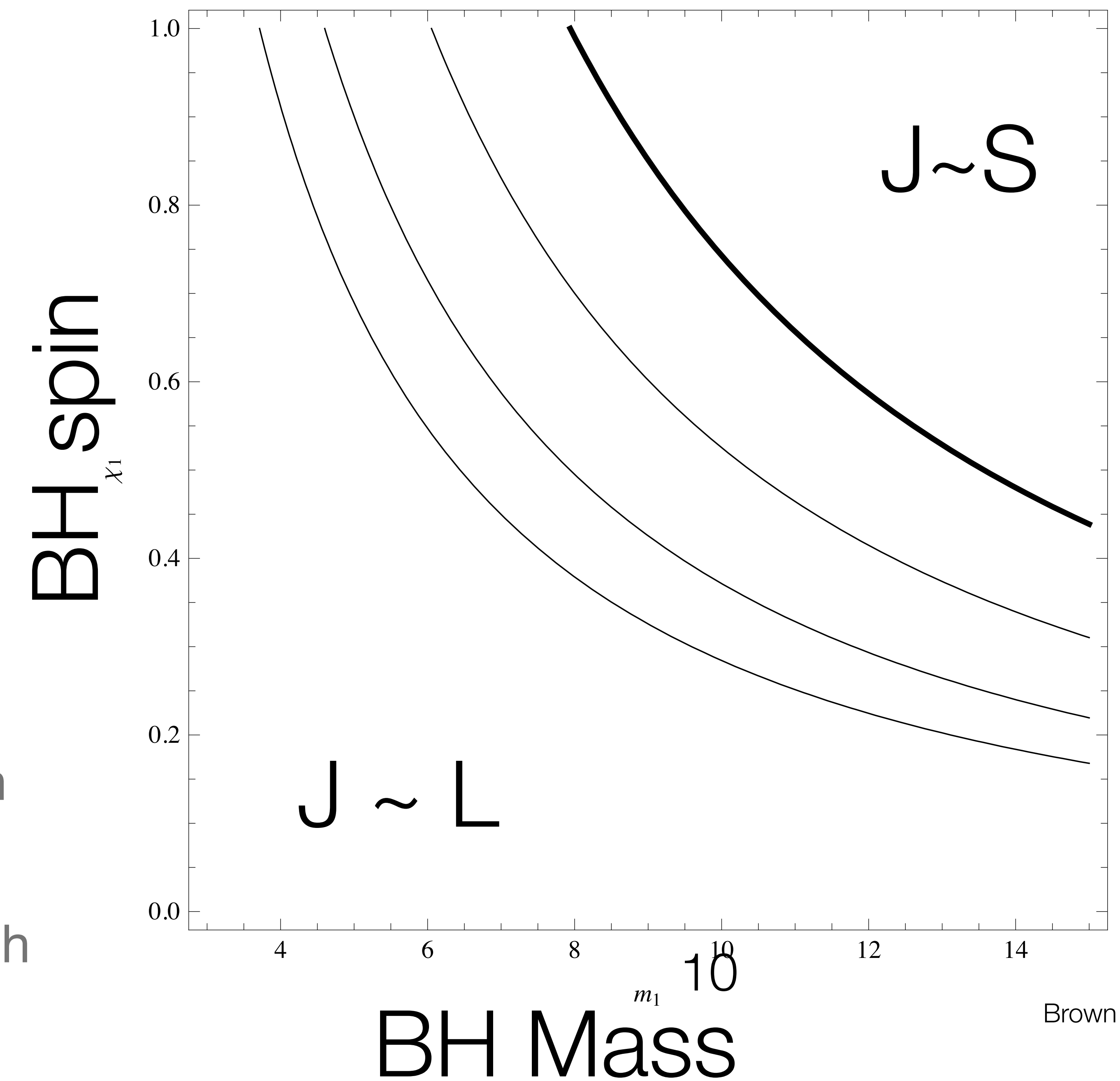
Abbott et al 2016 (1606.01262)



Marchant et al A&A 2016 (1601.03718)

Beyond the mass distribution: Power of spin

- Misalignments trace key kinematic effects (kicks or dynamics)
- “Single spin” (e.g., unequal mass or BH-NS binary):
  - Key misalignment is ~ *conserved* since past infinity.
    - Easy to interpret for astrophysics
    - Very many GW and precession cycles *possible*
    - Strong precession requires high mass ratio and BH spin
- “Two spin” (e.g., comparable mass):
  - both spins accessible

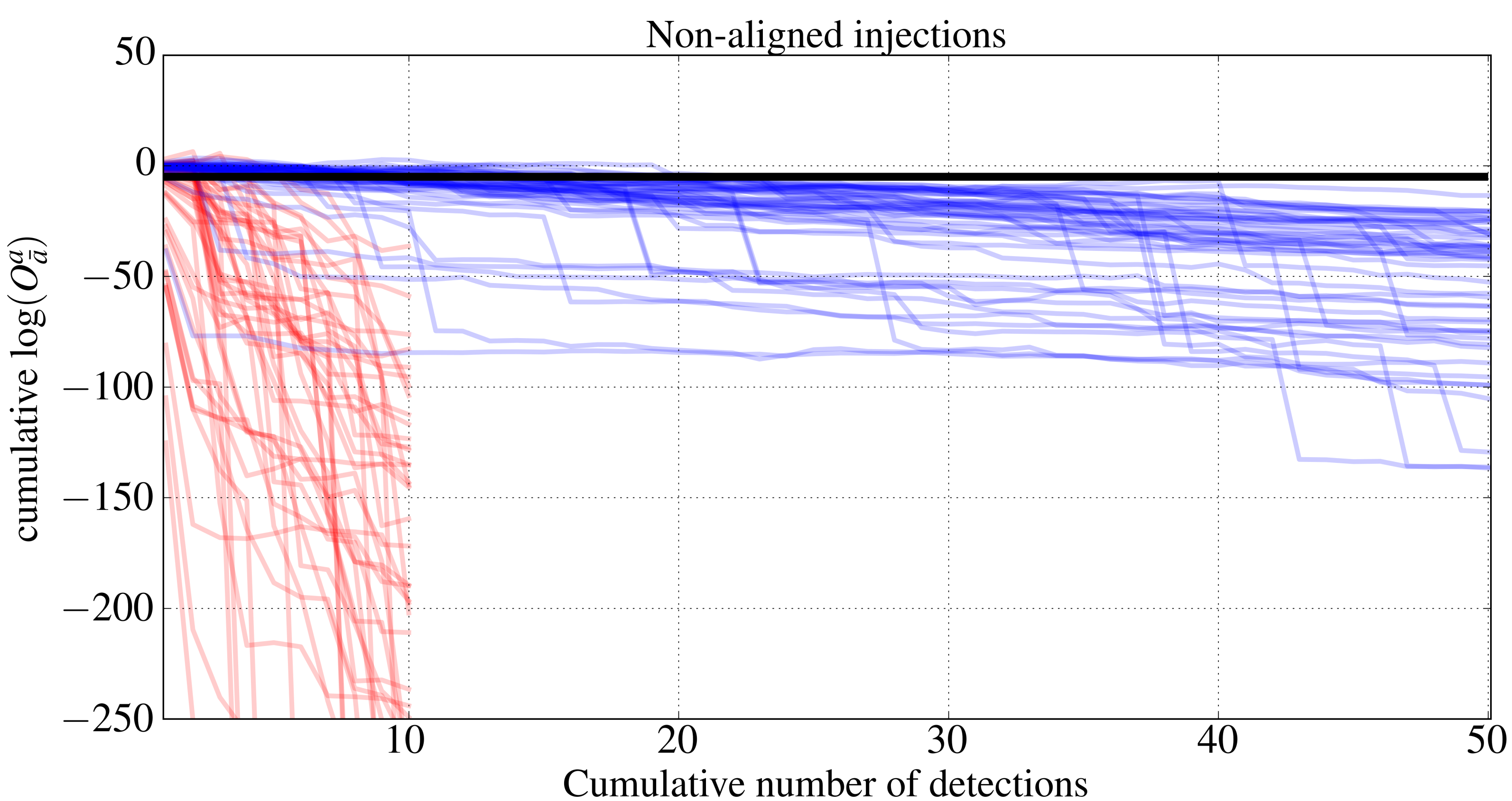


Brown et al 201

Example: Evidence for misalignment

- **Idea**: If almost all binaries are tightly spin-orbit aligned, then dynamical formation channels aren't consistent with the data

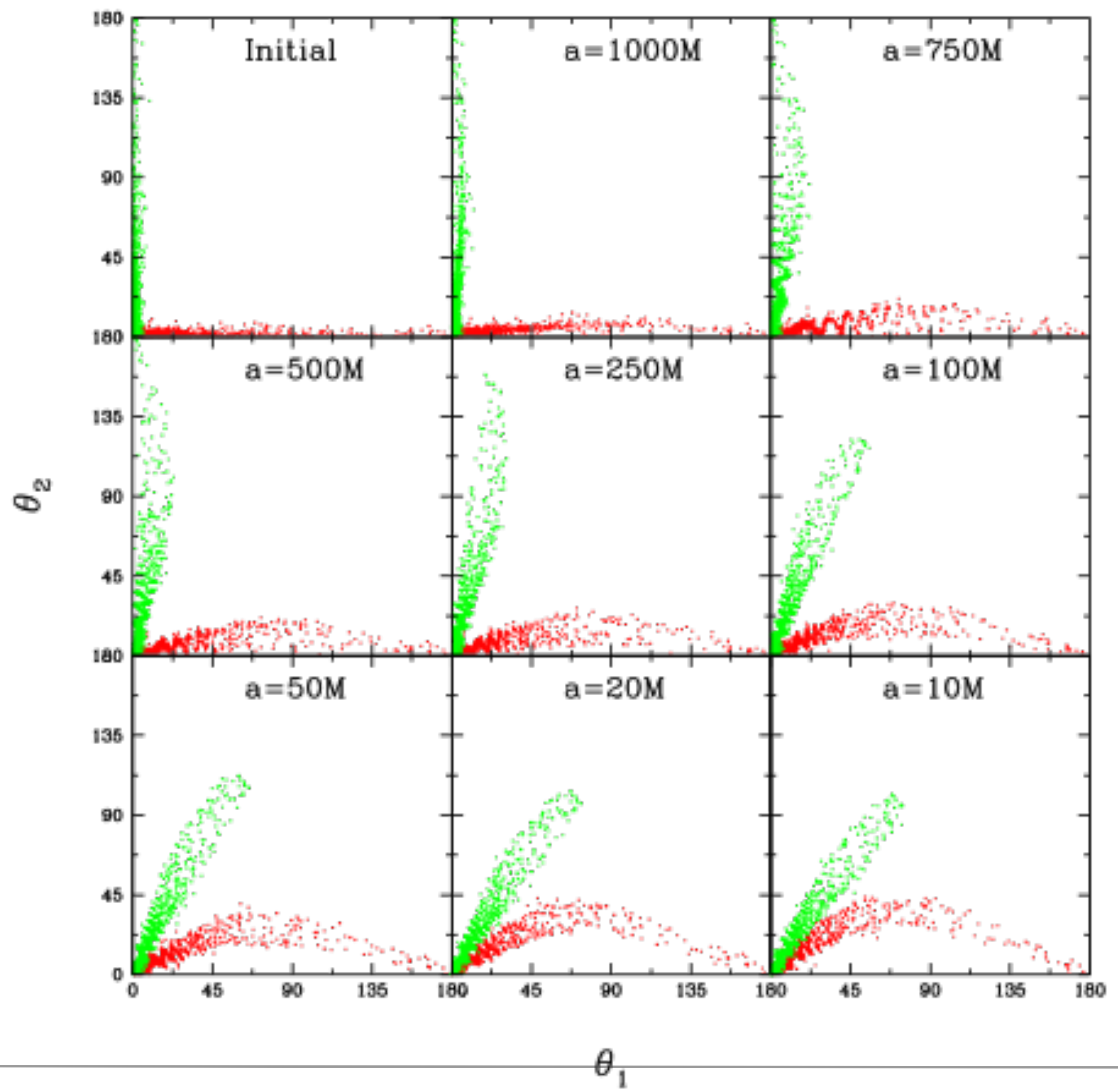
- **One realization of this idea**: odds ratio for aligned vs generic
  - Tight constraints on presence of misalignment, very quickly



Vitale et al 1503.04307; see also prior work

Interpreting spin misalignment

- **2-spin systems have strong spin-spin interactions**
- **Very significant, complicated** spin evolution since formation
  - Interpreting LIGO results: Be careful. Not reported at past infinity (yet)
  - Predicting what can be identified: Always evolve forward to LIGO band!

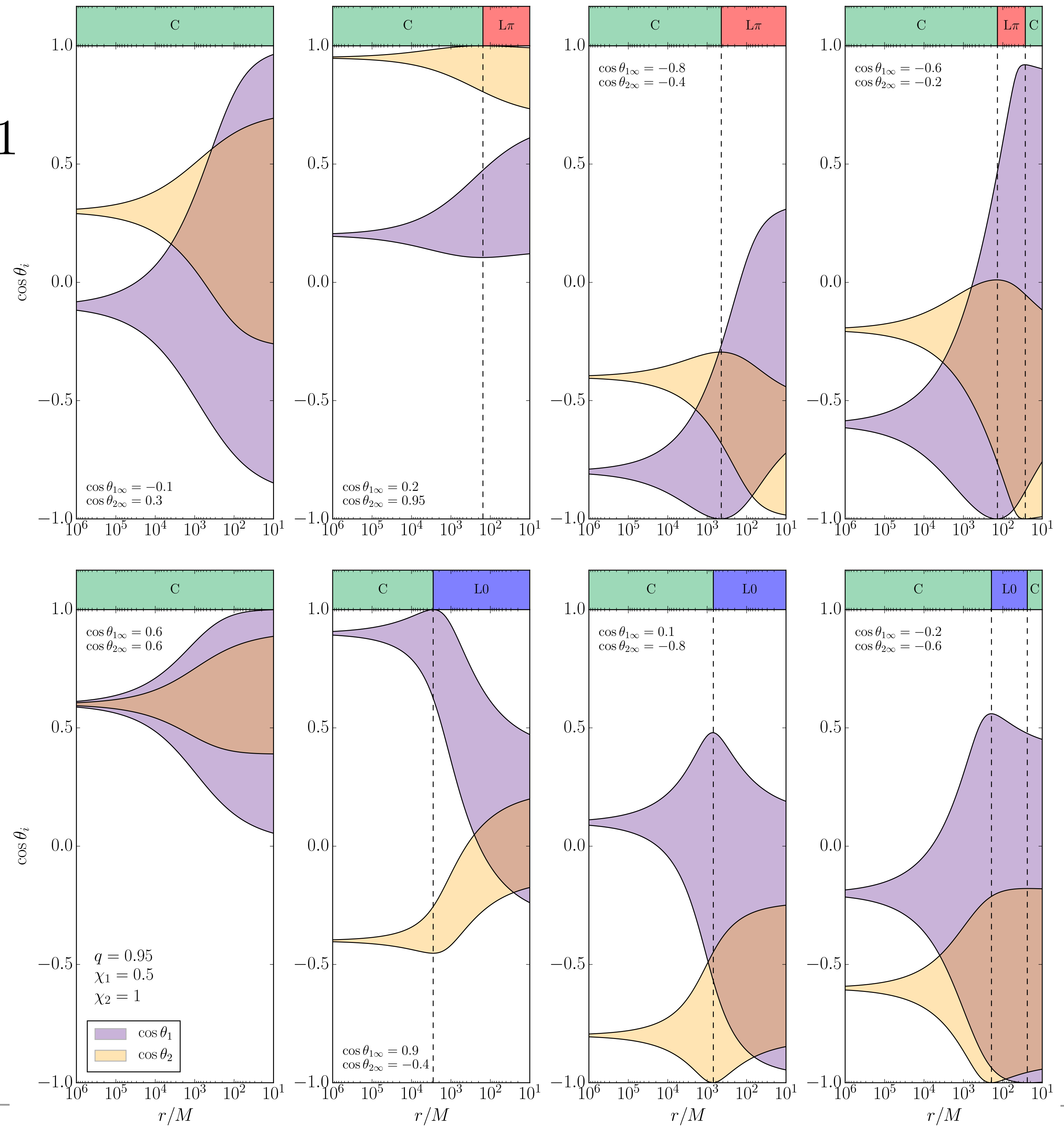


Gerosa et al

Interpreting spin misalignment

- Examples

$$q = 0.95, \chi_1 = 0.5, \chi_2 = 1$$



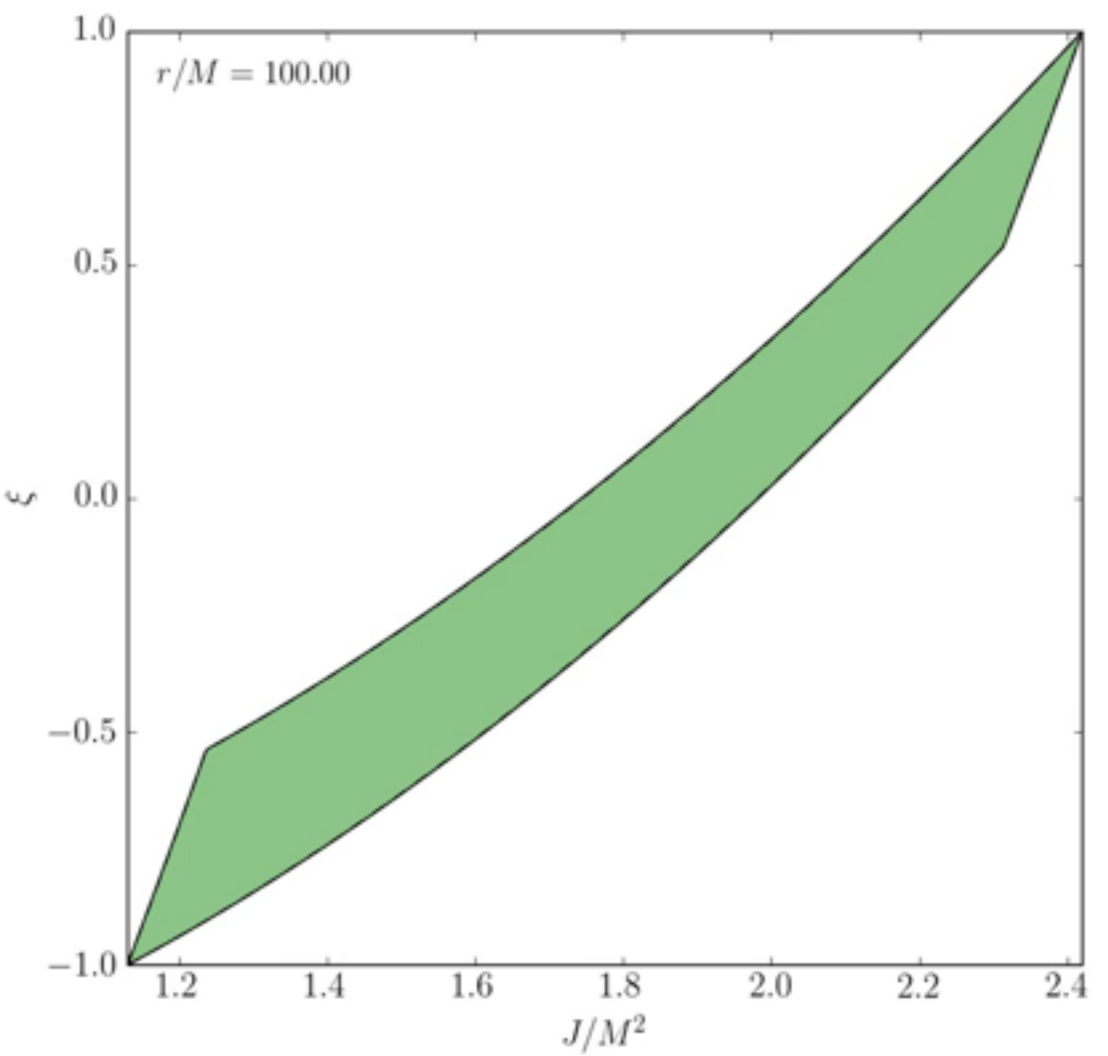
Gerosa et al (1506.30492)

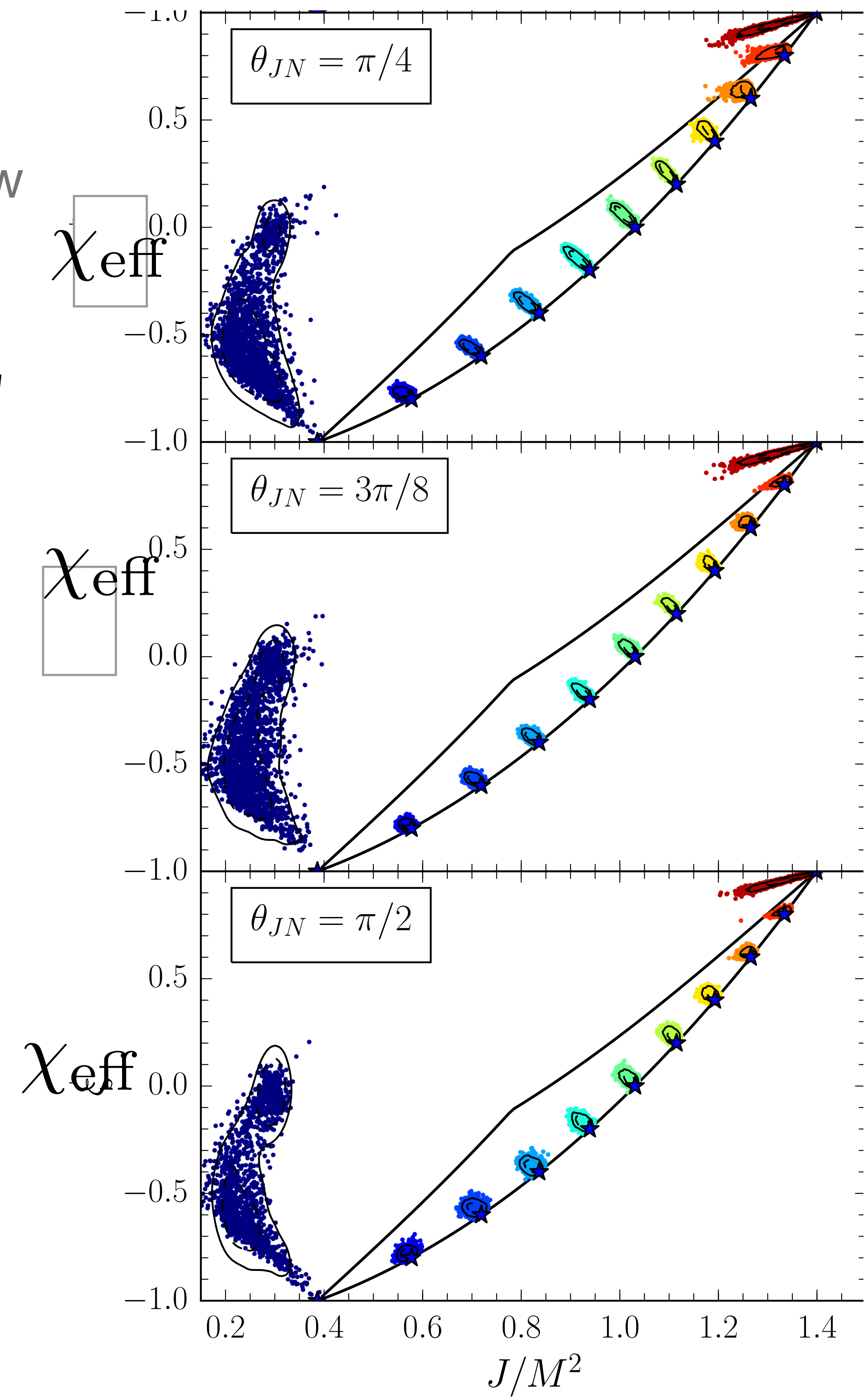
Interpreting spin misalignment

- **2-spin systems:**
  - relationship between tilt angles at infinity and now
  - defined in terms of constants J,L,

$$\chi_{\rm eff} = \frac{\boldsymbol{\chi}_1 m_1 + \boldsymbol{\chi}_2 m_2}{m_1 + m_2} \cdot \hat{\mathbf{L}}$$

  - influence of both spins often accessible





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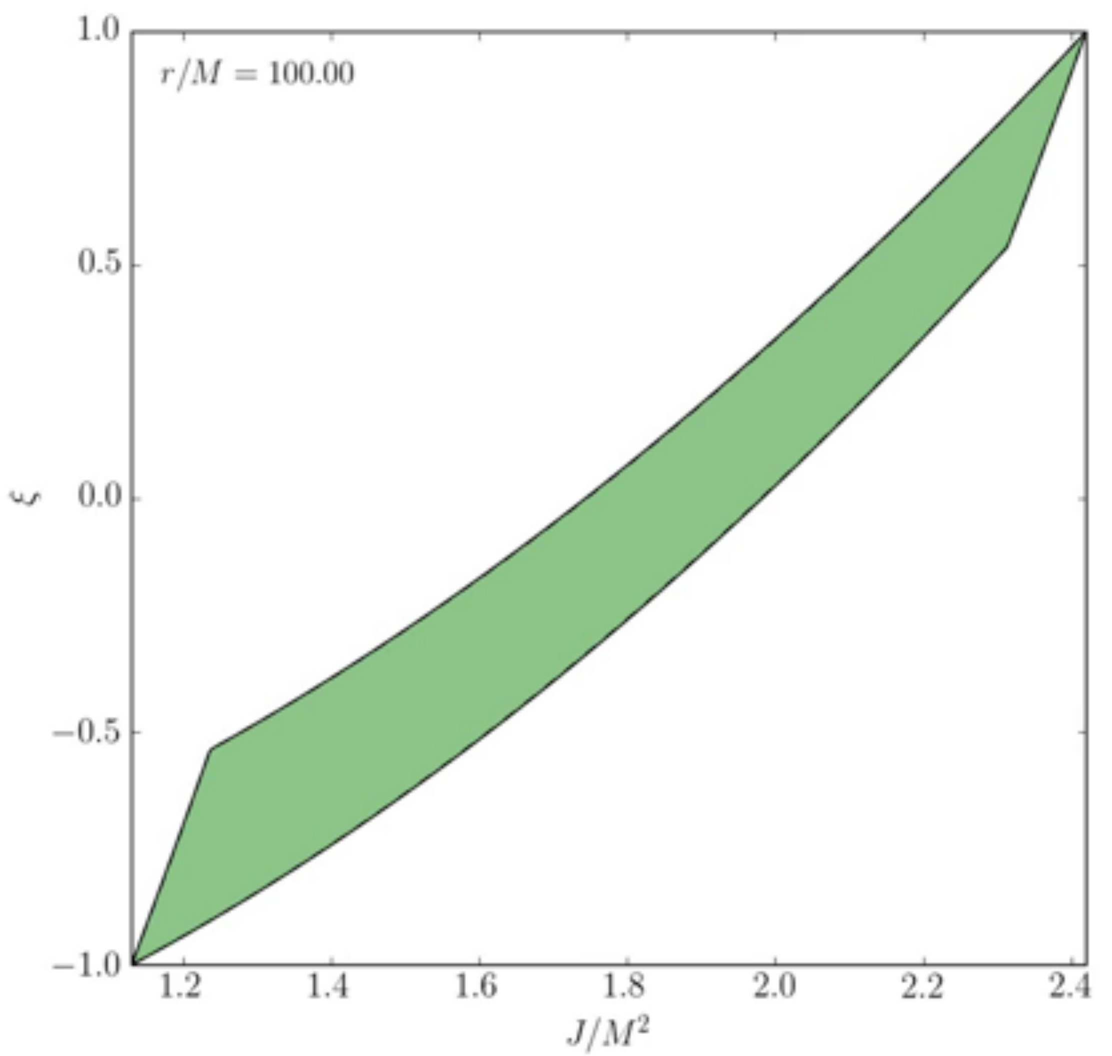
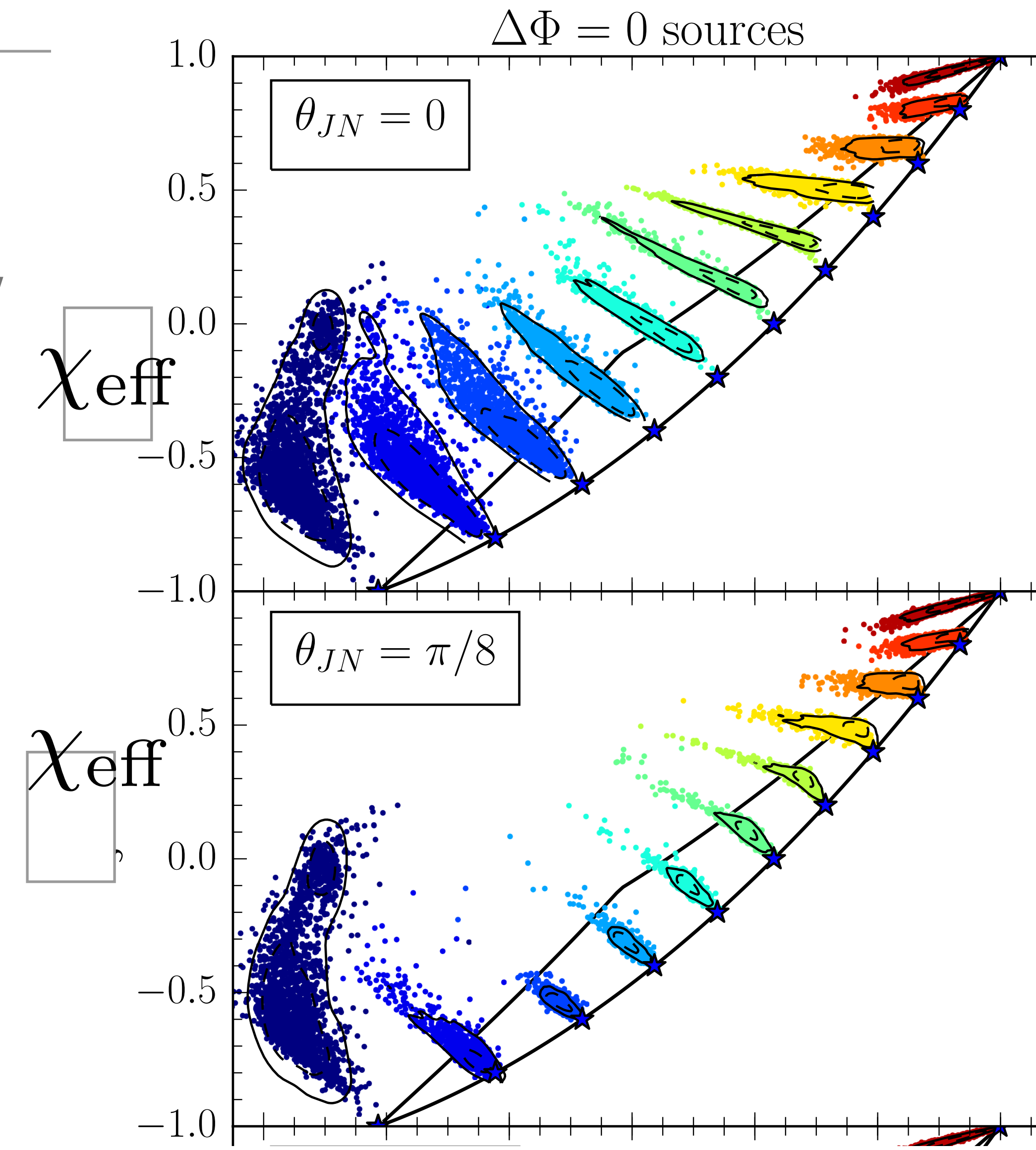




Interpreting eccentricity: Favata GWPAW 2016

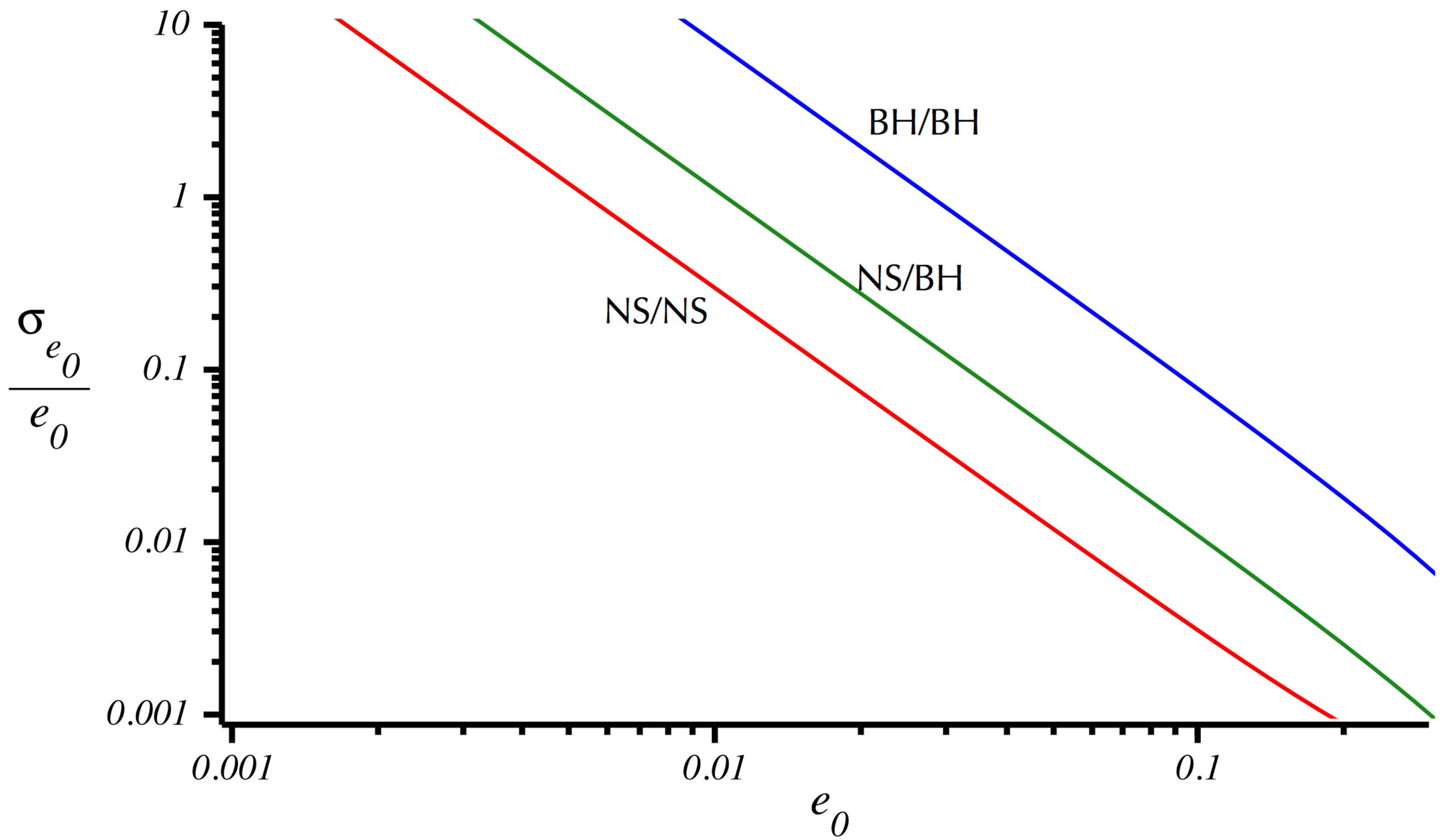
Parameter estimation: measurability of e_0

- We report ***preliminary*** results of a Fisher-matrix study.
- Advanced LIGO design-sensitivity; single-detector.
- Parameter set: $\theta_a = [A, t_c, \varphi_c, M_{tot}, \eta, \chi_2, e_0\,(10\text{Hz})]$

NS/NS 1.25+1.4 $M_\odot$
$\chi_2 = 0.01$
f = 10 Hz to 1000 Hz
SNR = 13.9 (100 Mpc)

BH/BH 10+15 $M_\odot$
$\chi_2 = 0.5$
f = 10 Hz to 372 Hz
SNR = 18.6 (500 Mpc)

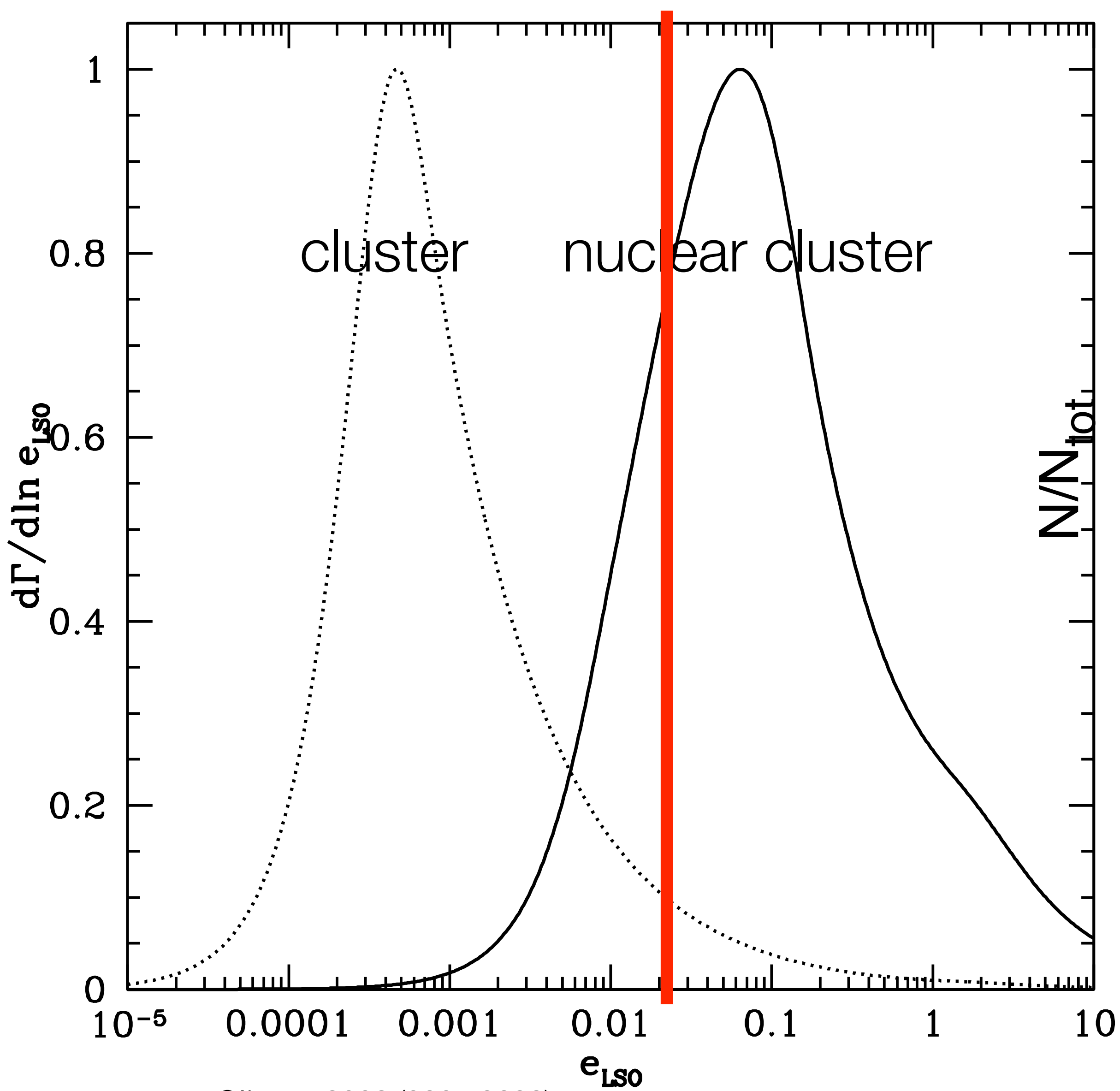
NS/BH 1.4+10 $M_\odot$
χ_2=0.5
f = 10 Hz to 616 Hz
SNR = 15.6 (200 Mpc)



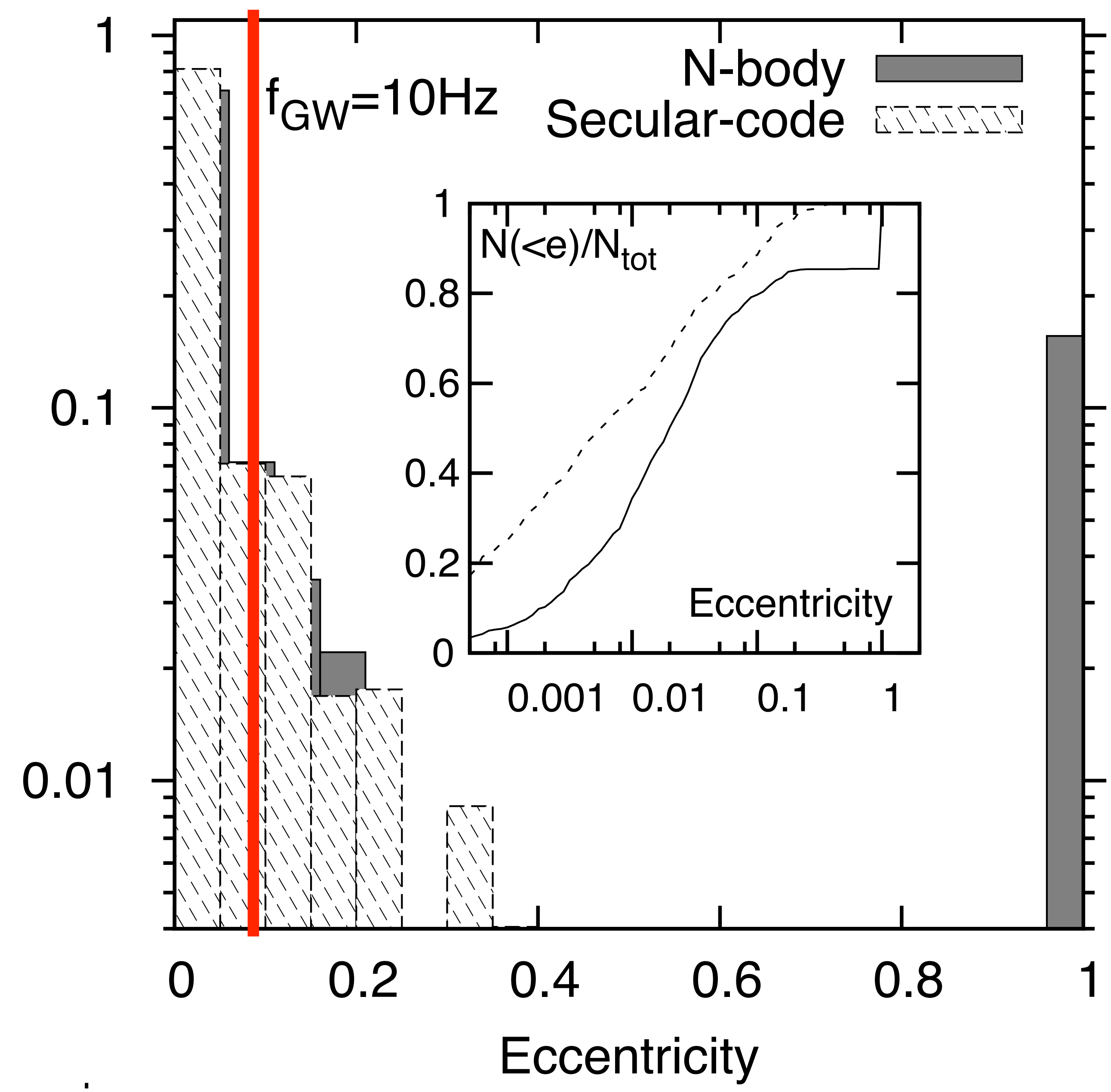
Favata, GWPAW 2016 (gwpaw2016.mit.edu)

Interpreting eccentricity

- Detectable eccentricities may be populated frequently enough



O'Leary 2009 (0807.2638)



Antonini and Rasio 2016

See also: Kozai
primordial (Bird et al PRL 2016)

Remarks on parameter estimation

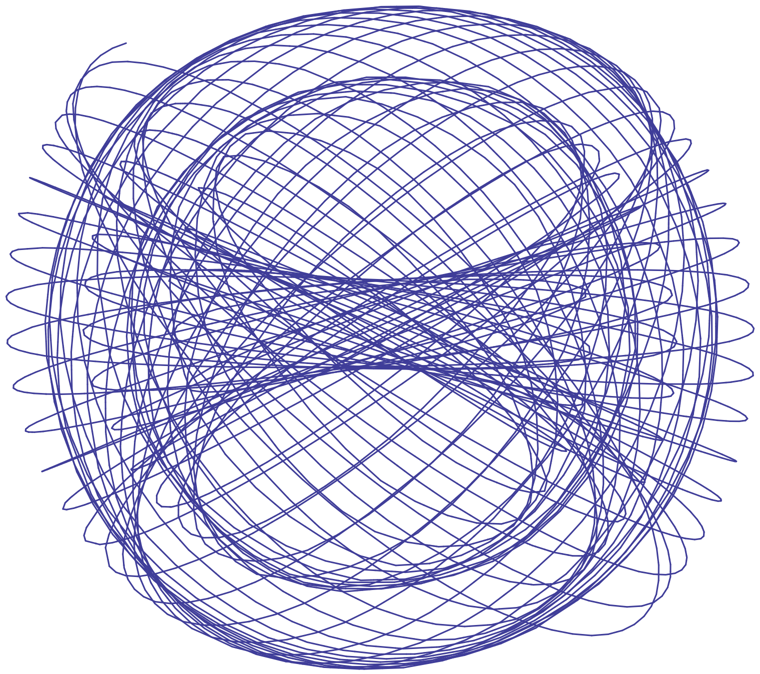
- Calibration error may limit utility of “golden”/exceptional binaries
- No complete model including eccentricity and other effects (spin, precession, IMR)
- Waveform model systematics for precessing spins need careful checking in this regime, biases possible for long signals (e.g., unknown PN terms)
  - Opening angle / evidence for precession can be robust
- If (enough) binaries detected with precession, spin distributions easy. But note many requirements (opportunity to precess in band [q,spin, large L cone], not face on)

Summary

- Enormous potential, and clear path to phenomenology
  - May directly constrain common features to multiple models
    - masses-> SN physics & isolated mass loss, for both field and clusters
  - May enable robust, detail-independent constraints in some cases
    - Spins: consistent spin alignment favors isolated evolution
- Theory challenges significant, but brute force may be possible
- The most easy-to-interpret parameters are hard to measure & use, and GR/astro systematics can limit their utility
- Independent corroboration critical
  - Galactic populations (XRBs, pulsars, WDs, massive star binaries, proper motions)
  - New resources (e.g., GAIA) and perspectives (e.g., ionizing photons; XRLF of low-Z galaxies)

Bonus slides: Supplementary discussions

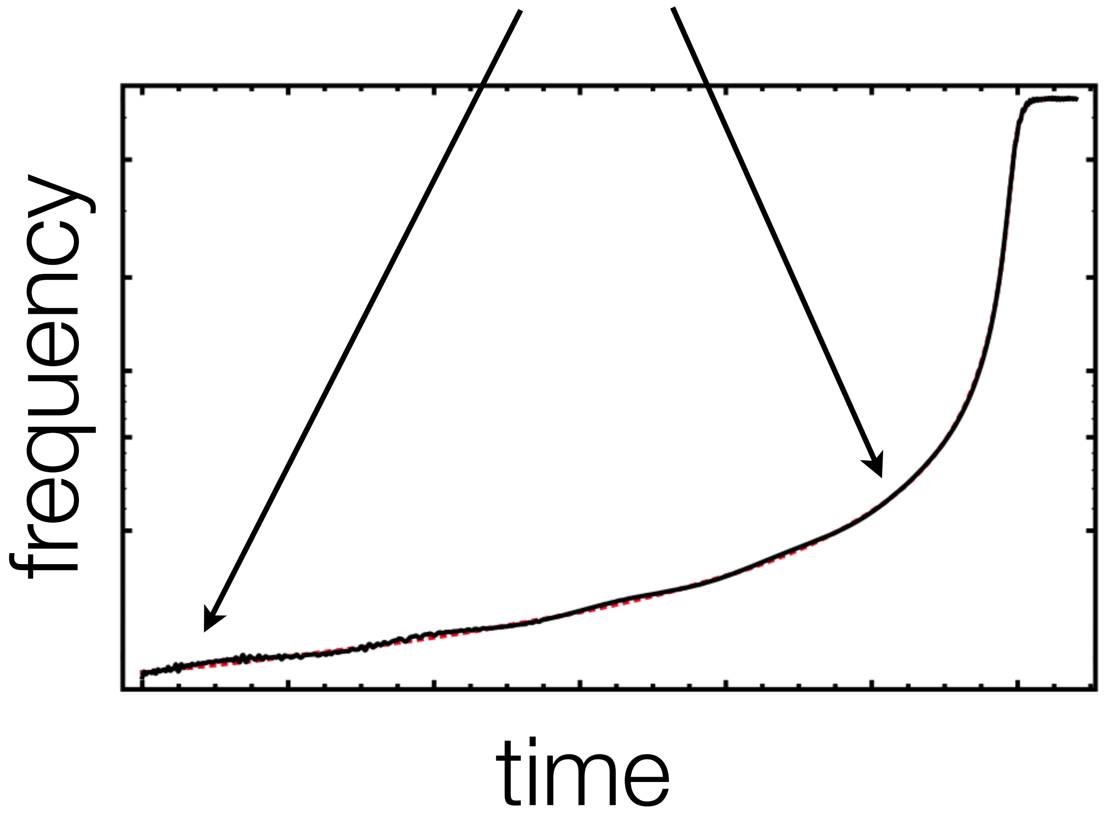
What about measuring spins in BH-NS binaries

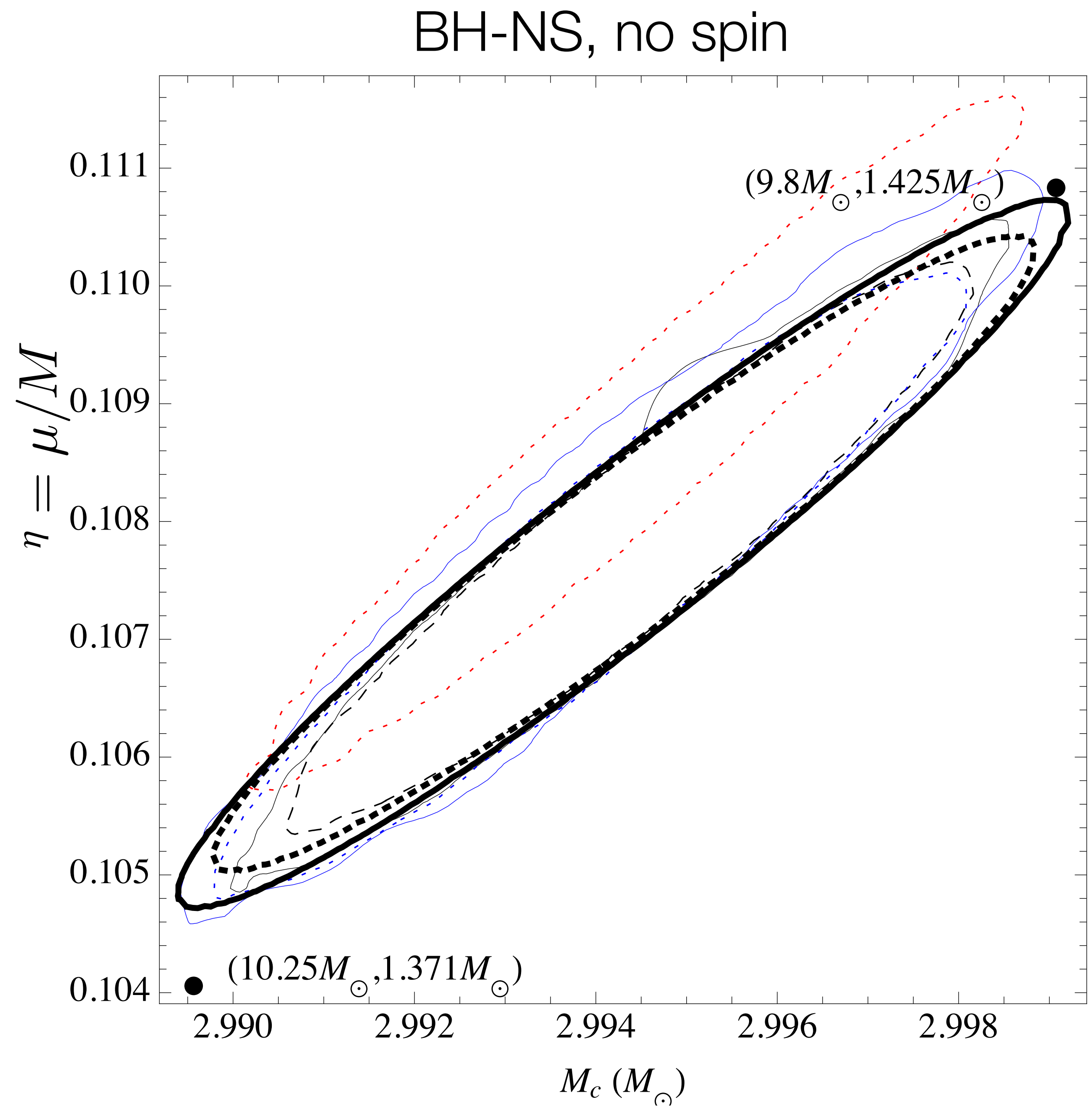


Based on: ROS et al arxiv:1403.0544
Prior work: ROS et al arxiv:1308.4704
Cho et al PRD 87, 24004 (2013)

One thing we measure reliably: “Chirp” mass

- Shrinking binary “chirps”





- Chirp rate (df/dt) set by “chirp mass”
  - “Exactly” measurable

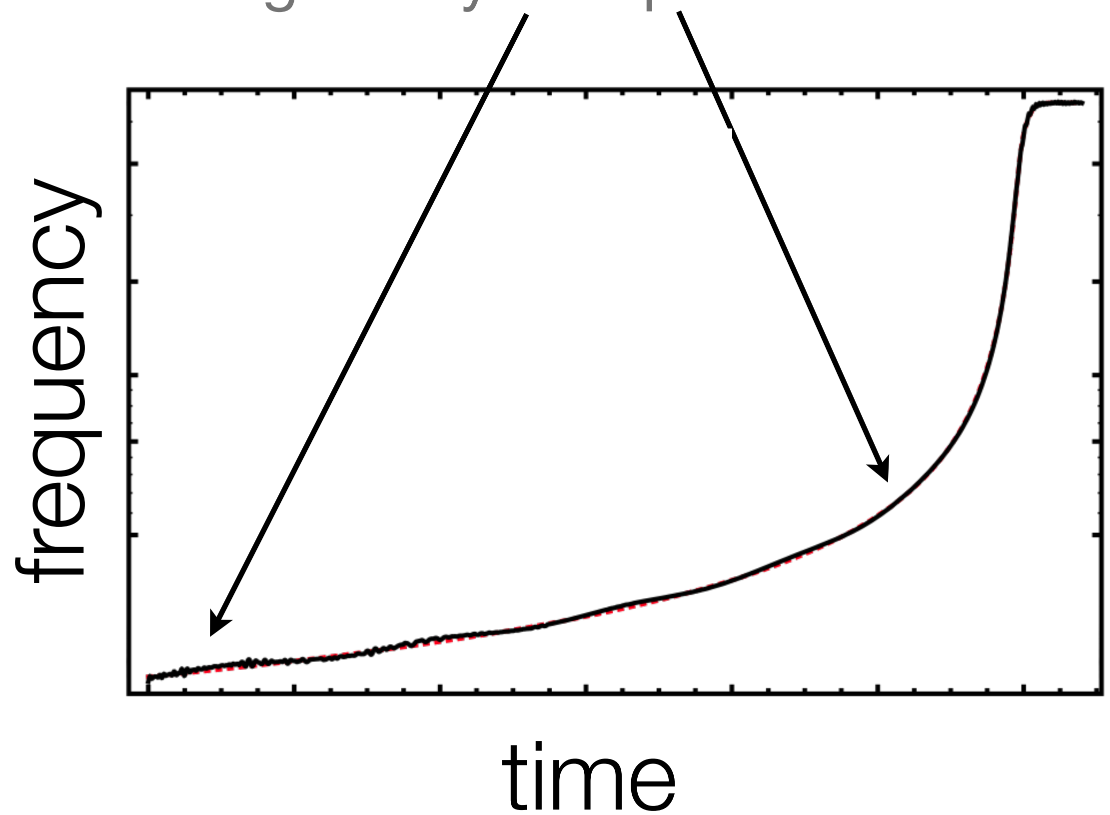
- Fisher matrix

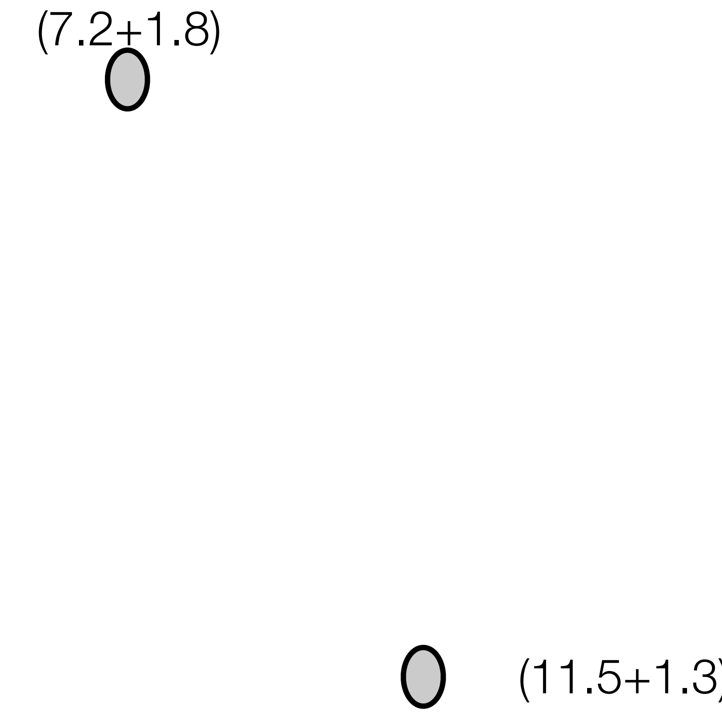
$$\Gamma_{ab} = 2\int_{-\infty}^{\infty} \frac{\partial_a h^* \partial_b h}{S_h} df$$

$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

What can we learn from the “chirp”?

- Shrinking binary “chirps”





- Measure masses, spins, tides, ...
  - Adding parameters (spin) degrades measurement accuracy
- Fisher matrix

$$\Gamma_{ab} = 2\int_{-\infty}^{\infty} \frac{\partial_a h^* \partial_b h}{S_h} df$$

ROS et al 2013

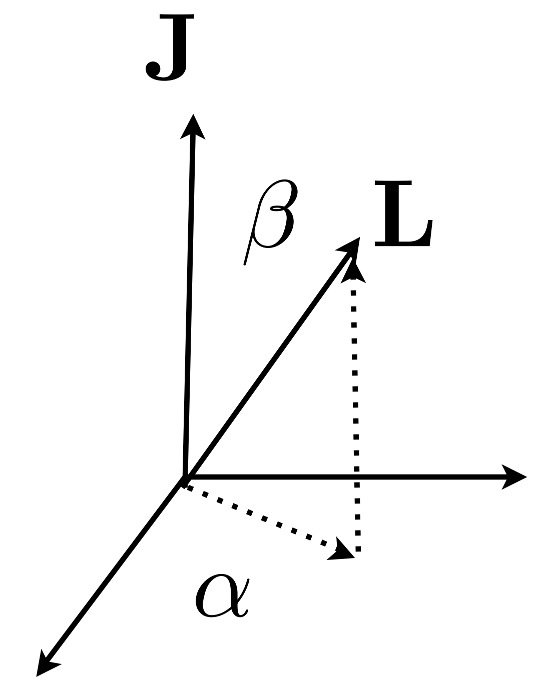
Approximate precessing kinematics

$$\partial_t \mathbf{X} = \mathbf{\Omega}_X \times \mathbf{X}, \quad \mathbf{X} = \mathbf{L}, \mathbf{S}_1, \mathbf{S}_2$$

- Example: one spin

$$\frac{d\hat{\mathbf{L}}}{dt} \simeq \frac{\mathbf{J}}{r^3}\left(2 + \frac{3m_2}{2m_1}\right) \times \hat{\mathbf{L}}$$

$$|\mathbf{J}| = |\mathbf{L} + \mathbf{S}|$$



[Apostolatos et al 1994]

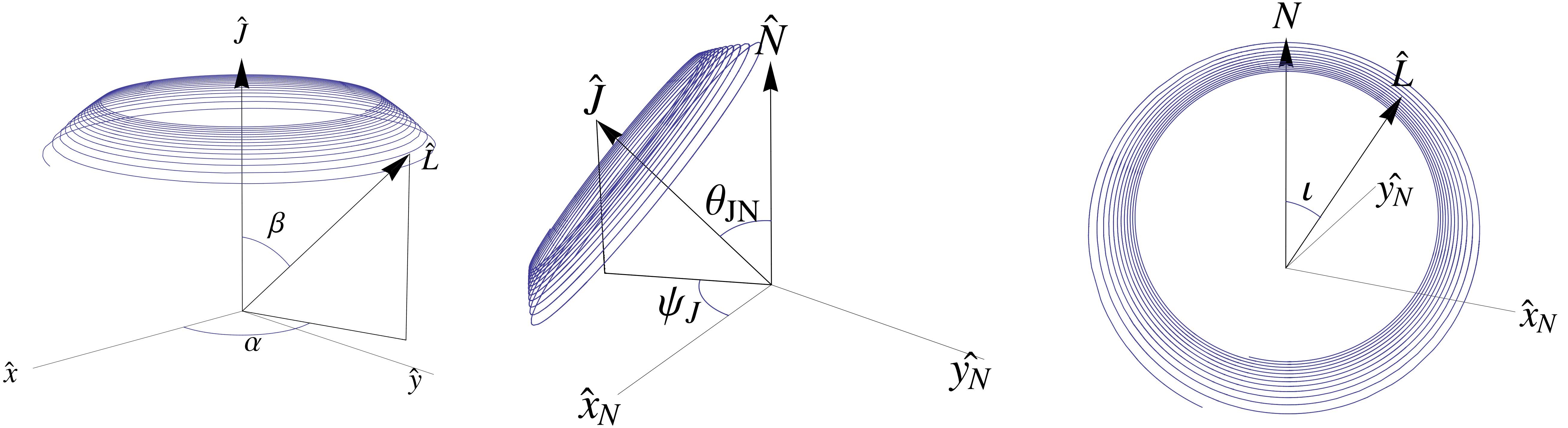
- Extend known single-spin precession solutions
  - $\beta(v)$: set by |L| and (conserved) L.S
  - α : precession phase

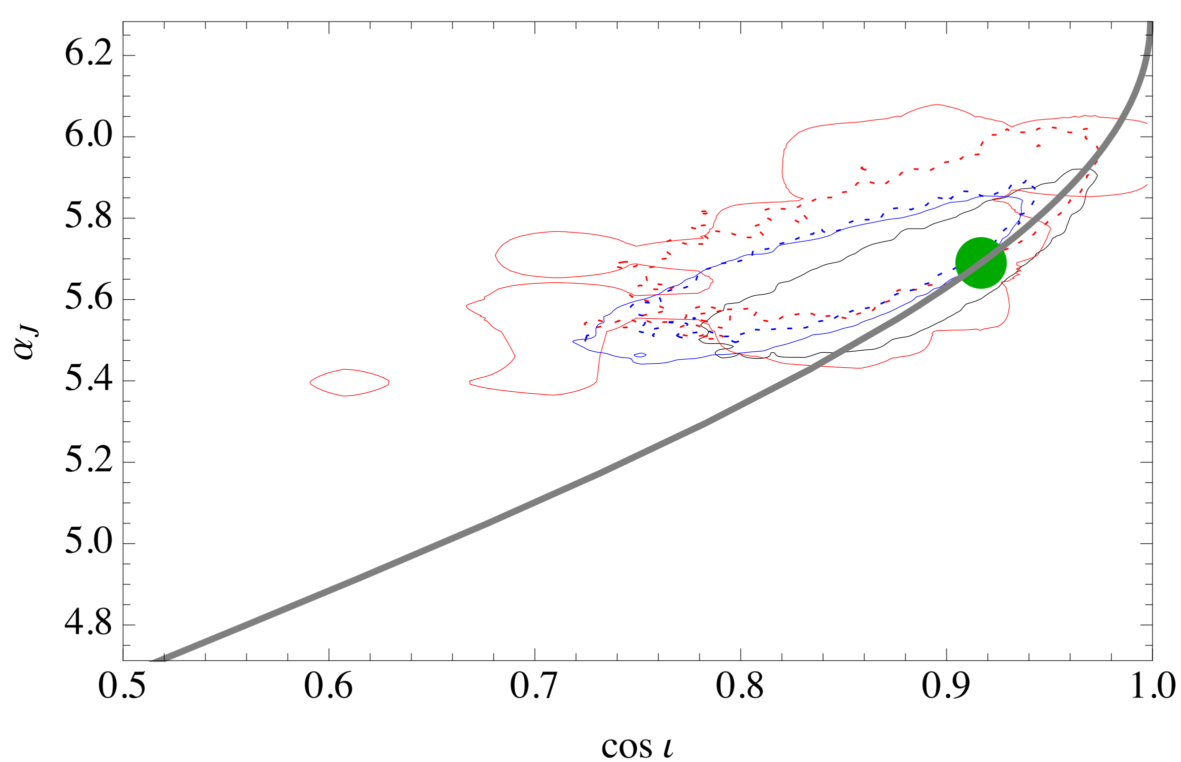
$$= \int \Omega_p \frac{dt}{dv} dv$$

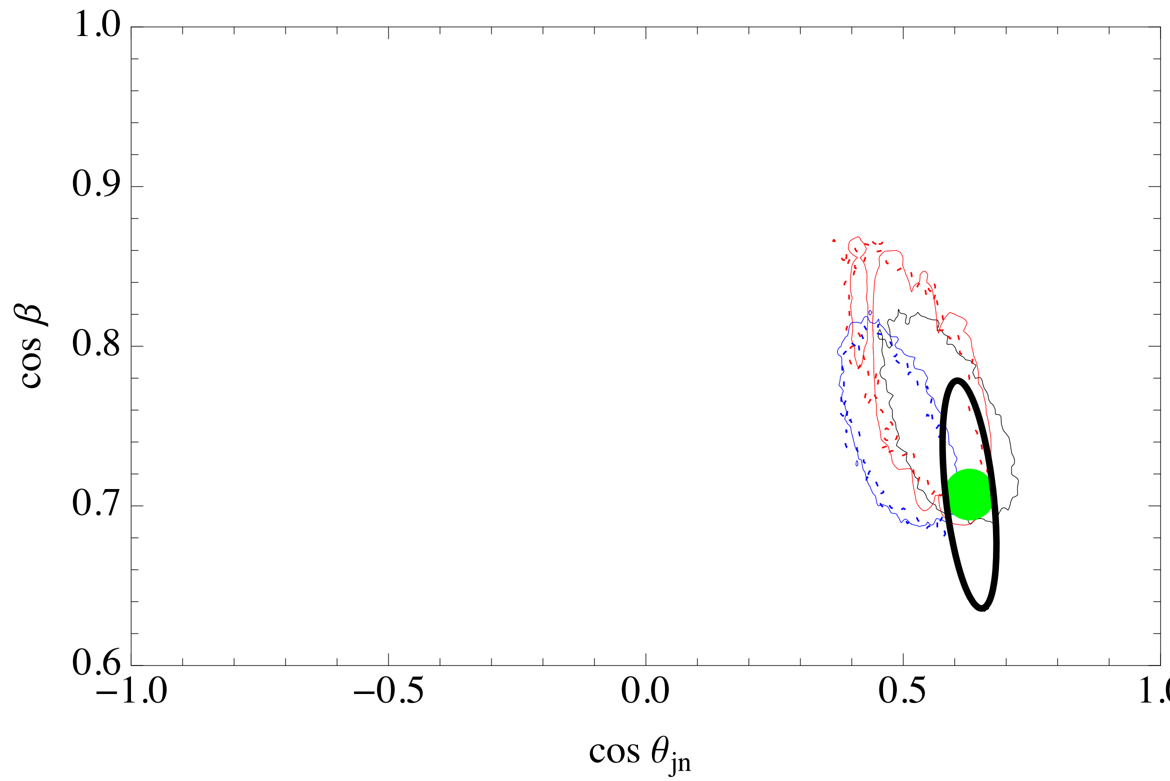
: **analytic approximations** exist

Apostolatos et al 1994; Lundgren and ROS 2013

Sample precessing geometry: BH-NS

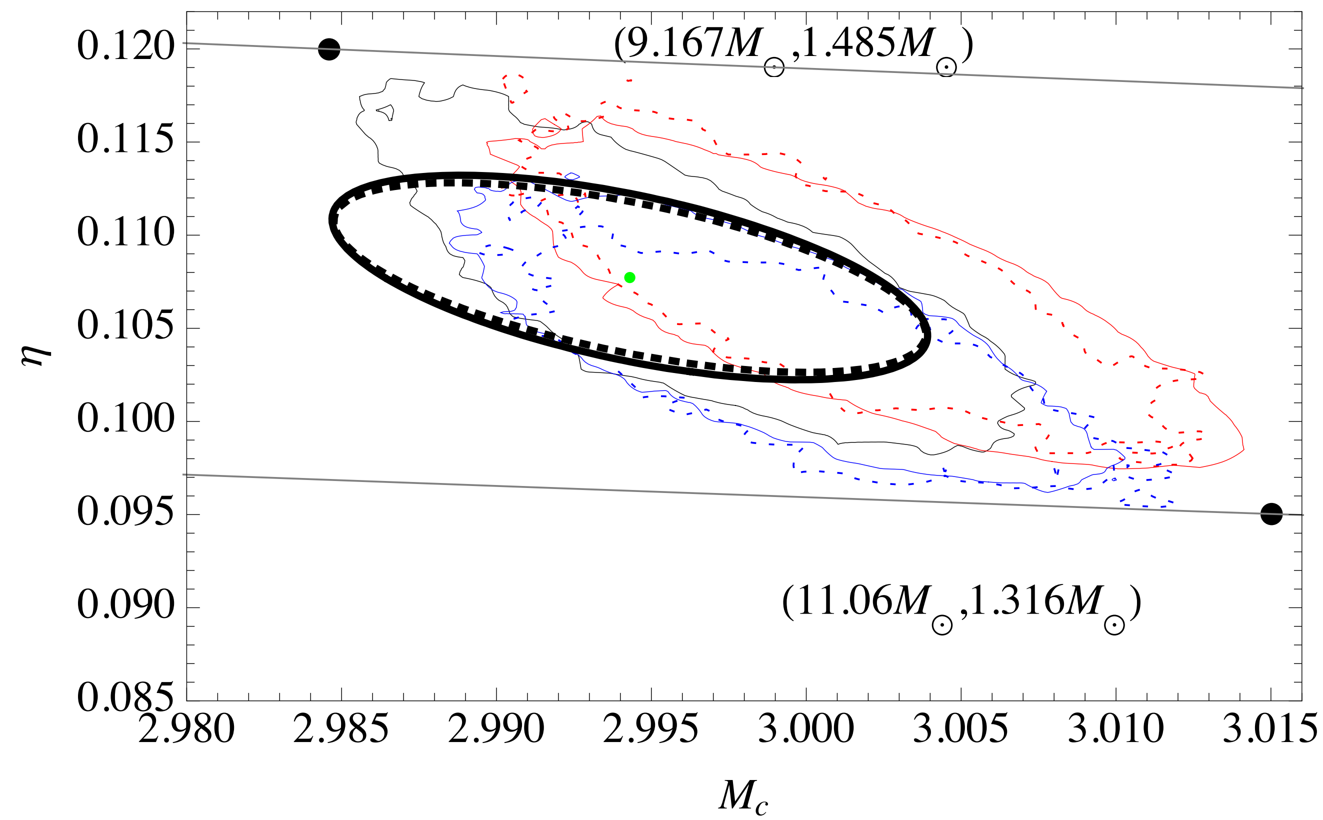


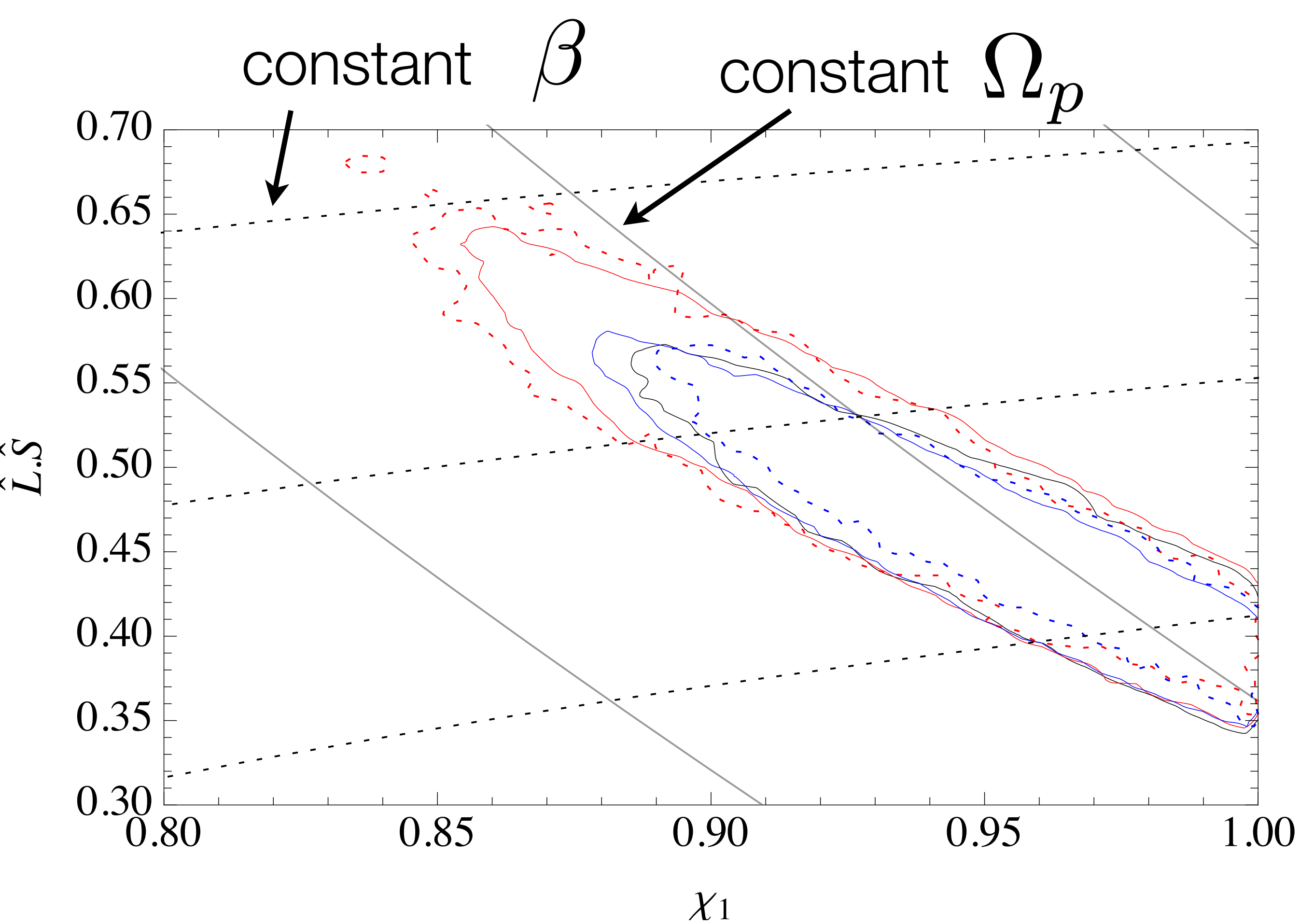


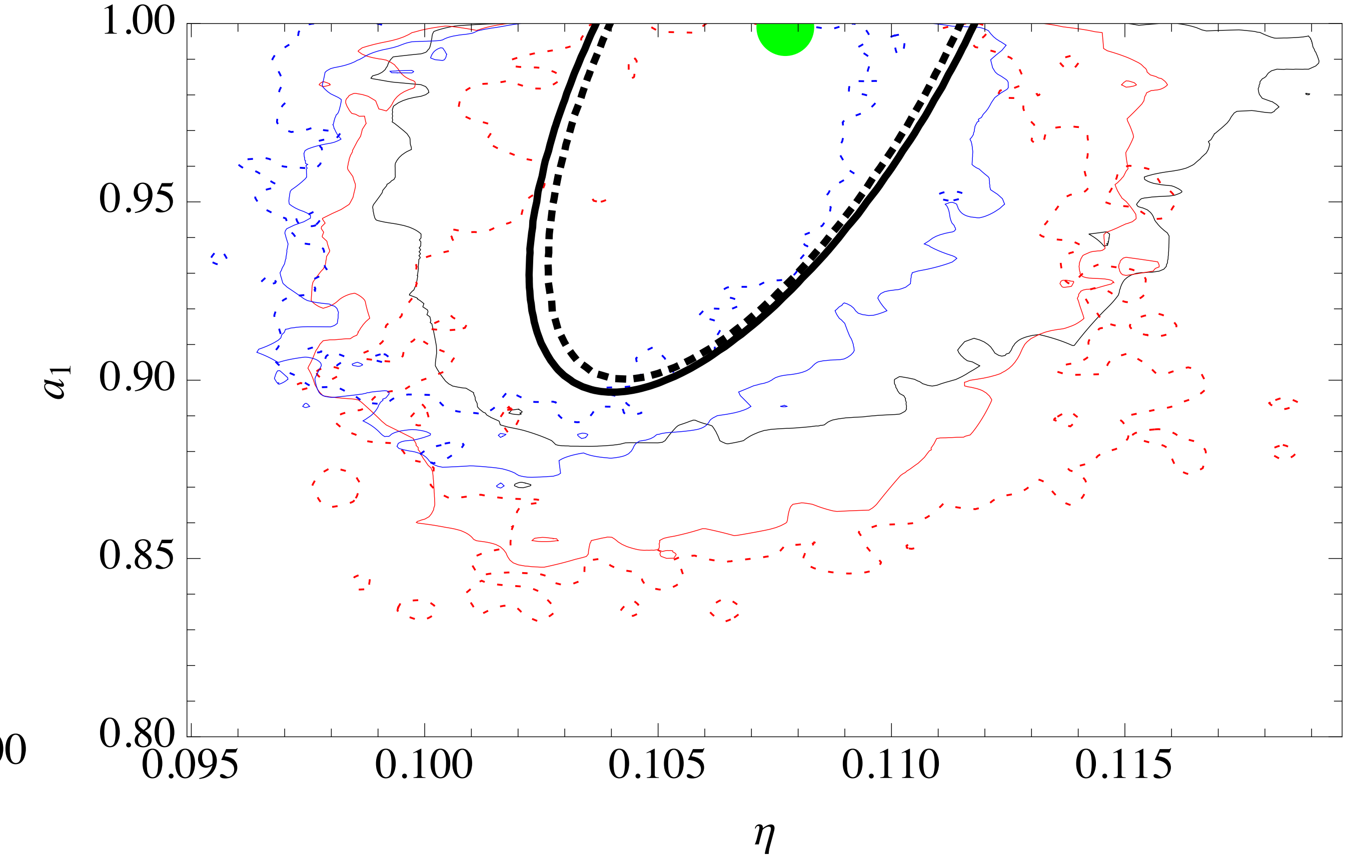


Results 2: Intrinsic parameters

- Chirp rate, precession rate set limits
  - More cycles -> more accuracy
- Precession enables measurements
  - Spin-orbit misalignment
  - Mass ratio $\times 3$ better







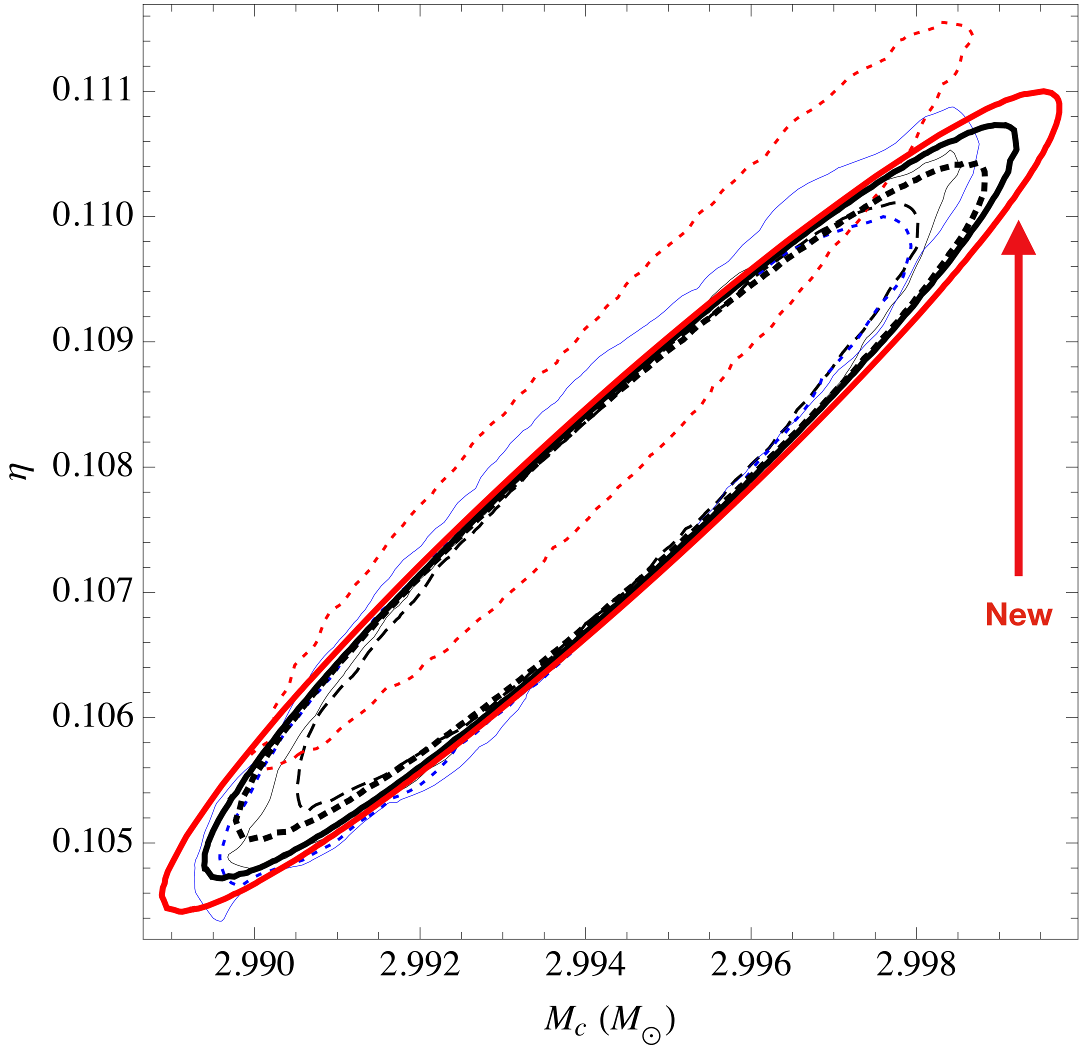
Simple approximate (intrinsic) Fisher matrix

$$\rho^2_{2ms} \equiv |{}_{-2}Y_{2m}(\theta_{JN}) d^2_{m,2s}(\beta)|^2 \int_0^\infty \frac{df}{S_h(f)} \frac{4(\pi \mathcal{M}_c{}^2)^2}{3d_L^2} (\pi \mathcal{M}_c f)^{-7/3}$$

- Amplitude
- Angular dependence
- Phase

$$\hat{\Gamma}^{(ms)}_{ab} = \frac{\int_0^\infty \frac{df}{S_h(f)} (\pi \mathcal{M}_c f)^{-7/3} \partial_a(\Psi_2 - 2\zeta - ms\alpha)\partial_b(\Psi_2 - 2\zeta - ms\alpha)}{\int_0^\infty \frac{df}{S_h(f)} (\pi \mathcal{M}_c f)^{-7/3}}$$

- Good:
  - Easy to calculate
  - Similar to nonprecessing (weighted average)
  - Intuition about separating parameters
- "Bad"
  - Ansatz / approximation
  - At best, retains all degeneracies of full problem (phases, …)



9d MCMC vs 4d Fisher
ROS et al 2014 (PRD 89 064048)

Simple approximate (intrinsic) Fisher matrix

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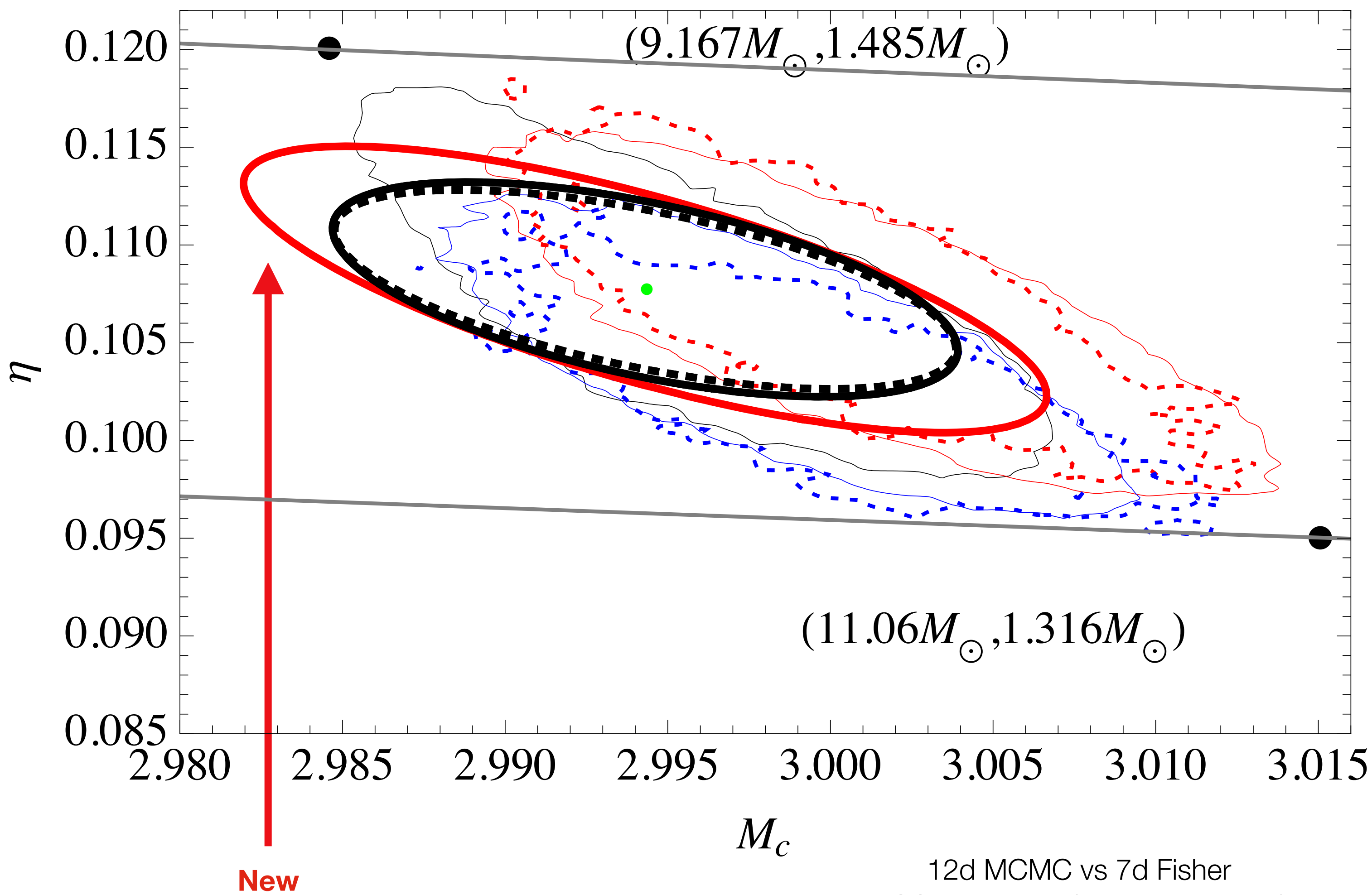

12d MCMC vs 7d Fisher
ROS et al 2014 (PRD 89 102005)