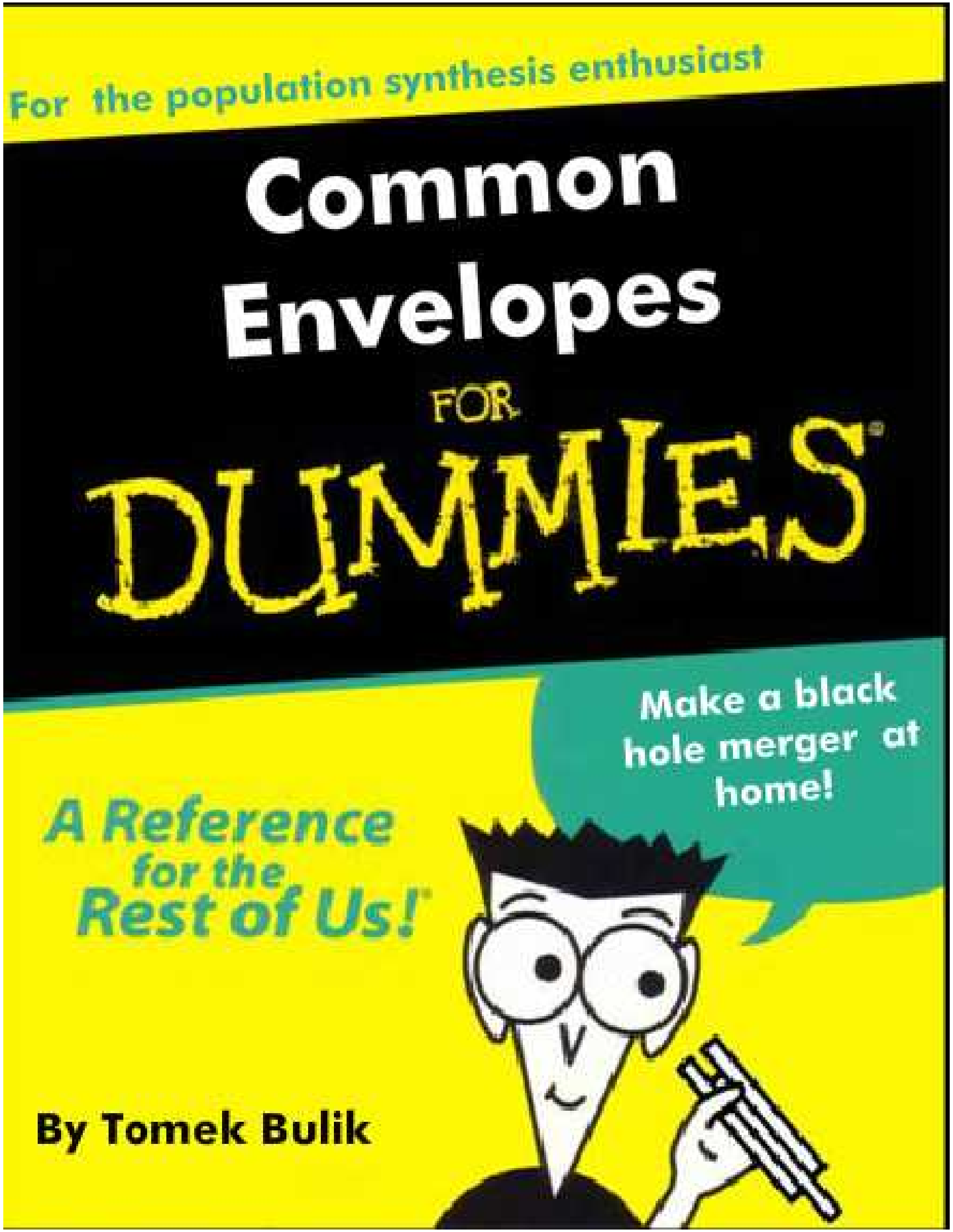
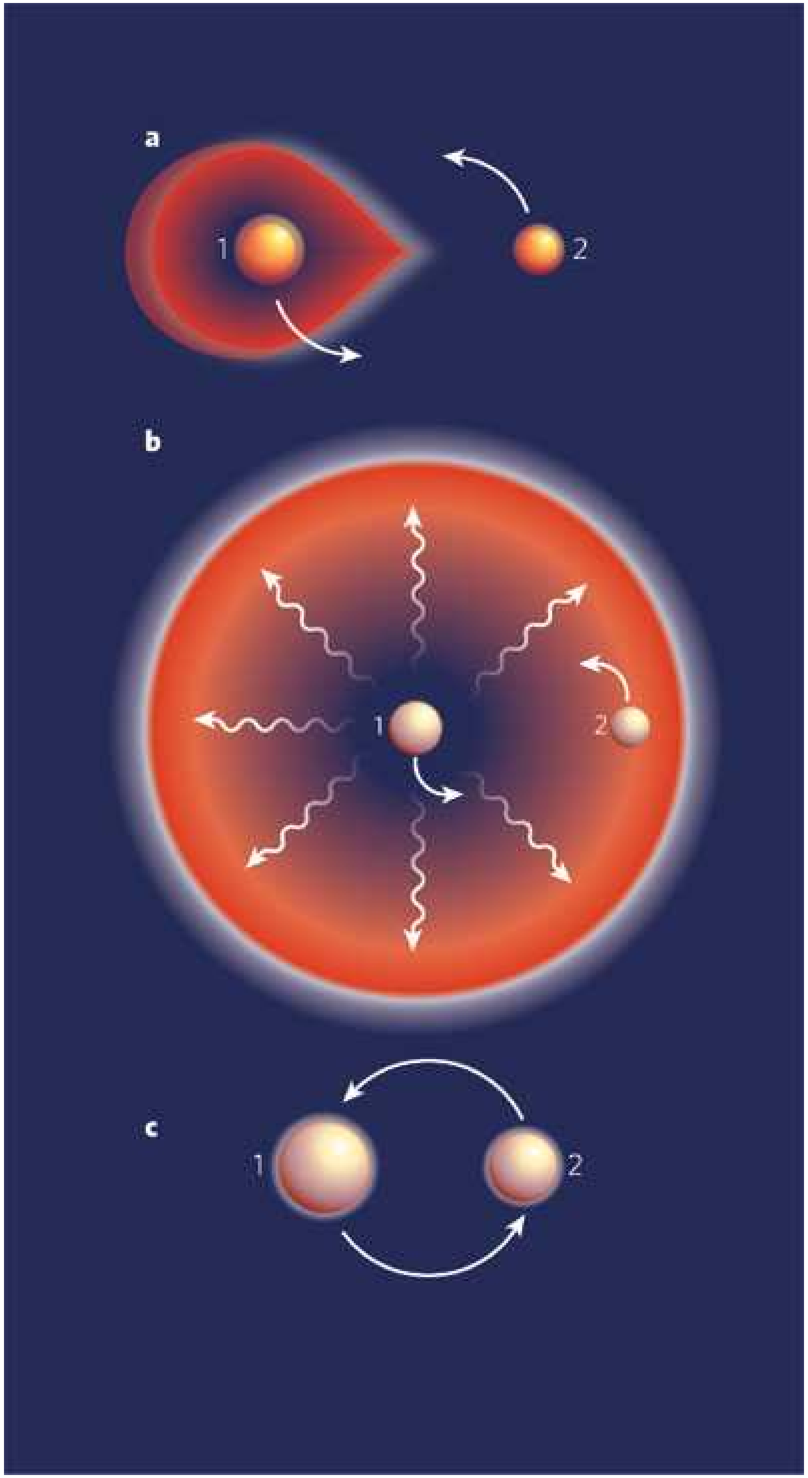
Overview of Common Envelope Evolution

Tomek Bulik


For the population synthesis enthusiast
Common Envelopes
FOR
DUMMIES
Make a black hole merger at home!
A Reference for the Rest of Us!
By Tomek Bulik


a
1
2
b
1
2
c
1
2

Outline

- Basic of mass transfer
- Stability of mass transfer
- How to treat the unstable cases?
- Application to compact object binary formation
- How to refine the treatment of common envelopes?

Basic formulae

Kepler's law

$$\left(\frac{2\pi}{P}\right)^2 A^3 = G(M_1 + M_2)$$

Angular momentum

$$J = \frac{M_1 M_2}{M_1 + M_2} \frac{2\pi A^2 \sqrt{1 - e^2}}{P}$$

Orbital energy

$$E = -\frac{G M_1 M_2}{2A}$$

Conservative mass transfer

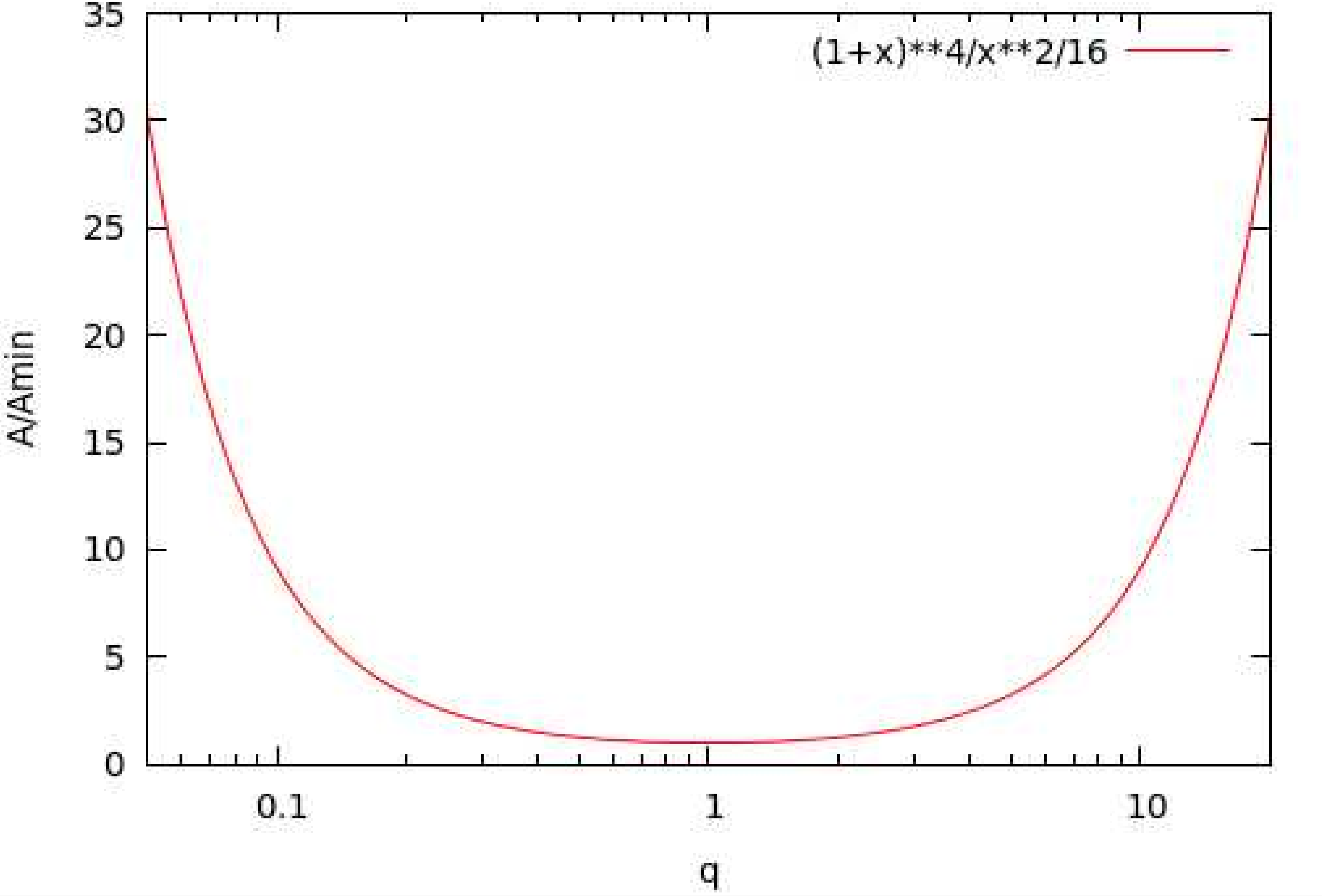
Mass and angular momentum conserved (use the Kepler's law!)

$$J = \frac{M_1 M_2}{(M_1 + M_2)^{1/2}} G^{1/2} A^{1/2} = \mathrm{const}$$

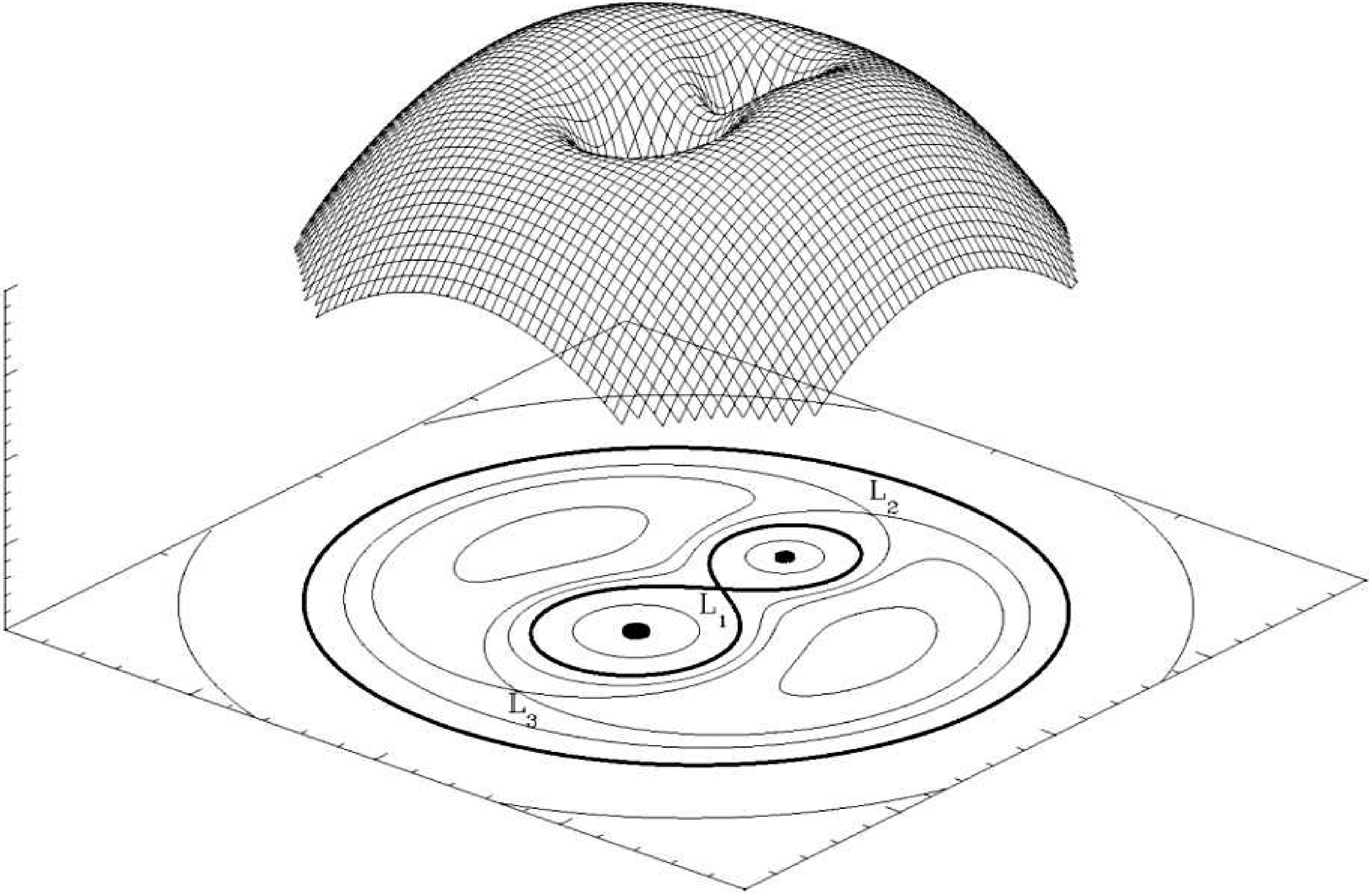
so

$$M_1^2 M_2^2 A = \mathrm{const}$$

$$A = \frac{J^2}{G} \frac{M}{M_1^2 M_2^2} = A_{min} \frac{(1+q)^4}{16 q^2}$$



Big to small → orbit contracts
Small to big → orbit expands



Roche robe radius – the radius of a sphere with the volume equal to the Roche lobe volume.

Roche lobe radius

$$\frac{R_L}{A} = r_L(q)$$

$$\frac{R_L}{A} = \frac{2}{3^{4/3}}(1+q)^{-1/3}$$

Kopal 1955

$$\frac{R_L}{A} = \frac{2}{3^{4/3}}\left(\frac{q}{1+q}\right)^{1/3}$$

Paczyński 1971

$$\frac{R_L}{A} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1+q^{1/3})}$$

$$\frac{R_L}{A} = 0.44\frac{q^{0.33}}{(1+q)^{0.2}}$$

Eggleton 1983

Stability

- Change of Roche lobe radius
- Change of stellar radius

$$\frac{d\log R_L}{d\log M_1} \quad > \text{ or } < \quad \frac{d\log R_1}{d\log M_1}$$

- Bottom line:
  - small mass → big mass, stable
  - big mass → small mass, unstable

Common envelope

$$\alpha_{CE}\left(E^{orb}_{ini} - E^{orb}_{fin}\right) = \lambda^{-1} E_{bind}$$

Donor envelope structure

Common emvelope efficiency

$$\alpha_{CE}\left(-\frac{GM_{1i}M_2}{2A_i} + \frac{GM_{1f}M_2}{2A_f}\right) = -\frac{GM_{1f}M_1^{env}}{\lambda A_i r_L}$$

czyli

$$\frac{A_f}{A_i} = \left(\frac{M_1^{env}}{\alpha_{CE}\lambda r_l M_2} + \frac{M_{1i}}{M_{1f}}\right)^{-1}$$

Note:

- α and λ enter as their product so they are essentially one parameter
- α can be greater than unity, if we assume additional energy source in the system
- In order for CE to stop the mass transfer must stop, so the donor must change its mass radius relation, or the mass ratio be reversed

Common envelope – alternative treatment

$$J_i - J_f = \gamma J_i \frac{\Delta M}{M_{tot}}$$

Efficiency of angular momentum ejection

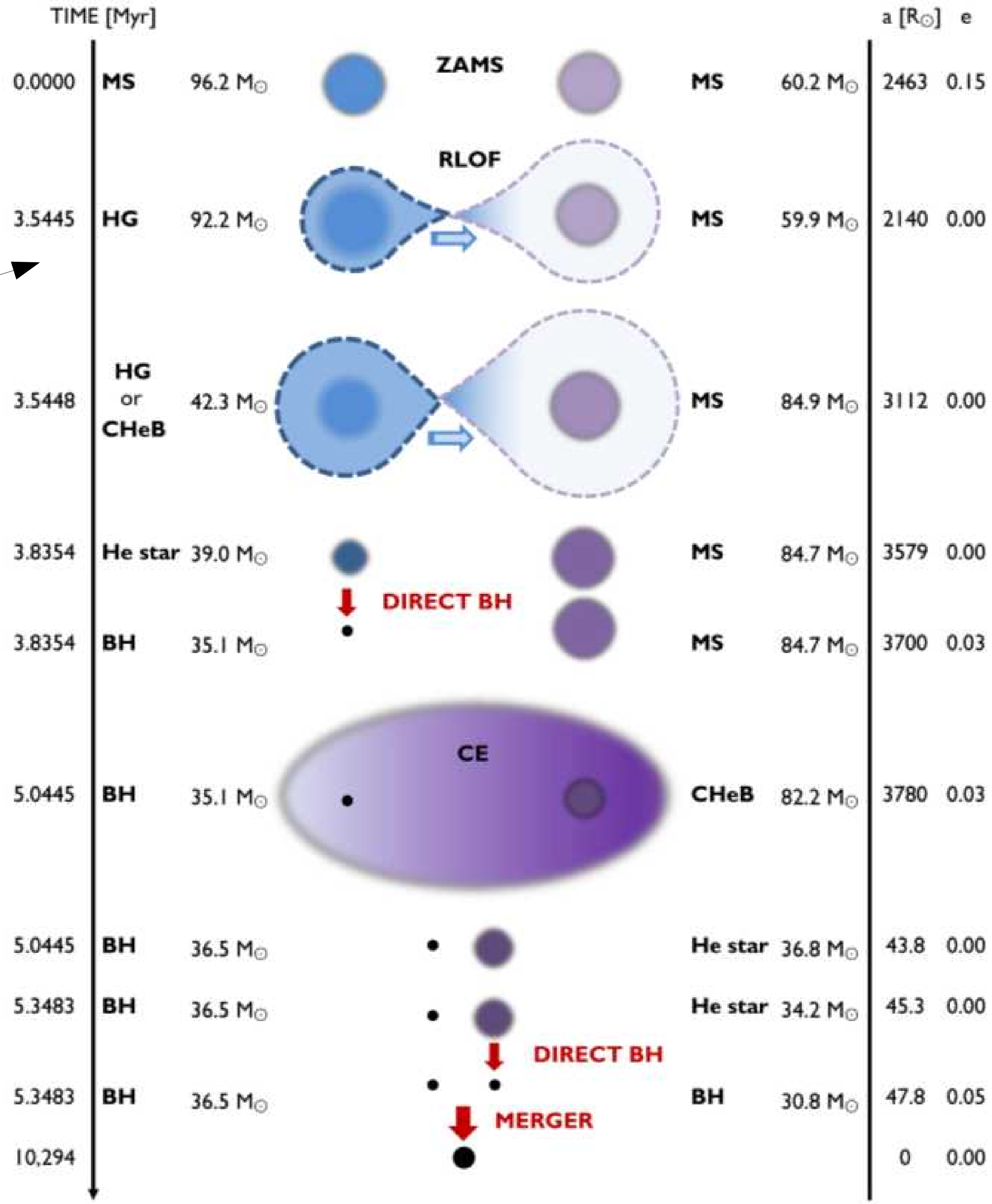
$$\frac{P_f}{P_i} = \left(\frac{M_{1i}M_{2i}}{M_{1f}M_{2f}}\right)^3 \frac{M_{1f}+M_{2f}}{M_{1i}+M_{2i}} \left(1-\gamma\frac{\Delta M}{M_{1i}+M_{2i}}\right)^3$$

Paczynski Ziolkowski 1967, Nelemens et al 2000, Nelemans Tout 2005

Evolutionary scenario

Look here

and here



Credit: Wojtek Gładysz

Impact of CE on compact object binary formation

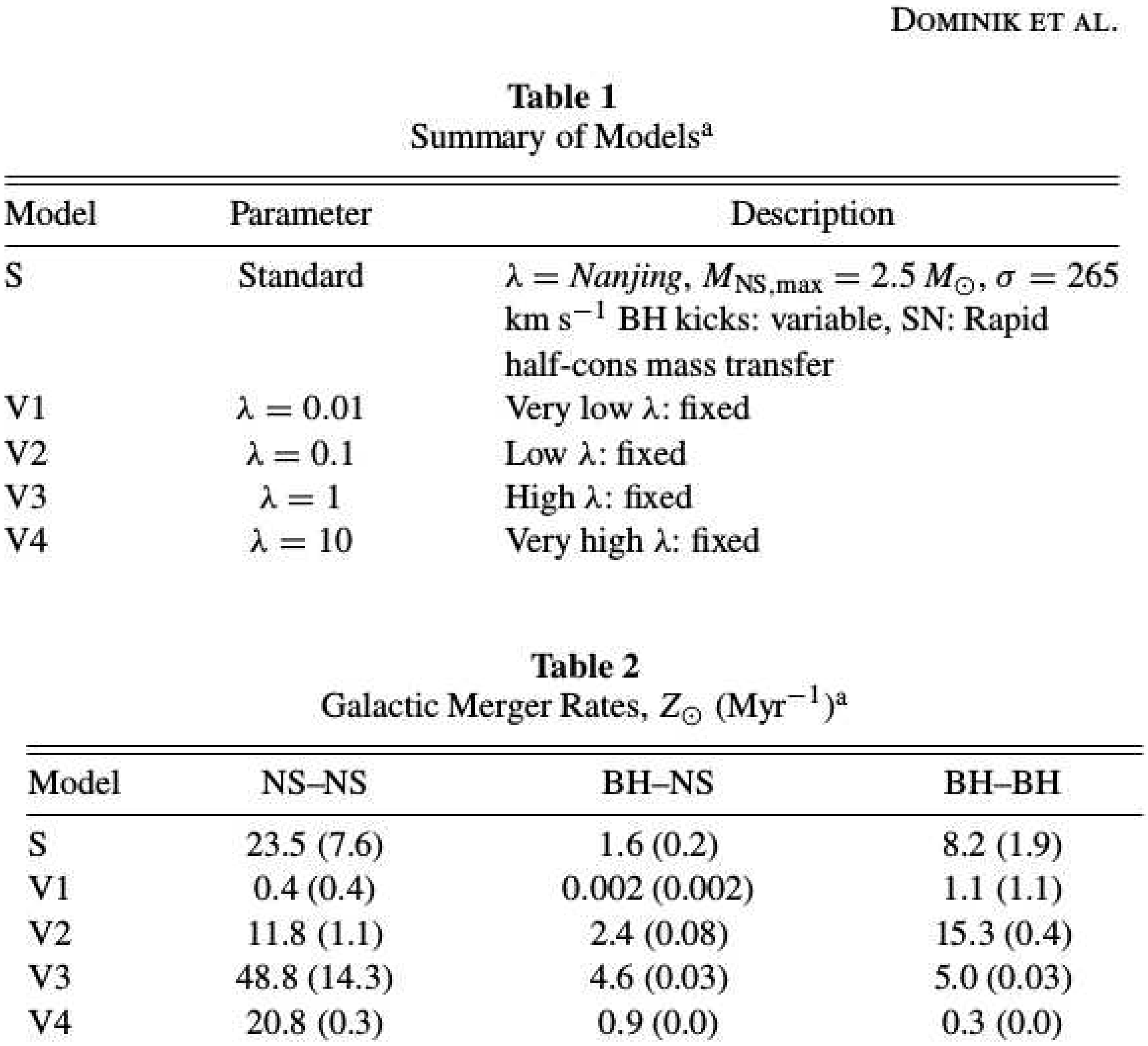


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Table 1
Summary of Models[a]

Model	Parameter	Description
S	Standard	$\lambda =$ *Nanjing*, $M_{\mathrm{NS,max}} = 2.5\,M_\odot$, $\sigma = 265$ km s^{-1} BH kicks: variable, SN: Rapid half-cons mass transfer
V1	$\lambda = 0.01$	Very low λ: fixed
V2	$\lambda = 0.1$	Low λ: fixed
V3	$\lambda = 1$	High λ: fixed
V4	$\lambda = 10$	Very high λ: fixed

Table 2
Galactic Merger Rates, $Z_\odot$ (Myr^{-1})[a]

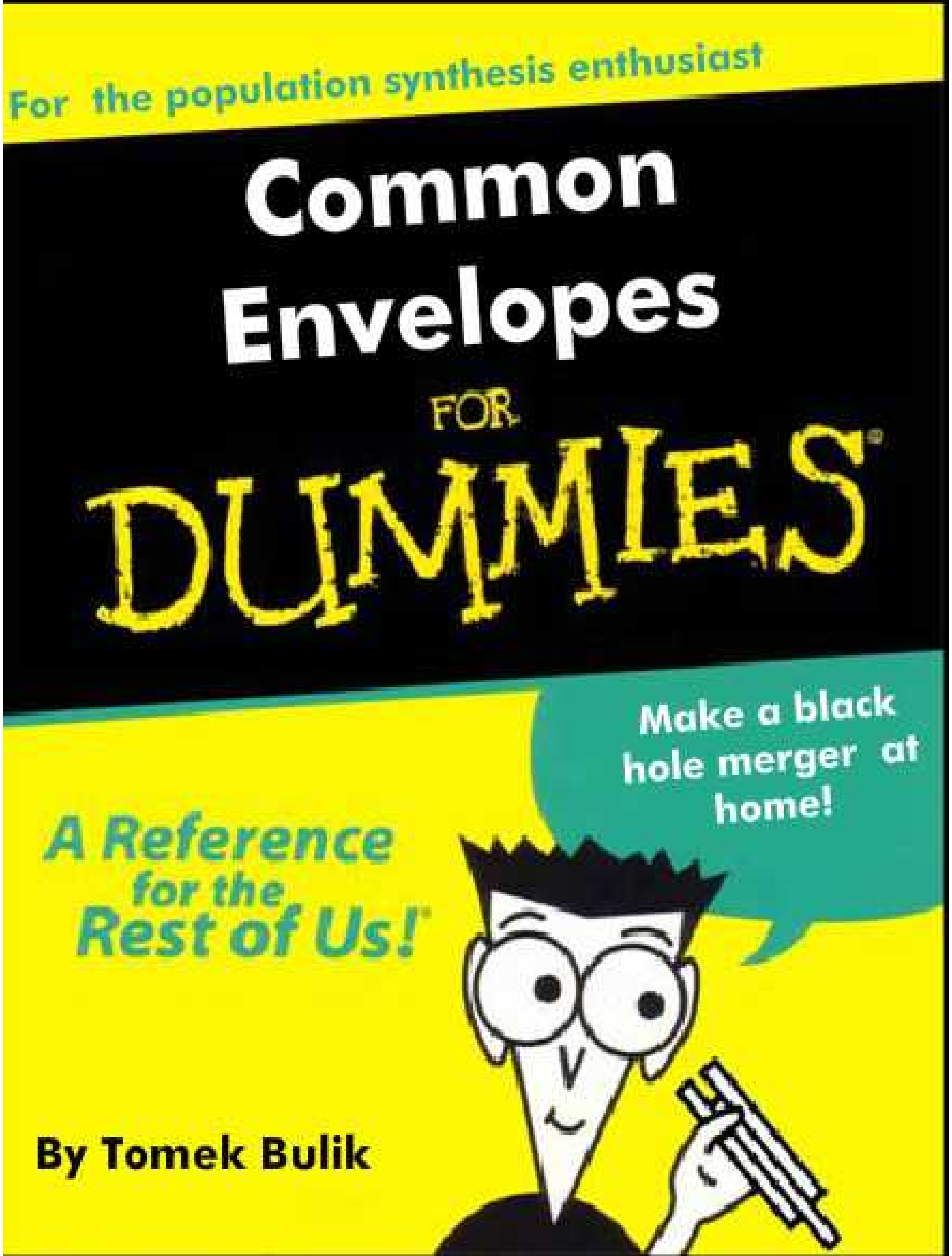
Model	NS–NS	BH–NS	BH–BH
S	23.5 (7.6)	1.6 (0.2)	8.2 (1.9)
V1	0.4 (0.4)	0.002 (0.002)	1.1 (1.1)
V2	11.8 (1.1)	2.4 (0.08)	15.3 (0.4)
V3	48.8 (14.3)	4.6 (0.03)	5.0 (0.03)
V4	20.8 (0.3)	0.9 (0.0)	0.3 (0.0)

Small λ – big binding energy difficult to get the two BHs together to merge in Hubble time

Big λ – small binding energy, system merges forming a Thorne-Zytkow object instead of a binary BH

Questions, problems

- How accurate is the approxmate treatment?
- When does the CE start?
- How do the CE parameters depend on a particular type of donor?
- When does the CE phase finish?



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