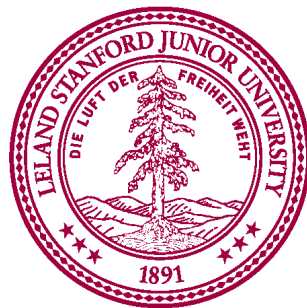


# A Twisted Ladder: Relating the Iron Superconductors and the High-Tc Cuprates

arXiv:0905.1096, To appear in New. J. Phys.

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Doug J. Scalapino<sup>2</sup>

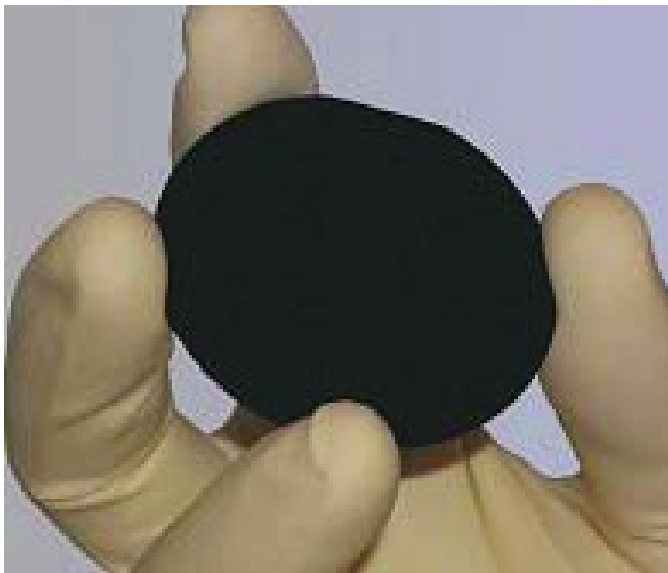
<sup>1</sup> Stanford University, <sup>2</sup> UC Santa Barbara



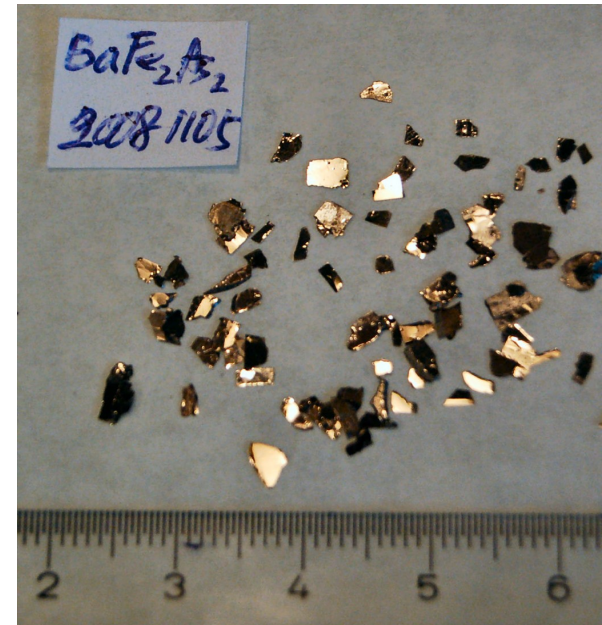
# The cuprates and the pnictides

Two "High Tc" families

$\text{Yba}_2\text{Cu}_3\text{O}_{7-y}$ ,  
Tc(max) ~ 90K



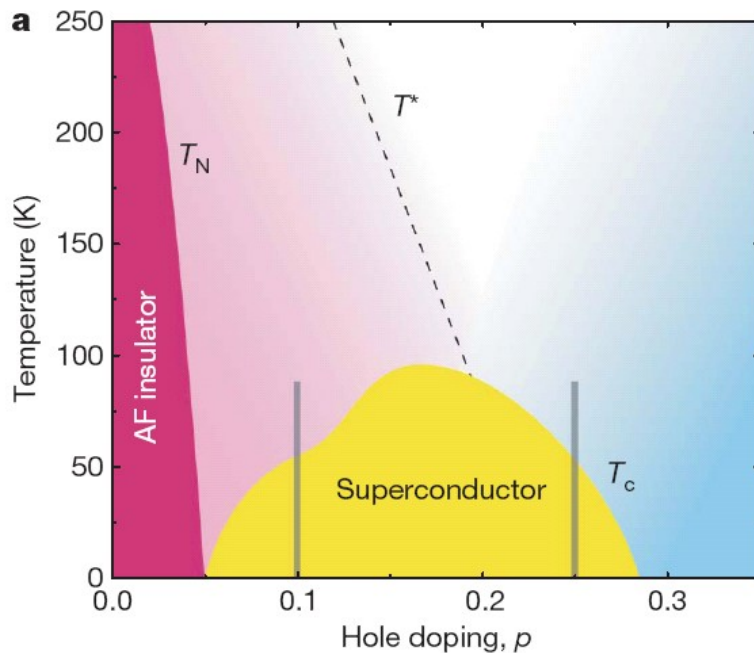
$\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$   
Tc(max) ~ 40K



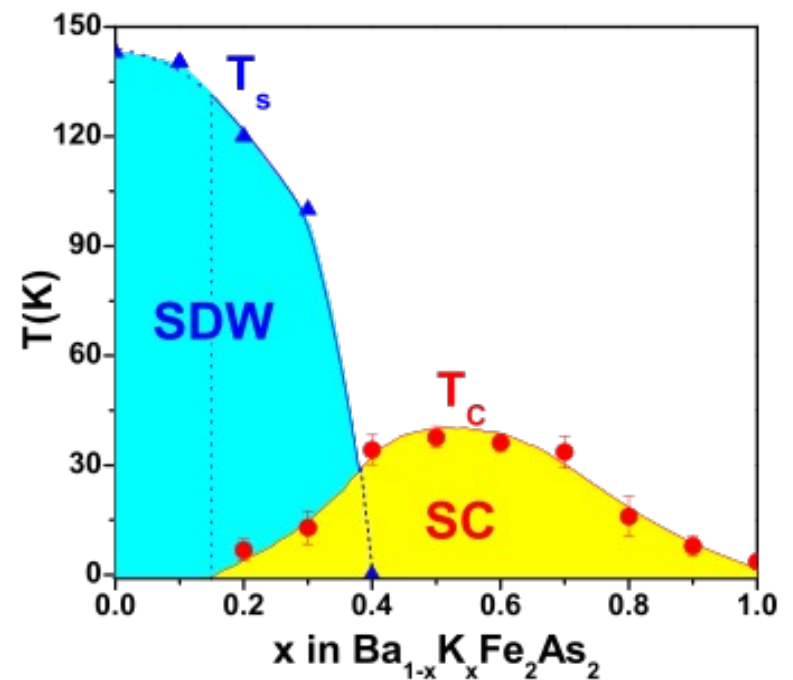
# The cuprates and the pnictides

- Striking similarity of the "topology" of the phase diagram: proximity of SC and magnetically ordered states

Cuprates ( $\text{YBa}_2\text{Cu}_3\text{O}_{7-y}$ )



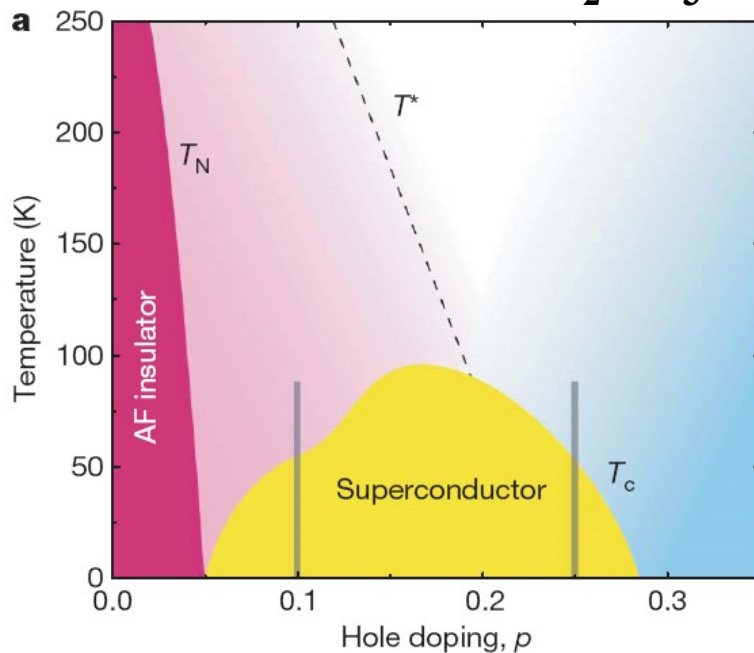
Iron pnictides ( $\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$ )



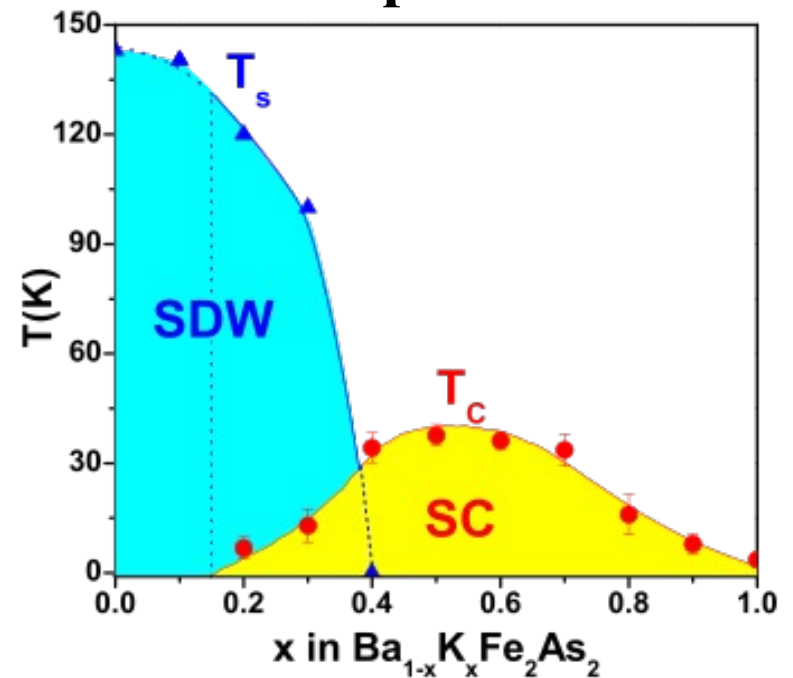
# The Question

- One mechanism / Two mechanisms?
- Cuprates vs. Pnictides:  
"Similar"/"Different"?

Cuprates ( $\text{YBa}_2\text{Cu}_3\text{O}_{7-y}$ )



Iron pnictides



# Experimentally: they are similar...

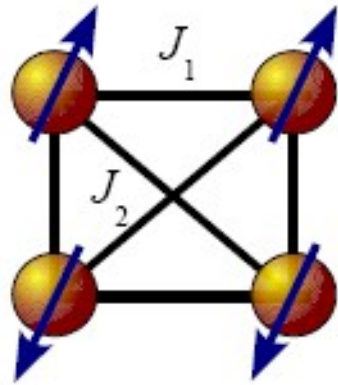
- Quasi-2D (Layered) materials
- Square lattice
- High resistivity in the normal state ("bad metal")
- Close-by magnetism
- "resonant peak" in  $\chi''(q, \omega)$   
at  $q \approx Q_{\text{mag}}$ ,  $\omega \approx \Delta$

## ...But also different

- **Single-band (cuprates) vs. multi-band (pnictides)**
- **Mott insulating vs. metallic parent compound**
- **d-wave vs. (possibly) s-wave order parameter**

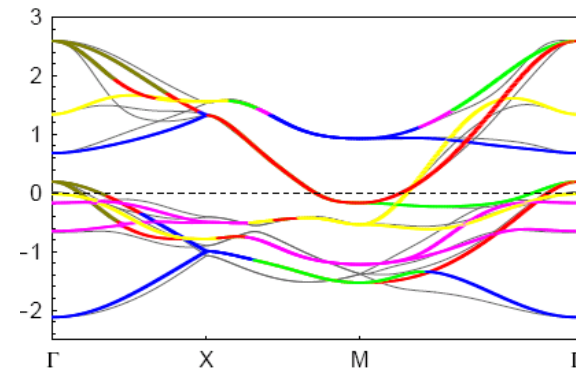
# A related question: weak/strong coupling?

**Strong coupling:  
local moments**



- Bad metal ( $K_f \sim 1$ )
- Magnetism is commensurate
- Large band renormalizations

**Weak coupling: energy bands**



From Graser et al. (2009)

- Metallic parent compound, small moments
- Rough agreement between ARPES and DFT
- $Q_{AF}$  close to a nesting vector

# What does theory have to say?

- **Needed: a theoretical model that can interpolate between the different regimes**
- **Numerical simulations on small clusters/one dimensional systems**
- **Can make sense since the coherence lengths are quite short**



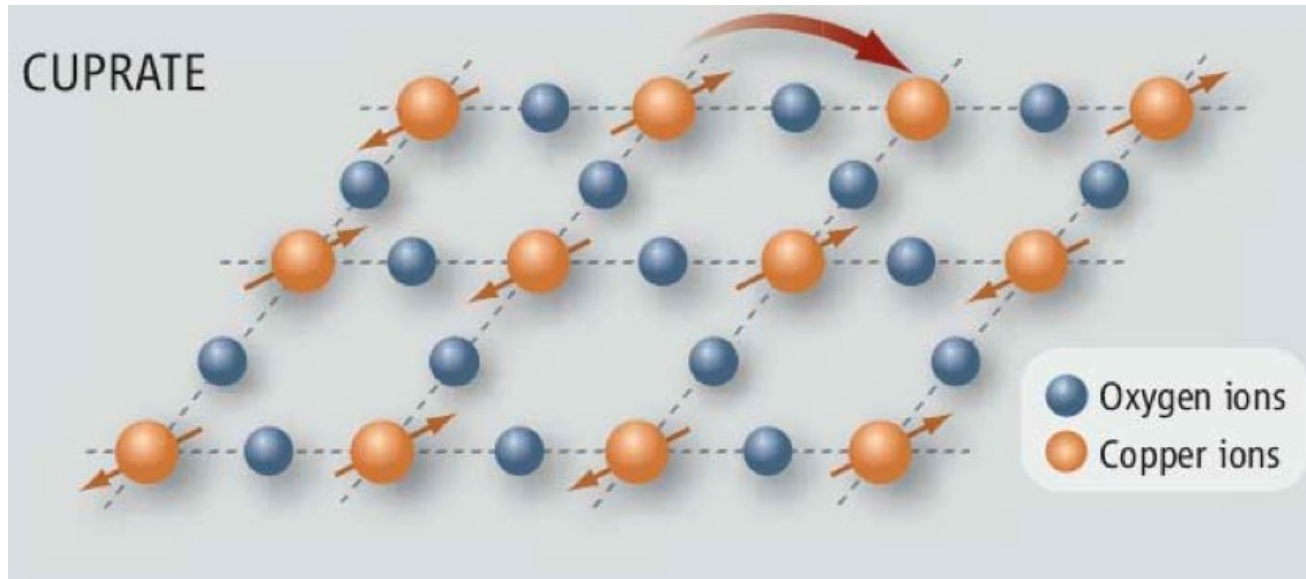
# Outline

Construction of 1D “Ladder” models for the Iron superconductors.

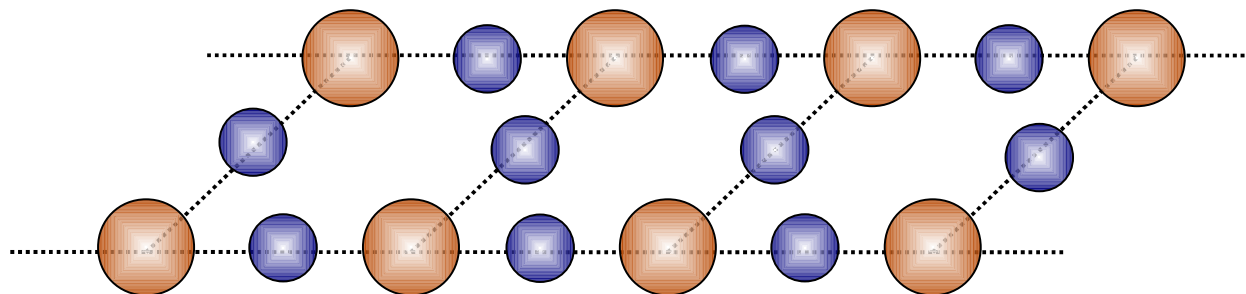
- Review of the Hubbard ladder model for the cuprates.
- The pnictide one-orbital “twisted ladder”
- A hidden relation to the cuprates?
- Beyond one orbital: preliminary results for the “diagonal ladder”

# The two-leg Hubbard model for the cuprates

Square CuO lattice:

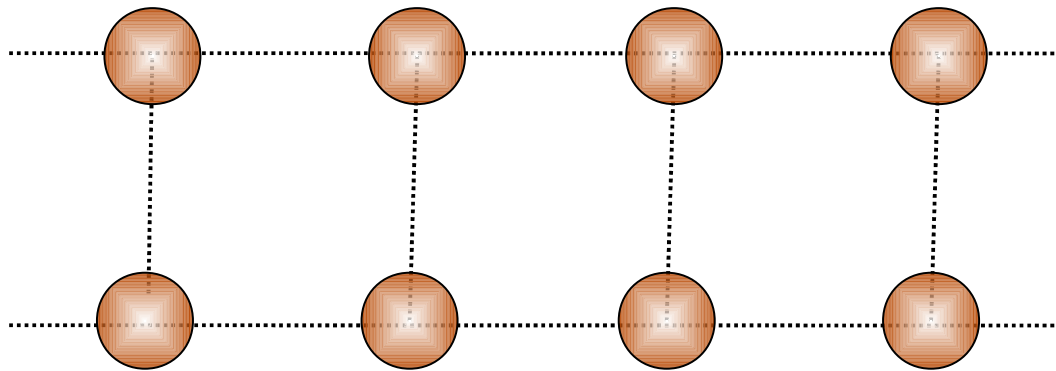


Cut out 2 chains:



# The two-leg Hubbard model for the cuprates (2)

Reduce to effective 1-band model (Zhang,Rice):



With on-site interactions: the Hubbard model

$$H = - \sum_{\langle i,j \rangle, \sigma} t_{ij} (c_{i,\sigma}^\dagger c_{j,\sigma} + \text{H.c.}) + \sum_i U_i n_{i,\uparrow} n_{i,\downarrow}$$

# Properties of the 2-leg ladder

Dagotto, Riera, Scalapino, White, Rice,...

## Undoped system:

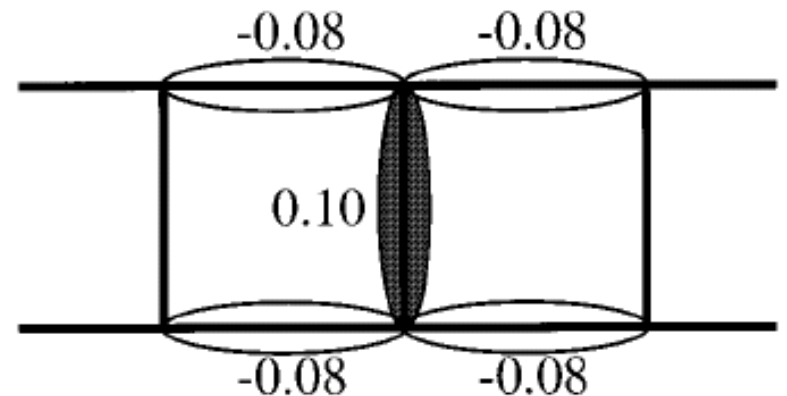
- Mott insulator for any  $U$
- Finite spin gap, short range  $(\pi, \pi)$  spin correlations

## Finite doping:

- Spin gap persists, d-wave like SC (and  $4k_F$  CDW) power-law correlations

## Pair structure:

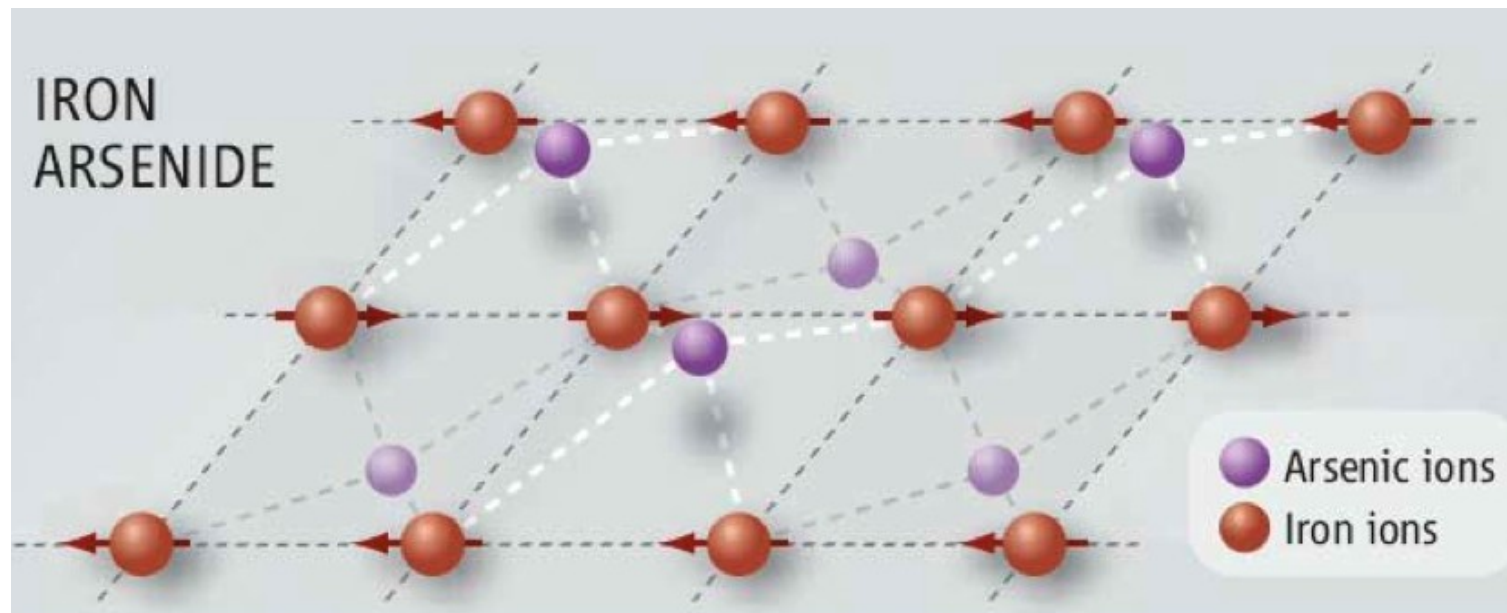
$$\Delta_{rr'} = \langle N_2 | (c_{\mathbf{r}\uparrow}^\dagger c_{\mathbf{r}'\downarrow}^\dagger - c_{\mathbf{r}\downarrow}^\dagger c_{\mathbf{r}'\uparrow}^\dagger) | N_1 \rangle$$



Noack et. al. (1997)

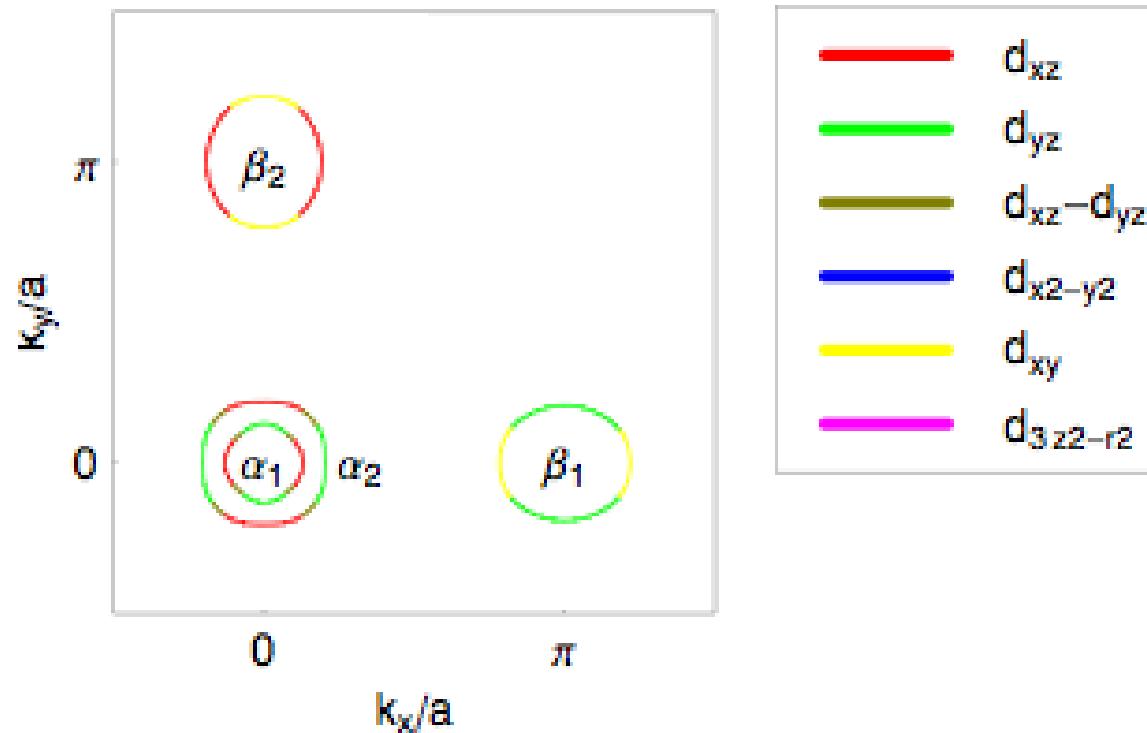
# Can we play a similar game for the iron superconductors?

FeAs square lattice:



# Can we play a similar game for the iron superconductors?

Tight binding fit to DFT  
and orbital structure near the FS

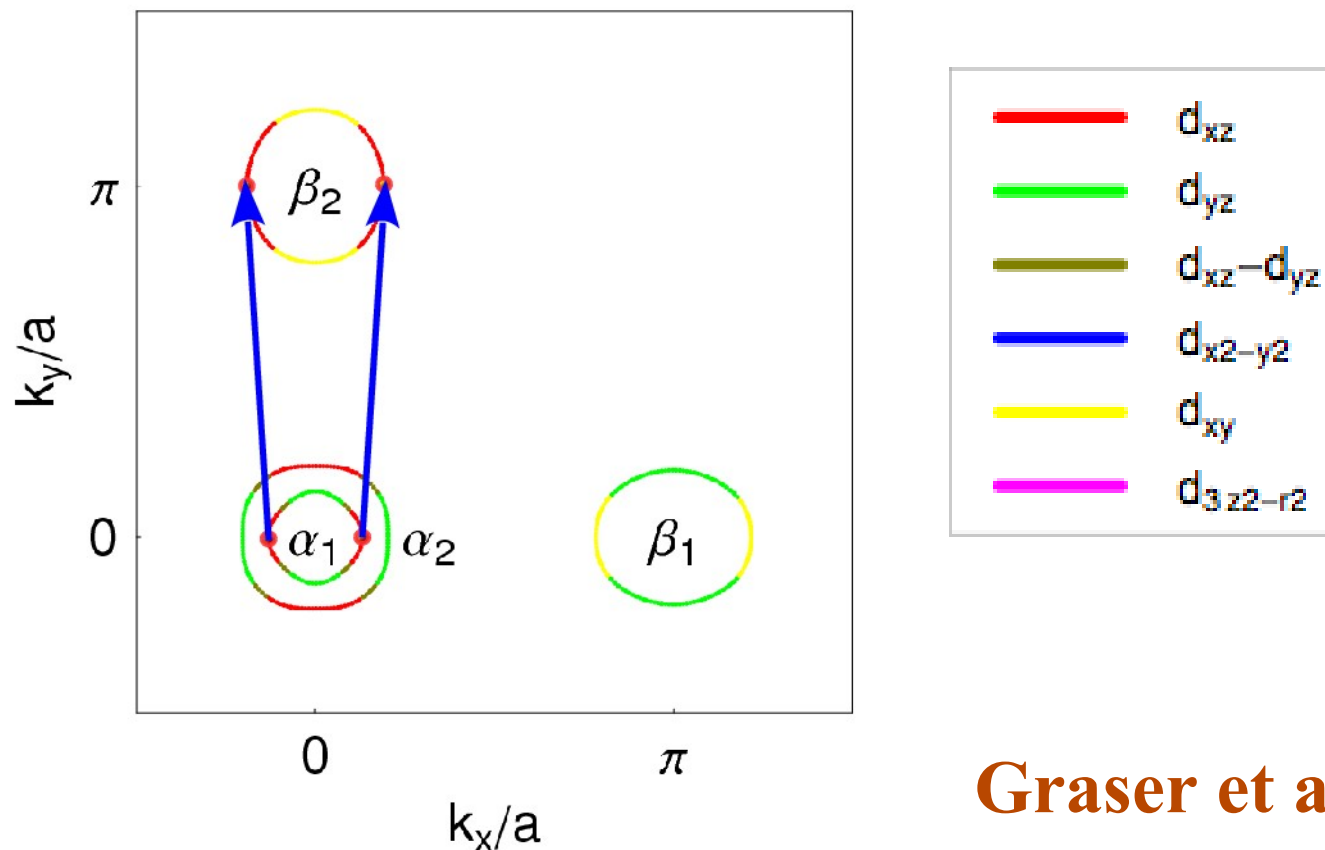


Graser et al. (2009), Cao et al. (2008)

# Can we play a similar game for the iron superconductors?

Weak coupling analysis (Graser et al.):

Large contribution to pairing from intra-orbital pair scattering by  $(0,\pi)$  or  $(\pi,0)$  spin fluctuations

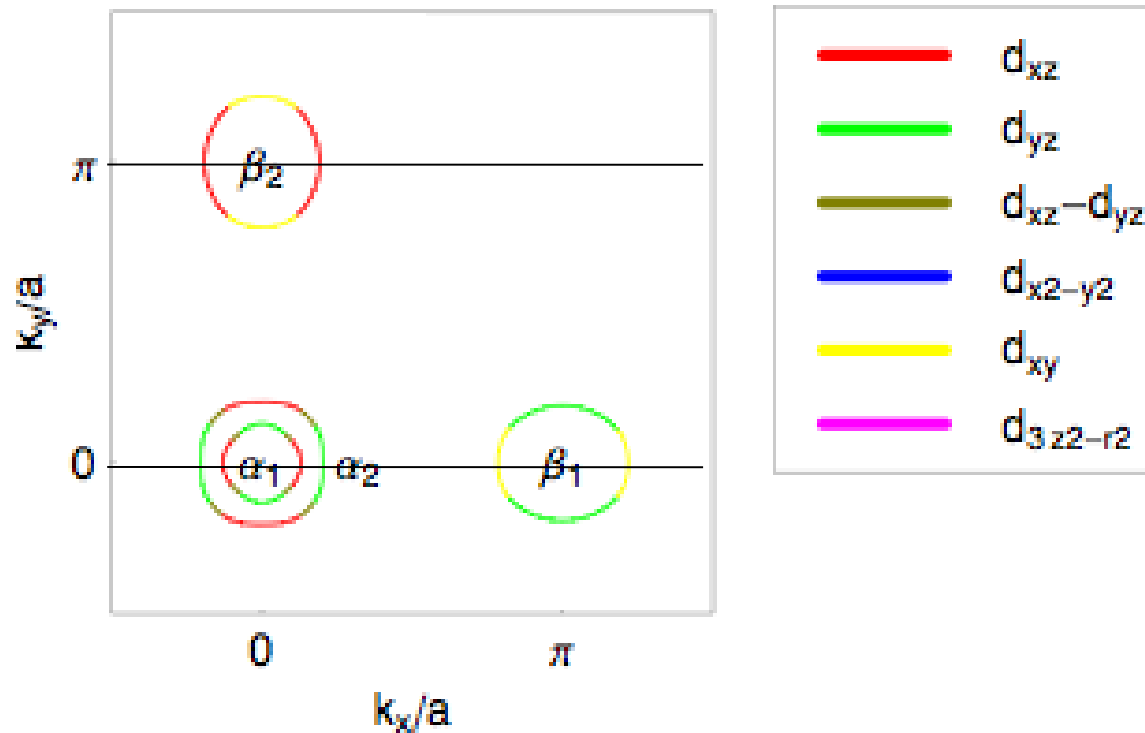


Graser et al. (2009)

# Can we play a similar game for the iron superconductors?

Two-leg ladder geometry:

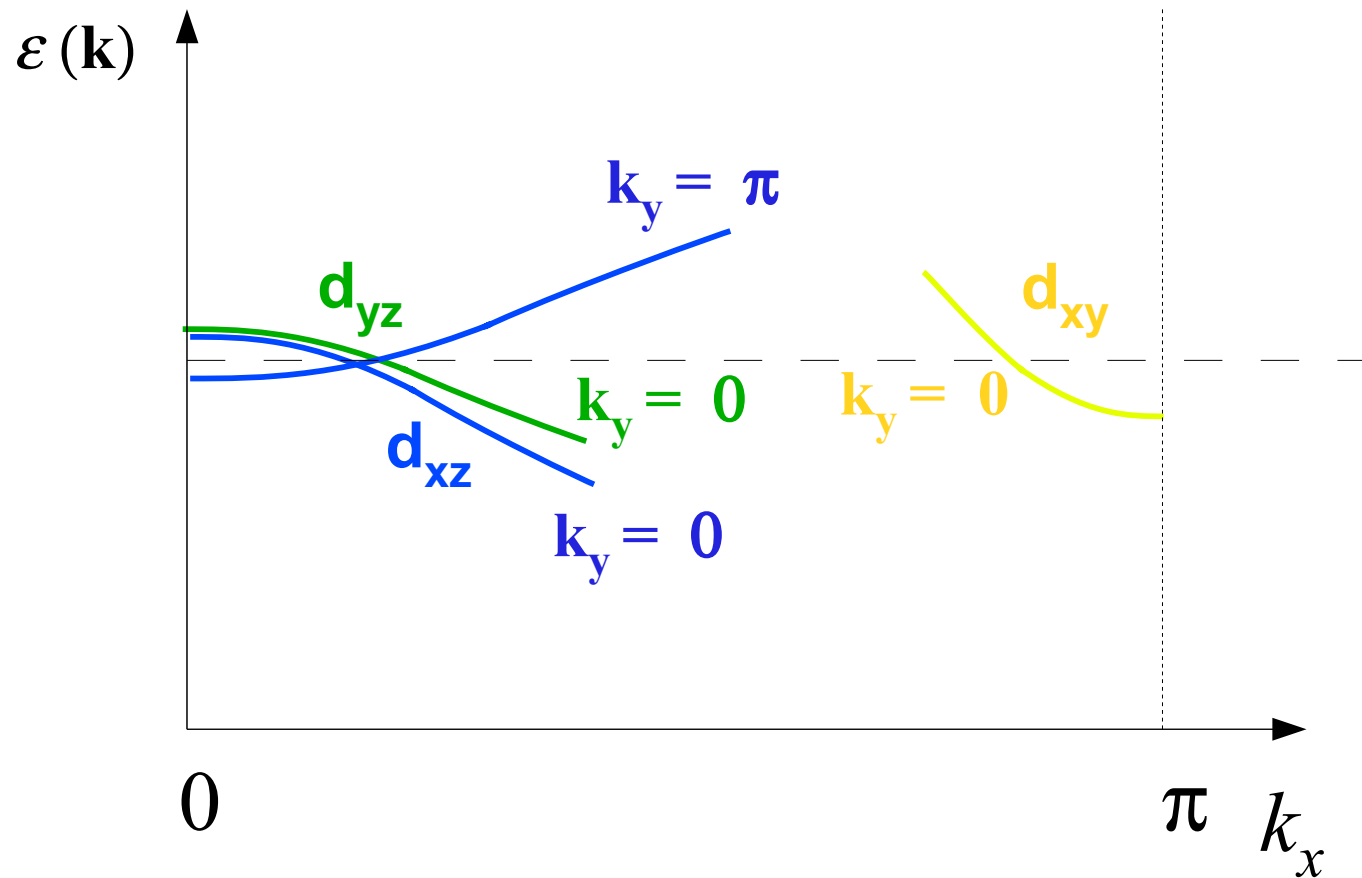
Cut the Brillouin zone through  $k_y = 0, \pi$





# Band structure

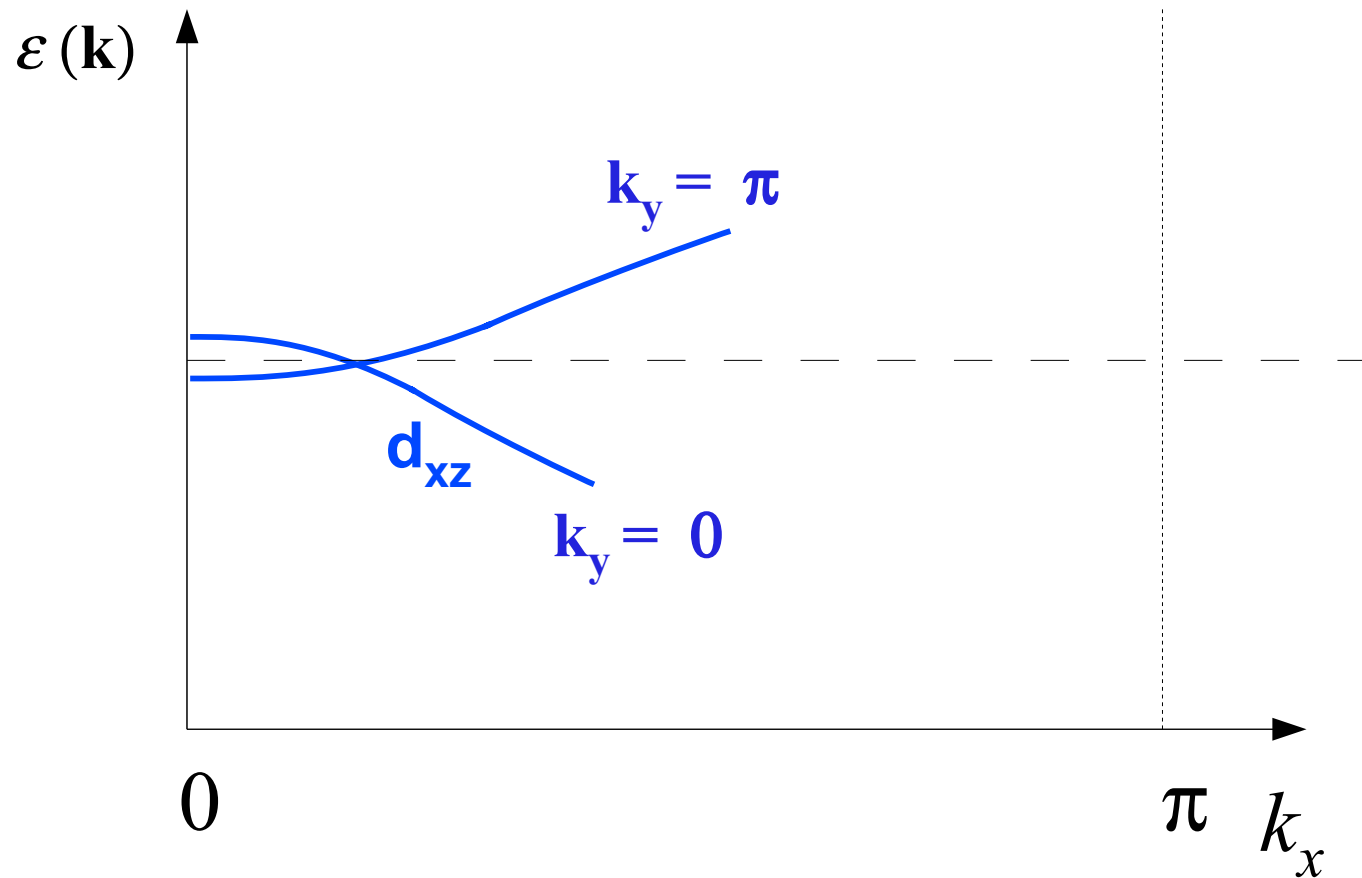
Ladder band structure near the Fermi level  
(schematic)



**DFT calculation: Cao, Hirschfeld, Cheng (2008)**

# Band structure

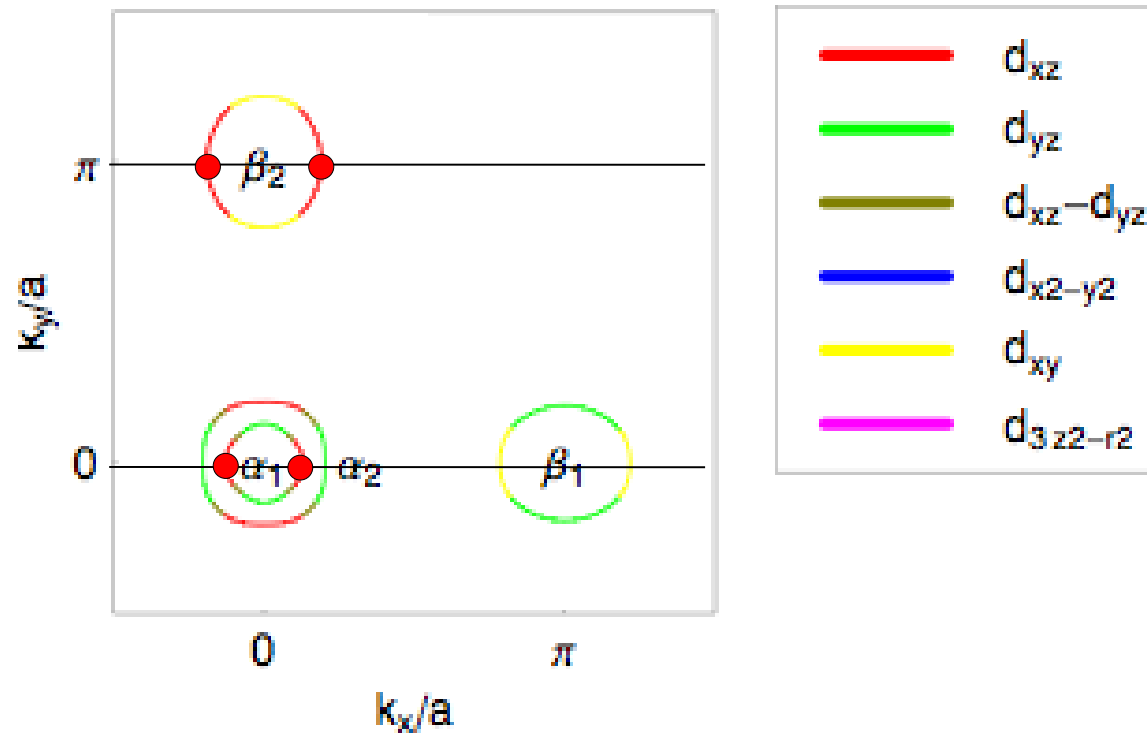
Keep only  $d_{xz}$ : ( $\alpha_1$  and  $\beta_2$  pockets)



**DFT calculation: Cao, Hirschfeld, Cheng (2008)**

# Can we play a similar game for the iron superconductors?

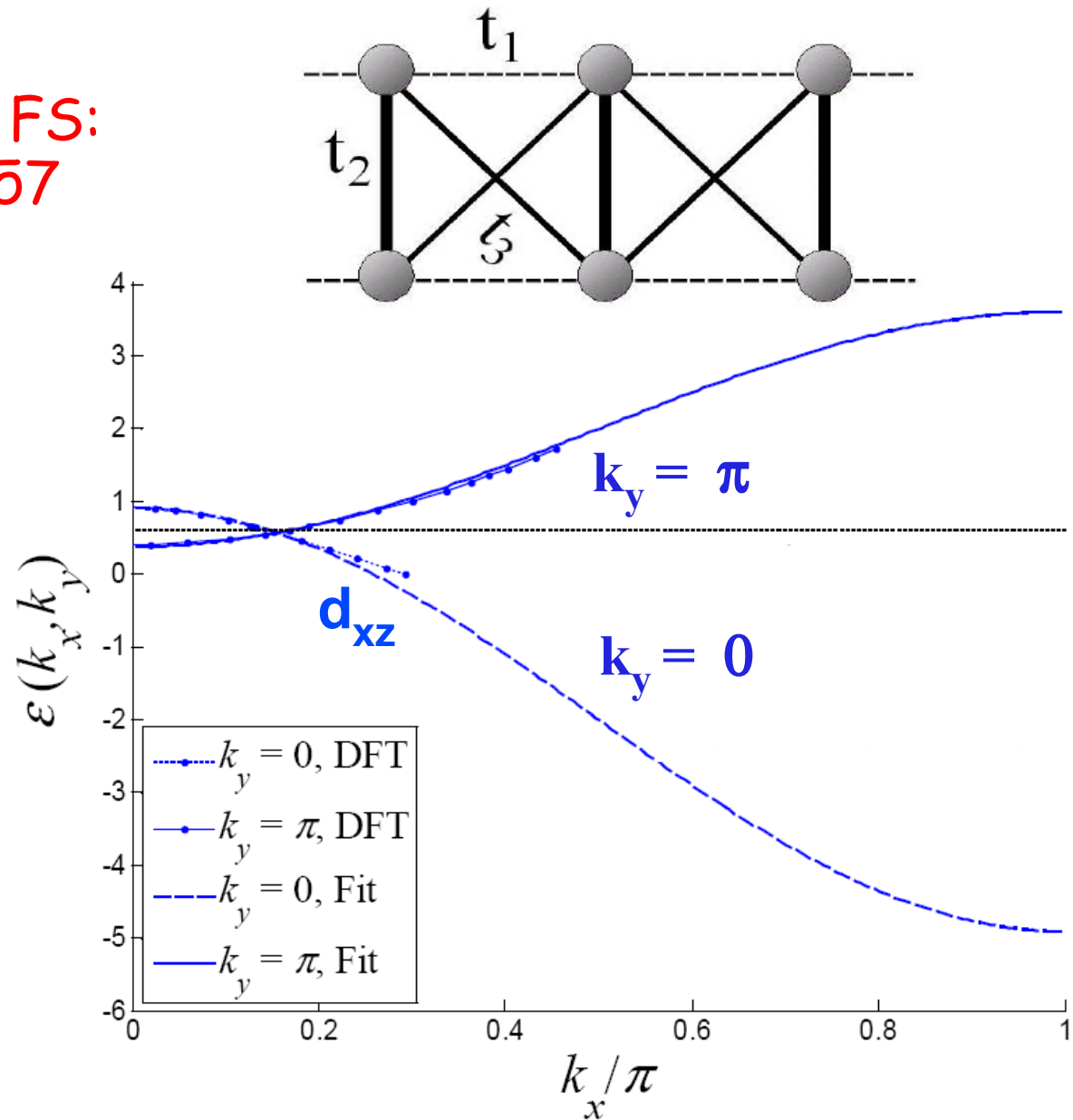
Four Fermi points with  $d_{xz}$  character



# Ladder band structure

Fit to LDA near the FS:  
 $t_1 = -0.32$ ,  $t_2 = 1$ ,  $t_3 = -0.57$

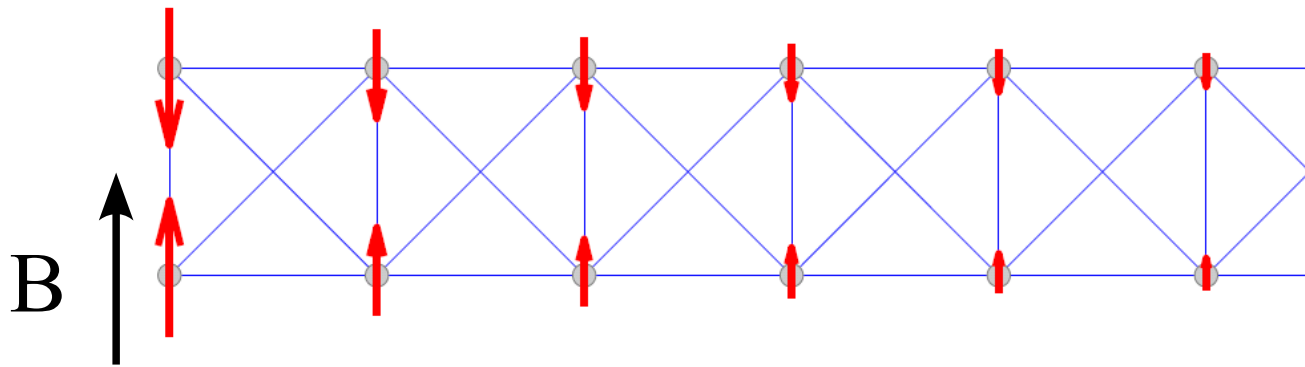
“Intermediate  
Coupling”:  $U = 3$



**DFT calculations: Cao, Hirschfeld, Cheng (2008)**

# Properties of the Fe ladder

DMRG results for the Undoped system: ( $n=1$  per site)



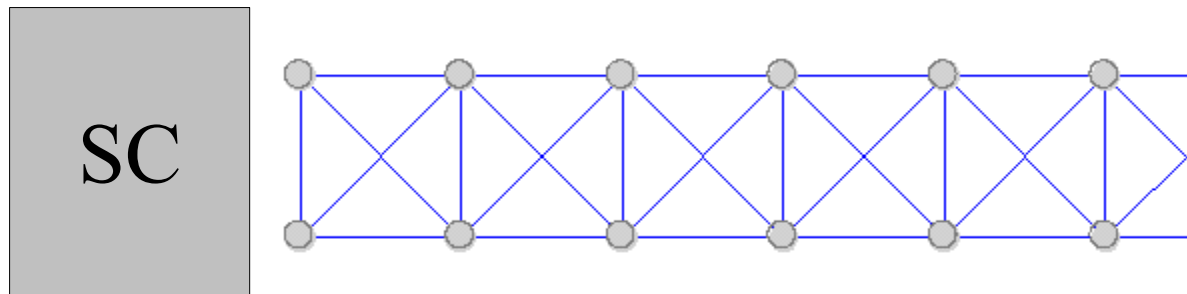
Short-range "stripe-like" magnetic correlations at  $(0, \pi)$

Insulator for any non-zero  $U$  at  $n=1$

**Berg, Kivelson, Scalapino, New J. Phys. (2009)**

# Properties of the Fe ladder (2)

Hole doped system ( $n=0.9375$  per site)  
proximity-coupled to a bulk superconductor

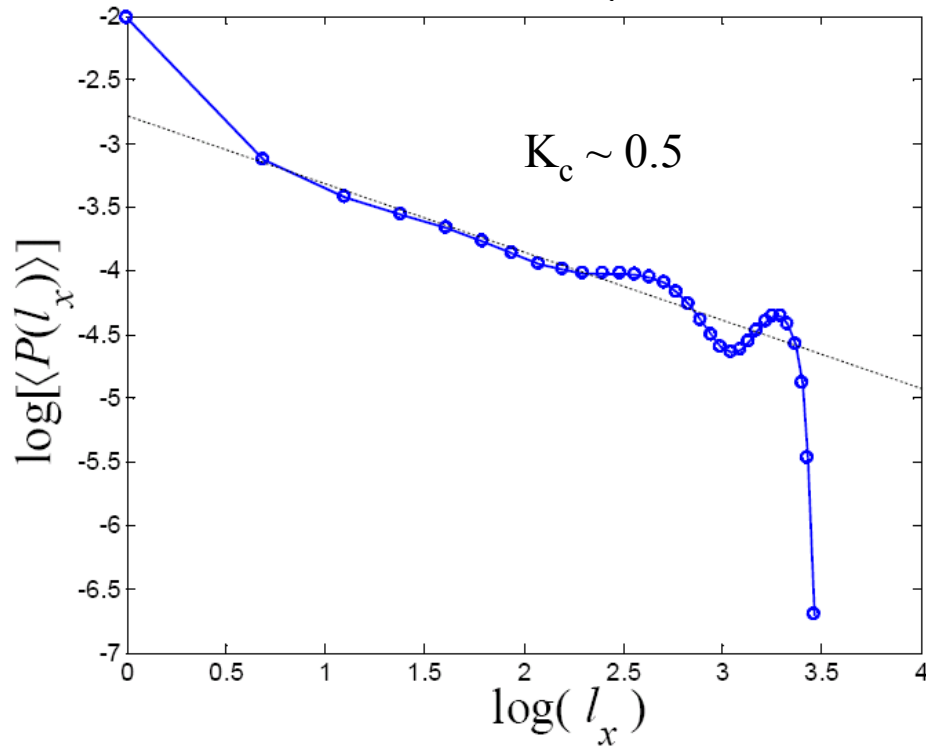


$$H_1 = \Delta_1 \left( P_1^\dagger + \text{h.c.} \right), \quad P_1^\dagger = \left( d_{1,1\uparrow}^\dagger d_{1,2\downarrow}^\dagger - d_{1,1\downarrow}^\dagger d_{1,2\uparrow}^\dagger \right)$$

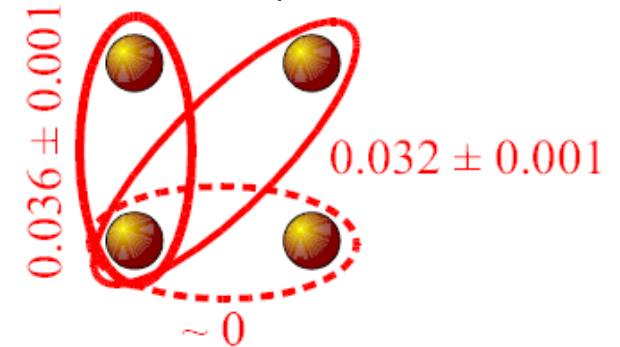
**Berg, Kivelson, Scalapino, New J. Phys. (2009)**

# Properties of the Fe ladder (3)

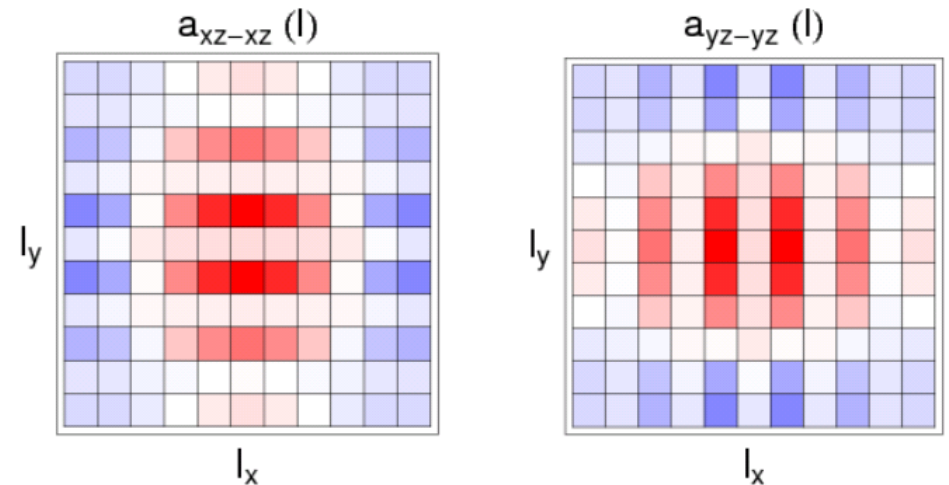
Induced rung SC order parameter for 32x2 system:



Pair structure near the middle of the system:



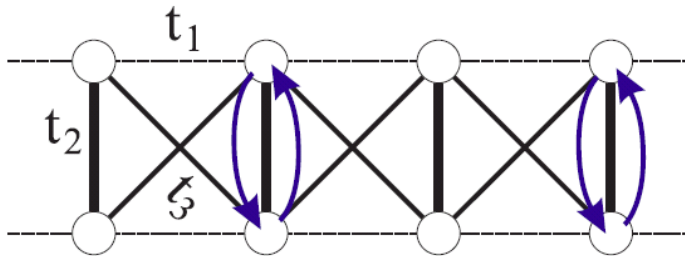
Compare to the RPA result for 5-band, 2D model (Graser et. al.) for an  $s_{\pm}$  gap



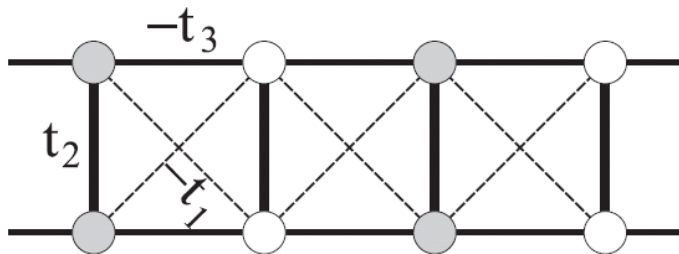
# Relation to the Hubbard ladder

Under a simple "twist", the Fe ladder becomes the usual Hubbard ladder used to model the cuprates.

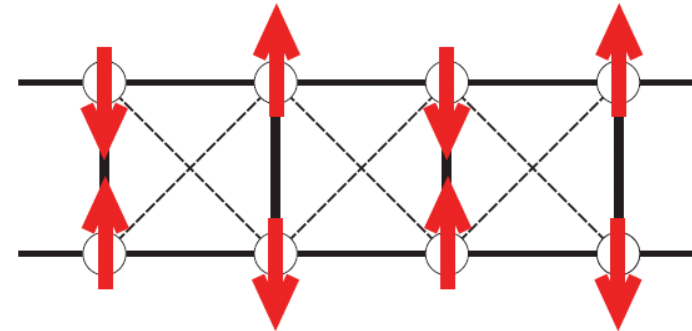
Interchange sites on every other rung



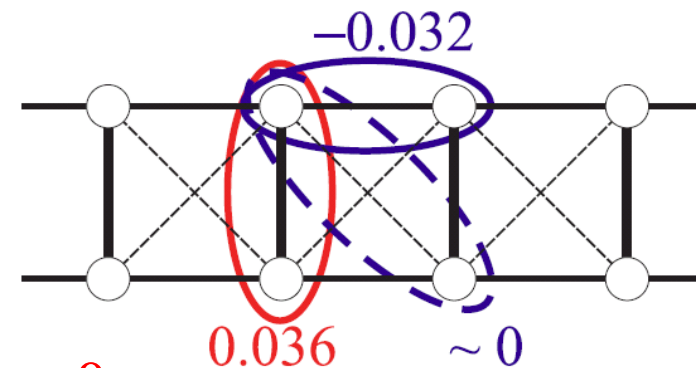
Change sign of orbitals of shaded sites  $c_i \rightarrow -c_i$



Short range  $(\pi, \pi)$  magnetic correlations



Power-law  $d_{x^2-y^2}$  like SC correlations



Hubbard ladder in the regime  $t_2 \approx 2t_1, t_3 > 0$ :

nearly optimal for pairing! (Noack et al., 1997)



# Beyond one orbital?

The "Twister Ladder" model suggests a relation between the pairing mechanisms of the pnictides and the cuprates.

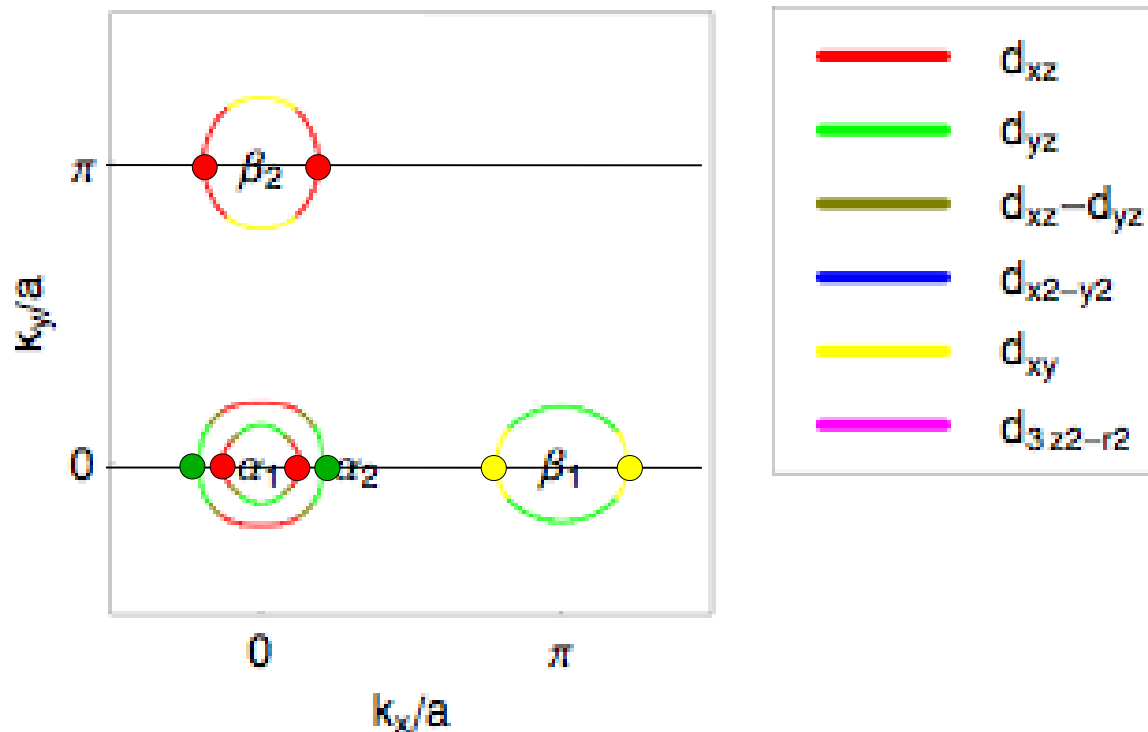
However, there are questions that it cannot address:

- d-wave vs. s-wave
- Nematic transition, orbital ordering

How to go beyond one orbital?

# Beyond one orbital?

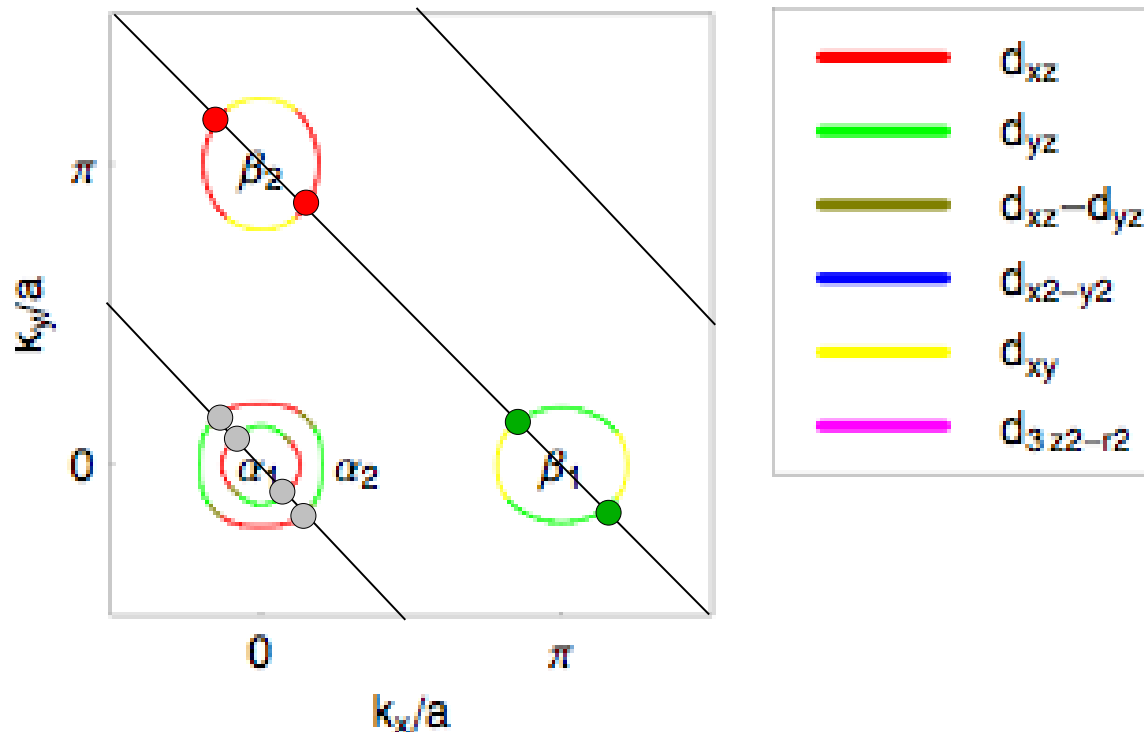
Natural extension: keep all the 2-leg ladder Fermi points (requires  $d_{xy}$ ,  $d_{yz}$  orbitals)



# Beyond one orbital?

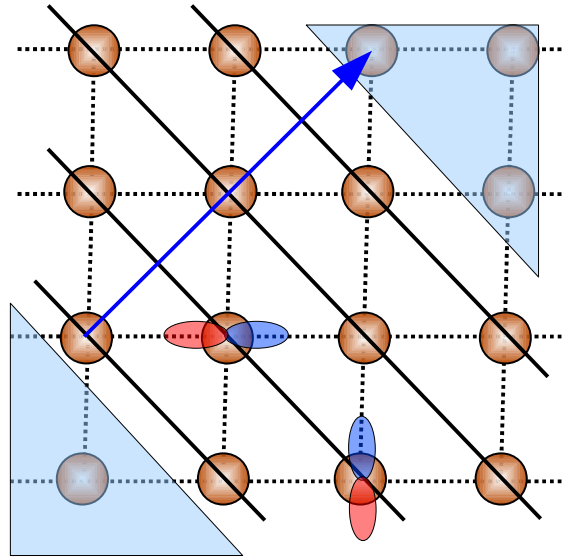
Alternatively, choose a diagonally oriented ladder

Treats the  $d_{xz}$  and  $d_{yz}$  orbitals symmetrically!



# The Diagonal Ladder

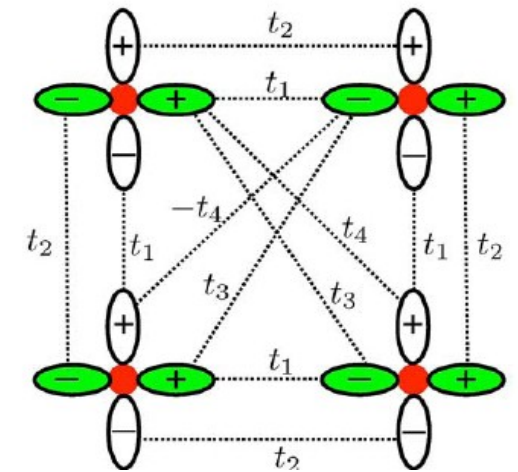
Extended along (1,-1), periodic with period (2a,2a)



Keep  $d_{xz}$ ,  $d_{yz}$  orbitals per site

Two band model: Raghu et. al. (2008)

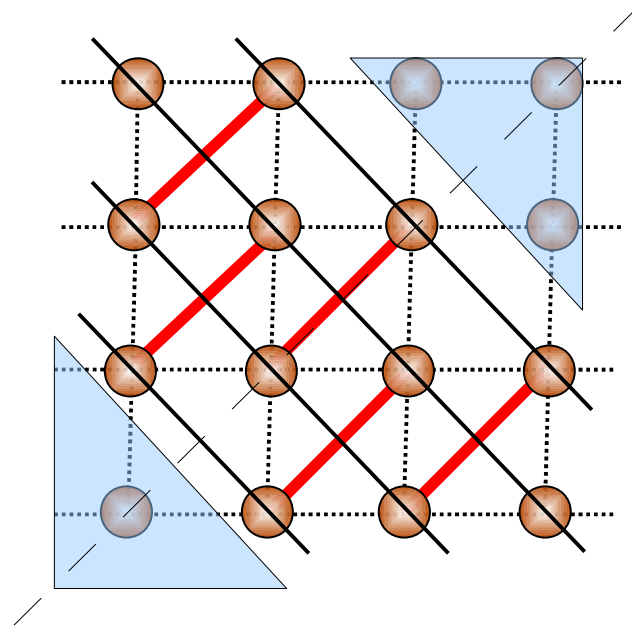
On-site Interactions:



$$H_{\text{int}} = \sum_{\mathbf{r}} \sum_{\alpha=xz,yz} [Un_{\alpha\uparrow\mathbf{r}}n_{\alpha\downarrow\mathbf{r}} + Vn_{x\mathbf{r}}n_{y\mathbf{r}} - JS_{x\mathbf{r}} \cdot \mathbf{S}_{y\mathbf{r}} + J' (d_{x\uparrow\mathbf{r}}^\dagger d_{x\downarrow\mathbf{r}}^\dagger d_{y\downarrow\mathbf{r}} d_{y\uparrow\mathbf{r}} + H.c.)]$$

# The Diagonal Ladder

Essentially, a 2-leg, 2-orbital model



Properties of the 2-orbital diagonal ladder:

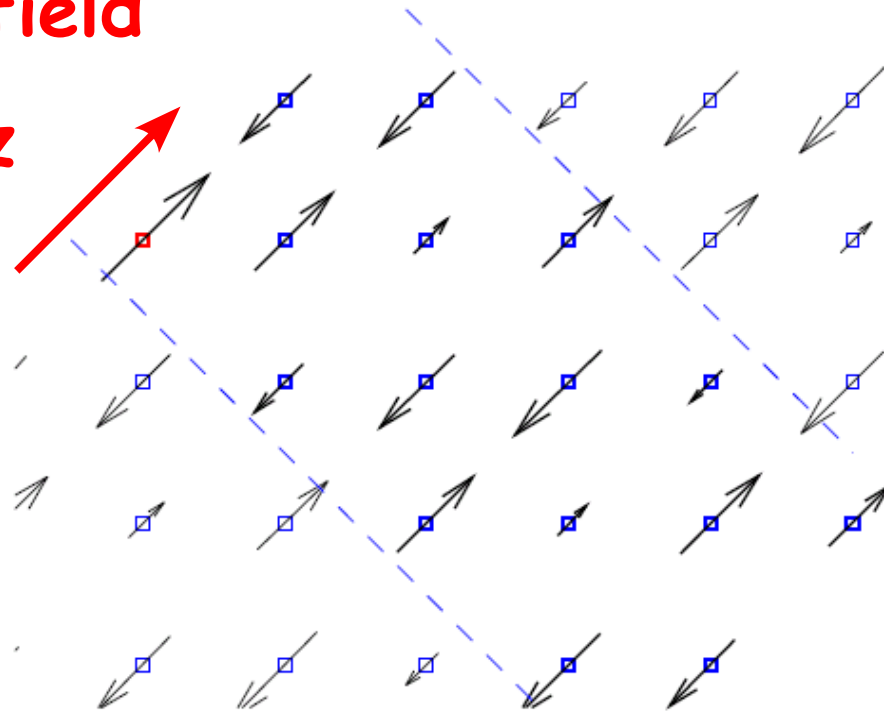
- Can be metallic at zero doping  
(Finite  $U_c$  for Mott insulaor)
- Reflection symmetry about the (1,1) direction:
  - Possible to have a nematic phase
  - Can distinguish d and s-wave pairing

# Preliminary results

$$U = 4, V = 3, J = U - V, J' = J/2$$

Magnetic correlations in the undoped system  
(2 el. per site, 8x2)

Magnetic field  
on the dxz  
orbital

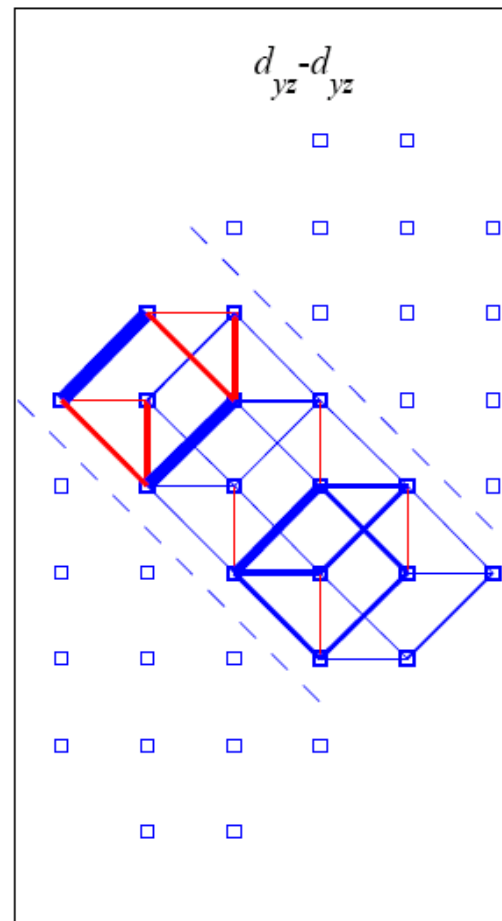
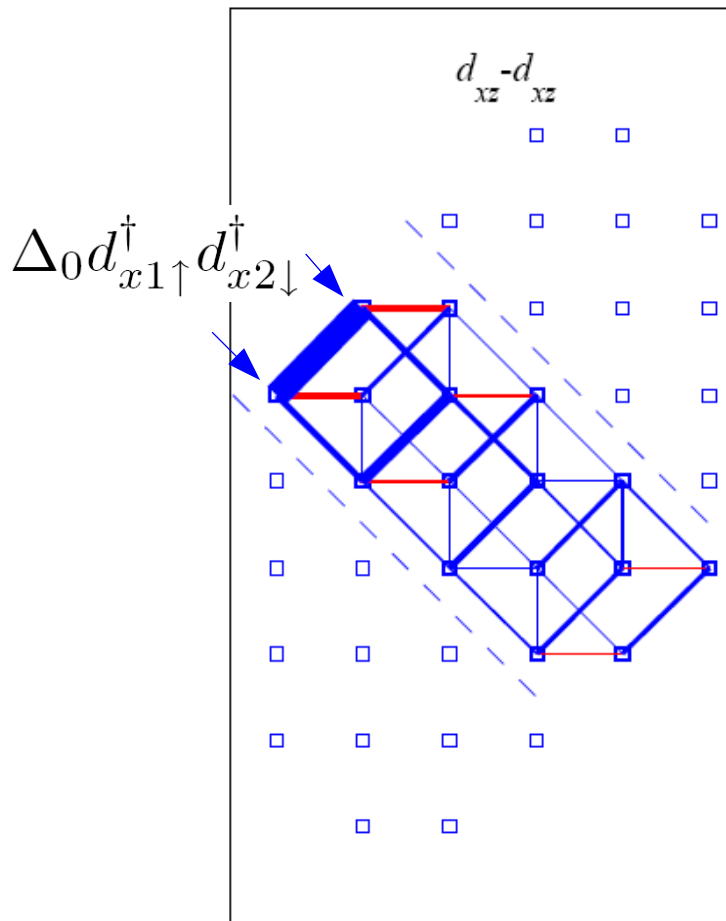


# Preliminary results

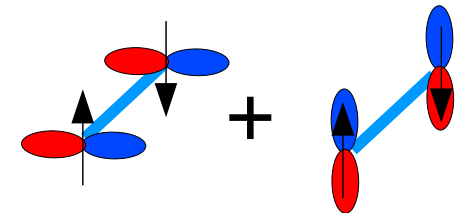
$$U = 4, V = 3, J = U - V, J' = J/2$$

Pairing correlations in a 8x2 system doped by 2 electrons: Proximity coupling to the last  $d_{xz}$  rung

Induced SC order parameter:



This calculation indicates s-wave!



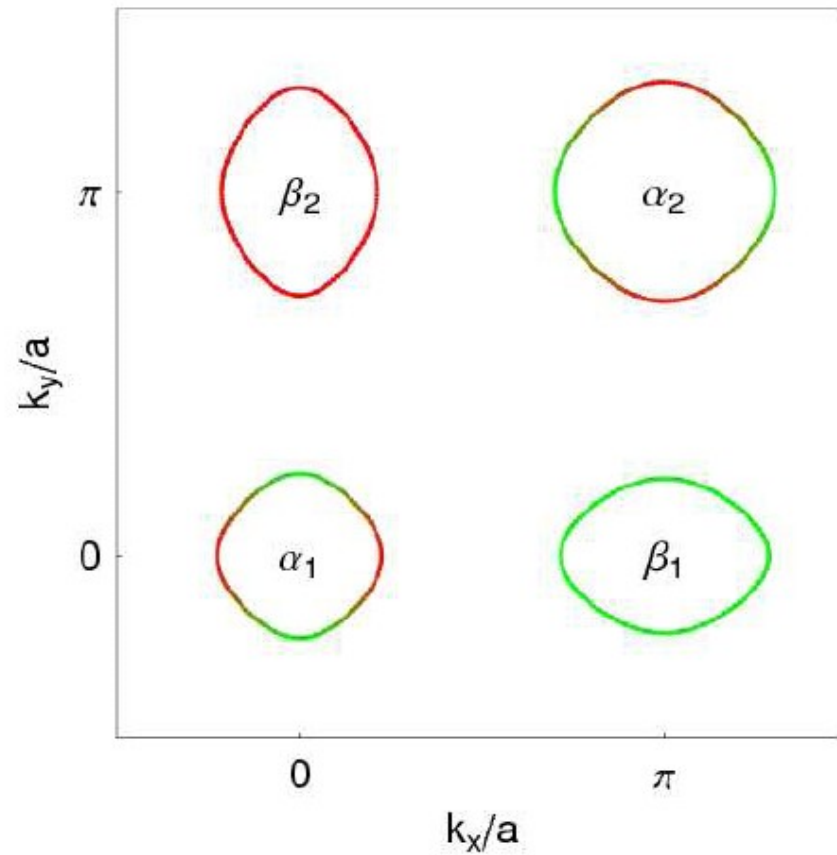
# Conclusions

- A 2-leg ladder model of the **Fe superconductors** was used to understand some of their magnetic and superconducting correlations.
- The model turns out to be just a “twisted” version of the 2-leg Hubbard model often used to describe the **Cuprates**, suggesting that the mechanism of superconductivity is closely related to that of the cuprates.
- To answer more detailed questions, **a diagonal two-orbital ladder model** is proposed. This model can elucidate the pairing symmetry and the existence of other correlations (e.g. nematic order).

**Studies of the diagonal ladder (DMRG and weak coupling RG) are under way.**

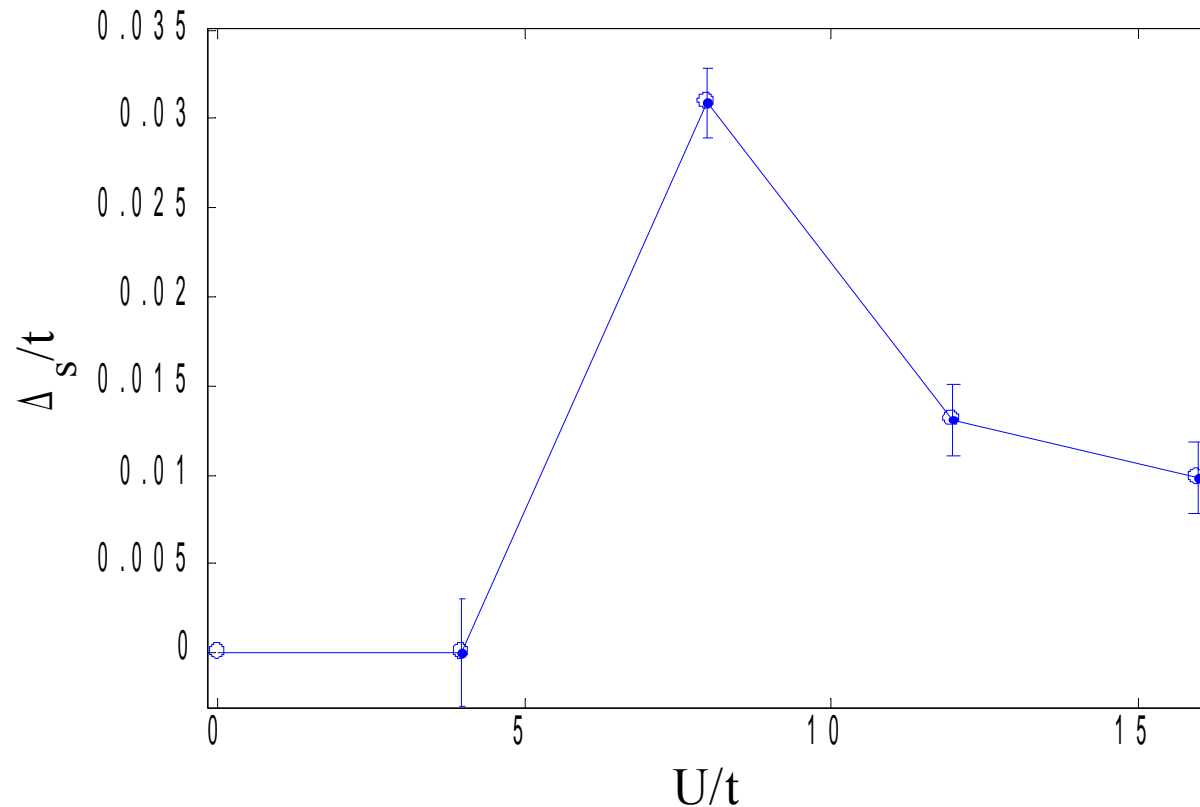


# 2-orbital model band structure



# Properties of the 2-leg ladder

Spin gap vs.  $U/t$ :



**Max. for  $U/t \approx 8$ : intermediate coupling!**

phase diagram, high value of the normal-state resistivity, magnetic resonance in the SC state) and differences (metallic/Mott insulating normal state, single/multi-orbital system, full gap/d-wave)

## Outline

- The questions: two superconducting mechanisms or one? Weak or strong coupling? (1-2)
- SC from repulsive interactions: general features
- 1D: the 2-leg ladder Hubbard model for the cuprates
- Models for the pnictides: strong/weak coupling
- Graser, Maier, Hirschfeld, Scalapino: band structure, importance of intra-orbital alpha-beta pocket scattering
- Fit to band structure. 2-leg model
- Results: spin correlations, spin gap, pairing correlations
- Relation to the Hubbard ladder
- The next step: treat two orbitals symmetrically. The diagonal ladder
- Preliminary results for the diagonal ladder
- Summary

# A related question: weak/strong coupling?

- Possible answer: cuprates - "strong", pnictides - "weak". (insulating vs. metallic when undoped)
- ...But there are indications that both are "intermediately coupled".
- Optical conductivity data from M. Qazilbash, D. Basov et al (Nature Phys. 2009):

