Fermionic atoms in optical lattices: Mott transition and metastable superconductivity

Benedikt Binz, Rolf Helmes, David Rasch, Akos Rapp,
Matthias Vojta, Achim Rosch,
Institute for Theoretical Physics, University of Cologne

Theo Costi
Institute for Solid State Research, Research Centre Jülich

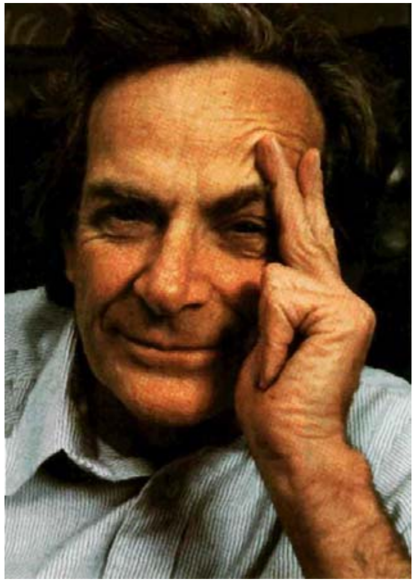
experiments:
U. Schneider, L. Hackermüller, S. Will, Th. Best,I. Bloch
University of Mainz

- Cold atoms as quantum simulators for solid state systems: Mott transition
- New physics out of equilibrium:
  exotic metastable superconductivity

Quantum simulations for higher Tc?

best: universal quantum computers

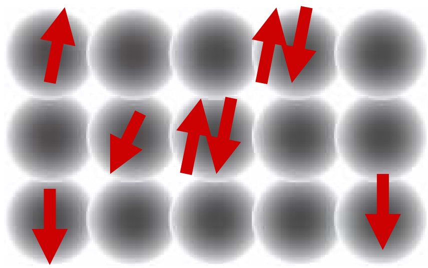
- more efficient to solve classical problems (prime number factorization, searches, …)
- useful to simulate **other** quantum systems (Feynman 1981)

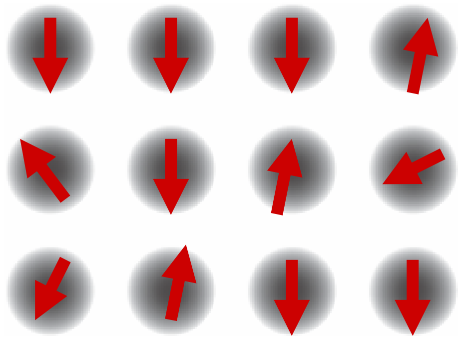


more modest goal (while waiting...):

- simulate quantum system A (e.g. **higher-Tc** superconductors) by quantum system B (e.g. cold atoms)
- useful if system B offers:
  - fewer complications (disorder, phonons, ...)
  - better control over parameters
  - other experimental probes
  - direct test of theories (!)
  - new physics (non-equilibrium)

Mott transition:





Sir Nevill Francis Mott
1905-1996

Hubbard model

$$H = -\mathbf{J} \sum_{\langle ij \rangle, \sigma=\uparrow\downarrow} c^\dagger_{i\sigma} c_{j\sigma} + \mathbf{U} \sum_i n_{i\uparrow} n_{i\downarrow}$$

kinetic energy ⟺ **interactions**

1 electron per unit cell: **metal** for $\mathbf{U} \ll \mathbf{J}$

Mott insulator for $\mathbf{U} \gg \mathbf{J}$

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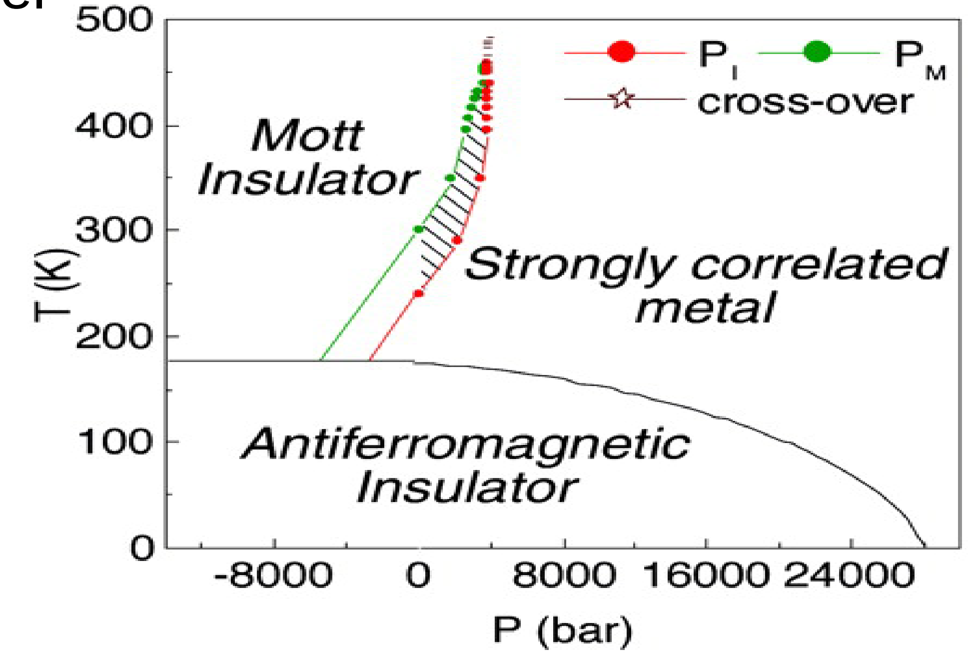
Mott transition

- example $(\mathbf{V}_{\mathbf{1-x}}\mathbf{Cr}_{\mathbf{x}})_{\mathbf{2}}\mathbf{O}_{\mathbf{3}}$
- but:
- **not** described by Hubbard model
  orbitals, crystal fields, disorder
  long-range interactions



from Limelette *et al.* 03

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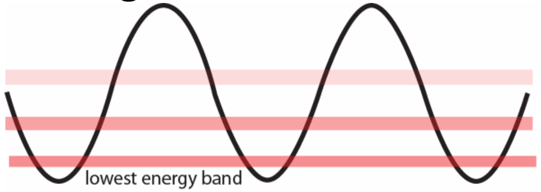
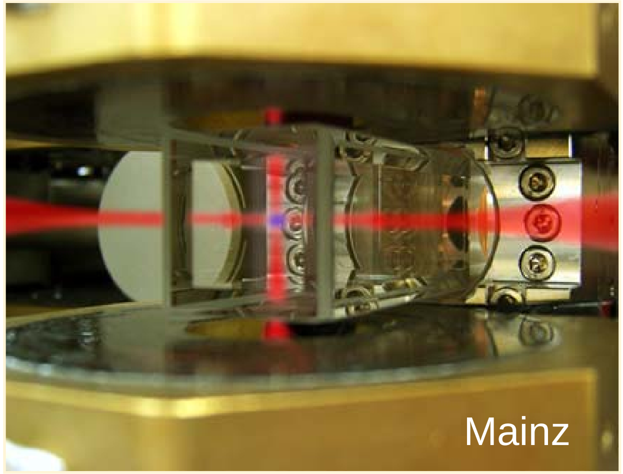
trapped atoms in lattices made of light

- trap & cool atoms in lattice made of light
- optical lattice from standing waves of laser
  effective potential



$$V(r) = \alpha(\omega)\langle E^2(r)\rangle$$
$$\propto \cos^2(kx) + \cos^2(ky) + \cos^2(kz)$$



- sufficiently high laser intensity:
  - only nearest neighbor hopping
  - only local interactions

  ⇨ **perfect realization of Hubbard model**

  in external parabolic potential (Jaksch et al. 98)
- all parameters (**J**, **U**, parabolic potential) known and fully controllable !!
- bosons or fermions (or arbitrary mixtures)

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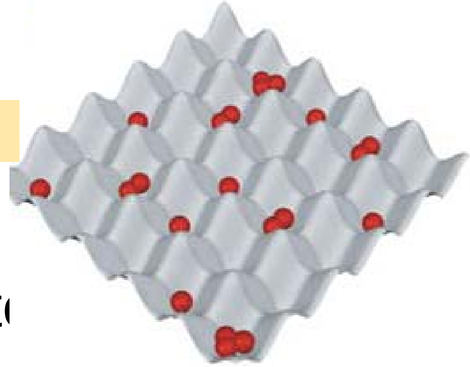
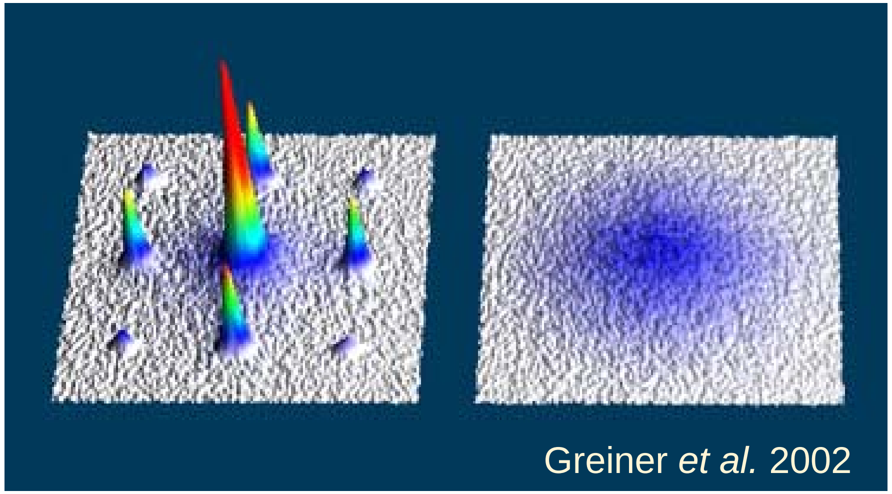
Mott transition of bosons

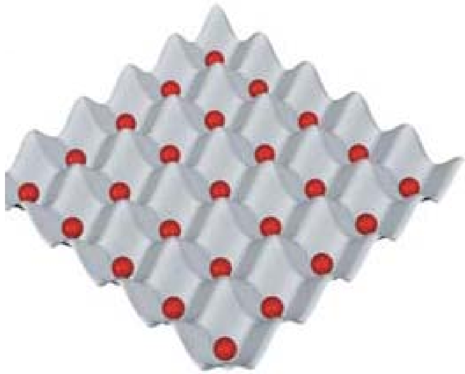
$$H \quad = -J \sum_{\langle ij \rangle} b_i^\dagger b_j + \mathbf{U} \sum_i n_i(n_i - 1) + \sum \mathbf{V_0} r_i^2 n$$



- **small *U*: *bose condensation & superfluidity***
- **large *U*:** integer number of localized atom per sit
  bosonic Mott insulator
- first realization: Greiner et al. 2002



superfluid Mott insulator



Bloch 05

- detections: **detect bose condensation**!

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Mott transition of fermions

fermionic Mott transition in optical lattices

more fun: magnetism, superconductivity, …

simulate materials

- problem 1: cooling (less scattering due to Pauli principle)
- problem 2: detection

experiments:

group of T. Esslinger (ETH)

R. Jördens, N. Strohmaier, K. Günter, H. Moritz, T. Esslinger, Nature 08

group of I. Bloch (Mainz)

U. Schneider, L. Hackermüller, S. Will, Th. Best,
I. Bloch, T. A. Costi, R. W. Helmes, D. Rasch,
A. Rosch, Science 08

theory needed !

Mott transition of fermions

- Theory for Mott transition?
- no symmetry breaking, no obvious order parameter

- method of choice: **dynamical mean field theory** (**DMFT**)

heavily used, e.g. for ab-initio description of correlated materials
only approximation of **DMFT**: self-energy purely **local**

$$\Sigma_{ij}(\omega) \approx \delta_{ij} \Sigma_i(\omega)$$

⇨ naturally generalizable to inhomogeneous systems

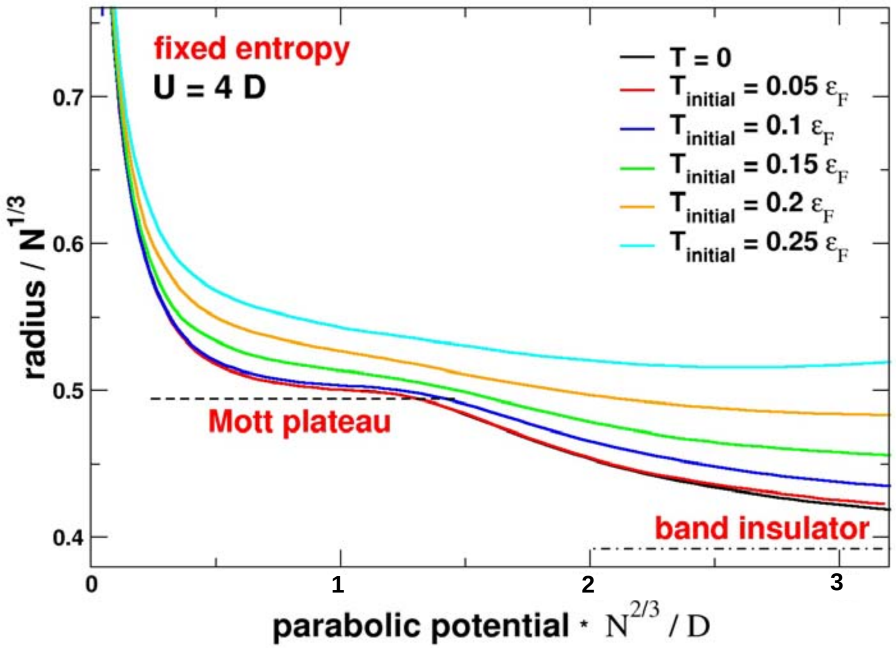
(Kotliar, Dobrosavljevic 97; Potthoff, Nolting 1999, Okamoto Millis 02, Freericks 04, Lee MacDonald 06, Snoek Hofstetter 08,….)

mapping to N single-impurity problems coupled by self-consistency equation

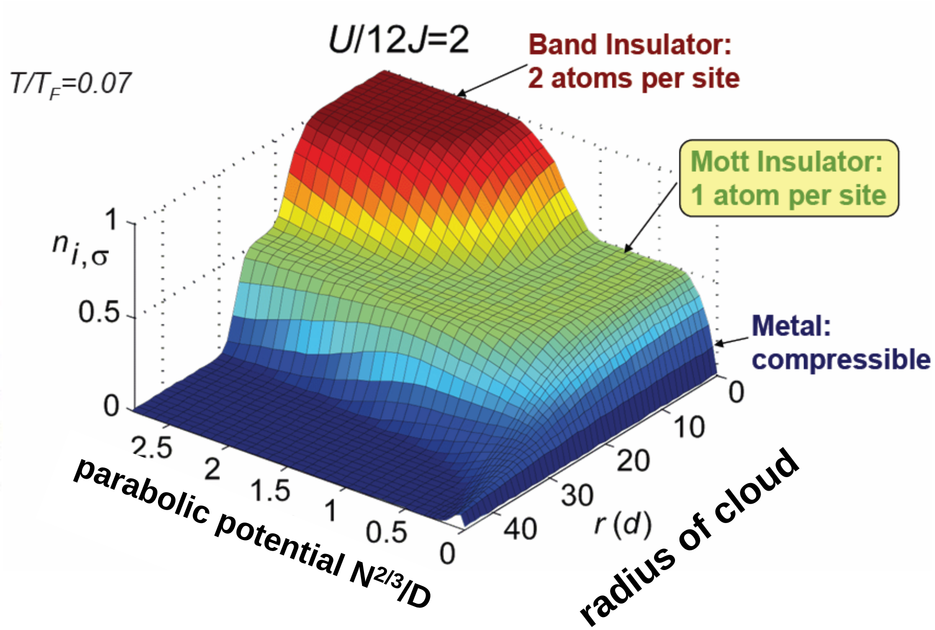
solved using NRG

R. Helmes, T. Costi, A. Rosch, PRL **100,** 056403 (2008)

- detecting Mott insulator: look for incompressible state
- ⇨ measure radius of cloud varying confining potential (I. Bloch)
- important: **fixed entropy *S***
- surprising: Mott plateau even visible for $\frac{S}{N} \gtrsim 2 \ln 2$, $T_{\text{initial}} \gtrsim 0.15\epsilon_F$
- **impossible** for homogeneous system (S< ln 2 for T<U)



Calculation: DMFT using 'LDA' approx.



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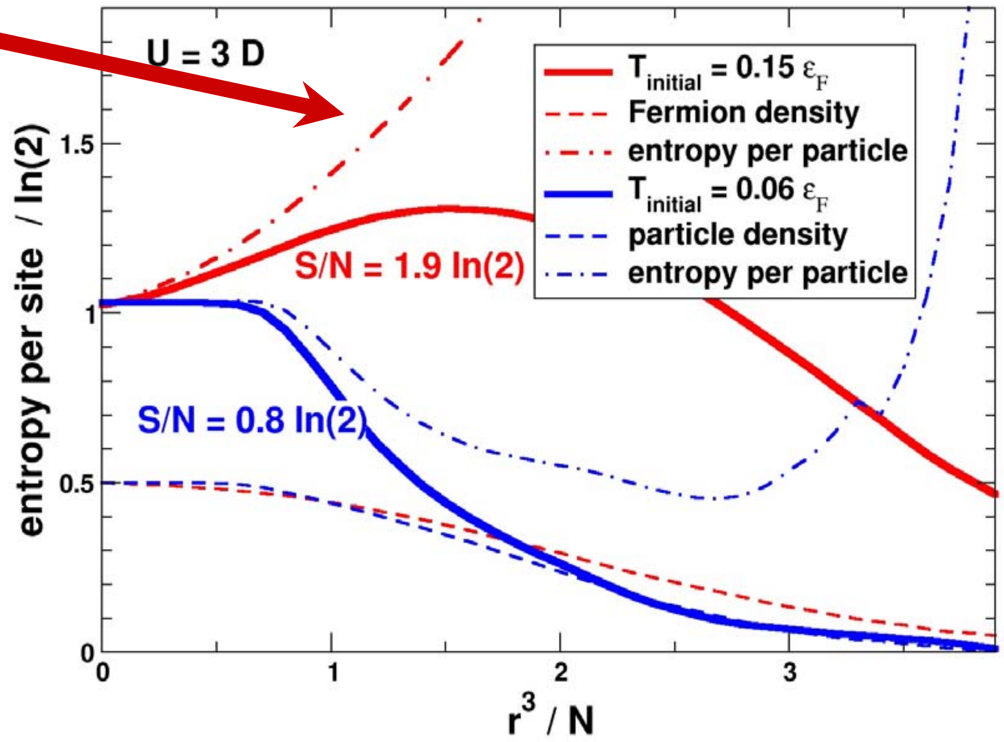
Mott transition of trapped atoms in optical lattices

Problem: large entropy per particle $\frac{\mathbf{S}}{N} \gtrsim 2 \ln 2$

Solution: dump entropy

in metallic belt



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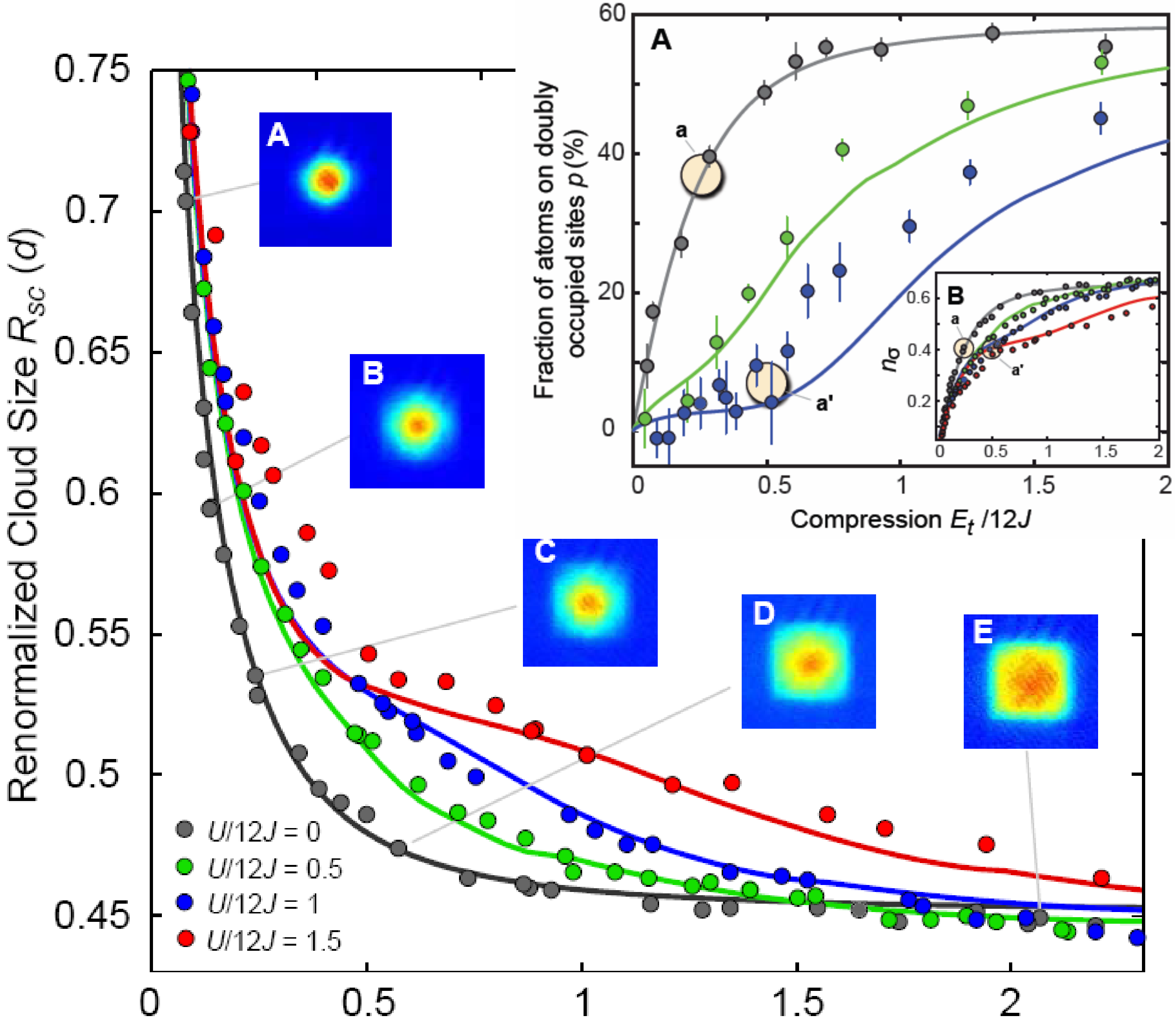
Mott transition of trapped atoms in optical lattices

- experimental results:
- U. Schneider, L. Hackermül S. Will, Th. Best, I. Bloch
- about 10^5 ^{40}K atoms in optical lattice
- initial T~0.15 E_F

- ➢ detection of fermionic Mott insulator
- ➢ real **experimental test** of DMFT without free parameters
- ➢ main experimental problem: adiabaticity



parabolic potential · $N^{2/3}$

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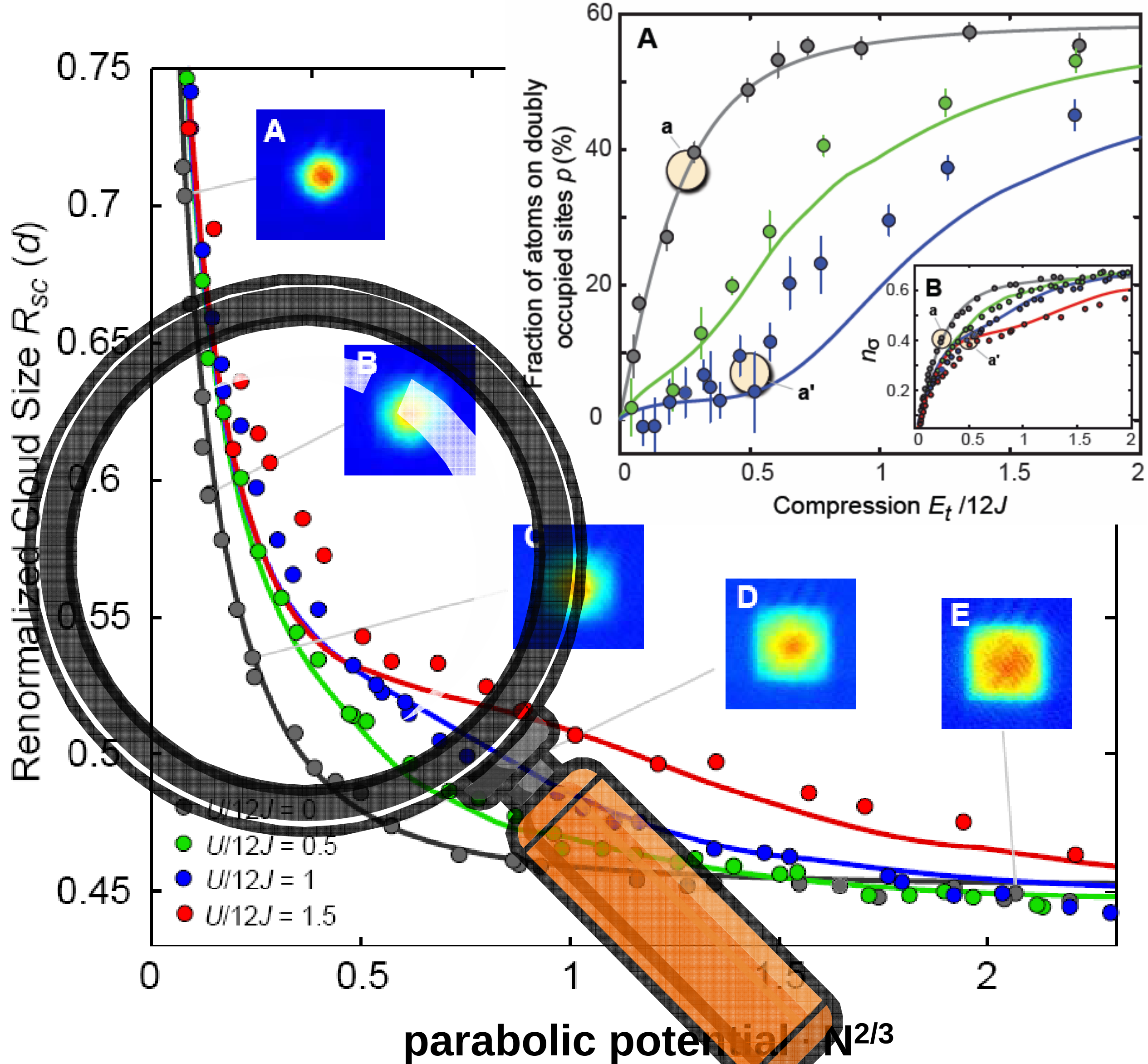
Mott transition of trapped atoms in optical lattices

- Deviations of theory and experiment for large U?

Experiment non-adiabatic for large U?



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Cold atoms as quantum simulator for Hubbard model?

- now: onset of Mott physics
- most needed: new cooling mechanism to reduce entropy by factor 4 for magnetism, factor 10 for d-wave superconductvity
  many ideas…
- **likely** within 2-3 years: detection of d-wave superconductivity in the 2d Hubbard model (if existent)
  not likely: any decent precision measurement (Tc as function of doping, T, …)
- already: useful as test for theories!
  see also: De Leo, Kollath, Georges, Ferrero, Parcollet, PRL 2008
  Snoek, Titvinidze, Toke, Byczuk, Hofstetter, New J. Phys. 2008
  Scarola, Pollet, Oitmaa, Troyer, PRL 2009,

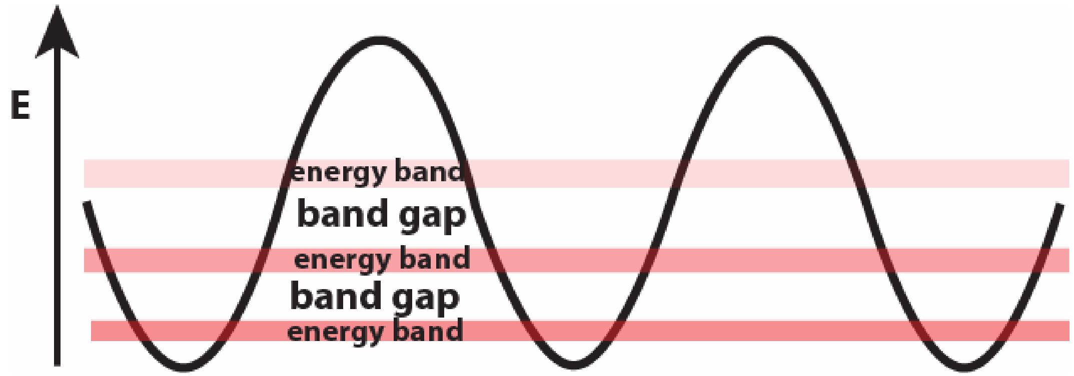
more fun with cold atoms: physics out of equilibrium

simple example: metastable finite momentum superconductivity

Lifetime of doubly occupied lattice sites

usually:
high energy states in interacting many body systems:
decay rapidely due to huge phase space
(exception: topologically stable states, solitions, domainwalls… of ordered phases)

periodic potential:
formation of bands and
band-gaps



⇨ regions **without** single-particle excitations

⇨ possiblity for (meta-) stable **high energy states** protected by energy conservation (not by topology)

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Lifetime of doubly occupied lattice sites

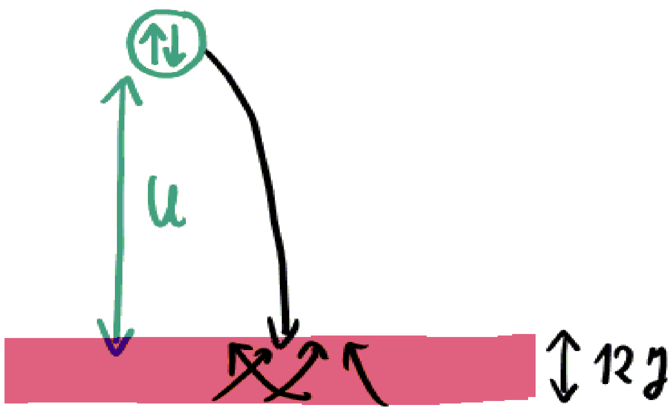
Hubbard model: band-gap $\gg \mathbf{U} \gg J$

$$H_h = -J \sum_{\langle ij\rangle,\sigma=\uparrow\downarrow} c^\dagger_{i\sigma} c_{j\sigma} + \mathbf{U} \sum_i n_{i\uparrow} n_{i\downarrow}$$

single particle states: $E_{\mathbf{k}} = -2J \sum_{i=x,y,z} \cos(k_i a)$

maximal single-particle energy: 12 ***J***

energy of doubly occupied site: ***U***



How can doubly occupied state decay?

necessary to create **many,** $\mathbf{N} \gtrsim \frac{\mathbf{U}}{12J}$,

single particles excitation to absorb energy

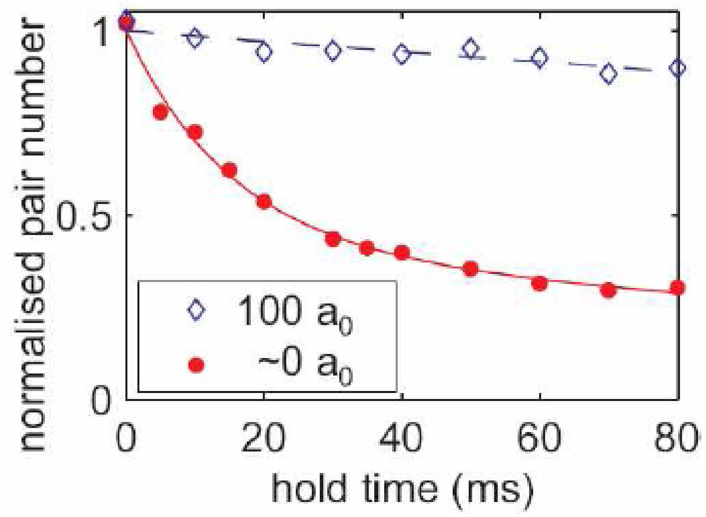
$$t_d \gtrsim (const.)^{\mathbf{N}} \sim \frac{1}{J} \exp[const.\mathbf{U}/J]$$

exponentially large life time

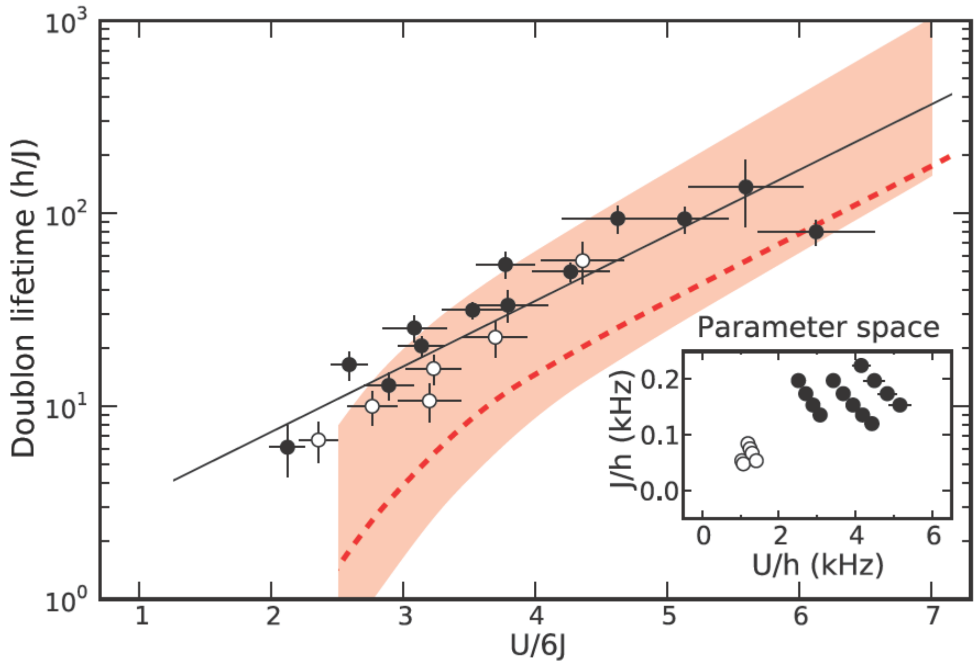
Exponentially large lifetime of doubly occupied lattice sites

Winkler, Thalhammer, Lang, Grimm, Denschlag, Daley, Kantian, Büchler, Zoller, Nature 2006

Strohmaier, Greif, Jördens,Tarruell, Moritz, Sensarma, Pekker, Altman, Demler, arXiv:0905.2963



lifetime 700 ms probably restricted by scattering from lattice photons



numerics in 1d: quasistationary state within time-dependent DMRG for large **U**: Kollath, Läuchli, Altmann, PRL 2007

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metastable superconductivity

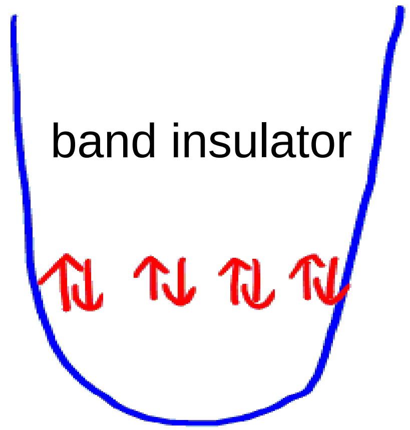
Include again trapping potential $\mathbf{H}_{\text{trap}} = \sum \mathbf{V_0} r_i^2 n_i$

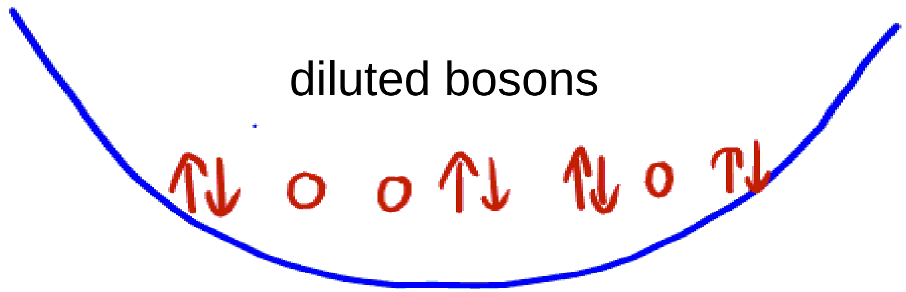
large $\mathbf{V_0}$ ⇨ band insulator with **large U**

reduce $\mathbf{V_0}$ **slowly** compared

to $1/J$ or $\mathbf{U}/J^2$

but **fast** compared to $\frac{1}{J} \exp[c\mathbf{U}/J]$





Bose condensation of diluted
doubly occupied sites

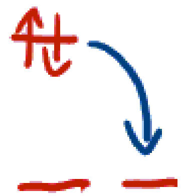
⇨ **s-wave superconductivity**

in strongly repulsive Hubbard model !

- Difference of repulsively from attractively bound states?
- Hopping via **virtual low-energy** rather than **high-energy** states
- ⇨ sign change of hopping rate of pairs

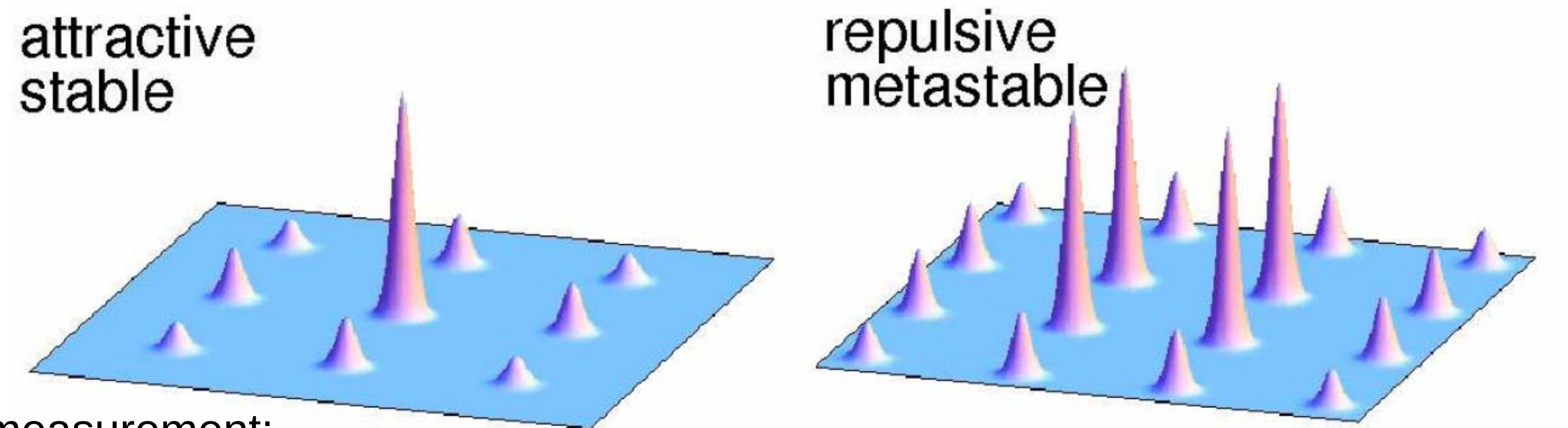
attractively bound pairs (***U*** <<- ***J***):

⇨ condensation at momentum (0,0,0)



repulsively bound pairs (***U*** >> ***J):***

⇨ condensation at momentum $\left(\frac{\pi}{a}, \frac{\pi}{a}, \frac{\pi}{a}\right)$



measurement:

convert to chemically bound pairs by rapid sweep through Feshbach resonace, measure momentum distribution of molecules (Regal, Greiner, Jin 04)

metastable superfluidity

alternative to bose-condensation of pairs:
phase separation (pairs clump together)

e.g. analog situation for Bosons analyzed by
D. Petrosyan, B. Schmidt, J. R. Anglin, M. Fleischhauer,
Phys. Rev. A 76, 033606 (2007)

phase separation seems to be unavoidable for bosons

controlled calculation needed for fermionic case !
usually difficult, here: symmetries help

Metastable superconductivity

Simple case: only doubly occupied and empty sites:

$$H_{\text{eff}} = \frac{2J^2}{\mathbf{U}}\sum_{\langle ij\rangle}(1-n_{i\uparrow})(1-n_{i\downarrow})n_{j\uparrow}n_{j\downarrow} + \frac{2J^2}{\mathbf{U}}\sum_{\langle ij\rangle} c^\dagger_{i\uparrow}c^\dagger_{i\downarrow}c_{j\downarrow}c_{j\uparrow}$$

Trick:
rewrite as spin Hamiltonian: $\downarrow$ empty site

$\uparrow$ doubly occupied site

$$S_i^+ = (-1)^i c^\dagger_{\uparrow i} c^\dagger_{\downarrow i}$$

$$H_{\text{eff}} = -\frac{2J^2}{\mathbf{U}}\sum_{\langle ij\rangle}\mathbf{S_i}\cdot\mathbf{S_j}$$

ferromagnetic Heisenberg model
due to SU(2) symmetry in the charge sector

exact groundstates for uniform systems:
magnetization in +z direction: band insulator
magnetization in –z direction: no particles
magnetization in x/y direction: s-wave superconductivity with momentum (π,π,π),
(η –paring, C.N. Yang 1989)

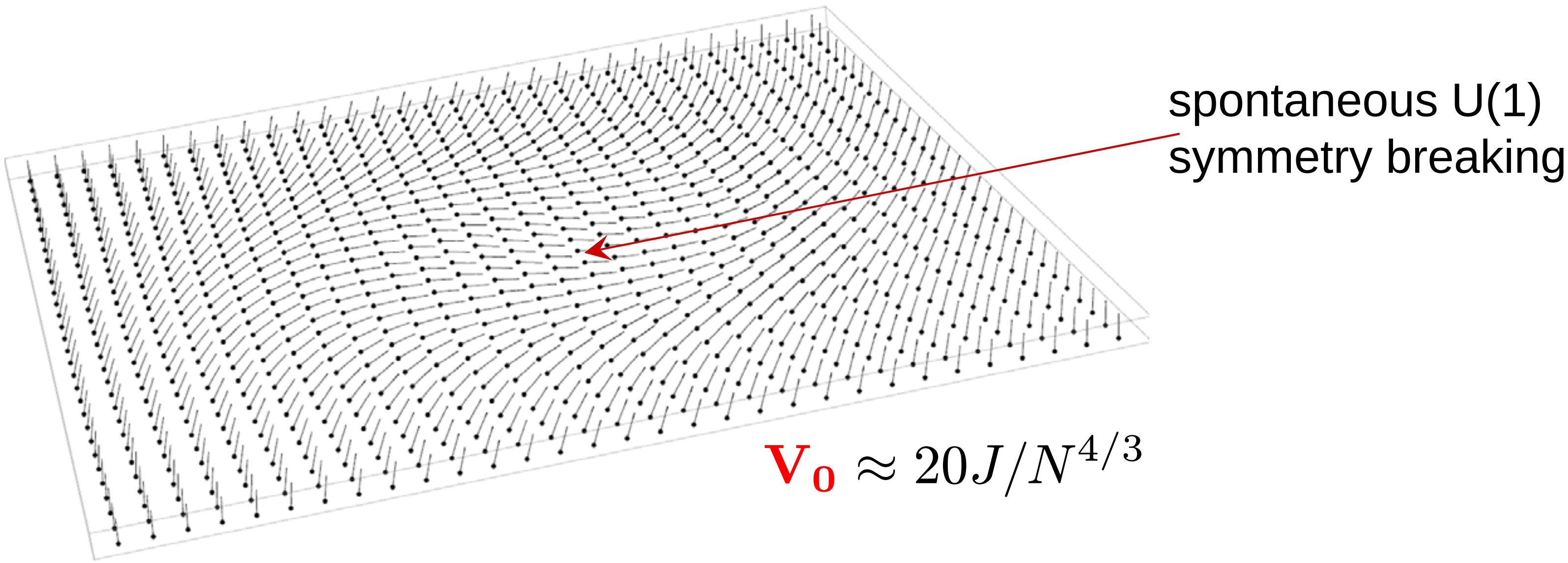
Metastable superconductivity

- with trapping and chemical potential:

$$H_{\text{eff}} = -\frac{2J^2}{\mathbf{U}}\sum_{\langle ij\rangle}\mathbf{S_i}\cdot\mathbf{S_j} + \sum_i(\mathbf{V_0}r_i^2 - \mu)S_i^z$$

- fixed by initial state: number of doubly occupied size $\sum(2S_i^z+1)$

variational solution in the 3d scaling limit: **superconductivity**



Metastable superconductivity

Is this a true superfluid with phase stiffness? **NO** (not yet)

ferromagnet: quadratic dispersion of spin-waves, $\omega \sim k^2$
superfluid: linear dispersion (necessary for superfluidity)

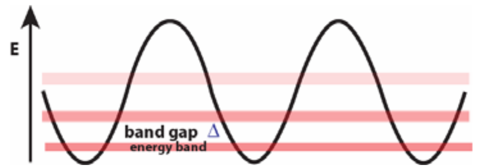
no superfluid ~ exact cancellation of attractions and hard-core repulsion of bosons

similarly:
up to domain wall energy
phase separation & superconductivity degenerate

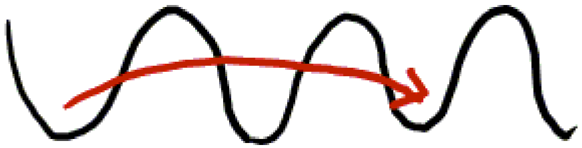
Leading correction can change physics!

Calculations: stabilized by corrections to Hubbard model (e.g. nearest neighbor interactions)

breaking of $SU_C(2)$ symmetry of Hubbard model

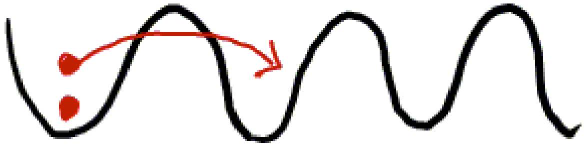


- Analyze leading corrections to **$SU_C(2)$** from **corrections** to Hubbard model



$O(J^2/\mathbf{gap})$

next nearest neighbor hopping favors phase separation but only to 2nd order $O(J^4/(\mathbf{gap}^2\mathbf{U}))$



assisted hopping

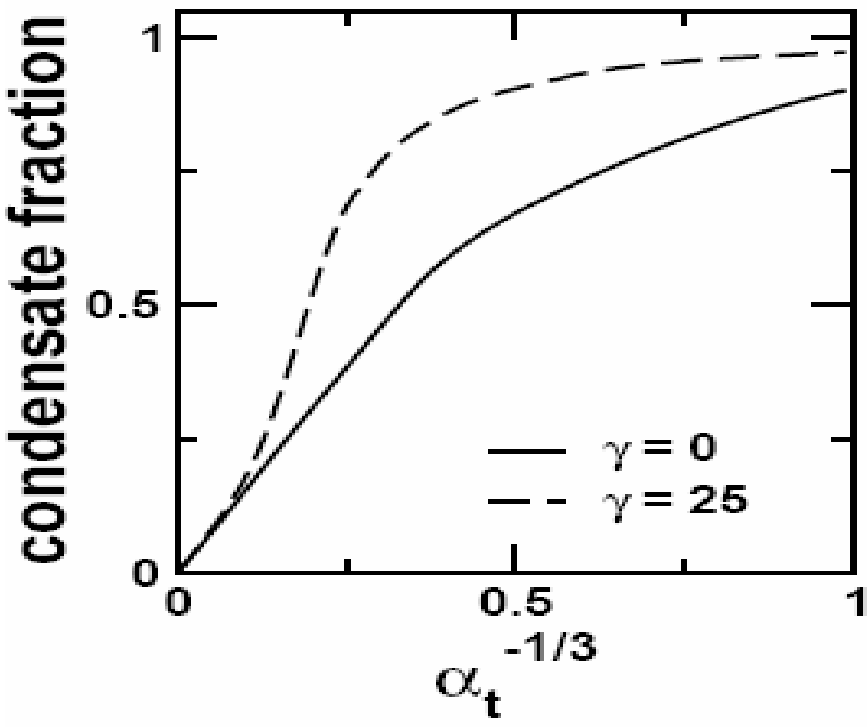
pair hopping

dominant:
nearest neighbor interaction

$$\mathbf{U}' \sim \mathbf{U}(J/\mathbf{gap})^2 \gg J^4/(\mathbf{gap}^2\mathbf{U})$$

breaking of $SU_C(2)$ symmetry of Hubbard model

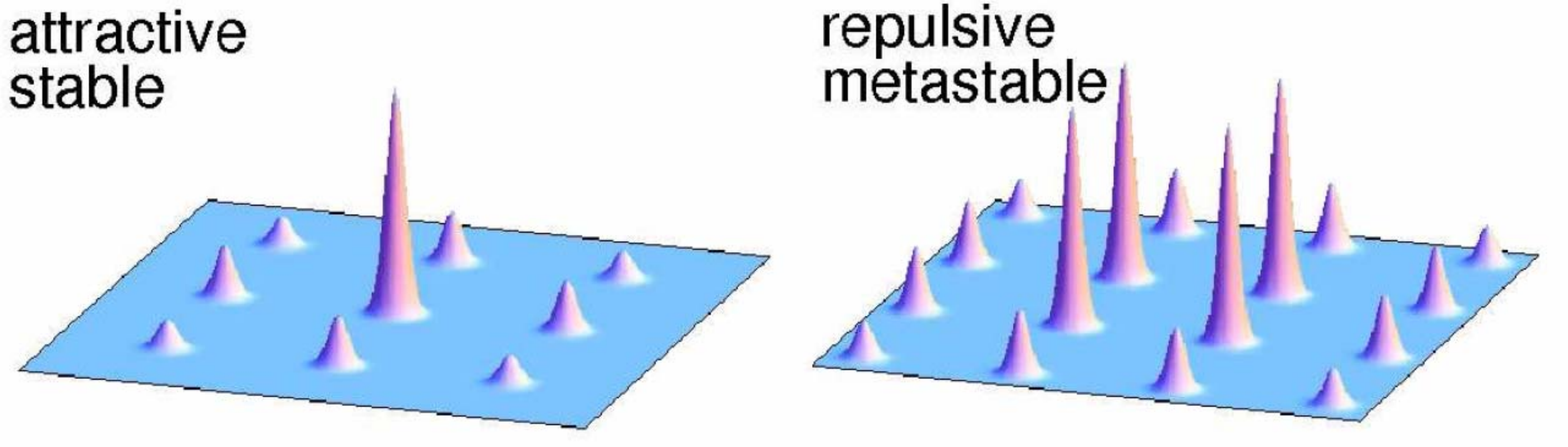
- trapping potential $\alpha_t = \mathbf{V} N^{4/3} \mathbf{U} / J^2$
- corrections to Hubbard model $\gamma = \mathbf{U}' \mathbf{U} N^{2/3} / J^2 \gg 1$



cloud expands
exotic metastable s-wave superfluidity stabilized in strongly repulsive Hubbard model

Metastable superfluidity

metastable superconductivity easy to realize and detect?



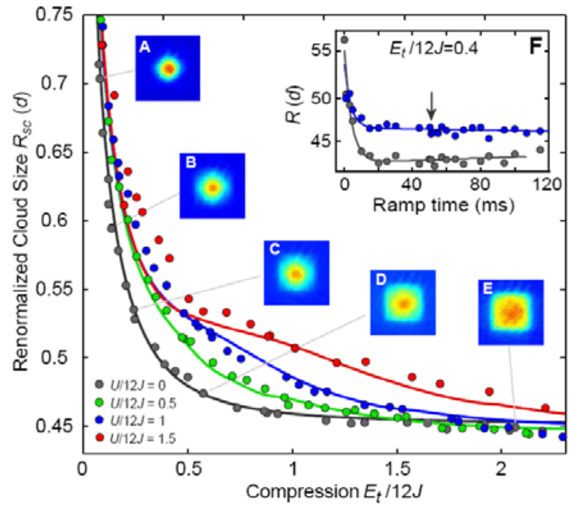
crude estimate:
Bose condensation in trap for entropie per boson $S<3.6\ k_B$

corresponding entropy per Fermion: $S<1.8\ k_B$
corresponding initial temperature before loading in trap: $0.22\ T_F$

presently reached (e.g. group of Bloch): $0.15\ T_F$
Main question: effect of singly occupied sites!

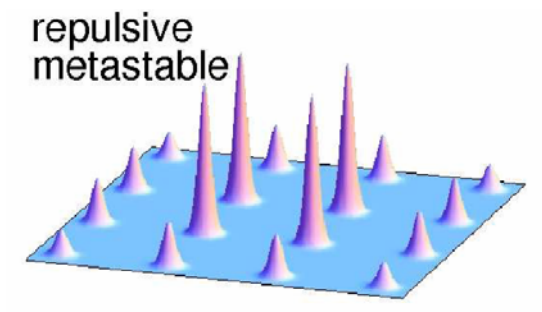
Quantum simulations with cold atoms

- first simulations of correlated fermions in lattices: Mott transition
- next steps:
  quantum magnetism (with 1/4th of entropy)
  d-wave superconductivity



new phenomena out of equilibrium:

- metastable superconductivity



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Schneider, Hackermüller, Will, Best, Bloch, Costi, Helmes, Rasch, A. Rosch, Science (2008)
Rosch, Rasch, Binz, Vojta, PRL **101**, 265301 (2008)

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