

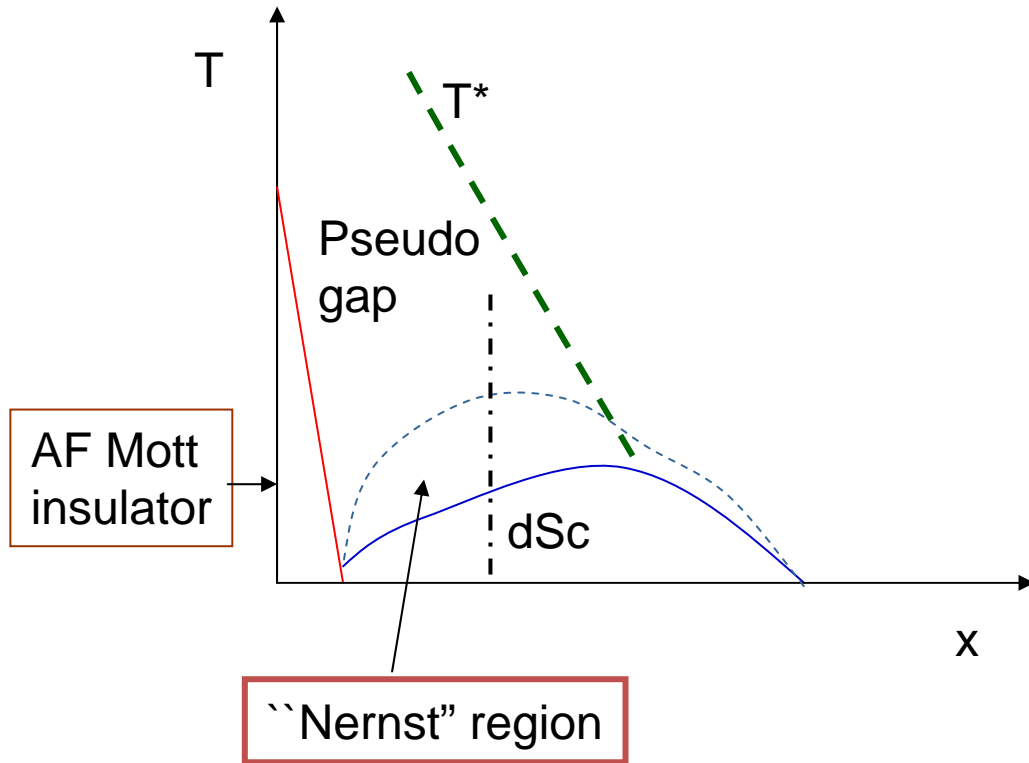
Physics of the underdoped cuprates: Phenomenological synthesis and a microscopic theory

T. Senthil (MIT)

1. [T. Senthil and P.A. Lee, PRB 09](#)

2. T. Senthil and P.A. Lee, arxiv 0904.1433

Cuprate phase diagram



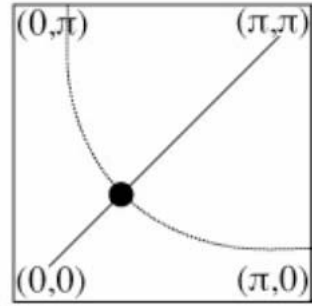
Focus on
pseudo gap state:
at not too low
doping.

Some important phenomena

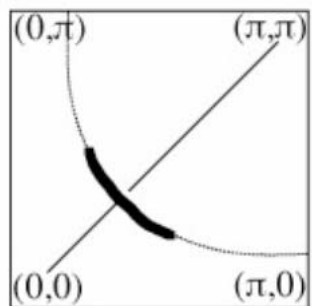
1. Antinodal gap (≈ 50 meV), gapless. T-dependent Fermi arcs near node.
2. "Landau" quasiparticles emerge only below a low "coherence" scale $T_{\text{coh}} \approx T_c$.
3. Persistence of SC amplitude without phase coherence above T_c (microwave, Ong Nernst/magnetization)
4. Other competing order (eg: SDW, CDW, ...)
Eg: At low-T SDW can be stabilized by magnetic field.

Gapless Fermi arcs

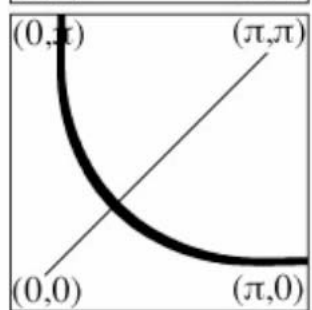
Arc length decreases with decreasing T at fixed x .
(possibly extrapolate to 0 at $T \rightarrow 0$)



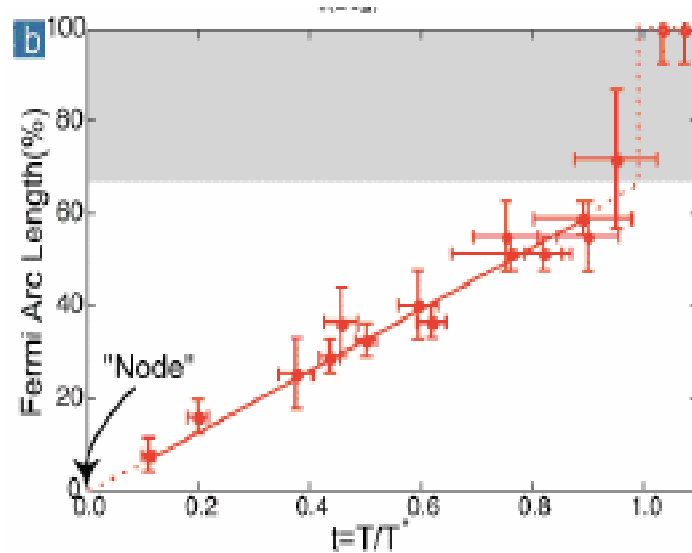
$$T < T_c$$



$$T_c < T < T^*$$



$$T > T^*$$

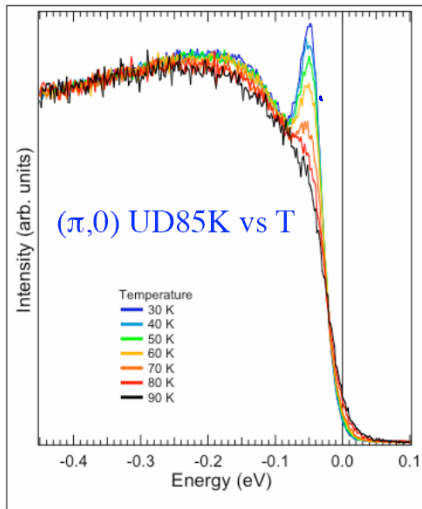


Kanigel et al, Nature Physics '06

Electron coherence crossover

Theory : fairly generic to proximity to Mott transition
(slave boson theory/DMFT)

Expt : 1. Evolution of antinodal spectra across T_c



2. Rapid suppression of scattering rate below T_c in microwave / thermal transport.
(Bonn et al.) (Ong et al.)

- also nodal ARPES across T_c (P. Johnson et al)

Field induced incommensurate magnetism

$H=0$: Dynamic incommensurate spin fluctuations in underdoped YBCO (Stock, Buyers et al '04)

Field induced SDW.

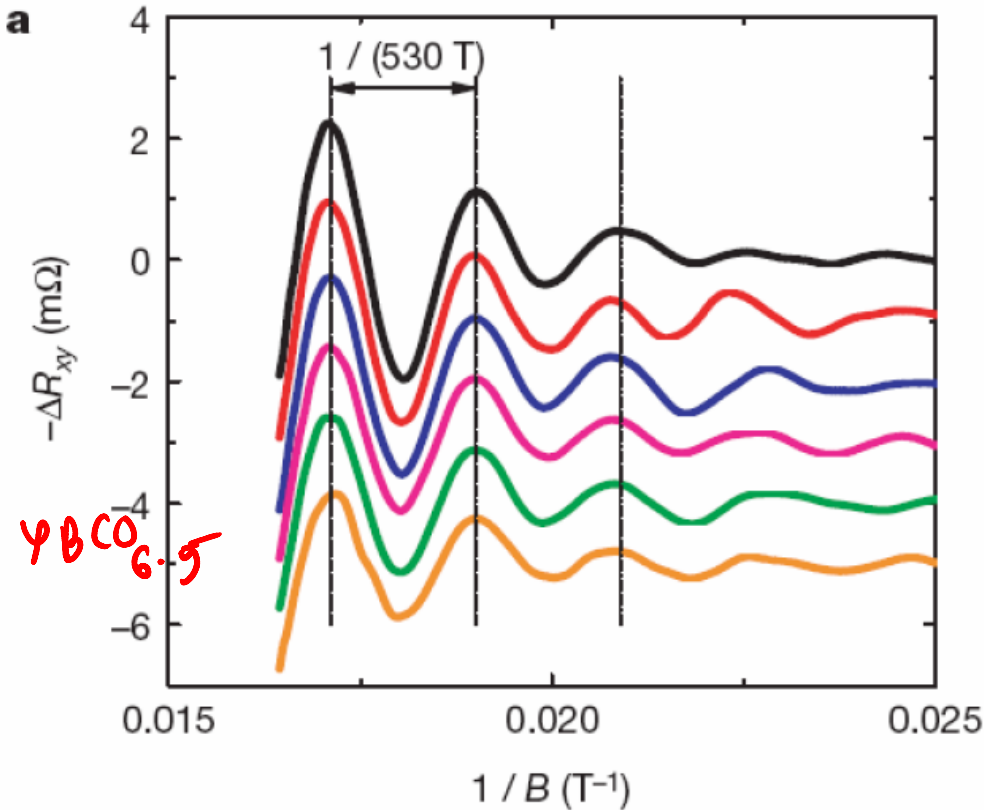
1. Direct evidence in LSCO family (Lake et. al.), and in YBCO_{6.45} (Keimer et. al. '09)

2. Indirect: No Zeeman splitting of high field quantum oscillations in YBCO_{6.5} (Sebastian, Harrison, et al '09)

Some important phenomena

1. Antinodal gap (≈ 50 meV), gapless. T-dependent Fermi arcs near node.
2. "Landau" quasiparticles emerge only below a low "coherence" scale $T_{\text{coh}} \approx T_c$.
3. Persistence of SC amplitude without phase coherence above T_c (microwave, Ong Nernst/magnetization)
4. Other competing order (eg: SDW, CDW, ...)
Eg: At low-T SDW can be stabilized by magnetic field.

New mystery: quantum oscillations in a magnetic field at low T



de Haas-van Alfen, Shubnikov-de Haas oscillations in ultra-pure YBCO_{6+x} ($x \approx 0.5$) and $\text{YBa}_2\text{Cu}_4\text{O}_8$...

Dominant frequency 530 T
 \Rightarrow small pocket.

(Proust, Taillefer, ... '07)

Other frequencies with lower amplitude

Eg: 1650 T (Sebastian et al. '08)

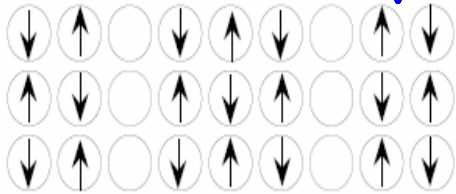
? Electron pockets?

Le Bouef, Taillefer et. al. '08 ($R_H < 0$ at low-T)

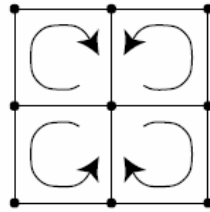
Antinodal electron pockets?

- various density wave orderings.

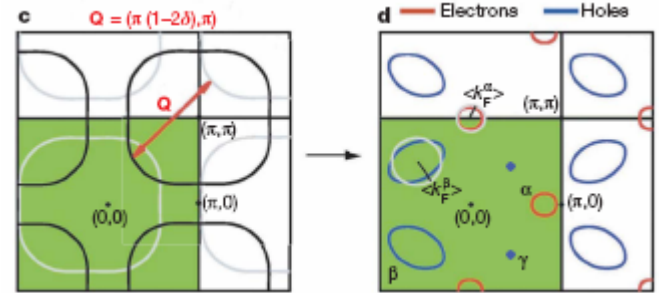
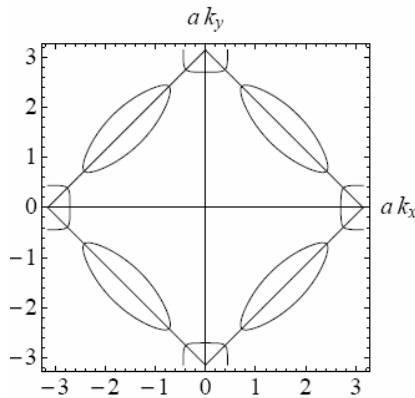
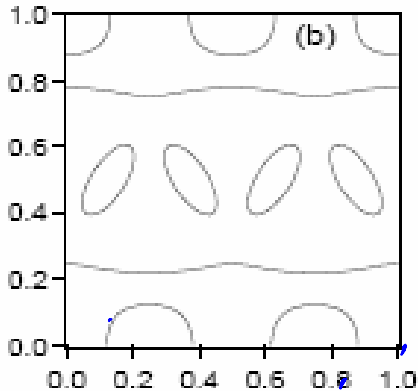
"Antiphase stripe"



"d-density wave"



Incommensurate spin density wave



Mullis, Norman '07

Chakravarty, Kee '08

Sebastian et. al. '08

How do all this fit together?

$T^* > T > T_c$, low H : "gapless Fermi arcs" that shrink as $T \searrow$, antinodal gap ≈ 50 meV

Low T , high H : closed Fermi pocket
(perhaps in antinodal region?)

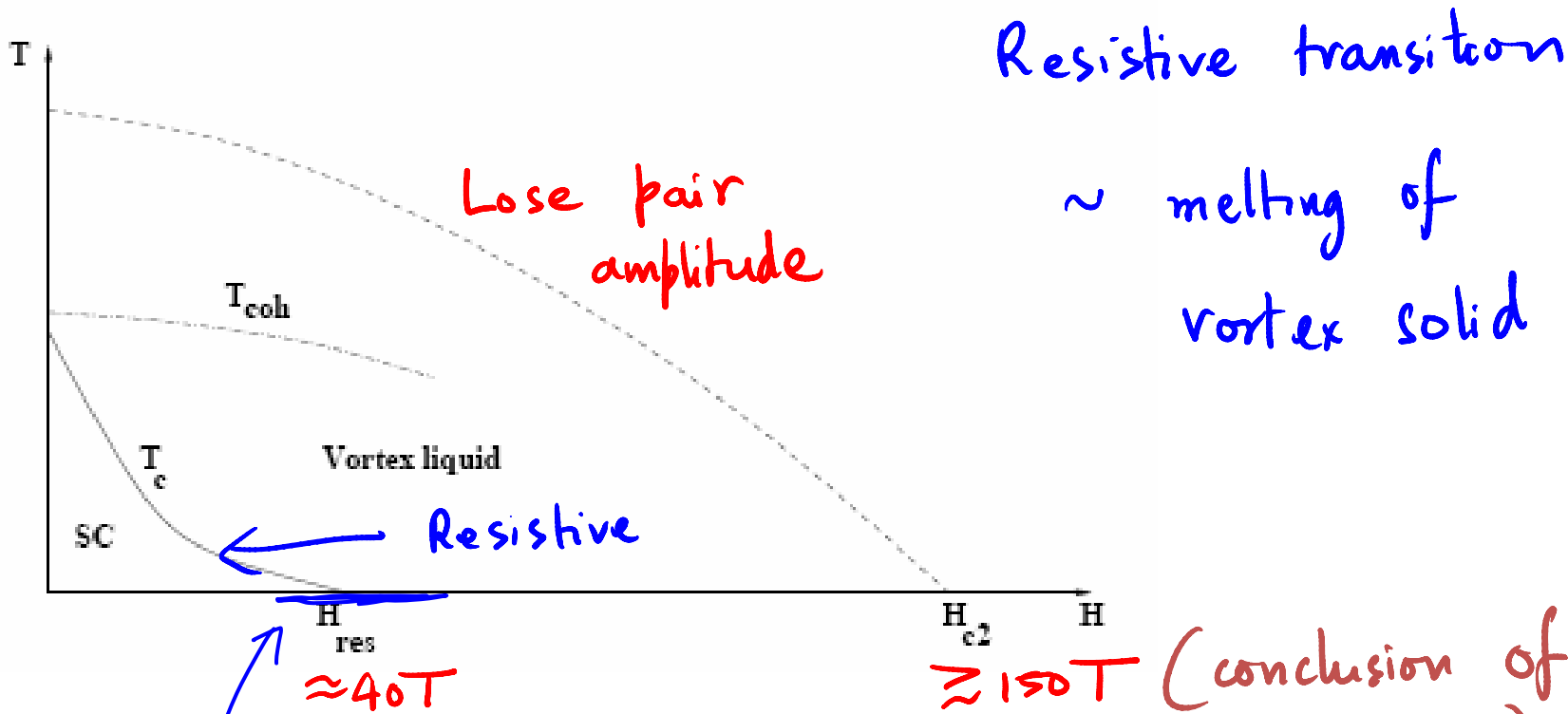
How can a closed Fermi surface emerge at low T ?

Can a field $\sim 50T$ really destroy the pseudogap?
(Antinodal gap ~ 50 meV)

Plan of this talk

1. A synthesis of the phenomenology
 - a coherent picture to reconcile ARPES, Nernst/magnetization with quantum oscillations.
2. A microscopic theory accessing key aspects of overall picture.

Ong 'high' field phase diagram



Resistive transition

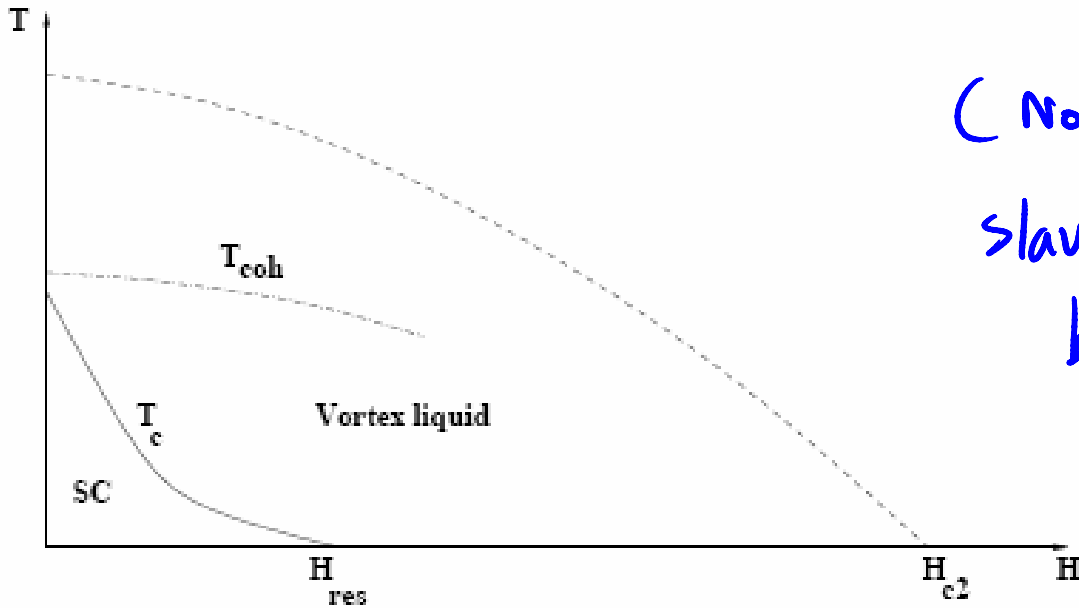
\sim melting of vortex solid.

Quantum oscillation expts done here

\Rightarrow Must understand within framework of "vortex liquid".

Key assumption: Electron coherence in a field

$H \sim 0$ (H_{res}) does not suppress T_{coh} to 0
but only T_c .



(Not valid in simplest
slave boson theory; need
better justification)

Exptl support: STM
tunneling into vortex core
(Hudson, Davis 2000)

Easier to weaken SC than to kill
coherence peak.

Low T, high H: emergence of large Fermi surface

Quasiparticle Hamiltonian

$T \ll T_{\text{cosh}} \Rightarrow$ effective quasiparticle Hamiltonian

Let q_{α}^{\dagger} create low energy quasiparticle.

Electron operator $c_{\alpha}^{\dagger} \approx \sqrt{Z_0} q_{\alpha}^{\dagger} + \dots$
($Z_0 \sim \alpha(x)$)

At $H=0$,

$$\mathcal{H}_{\text{eff}} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} q_{\mathbf{k}\Delta}^{\dagger} q_{\mathbf{k}\Delta} + \sum_{\mathbf{k}} \Delta_{\mathbf{k}} (q_{\mathbf{k}\uparrow}^{\dagger} q_{-\mathbf{k}\downarrow}^{\dagger} - q_{-\mathbf{k}\downarrow}^{\dagger} q_{\mathbf{k}\uparrow}^{\dagger}) + \text{h.c.}$$

$$\Delta_{\mathbf{k}} \sim \Delta_0 (\cos k_x - \cos k_y)$$

Model of a vortex liquid

In vortex liquid, d-wave pair order parameter

$$\rightarrow \Delta(\vec{R}, \tau) = \Delta_0 e^{i\phi(\vec{R}, \tau)}$$

($\Delta(\vec{R}, \tau)$ couples to d-wave singlet pair with center of mass at \vec{R}).

$$\text{Take } \langle \Delta^{\dagger}(\vec{R}, \tau) \Delta(0, 0) \rangle = \Delta_0^2 F(\vec{R}, \tau)$$

$$\text{with } F(0, 0) = 1$$

$$F(|\vec{R}| \rightarrow \infty, \tau) \sim e^{-|\vec{R}|/\xi_p}; \quad F(|\vec{R}|, \tau \rightarrow \infty) \sim e^{-|\tau|\Gamma}$$

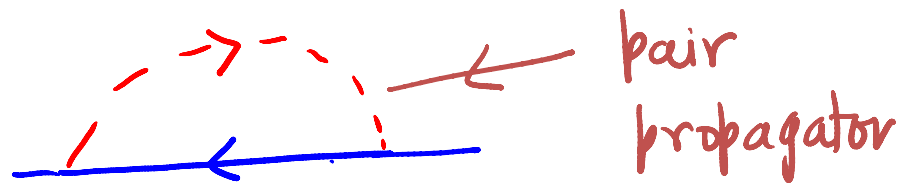
$$(\Gamma^{-1} \equiv \tau_p = \text{pair phase memory time})$$

Electronic structure of the vortex liquid

Quasiparticles scatter off fluctuating pair field.

Calculate self-energy in 2nd order perturbation theory

$$\Sigma(\vec{k}, \omega) =$$



⇒

$$\Sigma(\vec{k}=\vec{k}_F, \omega) = \frac{\Delta_0^2 \omega}{\pi \Gamma^2}$$

for small ω

(like in Fermi liquid)

$$\approx -\frac{\Delta_0^2}{\omega + \epsilon_k} \text{ for } |\omega| \gg \Gamma \text{ (like in dSC)}$$

Approximate self-energy

$$\Sigma(\vec{k}, i\omega) \approx \frac{\Delta_{0\mathbf{k}}^2 (-i\omega + \epsilon_{\mathbf{k}})}{-\omega^2 + \epsilon_{\mathbf{k}}^2 + \pi\Gamma^2}$$

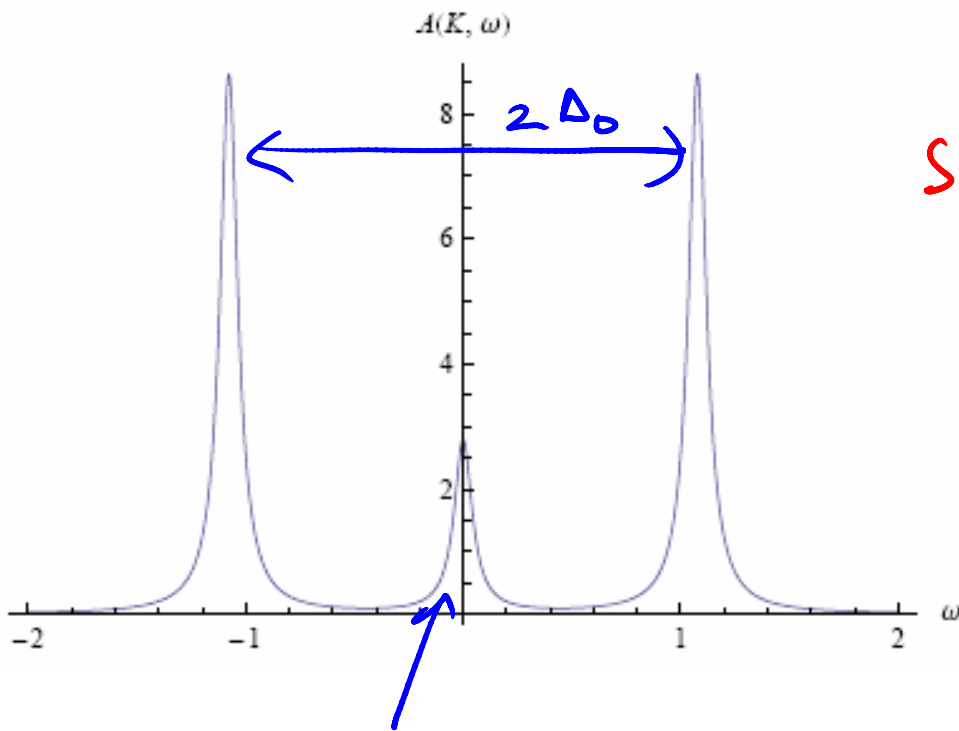
interpolates between
both limits.

Quasiparticle pole at "large" Fermi surface

with residue $Z_{\Delta} = \frac{1}{1 + \frac{\Delta_{0\mathbf{k}}^2}{\pi\Gamma^2}}$.

$$\Rightarrow Z_{\Delta}^{\text{nodal}} \approx 1 ; \quad Z_{\Delta}^{\text{anti-nodal}} \approx \frac{\pi\Gamma^2}{\Delta_0^2} \ll 1$$

Antinodal spectral function .



Quasiparticle
peak

Physical picture :

SC for time scales $\ll \tau_\phi$

length scales $\ll \xi_\phi$

but metal with large

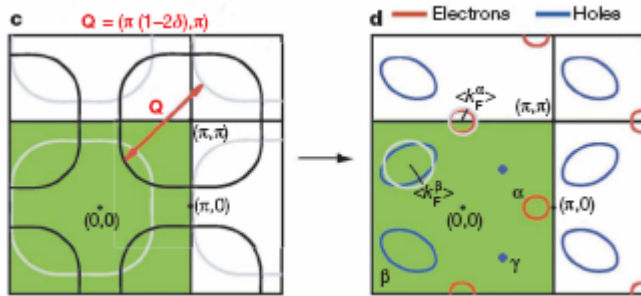
Fermi surface at

longer scales .

Effect of magnetic field

1. Usual Landau quantization of orbits.
2. Field induced SDW ordering

⇒ reconstruction of emergent large Fermi surface into electron & hole pockets



Can now follow previous papers (Mills, Norman, Sebastian,) to understand quantum oscillations.

Picture at low T, high H

Emergence of large Fermi surface metal in vortex liquid at low-T.

Pseudo gap does not close but mid-gap states with low spectral weight are produced.

Large Fermi surface metal — precondition for field induced SDW to do the job of reconstruction to produce electron/hole pockets.

Crucial question: how to reconcile with high T, low H phenomena?

Physics across T_c

Two things happen upon crossing T_c (at $H=0$).

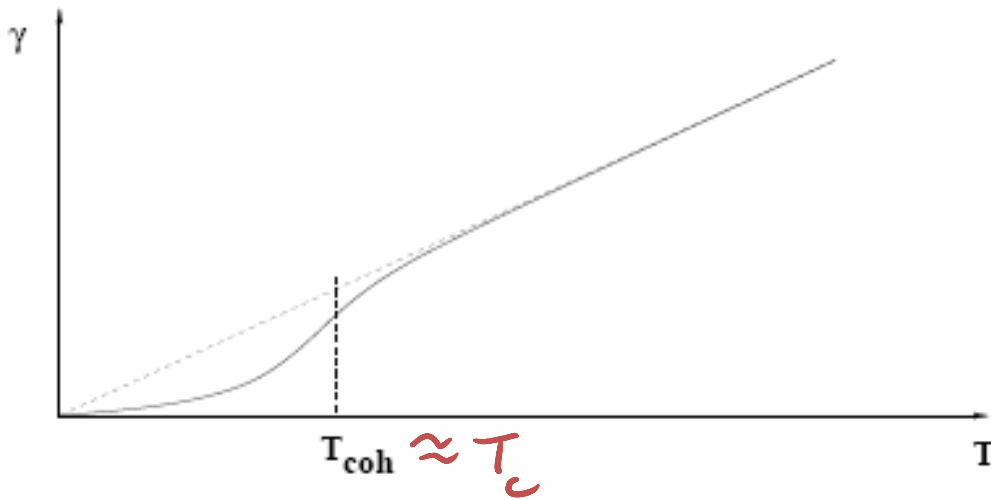
(i) Lose phase coherence of pair order parameter

BUT ALSO

(ii) Lose single particle coherence (as $T_{coh} \approx T_c$)

T_c - not just a phase disordering transition of SC but also a "coherence" transition for electrons.

Modeling single particle incoherence



Simplified model: take
single particle scattering
rate $\gamma = \text{large}, \propto T$
for $T > T_{\text{coh}}$
 $\approx T_c$

$\gamma = \text{small}, \lesssim T^2$ for
 $T < T_c$

Model SC phase disordering as before
with a phase decay rate $\Gamma \ll \Delta_0$.

Pseudogap and Fermi arcs

Take vortex liquid self energy from before and
let $\omega \rightarrow \omega + i\Gamma$

$$\Rightarrow \Sigma_R(\vec{k}, \omega) \approx \frac{\Delta_{0k}^2 (\omega - \epsilon_k + i\Gamma)}{(\omega + i\Gamma)^2 - \epsilon_k^2 - \pi\Gamma^2}$$

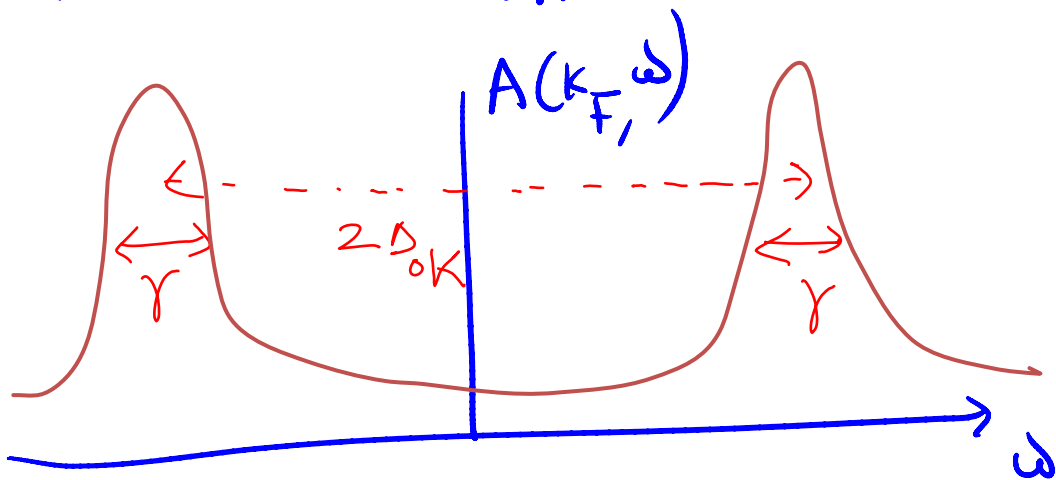
↑ electron decay rate ↑ Pair decay rate

For $\Gamma \ll \min(|\omega|, \gamma)$

$$\Sigma_R(\vec{k}, \omega) \approx \frac{\Delta_{0k}^2}{\omega - \epsilon_k + i\Gamma} = \text{self-energy similar to Norman et al '98, '07 to fit ARPES data.}$$

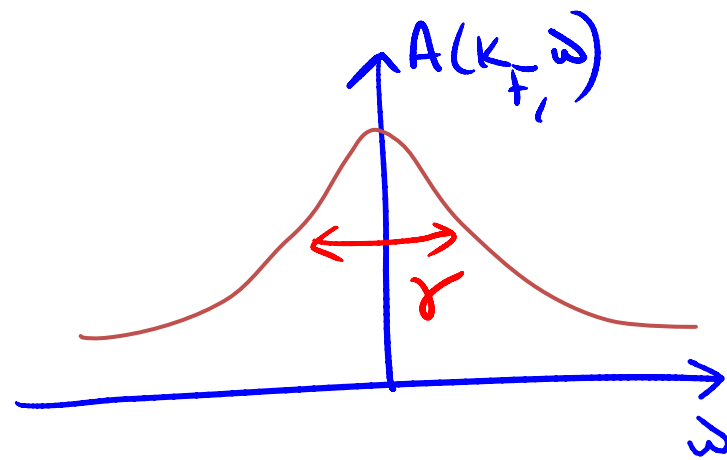
Pseudogap and Fermi arcs

For $\gamma \ll \Delta_0 k$



"Pseudogap" like

For $\gamma \gg \Delta_0 k$



"Fermi arc" like

$\gamma \gg \Delta_0 k$ always satisfied near nodal $\vec{k} \Rightarrow$ get Fermi arcs
AND pseudogap.

Arc length set by $\gamma \approx \Delta_0 k \Rightarrow$ decrease as $T \downarrow$

(Norman et. al. '98, '07; Chubukov et. al. '08)

Summary

1. Quantum oscillations in $T=0$ vortex liquid
 - emergence of large FS
 - reconstruction by field induced SDW
2. Pseudogap / Fermi arcs at $T > T_{cdh} \approx T_c$:
Incoherent single particle excitations + pairing / other order
fluctuations

KEY issue for microscopic theory: single particle (in)coherence & interplay with ordering.

Part 2: A microscopic theory

Revisit slave boson theory of doped t-J model.

$$H = -t \sum_{\langle ij \rangle} (c_i^\dagger c_j + h.c.) + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

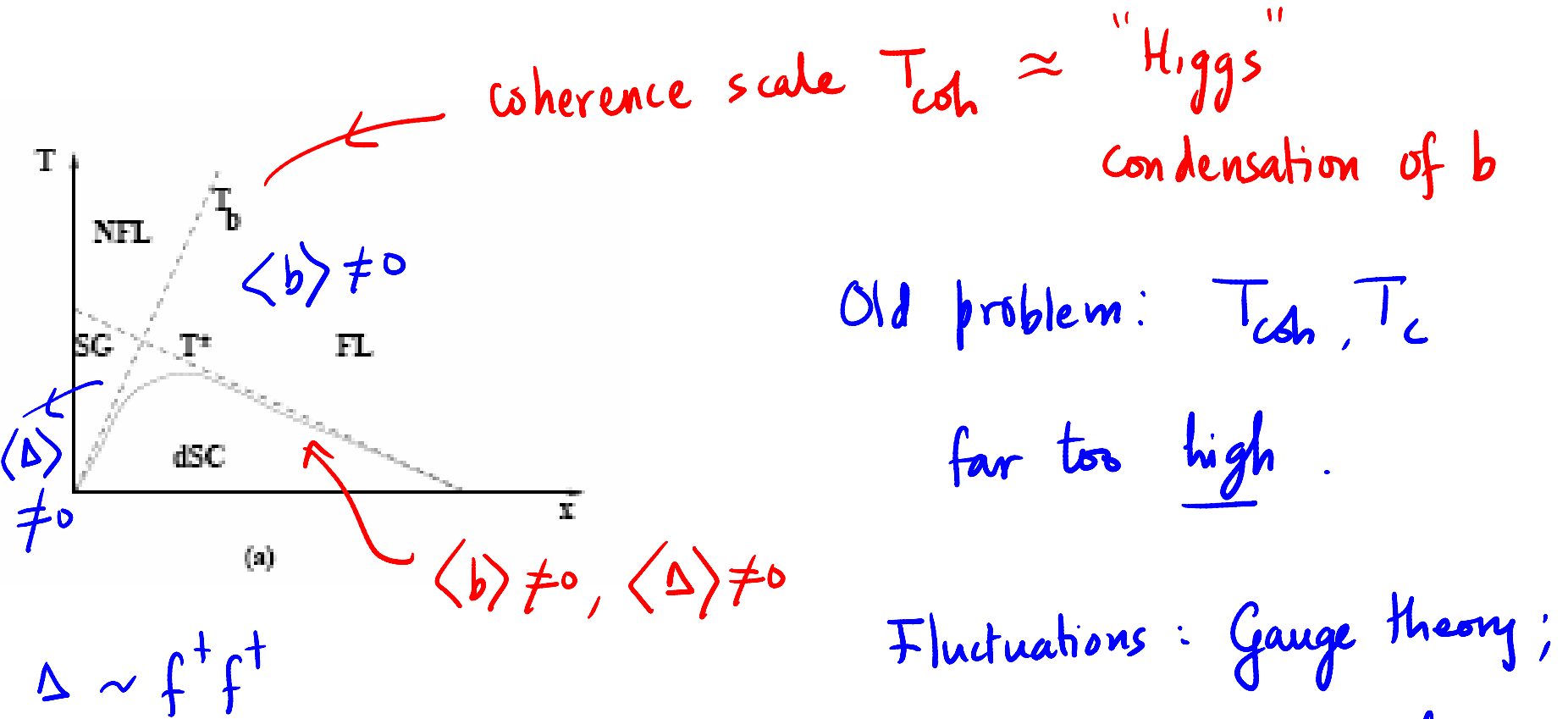
$$c_i^\dagger c_i \leq 1 \Rightarrow \text{solve by } c_{i\alpha} = \underset{\substack{\uparrow \\ \text{holon}}}{b_i^\dagger} \underset{\substack{\uparrow \\ \text{spinon}}}{f_{i\alpha}}$$

$U(1)$ phase redundancy

$$\Rightarrow \text{action } S = S[b, f, a_\mu]$$

\uparrow $U(1)$ gauge field

“Standard” slave boson RVB theory of doped Mott insulator



True coherence scale: Anderson is different

(TS '08)

Landau damped gauge dynamics

⇒ Anderson "plasmonization" scale of a_μ parametrically different from Higgs condensation scale.

Intermediate energies - holons "condensed" but a_μ "gapless"

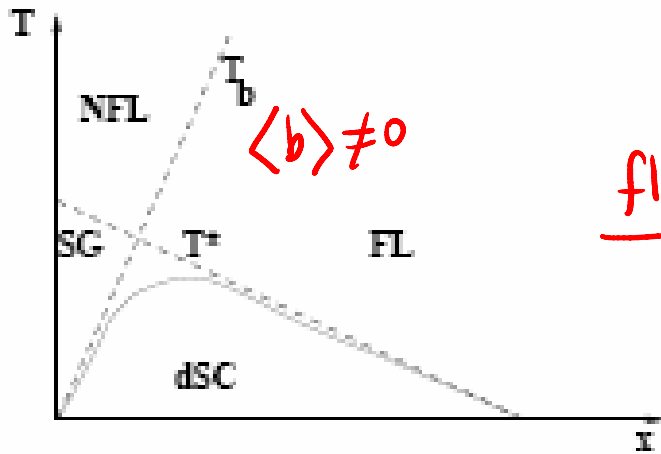
⇒ Electrons strongly scattered by a_μ fluctuations
⇒ $T_{coh} \sim T_b^{3/2} \ll T_b$

"INCOHERENT FERMI LIQUID"
(IFL)

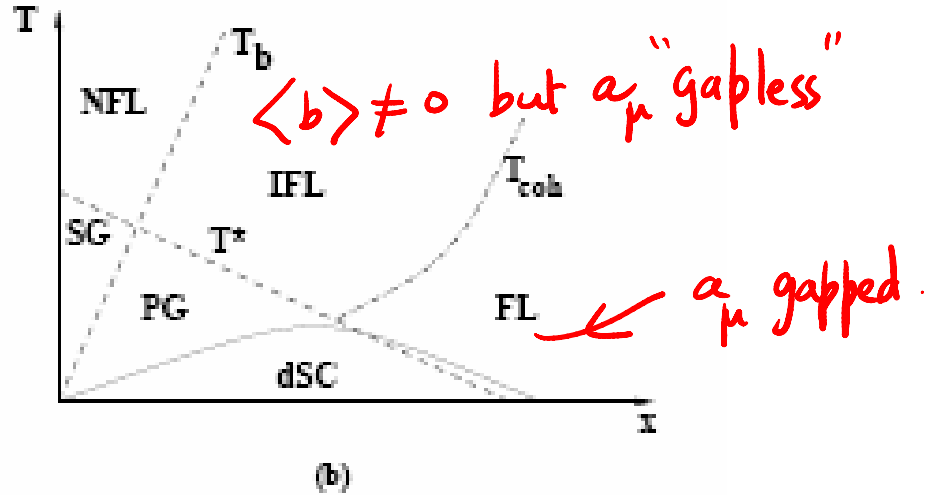
Modified slave boson gauge theory

(TS, Lee '09)

IFL regime: linear-T single particle scattering rate
 + other non-fermi liquid properties



fluctuations \rightarrow



Underdoped: IFL \rightarrow pseudogap (PG) state with gapless Fermi arcs, etc