Progress and Open Problems in the Physics of G_2-holonomy Spaces

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SIMONS COLLABORATION MEETING

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Broad Physics Picture

- M-theory on a compact G_2-holonomy space, a broad (X, Φ), framework for addressing $(3+1)$-dimensional particle physics and gravity (and more...)

- The interesting particle physics arises when (X, Q) has very particular kinds of singularities.

- These occur in codimension FOUR, SIX and SEVEN

- "Branes become light at these ... singularities"

- Understanding the physics of singularities $\implies$ MODELS of PARTICLE PHYSICS, DARK MATTER AND COSMOLOGY.

G_2-holonomy spaces provide "UV boundary conditions" on particle physics model.

Especially importat for HIDDEN SECTOR PHYSICS (DARK MATTER, AXIONS etc...)

Basic Problems:

1) What are physically reasonable singularities of (X, α)?

2) Given 1) what is the physics of reasonable singularities of (X, α)?

- OVER THE LIFETIME OF THIS SIMONS COLLABORATION PROGRESS IN ALL THREE CODIMENSIONS HAS BEEN MADE

- WILL TRY TO EMPHASISE OPEN PROBLEMS

$$SU(2) \subset SU(3) \subset G_2$$

CODIMENSION FOUR
$\Updownarrow$
NON-ABELIAN GAUGE SYMMETRY

CODIMENSION SIX
$\Updownarrow$
FIVE DIMENSIONAL, STRONGLY COUPLED CONFORMAL THEORIES

CODIMENSION SEVEN
$\Updownarrow$
CHIRAL FERMIONS + LOTS OF OTHER PHYSICS

LOCAL MODELS FOR THE SINGULARITIES ARE METRIC CONES,

$$ds^2(\mathbb{R}^+ \times \Sigma^{n-1}) = dr^2 + r^2 g(\Sigma^{n-1})$$

$$S^3 /_{\Gamma_{ADE}} \qquad n=4 \qquad n=6 \qquad n=7$$

SASAKI ~ EINSTEIN

Nearly Kähler

CODIMENSION FOUR

(X^7, Q) has a codim 4 orbifold singularity, along a 3-mfld Q^3. Near Q^3, X^7 looks like $\frac{\mathbb{C}^2}{\Gamma_{ADE}} \times Q^3_{\phi}$.

Physically, in M-theory on $X^7 \times \mathbb{R}^{3,1}$, we consider ADE super Yang-Mills on

$$\{0\} \times Q^3 \times \mathbb{R}^{3,1}$$

This has been studied over the years and continues to be understood further...

(BSA '98, '02; PANTEV/WIJNHOLT '05)

· The space of vacua

$$= \text{Moduli space of complex flat, stable ADE connections on } Q^3.$$

Within the collaboration studied

by - M.Hubner (development of
Morse theory
interpretation)

- A.Braun/S.Schafer-Nameki/M.Hubner/Sc.zed
(Interpretation for TCS G_2-mflds)

- M.Cvetic/J.Heckman/E.Torres/S.Zoccarato
(Local G_2 examples and extension to Spin7)
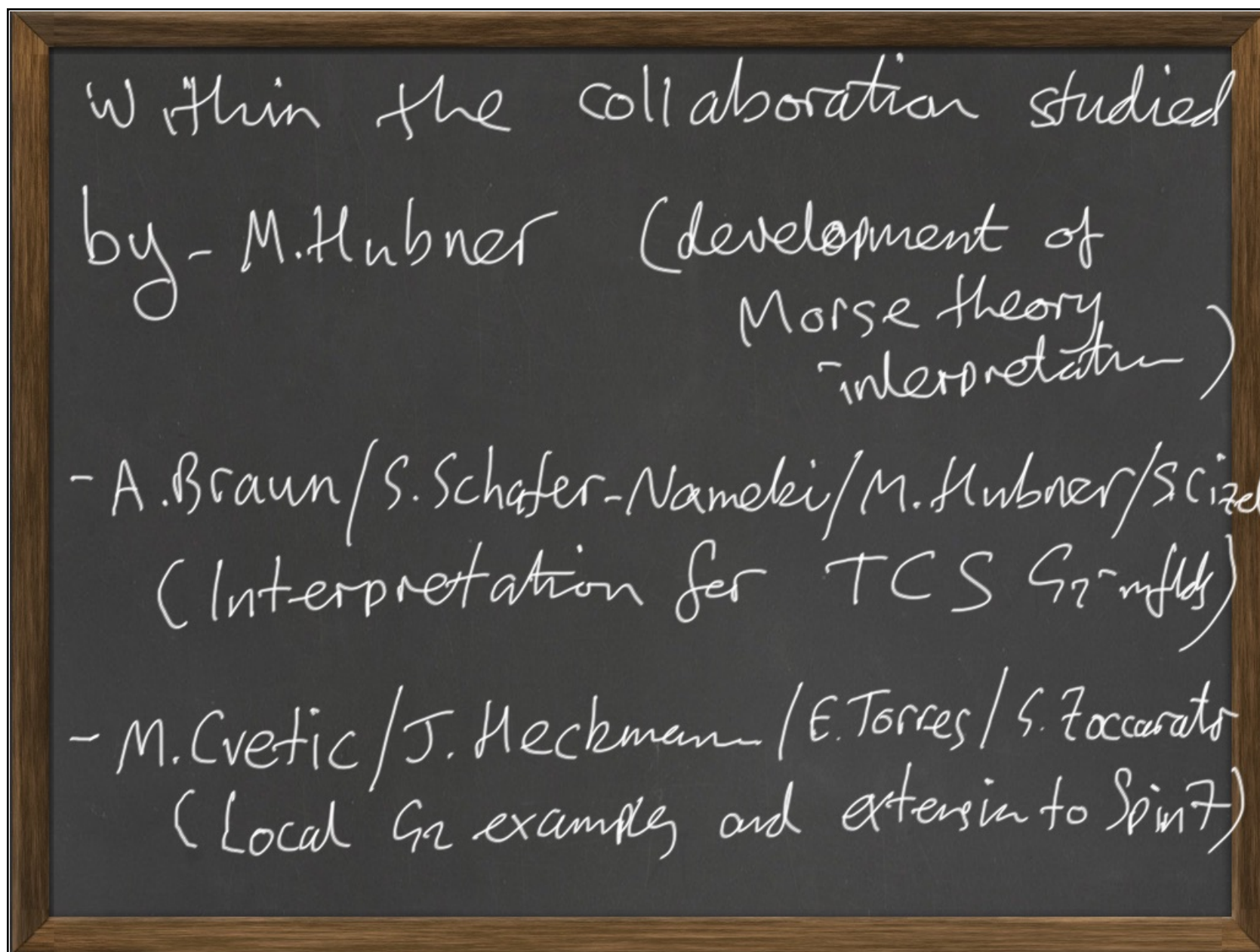
Yet more recently, D. Baldwin and I have explored the physics of the (twisted) Joyce-Karigiannis construction of compact G_2 mflds (BSA, D. Baldwin)

paper to appear

$$\implies \text{Higgs branches of gauge theories in } (3+1)\text{-dimensions}$$

Local Solvable models

(BSA, D. BALDWIN)

$$X_7 = \left(\frac{\mathbb{C}^2/_{\Gamma_{ADE}} \times T^3}{K} \right)$$

$(T^3/_K \text{ smooth})$

$$\varphi = \sum_i W_i^{HK} \wedge dx_i + dx_1 \wedge dx_2 \wedge dx_3$$

W_i^{HK} (moduli) are Hyper-Kahler structures with appropriate K action

(cf R. Barbosa)

Theorem: (Moduli space of $SU(2) \times K$ holonomy metrics, (suitably complexified) $\cong$ moduli space of complex ADE connections on T^3/K.

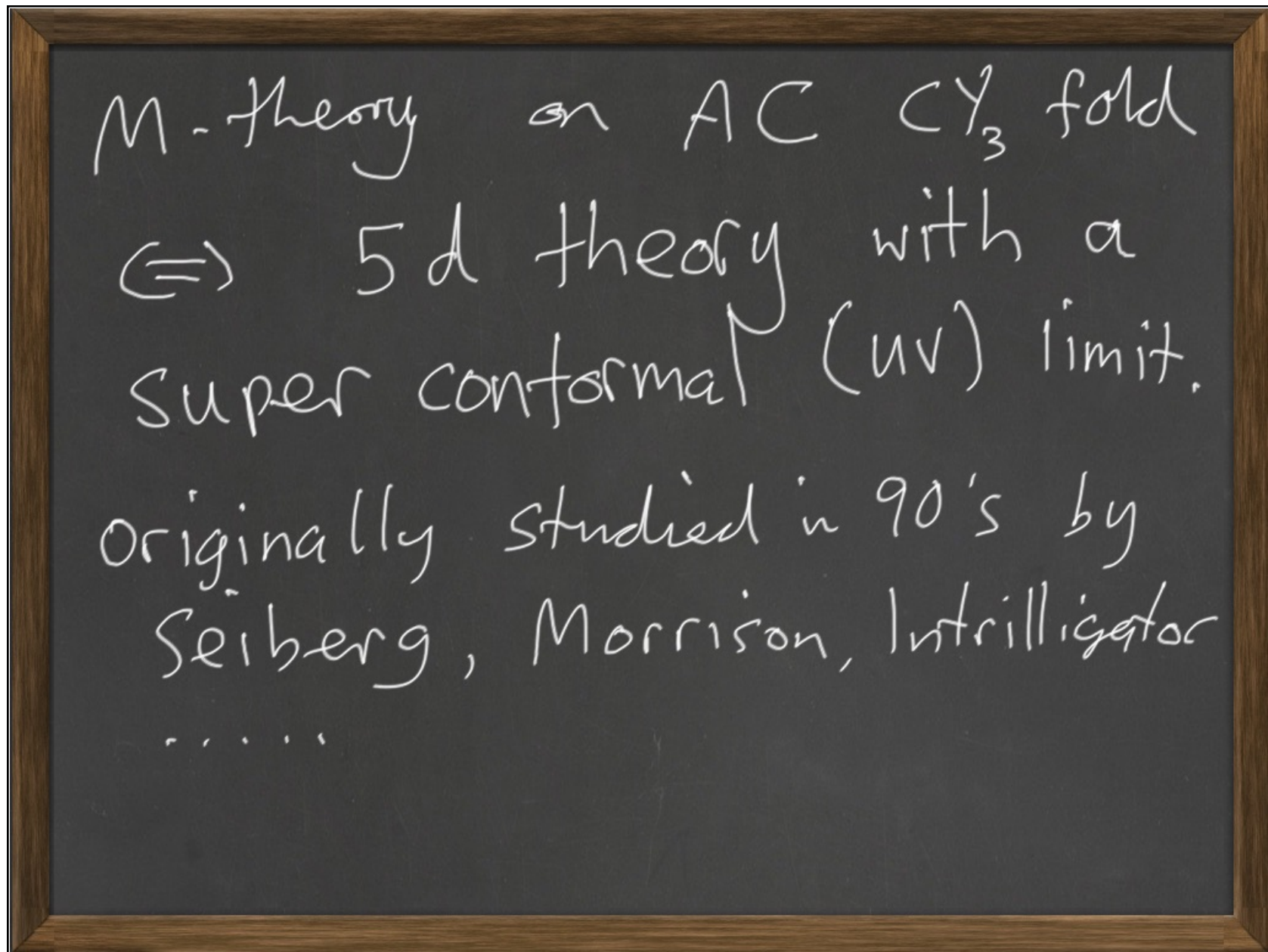– Interpret the different components as Coulomb, Higgs or mixed vacua.

Naturally suggests generalisations of Joyce-Kargiannis to $\dfrac{\bar{c}^2}{\Gamma_{AnE}}$ and to higher order twists.

TO EXPLORE:

- ALF generalisation...

- Allow K to act with fixed points $\Rightarrow$ codim SIX

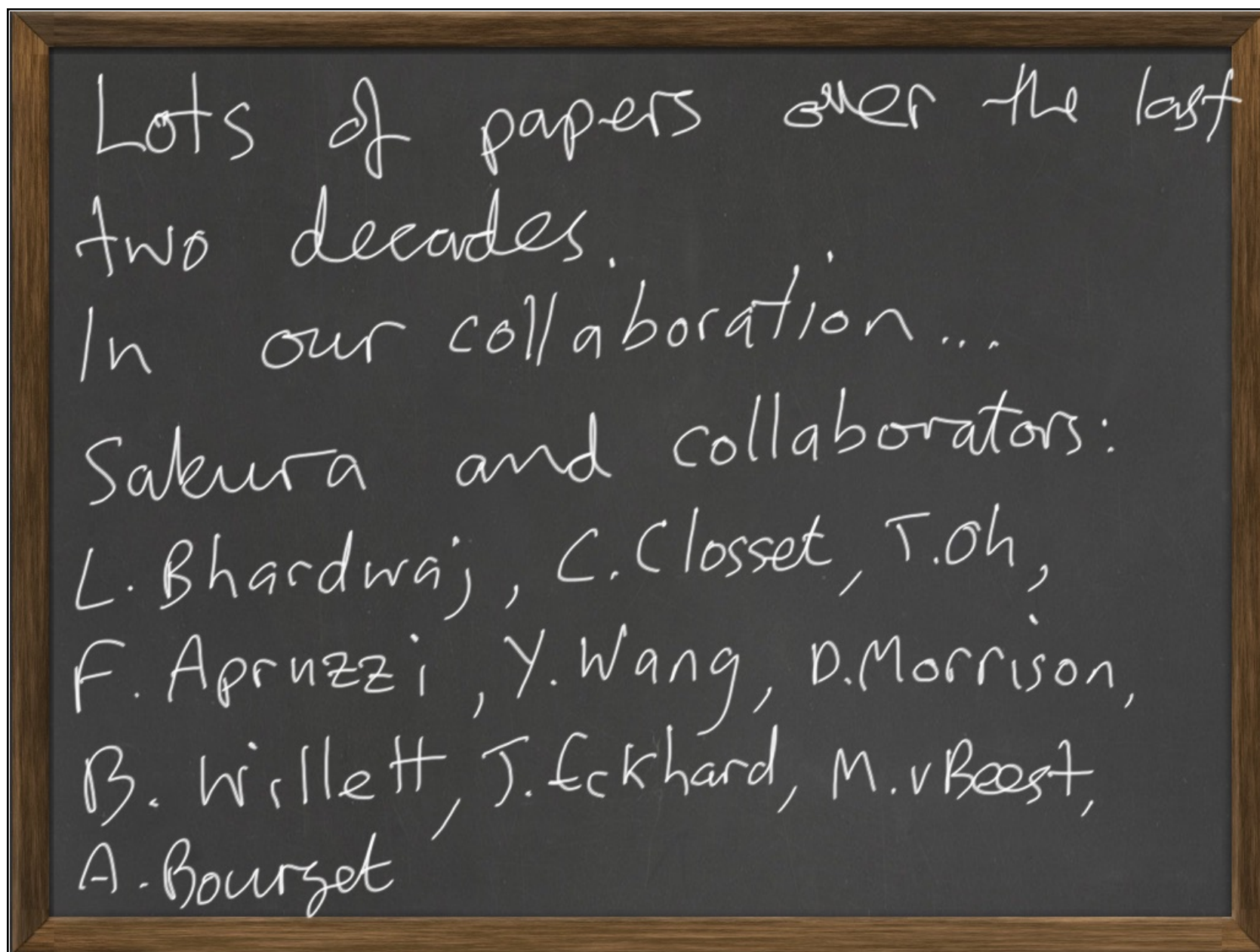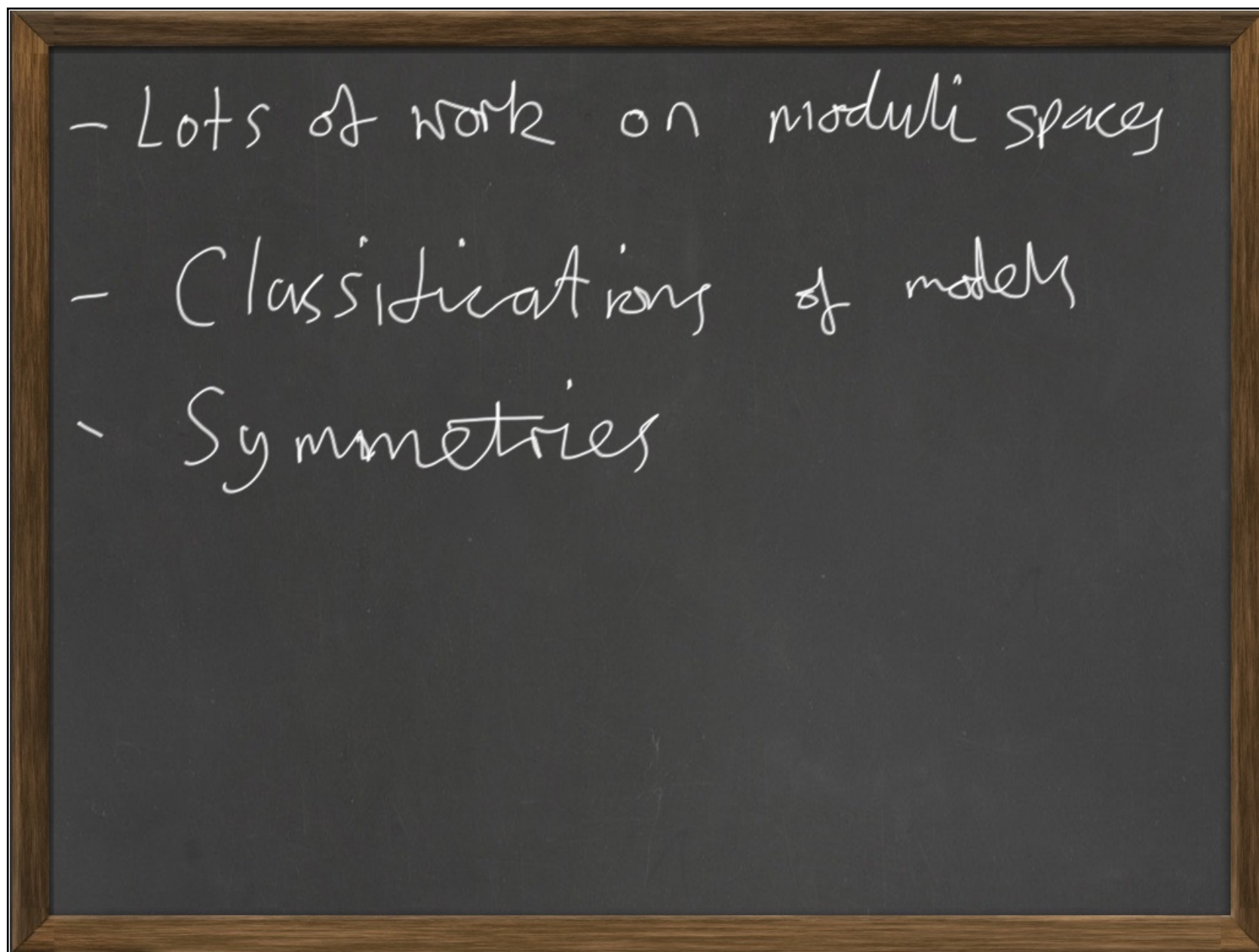- Physics of $\left(\dfrac{\mathbb{R}^7}{\Gamma}, \, d = flat \right)$ $\Gamma \subset \mathcal{G}_2$??
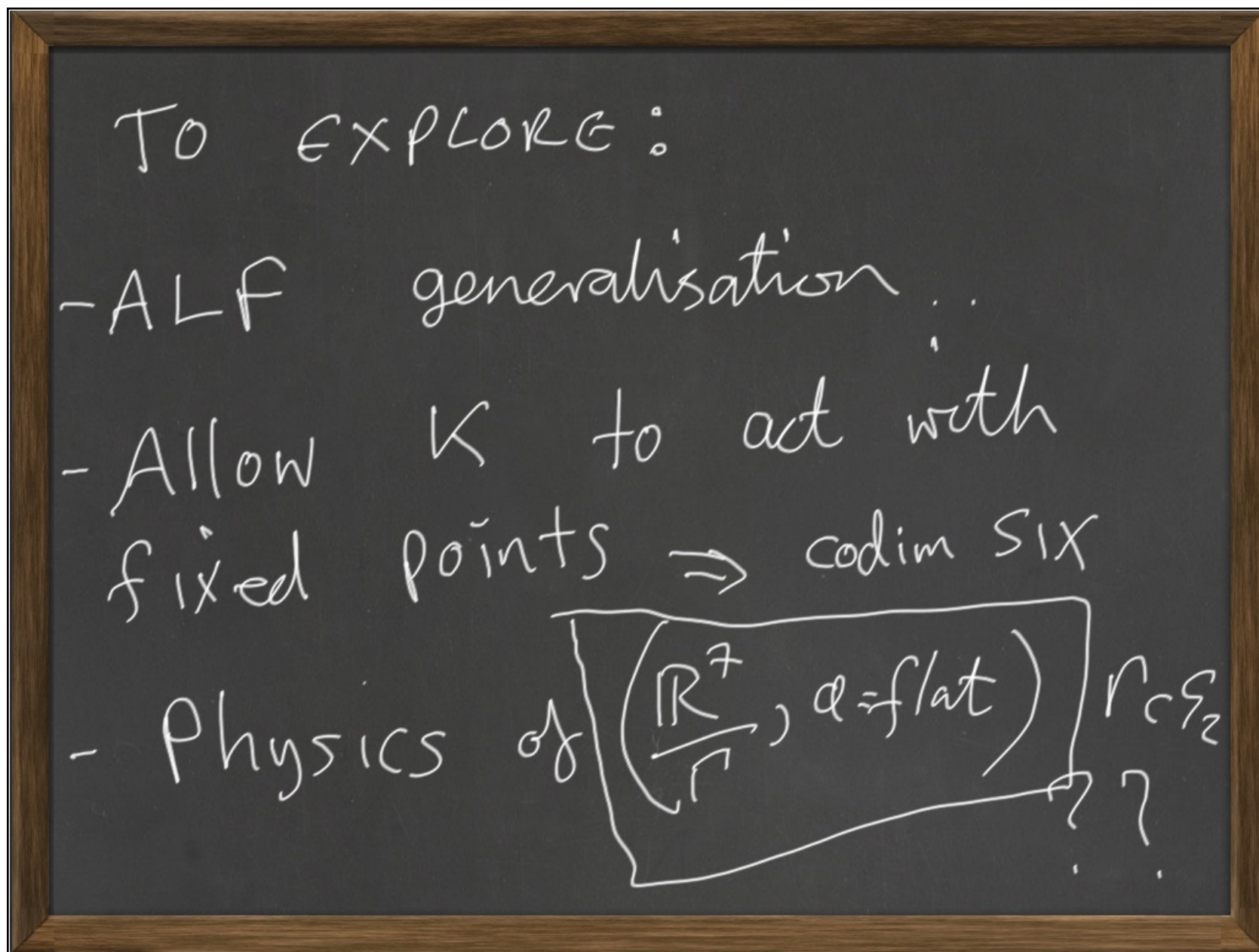
CODIMENSION SIX

Conical and Asymptotically Conical Calabi-Yau metrics.

NB. Much of the physics literature ignores the metric and considers only "topological, algebraic description. The geometry should play an important role!

M-theory on AC CY_3 fold

$\implies$ 5d theory with a super conformal (uv) limit.

originally studied in 90's by Seiberg, Morrison, Intrilligator

. . . . ,

Lots of papers over the last
two decades.
In our collaboration...
Sakura and collaborators:
L. Bhardwaj, C. Closset, T. Oh,
F. Apruzzi, Y. Wang, D. Morrison,
B. Willett, J. Eckhard, M. v Beest,
A. Bourget

- Lots of work on moduli spaces

- Classifications of models

- Symmetries

Other work:

(BSA, N. Lambert, M. Najjar, E. Svanes, J. Tian)

"Exceptional (E_6) symmetry in an infinite series of 5d SCFT's"

$$X = \mathbb{C}^3 / \Gamma_{3N^2} \qquad \Gamma \subset SU(3)$$

Also   (J. Tian, Y-N Wang)

CODIMENSION SEVEN

Early work on the physics of
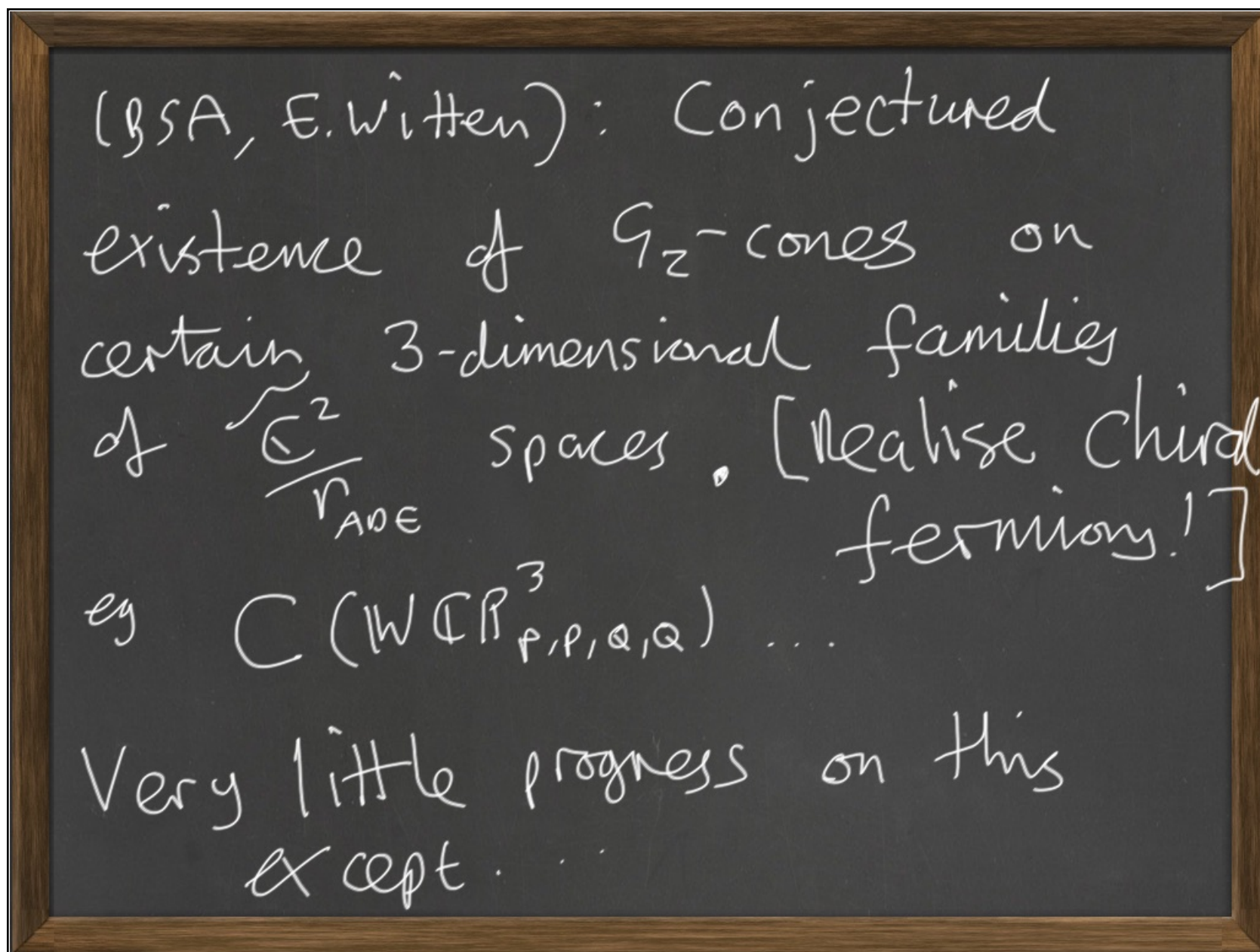Bryant-Salamon cones (or quotients)

(B.S.A. '2000, Atiyah, Maldacena, Vafa 2000):
$N=1$ SYM, CONFINEMENT, G_2-flops
(Atiyah-Witten 2001)

(BSA, E. Witten): Conjectured

existence of G_2-cones on certain 3-dimensional families of $\sqrt{\frac{\mathbb{C}^2}{\Gamma_{ADE}}}$ spaces. [realise chiral fermions!]

eg $C(W\mathbb{CP}^3_{P,P,Q,Q})$ ...

Very little progress on this except...

— According to (BSA, E. Witten) to realise $SU(N)$ with a chiral fermion in the $\Lambda^2 \mathbb{C}^N$ rep$\frac{n}{2}$, we take the 7d cong. to be the 3d family of $\mathbb{C}^2/\Gamma_{D_N}$ spaces containing an A_{N-1} singularity.

- In (BSA, L. Foscolo) we show that $\exists$ a G_2-holonomy cone on this space.

- Checked anomaly and symmetry breaking also.

- In fact the cone cross-section is twistor space of a class of Einstein 4-orbifolds due to Hitchin

- Recently: (BSA, M. del Zotto, J. Heckman, M. Hubner, E. Torres)

began exploration of more general conical G_2-holonomy orbifolds where codim FOUR
SIX
SEVEN
SINGULARITIES meet and "Interact"

Geometry and Physics

- Physics applications of known and future "G_2 theorems"

e.g. F-H-N ALC G_2-mflds as S^1-bundles over CY cong

- This could have lots of application to M-theory/$\overline{II}$ A duality . . . .

- CHNP TCS G_2

  — There have been physics papers (Braun/Schafer-Nameki, Morrison/Halverson) . . . . Ahira

~ I think there is a lot
of untapped P.E. there

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To relate to the "Enumerative
Invariants" investigations

- Physics could potentially
provide input

eg if CY_4 invariants make
sense, then

expect G_2 counting to also make sense

eg M-theory on $CY_4 \times T^2$

and $(X, \varphi) \times T^3$ give similar physical theories.