FREEZE-IN BARYOGENESIS AND ITS TESTS

Brian Shuve

based largely on work in progress

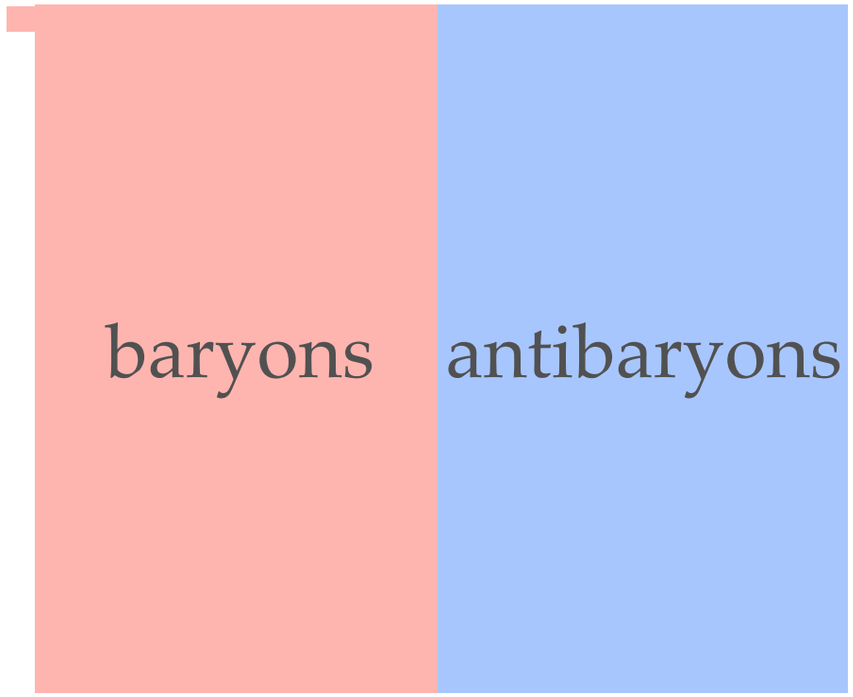
with David Tucker-Smith

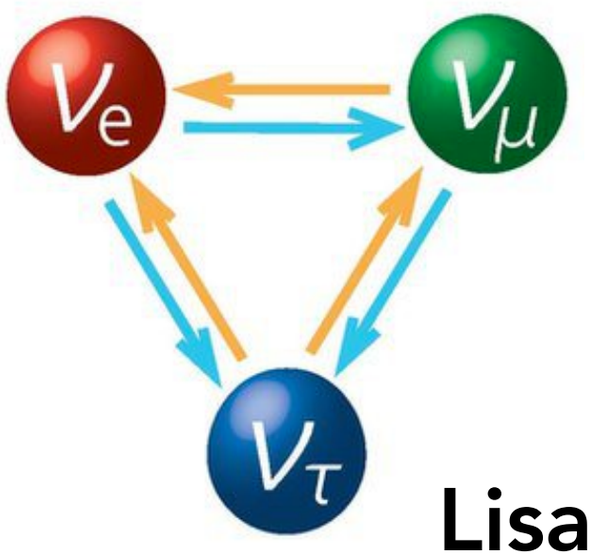
HARVEY MUDD COLLEGE

UCR

WHY BARYOGENESIS?

Yang's talk





Lisa's talk

WHY BARYOGENESIS?

- Lorentz-invariant theories must satisfy **Sakharov conditions** for baryogenesis. How does the SM do?

1. **Baryon number violation:** transitions between electroweak vacua can transform baryon number into lepton number

2. **CP-violation:** physical CP-violation requires all 3 generations, it is *too small* to explain observed baryon asymmetry ($\sim 10^{-10}$)

3. **Departure from equilibrium:** particle abundances must deviate from equilibrium value, *B*-violating processes should also be out of equilibrium

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MODELS OF BARYOGENESIS

10^{16} GeV	GUT baryogenesis Affleck-Dine baryogenesis Higgs relaxation baryogenesis
10^{10} GeV	standard thermal leptogenesis
10^{3} GeV	resonant leptogenesis freeze-in (oscillation) lepto/baryogenesis electroweak baryogenesis
10^{0} GeV	*B* meson oscillation baryogenesis

mass/energy

(warning: arbitrary conflation of mass & dynamical scales for purpose of illustration!)

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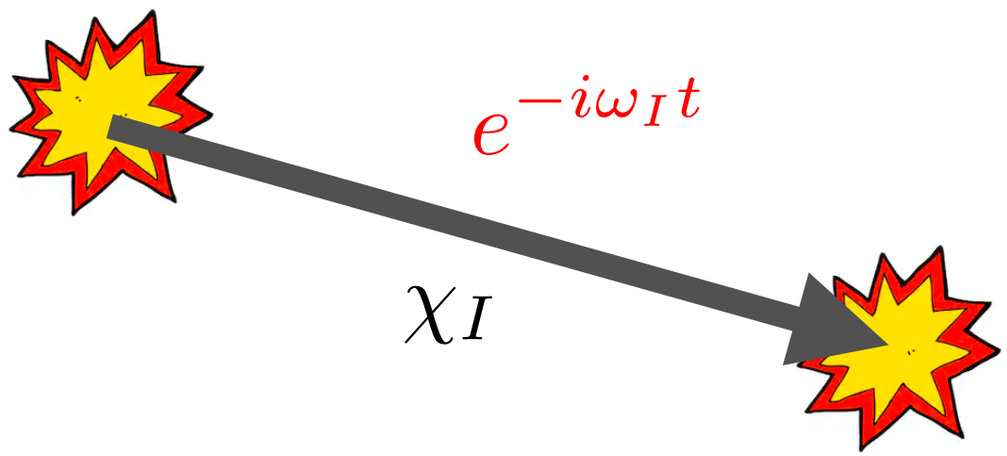
FREEZE-IN BARYOGENESIS



- Out-of-equilibrium production & scattering of singlets
- **Simple model:** same as many freeze-in DM models or see-saw mechanism
- **Understudied:** one limit (*ν*MSM) very well-studied, but other limits can give very different phenomenology/mechanisms
- **Multi-scale:** dynamics naturally links TeV, keV, and cm scales
- **Testable:** signals in colliders (potentially also cosmology)

FREEZE-IN BARYOGENESIS

$$\mathcal{L} = -F^a_{\alpha I}\overline{\psi}_\alpha \Phi_a \chi_I - \frac{M_I}{2}\bar{\chi}^{\rm c}_I \chi_I - V(\Phi_a)$$

ψ_α = SM quark or lepton, 3 flavours

χ_I = 2 or more singlet fields with Majorana masses

Φ_a = 1 or more scalar fields, same SM gauge charge as ψ_α

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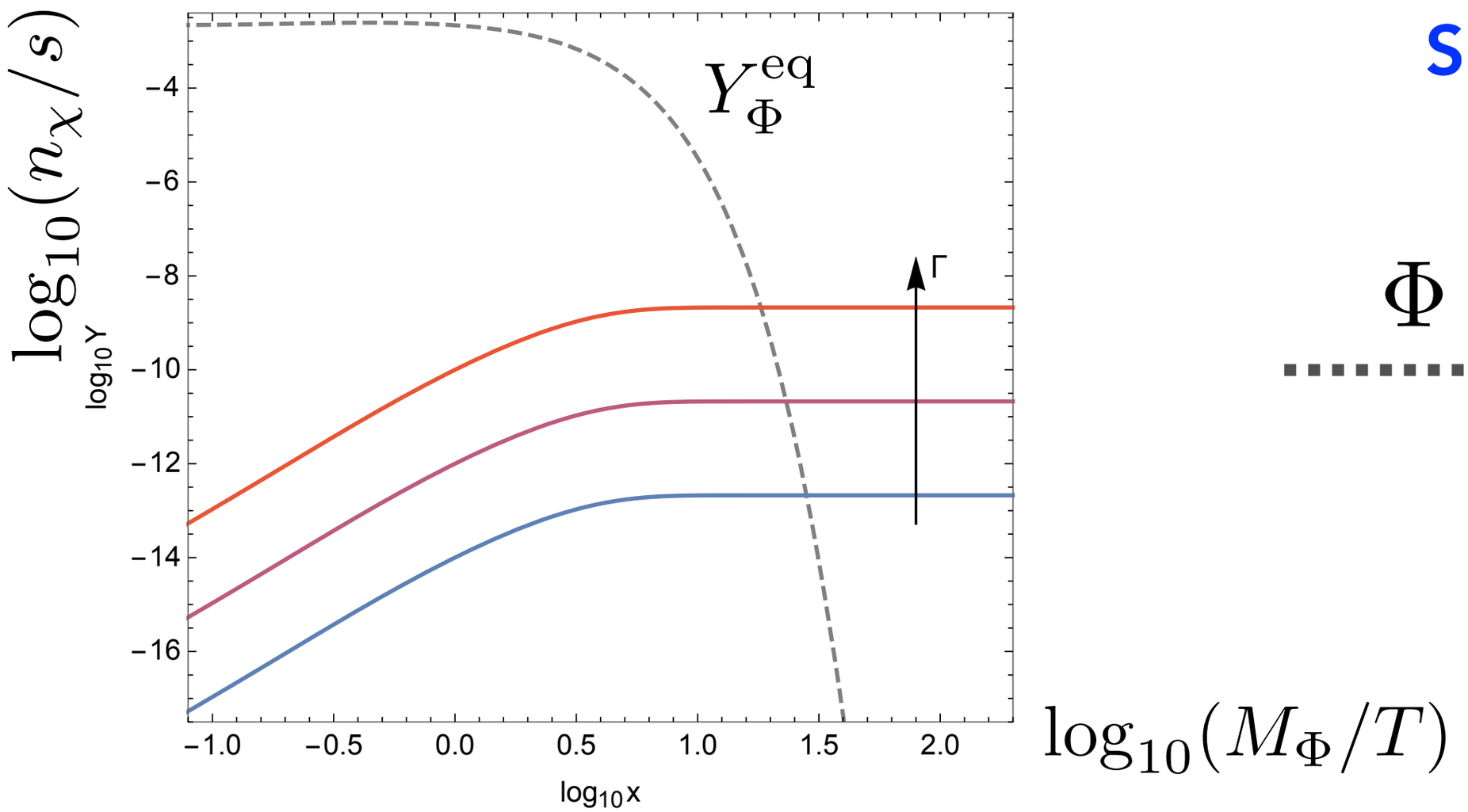
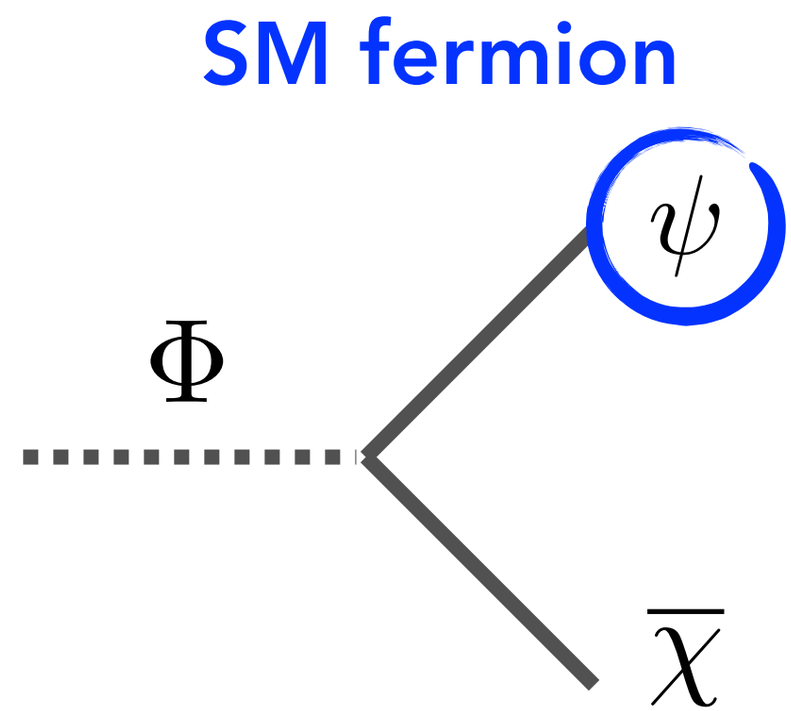
- There are many possible limits, but for now we choose the scenario where χ_I, Φ_a charged under unbroken **Z_2** symmetry

- χ_I are stable DM candidates, Φ_a carry baryon number +1/3

FREEZE-IN DARK MATTER





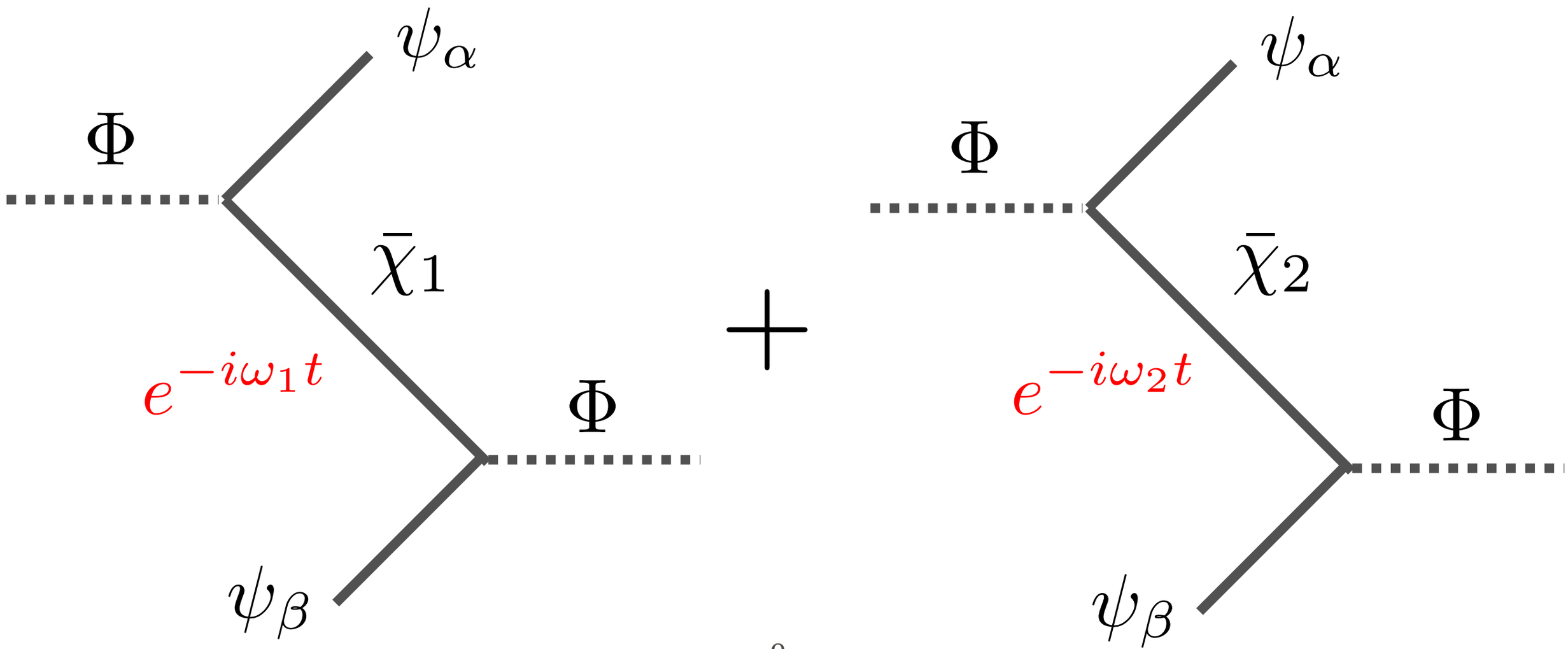
$$\Omega_\chi h^2 \sim 0.12 \left(\frac{|F|}{10^{-8}}\right)^2 \left(\frac{M_\chi}{10\ \text{keV}}\right) \left(\frac{1\ \text{TeV}}{M_\Phi}\right)$$

FREEZE-IN BARYOGENESIS

1. **Generation of asymmetry**
2. Survival of asymmetry
3. Size of asymmetry
4. Numerical results
5. Single-scalar baryogenesis

NEXT ORDER: BARYOGENESIS

- Some fraction of DM propagates coherently, re-scatters
- Inverse decay removes a quark from plasma
- Asymmetry in rate of re-scattering can lead to quark asymmetry!



CPV REFRESHER

$$\mathcal{M}(a \to b) = x\, e^{i\phi} \qquad \mathcal{M}(\bar{a} \to \bar{b}) = x\, e^{-i\phi}$$

$$\Delta|\mathcal{M}|^2 = 0$$

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$$\Delta|\mathcal{M}|^2 = -4x_1 x_2 \sin\phi \sin\theta$$

BARYOGENESIS

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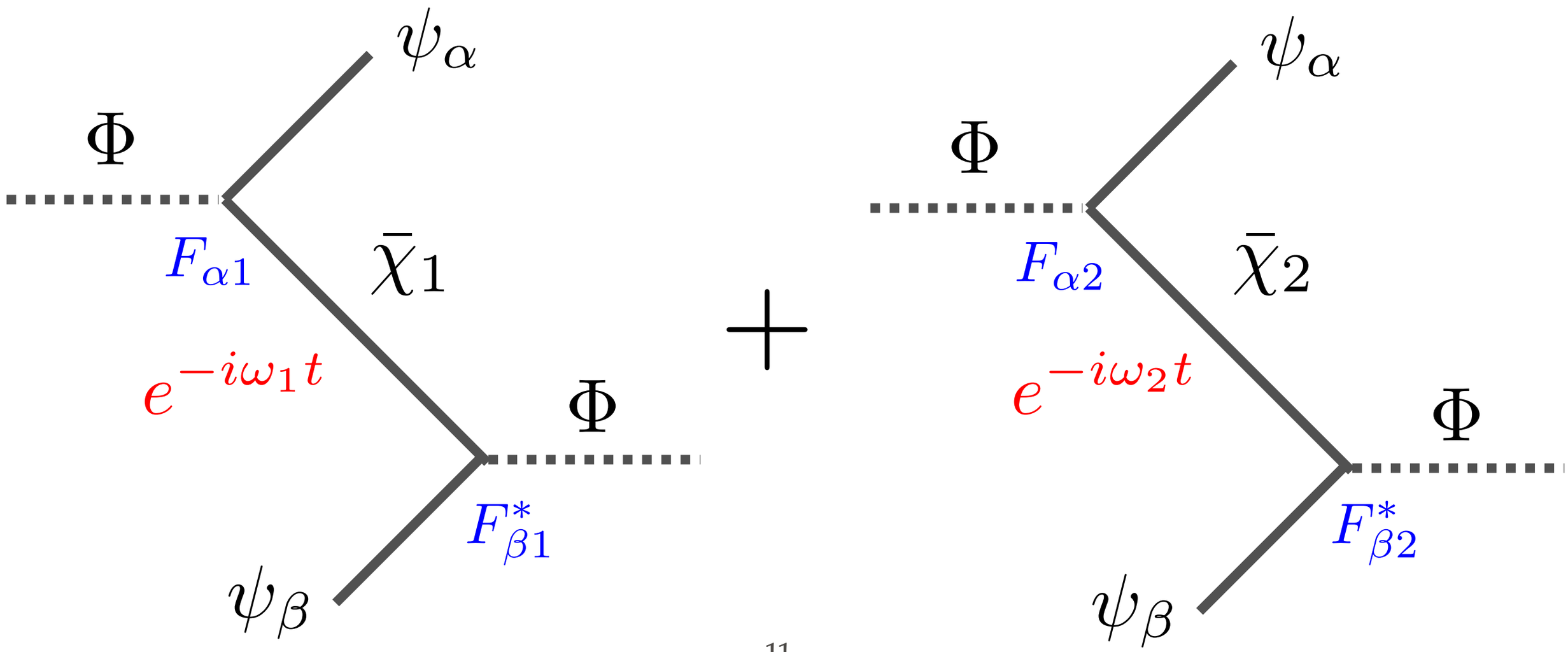

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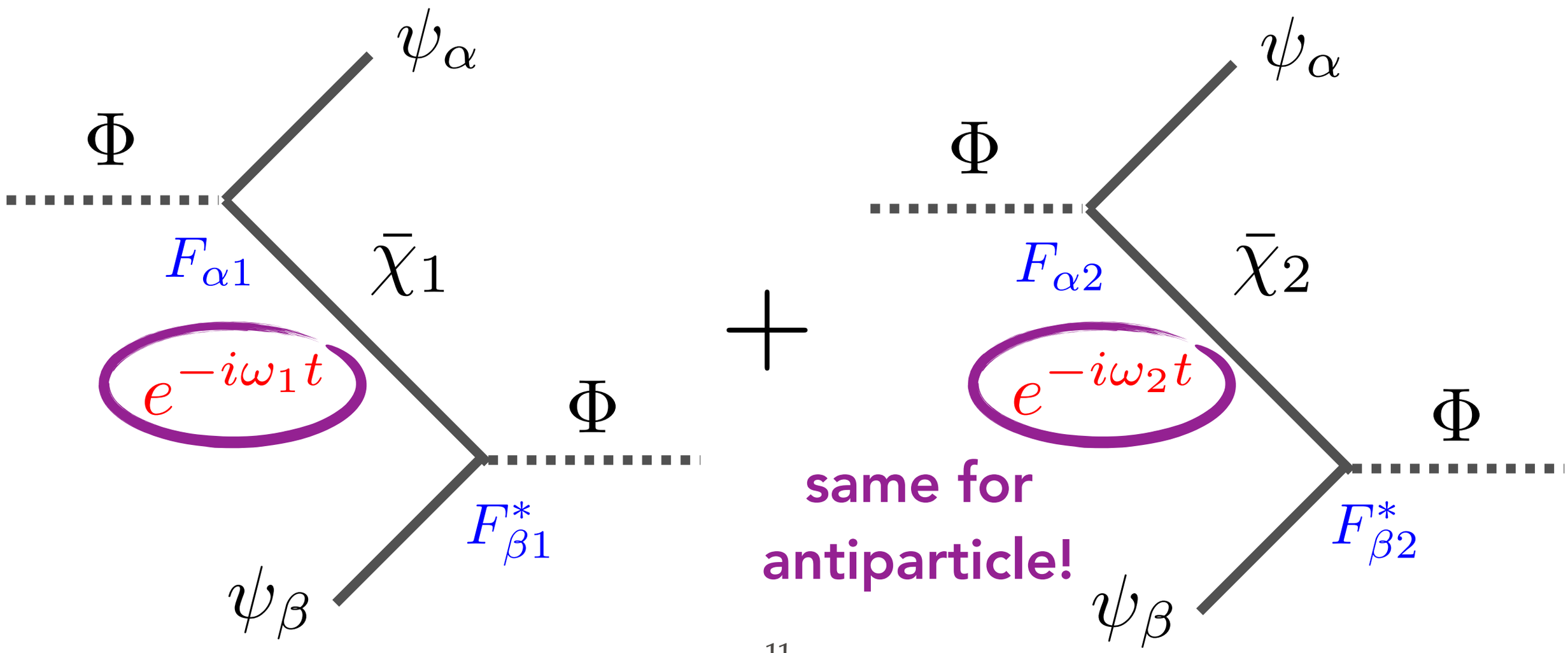
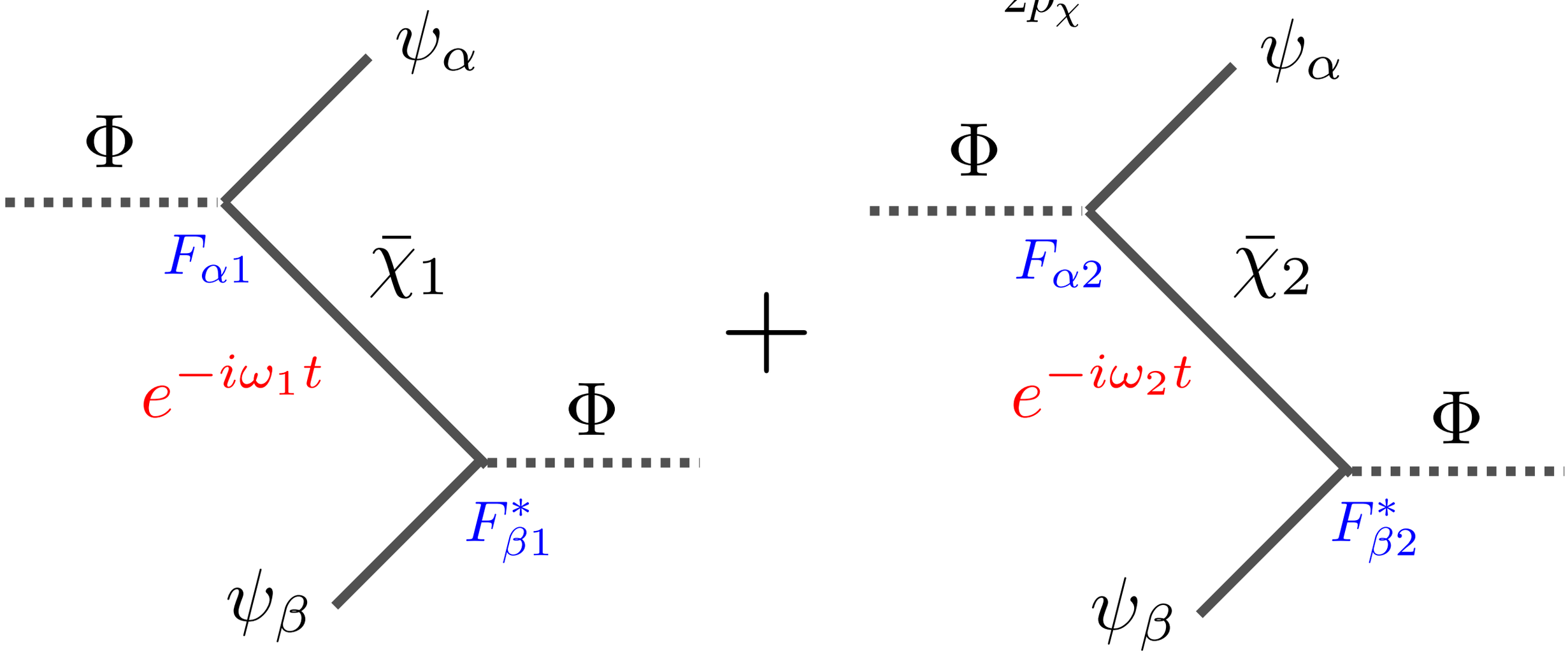


BARYOGENESIS

- For relativistic DM: $$\omega_2 - \omega_1 \approx \frac{M_2^2 - M_1^2}{2p_\chi}$$



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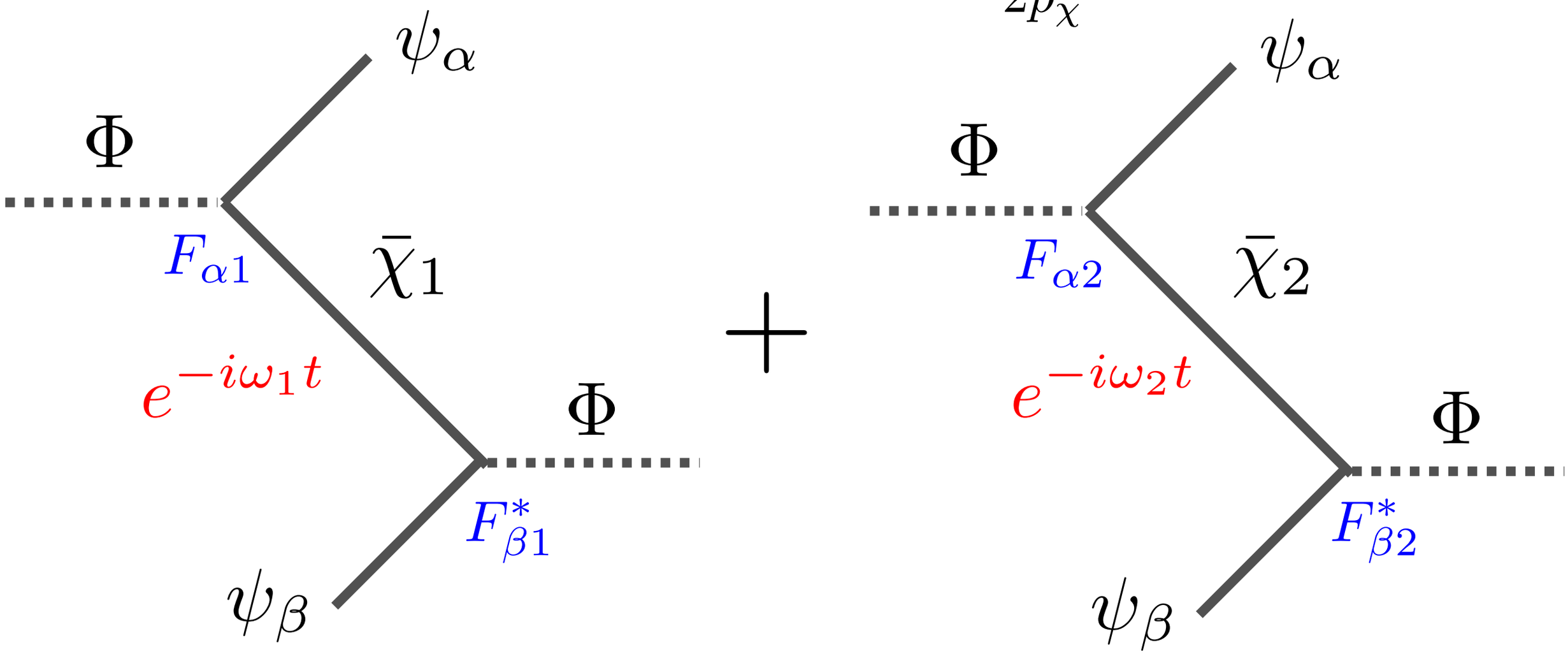
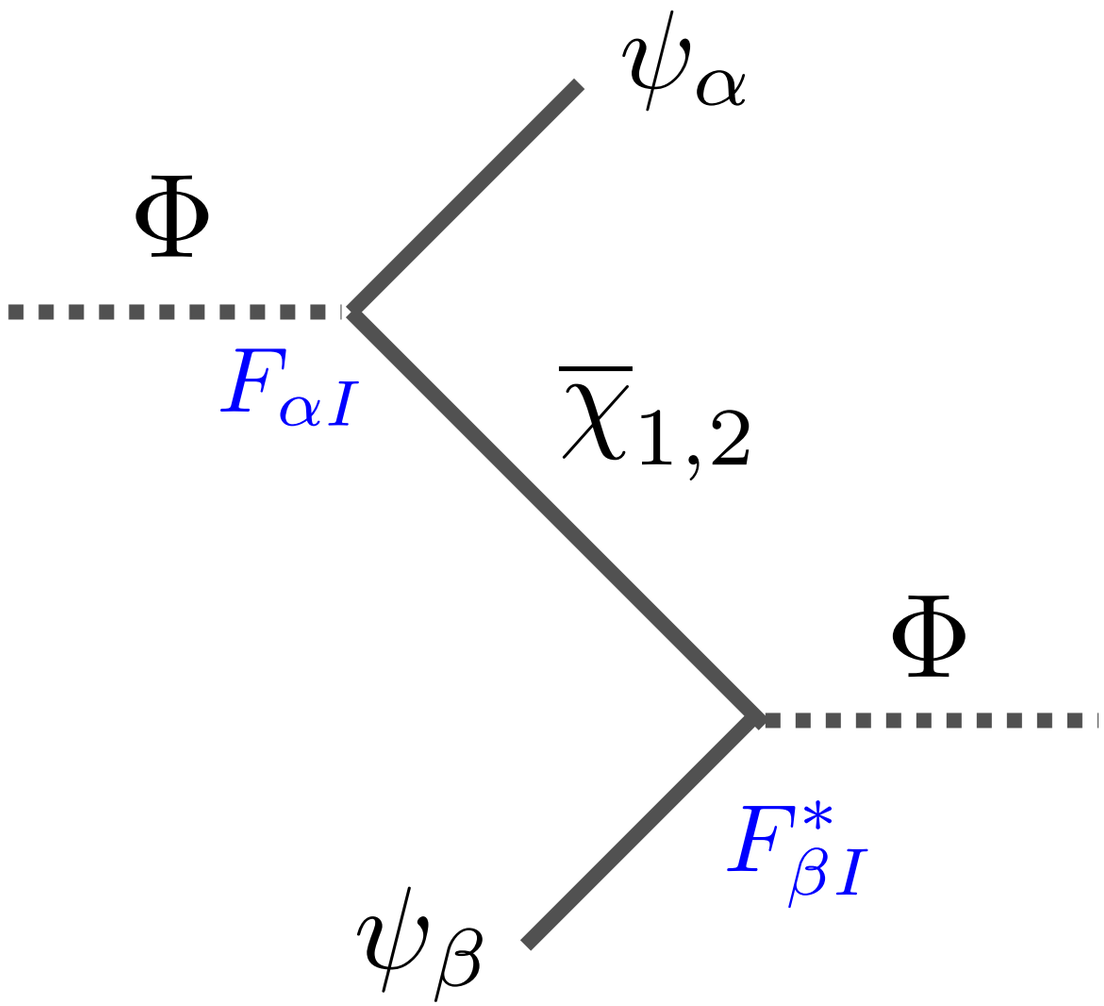
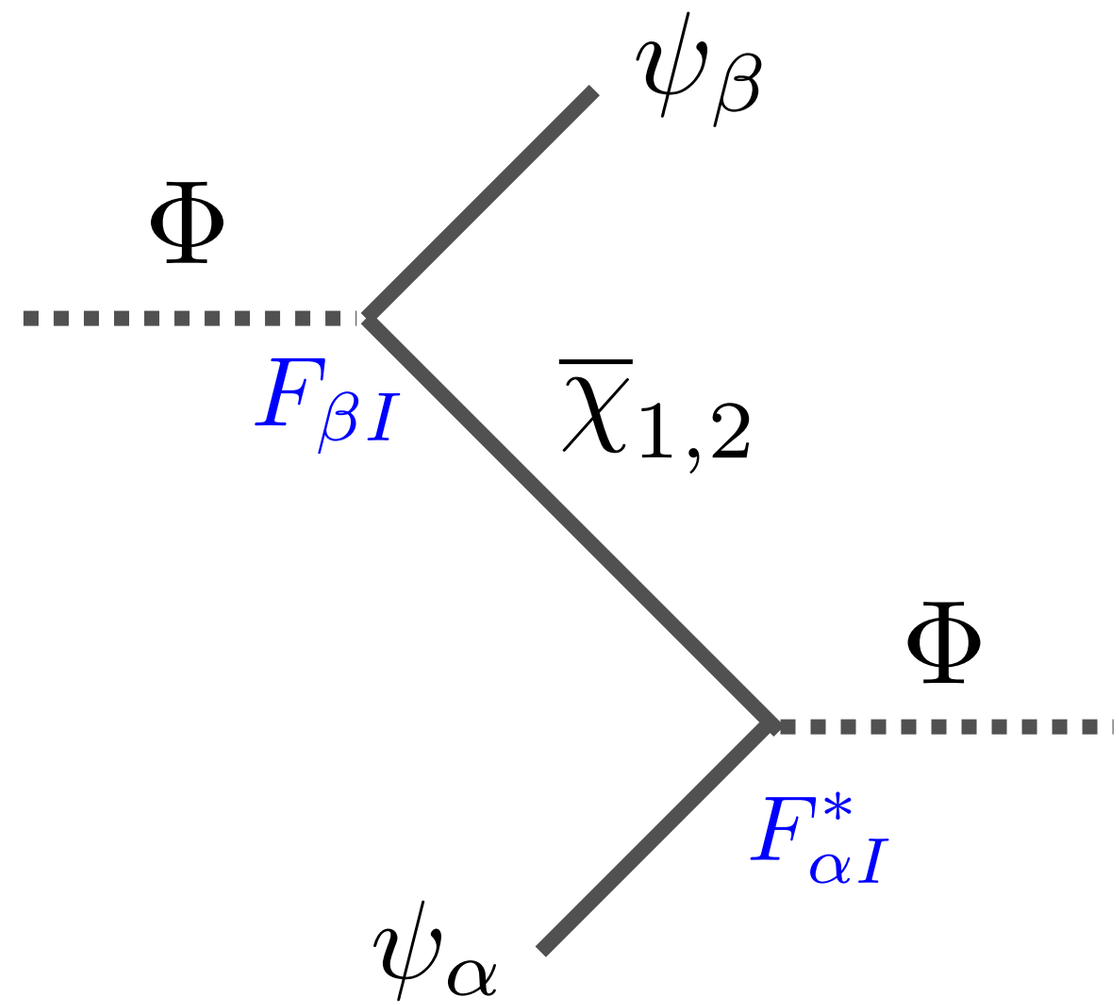


$$\Gamma(\psi_\beta \Phi \to \psi_\alpha \Phi) - \Gamma(\overline{\psi}_\beta \Phi^* \to \overline{\psi}_\alpha \Phi^*) \propto \mathrm{Im}(F_{\alpha 1} F^*_{\beta 1} F^*_{\alpha 2} F_{\beta 2}) \sin\left(\int dt \, \frac{\Delta M_{21}^2}{2p}\right)$$

BARYOGENESIS

- But wait…we don't actually have an asymmetry yet!



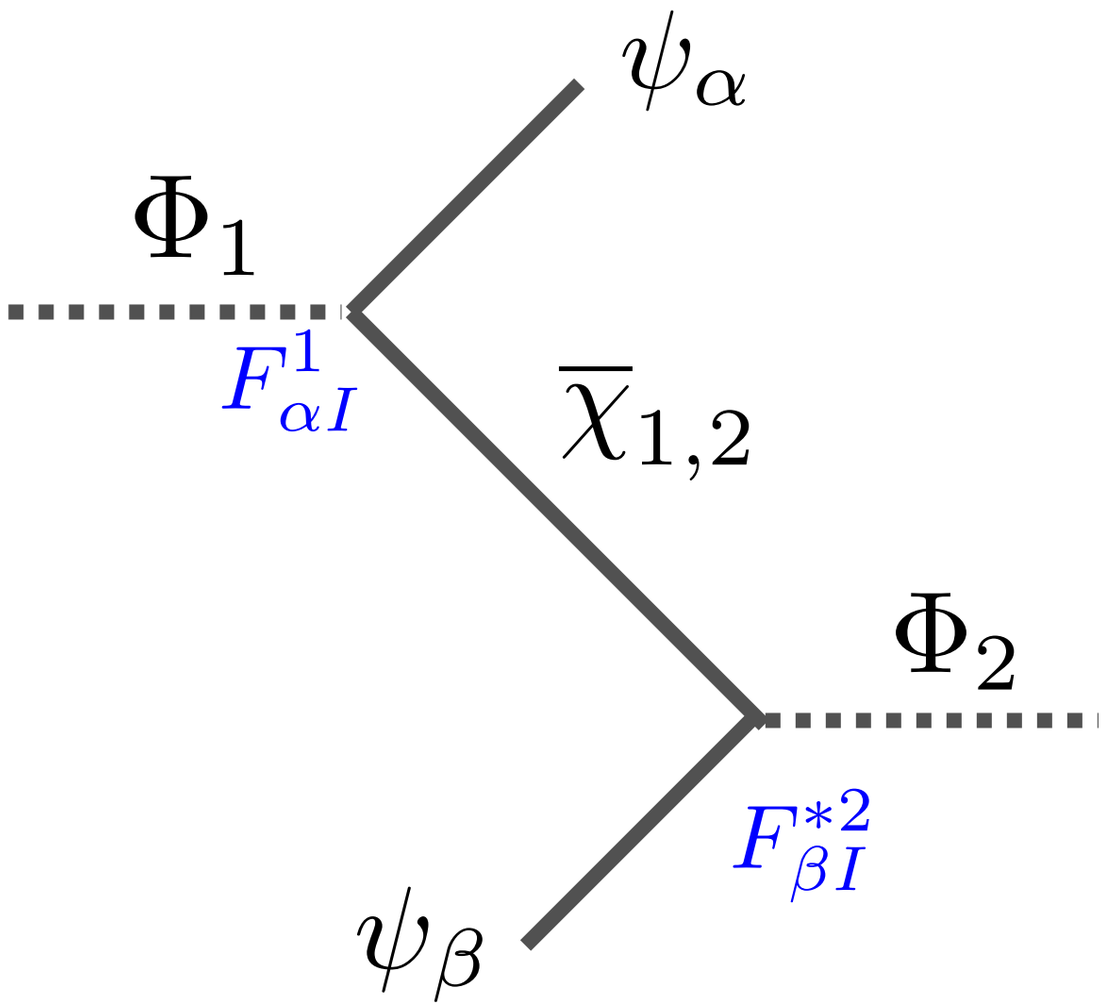


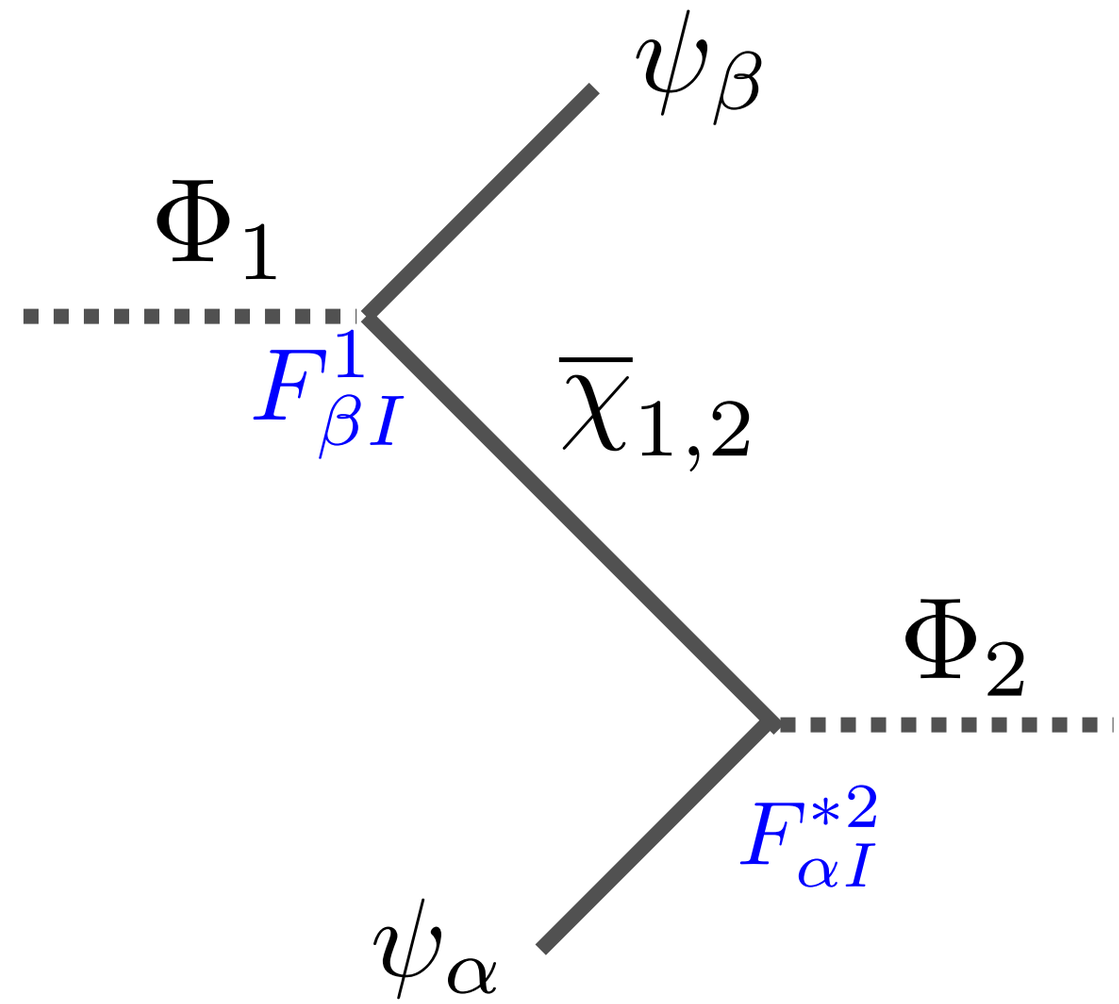
- If $M_{\psi_\alpha} = M_{\psi_\beta}$ (as is approx. true in early Universe), summed asymmetry is zero!

(there are other subtleties we will get to…)

TWO-SCALAR BARYOGENESIS

- However, an asymmetry arises at this order with **two scalars**

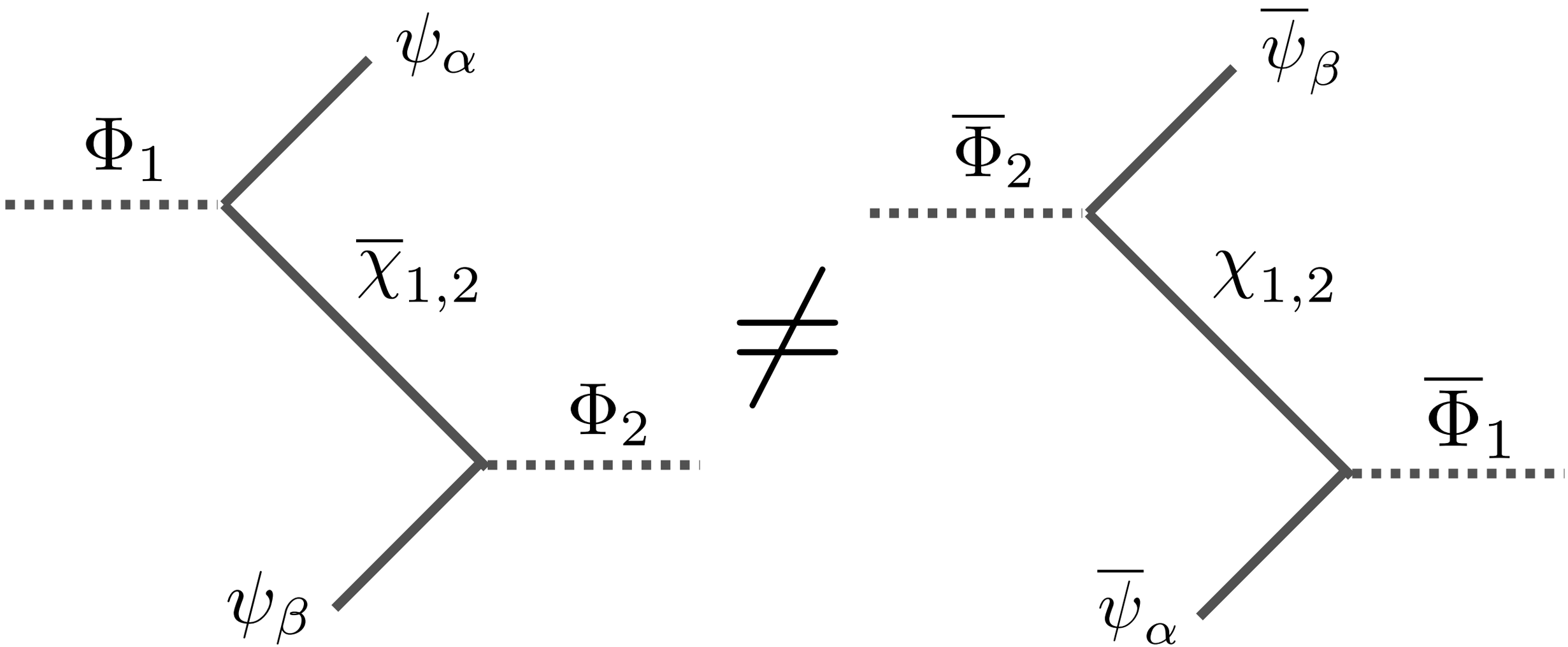




- Asymmetry no longer cancels when summed over flavours!

TWO-SCALAR BARYOGENESIS

- If the scalars have different masses, the rates differ between early- and late-time processes: *CPT* invariance no longer gives cancellation



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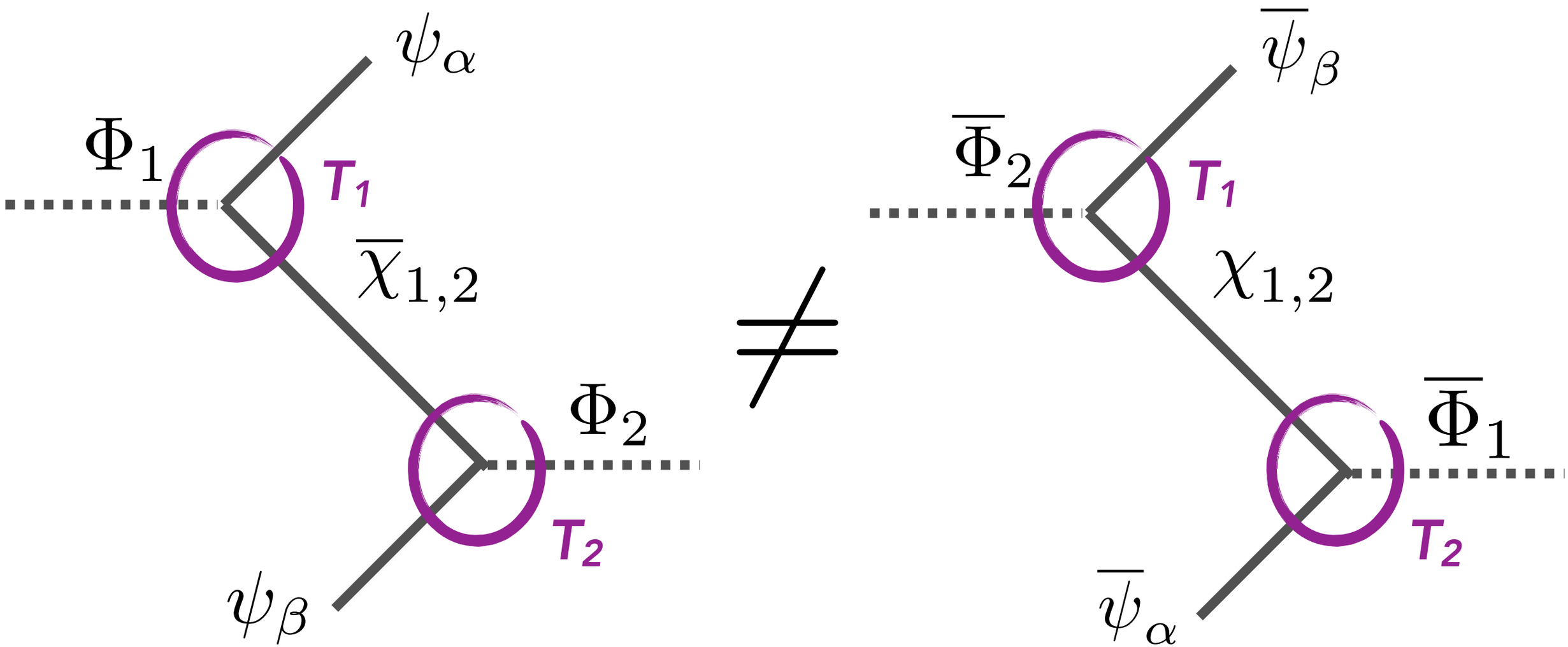

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1. Generation of asymmetry
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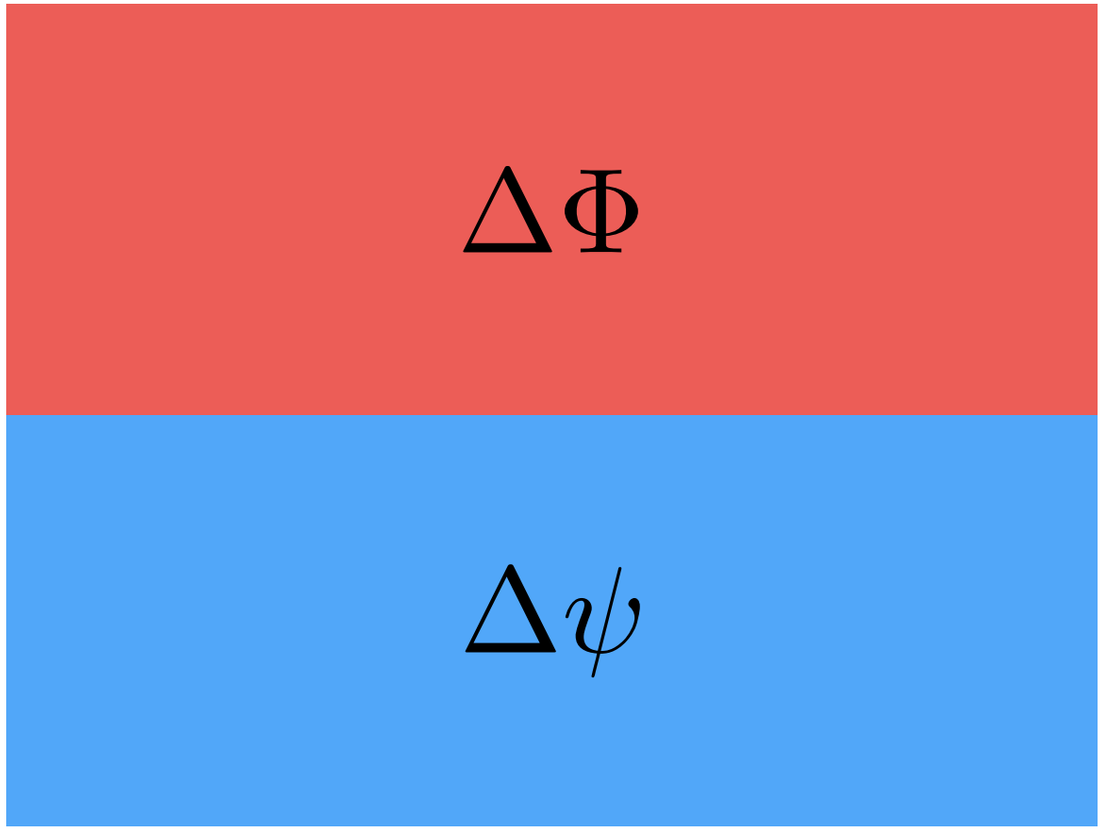
ASYMMETRY SURVIVAL

- There exists an exact *B-L* symmetry in the model!
- SM asymmetry exactly balanced by asymmetry in scalar
- But, SM asymmetry distributed between quarks & leptons



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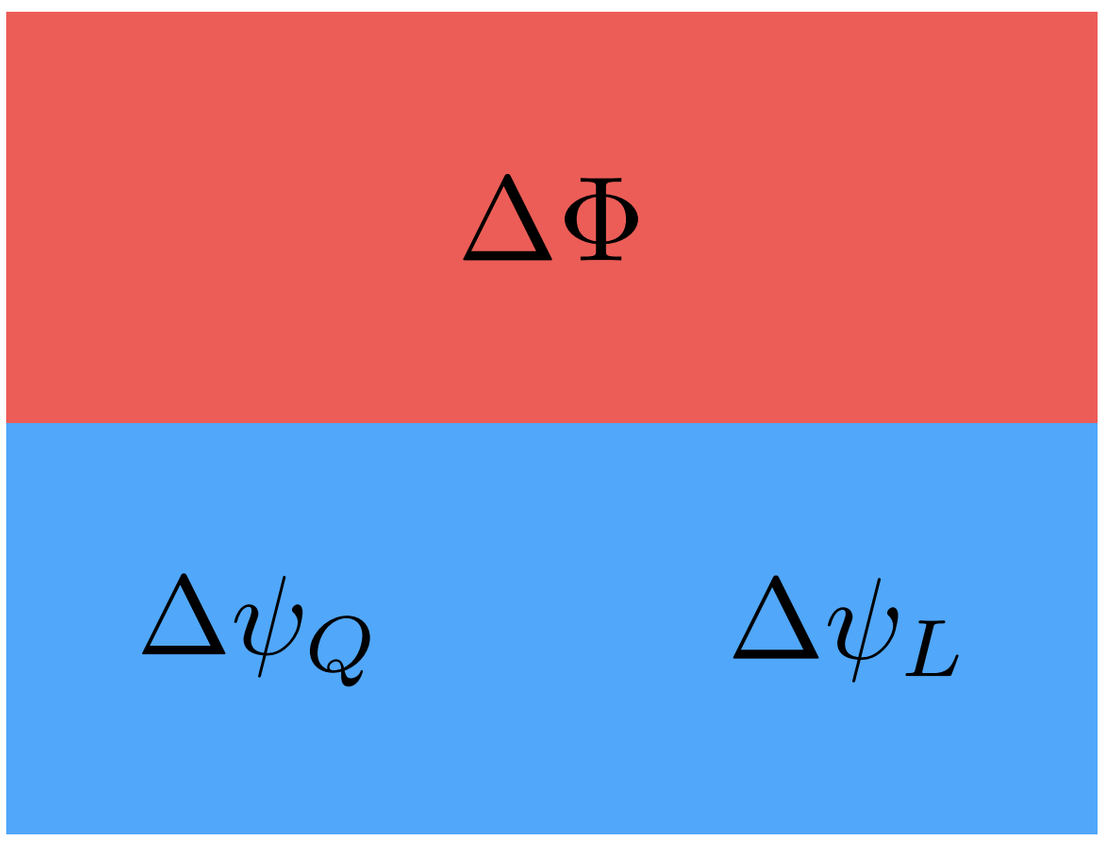

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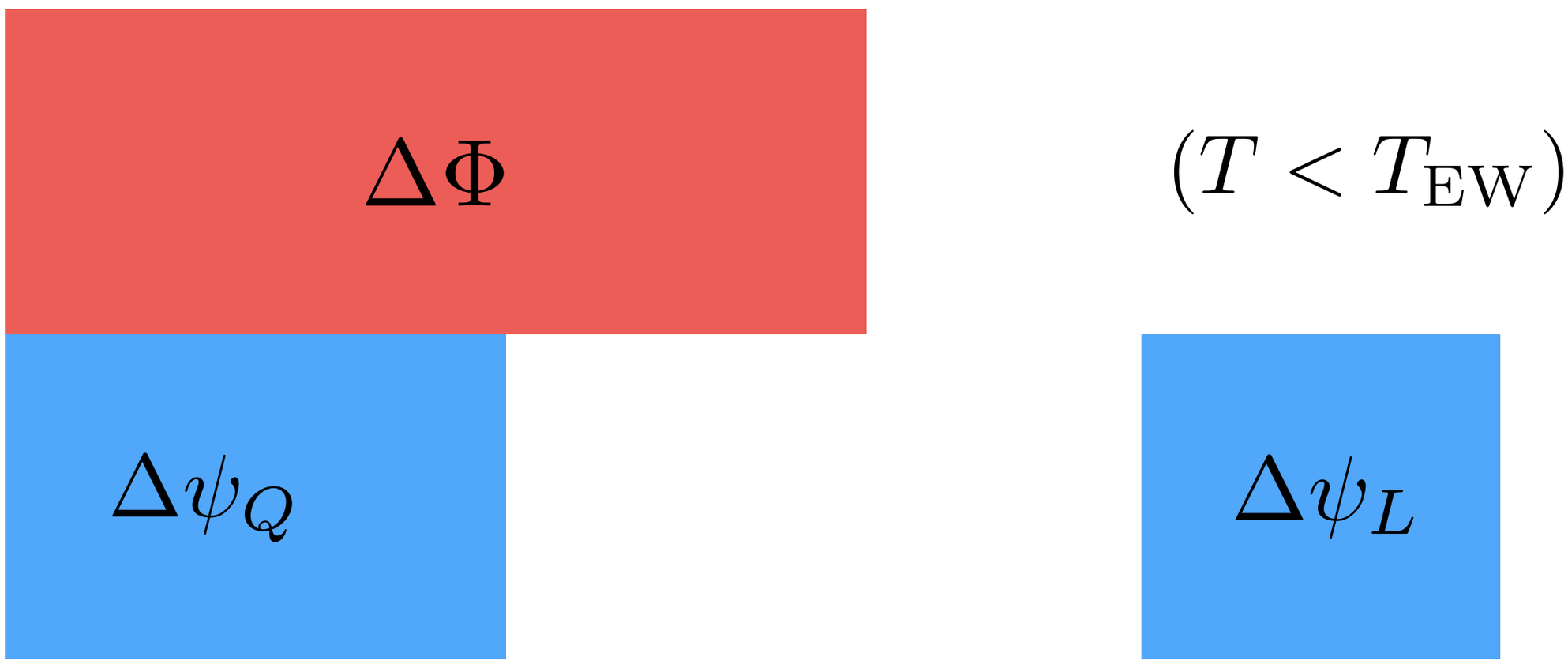

ASYMMETRY SURVIVAL

- If at least one scalar decays **after** electroweak phase transition, we are left with a baryon asymmetry

$$\Gamma_\Phi < H(T_{\rm ew})$$

$$c\tau_\Phi > \frac{M_{\rm Pl}}{1.66\sqrt{g_*}T_{\rm ew}^2} \sim 1 \text{ cm}$$

- For baryogenesis to work, at least one scalar must have a macroscopic lifetime: interesting implications for colliders

- Most direct link of particle lifetime to cosmological scale at electroweak phase transition that I know of

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ASYMMETRY & DM MASS

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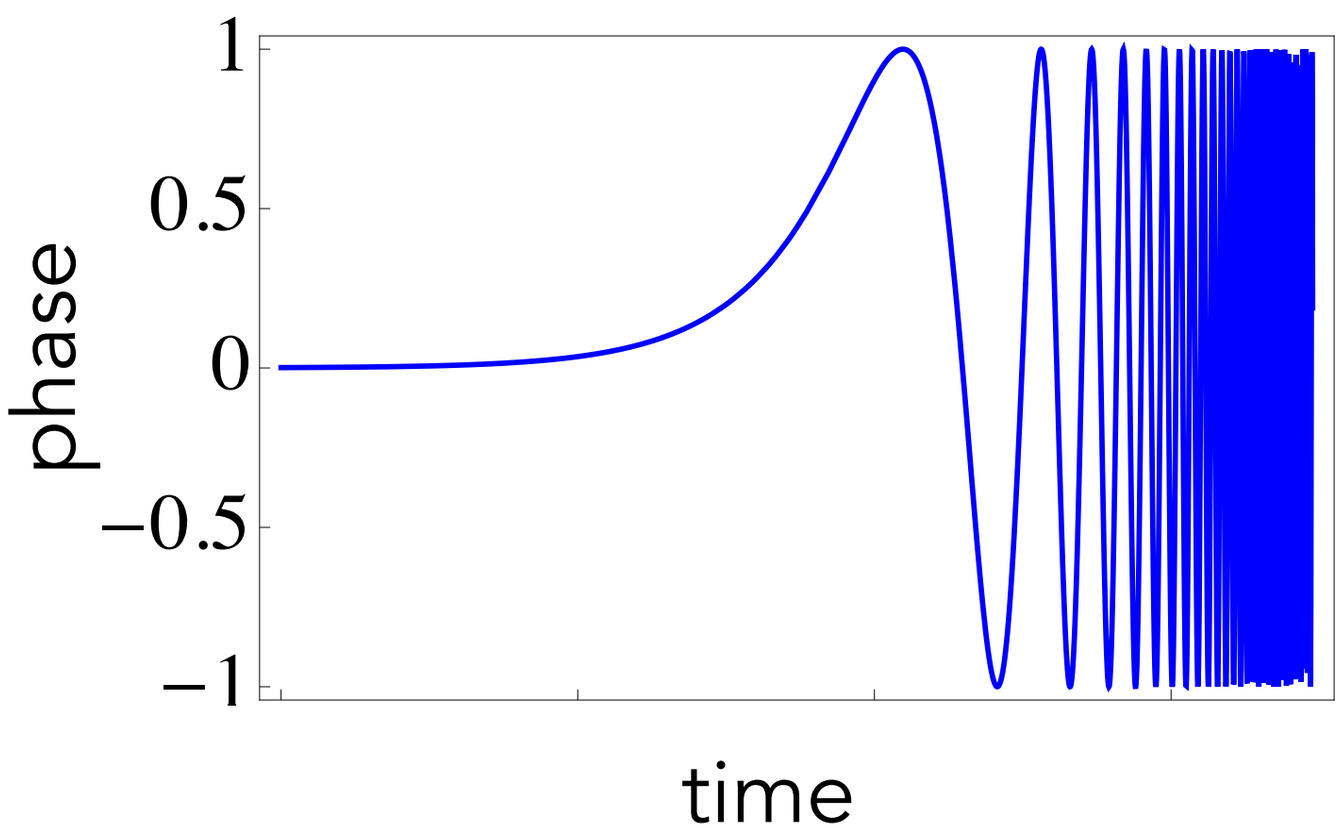
- Redshift causes increasing oscillation frequency for asymmetry generation rate



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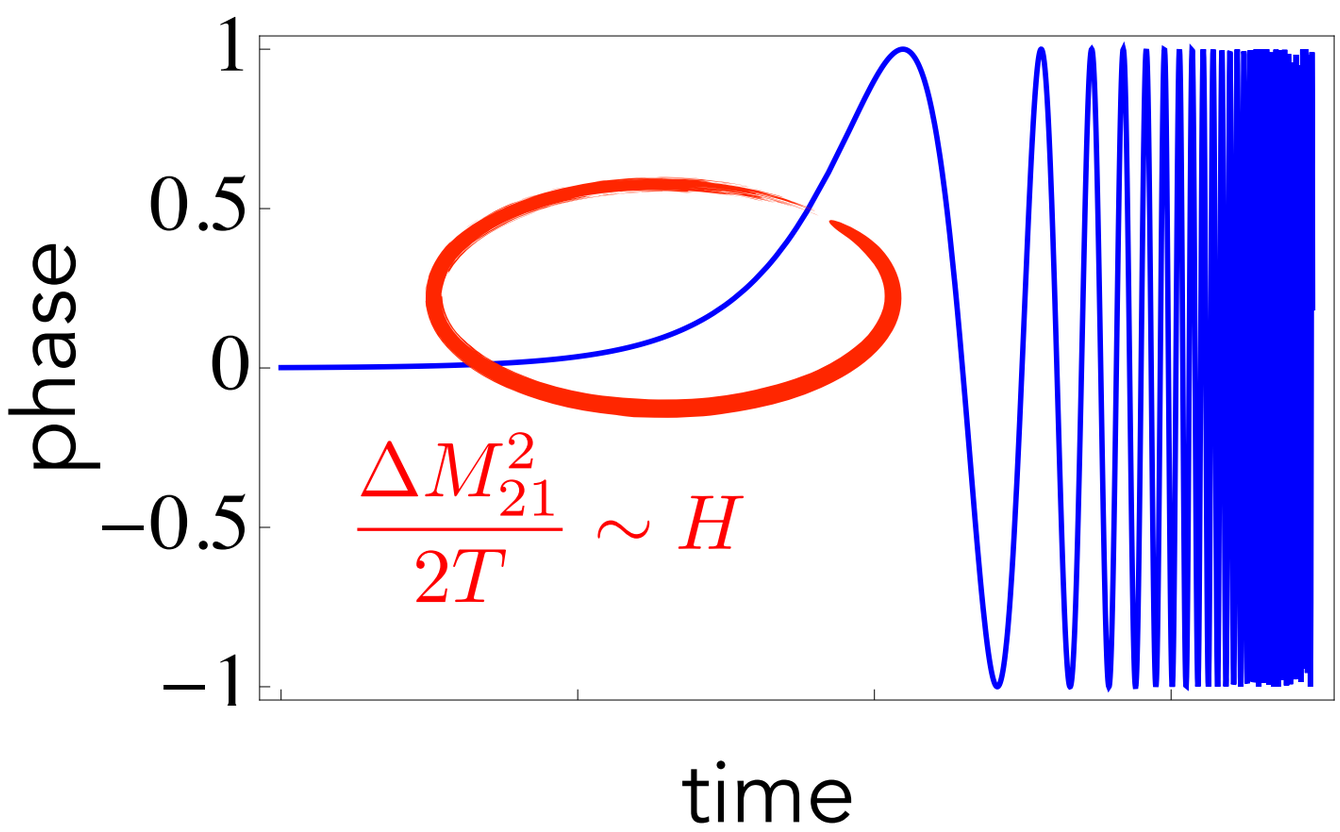

$$\mathrm{asym} \propto \frac{1}{(\Delta M^2_{21})^n}$$

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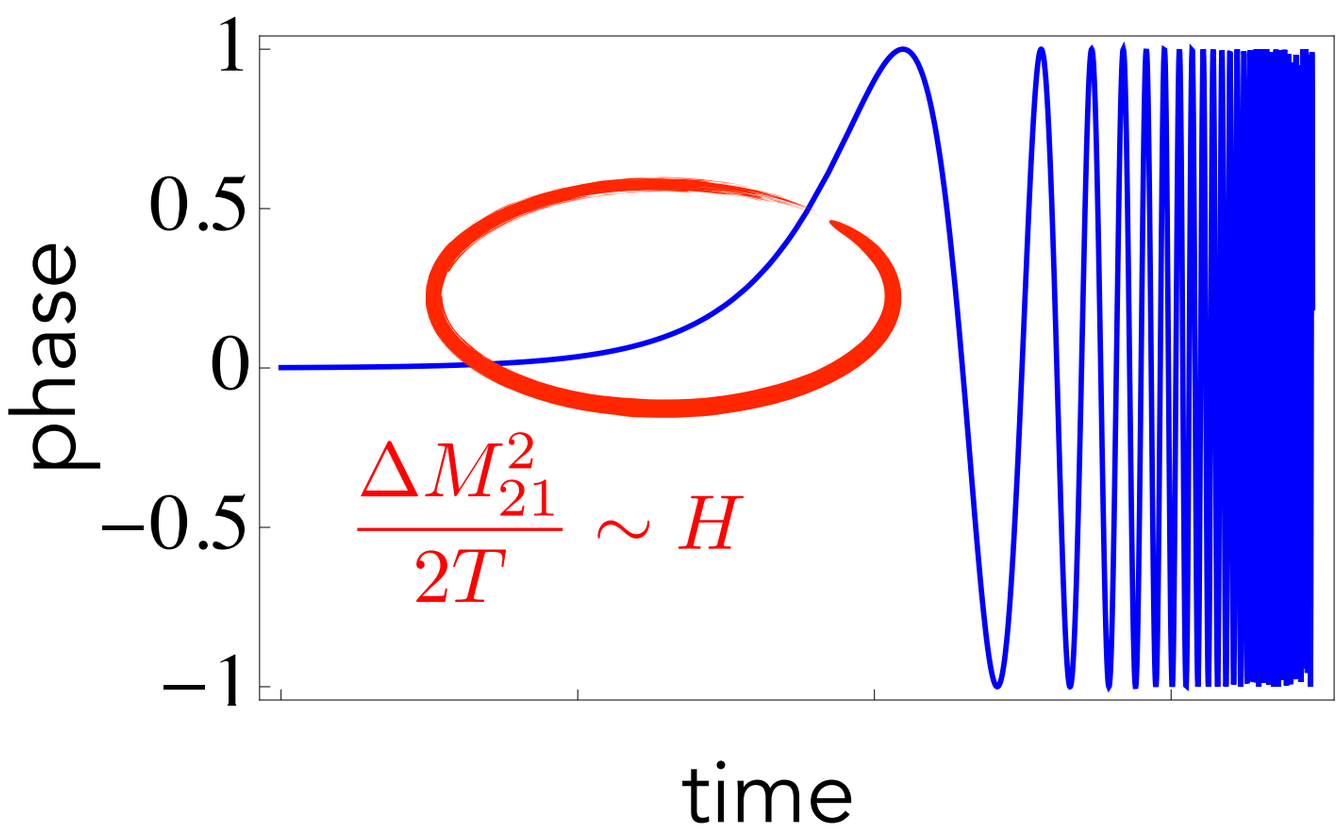

$$\mathrm{asym} \propto \frac{1}{(\Delta M^2_{21})^n}$$

- Optimal asymmetry when $\frac{\Delta M^2_{21}}{2T_{\mathrm{ew}}} \sim H(T_{\mathrm{ew}})$

$$\Delta M^2_{21} \sim (5 \text{ keV})^2$$

ASYMMETRY & DM MASS

$$\Delta M_{21}^2 \sim (5\ \mathrm{keV})^2$$

- This only tells us the mass-squared splitting, but is naturally realized when singlet mass is ~10 keV

- This is optimal for DM too: heavy enough to be consistent with small-scale structure, light enough to not have over-density

- **My takeaway:** the model is very simple, the mechanism is subtle, but it naturally generates several interesting and relevant mass scales: 100 GeV, 10 keV, and 1 cm

FREEZE-IN BARYOGENESIS

1. Generation of asymmetry
2. Survival of asymmetry
3. Size of asymmetry
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5. Single-scalar baryogenesis

RESULTS: ASYMMETRY + DM

- Concrete model (tends to give largest available parameter region):

$$\psi = u_{\mathrm{R}}$$

- In paper, we present transparent, perturbative calculation of results; confirmed with numerical solutions of density matrix evolution eq.

- Ignore thermal mass effects & washout (for now), quantum statistics, etc. (we have checked they are all small effects)

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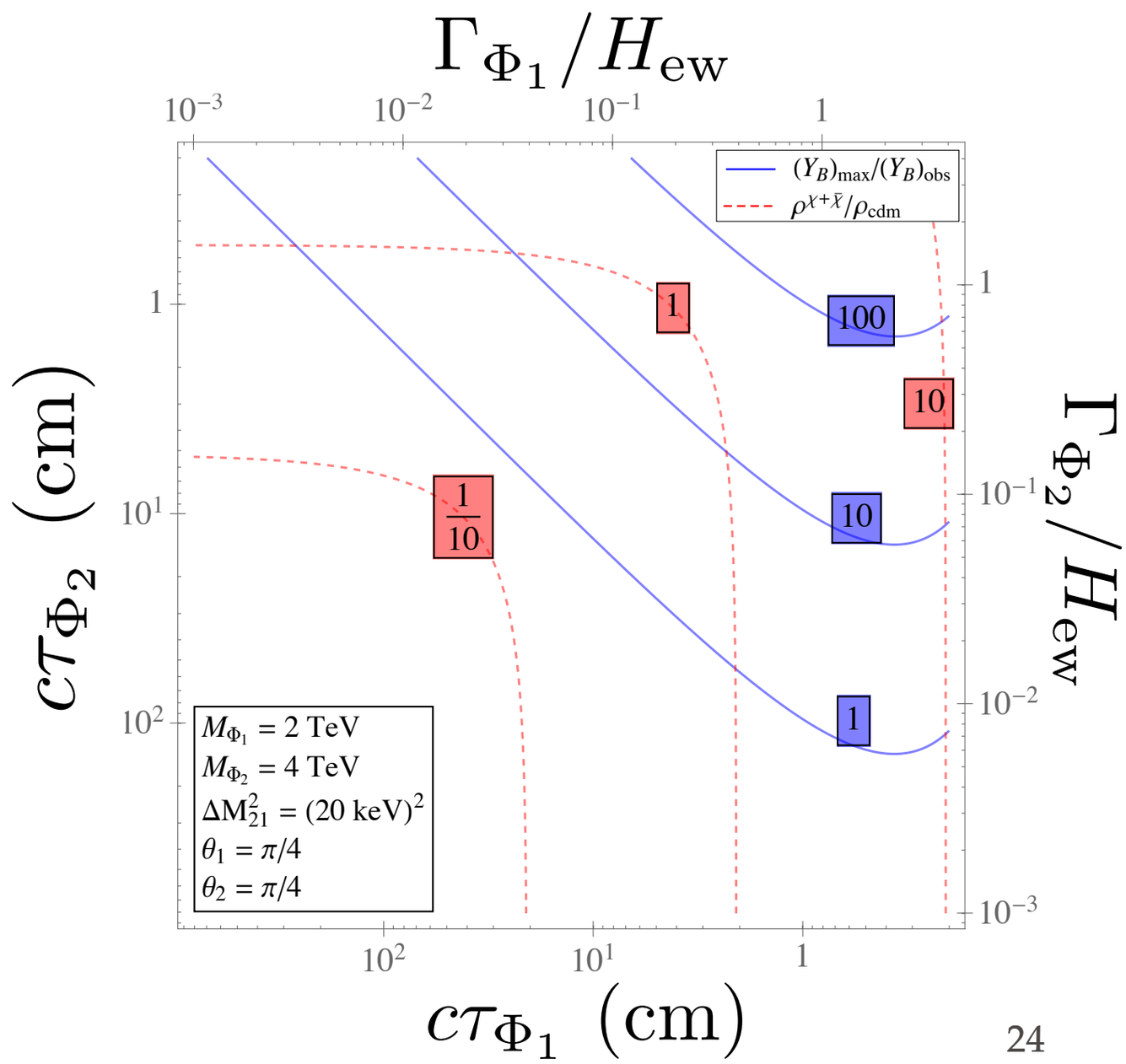

$$Y_B^{\rm max}/Y_B^{\rm obs}$$

$$\rho^{\chi+\bar{\chi}}/\rho_{\rm cdm}$$

$$M_{\Phi_1} = \frac{M_{\Phi_2}}{2} = 2 \text{ TeV}$$

$$M_{\chi_1} = 0,\ M_{\chi_2} = 20 \text{ keV}$$

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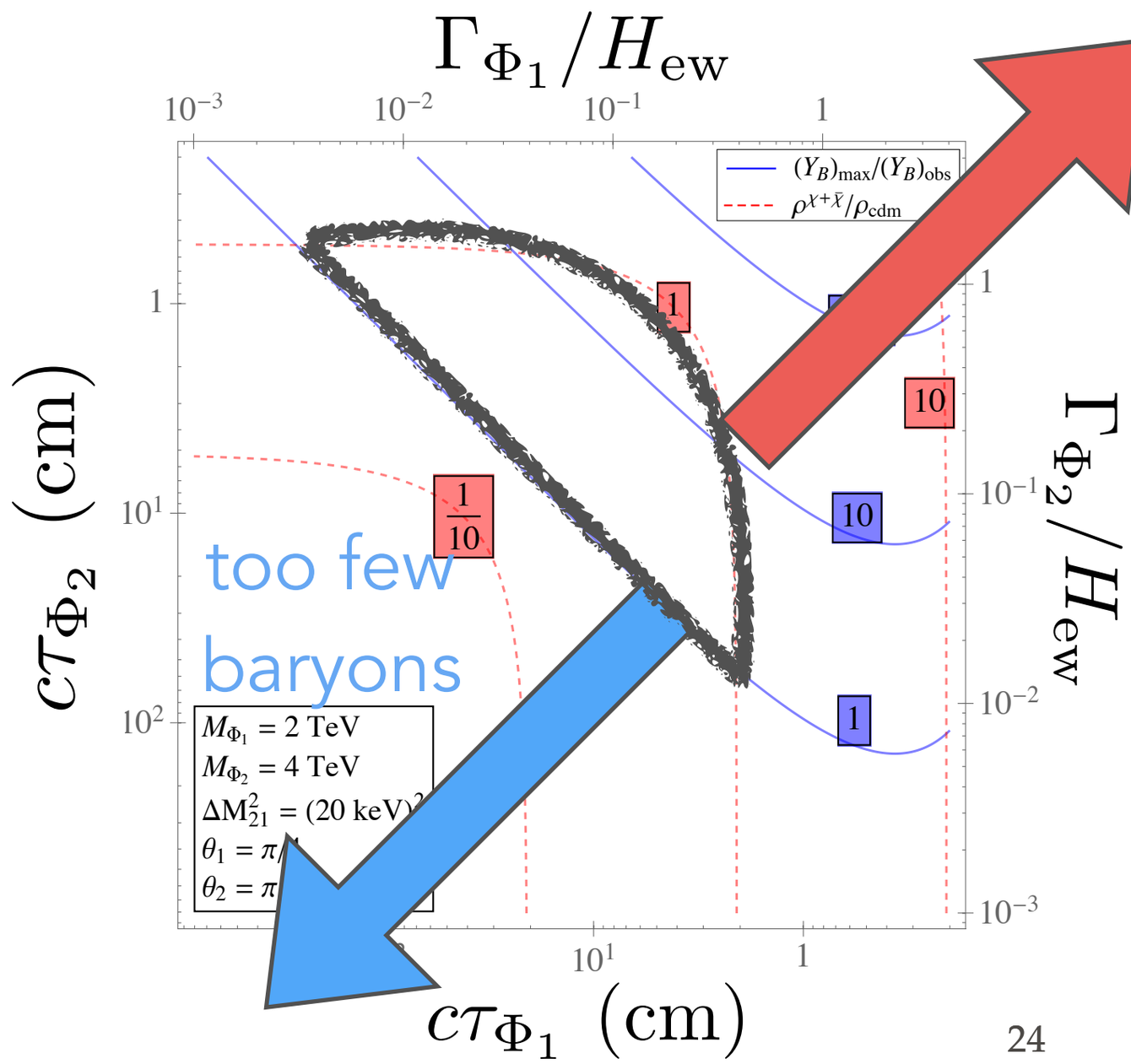

DM
overproduced

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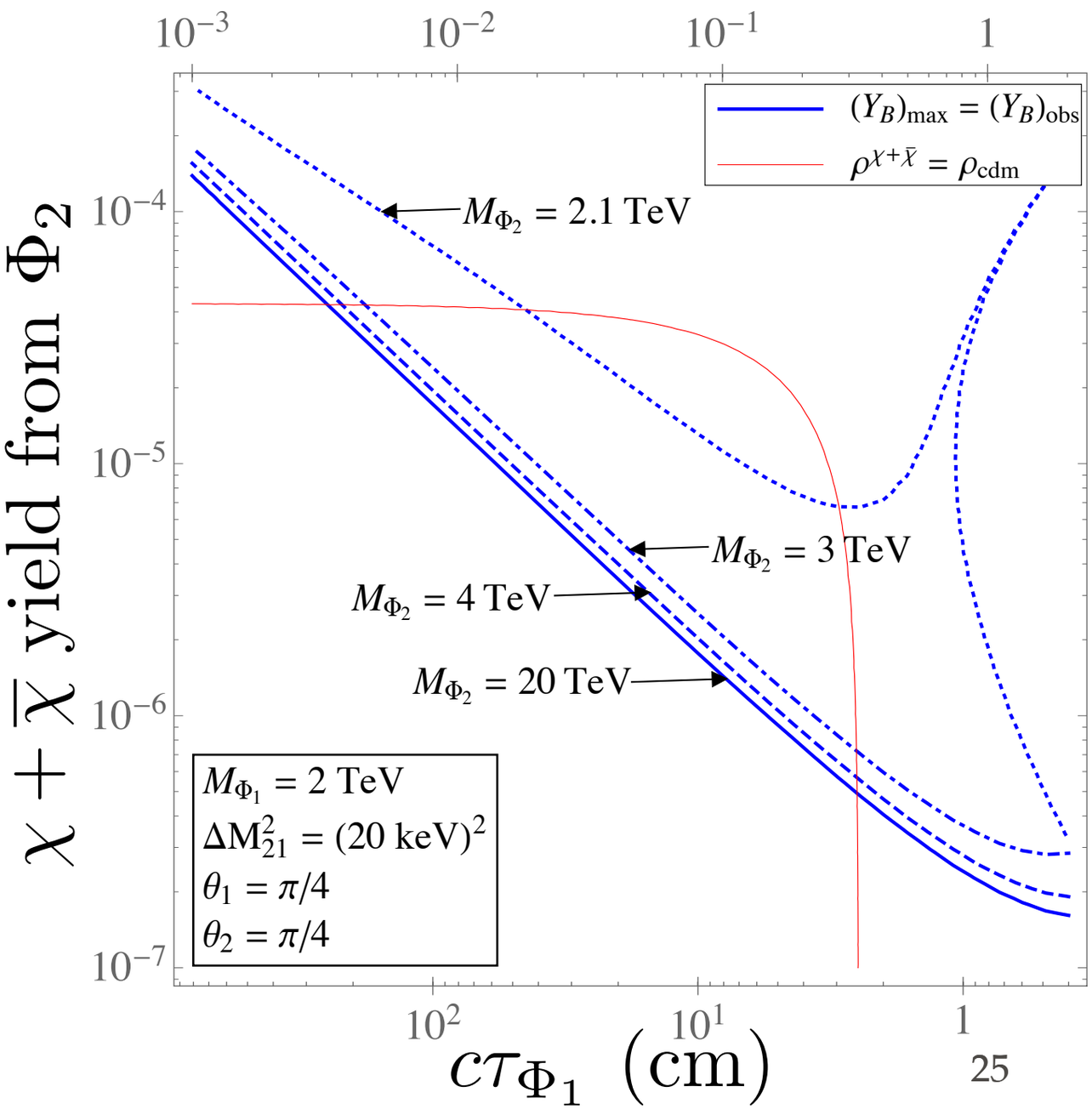
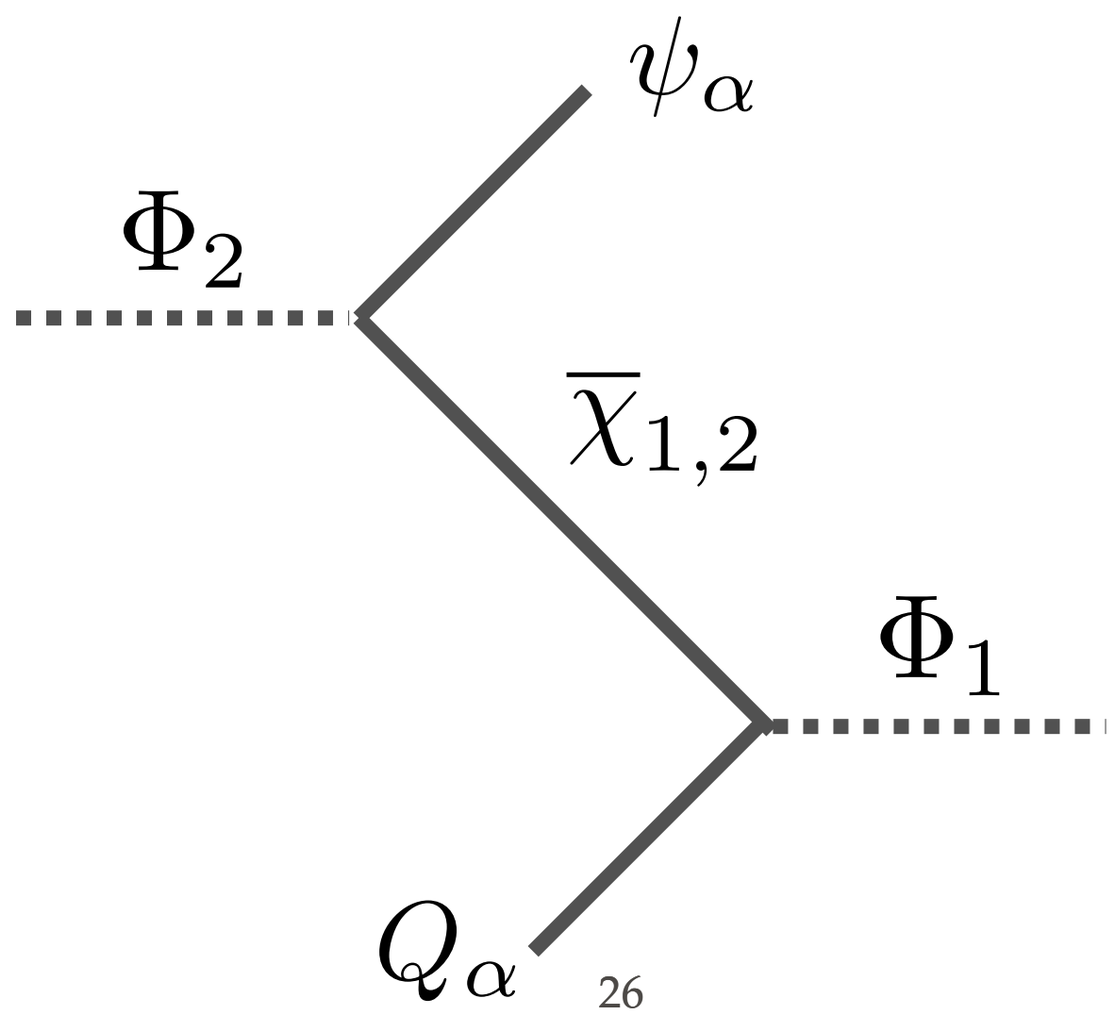
- Second scalar can actually be so heavy as to be decoupled!



- Can get appreciable asymmetry as long as heavy scalar decays to a large quantity of $\chi + \bar{\chi}$

ASIDE: COHERENT BKD BARYOGENESIS

- This gives us a new perspective: baryogenesis can actually result from **any** out-of-equilibrium states produced in early Universe!
- Common in (p)reheating models? Moduli decay? etc



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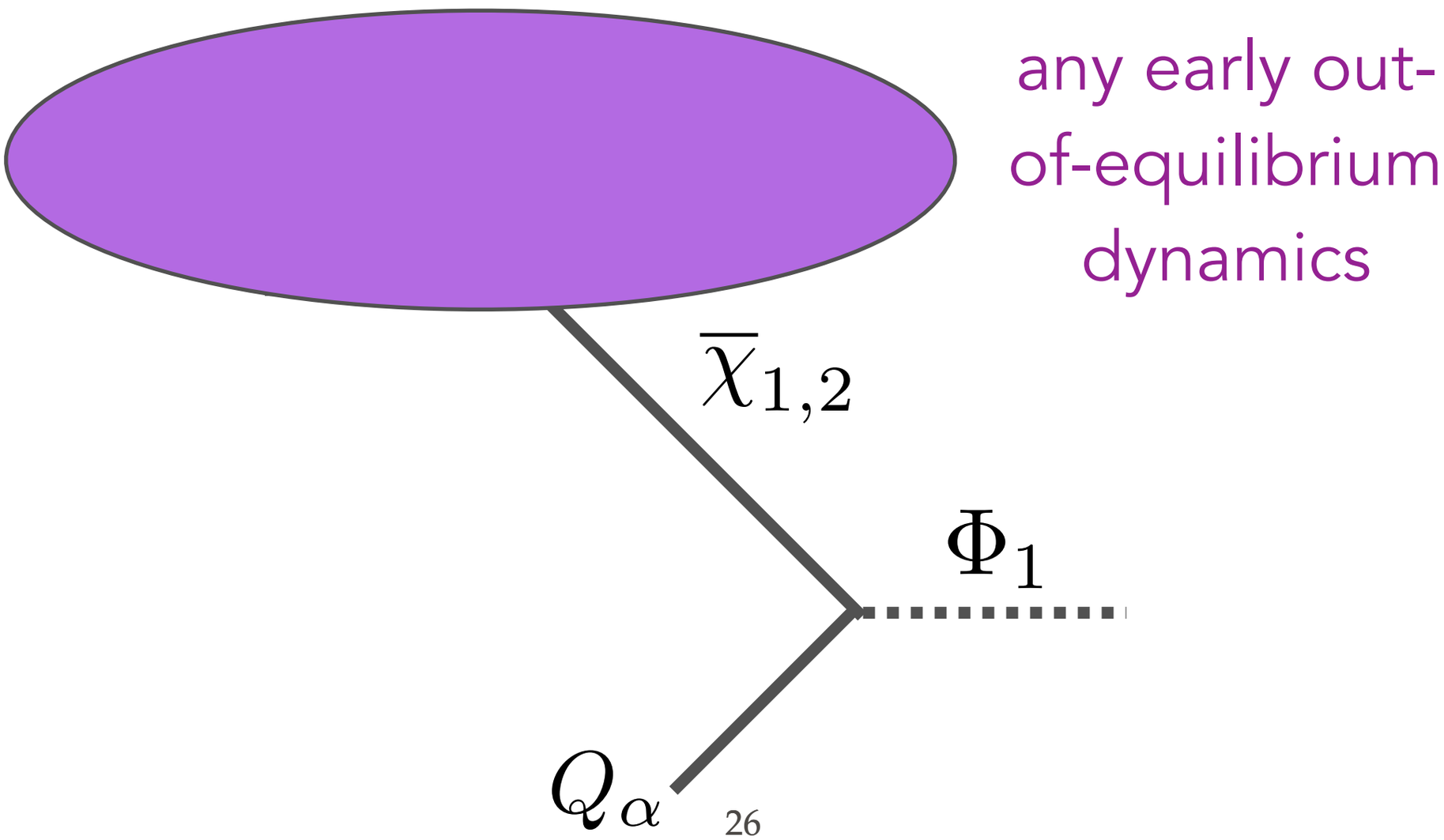
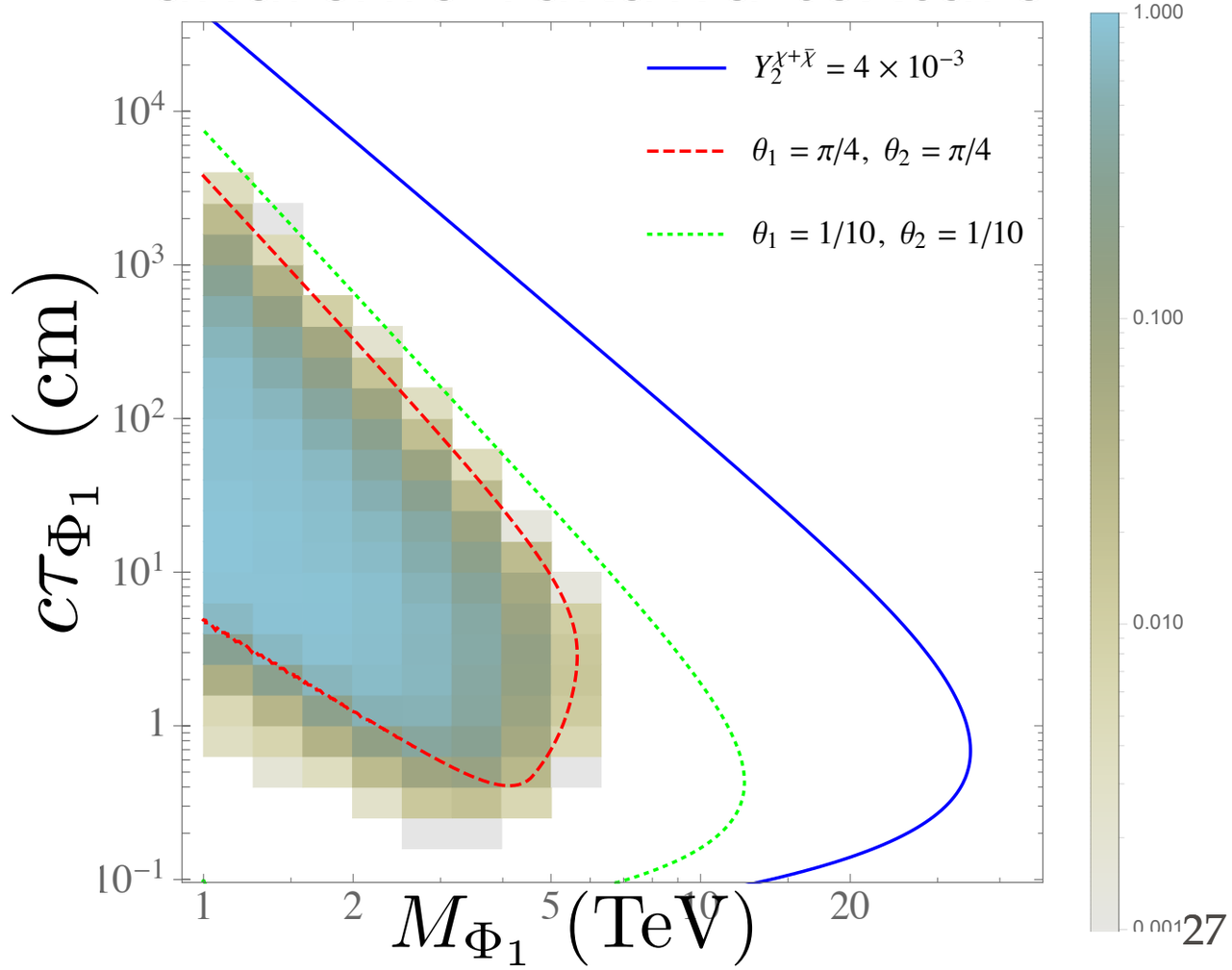


LARGEST PARAMETER SPACE

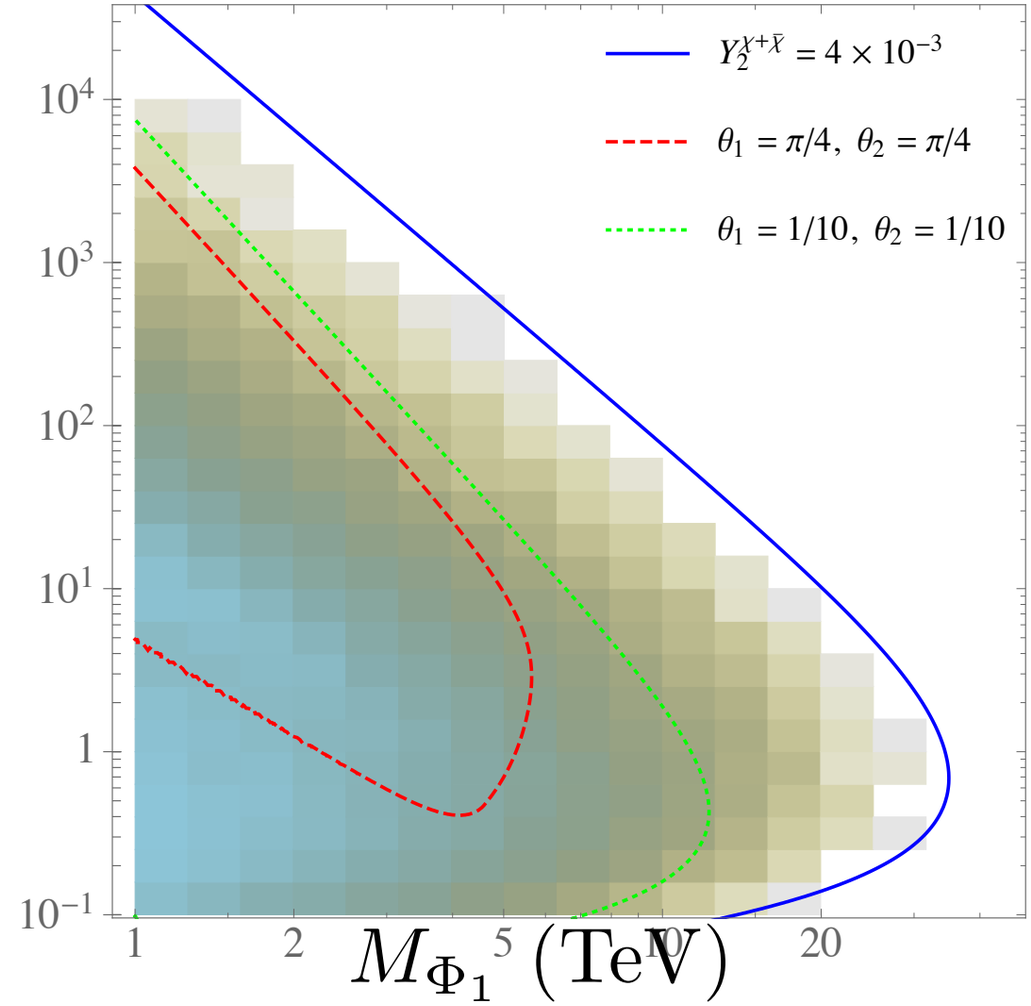
- In limit with decoupled heavy scalar; allows us to do simple scans
- Each point is consistent with baryogenesis + DM

$$M_{\chi_1} = 0,\ M_{\chi_2} > 10\ \mathrm{keV}$$

anarchic Yukawa texture

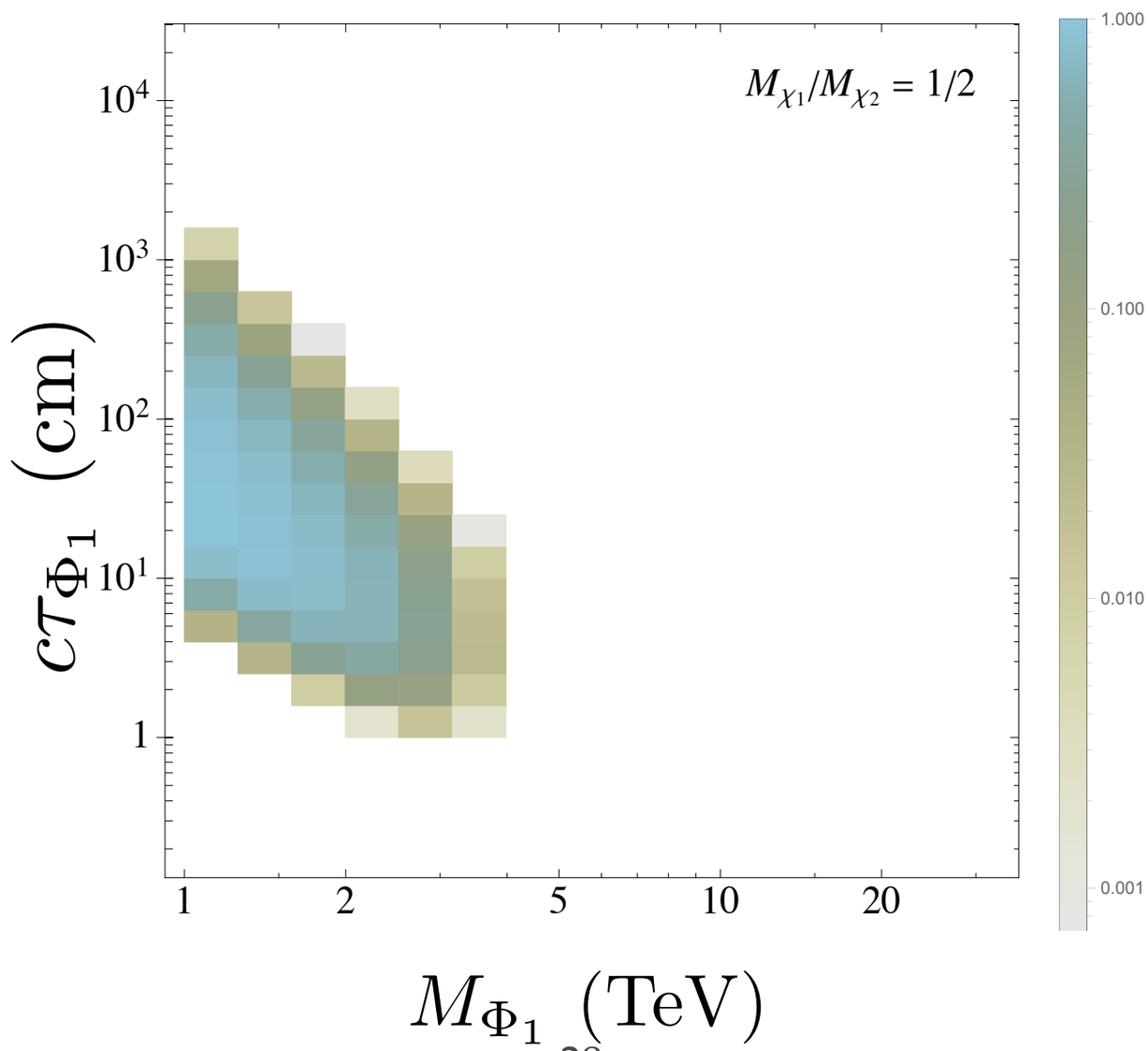


hierarchical Yukawa texture



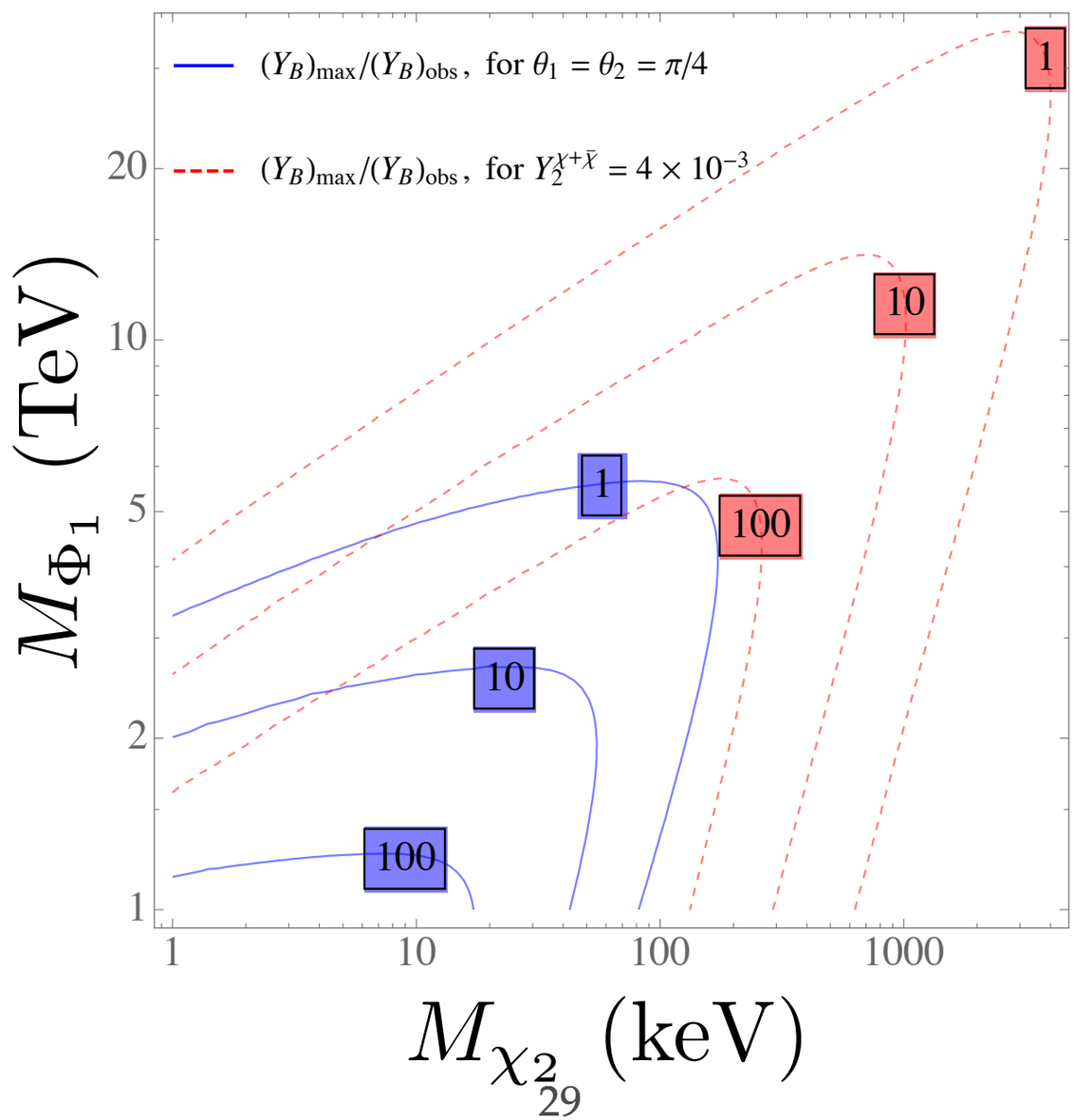
LARGEST PARAMETER SPACE

- If lightest DM particle is not massless, even tighter constraints
- If lightest DM < 10 keV, require it give < 35% of DM energy density



LARGEST PARAMETER SPACE

- DM masses up to 1 MeV allowed in most optimistic scenario
- Correlation between heavier scalar, larger DM mass



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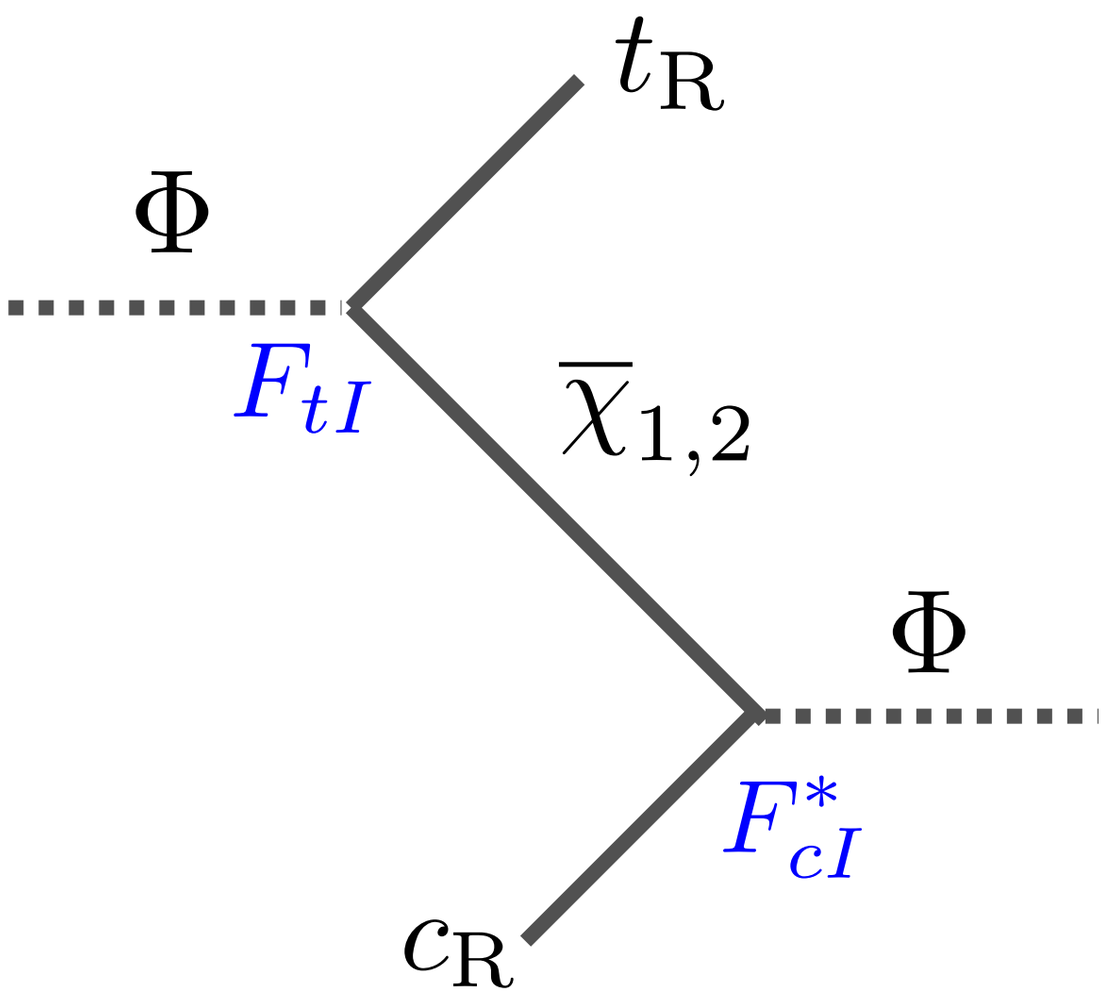
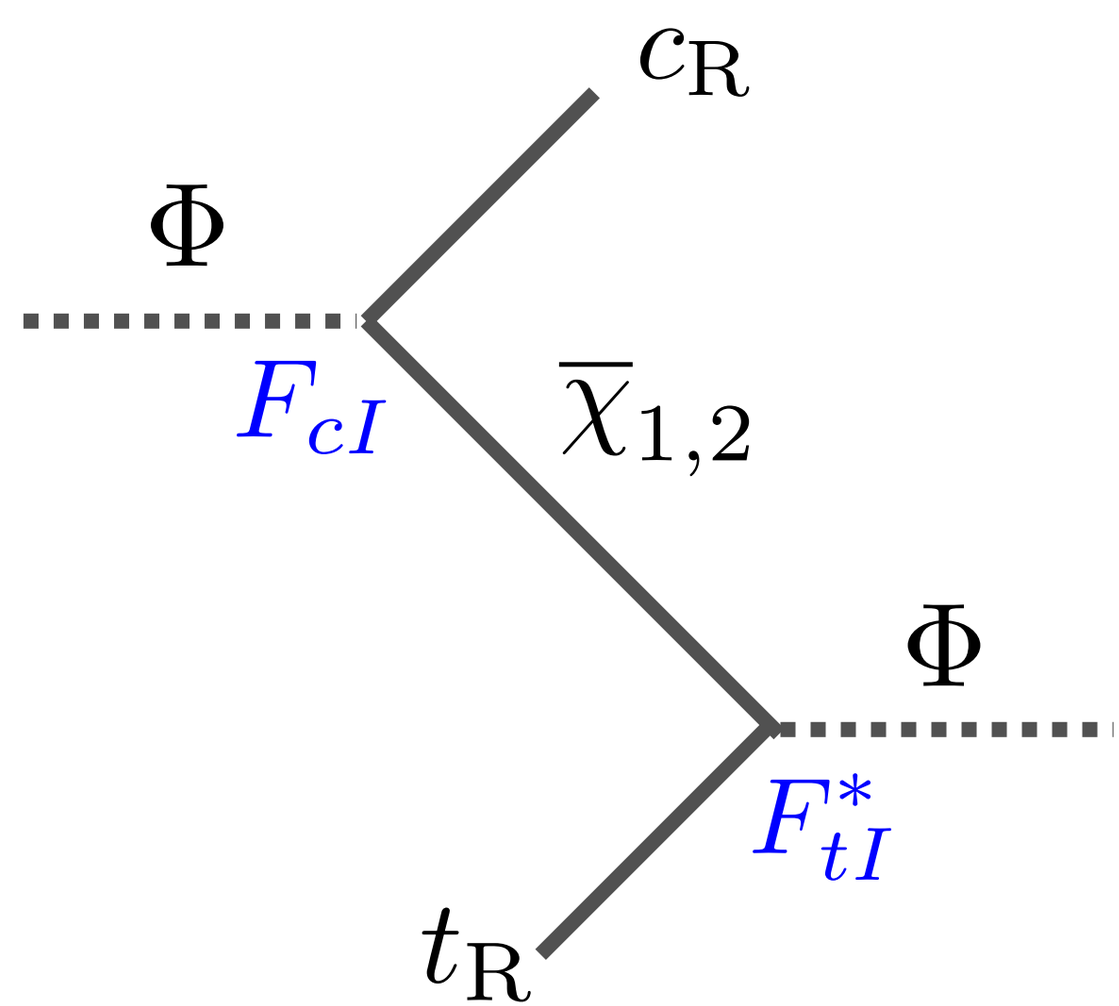
SINGLE-SCALAR BARYOGENESIS

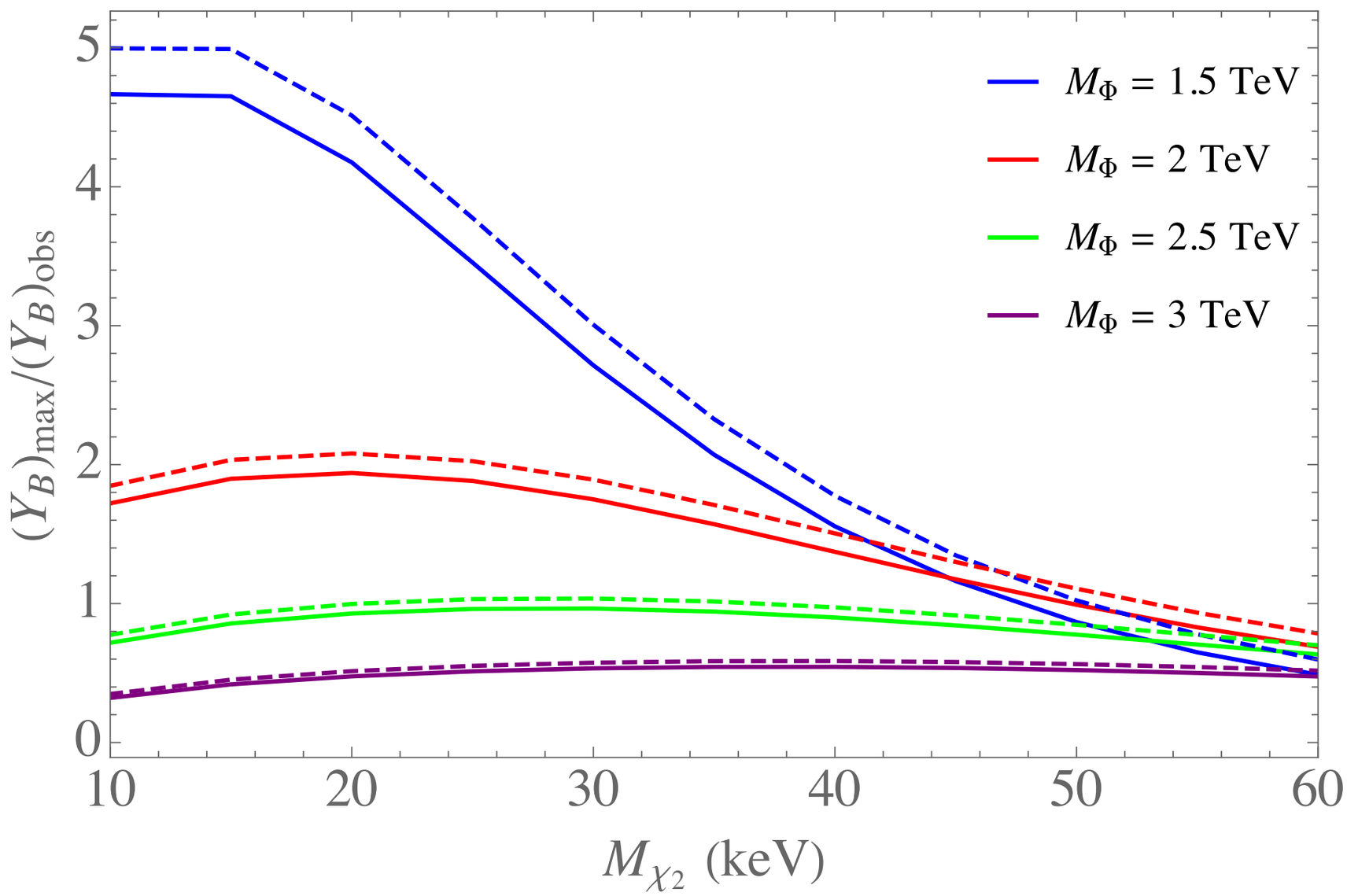
- If $M_{\psi_\alpha} = M_{\psi_\beta}$ (as is approx. true in early Universe), summed asymmetry is zero!
- But…**top quark** has larger thermal mass than other quarks





TOP-FLAVOUR SINGLE SCALAR

- Asymmetry is higher order: $\mathcal{O}(F^4 y_t^2)$
- Constrained parameter space, but it works!

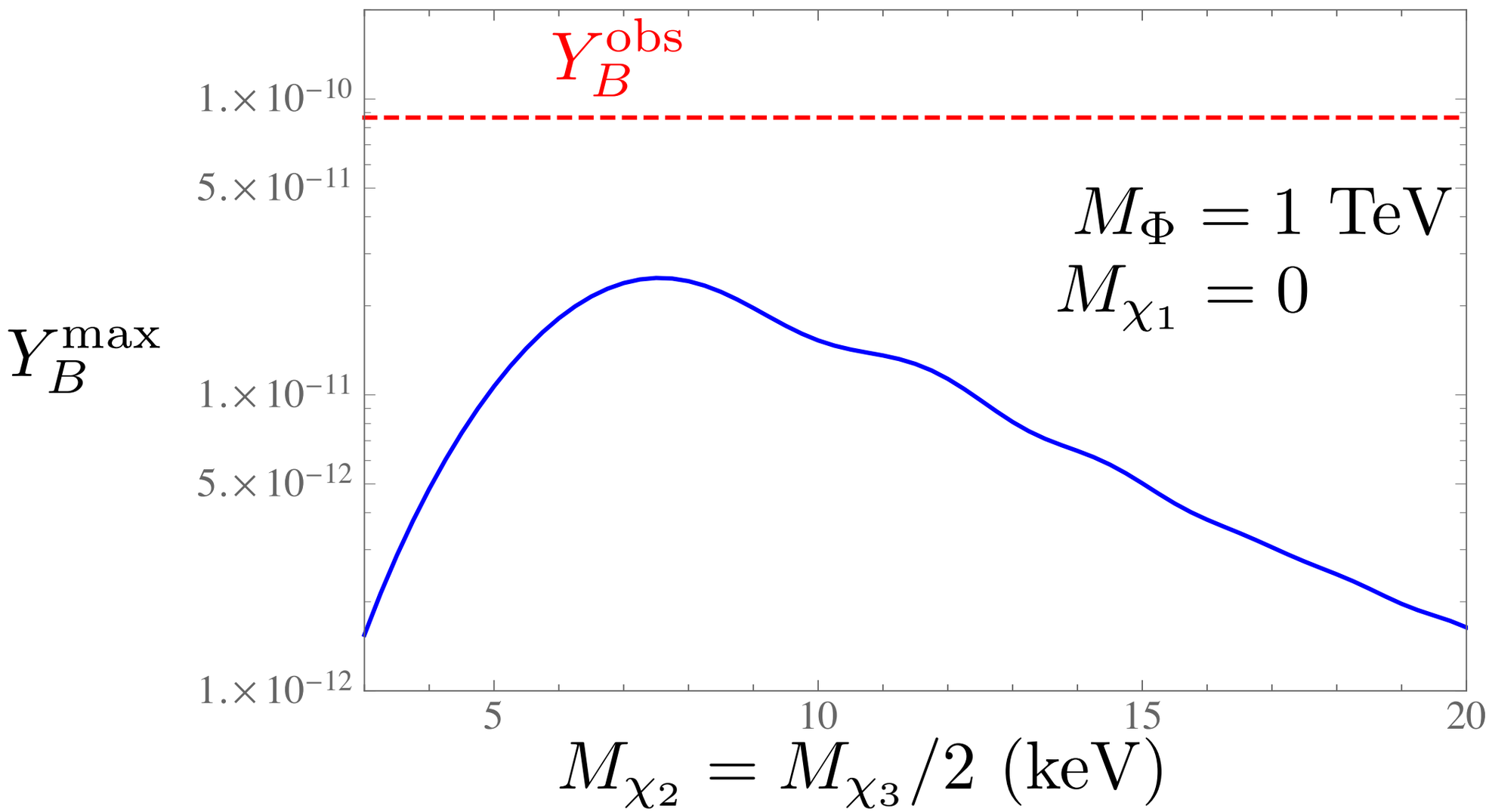


NO-TOP SINGLE SCALAR

- If we are unlucky and the top quark doesn't couple, an asymmetry arises at $\mathcal{O}(F^6)$ from a sequence of 3 scattering processes

A. Abada *et* al., arXiv:1810.12463

- Need 3 chi particles! Can't get baryon asymmetry + DM

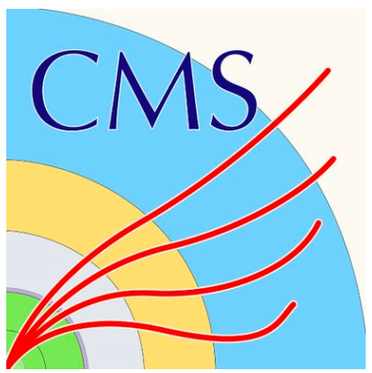


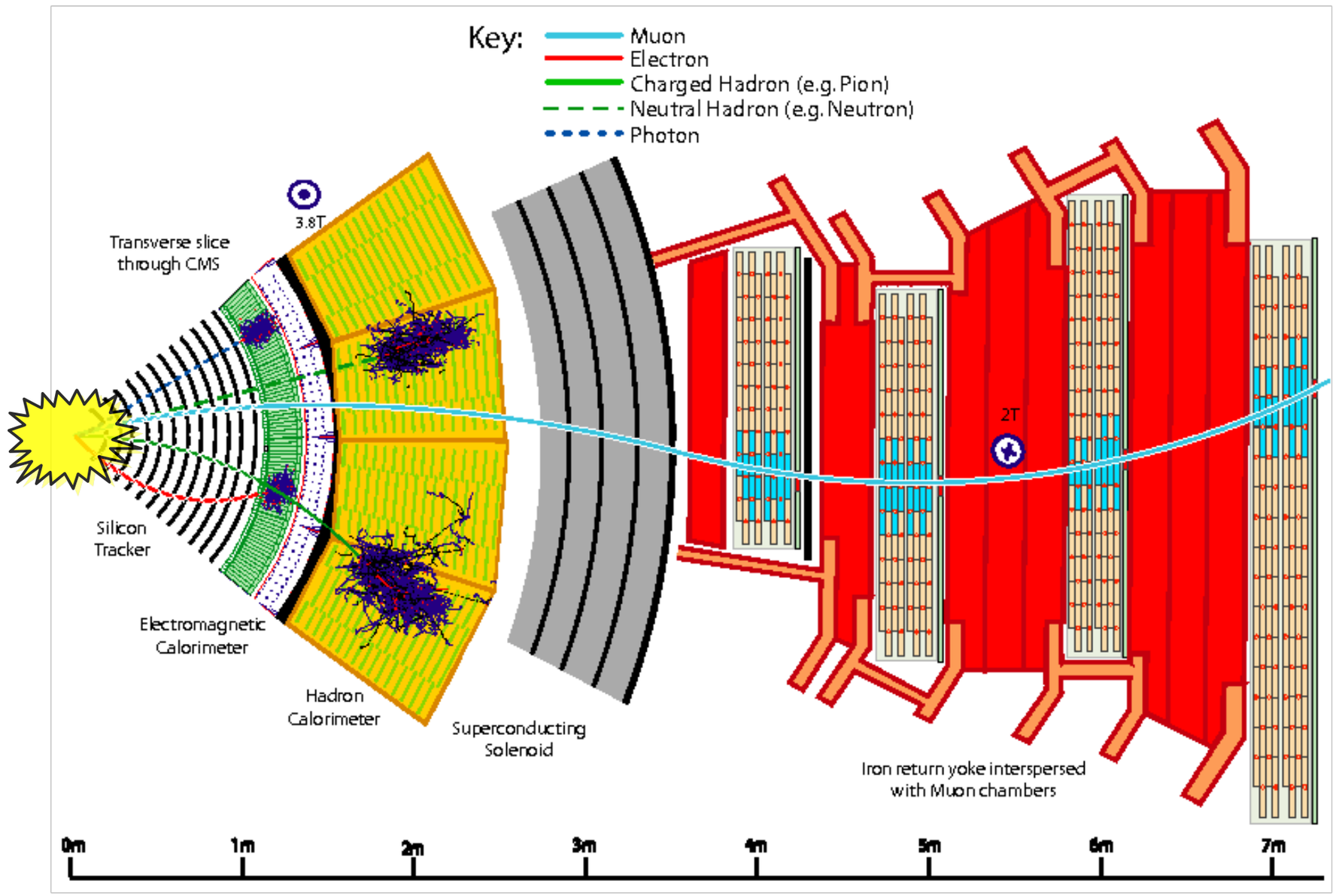
SIGNALS

LONG-LIVED PARTICLES

$$c\tau_\Phi \gtrsim 0.1 \text{ cm}$$

$$pp \to \Phi^* \Phi,\ \Phi \to Q_\alpha \bar{\chi}$$





LONG-LIVED PARTICLES

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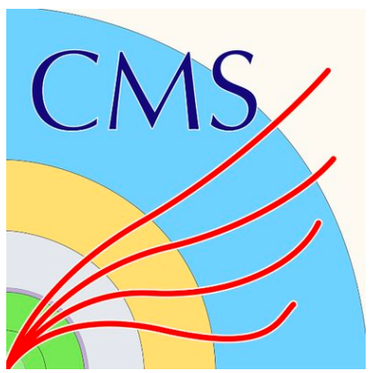
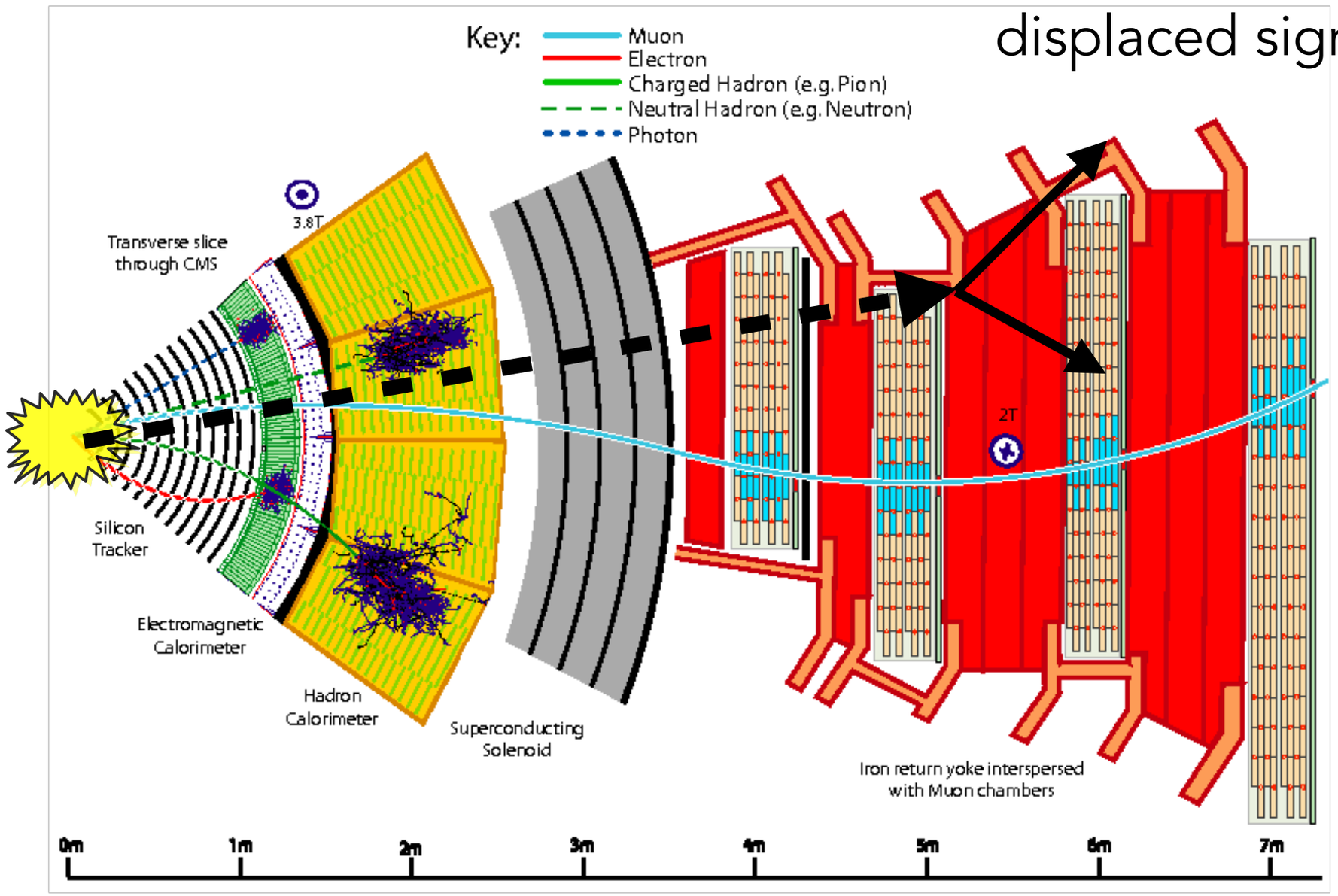
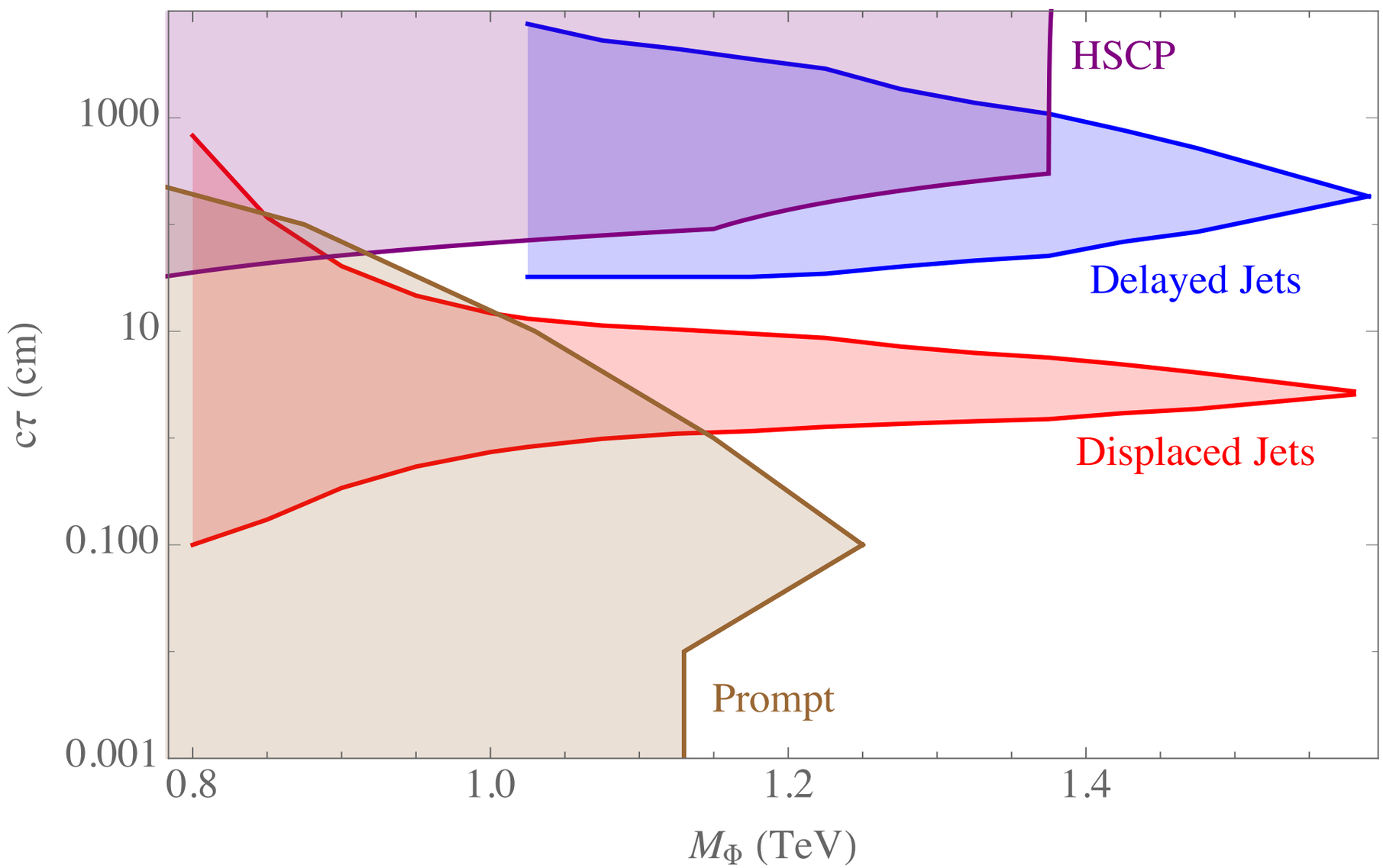




ATLAS, arXiv:1808.04095; CMS, arXiv:1811.07991; CMS, arXiv:1906.06441; CMS, arXiv:1908.04722

LONG-LIVED PARTICLES

- Any single fermion + missing energy signature well-motivated
- Experiments did well at providing material for interpretation!

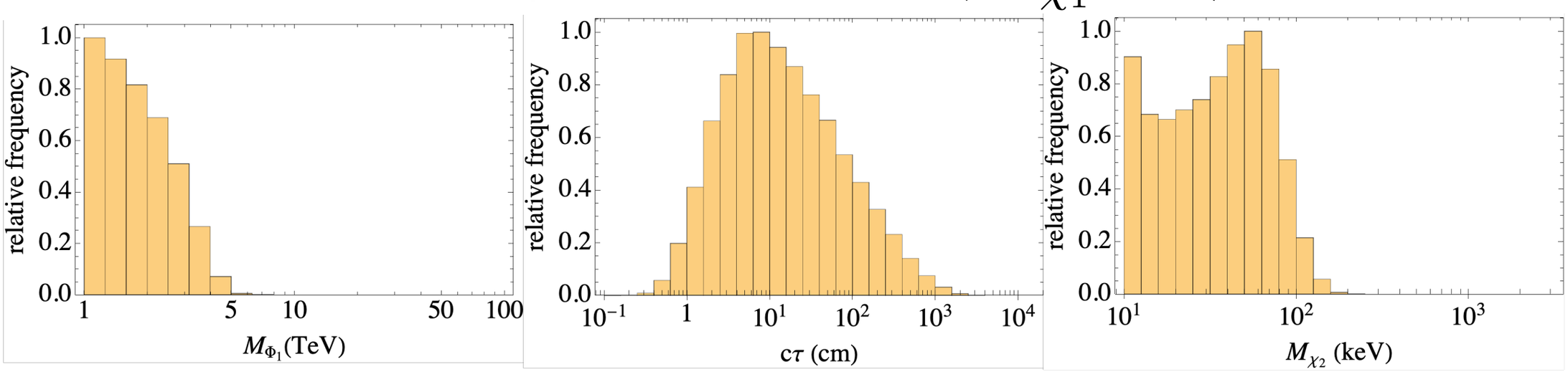


BS, D. Tucker-Smith, to appear

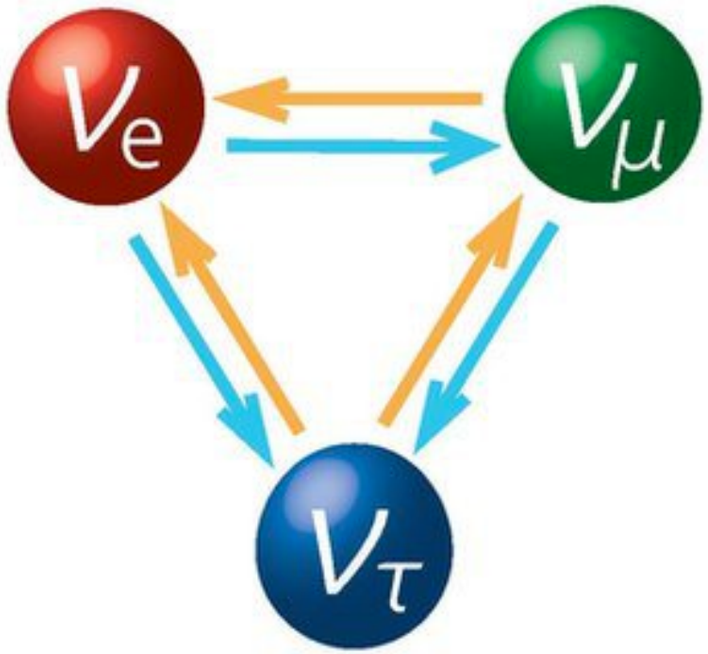
COSMOLOGY & ASTRO

- Low-mass DM: possible imprints in Lyman-alpha and other small-scale structure (DM masses close to 10 keV)
- Much of parameter space features an (almost) massless state with large abundance: affects $N_{\rm eff}$ (0.05 per Weyl fermion state)
- If $\mathbf{Z_2}$ very slightly broken, can get X-ray line signatures, but it is not a necessary prediction of the model

2 scalar, anarchic texture ($M_{\chi_1} = 0$)



FREEZE-IN LEPTOGENESIS (ARS)



ARS LEPTOGENESIS

$$\mathcal{L} = -F^a_{\alpha I}\overline{\psi}_\alpha \Phi_a \chi_I - \frac{M_I}{2}\bar{\chi}^c_I \chi_I - V(\Phi_a)$$

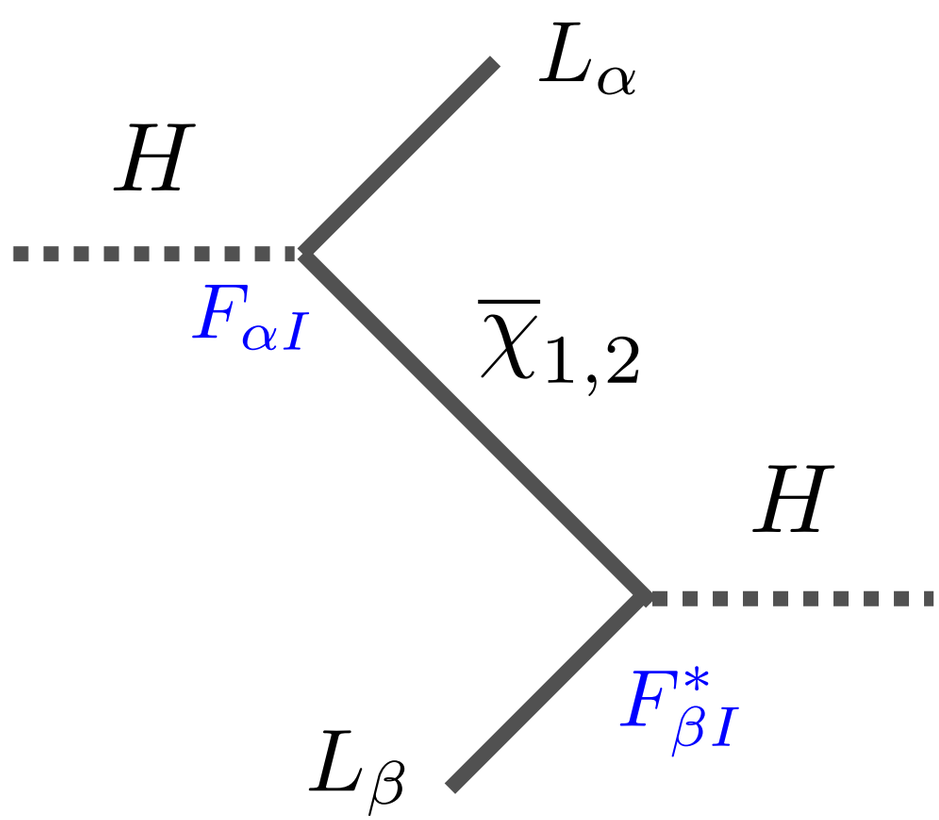
ψ_α = SM lepton doublet

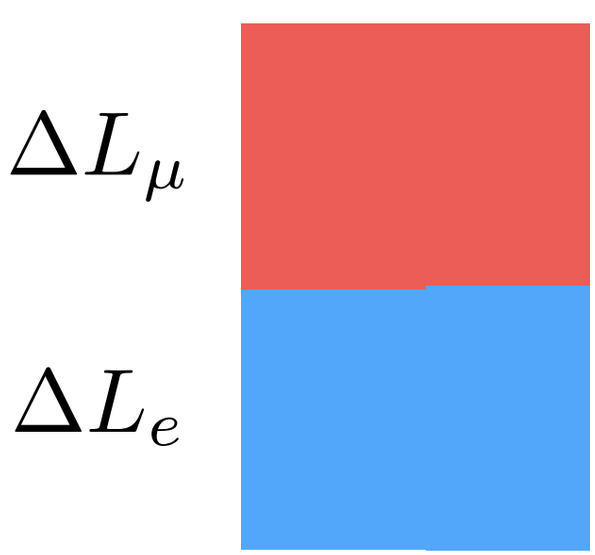
χ_I = 2 or more singlet fields with Majorana masses (RHNs)

Φ = SM Higgs, $\mathbf{Z_2}$ symmetry broken

- No DM, but singlets now play the role of the right-handed neutrinos
- Single scalar, so asymmetry comes in at $\mathcal{O}(F^6)$

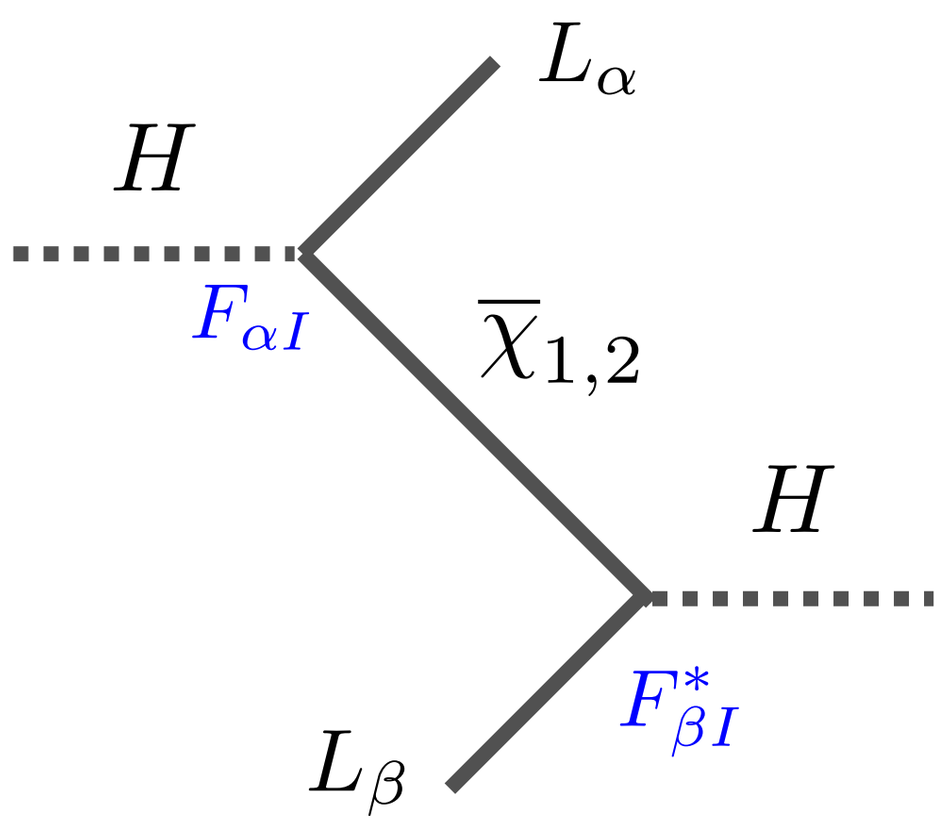
ARS LEPTOGENESIS





- No net asymmetry in leptons
- But, rate of $L_\beta \to L_\alpha$ can be different from rate of $\overline{L}_\beta \to \overline{L}_\alpha$
- Asymmetries in individual lepton flavours, cancel in sum
- Total asymmetry arises at $\mathcal{O}(F^6)$

ARS LEPTOGENESIS



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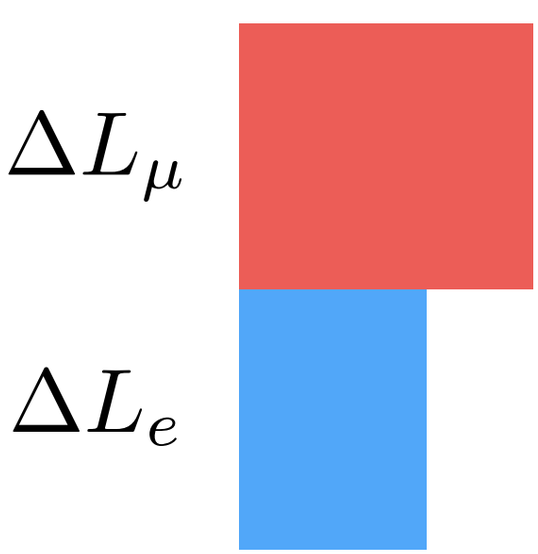
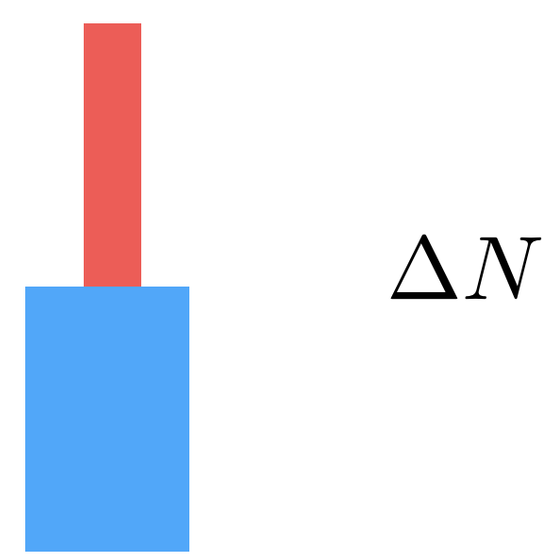




ARS LEPTOGENESIS

- Singlets typically called *N*

$$M_{\nu,\mathrm{SM}} \sim \frac{F^2 v^2}{M_N}$$

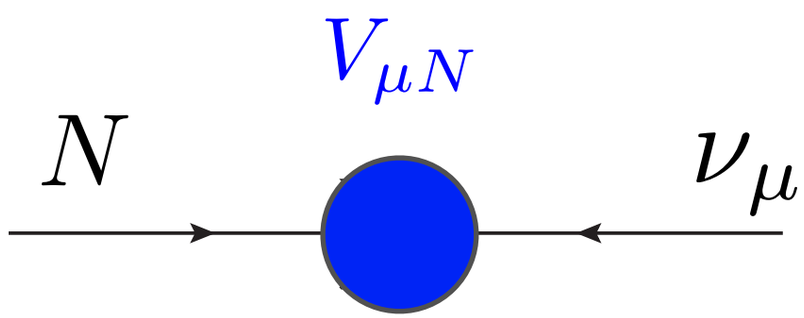
- To avoid coming into equilibrium, $F \lesssim 10^{-7}$
  To get sufficient asymmetry (single scalar), $F \gtrsim 10^{-9}$

$$M_N \sim 1 - 10 \text{ GeV}$$

$$\Delta M_N^2 \sim (5 \text{ keV})^2 \to \frac{\Delta M_N}{M_N} \lesssim 10^{-10}$$

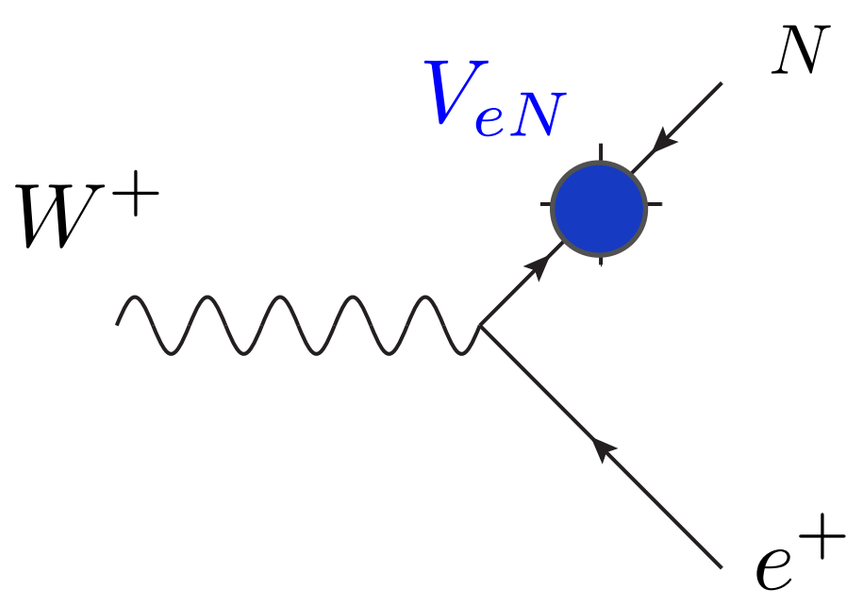
ARS LEPTOGENESIS

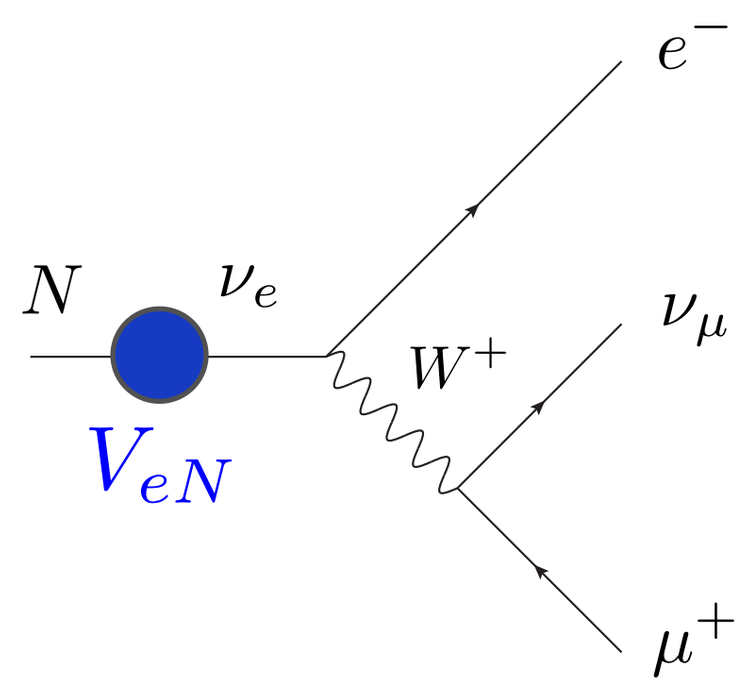
- In vacuum today, the RH and LH neutrinos mix



$$V_{\mu N}^2 \sim \frac{M_{\nu,\mathrm{SM}}}{M_N} \sim 10^{-10}$$

- Coupling can be **enhanced** in models with approximate symmetries (inverse see-saw, etc) *or* by fine-tuning the Yukawa couplings





ARS LEPTOGENESIS

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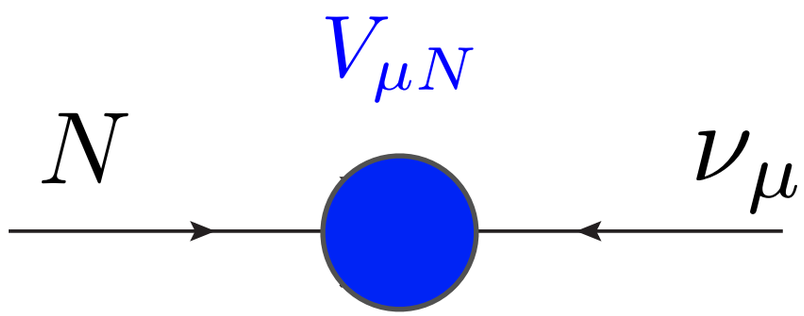

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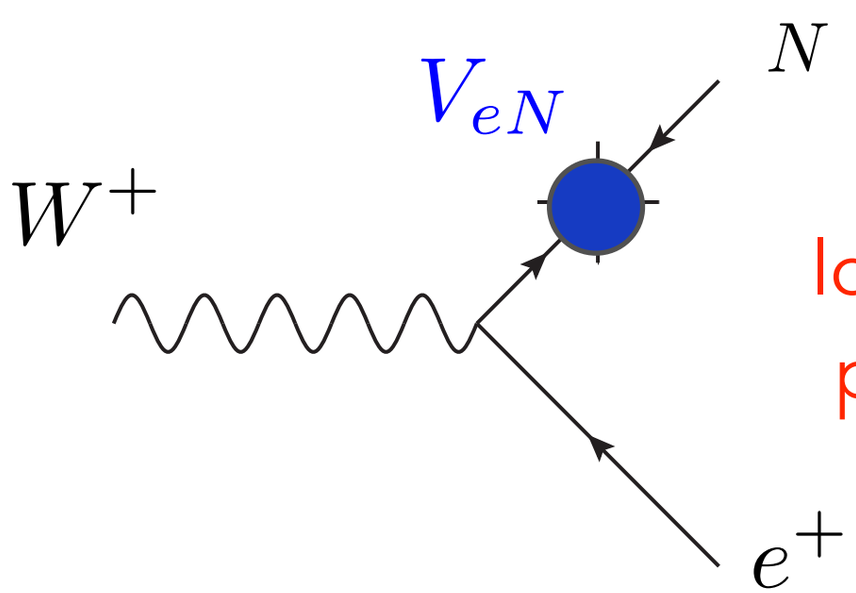
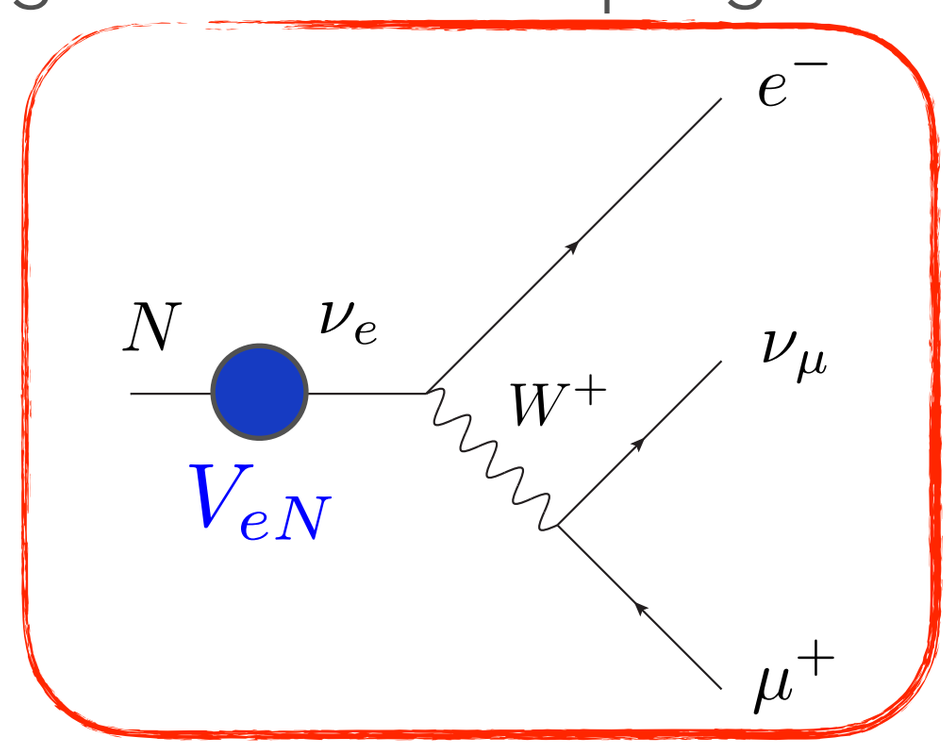
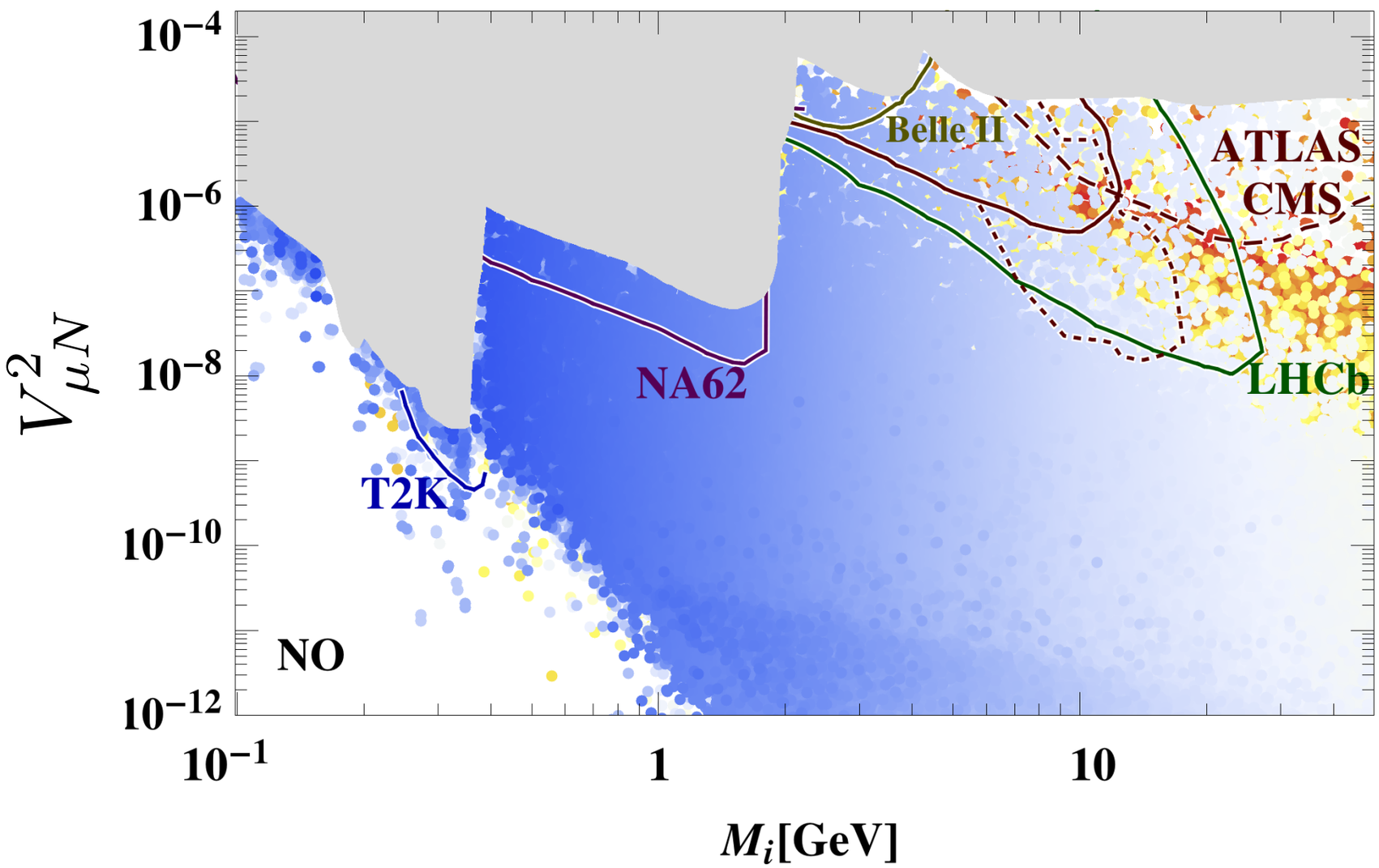


long-lived particle!!



ARS LEPTOGENESIS

- Parameter space for leptogenesis:
- Grey regions excluded by lab experiments



A. Abada *et* al., arXiv:1810.12463

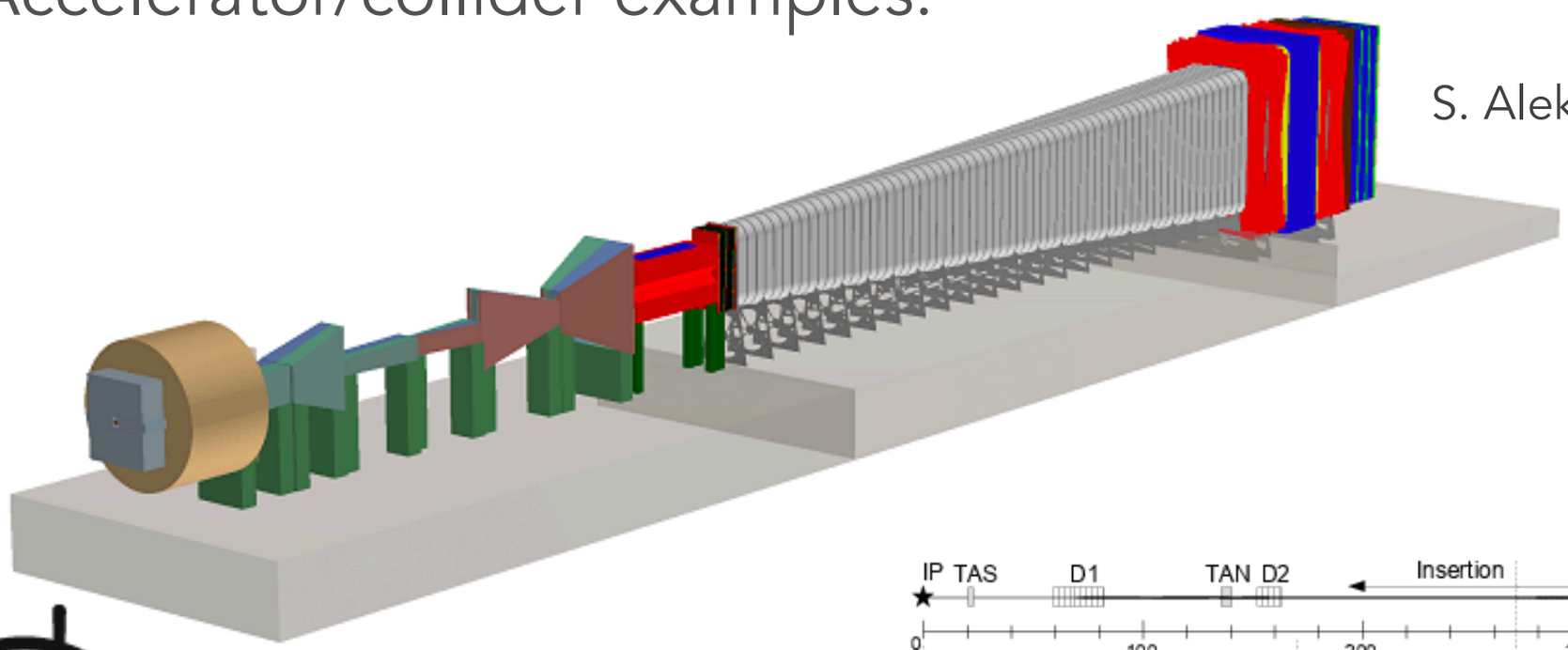
ARS LEPTOGENESIS: PROBES

- BBN: *N* cannot inject too much energy to disrupt BBN

O. Ruchayskiy and A. Ivashko, arXiv:1202.2841

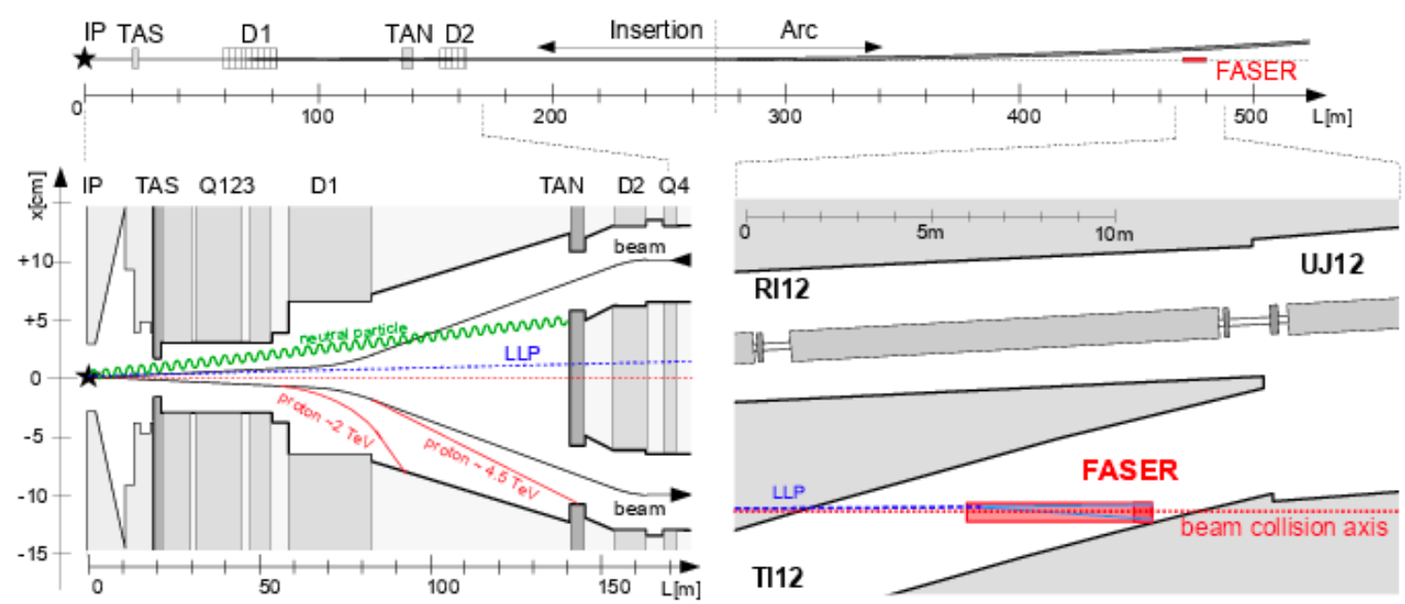
- Accelerator/collider examples:

S. Alekhin *et al.*, arXiv:1504.04855



FASER

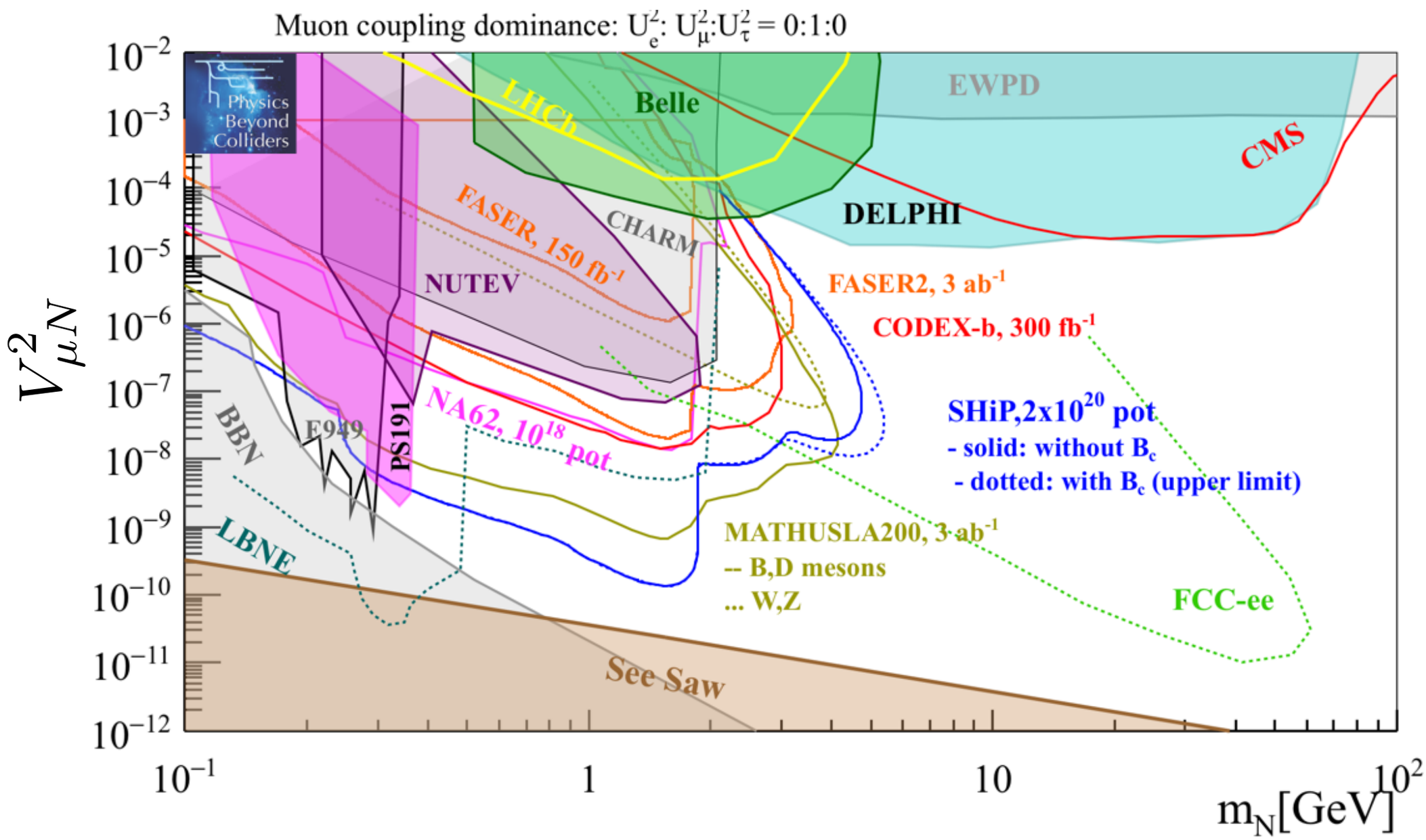
SHiP
Search for Hidden Particles



J. Feng *et al.*, arXiv:1708.09389

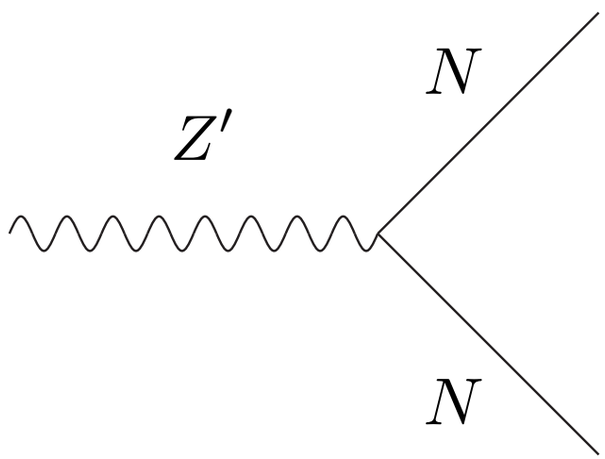
ARS LEPTOGENESIS: PROBES

- Also: recent first results from ATLAS & CMS

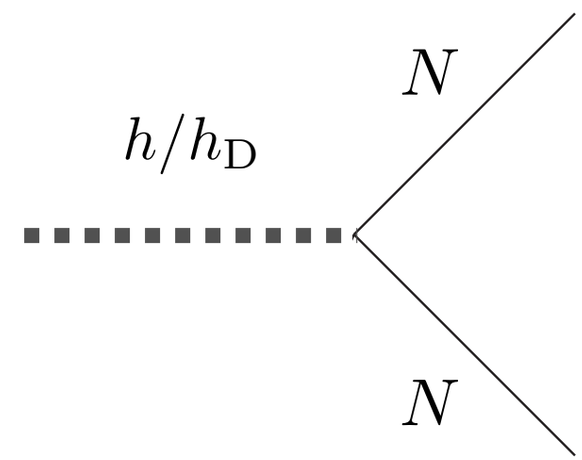


ARS LEPTOGENESIS: PROBES

- Discovery prospects are better in non-minimal models where there are additional *N* couplings



Basso *et al.*, 2008; Fileviez Perez, Han, Li, 2009; Batell, Pospelov, BS, 2016, …



Graesser, 2007; Shoemaker *et al.*, 2008; Gago *et al.*, 2015; Accomando *et al.*, 2016, …

ARS LEPTOGENESIS: PROBES

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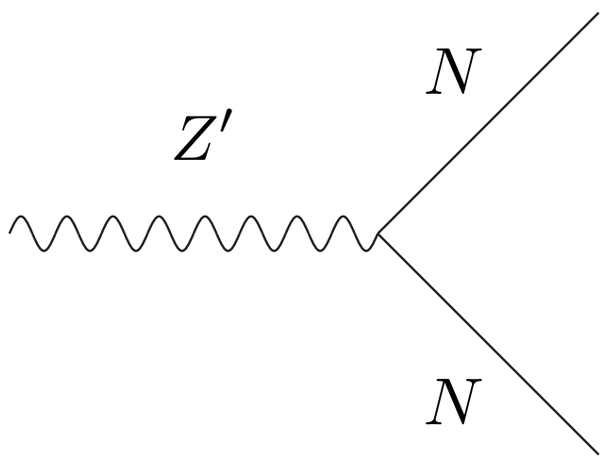

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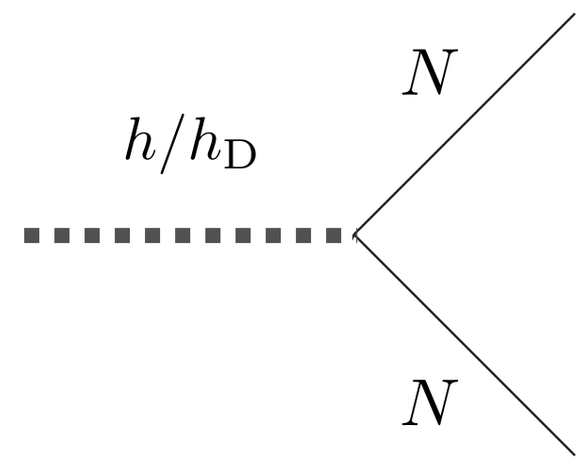


Graesser, 2007; Shoemaker *et al.*, 2008; Gago *et al.*, 2015; Accomando *et al.*, 2016, …

- But…by bringing *N* into equilibrium earlier, this can reduce the lepton asymmetry by many orders of magnitude

I. Flood (HMC '20), J. Schlesinger (HMC '21), BS, to appear

SUMMARY

- Freeze-in baryogenesis is a simple, predictive model for (some combination of) baryon asymmetry, dark matter, neutrino masses
- Asymmetry generation relies on initial condition after reheating
- Dynamics of decay, oscillation, and inverse decay point towards a multitude of interesting particle scales: keV, GeV, 100 GeV, cm
- Many signatures in colliders, accelerators, and cosmology
- I think we're just scratching the surface - what other regimes & signatures are there to uncover?

BACKUP SLIDES

(INCOMPLETE!) REFERENCES

- GUT baryogenesis

  Kolb, Wolfram, 1979; Kolb, Linde, Riotto, hep-ph/9606260

- Affleck-Dine Baryogenesis

  Affleck, Dine, 1985; Dine, Randall, Thomas, hep-ph/9507453

- Higgs relaxation baryogenesis

  Yang, Pearce, Kusenko, arXiv:1505.02461; Kawasaka *et al.*, 1701.02175

- Standard thermal leptogenesis

  Fukugita, Yanagida, 1986; Davidson, Nardi, Nir, arXiv:0802.2962

- Resonant leptogenesis

  Pilaftsis, Underwood, hep-ph/0309342; hep-ph/0506107; Dev *et al.*, arXiv:1404.1003

(INCOMPLETE!) REFERENCES

- Electroweak baryogenesis

  Kuzmin, Rubakov, Shaposhnikov, 1985; Cohen, Kaplan, Nelson, hep-ph/9302210; Morrissey, Ramsey-Musolf, arXiv:1206.2942

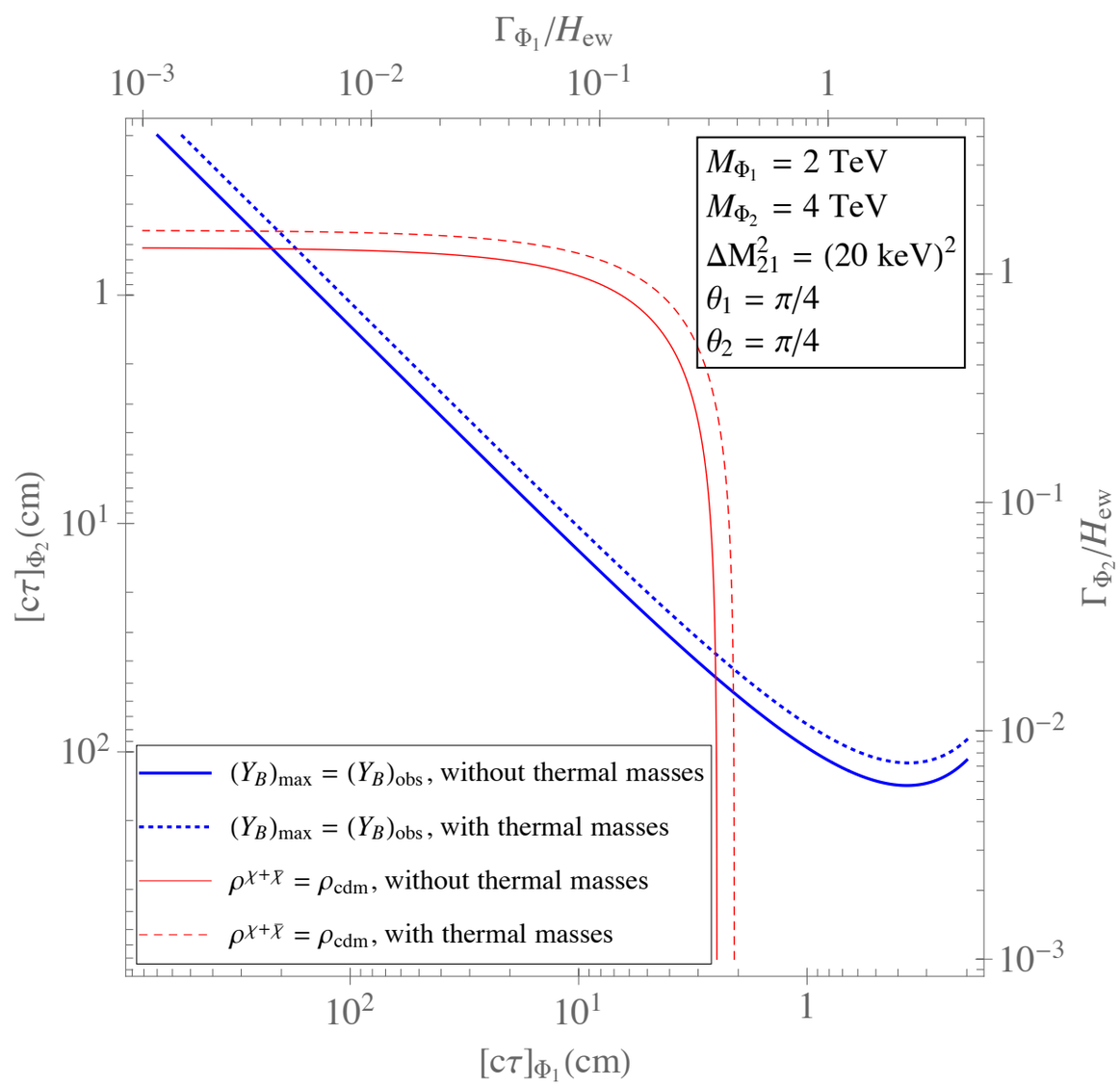
- Freeze-in leptogenesis

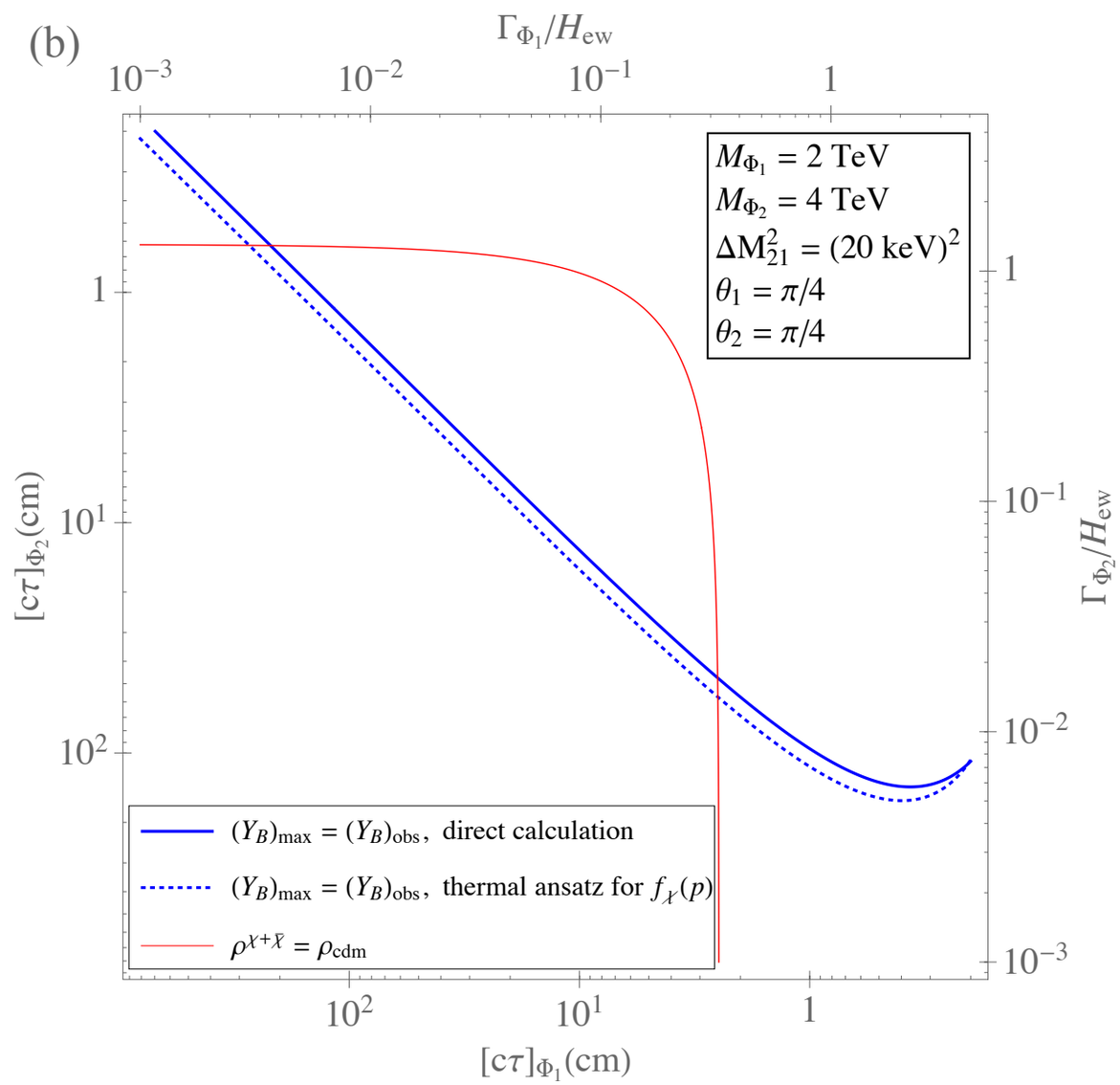
  Akhmedov, Rubakov, Smirnov, hep-ph/9803255; Asaka, Shaposhnikov, hep-ph/0505013; Drewes *et al.*, arXiv:1711.02862

- B meson oscillation baryogenesis

  McKeen, Nelson, arXiv:1512.05359; Ipek, March-Russell, arXiv:1604.00009; Elor, Escudero, Nelson, arXiv:1810.00880

CROSS-CHECKS





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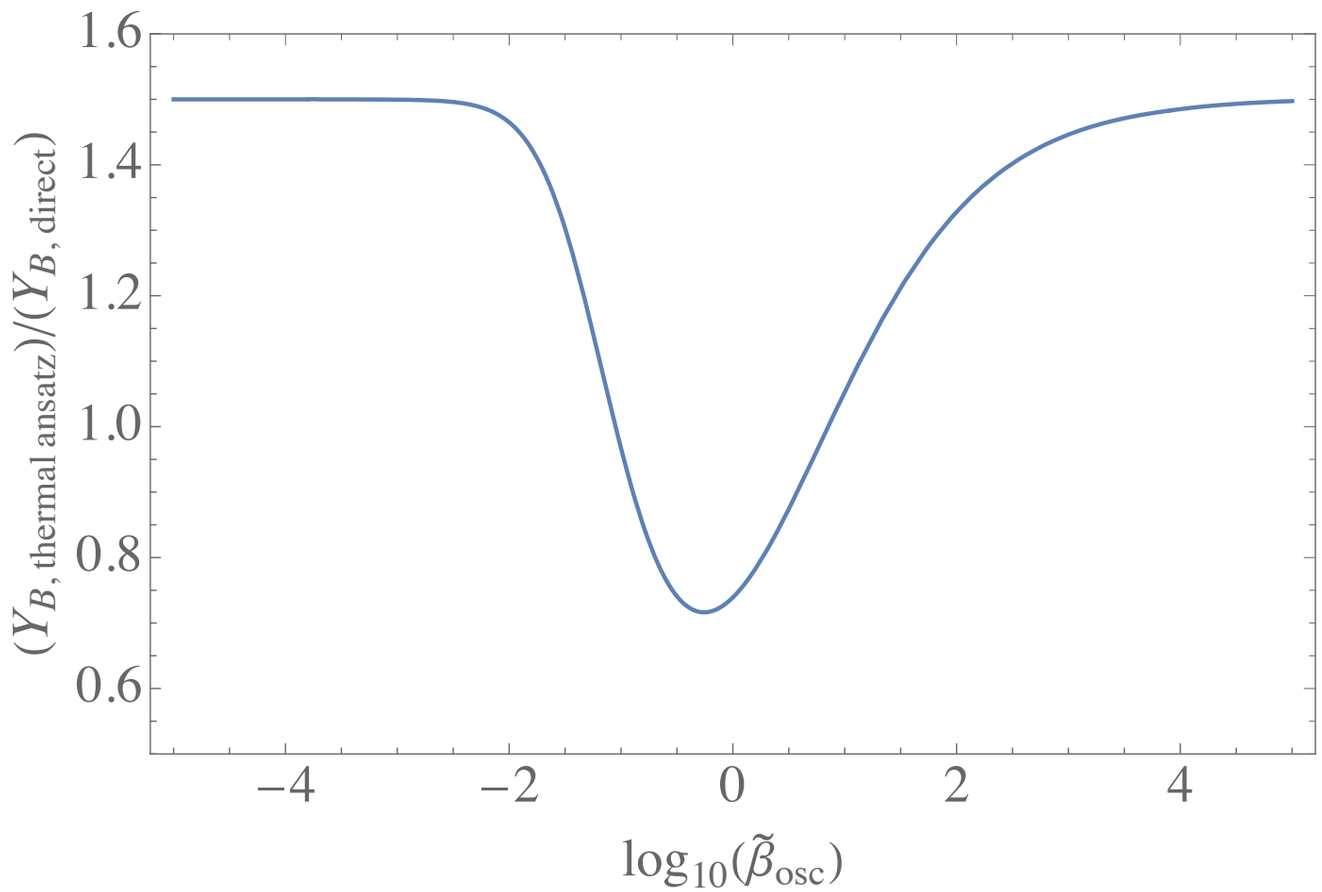

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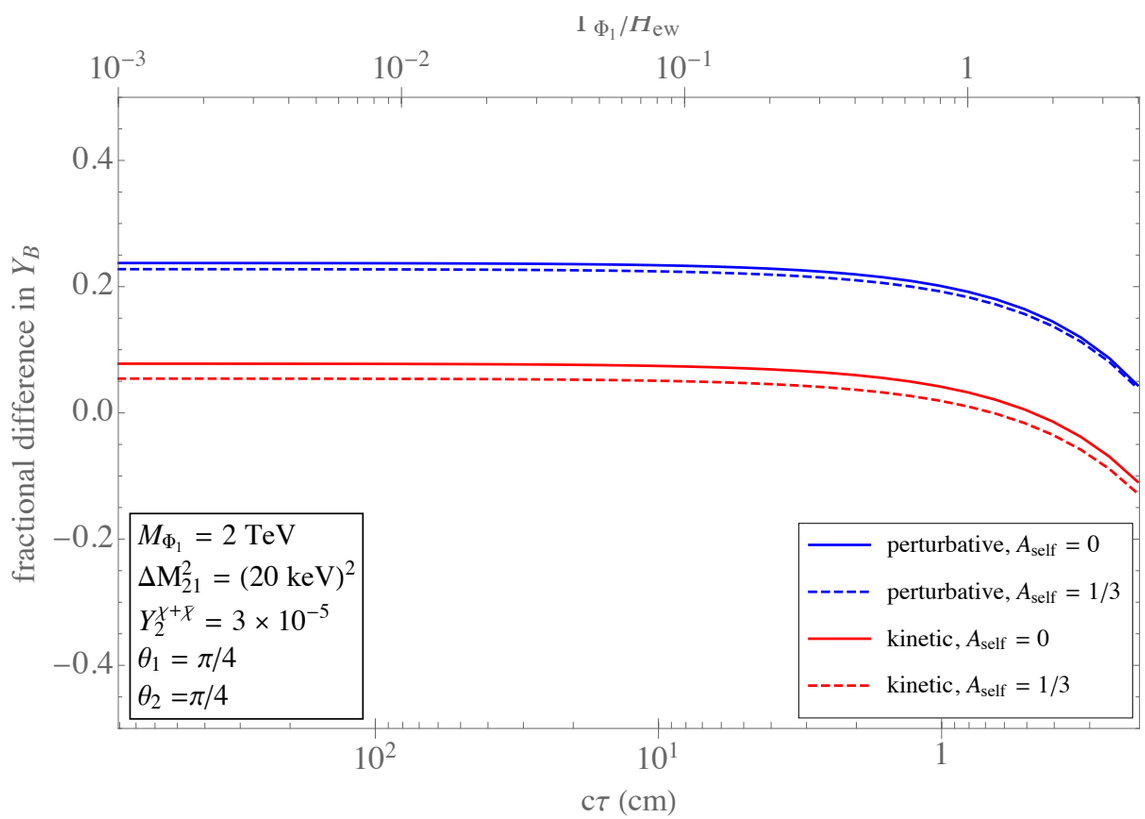
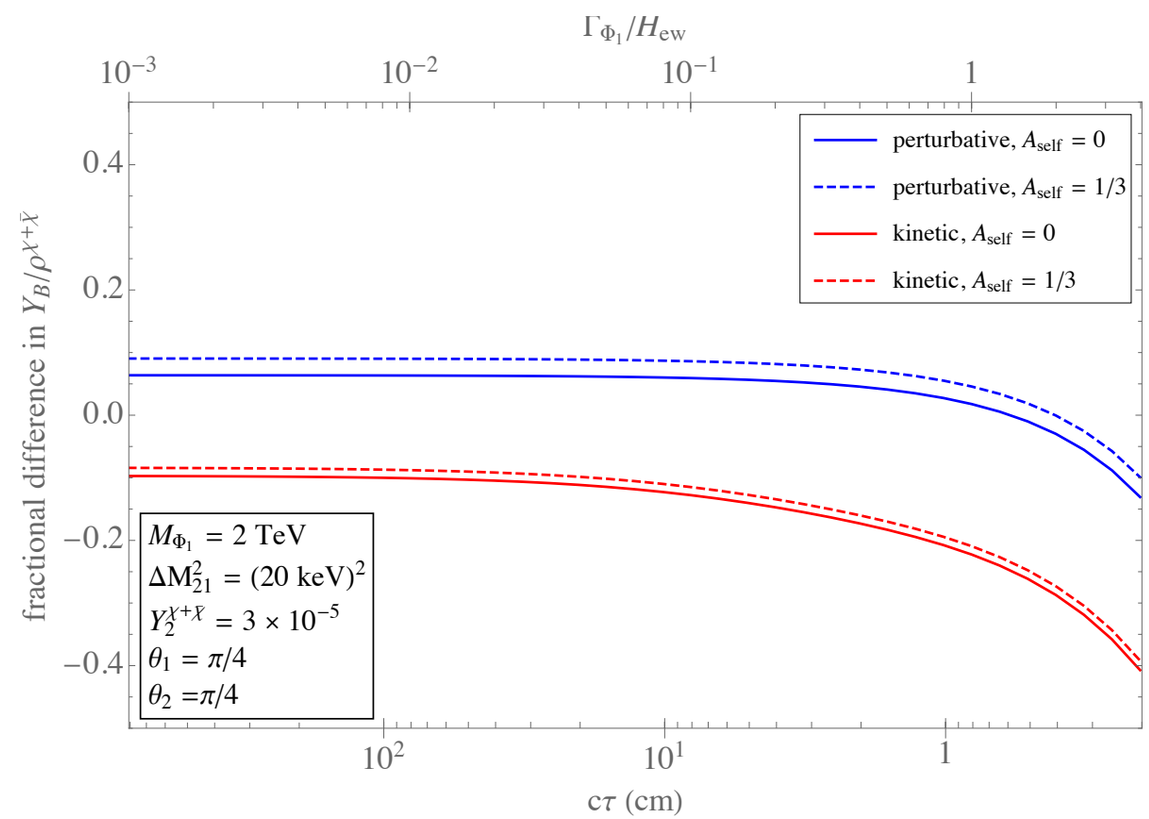




ANALYTIC RESULTS

$$Y_B = \frac{45 g_\Phi^2}{256 g_* \pi^6} \frac{\mathcal{K}_B}{\mathcal{K}_\Phi} \frac{M_{\Phi_1}^2 M_{\Phi_2}^2 M_0^2}{T_{\rm ew}^6}$$
$$\times \, {\rm Im}\left(F_1^{1\,*} F_1^2 F_2^{2\,*} F_2^1 \right) \left(I_{12} - I_{21} \right),$$

$$I_{ij} = \int_0^\infty dy \, \frac{e^{-y}}{y^2} \int_0^1 dz_1 \, z_1^2 \, S_{\Phi_i}(z_1) \, e^{-\alpha_i z_1^2/y}$$
$$\int_0^{z_1} dz_2 \, z_2^2 \, e^{-\alpha_j z_2^2/y} \sin\left[\frac{\beta_{\rm osc}}{y} (z_1^3 - z_2^3) \right]$$

$$S_{\Phi_i}(z) = \exp\left(-\frac{\Gamma_{\Phi_i}}{H_{\rm ew}} \int_z^1 dz' \, z' \, \frac{\mathcal{K}_1\left(\frac{M_{\Phi_i}}{T_{\rm ew}} z' \right)}{\mathcal{K}_2\left(\frac{M_{\Phi_i}}{T_{\rm ew}} z' \right)} \right)$$

$$\alpha_i \; = \; (M_{\Phi_i}/2T_{\rm ew})^2 \qquad\qquad \beta_{\rm osc} \; = \; M_0 \Delta M_{21}^2 / 6 T_{\rm ew}^3$$

MIXING PARAMETERS

$$\cos\theta_i = \sqrt{\frac{({F^i}^\dagger F^i)_{11}}{\mathrm{Tr}({F^i}^\dagger F^i)}},$$

$$\cos\rho_i = \frac{|({F^i}^\dagger F^i)_{12}|}{\sqrt{({F^i}^\dagger F^i)_{11}({F^i}^\dagger F^i)_{22}}}$$

$$\phi_i = \arg({F^i}^\dagger F^i)_{12}.$$

$$4\,\mathrm{Im}\left({F_1^1}^* F_1^2 {F_2^2}^* F_2^1\right) = \mathcal{J}\,\mathrm{Tr}\left[{F^1}^\dagger F^1\right]\mathrm{Tr}\left[{F^2}^\dagger F^2\right]$$

$$\mathcal{J} = \sin 2\theta_1 \sin 2\theta_2 \cos\rho_1 \cos\rho_2 \sin(\phi_1 - \phi_2)$$

KINETIC EQUATIONS

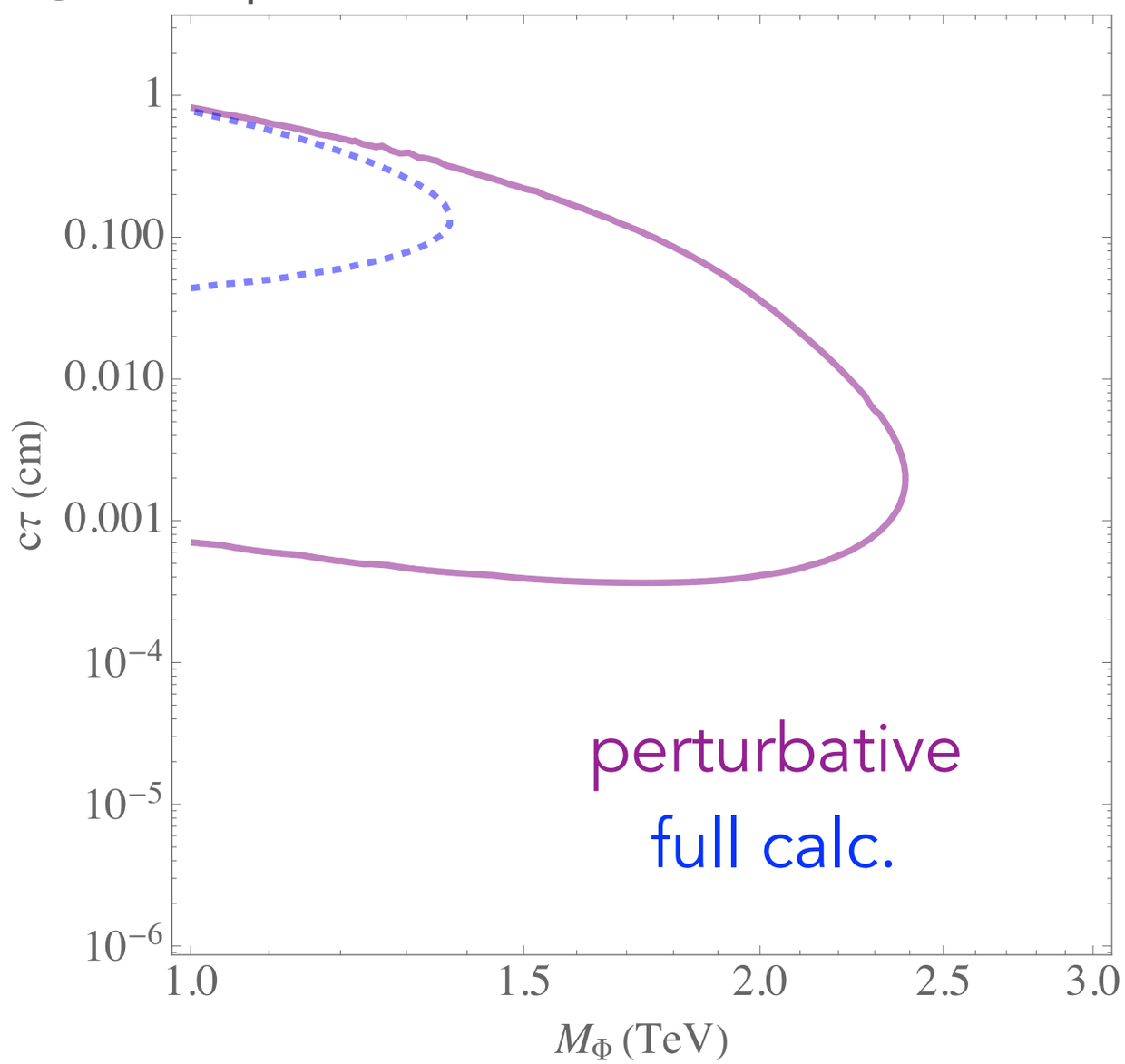
$$\frac{d\,Y^{\chi}_{IJ}}{d\ln z} = \sum_i \Bigg(-\frac{1}{2}\left\{\tilde{\gamma}_{0,i}\ ,\ Y^{\chi} - Y^{\chi}_{eq}\right\}$$

$$+\frac{\delta Y^{Q}}{2Y^{Q}_{eq}}\left[\tilde{\gamma}_{Q1,i}Y^{\chi}_{eq} + \frac{1}{2}\left\{\tilde{\gamma}_{Q2,i}\ , Y^{\chi}\right\}\right]$$

$$+\mathcal{G}\left(-\frac{\delta Y^{\Phi_i}}{2Y^{\Phi_i}_{eq}}\right)\left[\tilde{\gamma}_{\Phi 1,i}Y^{\chi}_{eq} - \frac{1}{2}\left\{\tilde{\gamma}_{\Phi 2,i}\ , Y^{\chi}\right\}\right]\Bigg)_{IJ}$$

NO-TOP SINGLE SCALAR

- Relax DM assumption, assume all 3 chi states decay to massless radiation through unspecified mechanism

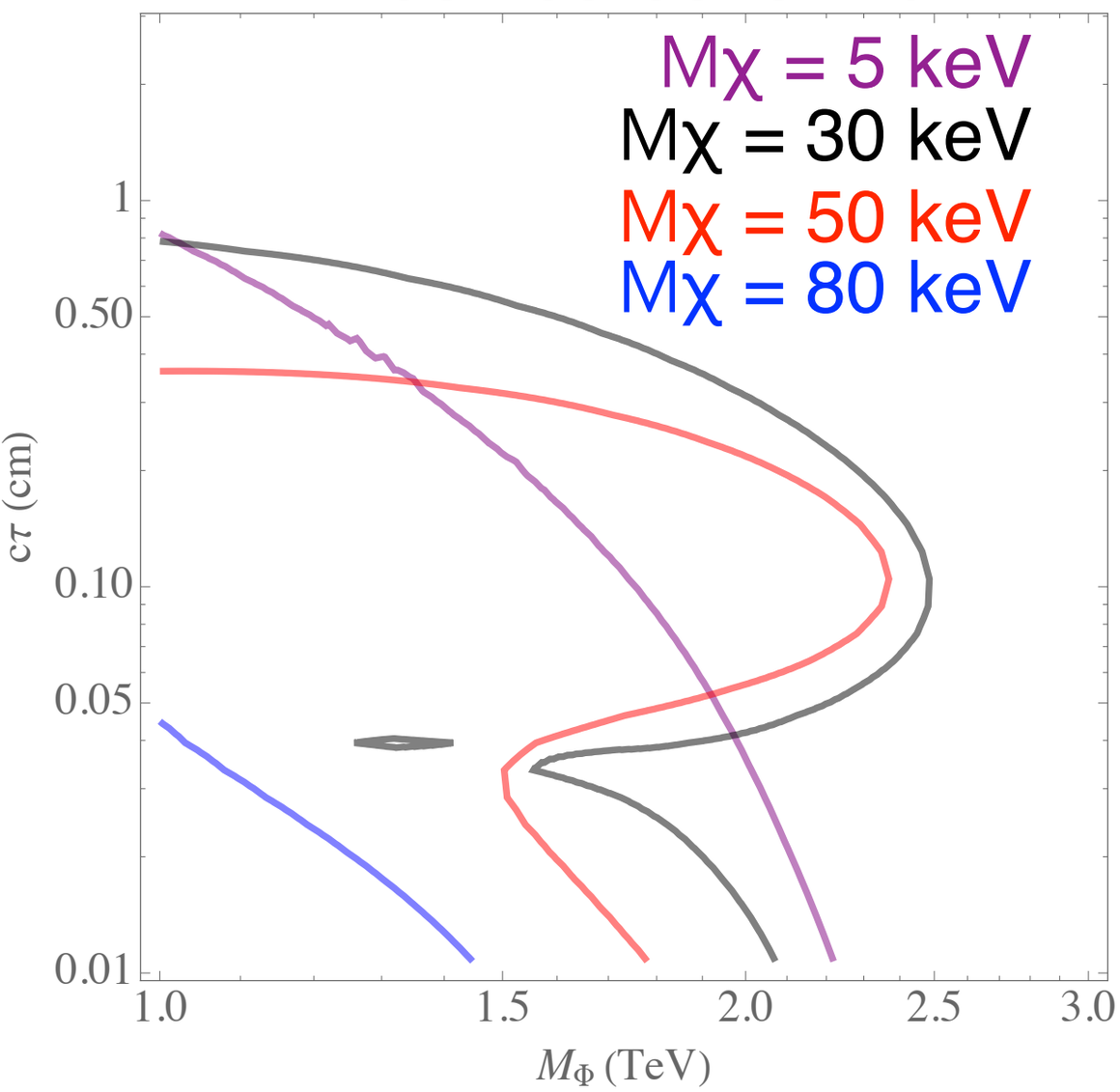


NO-TOP SINGLE SCALAR

- Relax DM assumption, assume all 3 chi states decay to massless radiation through unspecified mechanism

$$M_{\chi_3} = 2M_{\chi_2}$$

modified benchmark



$$c\tau_\Phi < 0.01 \text{ cm}$$