Effet of losses in interacting quantum gases : ultra-violet divergence and its regularization

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Open quantum systems

Quantum systems coupled to environements :

- Exchange of energy $\Rightarrow$ heat transfer
- Exchange of particles
  - Transport phenomena
  - Case of loss process
    - Zeno effect $\rightarrow$ strongly correlated gases
      F. Verstraete et al., Nature Physics 5, 633 (2009)
      M. Roncaglia et al., Phys. Rev. Lett. 104, 096803 (2010)
      N. Syassen et al., Science 320, 1329 (2008)

Effect of losses : still a lot to discover

Losses in a correlated system : a field at its begining
Here : one-body losses, gas with contact interactions

Outline

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1. UV catastrophy for Lindblad dynamics of a gas with contact interactions
2. Regularization for the 2-body problem
3. Many-body system : role of contact
4. Case of a BEC

Reservoir of vanishing correlation time : Lindblad equation

- System : density matrix ρ

Lindblad equation : used in all previous works in cold atoms domain

Lindblad equation for uniform 1-body losses

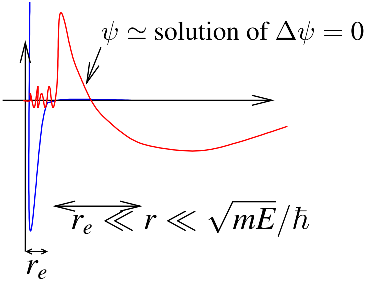
- Reservoir of correlation time $\ll$ system's dynamics time

$$\frac{d\rho}{dt} = -i[H,\rho] + \Gamma \int d^d\mathbf{r} \left(-\frac{1}{2}\{\psi_{\mathbf{r}}^{+}\psi_{\mathbf{r}},\rho\} + \psi_{\mathbf{r}}\rho\psi_{\mathbf{r}}^{+} \right)$$

- **Universal behavior** : single parameter Γ
- Effect **trivial if uncorrelated system** $\rho = \prod_a \rho_a$:
  $dN_a/dt = -\Gamma N_a$
- If quantum correlations present between atoms ?
  $\Rightarrow$ Difficult problem

Interactions between atoms $\Rightarrow$ correlations

Interactions between atoms : contact interaction



Potential $\Rightarrow$ boundary condition for $r \to 0$

For $r_e \ll r \ll \sqrt{mE}/\hbar$

$\psi \propto 1/r - 1/a$ (3D)

$\psi \propto \ln(r/a)$ (2D)

$\psi \propto 1 - r/a$ (1D)

Single parameter : scattering length a

Universal behavior

Pseudo-potential : $V = \delta^d(\mathbf{r})U$

$1/p^4$ momentum tail tail $\Rightarrow$ UV divergence of E_{kin}

For $p^2/m \gg E$, $\boxed{|\psi_p|^2 = \frac{m}{L^d}\,(U\psi)^2_{r=0}\,\frac{1}{p^4} = \frac{\alpha}{\mathbf{p}^4}}$

$E_{\text{kin}} \propto \int d^d\mathbf{p}\,\frac{p^2}{2m}|\psi_p|^2 \Rightarrow E_{\text{kin}} = \infty$ for $D > 1$

compensated by diverging interaction energy (E_{tot} finite)

UV divergence of the energy deposited by a loss event

Lindblad : loss event instantaneous

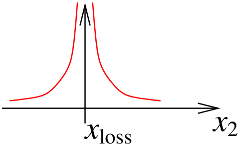
$\Rightarrow |\psi\rangle \to |\psi'\rangle = \psi_{\mathbf{r}_{\text{loss}}}|\psi\rangle \Rightarrow \psi'(\mathbf{r}_2, \mathbf{r}_3, \dots) \propto \psi(\mathbf{r}_1 = \mathbf{r}_{\text{loss}}, \mathbf{r}_2, \mathbf{r}_3, \dots)$

Singularity at loss event position :

$\Rightarrow$ momentum tails decreasing as $1/p^4$

$\Rightarrow E_{\text{kin}} = \infty$

No compensation by interaction term



Energy diverges in $D > 1$ after a loss event

UV catastrophe for Lindblad dynamics with contact interactions

$$dE/dt = \infty$$

Regularisation

- Finite interaction range
- Finite correlation time of the reservoir $\Rightarrow$ finite energy width

Outline

Homogeneous 1-body losses

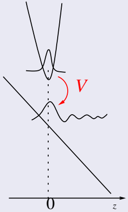
- 1-body loss mechanism : single-atom coupling to environement
- Homogeneous loss process : **undependent on r**

$\Rightarrow$ For each **r**, coupling to a continuum

Reduced dimension case

Frozen dimension(s) :

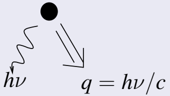
can serves as a continuum



3D case

continuum provided by extra particle

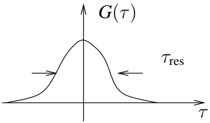
Desexcitation



No position-dependence $\rightarrow$ conservation of momentum

Homogeneous 1-body losses

Invariance by galilean transformation $\Rightarrow$ loss undependant on $\mathbf{p}$

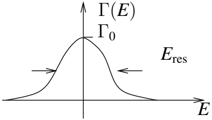


Model of losses

For each $\mathbf{p}$, continuum (label i), $E^{(\text{res})}_{\mathbf{p},i} = p^2/(2m) + E_i$

$$V = \sum_{\mathbf{p}} \sum_i V_i \Psi_{\mathbf{p}} B^+_{\mathbf{p},i} + h.c.$$

- Correlation function : $\boxed{G(\tau) = \sum_i |V_i|^2 e^{iE_i\tau/\hbar}}$
- Spectral density : $\Gamma(E) = \frac{1}{\hbar^2} \int d\tau e^{-iE\tau/\hbar} G(\tau)$
  $\Gamma(E) = \frac{2\pi}{\hbar} n(E) |V_i|^2$



Asumption made in Lindblad : $\begin{cases} \tau_{res} \to 0 (E_{res} \to \infty) \\ \Gamma(E) = \Gamma_0 \text{ for all } E \end{cases}$

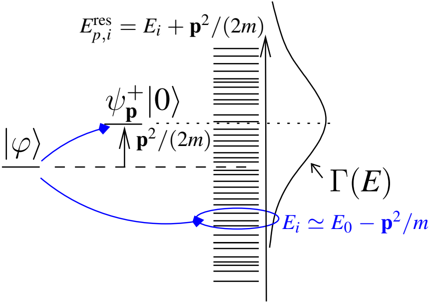
E_{res} : energy width of the reservoir

2-atoms phycis (For simplicity, consider Bosons)

Initial state :

$|\varphi\rangle = (1/2)\sum_{\mathbf{p}} \varphi(\mathbf{p})\Psi^+_{\mathbf{p}}\Psi^+_{-\mathbf{p}}|0\rangle$

$\langle \mathbf{p}, i|V|\varphi\rangle = \varphi(\mathbf{p})V_i$



Rate of events with final momentum of the remaining atom p

$$\gamma_{\mathbf{p}} = |\varphi(\mathbf{p})|^2\Gamma(E_0 - p^2/m)$$

Energy increase rate

$\frac{dE}{dt} = \sum_{\mathbf{p}} \gamma(\mathbf{p})\frac{|\mathbf{p}|^2}{2m}$ Contact interaction $\Rightarrow |\varphi(p)|^2 \underset{p>p_0}{\simeq} \frac{\alpha}{p^4}$

$$\left.\frac{dE}{dt}\right)_{p>p_0} = \alpha\Gamma_0\mathcal{B} \quad \text{with} \quad \mathcal{B} \simeq \begin{cases} 1/p_0 & \text{in 1D} \\ \ln(\sqrt{mE_{\rm res}}/p_0) & \text{in 2D} \\ \sqrt{mE_{\rm res}} & \text{in 3D} \end{cases}$$

Outline

Generalization to many-body systems

Role of the contact

- Contact C : quantifies the **number of pairs** in the system
- **Amplitude of $1/p^4$ momentum tails** :$C = \lim_{p\to\infty} p^4 n(p)$
- From 2-body to many-bodies : $\alpha \Rightarrow$ Contact paramater C

Energy increase rate for large E_{res}

$$dE/dt = \Gamma_0 \frac{C}{m} \mathcal{B} \qquad \text{with} \qquad \mathcal{B} \simeq \sqrt{mE_{\text{res}}} \qquad (3D)$$

$$dE/dt \to \infty \text{ when } E_{\text{res}} \to \infty \quad (\tau_{\text{res}} \to 0)$$

Outline

Case of a Bose-Einstein condensate

Macroscopic occupation of the ground state $\mathbf{p} = 0$.
Interactions between atoms $\rightarrow$ Bogoliubov tranformation :

$$H_{BG} = e_0 L^d + \sum_{\mathbf{p} \neq \mathbf{0}} \epsilon_p a_{\mathbf{p}}^+ a_{\mathbf{p}} \text{ , with } \begin{cases} \Psi_{\mathbf{p}} = u_p a_{\mathbf{p}} + v_p a_{-\mathbf{p}}^+ \\ \Psi_{-\mathbf{p}}^+ = v_p a_{\mathbf{p}} + u_p a_{-\mathbf{p}}^+ \end{cases}$$

Bogoliubov spectrum : $\epsilon_p = \sqrt{p^2/(2m)(p^2/(2m) + 2gn)}$
$u_p^2 - v_p^2 = 1$

Quantum deplection : $1/p^4$ tails recovered

$$\langle \Psi_{\mathbf{p}}^+ \Psi_{\mathbf{p}} \rangle = \sum_{\mathbf{p}} \{ (u_p^2 + v_p^2) \langle a_p^+ a_p \rangle + v_p^2 \} \qquad (\langle a_p a_{-p} \rangle = 0)$$

Large p behavior : $\langle a_p^+ a_p \rangle \simeq 0$ and $\boxed{v_p^2 \simeq (mgn)^2/\mathrm{p}^4}$

Contact : $C = L^d (mgn)^2 / (2\pi\hbar)^d$

Master equation under effect of losses

Coupling to the reservoir : $V = \sum_{\mathbf{p},\mathbf{i}} V_i B^+_{\mathbf{p}} (u_p a_{\mathbf{p}} + v_p a^+_{-\mathbf{p}}) + \ h.c.$

Master equation for Bogoliubov quasi-particles

- Second order in V
- Rotating wave approximation
- Born-Markov approximation

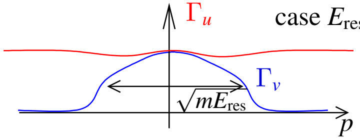
$$\frac{d\hat{\rho}}{dt} = \ -(i/\hbar)[H_0,\hat{\rho}] - \sum_{\mathbf{p}} \ \ \{\Gamma_u(p) u_p^2 \left(\tfrac{1}{2}\{a^+_{\mathbf{p}} a_{\mathbf{p}}, \rho\} - a_{\mathbf{p}} \rho a^+_{\mathbf{p}}\right)$$

$$+\Gamma_v(p) v_p^2 \left(\tfrac{1}{2}\{a_{\mathbf{p}} a^+_{\mathbf{p}}, \rho\} - a^+_{\mathbf{p}} \rho a_{\mathbf{p}}\right)\}$$

$\Gamma_u(p) = \Gamma(gn - (\frac{p^2}{2m} - \epsilon_p))$

$\Gamma_v(p) = \Gamma(gn - (\frac{p^2}{2m} + \epsilon_p))$

$\epsilon_p \underset{p\to\infty}{\simeq} p^2/(2m)$



Evolution of Bogoliubov mode's population

$$\frac{d\langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle}{dt} = -\Gamma_u(p) u_p^2 \langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle + \Gamma_v(p) v_p^2 \left(1 + \langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle\right)$$

- **Non interacting atoms :** $v_p = 0, u_p = 1, \Gamma_u = \Gamma_0$
  $\Rightarrow \boxed{d\langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle/dt = -\Gamma_0 \langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle}$ as expected
- **Reservoir of infinite energy width :** $\Gamma_u = \Gamma_v = \Gamma_0 \ \forall p$
  $\Rightarrow \boxed{d\langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle/dt = -\Gamma_0 \langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle + \Gamma_0 v_p^2}$ $\quad (u_v^2 - v_p^2 = 1)$
  $\Rightarrow$ UV divergence of dE/dt ($\epsilon_p \simeq p^2/(2m)$ and $v_p \propto 1/p^4$)

Evolution of energy in the system

$E = e_0 L^d + \sum_{\mathbf{p}\neq \mathbf{0}} \epsilon_p \langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle$

$$\Rightarrow \frac{dE}{dt} = A\frac{dn}{dt} + \sum_{\mathbf{p}} \epsilon_p \frac{d\langle a_{\mathbf{p}}^+ a_{\mathbf{p}}\rangle}{dt} \quad \text{with} \quad \frac{dn}{dt} \simeq -\Gamma_0 n$$

Evolution of the gas temperature

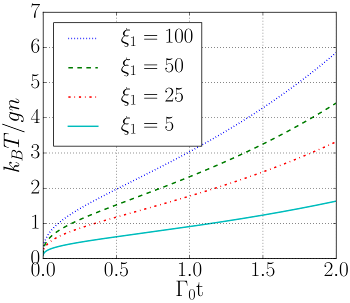
Relaxation towards thermal equilibrium

2D,3D Bose gases : cahotic systems

$\Rightarrow$ **Relaxation towards thermal equilibrium**

slow losses $\rightarrow$ at any time, thermal equilibrium at temperature $T(t)$

$T(t)$ computed using thermal equation of state and dE/dt



$$\Gamma(E) = \Gamma_0 e^{-E^2/(2E_{\text{res}})}$$

$$\xi_1 = \sqrt{E_{\text{res}}/(gn_0)}$$

Conclusion

This result (To appear in PRA)

- Two famous universal behaviors :
  - Lindblad losses (Γ_0)
  - contact interactions (a)
- UV divergence $\Rightarrow$ Γ_0 and a not sufficient to describe the physics
- Regularization with finite energy width of the reservoir
- Analytical results for BEC : $T(t)$ computed

Other results : 1D case

- No UV divergence : Lindblad and contact intercations OK
- Integrable system $\Rightarrow$ not thermalisation
- Losses produce non thermal state

I. Bouchoule et al., Scipost Phys. **9**,44 (2020), I. Bouchoule and J. Dubail, Phys. Rev. Lett. **126**, 160603 (2021), M. Schemmer and I. Bouchoule, Phys. Rev. Lett. **121**, 200401 (2018), I. Bouchoule and M. Schemmer, SciPost Physics **8**, 060 (2020).