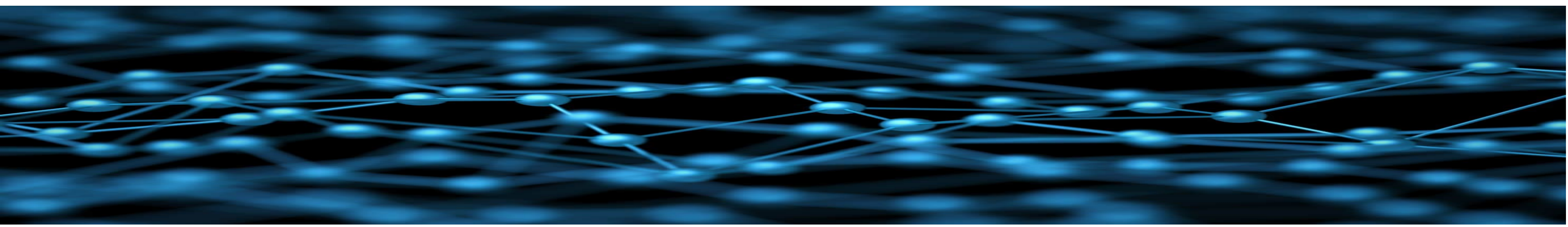
Probing topogical defect formation in a quantum annealer

Adolfo del Campo



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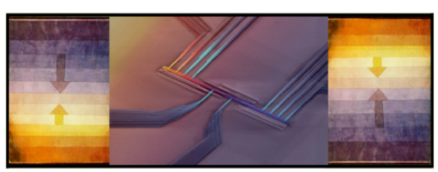


UC SANTA BARBARA
Kavli Institute for Theoretical Physics

Transport and Efficient Energy Conversion in Quantum Systems

Coordinators: Jean-Phillipe Brantut, Giulio Casati, Jorge Kurchan, and Heiner Linke

Testing a black box

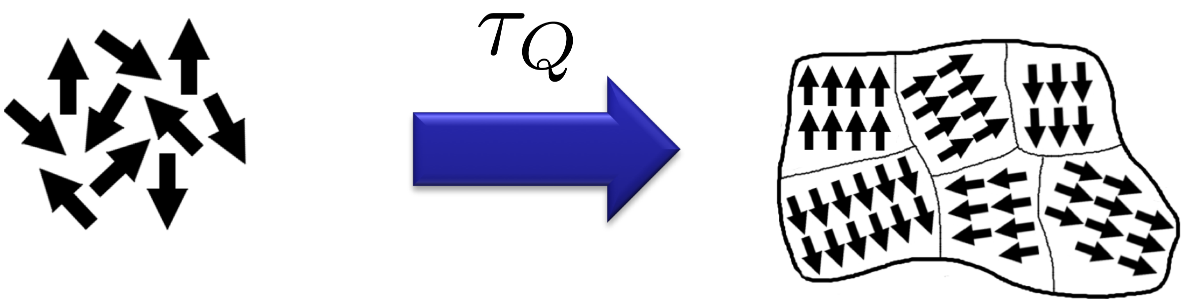
Probing the universality of topological defect formation in a quantum annealer: Kibble-Zurek mechanism and beyond

Yuki Bando, Yuki Susa, Hiroki Oshiyama, Naokazu Shibata, Masayuki Ohzeki, Fernando Javier Gómez-Ruiz, Daniel A. Lidar, Sei Suzuki, Adolfo del Campo, and Hidetoshi Nishimori
Phys. Rev. Research **2**, 033369 – Published 8 September 2020

Idea

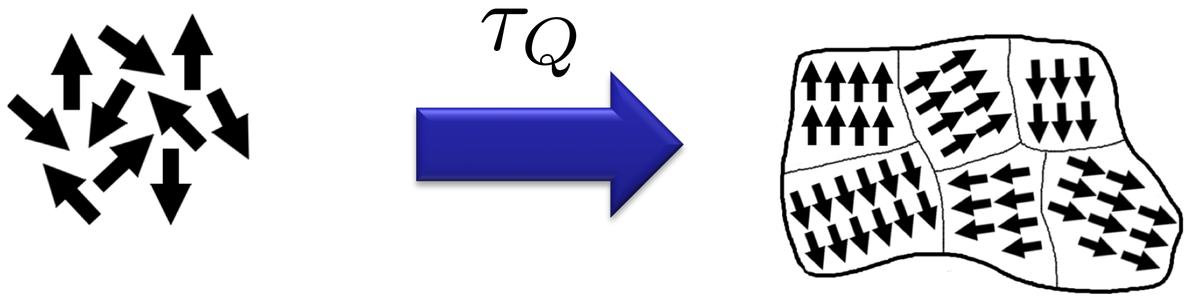
- Phase Transitions Dynamics & Kink Formation
- Tests in a Quantum Annealer

Dynamics of a phase transition



Spontaneous symmetry breaking leads to domain formation

Kibble-Zurek Mechanism



Domain size: **universal** power law scaling with quench time

$$\hat{\xi} = \xi(\hat{t}) = \xi_0 \left(\frac{\tau_Q}{\tau_0}\right)^{\frac{\nu}{1+z\nu}}$$

T. W. B. Kibble, JPA 9, 1387 (1976)
T. W. B. Kibble, Phys. Rep. 67, 183 (1980)
W. H. Zurek, Nature (London) 317, 505 (1985)
W. H. Zurek, Acta Phys. Pol. B. 1301 (1993)

KZM in quantum systems: Ising chain

Ising chain is equivalent to ensemble of two-level systems

$$\mathcal{H} = -J\sum_{m=1}^{N}(\sigma_m^z\sigma_{m+1}^z + g\sigma_m^x) = \sum_k E_k(\gamma_k^\dagger\gamma_k - 1/2)$$

Kink number operator: number of kinks = number of excited two-level systems

$$\hat{\mathcal{N}} = \frac{1}{2}\sum_{m=1}^{N}(1-\sigma_m^z\sigma_{m+1}^z) = \sum_k \gamma_k^\dagger\gamma_k$$

Sweep from paramagnet to ferromagnet: mean kink number

$$g(t) = -\frac{t}{\tau_Q} \qquad \langle n\rangle = Nd = N\frac{1}{2\pi}\sqrt{\frac{\hbar}{2J\tau_Q}}$$

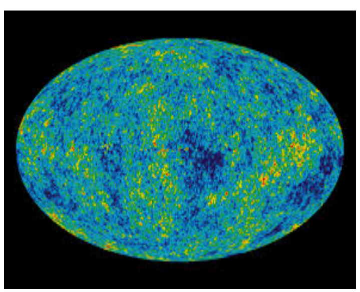
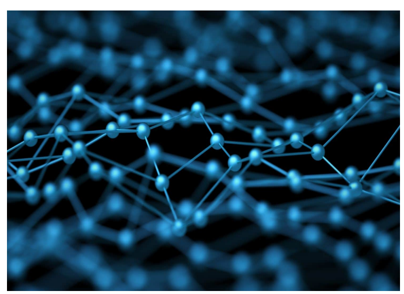
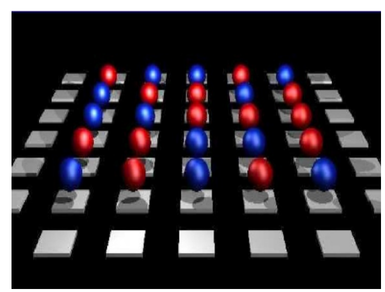
2005
Polkovnikov
Damski
Dziarmaga
Zurek, Dorner, Zoller

Universality of Kibble-Zurek Mechanism

The KZM is broadly applicable

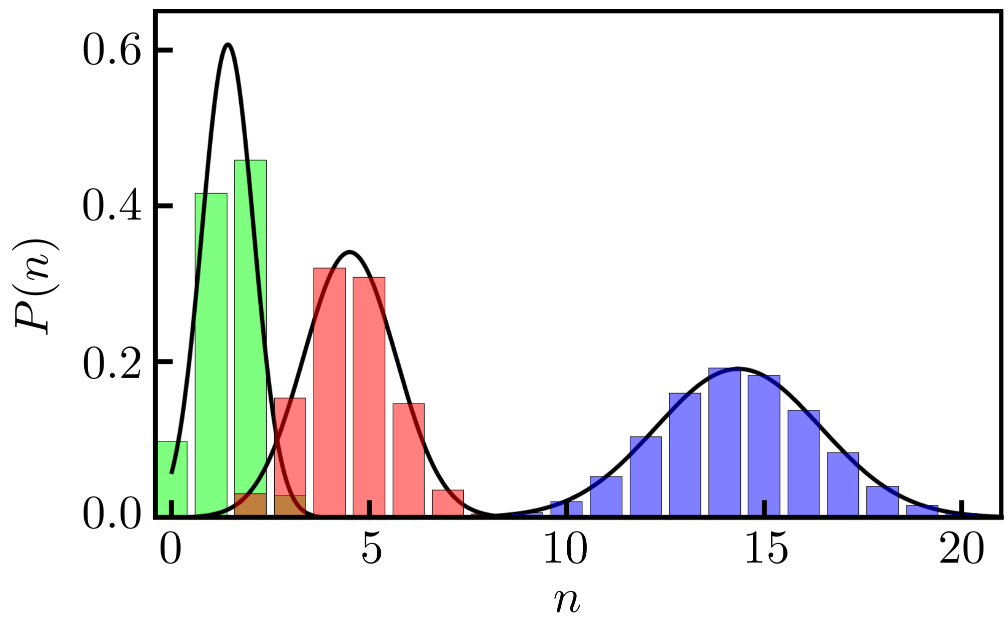
Tested numerically in integrable and nonintegrable models

Many experiments consistent with KZM


AdC & Zurek, Int. J. Mod. Phys. A 29, 1430018 (2014)

Beyond the Kibble-Zurek mechanism



AdC, PRL **121**, 200601 (2018) (theory)

J.M. Cui et al. Commun. Phys. 3, 44 (2020) (exp)

Beyond KZM: Full counting statistics of defects

Ising chain is equivalent to ensemble of two-level systems

$$\mathcal{H} = -J\sum_{m=1}^{N}(\sigma_m^z\sigma_{m+1}^z + g\sigma_m^x) = \sum_k E_k(\gamma_k^\dagger\gamma_k - 1/2)$$

Kink number operator: number of kinks = number of excited two-level systems

$$\hat{\mathcal{N}} = \frac{1}{2}\sum_{m=1}^{N}(1-\sigma_m^z\sigma_{m+1}^z) = \sum_k \gamma_k^\dagger\gamma_k$$

Probability to have "n" kinks

$$P(n) = \left\langle \delta[\hat{\mathcal{N}} - n] \right\rangle$$

L. Cincio, J. Dziarmaga, M. M. Rams, W. H. Zurek, PRA 75, 052321 (2007); AdC, PRL **121**, 200601 (2018)

Beyond KZM: Full counting statistics of defects

Kink number distribution and the characteristic function

$$P(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta \widetilde{P}(\theta) e^{-i\theta n}$$

Exact characteristic function

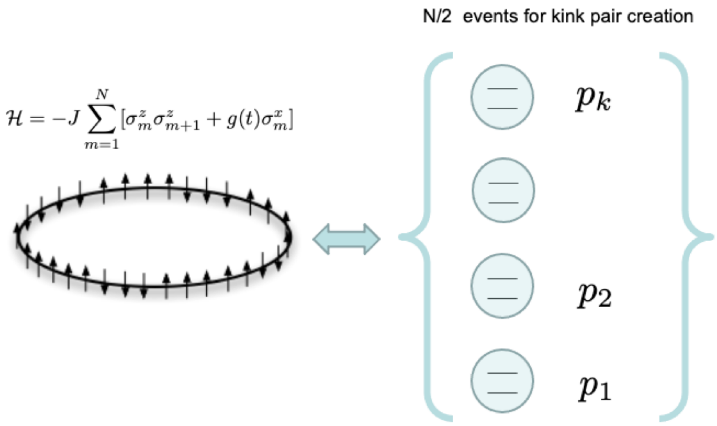
$$\widetilde{P}(\theta) = \left\langle e^{i\theta\hat{\mathcal{N}}} \right\rangle = \prod_k \left(1 - p_k + p_k e^{i\theta}\right)$$

Probabilities from Landau-Zener formula

$$p_k = \langle \gamma_k^\dagger \gamma_k \rangle = \exp\left(-\frac{1}{\hbar} 2\pi J \tau_Q k^2\right)$$

AdC, PRL **121**, 200601 (2018)

Beyond KZM: Full counting statistics of defects



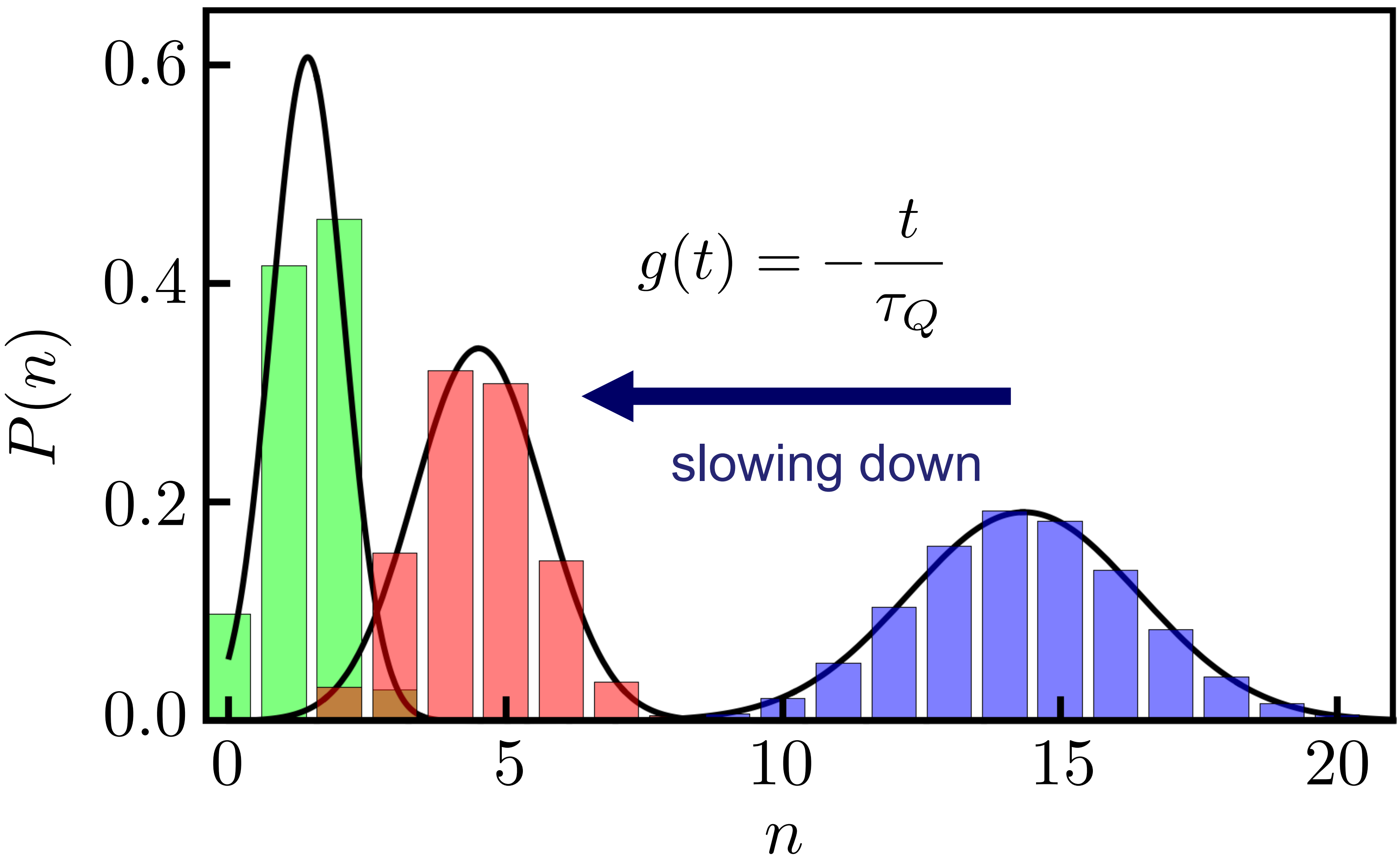
Kink number distribution = **Poisson binomial distribution**

- Ising chain = independent modes as two-level systems (TLS)
- Each TLS = discrete random variable yes/no=excited/ground
- Assign success probabilities from Landau-Zener formula

$$\widetilde{P}(\theta) = \left\langle e^{i\theta\hat{\mathcal{N}}} \right\rangle = \prod_k \left(1 - p_k + p_k e^{i\theta}\right) \qquad p_k = \exp\left(-\frac{1}{\hbar} 2\pi J \tau_Q k^2\right)$$

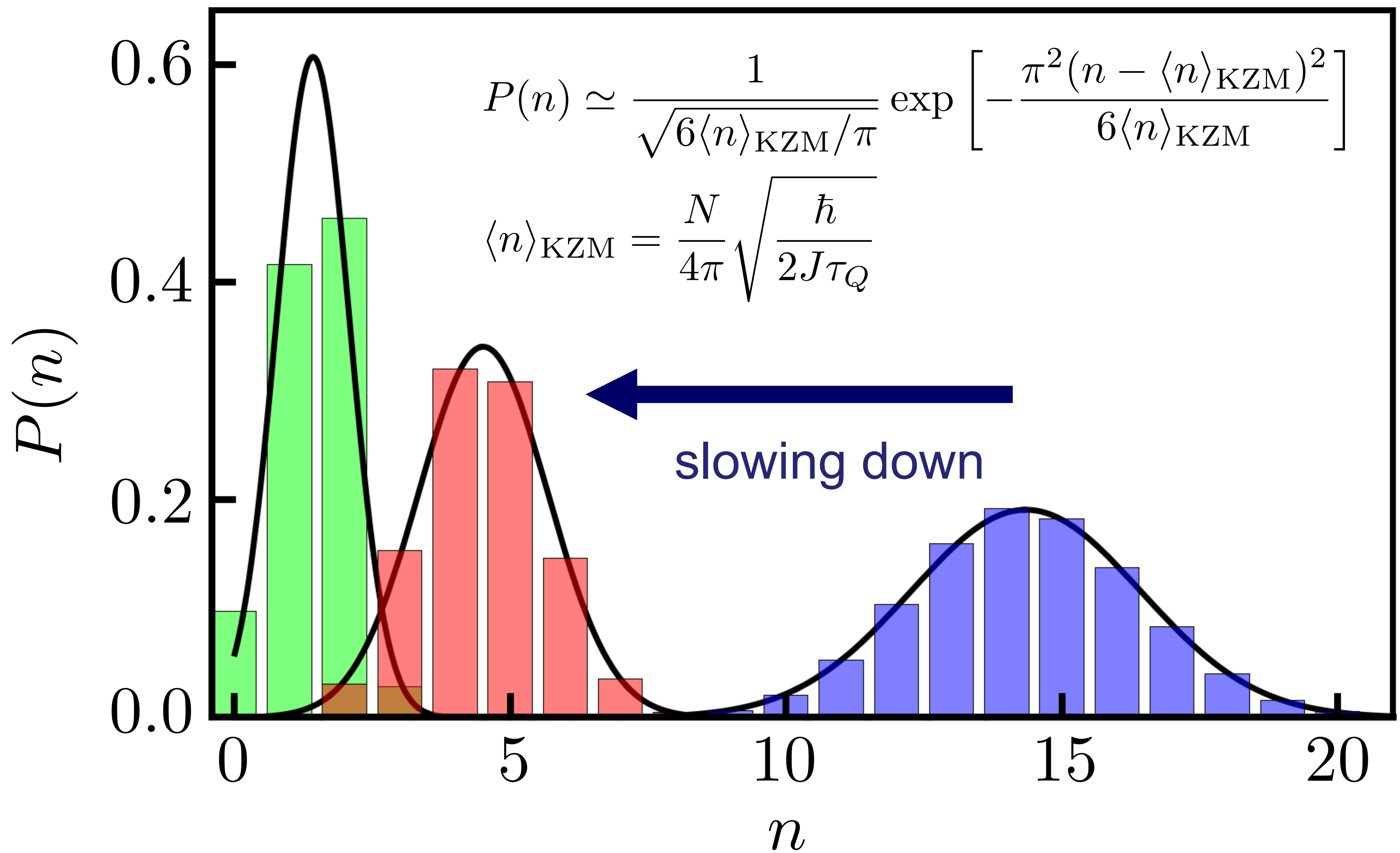
AdC, PRL **121**, 200601 (2018)

Beyond KZM: Full counting statistics of defects



AdC, PRL **121**, 200601 (2018)

Beyond KZM: Full counting statistics of defects



AdC, PRL **121**, 200601 (2018)

Beyond KZM: Full counting statistics of defects

Cumulant generating function

$$\log \widetilde{P}(\theta) = \sum_{q=1}^{\infty} \frac{(i\theta)^q}{q!} \kappa_q$$

Exact expression in the continuum limit

$$\log \widetilde{P}(\theta) = \frac{N}{2\pi} \int_0^{\pi} dk \log \left[1 + \left(e^{i\theta} - 1 \right) \exp\left(-\frac{1}{\hbar} 2\pi J \tau_Q k^2 \right) \right]$$

For slow quenches

$$\log \widetilde{P}(\theta) = -\langle n \rangle_{\rm KZM} \, {\rm Li}_{3/2}(1 - e^{i\theta}) \qquad {\rm Li}_{3/2}(x) = \sum_{p=1}^{\infty} x^p / p^{3/2}$$

AdC, PRL **121**, 200601 (2018)

Beyond KZM: Universal scaling of cumulants

First cumulant (pdf for number of kink pairs)

$$\kappa_1 = \langle n \rangle = \langle n \rangle_{\rm KZM} = \frac{N}{4\pi}\sqrt{\frac{\hbar}{2J\tau_Q}}$$

Second

$$\kappa_2 = \langle n^2 \rangle - \langle n \rangle^2 = \frac{2-\sqrt{2}}{2}\langle n \rangle_{\rm KZM}$$

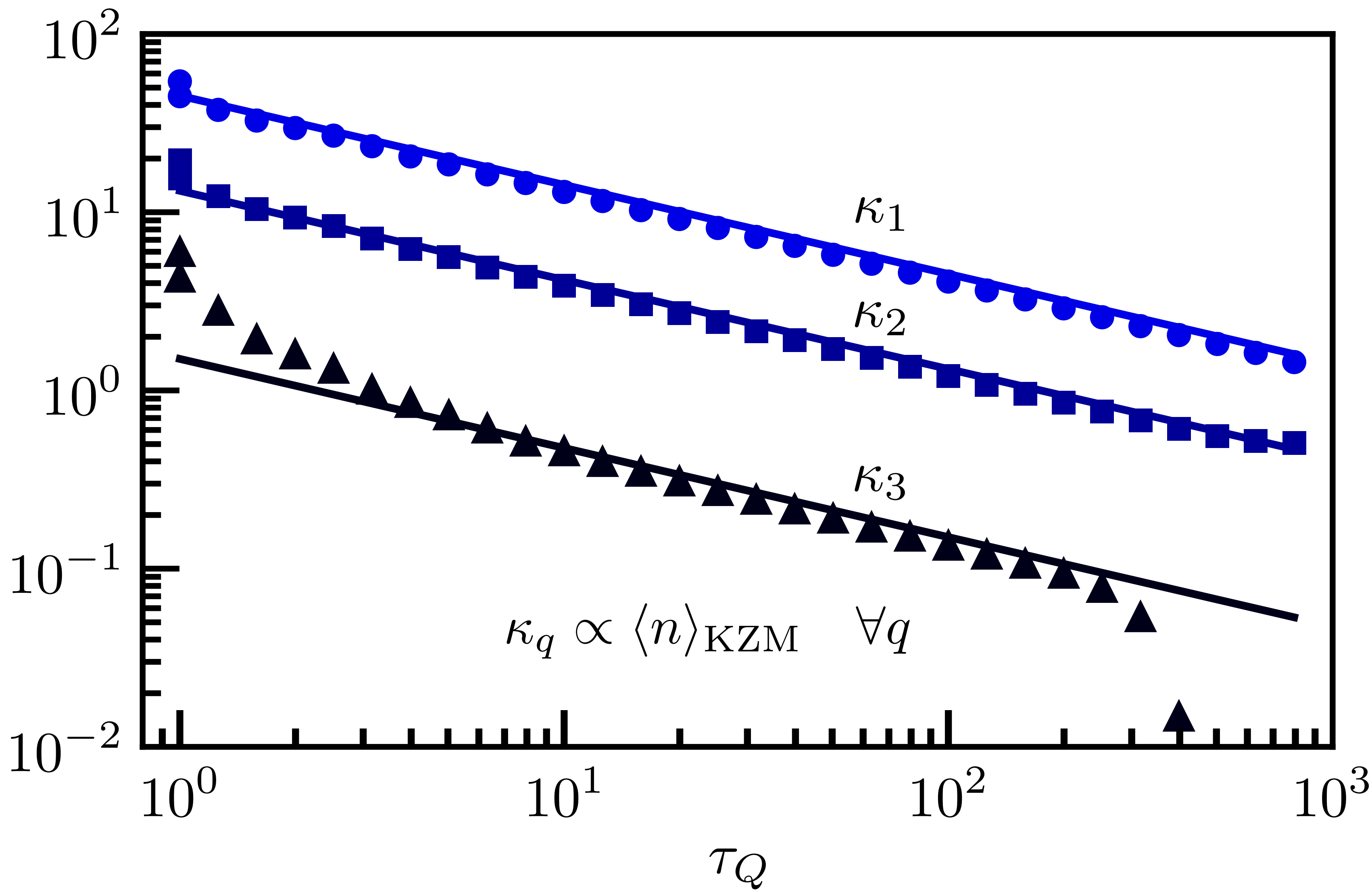
And third

$$\kappa_3 = \langle (n - \langle n \rangle)^3 \rangle = \left(1 - \frac{3}{\sqrt{2}} + \frac{2}{\sqrt{3}}\right)\langle n \rangle_{\rm KZM}$$

Non-normal/Gaussian distribution

AdC, PRL **121**, 200601 (2018)

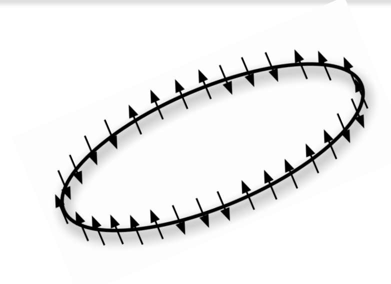
Beyond KZM: Universal scaling of cumulants



AdC, PRL **121**, 200601 (2018)

Universality of defect number distribution

1. AdC, PRL **121**, 200601 (2018);
 J-M. Cui, F. J. Gómez-Ruiz, Y.-F. Huang, C.F. Li, G. C. Guo, AdC, Commun. Phys. 3, 44 (2020)
 Exact P(n) in 1DTFIM and characterization



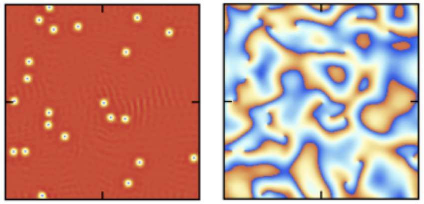
3. F. J. Gómez-Ruiz et al. PRL 124, 240602 (2020)
 J. J. Mayo et al. PRR 3, 033150 (2021)
 Continuous PTs in classical systems

$$V(\{\phi_i\}) = \sum_i \frac{1}{2}[\lambda(t)\phi_i^2 + \phi_i^4] + c\sum_i \phi_i\phi_{i+1}$$

$$\ddot{\phi}_i + \eta\phi_i + \partial_{\phi_i}V(\{\phi_i\}) + \zeta = 0$$

4. Y. Bando et al., PRR 2, 033369 (2020)
 Tests in D-Wave

5. AdC, FJ Gómez-Ruiz, ZH Li, CY Xia, HB Zeng, HQ Zhang, JHEP 06, 061 (2021)
 Numerical Tests in 2+1D holographic superconductor



Universality of defect formation

New testable prediction

Universal scaling of the mean number
and all cumulants

$$\kappa_q \propto \langle n \rangle_{\rm KZM} \quad \forall q$$

Test in a quantum annealer

Probing the universality of topological defect formation in a quantum annealer: Kibble-Zurek mechanism and beyond

Yuki Bando, Yuki Susa, Hiroki Oshiyama, Naokazu Shibata, Masayuki Ohzeki, Fernando Javier Gómez-Ruiz, Daniel A. Lidar, Sei Suzuki, Adolfo del Campo, and Hidetoshi Nishimori
Phys. Rev. Research **2**, 033369 – Published 8 September 2020

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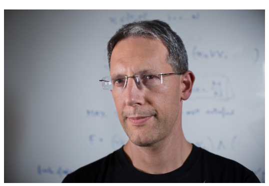

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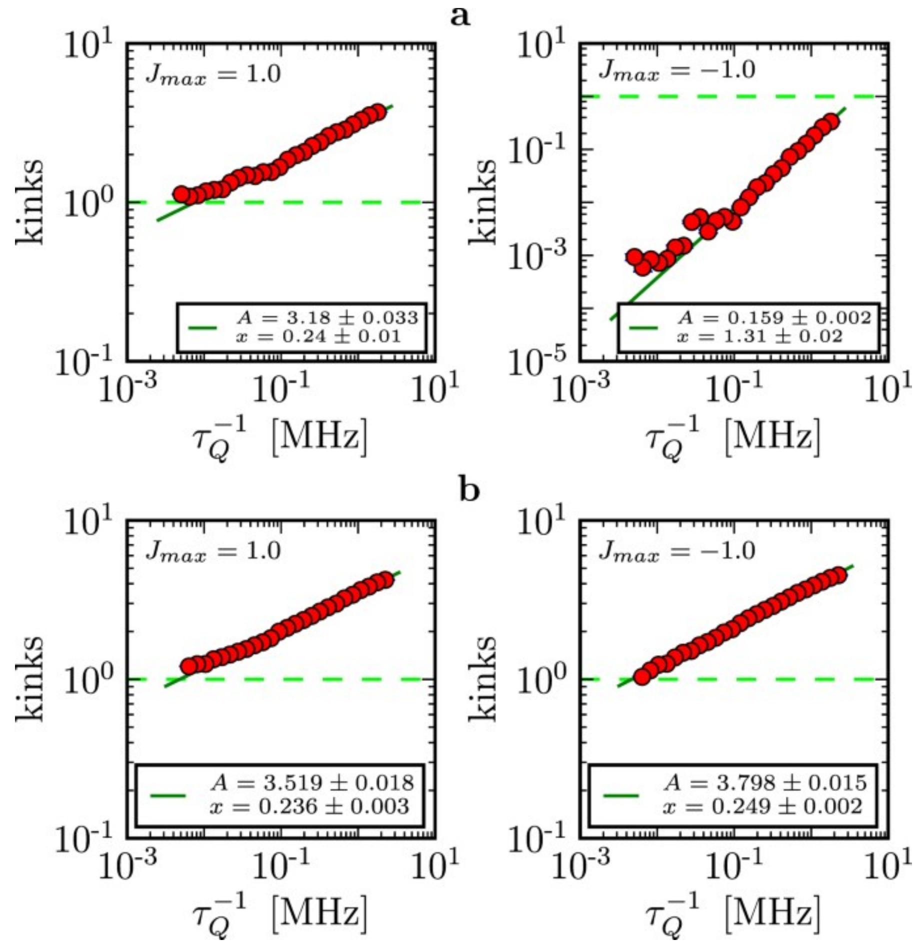
Previous D-Wave tests: 1D TFQIM

No quantitative agreement with KZM

DW2X (Los Alamos)

DW2X-SYS4 (Burnaby)

Unexplained fitted power-law exponents ~1 in LZ regime



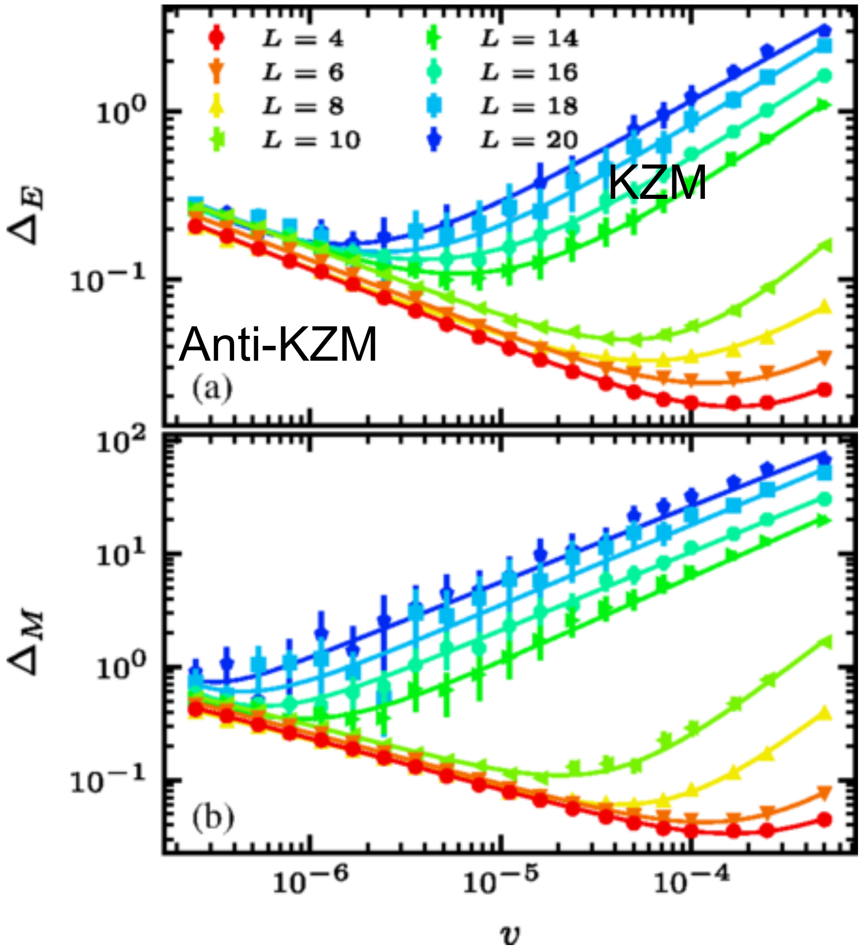
Article | Open Access | Published: 14 March 2018

Defects in Quantum Computers

Bartłomiej Gardas, Jacek Dziarmaga, Wojciech H. Zurek & Michael Zwolak

Scientific Reports **8**, Article number: 4539 (2018) | Cite this article

Previous D-Wave tests: 2D TFQIM



Power-law consistent with KZM

(2d quantum=3d classical)

$$d = 2, z = 1, \nu \approx 0.63$$

$$\beta_{\mathrm{KZM}} = \frac{d\nu}{1 + z\nu} \approx 0.77$$

$$\beta_{\mathrm{Dwave}} \approx 0.74 \pm 0.02$$

Scaling and Diabatic Effects in Quantum Annealing with a D-Wave Device

Phillip Weinberg, Marek Tylutki, Jami M. Rönkkö, Jan Westerholm, Jan A. Åström, Pekka Manninen, Päivi Törmä, and Anders W. Sandvik

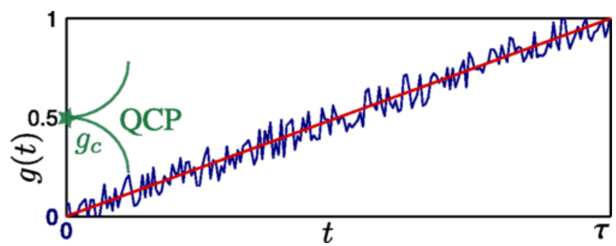
Phys. Rev. Lett. **124**, 090502 – Published 5 March 2020

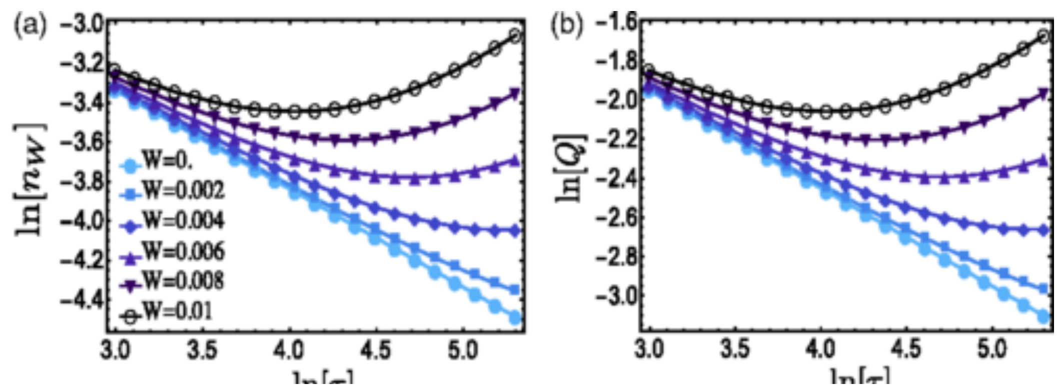
D-Wave 2000Q_2_1
L x L square lattice

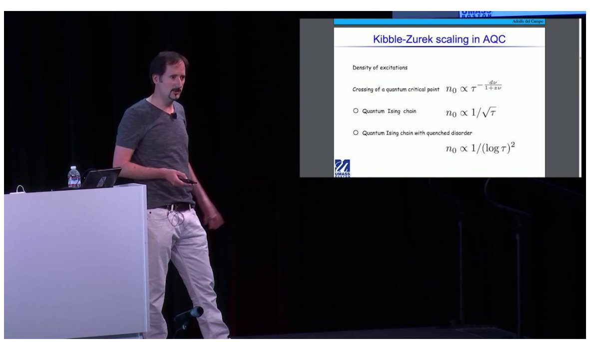
Previous D-Wave tests: KZM vs anti-KZM

Anti-KZM

heating induced by noise/dephasing dominates over KZM for slow quenches





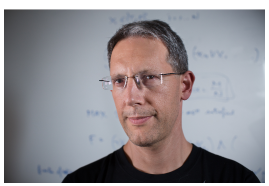


AQC2016 Venice, CA

Anti-Kibble-Zurek Behavior in Crossing the Quantum Critical Point of a Thermally Isolated System Driven by a Noisy Control Field

Anirban Dutta, Armin Rahmani, and Adolfo del Campo
Phys. Rev. Lett. **117**, 080402 – Published 17 August 2016

Test in a quantum annealer



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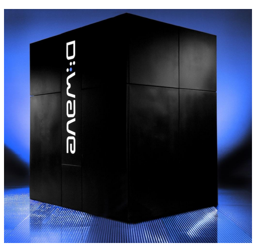
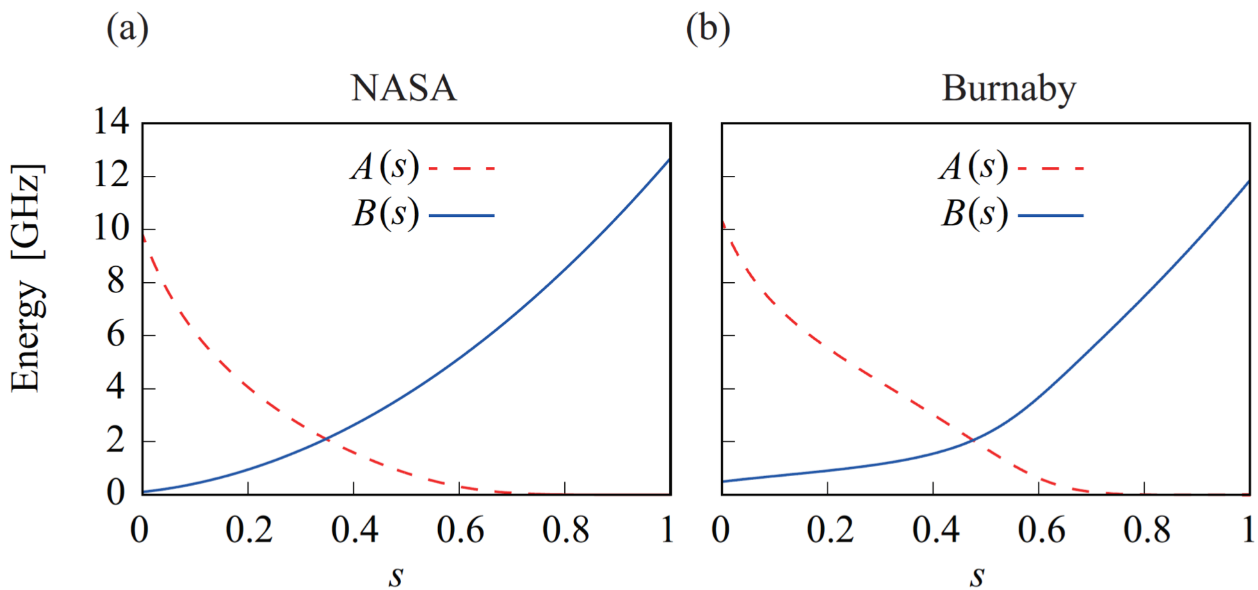
D-Wave test

D-Wave 2000Q: Embedding the Transverse-field 1D Ising model

$$H = -A(t/t_0)\sum_{i=1}^{L}\sigma_i^x + B(t/t_0)\sum_{i=1}^{L}J_i\sigma_i^z\sigma_{i+1}^z$$





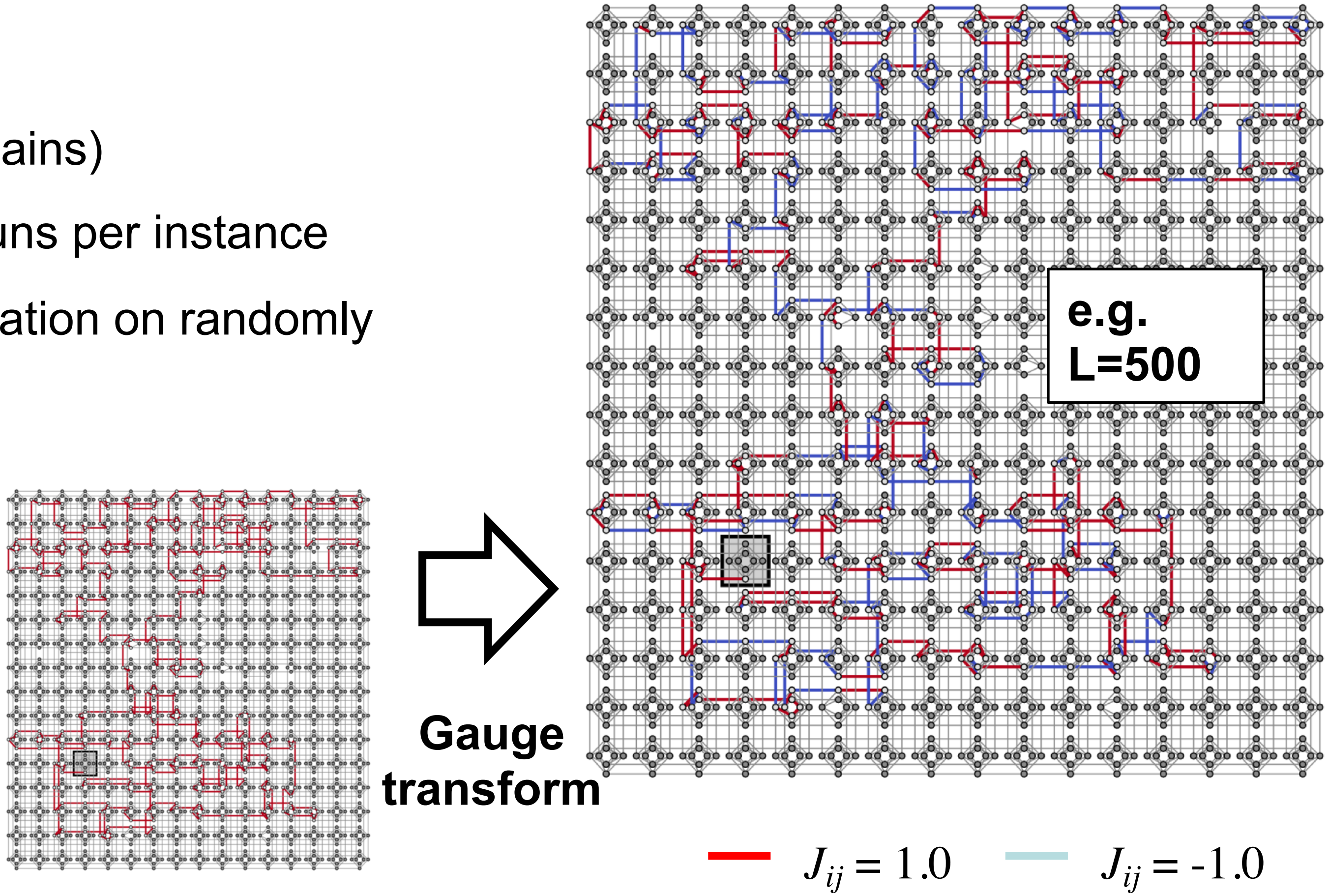
Low noise

Gauge averaging (avoid calibration errors)

Gauge-Averaged case

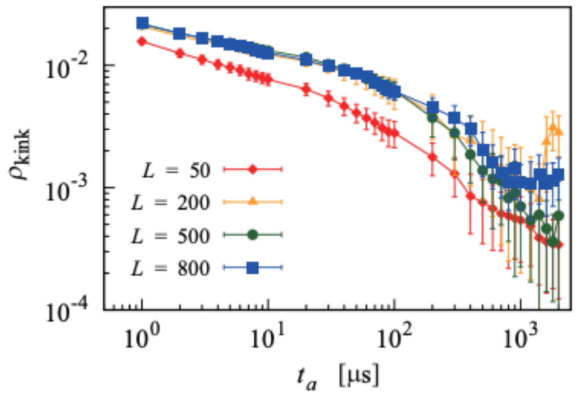
Sampling:

- 200 instances (chains)
- 1000 annealing runs per instance
- Gauge Transformation on randomly selected L/2 sites

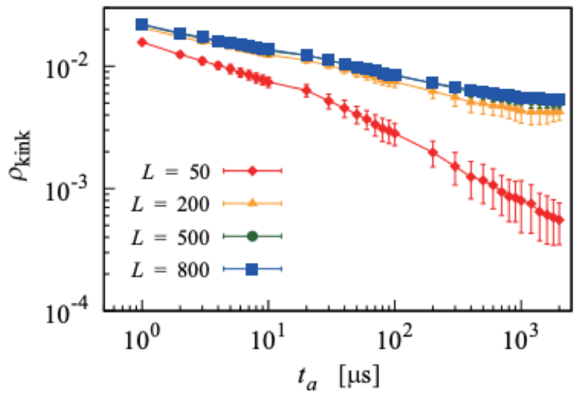


Collected density of kinks @ NASA

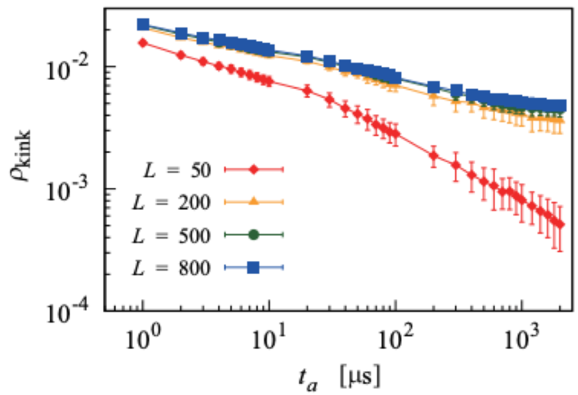
Ferromagnetic



AntiFerro



AF + random J flip



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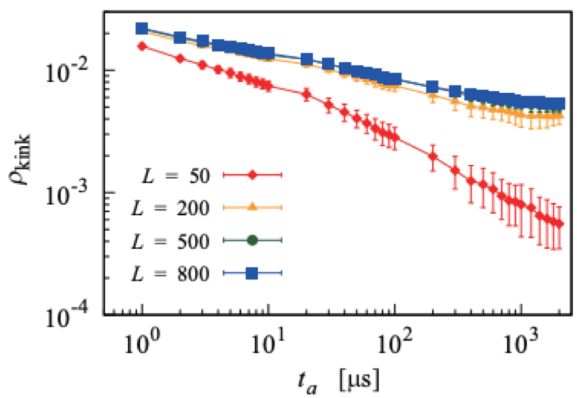
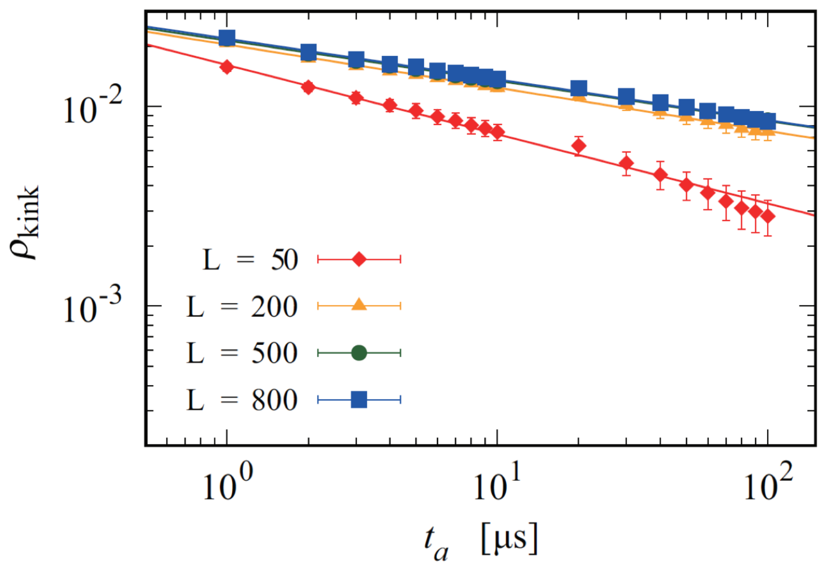
Ferromagnetic

AntiFerro

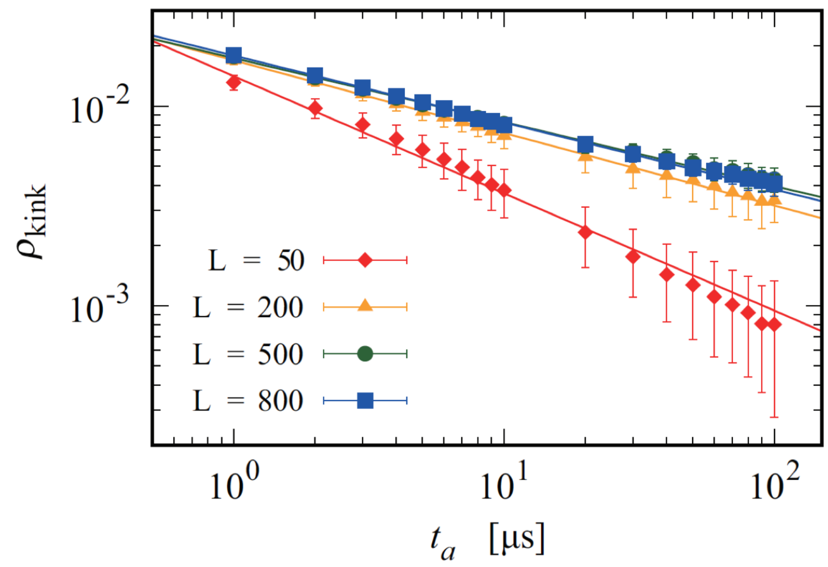


AF + random J flip

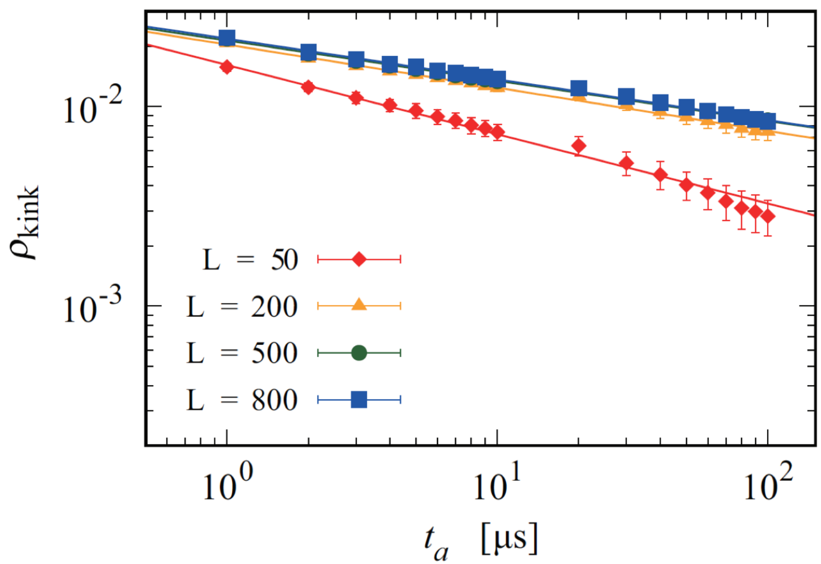
NASA (antiferromagnetic)



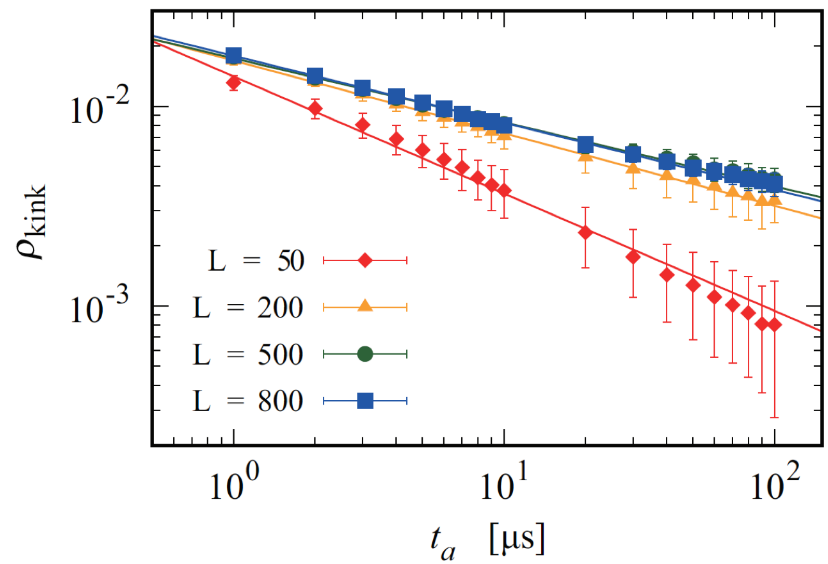
Burnaby (antiferromagnetic)



NASA (antiferromagnetic)



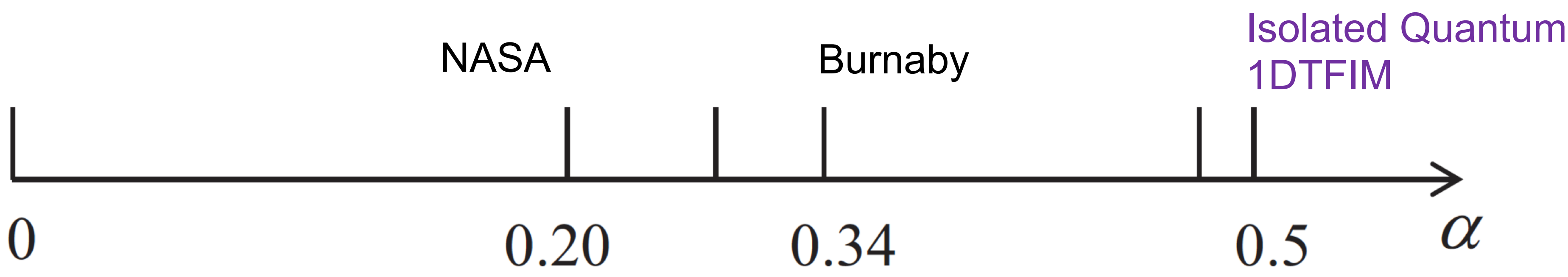
Burnaby (antiferromagnetic)



Fitted Power-law exponents

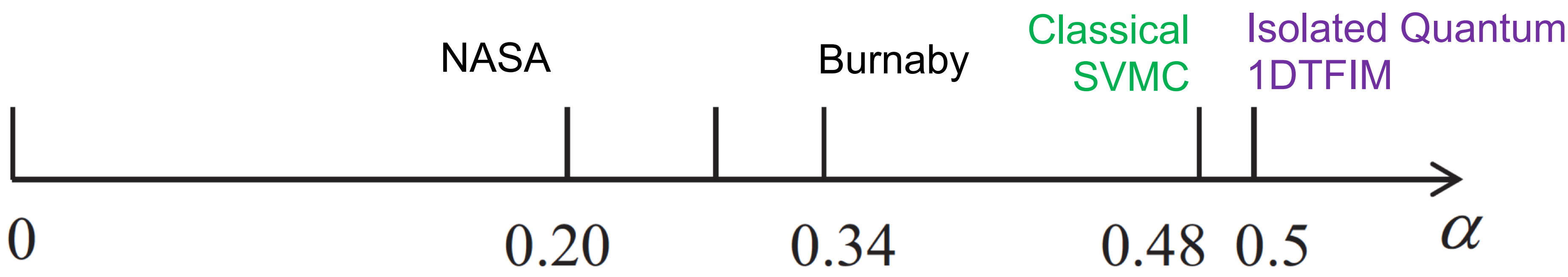
L	NASA	Burnaby
50	0.347 ± 0.008	0.587 ± 0.016
200	0.216 ± 0.003	0.363 ± 0.003
500	0.201 ± 0.003	0.320 ± 0.005
800	0.204 ± 0.002	0.335 ± 0.003

Power-law exponents



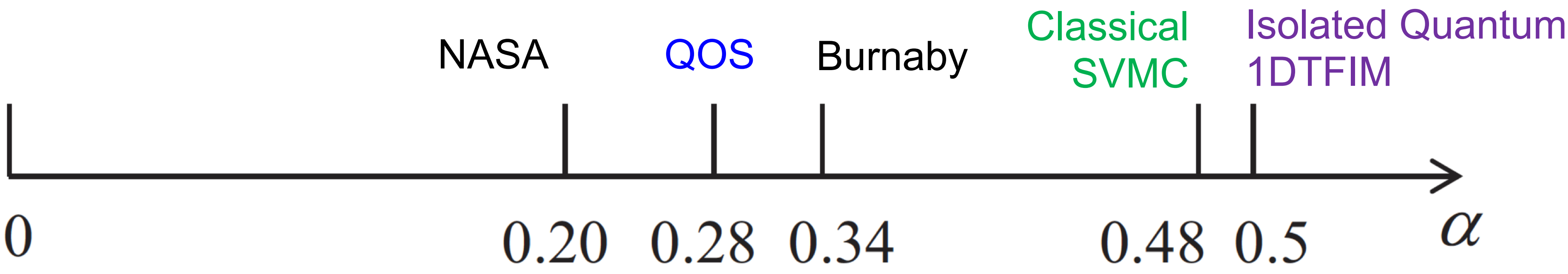
- Power laws exponent differs from isolated quantum case

Power-law exponents



- Power laws exponent differs from isolated quantum case
- Also differs from classical SVMC

Power-law exponents



- Power laws exponent differs from isolated quantum case
- Also differs from classical SVMC
- QOS: iTEBD calculation for open Ising chain with harmonic bath

First test of KZM in a Many-body Open Quantum System (?)

Power-law exponents: QOS

- QOS: open Ising chain with harmonic bath

$$H_{\rm total} = H(s) + \sum_{i,k} \left[C_k (a_{i,k}^\dagger + a_{i,k}) \sigma_i^z + \hbar \omega_{i,k} a_{i,k}^\dagger a_{i,k} \right]$$

$$J(\omega) = \frac{4\pi}{\hbar^2} \sum_i C_k^2 \delta(\omega - \omega_k) = 2\pi\eta\omega \quad (\omega < \omega_{\rm c})$$

$$\text{QMC/RG}: \quad \nu = 1 \rightarrow 0.64 \quad \& \quad z = 1 \rightarrow 1.99$$

Power-law exponents: QOS

- QOS: open Ising chain with harmonic bath

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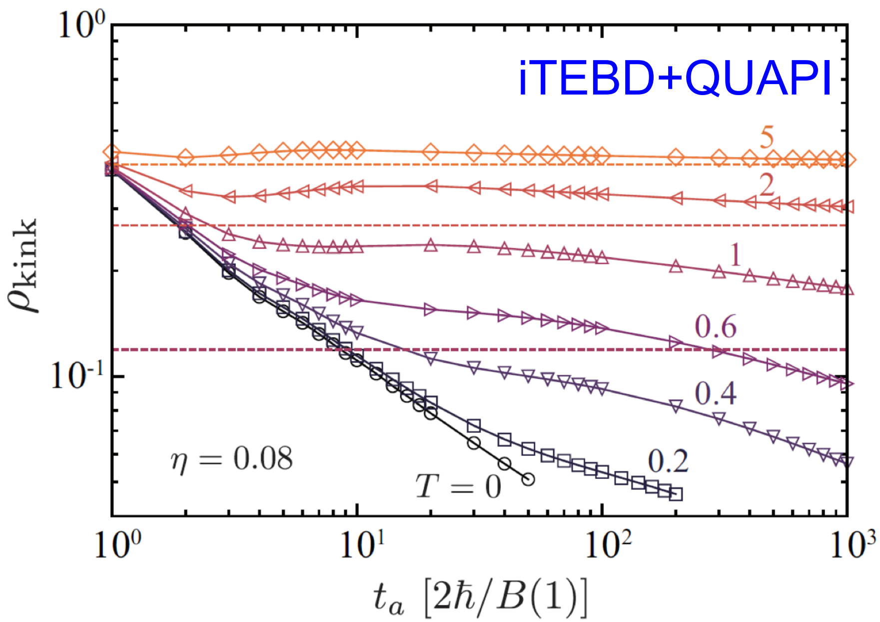

Power-law exponents: QOS

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$$J(\omega) = \frac{4\pi}{\hbar^2}\sum_i C_k^2 \delta(\omega - \omega_k) = 2\pi\eta\omega \quad (\omega < \omega_{\rm c})$$

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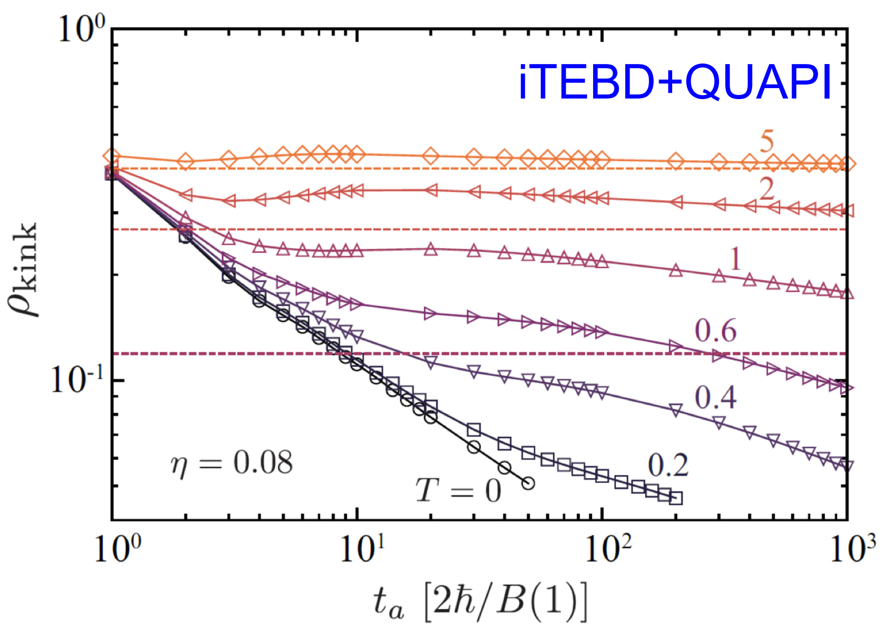

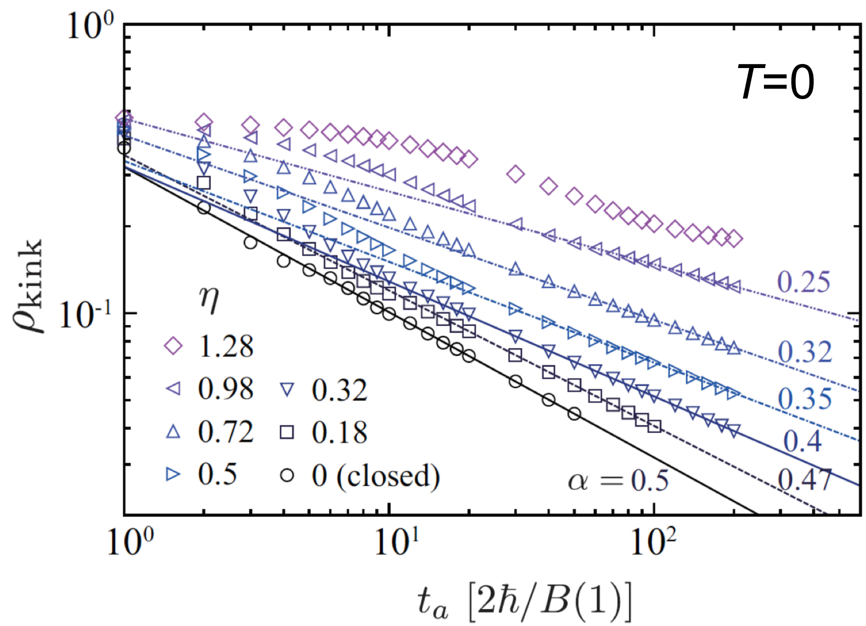

Power-law exponents: Spin-Vector Monte Carlo

- SVMC: spins mapped to classical planar rotors, dynamics by MC updates

$$H = \frac{B(s_n)}{2}\sum_{i=1}^{L-1}\cos\theta_i\cos\theta_{i+1} - \frac{A(s_n)}{2}\sum_{i=1}^{L}\sin\theta_i$$

$$s_n = \frac{n}{t'_a N_0}$$

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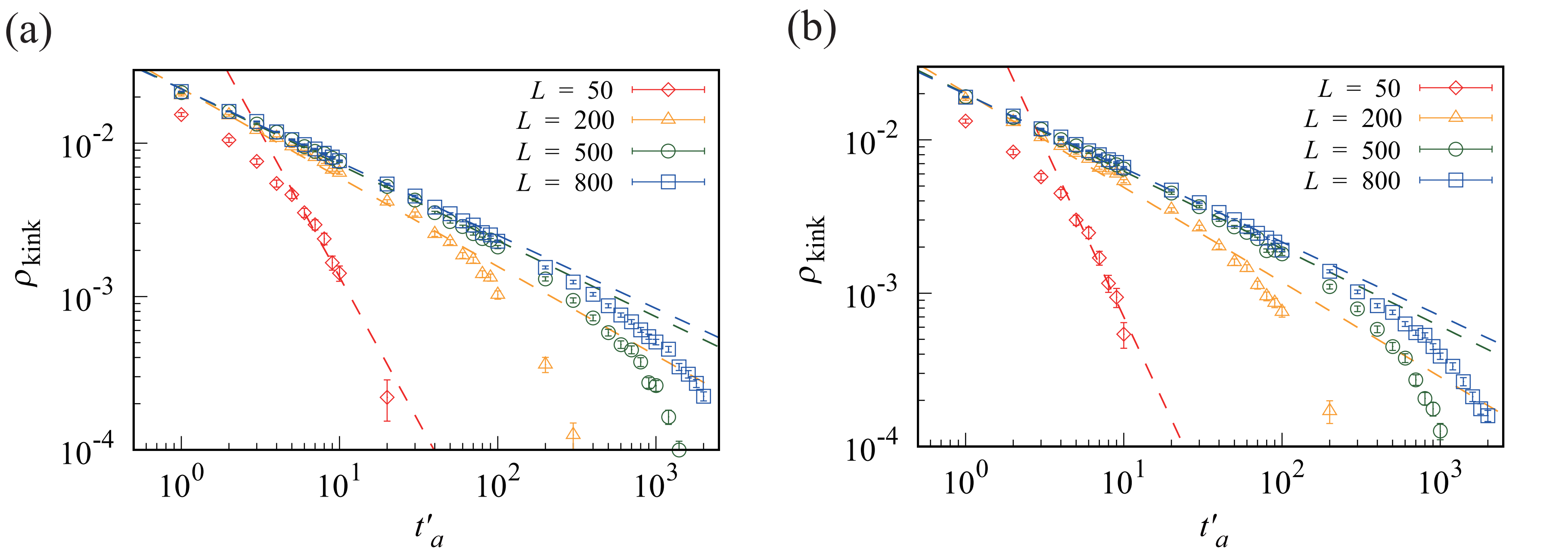

Power-law exponents: Spin-Vector Monte Carlo

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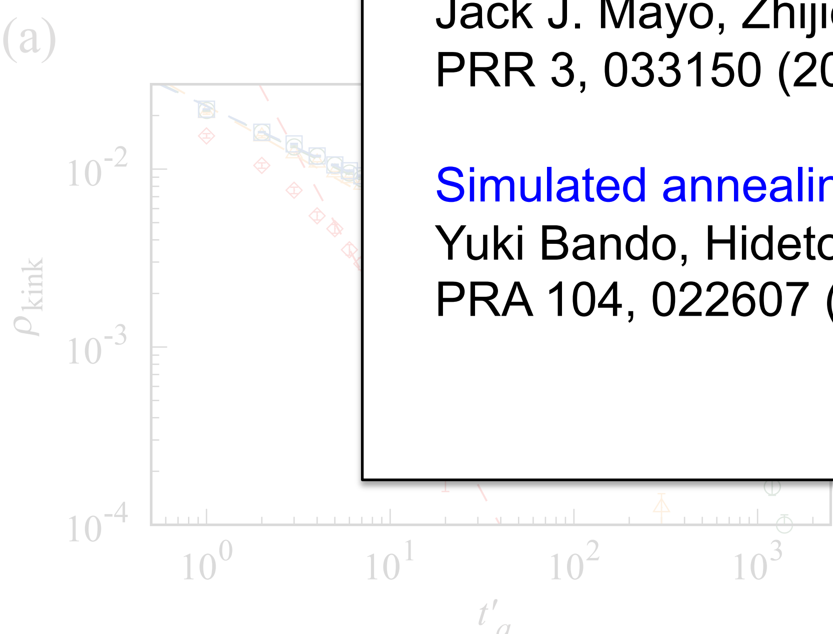
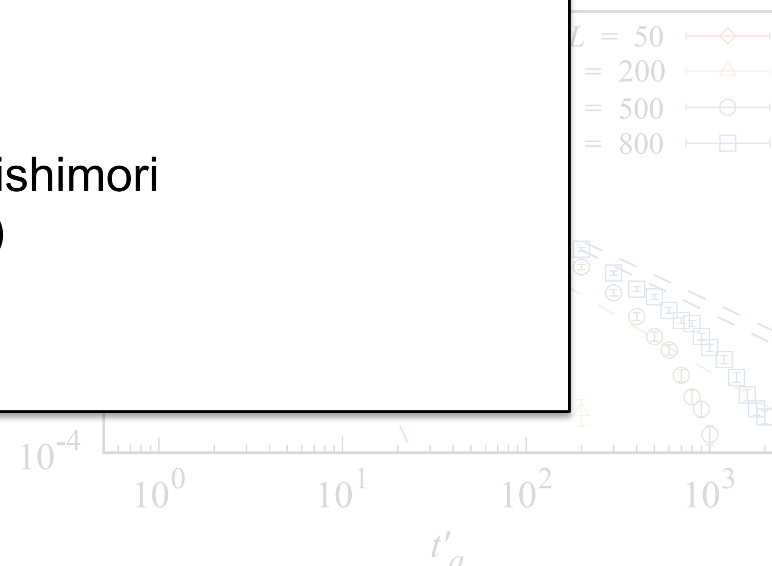


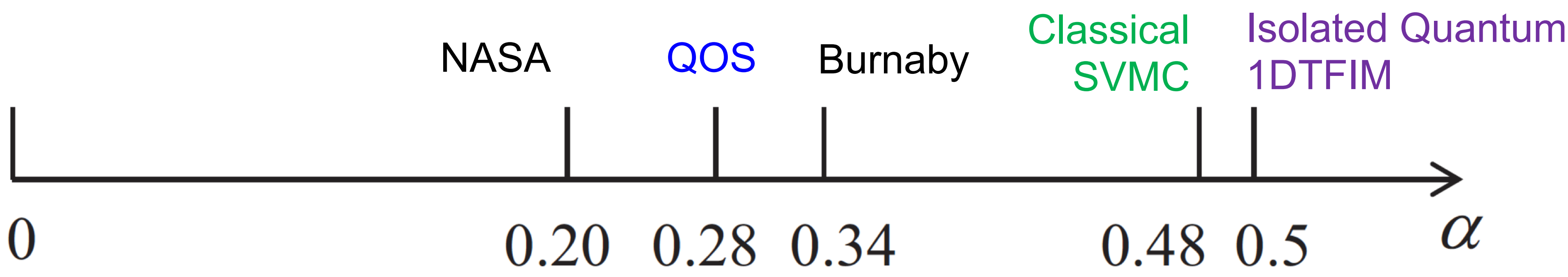


See other tests of classicality

Glauber dynamics
Jack J. Mayo, Zhijie Fan, Gia-Wei Chern, AdC
PRR 3, 033150 (2021)

Simulated annealing
Yuki Bando, Hidetoshi Nishimori
PRA 104, 022607 (2021)

Power-law exponents



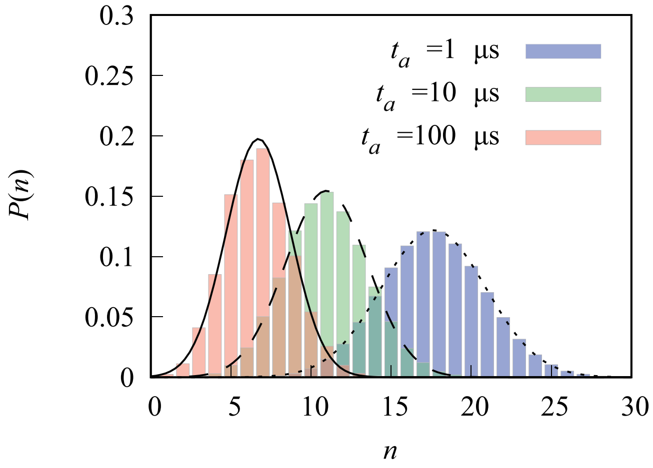
- Power laws exponent differs from isolated quantum case
- QOS: iTEBD calculation for open Ising chain with harmonic bath

First test of KZM in a Many-body Open Quantum System (?)

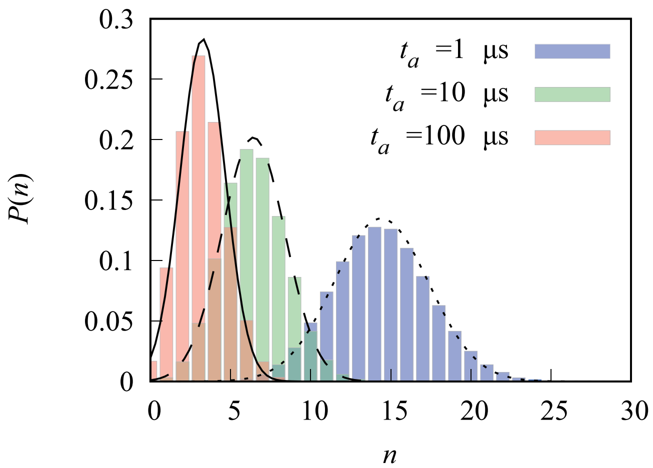
Beyond KZM in D-Wave

Kink Distribution Gauge-Ave. L=800

(a)



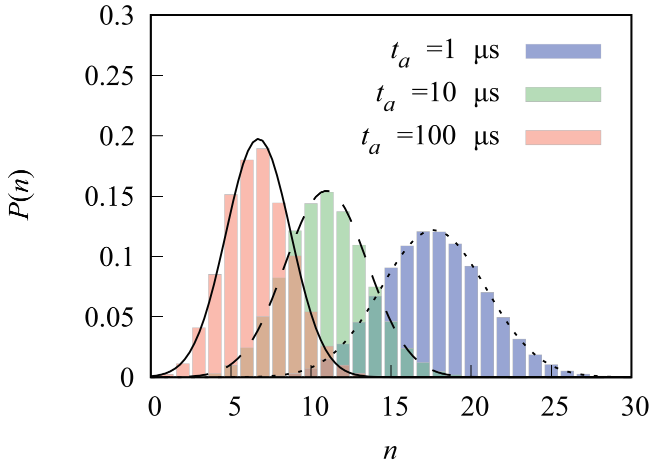
(b)



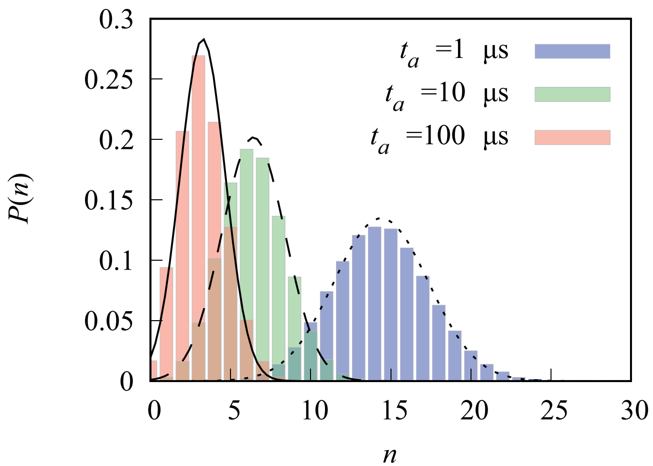
Beyond KZM in D-Wave

Kink Distribution Gauge-Ave. **L=800**

(a)



(b)



Normal/Gaussian approximation

$$P(n) \sim \mathrm{N}(\kappa_1, \kappa_2)$$

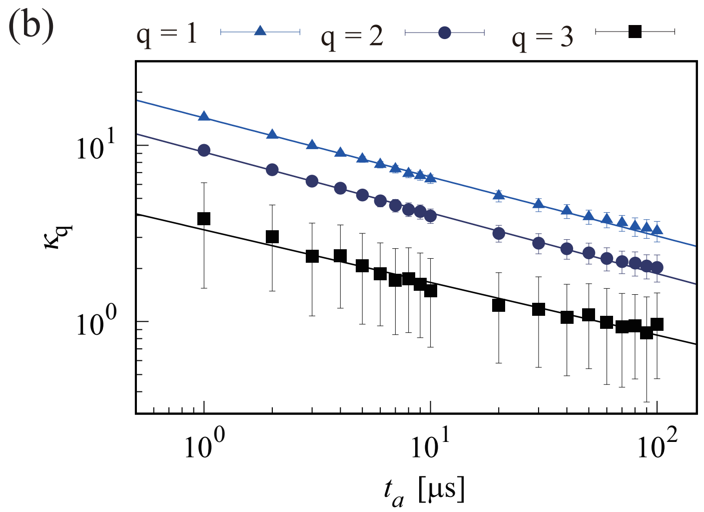
$$\kappa_1 = \frac{N}{2\pi}\sqrt{\frac{\hbar}{2Jt_a}}$$

$$\kappa_2 = (2-\sqrt{2})\kappa_2$$

Cincio et al., PRA 75, 052321 (2007)
AdC, PRL 121, 200601 (2018)
Cui et al. Commun. Phys. 3, 44 (2020)

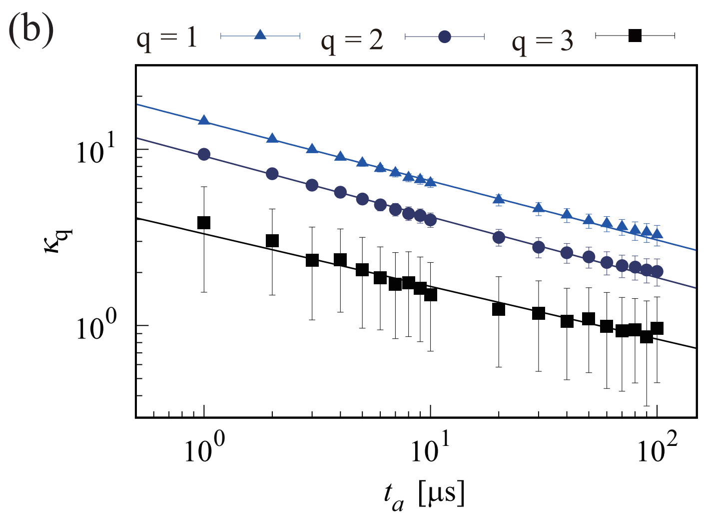
Beyond KZM in D-Wave

Universal scaling of cumulants

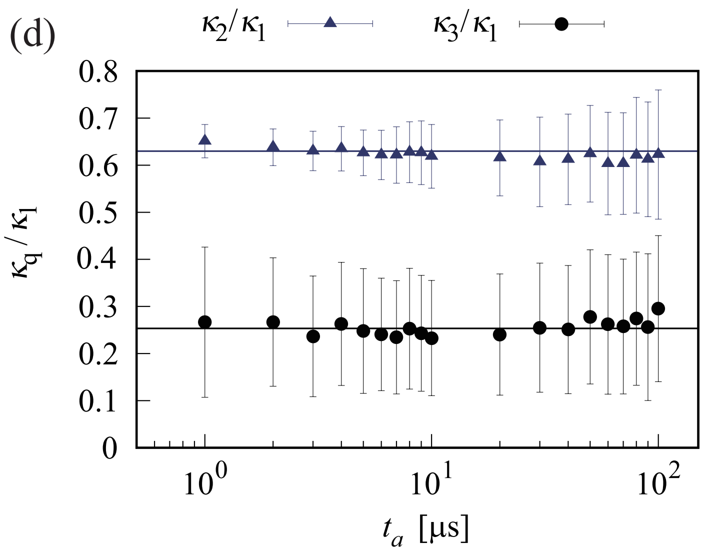


Beyond KZM in D-Wave

Universal scaling of cumulants



Cumulant ratios



DWave $\frac{\kappa_2}{\kappa_1} = 0.61 - 0.63$ $\frac{\kappa_3}{\kappa_1} = 0.23 - 0.25$

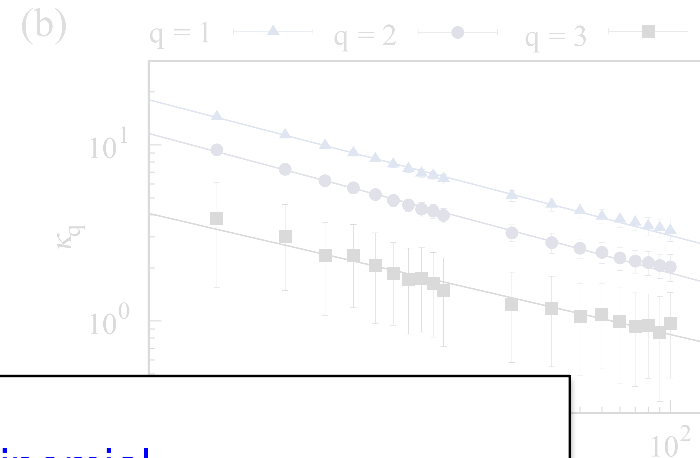
Theory $\frac{\kappa_2}{\kappa_1} = 0.586$ $\frac{\kappa_3}{\kappa_1} = 0.134$

Same distribution than in Isolation

AdC, PRL 121, 200601 (2018)

Beyond KZM in D-Wave

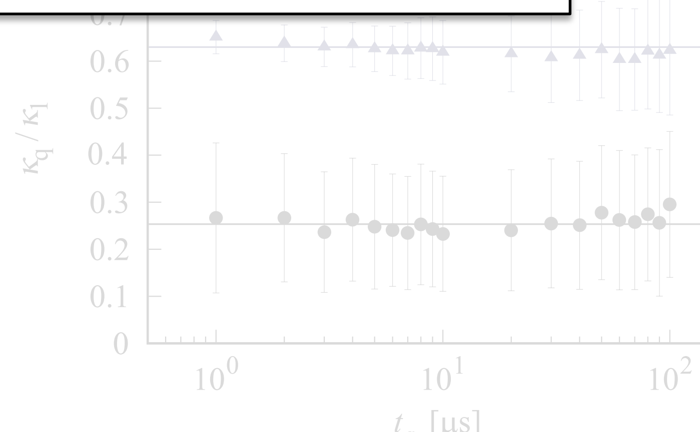
Universal scaling of cumulants



Universal distribution: Poisson-Binomial

First test of physics beyond KZM in a many-body system

Cumular



DWave $\frac{\kappa_2}{\kappa_1} = 0.61 - 0.63 \quad \frac{\kappa_3}{\kappa_1} = 0.23 - 0.25$

Theory $\frac{\kappa_2}{\kappa_1} = 0.586 \quad \frac{\kappa_3}{\kappa_1} = 0.134$

Consistent with values for Isolated System

AdC, PRL 121, 200601 (2018)

Closest Boltzmann distribution

- Kink number distribution from DWave

$$\text{NASA} \quad P_{\rm N}(n) \quad \text{Burnaby} \quad P_{\rm B}(n)$$

- Kink number distribution at thermal equilibrium (Boltzmann)

$$Q(n;\beta') = g(n)\frac{1}{Z}e^{-\beta' E(n)}$$

$$g(n) = \binom{L-1}{n}$$

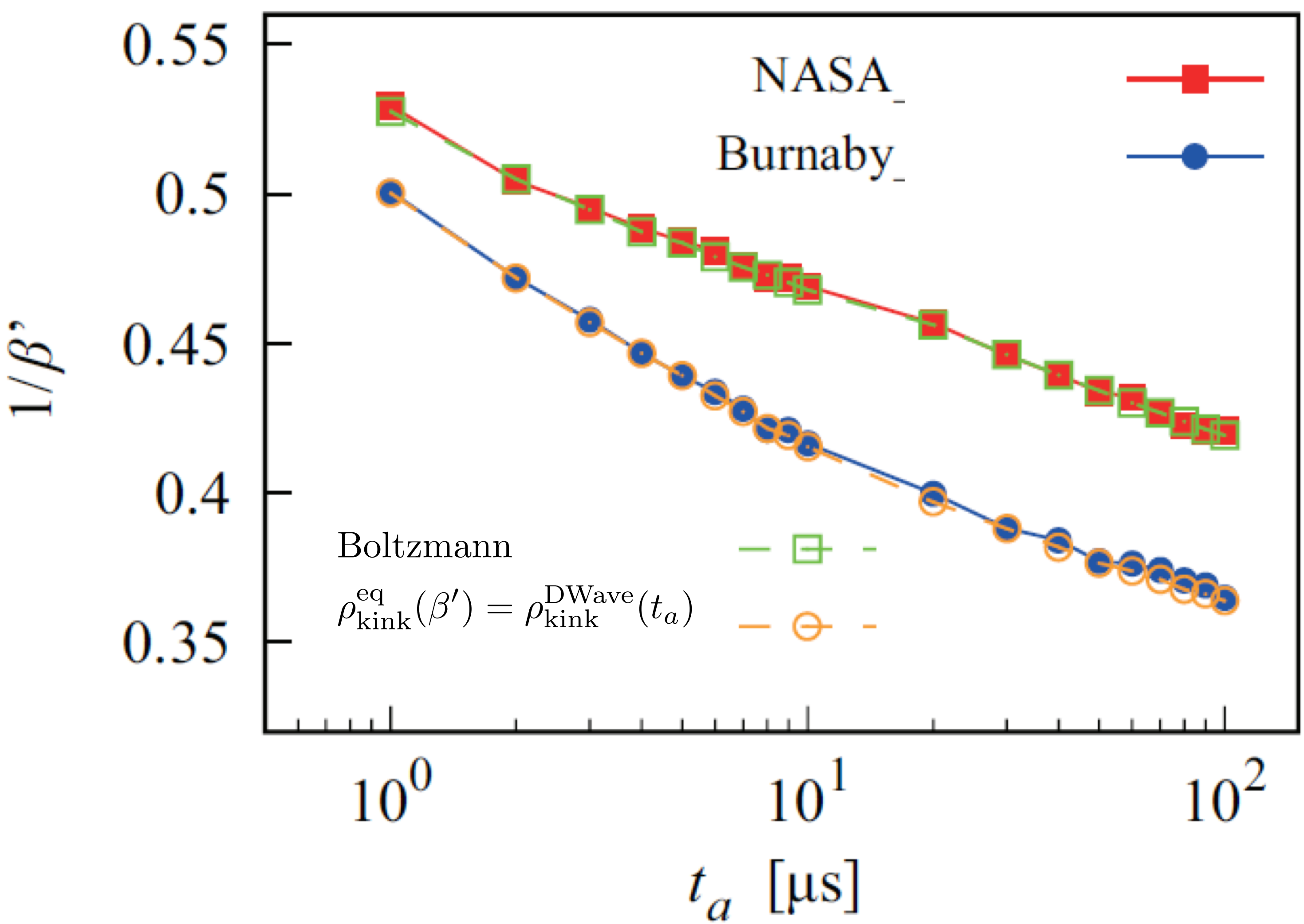
$$E(n) = 2n + 1 - L$$

- Minimize KL & TN distance

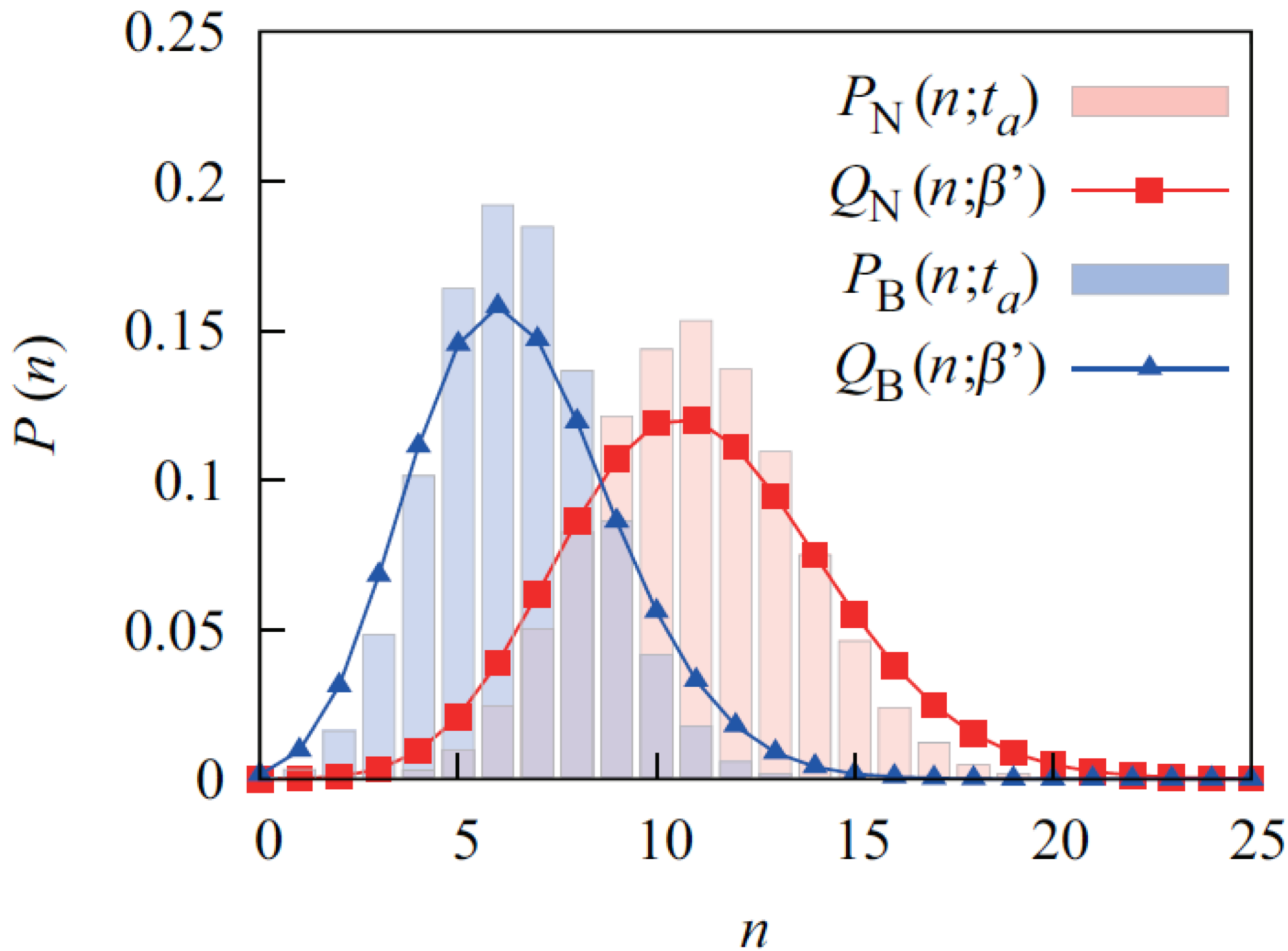
$$D_{\rm KL}(t_a) = \sum_n P(n;t_a)\ln\frac{P(n;t_a)}{Q(n;\beta')}$$

$$D_{\rm TN}(t_a) = \frac{1}{2}\sum_n |P(n;t_a) - Q(n;\beta')|$$

Closest Boltzmann distribution

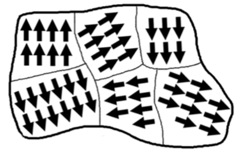


Non-thermal DWave distribution

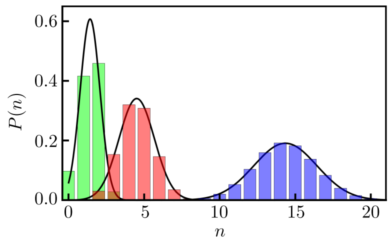


Summary

◆ Kibble-Zurek mechanism and Beyond



◆ Tests in Quantum annealers:



◆ **First test of KZM in an Open Quantum System**

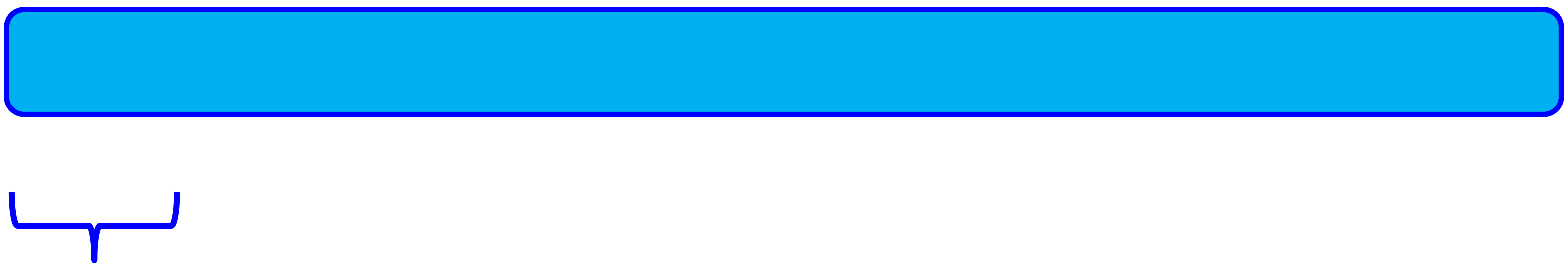
◆ **First test of physics beyond KZM (FCS defects) in a many-body system**

◆ Optimized Boltzmann and Nonthermal DWave distributions

Y. Bando et al. PRR 2, 033369 (2020);
F. J. Gómez-Ruiz et al PRL 124, 240602 (2020)
J. M. Cui et al. Commun. Phys. 3, 44 (2020)
AdC, PRL 121, 200601 (2018)

Beyond KZM: arbitrary systems

1D system



$$\hat{\xi} = \xi_0 \left(\frac{\tau_Q}{\tau_0} \right)^{\frac{\nu}{1+z\nu}}$$

Gómez-Ruiz et al PRL 124, 240602 (2020)   Mayo et al PRR 3, 033150 (2021)

Beyond KZM: arbitrary systems

1D system

Number of independent trials $\mathcal{N} = L/\hat{\xi}$

Gómez-Ruiz et al PRL 124, 240602 (2020)   Mayo et al PRR 3, 033150 (2021)

Beyond KZM: arbitrary systems

1D system

Number of independent trials $\mathcal{N} = L/\hat{\xi}$

Probability to form a kink at each boundary p

Binomial distribution, normal approx

$$P(n) \sim B(n, \mathcal{N}, p) \approx \frac{1}{\sqrt{2\pi(1-p)\langle n \rangle}} e^{-\frac{(n-\langle n \rangle)^2}{2(1-p)\langle n \rangle}}$$

Gómez-Ruiz et al PRL 124, 240602 (2020)   Mayo et al PRR 3, 033150 (2021)