General bounds on shot noise in the absence of currents: charge and heat transport

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KITP conference on "Transport and Efficient Energy Conversion in Quantum Systems", 31st of August 2021

Outline

- Short introduction: – Charge and heat currents and noise.

J. Eriksson, M. Acciai, L. Tesser, J. Splettstoesser: arXiv:2102.12988 (2021); accepted for publication in Phys. Rev. Lett.

Outline

- Short introduction: – Charge and heat currents and noise.
- Results: Charge and heat shot noise at zero currents in constant-transmission conductors.

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- Example for a concrete conductor.

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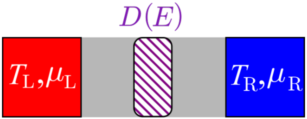
Introduction – Charge and heat currents and noise

Introduction: transport and noise

Charge and heat transport



- Thermoelectrics
- Transport spectroscopy

Charge current:

$$I_L = -\frac{2e}{h} \int dE\, D(E)[f_L(E) - f_R(E)]$$

Heat current:

$$J_L = \frac{2}{h} \int dE\, (E - \mu_L) D(E)[f_L(E) - f_R(E)]$$

G. Benenti, G. Casati, K. Saito, R. S. Whitney: Phys. Rep. **694**, 1 (2017)

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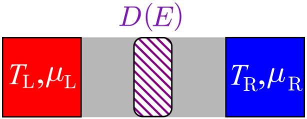

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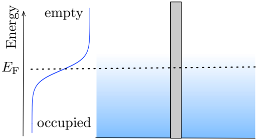
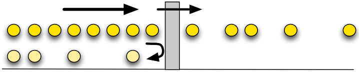
Noise

- Further spectroscopy
- Important for "non-macroscopic" thermodynamics
- TUR, FR,...

Thermal noise



Shot noise = partition noise



Ya. M. Blanter, M. Büttiker: Phys. Rep. **336**, 1 (2000)

Introduction: Shot noise (in the absence of currents)

Shot and thermal noise:

$$S^X_{\text{th}} = \int dE \frac{4x^2}{h} D(E) \sum_{\alpha=\text{L,R}} f_\alpha(E)[1 - f_\alpha(E)]$$ ⇒ Persists at equilibrium

$$S^X_{\text{sh}} = \int dE \frac{4x^2}{h} D(E)[1 - D(E)][f_{\text{L}}(E) - f_{\text{R}}(E)]^2$$ ⇒ **Nonequilibrium** effect!

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Nonequilibrium does not necessary imply an average current flow!!!

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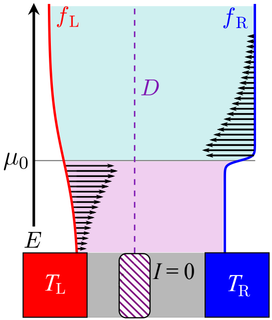


- **Delta-T-noise**

  E. V. Sukhorukov, D. Loss: Phys. Rev. B **59**, 13054 (1999)

- Recently measured!!

  O. S. Lumbroso, L. Simine, A. Nitzan, D. Segal, O. Tal: Nature **562**, 240 (2018)
  E. Sivre, H. Duprez, A. Anthore, A. Aassime, F. D. Parmentier, A. Cavanna, A. Ouerghi, U. Gennser, F. Pierre: Nat. Commun. **10**, 1 (2019)

- Limit for $T_\text{L} \gg T_\text{R} \approx 0$

$$S^I_{\text{sh}}/S^I_{\text{th}} \to \left(1 - D^0\right)(2\ln 2 - 1)$$

S. Larocque, E. Pinsolle, C. Lupien, B. Reulet: Phys. Rev. Lett. **125**, 106801 (2020)

Challenge to address

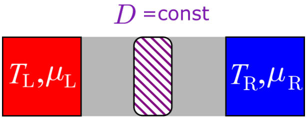
Generalize to:	
▸ Arbitrary energy-dependent transmission probabilities ▸ Generic nonequilibrium conditions ▸ Other types of currents: heat	Charge shot noise S^I_{sh} at thermovoltage μ_I Heat shot noise S^J_{sh} with $J = 0$ at μ_J

$\Rightarrow$ Find general statements on shot noise at zero currents

$\Rightarrow$ Particularly interesting for thermoelectrics

Zero-current charge and heat shot noise in constant-transmission conductors

Zero-current charge and heat shot noise at constant transmission



Charge transport:

Delta-T-noise

$\Delta T \neq 0$ and $\Delta\mu \equiv 0$:

$\Rightarrow I = 0$, but $S^I_\mathrm{sh} \neq 0$

Heat transport:

Heat conduction $\rightarrow$ voltage required

$$J_\mathrm{L} = D\frac{\Delta\mu^2}{2h} - D\frac{\pi^2 k_\mathrm{B}^2}{6h}\left[T_\mathrm{L}^2 - T_\mathrm{R}^2\right]$$

Heat current NOT conserved!

Consider vanishing zero-current heat shot noise in contact L...

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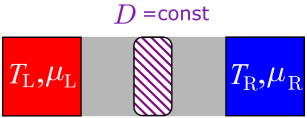
Zero-current charge and heat shot noise at constant transmission



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$$\left.\frac{S^I_{\text{sh}}}{S^I_{\text{th}}}\right|_{I=0} = R_I \rightarrow R_I^0$$
$$= (1-D)\,(2\ln 2 - 1) \approx 0.4$$

$$\left.\frac{S^J_{\text{sh}}}{S^J_{\text{th}}}\right|_{J=0} = R_J \rightarrow R_J^0$$
$$= 3(1-D)A(\pi/\sqrt{3})/\pi^2 \approx 0.45$$

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Zero-current charge and heat shot noise at constant transmission

D =const

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Can not be exceeded at any temperature difference if D =const.

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General bounds on zero-current charge and heat shot noise

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Maximal at constant transmission $D = 1/2$ — even if $I \equiv 0$ imposed!

Use variational principle as in: R. S. Whitney: Phys. Rev. B **91**, 115425 (2015)

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$$S^I_{\text{sh}} \leq \frac{4e^2}{h}\int dE D(E) f_{\text{L}}(E)[1 - f_{\text{R}}(E)] \leq S^I_{\text{th}}$$

$$\Rightarrow \quad R_I \equiv \left[S^I_{\text{sh}}/S^I_{\text{th}}\right]_{I=0} \leq 1$$

- Relies only on $0 \leq D(E) \leq 1$ and $0 \leq f_\alpha(E) \leq 1$
- Valid also for multi-channel contacts

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Which $D(E)$ maximizes R_I? Can the bound be reached?

$$R_I \lesssim 1$$

Any $D(E) \neq 0$ within an energy interval δ, with $\max(T_{\mathrm{L}}, T_{\mathrm{R}}) \gg \delta/k_{\mathrm{B}} \gg \min(T_{\mathrm{L}}, T_{\mathrm{R}})$ and $D(E) \ll 1$.

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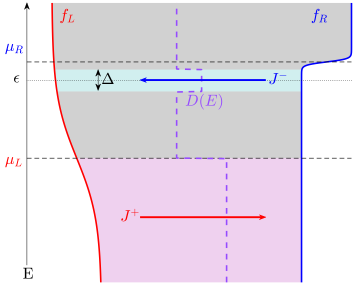
Heat shot noise is <u>not</u> bounded with respect to heat thermal noise!

<u>Counter example:</u>



- Counter example with finite transmission in **separate** energy intervalls!
  - ⇒ Major contributions to shot and thermal noise can be separately tuned!
- Different transport channels ⇔ different energy!

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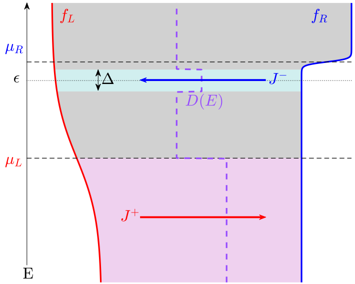

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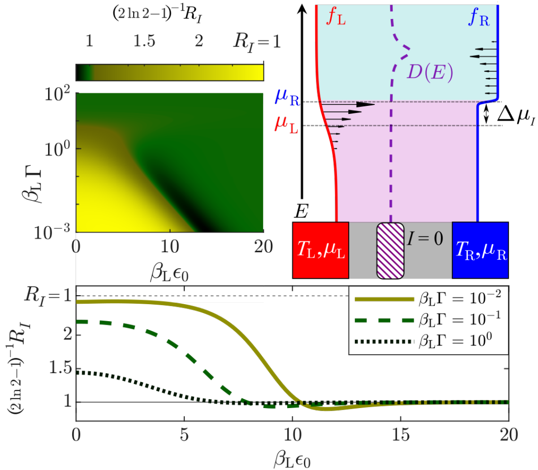
$$R_J = \left.\frac{S^J_{\text{sh}}}{S^J_{\text{th}}}\right|_{J=0}$$ **can become arbitrarily large!**

J. Eriksson, M. Acciai, L. Tesser, J. Splettstoesser: arXiv:2102.12988 (2021); accepted for publication in Phys. Rev. Lett.

Concrete example: Conductor with resonant (Lorentzian-shaped) transmission

Concrete example: Lorentzian transmission

Zero-current charge noise ratio:



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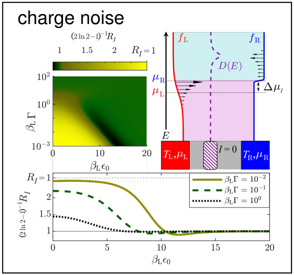
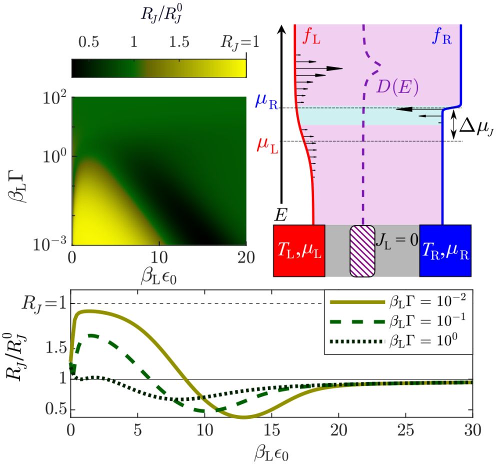


Zero-current **heat** noise ratio:



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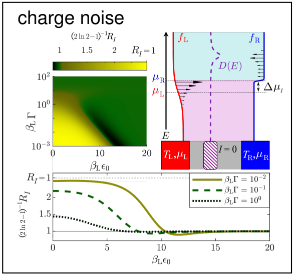

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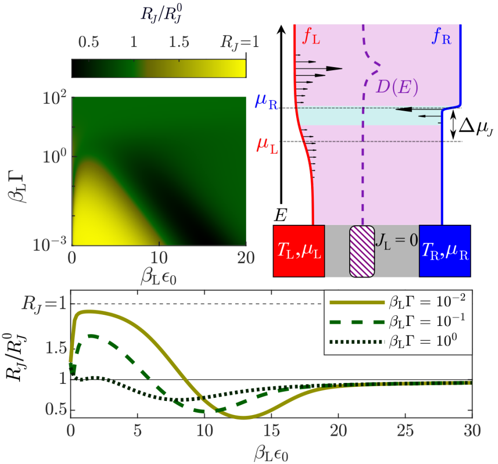

$R_J \leq 1$ for this specific transmission probability!

Test more complex transmissions to reach $R_J \geq 1$!

e.g. F. Hajiloo, P. T. Alonso, N. Dashti, L. Arrachea, J. Splettstoesser: Phys. Rev. B **102**, 155434 (2020)

J. Eriksson, M. Acciai, L. Tesser, J. Splettstoesser: arXiv:2102.12988 (2021); accepted for publication in Phys. Rev. Lett.

Conclusions

- **Zero-current shot noise extended to heat transport!**
  Generic nonequilibrium conditions (beyond Delta-T noise), arbitrary transmission probabilities
- **Simple expressions for limits of $R = S_{\text{sh}}/S_{\text{th}}$ at constant D**
- **General bounds on shot noise to thermal noise ratio at zero current**
  Zero-current charge shot noise bounded by thermal noise; heat shot noise behaves fundamentally differently!!
- **Lorentzian transmission as example**

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Outlook

- Many relevant cases in quantum thermoelectrics where $I, J = 0$ but $S_I, S_J \neq 0$

  R. Sánchez, J. Splettstoesser, R. S. Whitney: Nonequilibrium System as a Demon. Phys. Rev. Lett. **123**, 216801 (2019)
- Fluctuation dissipation theorem out of equilibrium?

  B. Altaner, M. Polettini, M. Esposito: Fluctuation-Dissipation Relations Far from Equilibrium. Phys. Rev. Lett. **117**, 180601 (2016)

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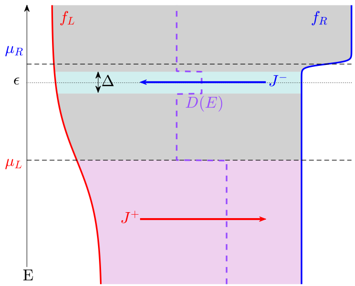

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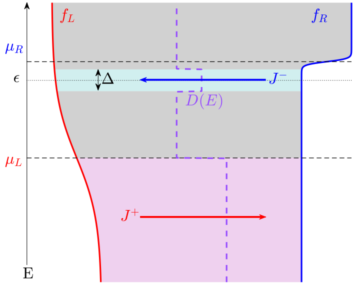

- Fulfill zero-current condition:

$$J = 0 \Rightarrow \Delta = \frac{hJ^+}{\epsilon}$$

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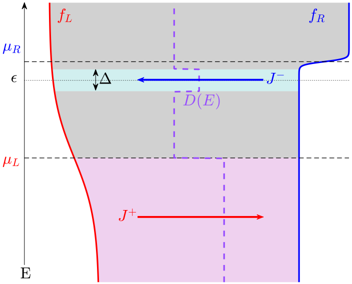

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- Shot noise contribution from **upper part**:

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$$\approx \frac{\epsilon^2 \Delta}{h} = \epsilon J^+$$

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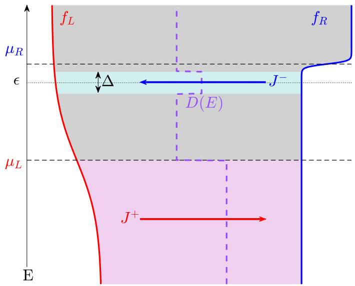

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- Thermal noise contribution from **lower part**: Independent of ϵ!!

$$S^J_{\mathrm{th}} \approx \frac{4}{h}\int_{-\infty}^{0} dE E^2 f_L [1 - f_L]$$

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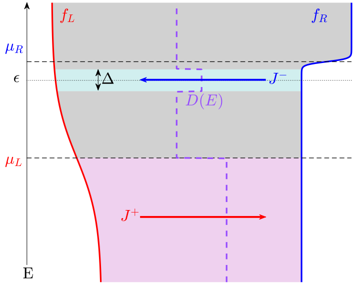

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